Technical Report No. 5

FINITE DIFFERENCE CALCULATION OF PRESSURE RISE IN SATURN S-IVB FUEL TANK

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ORA Project 07461

under contract with:

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
GEORGE C. MARSHALL SPACE FLIGHT CENTER
CONTRACT NO. NAS-8-20228
HUNTSVILLE, ALABAMA

administered through:

OFFICE OF RESEARCH ADMINISTRATION
ANN ARBOR

April 1969
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A - Area, ft$^2$

a - Cylindrical tank radius, ft; acceleration, ft/sec$^2$

b - Total tank height, ft

$C_p, C_v$ - Specific heats Btu/lbm-°F

$\frac{Df}{Dt}$ - Substantial derivative of function $f$

e - Specific internal energy, Btu/lbm

E - Total internal energy, Btu

g - Acceleration due to local gravity, ft/sec$^2$

h - Heat transfer coefficient, Btu/hr-ft$^2$-°F; enthalpy, Btu/lbm

$h_{fg}$ - Latent heat, Btu/lbm

k - Thermal conductivity, Btu/hr-ft-°F

P - Pressure, psia

q - Heat flux, Btu/hr-ft$^2$

r - Radial distance, ft

$\Delta r$ - Finite difference of radial distance, ft

R - Gas constant, lbf-ft/lbm-°R

t - Time, hr

T - Temperature, °R

$\Delta T_w = T - T_{sat}$, °R

$\Delta T_{w_{max}}$ - Specified maximum value of $\Delta T_w$, °R

u - Axial component of velocity, ft/sec
NOMENCLATURE (Continued)

U - Dimensionless axial velocity

v - Radial component of velocity, ft/sec

V - Volume, ft³; dimensionless radial velocity

w - Mass flow rate, lbm/sec

x - Axial distance, ft

X - Liquid height in container, ft

Δx - Finite-difference of vertical distance, ft

Z - Compressibility factor, [l]

Greek Letters

α - Thermal diffusivity, ft²/hr

β - Coefficient of thermal expansion, °R⁻¹

γ - Ratio of specific heats $C_p/C_v$, [l]

δ - Tank wall thickness, ft

η - Dimensionless radius, [l]

Δη - Finite difference of dimensionless radius

θ - Dimensionless temperature

ζ - Dimensionless axial distance, [l]

Δζ - Finite difference of dimensionless axial distance

ψ - Dimensionless stream function

ψ', ψ'' - Liquid and vapor stream functions

τ - Dimensionless time

Δτ - Finite difference of dimensionless time
NOMENCLATURE (Concluded)

\( \mu \) - Kinematic viscosity \( \text{ft}^2/\text{hr} \)

\( \nu \) - Dynamic viscosity, \( \text{lbm/ft-hr} \)

\( \rho \) - Density, \( \text{lbm/ft}^3 \)

\( \omega' \) - Vorticity, \( [\text{ft-sec}]^{-1} \)

\( \omega \) - Dimensionless vorticity

Subscripts

\( d \) - Discharge

\( f \) - Liquid or fluid

\( g \) - Gas or vapor

\( i \) - Liquid-vapor interface

\( l \) - Liquid

\( o \) - Initial state

\( p \) - Pressurization

\( r \) - Reduced

\( s \) - Saturation

\( w \) - Wall
ABSTRACT

The pressure rise of a two-phase system in a closed container subject to an external heat flux is related directly to the temperature of the liquid-vapor interface, which in turn depends on the heat and mass transfer interactions between the liquid, vapor, and container. The problem is formulated here for a cylindrical tank with an axial body force and a symmetrically imposed external heat flux in terms of the transport equations. The temperature and velocity distributions are determined using a finite-difference method, which is coupled with an integral form of the energy equation to determine the pressure rise. The procedure adopted takes into account the possibility of incipient and nucleate boiling.

Numerical computations are carried out for liquid hydrogen in a large, partially filled tank under low gravity. This system models the one orbital experiment conducted to date which provides the only available experimental data. These are the data on pressure rise and system temperatures telemetered from a Saturn LH₂ tank orbiting the earth, the AS-203 low gravity orbital experiment. A discussion of the modeling is included.

The numerical computations are carried out for various distributions of heat flux between the liquid and vapor, and with various container wall properties. The outputs of major interest are the pressure rise, and temperature and velocity distributions. Representative plots of the isotherms and streamlines are included.

The computations indicate that even though the container walls constitute an insignificant portion of the total heat capacity, less than 1% that of the liquid and vapor, variation of the wall heat capacity has a significant influence on the pressure rise rate. During a portion of the process, it is found that radial as well as axial stratification exists, with simultaneous evaporation and condensation occurring at various locations on the liquid-vapor interface.
I. INTRODUCTION

A number of space missions of current interest require the storage of liquid propellents for long periods of time, varying from hours to months. The ultimate goal is to maximize the quantity of useful propellant remaining at the end of this period of time. In the limit this maximum corresponds to the nonvented condition in which the original mass of propellant is retained. Whether nonventing is practical depends on the maximum internal pressure which results from the thermal interaction between the storage container, its contents, and its ambient. The prediction of pressure changes in a nonvented container is thus of major importance for purposes of feasibility determination and optimization.

In Ref. 3 the results of the initial attempt to calculate the pressure rise in a single-component, two-phase system were presented for a two-dimensional case with axial symmetry and an imposed heat flux. The governing equations were placed in finite difference form and solved with an IBM-7090 computer, using the MAD language, for an example case with liquid oxygen.

The measurements of pressure rise in a full size earth orbiting liquid hydrogen container on the Saturn IB vehicle AS-203, as reported in Ref. 17, makes possible a comparison with pressure rise calculated from this model. Certain deviations from the actual physical system are necessary in defining a model owing to the present stage of development of the computational procedures involved. These will be discussed at the appropriate places.

This report presents the results of the computations made on a model simulating the S-IV B stage on the Saturn IB vehicle AS-203, along with comparisons with the measurements. Since the IBM-7090 computer had been replaced by the IBM-360 system at The University of Michigan, it was expedient to transform the source program from MAD to FORTRAN IV, and the computer programs are so presented herein. For the sake of completeness the general formulation presented in Ref. 3 is reproduced here.

The pressure in a tank containing two phases, liquid and vapor, is related directly to the temperature at the liquid-vapor interface for a single component system. For a binary system, the pressure depends not only on the temperature at the interface but the relative liquid-vapor concentration of the two components as well. Pressure changes take place owing to heat and mass transfer interactions between the vapor, liquid, and container walls.

The processes of heat and mass transfer interactions between the gas and liquid phases of a single component in cylindrical containers with axial symmetry are considered in this study. In the general formulation presented first attention is given to the cases of external pressurization with and with-
out liquid discharge as well as to the nonvented case. Solutions are then presented, utilizing numerical finite-difference procedures for the nonvented case.

The initial conditions of the liquid and vapor are considered to be those at equilibrium, with uniform pressure $P$ and saturation temperature $T_0$. From these initial conditions, the walls of the container undergo a thermal perturbation, such as a change in temperature or an exposure to an external heat flux, either of which may be an arbitrary function of time and axial location. The perturbations in the boundary conditions lead to a series of nonequilibrium phenomena within the container. Natural convection currents are set up in the liquid and in the vapor spaces. The liquid-vapor system tends to adjust to the new nonequilibrium conditions within the container by transferring mass and energy across the interface by either evaporation or condensation. The conditions at the liquid-vapor interface couple simultaneous transport processes in the liquid and gas phases. In the case of self-pressurization of nonvented tanks the rate of pressure rise within the tank is governed by the rate of heat and mass transfer from the liquid and the rate of heat transfer from the wall to the gas phase. The interfacial temperature is essentially that of equilibrium (saturation) conditions corresponding to the system pressure. At the same time, the temperature of the liquid-vapor interface affects both the interfacial mass and heat transfer, as well as the convective processes in both phases. These latter processes influence the temperature gradients within both phases and in turn will have an effect on the rate of pressure rise in the ullage space. Indeed, all the processes of heat transfer from the ambient to both phases, the natural convection within the container, the interfacial phenomena and the rate of ullage pressure rise are all mutually coupled. Such interactions have been the subject of many experimental investigations.

Any analytical approach that would adequately describe the phenomena taking place in propellant tanks must take into account these interactions. This requires the calculation of the transient velocity and temperature profiles in both the vapor and liquid phases. Some work has been done along this line.$^{1-3}$ The difficulties in solution of this problem is further complicated with the presence of turbulence or boiling of liquid near the tank walls, as their nature is not yet fully understood nor adequately described.$^5$ Some assumptions are necessary at the present time for the construction of models which reasonably represent the practical situations.

A survey which reviewed much of the available literature in this area, such as pressurization, stratification, and interfacial phenomena in fuel tanks, is given in Ref. 5. Several studies which are pertinent to the present one will be mentioned. Thomas and Morse$^6$ and Knuth$^7$ have considered the phase change of suddenly pressurized single-component, liquid-vapor system. Thomas and Morse$^6$ presented both an exact solution and an approximate solution yielding an explicit expression for the mass transfer at the interface. Yang, et al.,$^8-10$ have solved the problem of phase change in a suddenly pressurized single-component and binary liquid-vapor system.
Thermodynamic equilibrium is assumed at the interface, i.e., the interfacial temperature is always at the saturation temperature corresponding to the system pressure. Experience shows that this is a reasonable assumption.\(^5\)

Epstein, et al.,\(^11\) used finite-differences for the calculation of the pressurization process. In this model, the axial variation of temperature is considered while the radial variation is neglected. The wall to fluid heat transfer was accounted for by introducing an effective heat transfer coefficient between the fluid and the wall. Also, effective thermal conductivities and mass diffusivities between the adjacent fluid layers were used to simulate the effects of random fluid motion. Provisions for variable tank cross-section as well as variable heat transfer coefficients were made in the program. The effect of boiling near the walls was not considered, and momentum effects were neglected. The principal advantage of this program is its simplicity and the relatively small amount of machine time required to advance the solution to a particular time level. However, the determination of the effective thermal conductivities and heat transfer coefficients and their variation with location require the determination of a large number of empirical constants, which require considerable experimental experience with a particular system.

Although some analytical models have been used to study liquid stratification in self-pressurized propellant containers, with the exception of Ref.\(^3\), none has considered the simultaneous interactions between the liquid and the vapor phases. Most of the available studies of the pressure and temperature transients in the vapor space are experimental in nature and have served to identify some factors influencing the rate of pressure rise within the containers. The studies of Huntley\(^12\) show that liquid stratification causes the pressure to rise at a higher rate than that calculated using the average or mixed mean liquid temperature. These also showed that stirring the liquid causes a smaller rate of pressure rise, while stirring the vapor resulted in a substantial increase in pressure. Higher rates of pressure rise were obtained with smaller ullage volumes. Liebenberg and Edescut have shown similar results.\(^13\)
II. FORMULATION

A. GENERAL FORMULATION

The formulation of the most general case will be given first. This involves external pressurization, liquid discharge, and the consideration of the vented and nonvented container. The heat capacity of the container side wall is also included in the formulation. The assumptions and limitations will be mentioned as appropriate and the results and problems encountered in the solution of the unvented, nondischarge case will be given.

A cylindrical container of diameter \(2a\), height \(b\) and with wall thickness \(\delta\), is partially filled with a liquid as shown in Fig. 1. The initial height of the liquid is \(X_0\), and that of the vapor is \(b-X_0\). The origin of the coordinate system is taken at the center of the container base with \(x\)-positive in the direction of the liquid. At any time \(t\), the location of the liquid-vapor interface is given by \(X = X(t)\). The fluid in both phases is initially at rest in an equilibrium state at a uniform temperature \(T_0\) and pressure \(P_0\). From these initial conditions the tank side walls are subjected to an arbitrary heat flux, \(q_w(x,t)\). The tank ends are assumed adiabatic with zero heat capacity, but this restriction can be relaxed if desired. The differential equations describing the physical behavior are listed separately below for the various regions.

1. Tank Side Wall

The differential equation describing the temperature-time history of the tank wall, considering the wall to be lumped radially but not axially, is

\[
(\rho c_p \frac{\delta}{\delta t})_w = - (k \frac{\partial T}{\partial x})_f + (\delta k \frac{\partial^2 T}{\partial x^2})_w + q_w(x,t),
\]

where the subscripts \(w\) and \(f\) refer to the wall and the fluid, respectively. The first term on the right-hand side of Eq. (1) is the rate of heat flow from the wall to the fluid, while the second term accounts for the net axial conduction.

Initial and Boundary Conditions

\[
T_w(x,0) = T_0
\]

\[
\frac{\partial T_w(0,t)}{\partial x} = \frac{\partial T_w(b,t)}{\partial x} = 0
\]

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Fig. 1. Container configuration and coordinate system.
2. Liquid Region

The following assumptions are made:

(a) Thermal conductivity and viscosity are constant.

(b) Fluid is incompressible. Density variations are introduced only in the body force term of the momentum equation. These variations are described by

\[ \rho = \rho_0 [1 + \beta(T_o - T)] \]  

(c) The influence of viscous dissipation is neglected.

Governing Equations

(i) axial-momentum

\[ \rho \frac{Du}{Dt} = -\rho g - \frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2} \right) \]

(ii) radial momentum

\[ \rho \frac{Dv}{Dt} = -\frac{\partial p}{\partial r} + \mu \left( \frac{\partial^2 v}{\partial x^2} - \frac{v}{r^2} + \frac{1}{r} \frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial r^2} \right) \]

(iii) continuity

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial r} + \frac{v}{r} = 0 \]

(iv) energy

\[ \frac{DT}{Dt} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} \right) \]
Initial Conditions

\[ T(x, r, o) = T_o \quad , \quad u(x, r, o) = 0 \quad , \quad v(x, r, o) = 0 \]  \hspace{1cm} (8)

Boundary Conditions

Velocity

\[ u(o, r, t) = 0 \]  \hspace{1cm} (9)

\[ u(x_{-o}, r, t) = - \frac{w_{ld}}{\rho \pi a^2} \]  \hspace{1cm} (10)

where \( w_{ld} \) is the rate of liquid discharge.

\[ u(x, a, t) = 0 \]  \hspace{1cm} (11)

\[ \frac{\partial u(x, a, t)}{\partial r} = 0 \]  \hspace{1cm} (12)

\[ v(x, a, t) = 0 \]  \hspace{1cm} (13)

\[ v(x, o, t) = 0 \]  \hspace{1cm} (14)

\[ v(o, r, t) = 0 \]  \hspace{1cm} (15)

\[ \frac{\partial v(x, r, t)}{\partial x} = 0 \]  \hspace{1cm} (16)

Temperature

\[ T(x, r, t) = T_s(t) \]  \hspace{1cm} (17)

\[ T(x, a, t) = T_w(x, t) \]  \hspace{1cm} (18)
\[ \frac{\partial T(o,x,r,t)}{\partial x} = 0 \]  \hspace{1cm} (19)  
\[ \frac{\partial T(x,o,t)}{\partial r} = 0 \]  \hspace{1cm} (20)  

where \( T_s(t) \) is the saturation temperature corresponding to the system pressure and \( T_w \) is the wall temperature, Eq. (1).  

3. Vapor Region  
The differential equations for the vapor region are obtained assuming negligible changes in the viscosity and thermal conductivity within the vapor phase. The compressibility of the vapor is considered.  

Governing Equations  

(i) axial momentum

\[ \rho \frac{Du}{Dt} = -\rho g - \frac{\partial \sigma}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2} \right) + \frac{1}{3} \mu \frac{\partial}{\partial x} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial r} + \frac{v}{r} \right) \]  \hspace{1cm} (21)  

(ii) radial momentum

\[ \rho \frac{Dv}{Dt} = -\frac{\partial \sigma}{\partial r} + \mu \left( \frac{\partial^2 v}{\partial x^2} - \frac{v}{r^2} + \frac{1}{r} \frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial r^2} \right) + \frac{1}{3} \mu \frac{\partial}{\partial r} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial r} + \frac{v}{r} \right) \]  \hspace{1cm} (22)  

(iii) continuity

\[ \frac{D\rho}{Dt} + \rho \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial r} + \frac{v}{r} \right) = 0 \]  \hspace{1cm} (23)  

(iv) energy

\[ \rho c_v \frac{DT}{Dt} + \rho \frac{D(P/\rho)}{Dt} = \frac{D\rho}{Dt} + k \left( \frac{\partial^2 T}{\partial x^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} \right) \]  \hspace{1cm} (24)  

(v) equation of state

\[ p = z_0 \rho RT \]  \hspace{1cm} (25)
where \( z \) is the compressibility factor which can be evaluated from \( P, V, T \) data.

**Initial Conditions**

\[
T(x, r, o) = T_o \\
\rho(x, r, o) = \rho_o \\
u(x, r, o) = 0 \\
v(x, r, o) = 0
\]  
(26)

**Boundary Conditions**

**Velocity**

\[
u(b, r, t) = 0
\]  
(27)

or

\[
u(b, r, t) = u_{\text{gp}} = -\frac{w_{\text{gp}}}{\rho_{\text{gp}}} \quad ,
\]  
(28)

where \( w_{\text{gp}} \) is the rate of mass flow of the pressurant and \( \rho_{\text{gp}} \) is its density. Equation (28) is written for the case of uniform velocity at the inlet diffuser of the tank as in Fig. 2.

\[
u(x_{+o}, r, t) = u_{\text{gi}}
\]  
(29)

\[
\frac{\partial u(x, o, t)}{\partial r} = 0
\]  
(30)

\[
u(x, a, t) = 0
\]  
(31)
\[ v(b,r,t) = 0 \]  
\[ v(x,a,t) = 0 \]  
\[ v(x,o,t) = 0 \]  
\[ \frac{\partial v(x,r,t)}{\partial x} = 0 \]  
\[ \text{where } u_{gi} \text{ is the velocity of the vapor at the interface caused by simultaneous phase change and liquid discharge, Eq. (45). The boundary conditions given by Eqs. (16) and (35) assume a zero shear stress in both the liquid and vapor at the interface. These are reasonable and are made for the sake of simplification. Another approximation would be to take the vapor velocity at the interface equal to that of the liquid at that point. The interfacial shear stress would then be determined by the analysis. For flows with large radial gas velocities at the interface, this latter boundary condition becomes more realistic.} \]

**Temperature**

\[ T(x,r,t) = T_s(t) \]  
\[ T(x,a,t) = T_w(x,t) \]  
\[ \frac{\partial T(x,o,t)}{\partial r} = 0 \]  
\[ \frac{\partial T(b,r,t)}{\partial x} = 0 \]  
\[ \text{or} \]

\[ T(b,r,t) = T_{gp}(r,t) \]  

Equation (39) applies for the case with no heat transfer across the top surface and no external pressurization, and Eq. (40) for arbitrary external pressurization.
Fig. 2. Control volume of vapor region.
4. Interfacial Heat and Mass Transfer

The processes in the liquid and vapor regions are coupled by $T_s$ in Eqs. (17) and (36) and by $u_{gi}$ in Eq. (29), which is related to the rate of mass transfer at the interface.

The rate of mass transfer by evaporation or condensation across the liquid-vapor interface depends on the relative rates of heat transfer by diffusion from each phase at the interphase. Conservation of energy at the interface determines the rate of phase change as well as the interfacial velocity of displacement, according to:

$$h_{fg} w_i = \int_0^a \left( k \frac{\partial T(X,r,t)}{\partial x} \right)_l - \left( k \frac{\partial T(X,r,t)}{\partial x} \right)_g \ 2\pi r \ dr , \quad (41)$$

where $w_i$ is the rate of interfacial phase change. According to Eq. (41), $w_i$ will be positive if condensation takes place. The interfacial displacement with no liquid discharge is then given by:

$$\rho_g \pi a^2 \frac{dX_i}{dt} = w_i \quad (42)$$

Should it be desirable to include the process of liquid discharge, Eq. (42) would be written

$$\rho_g \pi a^2 \frac{dX_d}{dt} = -w_d \quad (43)$$

where $w_d$ is positive for liquid discharge, and

$$u_i = \frac{dX_i}{dt} = \frac{dX_i}{dt} + \frac{dX_d}{dt} \quad (44)$$

where

$$\frac{dX_i}{dt} = \text{rate of interfacial displacement due to phase change},$$

$$\frac{dX_d}{dt} = \text{rate of interfacial displacement due to liquid discharge},$$

$$\frac{dX}{dt} = \text{combined rate of interfacial displacement due to phase change and liquid discharge}.$$
Equation (44) is obtained assuming that the liquid surface remains flat during the discharge process. This is an approximation which neglects the influence of viscosity near the walls and surface tension effects. Such effects are negligible except for very low gravity levels and small Bond numbers.

The vapor velocity \( u_{gi} \) at the interface is related to the rate of interfacial displacement due to phase change and discharge by,

\[
\frac{dX_i}{dt} = -\frac{\rho_f - \rho_{gs}}{\rho_{gs}} \frac{dX_d}{dt}
\]

(45)

B. TRANSFORMATION OF THE PARTIAL DIFFERENTIAL EQUATIONS

The momentum and the continuity equations are combined to obtain the vorticity equation and an elliptic equation relating the vorticity and the stream function. This is accomplished in the same manner as described in Refs. 1-3. The x-momentum is differentiated with respect to \( r \), the \( r \)-momentum equation is differentiated with respect to \( x \) and the two combined to eliminate the pressure terms.

1. Liquid Region

Equations (4) and (5) reduce to:

\[
\frac{D\omega'}{Dt} = -\frac{1}{\rho r} \frac{\partial p}{\partial r} + \nu \left[ \frac{\partial^2 \omega'}{\partial x^2} + \frac{3}{r} \frac{\partial \omega'}{\partial r} + \frac{\partial \omega'}{\partial x} \right]
\]

(46)

where \( \omega' \) is given by:

\[
\omega' = \frac{1}{r} \left( \frac{\partial u}{\partial x} - \frac{\partial v}{\partial x} \right)
\]

(47)

Introducing the stream function \( \psi' \), defined by

\[
u = \frac{1}{r} \frac{\partial \psi'}{\partial r} ; \quad \psi = -\frac{1}{r} \frac{\partial \psi'}{\partial x}
\]

(48)

Equations (7), (46), and (6) are transformed respectively to:

\[
\frac{\partial \psi'}{\partial t} + \frac{1}{r} \frac{\partial \psi'}{\partial r} \frac{\partial \psi'}{\partial x} - \frac{1}{r} \frac{\partial \psi'}{\partial x} \frac{\partial \psi'}{\partial r} = \alpha \left( \frac{\partial^2 \psi'}{\partial x^2} + \frac{1}{r} \frac{\partial \psi'}{\partial r} + \frac{\partial^2 \psi'}{\partial r^2} \right)
\]

(49)

13
\[
\frac{\partial \omega'}{\partial t} + \frac{1}{r} \frac{\partial \psi'}{\partial r} \frac{\partial \omega'}{\partial x} - \frac{1}{r} \frac{\partial \psi'}{\partial x} \frac{\partial \omega'}{\partial r} = \frac{\rho \omega'}{r} \frac{\partial \omega'}{\partial r} + \nu \left( \frac{\partial^2 \omega'}{\partial x^2} + \frac{2}{r} \frac{\partial \omega'}{\partial r} + \frac{\partial^2 \omega'}{\partial r^2} \right),
\]

(50)

\[
\frac{\partial \psi'}{\partial x^2} - \frac{1}{r} \frac{\partial \psi'}{\partial r} + \frac{\partial^2 \psi'}{\partial r^2} = \omega' r^2
\]

(51)

The initial and boundary conditions must also be transformed appropriately. Equations (49), (50), and (51) are solved numerically by finite differences to obtain the temperature, vorticity, and stream function distributions. From Eq. (48) the velocity distributions can then be computed.

2. Vapor Region

Combining the momentum Eqs. (21) and (22) in the manner described above, the following is obtained:

\[
\frac{D\omega'}{Dt} = \frac{\gamma'}{\rho} \frac{D\rho}{Dt} - \frac{1}{\rho r} \frac{D\gamma}{Dt} \frac{\partial \omega'}{\partial r} + \frac{1}{\rho r} \frac{D\gamma}{Dt} \frac{\partial \omega'}{\partial x} - \frac{\gamma}{\rho r} \frac{\partial \omega'}{\partial r} + \nu \left( \frac{\partial^2 \omega'}{\partial x^2} + \frac{2}{r} \frac{\partial \omega'}{\partial r} + \frac{\partial^2 \omega'}{\partial r^2} \right)
\]

(52)

where \( \omega' \) is given by Eq. (47).

Equation (52) is the vorticity equation for this case. The first terms within the brackets on the right-hand side of this equation account for the compressibility effects due to density variation. Should the compressibility effects be neglected, Eq. (52) will reduce to Eq. (46). It should be noted that the presence of these terms does not introduce any additional difficulties as far as the solution of the vorticity equation is concerned. The difficulty will be in combining the continuity Eq. (23) and Eq. (47), in order to obtain an equation similar to Eq. (51). However, this difficulty can be overcome if the term \( \frac{\partial \gamma}{\partial t} \) is neglected in combining the continuity Eq. (23) with the definition of vorticity. The rigorous justification for this approximation has not yet been established. In this case Eq. (23) will be rewritten as:

\[
\frac{\partial (\rho u)}{\partial x} + \frac{1}{r} \frac{\partial (\rho v)}{\partial r} = 0.
\]

(53)

The continuity Eq. (53) is combined with Eq. (47) by introducing the stream function \( \psi'' \), which satisfies (53). This stream function \( \psi'' \) is defined by:

\[
u = \frac{1}{\rho r} \frac{\partial \psi''}{\partial r}; \quad v = -\frac{1}{\rho r} \frac{\partial \psi''}{\partial x}
\]

(54)
Substituting (54) and (47) into (53), the following equation which relates \( \psi'' \) to \( \omega' \), is obtained,

\[
\frac{\partial^2 \psi''}{\partial \alpha^2} - \frac{1}{r} \frac{\partial \psi''}{\partial \alpha} + \frac{\partial^2 \psi''}{\partial \alpha^2} = \rho \omega' r^2 + r \left[ u \frac{\partial \psi}{\partial \alpha} - v \frac{\partial \psi}{\partial \alpha} \right]
\]  \( (55) \)

The last terms within the brackets in Eq. (55) account for the density changes in the vapor region. The terms \( \partial \rho / \partial \alpha \) and \( \partial \rho / \partial \alpha \) are calculated in the finite-difference procedure using the equation of state and the temperature distribution from the previous time step. The coefficients \( u \) and \( v \) in \( u \partial \psi / \partial \alpha \) and \( v \partial \psi / \partial \alpha \) will be taken equal to their values at the previous time step, which is the same procedure used in calculating the nonlinear terms \( u \partial T / \partial \alpha \), \( v \partial T / \partial \alpha \), \( u \partial \omega' / \partial \alpha \), and \( v \partial \omega' / \partial \alpha \), thus in effect linearizing these terms.

The spatial variation of pressure in the vapor region can be neglected. The density terms in Eqs. (52) and (55) are evaluated using the equation of state (25), and the derivatives given by:

\[
\frac{\partial \rho}{\partial \alpha} = - \frac{P}{R \zeta T} \left( \frac{\partial (ZT)}{\partial T} \right)_P \frac{\partial T}{\partial \alpha} \]  \( (56) \)

\[
\frac{\partial \rho}{\partial \alpha} = - \frac{P}{R \zeta T^2} \left( \frac{\partial (ZT)}{\partial T} \right)_P \frac{\partial T}{\partial \alpha} \]  \( (57) \)

Substitution in Eq. (52) gives:

\[
\frac{D \omega'}{D t} - \omega' \frac{d p}{dt} + \omega' \left( \frac{\partial (ZT)}{\partial T} \right)_P \frac{d T}{d t} = - \frac{g}{rZT} \left( \frac{\partial (ZT)}{\partial T} \right)_P \frac{\partial T}{\partial \alpha} + \frac{1}{rZT} \frac{\partial T}{\partial \alpha} \left( \frac{\partial (ZT)}{\partial T} \right)_P
\]

\[
\cdot \cdot \cdot \frac{D u}{D t} = - \frac{1}{rZT} \left( \frac{\partial (ZT)}{\partial T} \right)_P \frac{\partial T}{\partial \alpha} \frac{D v}{D t} + v \left[ \frac{\partial \omega'}{\partial \alpha} + \frac{3}{r} \frac{\partial \omega'}{\partial \alpha} \right]
\]  \( (58) \)

Substitution of Eqs. (25), (56), and (57) into Eq. (55) gives:

\[
\frac{\partial^2 \psi''}{\partial \alpha^2} - \frac{1}{r} \frac{\partial \psi''}{\partial \alpha} + \frac{\partial^2 \psi''}{\partial \alpha^2} = \frac{P}{R \zeta T} \omega' r^2 - \frac{Pr}{R \zeta T^2} \left( \frac{\partial (ZT)}{\partial T} \right)_P \left[ u \frac{\partial T}{\partial \alpha} - v \frac{\partial T}{\partial \alpha} \right]
\]  \( (59) \)

The terms \( u \) and \( v \) in Eqs. (58) and (59) could also be expressed in terms of \( \psi'' \) by Eq. (54). Substitution of the equation of state (25) into the energy Eq. (24) gives:
\[ \rho \left[ C_v + R \frac{\partial \left( \frac{\alpha(T)}{\partial T} \right)}{\partial T} \right] \frac{DT}{Dt} = \frac{dP}{dt} + k \nabla^2 T \] (60)

Taking the thermodynamic relation

\[ \frac{C_p}{C_v} = T \frac{\partial V}{\partial T} \frac{\partial P}{\partial T} \frac{\partial T}{\partial V} \] (61)

and the ratio of specific heats

\[ \frac{C_p}{C_v} = \gamma \] (62)

and substituting into Eq. (60) and rearranging, another form of the energy equation is:

\[ \frac{DT}{Dt} = \frac{1}{1 + (\gamma - 1)Z} \frac{R \beta T}{C_v} \frac{dP}{dt} + \frac{\gamma}{1 + (\gamma - 1)Z} \nabla^2 T \] (63)

It might be noted that the body force term in Eq. (58) could also be expressed as:

\[ \frac{g}{rZT} \left( \frac{\alpha(T)}{\partial T} \right) \frac{\partial T}{\partial x} = \frac{g}{r} \beta \frac{\partial T}{\partial x} \] (64)

For the case where the behavior of the vapor is approximated adequately by an ideal gas, Eqs. (58), (59), and (63) reduce, respectively, to:

\[ \frac{D\omega'}{Dt} - \frac{\omega'}{P} \frac{dP}{dt} + \omega' \frac{DT}{Dt} = - \frac{\alpha(T)}{rT} \frac{\partial T}{\partial x} + \frac{1}{rT} \frac{\partial T}{\partial r} \frac{Du}{dt} - \frac{1}{rT} \frac{\partial T}{\partial x} \frac{Dv}{dt} + \nu \left[ \frac{\partial^2 \omega'}{\partial x^2} + \frac{2}{r} \frac{\partial \omega'}{\partial r} + \frac{\partial^2 \omega'}{\partial r^2} \right] \] (65)

\[ \frac{\partial^2 \psi''}{\partial x^2} - \frac{1}{r} \frac{\partial \psi''}{\partial x} + \frac{\partial^2 \psi''}{\partial r^2} = \frac{P}{RT} \omega'r^2 - \frac{Pr^2}{RT^2} \left[ u \frac{\partial T}{\partial x} - \nu \frac{\partial T}{\partial x} \right] \] (66)
\[
\frac{DT}{Dt} = \frac{\gamma - 1}{\gamma} T \frac{dP}{dt} + \alpha V T
\] (67)

Equations (58), (59) and (63) (or (65)-(67) for ideal gases) together with the transformed boundary conditions constitute a system of 3 equations with 4 unknowns for the vapor space, \(\omega'(x,r,t)\), \(\psi''(x,r,t)\), \(T(x,r,t)\), and \(P(t)\). The finite-difference solution of Eqs. (58), (59), and (63) for \(\omega', \psi'',\) and \(T\) requires that a function for \(dP/dt\) be available. This is obtained from the First Law of Thermodynamics written as an instantaneous rate equation, taking the vapor space as the control volume. Referring to Fig. 2 this formulation is written:

\[
\frac{d}{dt} \int_V \left( \rho \frac{dV}{dA} \right) - \int_{A_1} h_{gs} \rho_{gs} (u_i - u) dA + \int_{A_p} h_{gp} \rho_{gp} u dA = \int_{A_{c.s.}} q dA - P \frac{dV}{dt}
\] (68)

This formulation includes the following generalized variation:

\[
\begin{align*}
e_{g} & = e(x,r,t) \\
\rho_{g} & = \rho(x,r,t) \\
u_{gi} & = u(r,t)
\end{align*}
\]

\(h_{gs}, \rho_{gs}\) are uniform over the liquid-vapor interface

\(u_i\) is uniform for a flat interface

\(h_{gp}, \rho_{gp}, u_p\) are specified by the pressurization process

\(q\) is the local heat flux on the control surface, and its integral includes that from the tank wall and through the liquid-vapor interface.

Given property information on \(e_{g} = f_1(P,T), \rho_{g} = f_2(P,T), h_{gs} = f_3(P), \rho_{gs} = f_4(P)\), the desired term \(dP/dt\) can be extracted from the first term of Eq. (68). The necessary properties can come either from specific heat data plus an equation of state, or from tabulated values. It may be anticipated that different computational procedures would be required for each of these.
The procedure will be demonstrated for the case of self-pressurization, that is with no liquid discharge, no external pressurization, and a flat interface. Thus, \( w_d = 0 \), \( w_p = 0 \), and \( \frac{\partial p}{\partial t} \) = 0. Then,

\[
\frac{dX}{dt} = u_i = \frac{dx_i}{dt} \tag{69}
\]

and from Eq. (45), assuming a uniform vapor velocity across the interface

\[
u_{gi} - u_i = \frac{\rho_g}{\rho - \rho_g} u_i \tag{70}
\]

For the coordinate frame selected

\[
\frac{dV_g}{dt} = -A_{i} \frac{dx_i}{dt} = -A_{i} u_i \tag{71}
\]

For brevity

\[
q_{tot} = \int_{A_{c,s}} q dA \tag{72}
\]

Substituting Eqs. (70)-(72) into Eq. (68)

\[
\frac{d}{dt} \int_{V_g} (\rho g e) dV = q_{tot} + h \frac{\rho_g}{\rho - \rho_g} u_i A_i + PA_i u_i \tag{73}
\]

Using Leibnitz's rule for differentiation of integrals having variable limits the first term of Eq. (73) can be written as

\[
\frac{d}{dt} \int_{V_g} (\rho g e) dV = \int_{V_g} \rho g \frac{\partial e}{\partial t} dV + \int_{V_g} e \frac{\partial g}{\partial t} dV - \rho_g e u_i A_i \tag{74}
\]

Substituting Eq. (74) in Eq. (73) and rearranging with the use of Eq. (45) gives,

\[
\int_{V_g} \rho g \frac{\partial e}{\partial t} dV + \int_{V_g} e \frac{\partial g}{\partial t} dV = q_{tot} + h A_i u_i (\rho_g - \rho) \tag{75}
\]
For the case in which the use of an equation of state (25) is desired, Eq. (75) can be expressed as the following by means of general thermodynamic property relations:

\[
\frac{C_v}{R} \frac{dP}{dt} = \int_v \frac{dV}{g} \left[ \left( \frac{\partial (ZT)}{\partial T} \right)_P \right] + \int_v \left( \frac{pC_v}{R} + \frac{1}{ZT} \right) \frac{dV}{g} + \int_v \left( \frac{pC_v}{R} + \frac{1}{ZT} \right) \frac{dV}{g} \\
- \frac{P}{R} \int_v \frac{h}{ZT} \left[ \frac{dV}{g} \right] \frac{dV}{g} = q_{tot} + \frac{h_{gs} A_i u_i}{\rho_{gs} - \rho_{li}} 
\]

(76)

The enthalpy terms in Eqs. (75) and (76) can be expressed by:

\[
h - h_R = \int^{T}_{T_R} \frac{C}{P} dT + \int^{P}_{P_R} \left\{ Z - \left[ \frac{dV}{g} \right] \frac{dV}{g} \right\} 
\]

(77)

where the subscript \( R \) refers to a low pressure reference condition.

For an ideal gas with constant specific heat Eq. (76) reduces to:

\[
\frac{C_v}{R} \frac{dP}{dt} = q_{tot} + \frac{h_{gs} A_i u_i}{\rho_{gs} - \rho_{li}} 
\]

(78)

Expressing the enthalpy in terms of specific heat and rearranging, Eq. (78) reduces to:

\[
\frac{dP}{dt} = \frac{R}{C_v} q_{tot} + \frac{RT_{gs}}{V} A_i u_i (\rho_{gs} - \rho_{li}) 
\]

(79)

For the case far from the region of the critical state, such that \( \rho_{li} \gg \rho_{gs} \), Eq. (79) can be written as

\[
\frac{dP}{dt} = \frac{R}{C_v} q_{tot} - \frac{w_{iR} T_s}{V} 
\]

(80)

where \( w_i \) is the rate of interfacial mass transfer (having a negative value for evaporation) and is given by

\[
w_i = -\frac{\rho_{gs} A_i (u_i g_i - u_i)}{\rho_{li} - \rho_{gs}} A_i u_i 
\]

(81)
C. DIMENSIONLESS FORM OF THE EQUATIONS

The governing equations for the wall, liquid, and vapor regions (1), (49), (50), (51), (58), (59), and (63), along with the boundary conditions, are restated in dimensionless forms for convenience.

The substitution used to nondimensionalize these are:

\[
\begin{align*}
\nu &= \frac{a^2}{\alpha} \frac{b}{U}; & V &= \frac{a}{b} V; & T - T_o &= \frac{V}{\beta \rho \sigma a^2} \theta \\
t &= \frac{a^2}{\alpha} \tau; & x &= b \zeta; & r &= \alpha \eta \\
\omega'' &= \frac{a^2}{\alpha} \omega; & \psi' &= \alpha b \psi
\end{align*}
\]

The resulting dimensionless equations are given below.

1. Tonic Side Wall

The dimensionless equations describing the wall temperature, from Eq. (1), are given by:

\[
(i) \quad 0 \leq \zeta \leq \frac{x(t)}{b}
\]

\[
\frac{\partial \theta}{\partial \tau} = - \frac{a}{\delta_w} \frac{\rho C_v}{\rho C_p} \frac{k}{k_w} \left( \frac{\partial \theta}{\partial \eta} \right) \eta = 1 + \frac{a}{\delta_w} \frac{\rho C_v}{\rho C_p} \frac{k}{k_w} \frac{\partial^2 \theta}{\partial \xi^2} + \frac{a}{b} \frac{\rho C_v}{\rho C_p} \frac{k_w}{k_w} \Pr \cdot Gr_w^* (\zeta, t) \quad (83)
\]

\[
(ii) \quad \frac{x(t)}{b} \leq \zeta \leq 1
\]

\[
\frac{\partial \theta}{\partial \tau} = - \frac{a}{\delta_w} \frac{\rho C_v}{\rho C_p} \frac{k}{k_w} \left( \frac{\partial \theta}{\partial \eta} \right) \eta = 1 + \frac{a}{\delta_w} \frac{\rho C_v}{\rho C_p} \frac{k}{k_w} \frac{\partial^2 \theta}{\partial \xi^2} + \frac{a}{b} \frac{\rho C_v}{\rho C_p} \frac{k_w}{k_w} \Pr \cdot Gr_w^* (\zeta, t) \quad (84)
\]

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where \( \text{Gr}_w^*(\xi,t) \) is the modified Grashof number and is given by:

\[
\text{Gr}_w^*(\xi,t) = \frac{g\beta a^4}{k \nu^2} q_w(\xi,t)
\]  

(iii) Boundary and initial conditions

\[
\frac{\partial \theta_w^*}{\partial \xi}(0,\tau) = \frac{\partial \theta_w^*}{\partial \xi}(1,\tau) = 0
\]
\[
\theta(t,0) = 0
\]

2. Liquid Region

(i) The energy equation, Eq. (49)

\[
\frac{\partial \theta}{\partial t} + \frac{1}{\eta} \frac{\partial \psi \frac{\partial \theta}{\partial \eta}}{\partial \eta} - \frac{1}{\eta} \frac{\partial \psi \frac{\partial \theta}{\partial \eta}}{\partial \eta} = \frac{a^2}{b^2} \frac{\partial^2 \theta}{\partial \xi^2} + \frac{1}{\eta} \frac{\partial \theta}{\partial \eta} + \frac{\partial \theta}{\partial \eta}^2
\]

(ii) The vorticity equation, Eq. (50)

\[
\frac{\partial \omega}{\partial t} + \frac{1}{\eta} \frac{\partial \psi \frac{\partial \omega}{\partial \eta}}{\partial \eta} - \frac{1}{\eta} \frac{\partial \psi \frac{\partial \omega}{\partial \eta}}{\partial \eta} = \text{Pr} \left[ \frac{1}{\eta} \frac{\partial \theta}{\partial \eta} + \frac{a^2}{b^2} \frac{\partial \psi \omega}{\partial \xi^2} + \frac{3}{\eta} \frac{\partial \omega}{\partial \eta} + \frac{\partial \omega}{\partial \eta}^2 \right]
\]

(iii) The vorticity-stream function equation, Eq. (51)

\[
\frac{1}{\eta^2} \left( \frac{a^2}{b^2} \frac{\partial \psi}{\partial \xi^2} - \frac{1}{\eta} \frac{\partial \psi}{\partial \eta} + \frac{\partial \psi}{\partial \eta} \right) = \omega
\]

\[
U = \frac{1}{\eta} \frac{\partial \psi}{\partial \eta} \; ; \; V = - \frac{1}{\eta} \frac{\partial \psi}{\partial \xi}
\]

(iv) The boundary conditions

(1) Stream function boundary conditions

\[
\psi(0,\eta,\tau) = \frac{\partial \psi(0,\eta,\tau)}{\partial \xi} = 0
\]
\[
\psi(\xi,\eta,\tau) = \frac{\partial^2 \psi(\xi,\eta,\tau)}{\partial \xi^2} = 0
\]
\[
\psi(\xi, 0, \tau) = \partial_{\eta} \left[ \frac{1}{\eta} \frac{\partial \psi(\xi, 0, \tau)}{\partial \eta} \right] = 0 \quad (94)
\]

\[
\psi(\xi, 1, \tau) = \frac{\partial \psi(\xi, 1, \tau)}{\partial \eta} = 0 \quad (95)
\]

(2) Vorticity boundary conditions

\[
w(1, \eta, \tau) = 0 \quad (96)
\]

\[
w(\xi, 0, \tau) = 0 \quad (97)
\]

Two additional vorticity boundary conditions are required at the tank wall and bottom. An explicit expression for the vorticity at any of these locations is difficult to obtain. The method of solution used in this work, which follows that of Refs. 1-3, overcomes this difficulty.

(3) Thermal boundary conditions

\[
\theta(\xi, 1, \tau) = \theta_w(\xi, \tau) \quad (98)
\]

\[
\theta(\xi, \eta, \tau) = \theta_s(\tau) \quad (99)
\]

\[
\frac{\partial \theta(0, \eta, \tau)}{\partial \xi} = 0 \quad (100)
\]

\[
\frac{\partial \theta(\xi, 0, \tau)}{\partial \eta} = 0 \quad (101)
\]

where \( \theta_s \) is given by:

\[
\theta_s(\tau) = \frac{a}{b} Pr \left[ T_s(t) - T_0 \right] \frac{g B_0 a^3}{v^2} \quad (102)
\]

(4) Initial conditions

\[
\theta(\xi, \eta, 0) = U(\xi, \eta, 0) = V(\xi, \eta, 0) = w(\xi, \eta, 0) = \psi(\xi, \eta, 0) = 0 \quad (103)
\]
3. Vapor Region

(i) The energy equation, Eq. (63)

\[
\frac{\partial \Theta}{\partial \tau} + \frac{1}{\rho \eta} \frac{\partial \psi}{\partial \eta} \frac{\partial \Theta}{\partial \zeta} - \frac{1}{\rho \eta} \frac{\partial \psi}{\partial \zeta} \frac{\partial \Theta}{\partial \eta} = \frac{1}{1 + (\gamma - 1)Z/\langle \chi \chi T \rangle \chi \chi T \langle \partial \chi \chi T \rangle} \frac{\partial}{\partial \tau} \left[ \frac{RZ(\Theta + \Theta_o)}{\rho} \frac{\partial P}{\partial \tau} \right] \\
+ \frac{\gamma}{1 + (\gamma - 1)Z/\langle \chi \chi T \rangle \chi \chi T \langle \partial \chi \chi T \rangle} \frac{\alpha}{\alpha} \left[ \frac{a^2}{b^2} \frac{\partial \Theta}{\partial \zeta} + \frac{1}{\eta} \frac{\partial \Theta}{\partial \eta} + \frac{\partial \Theta}{\partial \eta} \right]
\]

(104)

where

\[
\Theta_o = \frac{g \eta \chi^4}{\nu \alpha^2} T_o
\]

(105)

(ii) The vorticity equation, Eq. (58)

\[
\frac{\partial \omega}{\partial \tau} + \frac{1}{\rho \eta} \frac{\partial \psi}{\partial \eta} \frac{\partial \omega}{\partial \zeta} - \frac{1}{\rho \eta} \frac{\partial \psi}{\partial \zeta} \frac{\partial \omega}{\partial \eta} = Pr \frac{\nu}{\nu} \left[ \frac{a^2}{b^2} \frac{\partial \omega}{\partial \zeta} + \frac{3}{\eta} \frac{\partial \omega}{\partial \eta} + \frac{\partial \omega}{\partial \eta} \right] + \frac{ga^4}{\alpha^2 b z \eta (\Theta + \Theta_o)} \frac{\langle \chi \chi T \rangle}{\rho} \frac{\partial \Theta}{\partial \eta} \\
+ \frac{1}{\eta \chi (\Theta + \Theta_o)} \frac{\langle \chi \chi T \rangle}{\rho} \frac{\partial \Theta}{\partial \eta} \frac{\partial U}{\partial \tau} - \frac{1}{\eta \chi (\Theta + \Theta_o)} \frac{\langle \chi \chi T \rangle}{\rho} \frac{\partial \Theta}{\partial \eta} \frac{\partial V}{\partial \tau} + \frac{\partial P}{\partial \tau}
\]

(106)

where \( \beta \) has been replaced by the term:

\[
\beta_g = \frac{1}{\langle \chi \chi T \rangle} \frac{\partial \langle \chi \chi T \rangle}{\partial \rho}
\]

(107)

as in Eq. (64).

(iii) The vorticity-stream function equation, Eq. (59)
\[
\frac{\partial^2}{\partial \xi^2} \frac{\partial \psi}{\partial \xi} - \frac{1}{\eta} \frac{\partial \psi}{\partial \eta} = \frac{P}{ZR(\theta + \theta_0)} \left( \frac{g_\ell a^4}{\nu \alpha_\ell b} \right) \omega \eta^2
\]

\[
- \left( \frac{g_\ell a^4}{\nu \alpha_\ell b} \right) \frac{P}{ZR(\theta + \theta_0)} \left( \frac{\partial (ZT)}{\partial T} \right)_p \eta \left[ U \frac{\partial \theta}{\partial \eta} - \frac{a^2}{P^2} V \frac{\partial \theta}{\partial \xi} \right]
\]

(108)

(iv) Boundary conditions

1. Stream-function boundary conditions

\[
\psi(1, \eta, \tau) = \frac{\partial \psi(1, \eta, \tau)}{\partial \xi} = 0
\]

(109)

\[
\psi(\xi_1, \eta, \tau) = \frac{\rho g_s U_g i \eta^2}{2} = (\rho_\ell - \rho g_s) \frac{w_i}{\rho_\ell \alpha_\ell 2nb} \eta^2
\]

(110)

\[
\psi(\xi, 0, \tau) = \frac{\partial}{\partial \eta} \left( \frac{1}{\rho g i \eta} \frac{\partial \psi(\xi, 0, \tau)}{\partial \eta} \right) = 0
\]

(111)

\[
\psi(\xi, 1, \tau) = \frac{\partial \psi(\xi, 1, \tau)}{\partial \eta} = 0
\]

(112)

2. Vorticity boundary conditions

\[
\omega(\xi, 0, \tau) = 0
\]

(113a)

\[
\omega(\xi_1, \eta, \tau) = 0
\]

(113b)

3. Temperature boundary conditions

\[
\Theta(\xi_1, \eta, \tau) = \Theta_s(\tau)
\]

(114)

\[
\frac{\partial \Theta(\xi_1, \eta, \tau)}{\partial \xi} = 0
\]

(115)
\[ \theta(\xi, 1, \tau) = \theta_w(\xi, \tau) \]  
(116)

\[ \frac{\partial \theta(\xi, o, \tau)}{\partial \eta} = 0 \]  
(117)

(4) Initial conditions

\[ \theta(\xi, \eta, 0) = U(\xi, \eta, 0) = V(\xi, \eta, 0) = \psi(\xi, \eta, 0) = \alpha(\xi, \eta, 0) = 0 \]  
(118)

From the above results, it is clear that the temperature, pressure, and velocity within the container are functions of the parameters Gr^*, Pr, \((\rho_C p)_{L}/(\rho_C p)_{W}\), \((k_L/k_F)\), \((\alpha_L/\alpha_F)\), \((\nu_L/\nu_F)\), \((\beta_L/\beta_F)\), \((a/b)\), and \((a/b)\). Gr^* is defined in Eq. (83).
III. METHOD OF SOLUTION

A. FINITE-DIFFERENCE FORMS

The finite-difference method of solution used in Refs. 1-2 is adopted here. A complete discussion of the application of difference methods for the solution of the energy and vorticity equations is given in these references, along with the problem of stability of the difference equations. A brief discussion of the method of solution will be made here.

The basic technique in the application of finite-difference methods for the solution of partial differential equations is the use of Taylor Series Expansion to approximate the derivatives at a point in terms of the value of the function at that point and/or at its neighboring points. This may be demonstrated as follows:

1. The time derivative is represented by

\[ \frac{\partial f}{\partial \tau} = \frac{f(\tau + \Delta \tau) - f(\tau)}{\Delta \tau} + O(\Delta \tau) \]  \hspace{1cm} (119)

2. The first order derivatives \( \partial f/\partial \zeta \) and \( \partial f/\partial \eta \) can be approximated by

(i) \[ \frac{\partial f}{\partial \zeta} = \frac{f(\zeta + \Delta \zeta, \eta, \tau) - f(\chi, \eta, \tau)}{\Delta \zeta} + O(\Delta \zeta) \] \hspace{1cm} (forward differences) \hspace{1cm} (120)

or

(ii) \[ \frac{\partial f}{\partial \zeta} = \frac{f(\zeta, \eta, \tau) - f(\zeta - \Delta \zeta, \eta, \tau)}{\Delta \zeta} + O(\Delta \zeta) \] \hspace{1cm} (backward differences) \hspace{1cm} (121)

or

(iii) \[ \frac{\partial f}{\partial \zeta} = \frac{f(\zeta + \Delta \zeta, \eta, \tau) - f(\zeta - \Delta \zeta, \eta, \tau)}{2\Delta \zeta} + O(\Delta \zeta)^2 \] \hspace{1cm} (central differences) \hspace{1cm} (122)

3. The second order derivatives are replaced by finite-differences according to the formula.

\[ \frac{\partial^2 f}{\partial \zeta^2} = \frac{f(\zeta + \Delta \zeta, \eta, \tau) - 2f(\zeta, \eta, \tau) + f(\zeta - \Delta \zeta, \eta, \tau)}{(\Delta \zeta)^2} + O(\Delta \zeta)^2 \] \hspace{1cm} (123)
The function \( f \) represents either \( \Theta \) or \( \omega \), and \( \Delta \zeta \) is the spatial increment in the \( \zeta \)-direction. The last term on the righthand side of Eqs. (119) through (123) indicates the order of the truncation error involved in replacing the derivatives by finite-differences. It is clear that the central differences offer a better representation of the first order derivative \( \partial \Theta / \partial \zeta \) than the forward or backward differences. However, the form used to approximate the first order derivatives is usually determined by stability considerations. Similar formulations can be written for the derivatives \( \partial \Phi / \partial \eta \) and \( \Phi^2 / \partial \eta^2 \).

The substitution of the above formulae in the energy and vorticity equations produces a set of explicit difference equations. However, if the values of the function \( f \) in Eqs. (120) through (123) are taken at the time level \( \tau + \Delta \tau \) instead of being taken at time level \( \tau \), the resulting finite-difference methods may require the use of small time increments and consequently large machine time. Certain implicit formulations may permit the use of large increments. The application of both explicit and implicit methods to the present problem has been extensively investigated in Ref. 2. It was concluded that the lack of explicit boundary conditions for the vorticity at the solid boundaries prevent the use of large time increments, i.e., implicit methods. Other than that, it was decided to employ explicit methods.

It is clear from Eqs. (120) through (122) that more than one explicit finite-difference formulation can be constructed for each of the vorticity and energy equations. The finite-difference formulation chosen for the solution of the present problem is dictated by stability, as well as practical considerations, which will be shown below in studying the stability of the finite-difference equations.

The method of solution used in the present problem can be summarized as follows:

1. The time derivatives \( \partial \Theta / \partial \tau \) and \( \partial \omega / \partial \tau \) are approximated by Eq. (119).

2. The nonlinear terms \( U \partial \Theta / \partial \zeta \), \( V \partial \Theta / \partial \eta \), \( U \partial \omega / \partial \zeta \), and \( V \partial \omega / \partial \eta \) are linearized by considering the velocity components \( U \) and \( V \) to be known and are taken equal to their values at time level \( \tau \). The order of the error introduced by this linearization can be obtained from Taylor Series Expansion. If \( U_0 \) and \( U \) are the values of the axial velocity component at time levels \( \tau_0 \) and \( \tau_0 + \Delta \tau \), respectively, then

\[
U \frac{\partial \Theta}{\partial \zeta} = U_0 \frac{\partial \Theta}{\partial \zeta} + \left[ \frac{\partial \Theta}{\partial \tau} \Delta \tau + O((\Delta \tau)^2) \right] \frac{\partial \Theta}{\partial \zeta} = U_0 \frac{\partial \Theta}{\partial \zeta} + O(\Delta \tau) \frac{\partial \Theta}{\partial \zeta} \tag{124}
\]

The second term of the right side of Eq. (124) represents the linearization error.

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3. The nonlinear terms $U \frac{\partial \Theta}{\partial \xi}$ and $U \frac{\partial \omega}{\partial \xi}$ are approximated by backward differences, Eq. (121), if the coefficient velocity $U$ is positive and by forward differences, Eq. (120), if $U$ is negative. The same procedure is followed for approximating the terms $V \frac{\partial \Theta}{\partial \eta}$ and $V \frac{\partial \omega}{\partial \eta}$ according to the sign of the velocity component $V$.

4. Central differences are used to approximate the first order terms $1/\eta \frac{\partial \Theta}{\partial \eta}$, $3/\eta \frac{\partial \omega}{\partial \eta}$, and $1/\eta \frac{\partial \psi}{\partial \eta}$. No stability problems will be encountered in this case. At the centerline where both $\eta$ and $\partial \Theta/\partial \eta$ are zero, $1/\eta \frac{\partial \Theta}{\partial \eta}$ is replaced by its limit according to L'Hopital's rule

$$\text{Limit } \frac{1}{\eta} \frac{\partial \Theta}{\partial \eta} = \frac{\partial^2 \Theta}{\partial \eta^2} \bigg|_{\eta=0}$$ (125)

5. The second order term $\frac{\partial^2 \Theta}{\partial \xi^2}$, $\frac{\partial^2 \Theta}{\partial \eta^2}$, $\frac{\partial \omega}{\partial \xi^2}$, $\frac{\partial \omega}{\partial \eta^2}$, $\frac{\partial \psi}{\partial \xi^2}$, and $\frac{\partial \psi}{\partial \eta^2}$ are represented by Eq. (123).

6. Although the first order derivative $\frac{\partial \Theta}{\partial \eta} \bigg|_{\eta=1}$ in Eqs. (83) and (84) can be represented by any of Eqs. (120), (121), or (122), the following formula, which has a higher order truncation error, is used,

$$\frac{\partial \Theta}{\partial \eta} \bigg|_{\eta=1} = \frac{11 \Theta(\xi, \eta) - 18 \Theta(\xi, \eta - \Delta \eta) + 9 \Theta(\xi, \eta - 2\Delta \eta) - 2 \Theta(\xi, \eta - 3\Delta \eta)}{6 \Delta \eta}$$ (126)

Similarly,

$$\frac{\partial \Theta}{\partial \xi} \bigg|_{\xi=\xi_1} = \frac{11 \Theta(\xi_1, \eta) - 18 \Theta(\xi_1 - \Delta \xi, \eta) + 9 \Theta(\xi_1 - 2\Delta \xi, \eta) - 2 \Theta(\xi_1 - 3\Delta \xi, \eta)}{6 \Delta \xi}$$ (127)
In determining the side wall temperature at the location of the interface by the finite-difference procedure, a special difference equation must be used in order to take proper physical account of the possible axial variation of heat flux imposed on the wall, the difference in thermal conductivity between the liquid and vapor, and the possible difference in axial grid spacing in the liquid and vapor regions.

The energy equation for the nodal point in the wall at the liquid-vapor interface is:

\[
\frac{Q_{wi}(\tau+\Delta \tau) - Q_{wi}(\tau)}{\Delta \tau} \left( \frac{\Delta \xi_f + \Delta \xi_g}{2} \right) = - \left( \frac{a(kL^g + kL^g)}{2 \rho_w c_{pw} \delta \alpha_L} \right) \\
\frac{11\theta(\xi_i, \eta_w) - 18\theta(\xi_i, \eta_w - \Delta \eta) + 9\theta(\xi_i, \eta_w - 2\Delta \eta) - 2\theta(\xi_i, \eta_w - 3\Delta \eta)}{6\Delta \eta} \\
+ \left( \frac{a^2 \alpha_w}{\beta_w} \right) \left( \frac{\Delta \xi_f + \Delta \xi_g}{2} \right) \theta(\xi_i + \Delta \xi, \eta_w) - \left( 1 + \frac{\Delta \xi_f}{\Delta \xi_g} \right) \theta(\xi_i, \eta_w) + \frac{\Delta \xi_f}{\Delta \xi_g} \theta(\xi_i - \Delta \xi, \eta_w) \\
+ \left( q_{wi} \Delta \xi_f + q_{wg} \Delta \xi_g \right) \left( \frac{\beta_L g a^g}{2 \rho_w c_{pw} \delta} \right) (128)
\]

7. As mentioned earlier, two vorticity boundary conditions each are required in the liquid and vapor regions in addition to those given by Eqs. (96) and (97) for the liquid and Eqs. (113a) and (113b) for the vapor. An explicit expression is difficult to find, but the procedure followed in the numerical computations is described here. The step-by-step explicit computations of vorticity allow progressing from one vorticity distribution to the next at all grid points except those on the boundary, using the values of the vorticity from the previous time step. The new values of the vorticity are used to determine the stream function, which in turn are used to compute the vorticity at the solid boundaries.

Using the Taylor series expansion together with the stream function boundary conditions, the following expressions are obtained for the vorticity at the solid boundaries:

**Sidewall**

\[
\omega(\xi, \eta_w) = \frac{8\psi(\xi, \eta_w - \Delta \eta) - \psi(\xi, \eta_w - 2\Delta \eta)}{2(\Delta \eta)^2} \quad (129)
\]
\[ \omega(\alpha, \eta) = \frac{a^2}{2b^2} \frac{8\psi(\alpha+\Delta\xi, \eta) - \psi(\alpha, \eta)}{\eta^2(\Delta\xi)^2} \]  

(130)

B. STABILITY CRITERIA

The stability of the finite-difference equations is an important consideration in establishing the size of the grid and the time steps, and the form of difference. The details are given in Ref. 2. As might be anticipated, the size of the grid and time steps will also be governed by the storage capacity of the machine and limitations of time and cost. The necessary requirements for stability are given by the following:

\[ \Delta\tau \left( \frac{2a^2}{b^2(\Delta\xi)^2} + \frac{2}{(\Delta\eta)^2} + \frac{|U_{i,j}|}{\Delta\xi} + \frac{|V_{i,j}|}{\Delta\eta} \right) \leq 1 \]  

(131)

\[ \Delta\tau \left( \frac{2a^2\Pr}{b^2(\Delta\xi)^2} + \frac{2\Pr}{(\Delta\eta)^2} + \frac{|U_{i,j}|}{\Delta\xi} + \frac{|V_{i,j}|}{\Delta\eta} \right) \leq 1 \]  

(132)

For the centerline, where \( V = 0 \),

\[ \Delta\tau \left( \frac{2a^2}{b^2(\Delta\xi)^2} + \frac{4}{(\Delta\eta)^2} + \frac{|U_{i,1}|}{\Delta\xi} \right) \leq 1 \]  

(133)

\[ \Delta\tau \left( \frac{2a^2\Pr}{b^2(\Delta\xi)^2} + \frac{4\Pr}{(\Delta\eta)^2} + \frac{|U_{i,1}|}{\Delta\xi} \right) \leq 1 \]  

(134)

whichever one of the above is the most restrictive must be used in calculating the maximum value of \( \Delta\tau \) permitted.

C. COMPUTATIONAL PROCEDURES

The sequence of steps in establishing the numerical calculation is as follows:

(1) A convenient grid size is selected, limited by the machine storage capacity.

(2) A suitable time increment is chosen. This may be adjusted during the process of computation as necessary to maintain numerical stability.
(3) The temperature distribution is computed using velocities, temperatures, and pressure from the previous time step.

(4) These temperatures are used to compute the vorticity at the interior nodal points at the current time step.

(5) The stream function is computed at the interior nodal points.

(6) The vorticities at the solid boundaries are calculated using computed stream functions.

(7) The velocity components are calculated.

(8) The rate of phase change at the interface is determined using the computed temperature distribution, with Eq. (41).

(9) The pressure rise is computed with an equation such as (80) using the parameters from the previous time step.

(10) The above procedures are repeated successively.

A problem inherent in the use of numerical methods is the accurate determination of temperature gradients. In the present application temperature gradients are computed in the liquid and vapor at the liquid-vapor interface to determine the rate of phase change, from Eq. (41), and in the liquid and vapor at the wall to determine the heat transfer to the bulk liquid and vapor, respectively.

The general problem of the effect of grid size on the interfacial heat and mass transfer is presented in some detail here.

The formulation with Eq. (41) was used by a number of investigators\textsuperscript{6-11} to determine the rate of interfacial phase change for the case of a suddenly pressurized one-dimensional model. However, the determination of the temperature gradients by numerical representation of the calculated temperature distribution in the case of self-pressurized containers may be difficult for two-dimensional cases because of the increased machine storage requirements combined with large temperature gradients near the interface, which cause the phase changes. These large temperature gradients exist in a very thin layer near the interface, as has been shown by experimental measurements.\textsuperscript{14,15} This requires the use of a very small grid size in order to obtain an acceptable approximation for the temperature gradients near the interface.
In the case of the temperature gradients in the fluid adjacent to the wall two aspects must be considered when either the heat flux or the wall temperatures are imposed. One is similar to the liquid-vapor interface problem in that the grid spacing should again be small enough to permit sufficiently accurate representation of the temperature distribution in the fluid, either liquid or vapor, and in general dictates that the grid spacing be as small as possible. The degree of being sufficiently small will depend upon the magnitude of the imposed heat flux or the imposed temperature, and the response of the transient convective process to these disturbances under the prevailing effective gravity level. The only true test available is to vary the grid size for given conditions and compare the coupled results.

The other aspect to be considered is a physical one involving only the liquid, but is also related to the problem of grid spacing. If the temperature of the solid wall in contact with the liquid exceeds the saturation temperature by some amount, dependent upon various parameters including liquid and solid properties and the configuration, nucleate boiling will be initiated. This particular heating surface superheat might be called the incipient boiling point. If information on the incipient boiling point is available for the prevailing conditions, an imposed wall temperature below this point then represents no additional problem beyond that of having sufficiently small grid sizes, as discussed above. For the case of a heat flux imposed on the outer surface of the container, however, the resulting wall temperature is a variable dependent upon a number of parameters such as wall thickness and heat capacity, fluid properties, acceleration level, and container geometry. Whether the wall temperature will exceed the incipient boiling point will not be known a priori, since the wall temperature is computed during the course of the computations.

The procedure by which the possibility of nucleate boiling is taken into account is based on the following physical assumptions:

(1) should nucleate boiling begin, further increases in heat flux generally result in relatively small increases in surface temperature as compared to non-boiling convection. This has been observed widely (e.g., Ref. 16).

(2) The vapor bubbles formed are transported by buoyant forces to the ullage volume quite rapidly. The extent to which this may occur is as yet uncertain, but may be anticipated to depend upon the degree of subcooling present, the pressure and the effective gravity level. These are implemented in the computational procedure by considering that should the tank wall temperature, and hence the liquid adjacent to the wall, exceed the existing saturation temperature by some arbitrary amount, this excess is eliminated by the evaporation of the appropriate amount of liquid directly into the ullage space. In effect, then, a portion of the vapor bypasses the liquid-vapor interface. The arbitrary amount referred to above, the symbol for which is given as $\Delta T_{\text{wmax}}$ might be considered as the incipient boiling point, and no longer will be arbitrary when sufficient information on its behavior is available.
The physical phenomena described above is simulated in the computer program as follows:

(1) The container wall temperature is calculated using Eq. (1).

(2) The liquid temperature is obtained using Eq. (149).

(3) The calculated wall temperature in the liquid region is examined. If it exceeds the saturation temperature by more than the prescribed temperature difference \( \Delta T_{\text{wmax}} \), it then is reduced such that it equals the saturation temperature plus the prescribed temperature difference.

(4) Part of the heat added to the liquid region appears as enthalpy in the wall and liquid and the rest is used for evaporating some of the liquid. The portion of the heat transferred to the liquid and resulting in evaporation is determined by setting an energy balance according to

\[
\int_0^X 2\pi a q_w(x,t) \, dx \, dt + \int_0^a 2\pi r q_{iL} \, dr = \int_0^a \int_0^X 2\rho c_p \frac{\partial T}{\partial t} \, dx \, dr + \int_0^X 2\pi a \delta_0 \frac{\partial T}{\partial t} \, dx + w_1 h_{fg}
\]

(135)

where \( q_{iL} \) is the rate of heat flow from the interface to the liquid and is given by,

\[
q_{iL} = k \frac{\partial T}{\partial x} \bigg|_{x=X}
\]

(136)

Equations (135) and (136) are used to determine the rate of evaporation from the interface, \( w_1 \).

If the difference between the wall temperature and the saturation temperature is less than the specified maximum, \( \Delta T_{\text{wmax}} \), then the procedure above is bypassed and computations proceed as described earlier. An implicit assumption in the use of this procedure is that the laminar flow conditions described by the momentum and energy equations are not affected. This may be reasonable if the container is relatively large compared to the "bubble boundary layer" region next to the wall. In other words, if the vapor bubbles remain in the vicinity of the wall, the major bulk laminar motion of the liquid will not be influenced by their movement to the ullage space. An accurate physical description of this behavior requires additional analytical and experimental investigation in incipient boiling and the departure and motion of vapor bubbles.
under low gravity fields with various patterns of subcooling.

D. COMPUTER PROGRAM

The simplified computer program flow chart based on the formulation presented is given in Appendix A.

Appendix B shows the computer program statement list for the case with liquid hydrogen and with the assumption of ideal gas behavior for the vapor. FORTRAN IV language is used here. In Appendix C are listed the meanings and units of the computer symbols employed. Appendix D lists the inputs (variables) required for the program, along with the units, and Appendix E presents a typical output. The compiled results for a number of computer runs covering several variables are presented and discussed in Section IV for liquid hydrogen.
IV. RESULTS

Computations were carried out with the program listing of Appendix B, using values of inputs calculated from the telemetered flight data of the Saturn IB vehicle AS-203, as reported in Ref. 17, so as to compare the calculated pressure rise with the measurements. Not only do differences exist between the actual physical system and the model which serves as the basis for the computer solution, but uncertainties exist in the accuracy of the necessary computer inputs given in Ref. 17, thus making a realistic comparison between the measured and predicted pressure rises difficult at this time. In order to indicate the possible influence of the more significant uncertain inputs, these are treated as variables in several cases.

A. GENERAL ASSUMPTIONS

The general assumptions incorporated into the particular program of Appendix B are listed below:

1. The tank is cylindrical with flat ends.

2. Acceleration or body forces act along the axis.

3. Two-dimensional conditions prevail, with variations only along the axis and radially.

4. Flow conditions are laminar within the entire container.

5. The vapor behaves as an ideal gas.

6. The liquid has constant properties except in the body force terms.

7. The tank side wall is uniform in thickness and has constant properties. The wall is lumped in the radial direction but axial conduction is taken into consideration.

8. The imposed heat flux on the outside of the tank wall is uniform, but differs in those portions in contact with the liquid and vapor.

9. The ends of the tank are adiabatic, and the heat capacity of the ends is neglected.

10. The grid size varies in the liquid and vapor region as liquid fraction changes, in order that a nodal point always exist at the liquid-vapor interface.

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11. Initial conditions of uniform temperature and zero velocity exists within the container.

12. The Bond number is sufficiently large that a reasonable approximation to a flat liquid-vapor interface exists.

B. VARIATIONS POSSIBLE IN PROGRAM

By relatively minor modifications to the program, some of the above listed general assumptions can be relaxed, providing additional flexibility. In any case, however, axial symmetry must be maintained in order that the problem be two-dimensional.

1. The imposed heat flux can be varied axially and with time.

2. Specified initial conditions of temperature and velocity can be utilized.

3. Specified heat flux to the tank ends, including variations with radius and time, and including heat capacity can be incorporated.

4. If the description of the physical process warrants, imposed temperatures of the container walls can be utilized, with variation axially and with time.

5. Axial variation of tank side wall thickness and variation of specific heat with temperature can be accounted for.

6. Radial variations in the tank wall temperature can be taken into consideration. This may be particularly desirable if the wall is of composite construction.

Although the influence of variations in liquid properties with temperature and pressure can be incorporated with minor changes, the use of real gas properties in the vapor space will require major modifications to the program. This would be necessary should the pressure variation be large or should the pressure approach the critical state.

Major modifications also are necessary to handle the spacewise variation of grid size in either the liquid or vapor domains.

C. INPUT PARAMETERS

Appendix D lists the input requirements for the program as presented in Appendix B. For purposes of presentation and discussion the inputs are subdivided by categories of geometry, heat flux, acceleration level (for body
forces), fluid properties, and miscellaneous.

1. Geometry

The S-IVB LH₂ tank is cylindrical with hemispherical ends outward at the upper end and inward at the lower end. The computer model here consists of a cylindrical tank with flat ends. For purposes of simulation it was decided to maintain the diameter, total volume, and ullage volume approximately the same between the physical system and the model. This makes the wetted area of the model considerably less than in the physical system, and cognizance of this should be taken in interpreting the results. The significant geometrical comparisons are shown in Table I.

<table>
<thead>
<tr>
<th>TABLE I</th>
</tr>
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<tbody>
<tr>
<td>COMPARISON OF GEOMETRY FOR LH₂ TANKS</td>
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<table>
<thead>
<tr>
<th></th>
<th>S-IVB</th>
<th>Computer Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
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<td>22.0 ft</td>
</tr>
<tr>
<td>Total volume</td>
<td>10,500 ft³</td>
<td>11,050 ft³</td>
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<tr>
<td>Ullage fraction</td>
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<tr>
<td>Gas H₂ mass (initial)</td>
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<td>≈ 500 lbm</td>
</tr>
<tr>
<td>Liquid volume</td>
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<td>3,570 ft³</td>
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<td>Height of tank</td>
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<td>29.2 ft</td>
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<tr>
<td>Wetted side wall area</td>
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<td>651 ft²</td>
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<tr>
<td>Wetted bottom area</td>
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<td>379 ft²</td>
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<td>Total wetted area</td>
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<td>1,030 ft²</td>
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<td>Unwetted side wall area</td>
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<td>379 ft²</td>
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<td>Total unwetted area</td>
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<td>1,744 ft²</td>
</tr>
<tr>
<td>Total skin area</td>
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<td>2,774 ft²</td>
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</tbody>
</table>

The tank side wall of the S-IVB consists (Ref. 22) of a lamination of polyurethane foam (with glass fibers and glass cloth on inside) and 0.134 in. thick aluminum on the outside, with reinforcing ribs 0.10 in. thick by 0.650 in. high on a 9.5 in.² waffle pattern. Since the model as presently constituted considers only a radially lumped (thermally) wall, the composite properties of the physical system cannot properly be taken into account, and requires a modification of the program. For purposes of this study, a radially lumped wall was assumed, using in one case the properties of the insulating layer of polyurethane foam alone, and in the other case the uninsulated aluminum skin alone, giving due regard for the axial conductance path contribution of the
reinforcing ribs. The effect of each of these models is to make the wall temperature more nonuniform in the axial direction than would be the case with a composite wall model, and results in a different distribution of energy input rate between the liquid and vapor. The relevant properties used are listed in Table II. As a further test of the influence of wall material, one computation was conducted for stainless steel with the same thickness as the aluminum.

**TABLE II**

**TANK WALL PROPERTIES**

<table>
<thead>
<tr>
<th>Material</th>
<th>Polyurethane</th>
<th>Aluminum</th>
<th>Stainless Steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness, ( \delta )</td>
<td>0.71 in.</td>
<td>0.140 in.</td>
<td>0.140 in.</td>
</tr>
<tr>
<td>Density, ( \rho_w )</td>
<td>10 lbm/ft(^3)</td>
<td>169 lbm/ft(^3)</td>
<td>488 lbm/ft(^3)</td>
</tr>
<tr>
<td>Specific heat, ( C_{pw} )</td>
<td>0.33 Btu/lbm-°R</td>
<td>0.20 Btu/lbm-°R</td>
<td>0.11 Btu/lbm-°R</td>
</tr>
<tr>
<td>Thermal conductivity, ( k_w )</td>
<td>0.025 Btu/hr-ft-°R</td>
<td>219.5 Btu/hr-ft-°R</td>
<td>8.0 Btu/hr-ft-°R</td>
</tr>
<tr>
<td>Thermal diffusivity, ( \alpha_w )</td>
<td>0.1985x10(^{-5}) ft(^2)/sec</td>
<td>1.67x10(^{-2}) ft(^2)/sec</td>
<td>4.16x10(^{-5}) ft(^2)/sec</td>
</tr>
<tr>
<td>Total heat capacity, ( \rho C_p \delta )</td>
<td>0.207 Btu/°F-ft(^2)</td>
<td>0.394 Btu/°F-ft(^2)</td>
<td>0.626 Btu/°F-ft(^2)</td>
</tr>
</tbody>
</table>

The properties listed above apply for near room temperature, which is valid if a polyurethane layer of insulation lies between the cryogenic liquid and the outer metal skin.

A number of computer runs were also conducted with the aluminum wall case, but using a heat capacity equal to 1/10 of that tabulated above. This in effect makes the heat transfer rate to the tank contents more nearly equal to that on the exterior surface of the tank, but retains the axial conduction effects of the tank side walls. The desirability of minimizing the tank wall heat capacity arises because the heating rate is given to the tank contents, excluding the tank walls.

2. Heat Flux

In Section VII of Ref. 17, the mean rates of energy input to the liquid and gaseous contents of the fuel tank are computed from the physical measurements by four different procedures. These heating rates are summarized in Table III and the computation procedures discussed are below.

The values by procedure A are based on an energy balance using the temperature measurements in the liquid and vapor during the closed tank experiment. In this procedure it was assumed that all of the energy required for evaporation during this period comes from the liquid alone.
TABLE III

S-IVB FUEL TANK HEATING RATES

<table>
<thead>
<tr>
<th>Procedure</th>
<th>Liquid, Btu/sec</th>
<th>Ullage, Btu/sec</th>
<th>Total, Btu/sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>29.9</td>
<td>5.9</td>
<td>35.8</td>
</tr>
<tr>
<td>B</td>
<td>23.5</td>
<td>11.5</td>
<td>35.0</td>
</tr>
<tr>
<td>C</td>
<td>21.0</td>
<td>6.6</td>
<td>27.6</td>
</tr>
<tr>
<td>D</td>
<td>13.5</td>
<td>17.0</td>
<td>30.5</td>
</tr>
</tbody>
</table>

Procedure B is based on the continuous vent data, using the mean values from Figs. VII-2 and VII-3 in Ref. 17. It might be anticipated that some difference in the pressure rise would arise between using a mean value and a periodic heating rate as in Fig. VII-2 owing to the nonlinearity of the system.

Procedure C is based on a combination of the continuous vent data and the fluid temperatures measured during a transient period between the first and second blowdown.

The heating rates with procedure D are computed by using the temperature measurements across the tank walls to calculate the wall heat flux, using previously determined values of the thermal conductivity of the wall material.

It is noted that these different procedures give results ranging from approximately equal energy input rates to the liquid and vapor, to liquid rate five times that to the vapor. If the primary source of energy is solar radiation, the heat flux should be distributed almost uniformly on the tank exterior, in the axial direction. The higher heating rate to the liquid might then be considered as due to axial conduction from the vapor region to the liquid region, as a result of the lower heat transfer rate between the inner tank wall and the vapor compared to that between the tank wall and the liquid.

In the computer model the total energy input rate of 35.8 Btu/sec, corresponding to procedure A in Table III was used. In one case this is distributed between the liquid and vapor as in procedure A, and in the other case it is distributed such that the heat flux is uniform over the entire exterior of the container wall. Using the side wall areas of Table I, these result in the distributions as shown in Table IV. The heat fluxes listed are exterior to the surface, and owing to the heat capacity of the tank walls it can be expected that the energy input rate to the contents of the tank will be much lower than that of the actual case which this model simulates, with a lower pressure rise.
TABLE IV
COMPUTER MODEL HEAT FLUX DISTRIBUTION

<table>
<thead>
<tr>
<th>Case</th>
<th>Heating Rate, Btu/sec</th>
<th>Heat Flux, Btu/hr-ft²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Liquid</td>
<td>Ullage</td>
</tr>
<tr>
<td>A</td>
<td>29.9</td>
<td>5.9</td>
</tr>
<tr>
<td>E</td>
<td>11.6</td>
<td>24.3</td>
</tr>
</tbody>
</table>

3. Acceleration

From Fig. IV-13 of Ref. 17 it is noted that during the closed tank experiment, the vehicle acceleration decreased approximately exponentially from \( a/g_o = 3.7 \times 10^{-4} \) at the beginning to \( a/g_o = 0.8 \times 10^{-4} \) at the end. For determining the influence of other parameters an approximate average value of \( a/g_o = 1.7 \times 10^{-4} \) was used. However, to provide an indication of the significance of the body forces, computations were made with values of \( a/g_o \) above and below this level at \( 2.5 \times 10^{-4} \) and \( 1.0 \times 10^{-4} \).

4. Fluid Properties

The properties of hydrogen used in the computer model were obtained in Refs. 17-19. Those listed here are taken as constant, corresponding to the saturation temperature at the initial pressure of 12.4 lbf/in.².

Liquid Hydrogen:

- Thermal conductivity, \( k_l = 1.89 \times 10^{-5} \) Btu/sec-ft-°R
- Coefficient of expansion, \( \beta_l = 0.862 \times 10^{-2} \) °R⁻¹
- Kinematic viscosity, \( \nu_l = 2.065 \times 10^{-6} \) ft²/sec
- Specific heat, \( c_p l = 2.6 \) Btu/lbm-°R
- Thermal diffusivity, \( \alpha_l = 16.6 \times 10^{-7} \) ft²/sec
- Density, \( \rho_l = 4.39 \) lbf/ft³
Gaseous Hydrogen:

Thermal conductivity, \( k_v = 2.5 \times 10^{-6} \text{ Btu/sec-ft-°R} \)

Kinematic viscosity, \( \nu_v = 0.85 \times 10^{-5} \text{ ft}^2/\text{sec} \)

Specific heat, \( C_{pv} = 2.85 \text{ Btu/lbm-°R} \)

Specific heat, \( C_{vv} = 1.492 \text{ Btu/lbm-°R} \)

Thermal diffusivity, \( \alpha_v = 1.06 \times 10^{-5} \text{ ft}^2/\text{sec} \)

Gas constant, \( R = 766.4 \text{ ft-lbf/lbm-°R} \)

To evaluate the saturation temperature corresponding to the system pressure the following correlation and constants (Ref. 20) are used.

\[
\log_{10} P = A + \frac{B}{C + T} + DT \quad (137)
\]

where

\[
P = \text{vapor pressure in atm} \\
T = \text{temperature in °K} \\
A = 2.000620 \\
B = -50.09708 \\
C = 1.0044 \\
D = 1.74849 \times 10^{-2}
\]

Data on the latent heat of vaporization in the range of interest are read in tabular form.\(^{21}\)

5. Miscellaneous

Within the limitations on the total number of nodal points, dictated by the machine storage capacity and other economic aspects of the computational process, one can adjust the distribution of nodal points between the liquid and vapor region somewhat arbitrarily. For most effective use, including an optimum accuracy, the nodal points should be more concentrated in regions of greater velocity and/or temperature gradients. In the present case, with 31 axial nodes present, and on ullage fraction of 2/3, the axial grid spacing was selected which gave a spacing in the liquid region 1/4 that in the vapor region, since it was felt that the largest changes would take place in the liquid region. In one case a run was repeated with equal axial grid spacing to test the influence of this parameter alone. The resulting differences were negligibly small.
An additional parameter, which must be specified is the value of $\Delta T_{w,max}$, the arbitrary amount by which the side wall temperature cannot exceed the saturation temperature. This can be considered as the incipient boiling point.

Coeing$^4$ studied incipient pool boiling for cryogenic fluids, and reported that the initial vapor formation in liquid hydrogen and liquid nitrogen is primarily a function of the surface superheat, for a given surface—fluid combination, and is not a strong function of orientation or heat flux. The results of the measurements for $1/4$ surface-orientation combinations were plotted and it was found that the heat flux and surface superheat of each individual observation was generally located along the natural convection correlations, $Nu = 0.14 (Gr Pr)^{1/3}$, for all horizontal surface-liquid combinations. From this correlation, surface superheat is calculated as 2.3°F at 166 Btu/hr-ft$^2$ for LH$_2$.

Due to the uncertainty, $\Delta T_{w,max}$ was assigned 1°F for the present work. On a surface consisting of a glass fiber material coated with an epoxy cement, similar to the inner lining of the LH$_2$ tank on the S-IVB stage, it was reported$^4$ that the surface became active at approximately 0.5°F superheat.

D. RESULTS OF COMPUTATIONS

In Table V are listed the various computational runs made, with a listing of the different independent variables used, and the significant dependent variables, or outputs, at different values of real time. These latter are the system pressure and the total mass of liquid evaporated. Reference to these runs in the following comparisons will be made in terms of the B-H numbers listed. Most of the column headings in Table V are self-explanatory. The Max. Real Time gives the maximum time for which a run was made, but is not the computer time. The Max. Time Steps gives the maximum number of time steps used in the numerical marching procedure, and gives an indication of the computer time used.

1. Typical Plots of Streamlines and Isotherms

In Figs. 3a, b, and c are shown computer output plots of the non-dimensional isotherms in the liquid and vapor regions at three different times during the transient heating process in Run B-H 47. These dimensionless temperatures can be converted to real temperatures by means of Eq. (82). Although the computer grid contained 21 x 31 nodal points, for these graphical presentations these were increased by a factor of three in each direction by linear interpolation of the results such as shown in Appendix E.

Each digit in the body of the plot corresponds to a range of temperatures within which the temperature at that point lies. The key for the ranges is given in the table at the bottom of each figure. Except for the digits 1 and 9,
<table>
<thead>
<tr>
<th>Column 1</th>
<th>Column 2</th>
<th>Column 3</th>
<th>Column 4</th>
<th>Column 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data 1</td>
<td>Data 2</td>
<td>Data 3</td>
<td>Data 4</td>
<td>Data 5</td>
</tr>
<tr>
<td>Data 1</td>
<td>Data 2</td>
<td>Data 3</td>
<td>Data 4</td>
<td>Data 5</td>
</tr>
<tr>
<td>Data 1</td>
<td>Data 2</td>
<td>Data 3</td>
<td>Data 4</td>
<td>Data 5</td>
</tr>
<tr>
<td>Data 1</td>
<td>Data 2</td>
<td>Data 3</td>
<td>Data 4</td>
<td>Data 5</td>
</tr>
<tr>
<td>Data 1</td>
<td>Data 2</td>
<td>Data 3</td>
<td>Data 4</td>
<td>Data 5</td>
</tr>
<tr>
<td>Data 1</td>
<td>Data 2</td>
<td>Data 3</td>
<td>Data 4</td>
<td>Data 5</td>
</tr>
</tbody>
</table>

Note: The table continues with similar entries.
each digit, and the blank spaces between, represent equal ranges of temperature. To provide the maximum information it was necessary to change ranges between Figs. 3a and b. Thus, the digit 7 in Fig. 3a corresponds to the same range of temperatures as the digit 6 in Figs. 3b and c.

In the liquid regions of Figs. 3a and b one can note the motion of the heated liquid adjacent to the wall as it rises toward the interface, and then turns inward, and then downward. As a result "stratification" can exist not only in the axial direction but radially as well, owing to the very low level of axial body force used. In the vapor region one notes the formation of one and then two vortices. From Fig. 3c, it is possible to have quite large temperature differences in the vapor at a given axial level, depending on the radius.

These figures indicate that the liquid is superheated to some extent almost throughout the entire domain, and axial stratification is almost non-existent for these conditions. The pressure rise rate, which is related to the temperature of the liquid-vapor interface, is thus much lower than if axial stratification were present, since the heat capacity of the liquid is being effectively utilized. It is felt that the lack of axial stratification in the present case is a consequence of the larger initial ullage fraction used. This was demonstrated in results presented in Ref. 3, which showed a large stratification for small ullage fractions, even with a lower level of a/g.

In Figs. 4a, b, and c the dimensionless stream functions corresponding to the isotherms of Figs. 3a-c are plotted. Lines of constant stream function correspond to streamlines and shows the flow patterns, and the succession of the three figures indicate the transient development of the flow patterns. The table at the bottom of the figures show the range of stream functions corresponding to each digit, and are the same in the three figures.

The real velocity components at each location can be calculated by Eqs. (82) and (91) for the liquid region, and by Eq. (82) and the dimensionless form of Eq. (54) for the vapor region. Increasing gradients in the stream function correspond to increasing velocities in the direction perpendicular to the gradients. It is thus noted in Figs. 4a-c that the velocities are increasing with time, and that the velocities in the liquid and vapor regions adjacent to the interface are in opposite directions. Regions of uniform stream functions completely enclosed indicate the existence of vortices, and are noted in all cases in both the liquid and vapor regions, while Fig. 4c shows a double vortex in the vapor region. Dimensional stream functions (units in ft$^2$/sec) are plotted in Figs. 5a-d for the same conditions but at slightly different times as those in Figs. 4a-c.

2. Influence of Change of Grid Size

In one case the grid size in the axial direction was changed between the liquid and vapor region, indicated in B-H 47 and B-H 53 in Table V. The sig-
Fig. 5. Streamlines—Run B-H 47.
significant differences are summarized in Table VI below. The tank height is 29.2 ft, and the liquid depth is 9.4 ft. The difference in pressure rise amounts to 5% of the computed pressure rise and may be neglected in light of other present uncertainties. One difference that can be significant is in the number of computational steps required, which is related to the computer operating time. Smaller nodal spacing results in smaller time steps because of computational stability requirements, and an increase in running time of almost 50% is noted above. This may be unwarranted in terms of justifiable benefits.

TABLE VI

EFFECT OF AXIAL GRID SIZE

<table>
<thead>
<tr>
<th>B-H No.</th>
<th>Liquid</th>
<th>Vapor</th>
<th>Nodal Spacing, ft</th>
<th>System Pressure After 4600 sec, 1b/in.²</th>
<th>Maximum Number of Time Steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>47</td>
<td>20</td>
<td>10</td>
<td>0.47</td>
<td>18.38</td>
<td>1644</td>
</tr>
<tr>
<td>53</td>
<td>10</td>
<td>20</td>
<td>0.94</td>
<td>18.70</td>
<td>1183</td>
</tr>
</tbody>
</table>

3. Influence of Heat Flux Distribution

The influence of the distribution of the heat flux on the pressure rise is shown in Fig. 6 for two materials, where the total heat capacities of the wall differ by a factor of 5. For a given material, the slopes of the pressure-time curve, giving the rate of pressure rise, become asymptotically the same, regardless of the heat flux distribution. This might be expected since for a given material the total heat capacities are the same, and the total energy input rate is also the same. Also, the slopes for the wall with higher heat capacity are somewhat less than for the aluminum. This is also what one might expect, unless one examines the relative heat capacities of the walls and the fluids. The heat capacity of the liquid in the tank is 40,800 Btu/°F, and that of the vapor is 1400 Btu/°F. The ratio of the wall to the total fluid heat capacity is tabulated in Table VII for the different cases. From Table VII, the highest wall heat capacity, with polyurethane, is only 1% of that of the fluid contents, and only 0.2% with the aluminum used (C_p = 0.02). This being the case, one would hardly expect a difference in behavior to be evident, for a given heat flux. Such is not the case, as is seen in Fig. 6, where a distinct difference in slope between the two materials is seen. The explanation lies in that not all of the heat capacity of the liquid is brought into effective use, because of the time lag associated with the dynamics of the fluid motion taking place from the walls to the interior.
Fig. 6. Effect of heat flux distribution on pressure rise.
<table>
<thead>
<tr>
<th>Material</th>
<th>Polyurethane</th>
<th>Aluminum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific heat, $C_p \text{Btu/lbm}^\circ\text{F}$</td>
<td>.35</td>
<td>.2</td>
</tr>
<tr>
<td>Wall heat capacity, $\rho C_p \delta, \text{Btu/}^\circ\text{F-ft}^2$</td>
<td>.200</td>
<td>.400</td>
</tr>
<tr>
<td>Total wall heat capacity, Btu/°F</td>
<td>400</td>
<td>800</td>
</tr>
</tbody>
</table>

Ratio—total wall to fluid heat capacity

With the polyurethane, the pressure rise rate is more rapid, initially, with the higher heat flux to the liquid. Since the total energy input rate is the same in both cases, it appears that heat transfer to the liquid has the primary influence on the pressure rise. This is reasonable on physical grounds. The system pressure is directly related to the liquid-vapor interface temperature, and the pressure rise rate is greatest when the transport rate of energy from the container walls to the interface is the greatest. In light of the existing axial body forces, this is the case with greater heat transfer to the liquid region. Examination of the computed temperature distributions shows that the liquid temperatures are greater for B-H 47 than for B-H 52 throughout.

With the aluminum walls of low heat capacity, the initial pressure rise rate is more rapid with the low heat flux to the liquid, opposite to that noted above, although the difference between the two heat fluxes is not so great. Detailed examination of the computer output results shows that this apparent reversal of the effect of the heat flux distribution is a consequence of the very low heat capacity of the aluminum wall coupled with the provision in the program to limit the superheat of the wall, as designated by the "incipient boiling point," $\Delta T_{\text{max}}$. With the high heat flux to the liquid, the wall temperature rises very rapidly to this point. Any further rise above this level (which is imposed as constant above the instantaneous saturation temperature) is suppressed by evaporation of the appropriate amount of liquid from the vicinity of the wall directly into the ullage space. Because of the large ullage fraction this in itself has negligible effect on the pressure in the ullage space. Because of the continual transport of this excess energy to the ullage, by way of the latent heat, the energy available for heating the bulk liquid is less, even though the heat flux to the outside of the walls in the liquid region is greater. Hence the pressure rise is less. In a real situation, the vapor generated by boiling at the wall would pass to the ullage space by motion through the liquid, and in so doing would be transferring part of the latent heat energy to the liquid, and stir the liquid additionally. This would serve to increase the rate of pressure rise for a given heat flux. Insofar as the present model does not take into account the transfer of heat between the vapor bubbles and the bulk liquid, nor the stirring action near the wall induced by the bubbles, the model
must be considered incomplete.

4. Influence of Wall Material

For the same total energy input rate, the influence of the container wall on the pressure rise is indicated in Fig. 7. For convenience, the wall properties of specific heat and total heat capacity are included.

The stainless steel wall has the largest total wall heat capacity and the lowest pressure rise rate. One notes in Fig. 7 that for the same heat flux distribution, equal in the liquid and vapor regions, that the pressure rise rate increases directly as the total wall heat capacity decreases successively with Runs B-H 55, 57, 52, and 56. A zero wall heat capacity would correspond to the case where the imposed heat flux is applied directly to the fluid contents. In these four runs the wall thickness and material vary, hence the possible compound influence of axial conduction is present.

For the same material, wall thickness, and equal heat flux, one can observe the effect of the specific heat of the wall, \( C_p \), alone in Runs B-H 56 and 57, with the same axial conduction in both cases. A very large increase in pressure rise rate occurs, and were the heat capacity decreased further, further increases in pressure rise would take place. This was done for one case with the heat flux being higher to the liquid than to the vapor, Runs B-H 59 and 63 in Fig. 7.

An operational difficulty arises as the wall heat capacity decreases, which in effect increases the net heat flux to the fluid. Owing to the larger temperature differences in the fluid the velocity increases, requiring smaller computational time steps because of the stability limitations imposed by Eqs. (131)-(134). The result is an increase in the ratio of computer time to real time. As an example, in Fig. 7, Run B-H 63 required approximately 20 min of computer time to cover 10 min of real time, while Run B-H 59 required 20 min of computer time for 100 min of real time.

5. Influence of Body Forces

The influence of changing the axial body force on the pressure rise is shown in the lower part of Fig. 8, for one set of conditions of high liquid heat flux with the aluminum walls. The effect of the different body forces is almost negligible for this case, and arises because of the predominant role that the process of bulk boiling plays in suppressing the wall temperature rise for the particular conditions selected. The vapor generation taking place at the liquid vapor interface is negligibly small compared to that associated with bulk boiling. Changing the body forces should have no direct effect on the vapor generation of bulk boiling, but should influence the phase change taking place at the liquid-vapor interface since body forces affect the natural convec-
Fig. 7. Effect of wall material on pressure rise.
$a/g = 1.04 \times 10^{-4}$
$= 1.7 \times 10^{-4}$
$= 2.5 \times 10^{-4}$

$(q/A)_L = 165.5$ BTU/HR-FT$^2$
$(q/A)_V = 15.55$ BTU/HR-FT$^2$

<table>
<thead>
<tr>
<th>Symbol</th>
<th>B-H No.</th>
<th>$a/g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>□</td>
<td>64</td>
<td>$1.0(10)^{-4}$</td>
</tr>
<tr>
<td>△</td>
<td>59</td>
<td>$1.7(10)^{-4}$</td>
</tr>
<tr>
<td>○</td>
<td>65</td>
<td>$2.5(10)^{-4}$</td>
</tr>
</tbody>
</table>

Fig. 8. Effect of body force on pressure rise.
tive process by which heated liquid is brought to the vicinity of the inter-
face. That this is indeed the case is noted from the upper part of Fig. 8,
which shows that increasing the body force increases the rate of evaporation of
the liquid. Were the proper combination of parameters selected such that bulk
boiling did not occur, it could then be expected that a more pronounced effect
of a/g on the pressure rise would be present.

6. Influence of Ullage Fraction

Figure 9 indicates that changing the ullage fraction from a value of 2/3
to 1/3 has negligible influence on the pressure rise. The flux to the liquid
and vapor regions are made equal in order that the total energy input rate be
the same for both cases. Even though the pressure is changed negligibly, a
significant difference in the mass evaporated exists, with a larger mass evapo-
rated associated with the larger ullage fraction. The trends observed are
similar to that obtained with liquid oxygen (Ref. 3). Were the heat flux to the
liquid to be different from that to the vapor, then differences in pressure
rise rate would occur with different values of ullage fraction, since the total
energy input rates would also differ. Also, differences in behavior can be
expected if the ullage fractions are very large or very small, for the same
heat flux. The physical explanation as to why the pressures do not change for
the present case in Fig. 9 lie in counteracting effects. As the ullage frac-
tion decreases, less vapor is needed for a given pressure rise, hence less
energy is stored as latent heat. However, the increased heat capacity asso-
ciated with increased amounts of liquid compensates approximately for this.

E. DISCUSSION

The pressure and total mass evaporated from computer Run B-H 47 are re-
peated in Fig. 10 as a function of time. The program listing in Appendix B
incorporates the ability to store and/or punch intermediate results for rein-
sertion at a later time to continue the computations. In the case where the
results are punched, only distributions of the stream functions and temperature
are retained, along with the system pressure (i.e., the U and V velocity com-
ponents are discarded). To observe the influence of not using the velocities
at the start of each section, computer Run B-H 47 was repeated by starting and
stopping the computational process a number of times, as indicated at the bot-
tom of Fig. 10. The general overall trend is affected only in a minor way if
one neglects the "starting transients," and the comparison with the continuous
run is quite good.

Included in Fig. 10 is the final measured pressure in the LH₂ tank on the
AS-203 flight, as taken from Ref. 17, and is considerably higher than the re-
results predicted by the particular computer model included. A number of factors
which contribute to this difference have already been discussed, and the more
significant of these will be reviewed here.
### Table

<table>
<thead>
<tr>
<th>Symbol</th>
<th>B-H</th>
<th>Ullage</th>
<th>(q/A)</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>○</td>
<td>56</td>
<td>.667</td>
<td>64.</td>
<td>Aluminum</td>
</tr>
<tr>
<td>□</td>
<td>66</td>
<td>.333</td>
<td>64.</td>
<td>Aluminum</td>
</tr>
</tbody>
</table>

### Figure

Fig. 9. Effect of ullage fraction on pressure rise.
Fig. 10. Composite computed pressure rise.
The geometrical difference between the actual system and the model is obvious. Although the total volumes and total energy input rates were maintained the same, the wall heat flux could not be modeled simultaneously because of the geometrical differences in wetted and nonwetted heat transfer surface areas. These differences are shown in Table VIII. Cases A and E correspond to the heat fluxes for the actual and computer models listed in Table IV, each case having the same respective energy input rates to the liquid and vapor regions. For case F, the heat flux is considered uniform over the entire actual tank surface area. The heat flux for this case is almost one-half that of the smallest used for the computer model. In all cases the total energy input rate is 35.8 Btu/sec. If the heat flux for the computer model were maintained the same as that based on the actual geometry, then the total energy input rates would differ between the two cases.

<table>
<thead>
<tr>
<th>Case</th>
<th>Computer Model, Btu/hr-ft$^2$</th>
<th>Based on Actual Geometry, Btu/hr-ft$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(q/A)$_L$</td>
<td>(q/A)$_V$</td>
</tr>
<tr>
<td>A</td>
<td>165.5</td>
<td>15.6</td>
</tr>
<tr>
<td>E</td>
<td>64</td>
<td>64</td>
</tr>
<tr>
<td>F</td>
<td>37.8</td>
<td>37.8</td>
</tr>
</tbody>
</table>

The greater heat flux to the wall in the liquid region, in the computer model, results in a large rate of increase in wall temperature. The combination of conduction and convection in the liquid is not sufficient to transport the energy away from the wall, and the incipient boiling point is quickly reached. In the results presented thus far, the incipient boiling point was included as an input parameter. Since in the physical process of nucleate boiling changes in the wall temperatures are relatively small, this was simulated by suppressing the wall temperature with an appropriate release of vapor to the ullage space, computed with Eq. (135). The generation of this vapor in effect reduces the rate of temperature rise of the bulk liquid that might otherwise take place, and with a large ullage fraction the pressure rise rate would also be smaller.

For one case only, a computer run was made in which the suppression of the liquid wall temperatures did not take place (i.e., no nucleate boiling was permitted). The resulting system pressure behavior is plotted in Fig. 11 as B-H 71. Included for reference purposes, as a basis for comparison is the otherwise identical case B-H 56. Computations were carried out for only 2000 sec, which corresponds to 20 min of computer time. However, if the pressure behavior in Run B-H 71 were extrapolated linearly to 9500 sec, the predicted
WALL MATERIAL: ALUMINUM
WALL THICKNESS: .01165 FT
UNIFORM q/A = 64 BTU/FT²-HR.
Cpw = .02 BTU/LBM-°F
a/g = 1.7 x 10⁻⁴

<table>
<thead>
<tr>
<th>Symbol</th>
<th>B-H No.</th>
<th>Nucleate Boiling</th>
</tr>
</thead>
<tbody>
<tr>
<td>○</td>
<td>56</td>
<td>Yes</td>
</tr>
<tr>
<td>△</td>
<td>71</td>
<td>No</td>
</tr>
</tbody>
</table>

Fig. 11. Effect of nucleate boiling on pressure rise.
pressure would be 38.6 lbf/in.², as compared with 37.7 lbf/in.² in the actual case (Fig. 10).

Although the pressure comparison is quite good, the computed bulk liquid temperatures become unrealistically high. For Run B-H 56, with nucleate boiling, at the end of 2000 sec the saturation temperature was 38.2°C with a mean bulk temperature of 38.8°C, whereas for Run B-H 71 the saturation temperature was 39.4°C with a mean bulk temperature of 387.9°C. This is a result of the high heat flux to the liquid, case E in Table VIII, and in effect states that, in a real system patterned after this computer model, nucleate boiling would most certainly occur at the wall. In Section VII-A of Ref. 17, statements are made that nucleate boiling indeed occurred on the AS-203 flight, with the vapor apparently recondensing in the bulk liquid.

The computer model used here considers that the vapor generated by nucleate boiling at the wall passes directly to the ullage space. Vapor bubbles traveling from the vicinity of the wall to the ullage space would in fact introduce two effects, both of which must be included in the computer model in order to have a more complete description. The motion of the bubbles would tend to increase the velocity of the liquid, and also would tend to heat the liquid as it travels toward the liquid-vapor interface, if collapse takes place. The effect of each of these would be to increase further the rate of pressure rise.

In the course of the transient processes occurring in the closed container, both with and without simulated nucleate boiling, it was observed that simultaneous evaporation and condensation occurs at the liquid-vapor interface during certain early intervals. This is demonstrated in Fig. 12, which includes the same two cases presented in Fig. 11. The ordinate is the local relative mass flux, either evaporation or condensation, at the liquid-vapor interface as a function of radius and time. The logarithmic ordinate scale increases in both the evaporation and condensation directions.

With nucleate boiling, Run B-H 56, condensation near the centerline persists for a considerable period. Since the boiling adds a great deal of vapor to the ullage and the bulk liquid temperature rise is relatively slow, the compression of the ullage vapor tends to increase the condensation and reduces the tendency for boiling as the system pressure increases, which is related to the temperature at the liquid-vapor interface. The evaporation rate near the side wall of the tank quickly reaches a steady value, which indicates that the temperature gradients in the liquid and vapor in that region remain constant.

With no nucleate boiling permitted, Run B-H 71, condensation also occurs near the centerline, but for a short period only. The evaporation rate becomes quite large, due to the establishment of large temperature gradients in the liquid, which is possible when no restriction on the level of wall temperature is present.
Fig. 12. Effect of nucleate boiling on local liquid-vapor interface mass transfer.
An additional improvement in the computer model could be made by the use of nonlumped walls, as discussed earlier in Sections C.1 and C.2. In the actual case, aluminum is on the outer side with an insulating material on the inner side. This tends to make the temperature on the outer side more uniform because of axial conduction. The boundary condition of an imposed heat flux can result in somewhat artificial results if the axial conduction is not included, and is particularly important in the vicinity of the liquid-vapor interface where the axial gradients in the wall can become large.

One additional observation on the numerical procedure should be made. On beginning the computational process, the changes take place from the region where the disturbance takes place, and progress one node per computational time step. This means that one must not attribute undue significance to the results until sufficient computational steps have taken place to influence all of the nodes. This is particularly important when one is attempting to model a physical system which itself is not stagnant at the outset by a system initially at a uniformly stationary condition.
APPENDIX A

COMPUTER PROGRAM FLOW CHART
APPENDIX B

LH₂ COMPUTER PROGRAM LISTING
PROGRAM FOR CYLINDRICAL CONTAINER FOR IDEAL GAS-HYDROGEN

DIMENSION U(31,21), V(31,21), T(31,21), W(31,21), T0(31,21), 
1 W0(31,21), SF(31,21), EJ(31), F(31), D(31), D1(31), DR2(31), DR3(31), 
2 DR4(31), R(31), R2(31), R3(31), R4(31), R5(31), R6(31), 
3 A(31), A2(31), A3(31), 
4 C2(31), CI(31), C1(31), C2(31), FJ(31), D1(31), D2(31), D3(31), R6(31), 
5 A(31), A4(31), V(31), 
6 U(31,21), V(31,21), T(31,21), W(31,21), T0(31,21), D(31), 
7 U0(31,21), W0(31,21), D(31), D(31), 
8 DR7(31), DR8(31), 
9 A4(31), V(31), 
10 6HK(31), C(31), CT(31), RT(31,21), ZTT(12), HHHF(12), TT(12), HHHF(12)

INTEGER CODE, H, P, TAU1, TAU2

REAL KG, KL, M, L, M2, M3, M4, M5, M6, MT, MP, N, NP1, NEW, NV, NUL, M1, M2, MP, NP2

FORMAT(3F12.4, F12.8, I4, F12.8, E16.8, E16.8, /, 15E16.8)

NAMELIST DATA /* DATA */

2RGA, ALPHAV, CPW, ROWN, DELTA, TAU1, N1, VE, EPSLON, NC, NR, B1, B2, B3, B4, Z

3ZTT, HHHF, NDIM, TAU2

4/DATA/CONST, CONST2, CONST1, CONST3, PR, PRV

READ(5, DATA1, END=5001)

WRITE(6, DATA1)

K1=M+1

K2=N+1

K3=P+1

K4=M+2

K5=N-1

K6=M+3

K7=P-1

K8=M-1

DATA A1, A2, A3, A4, C2, D1, D2, D3, D4, D5, D6, D7, D8,
1 D9, D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
2 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
3 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
4 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
5 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
6 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
7 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
8 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
9 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
10 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
11 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
12 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
13 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
14 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
15 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
16 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
17 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
18 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
19 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
20 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
21 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
22 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
23 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
24 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
25 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
26 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
27 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
28 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
29 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
30 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
31 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
32 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
33 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
34 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
35 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
36 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
37 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
38 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
39 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,
40 D10, D11, D12, D13, E1, E2, E3, E4, E5, E6, E7, E8, E9,

T1=N+2+1

Y=ADVERB

B=A/ADVERB

PR=NEW/ALPHA

PRV=NEW/ALPHA

RNEW=NEW/NEW

RALPHA=ALPHA/ALPHA

DR=1.0/N

ER2=DR*DR

GAM=CPW/CV

DY=2.0*ER2

ER6=ER2/2.0

PRESS=P0

BB=(B3*B4-ALOG10(PRESS)+B1)

C=(B2-B3*ALOG10(PRESS)+B1*B3)

D=BB*B8 -4.*B4*C

TSAT=1-BB*SQRT(DT))/10**2.84

TINIT=TSAT

CONST = A*A/ALPHA

CONST1=Y*PRG*BEITA*(A**3)/NEW/NEW

CONST2=1.0/CONST

CONST3=1.0/CONST1

WRITE(6, DATA2)

M1=ADVERB*QSUFL*(A**5)*BEITA/G/ALPHA/ALPHA/NEW/ROW/CPW
1 /DELTA
0041 M2=KL*A/ALPHA/ROW/CPW/DELTA
0042 M3=KG*A/ALPHA/ROW/CPW/DELTA
0043 M1=AVERTB*QURFG*(A**5)*BEITA*G/ALPHA/ALPHA/NEW/ROW/CPW
0044 1 /DELTA
0045 M5=CONST1*TINIT
0046 DZR=DR/2.0
0047 AOVRB2=AOVRB*A/OAVERB
0048 DO 11 J=2,K2
0049 AI(J)=ER2*ER2*(J-1.)*(J-1.)
0050 C2(J)=1.+1./((2.*J-1.))
0051 AT(J)=1.0/(J-1.0)/DR/4.0
0052 DO 11 J=1,K3
0053 IF(CODE.NE.1) GO TO 14
0054 READ (5,2041) A PRESS,PO,X,NT,TIME,OO,DOMBT,DOMTT,({T(I,J),J=1,K2),
0055 11=1,K3}),(SF(I,J),J=1,K2),T=I,K3)
0056 DO 13 J=1,K2
0057 13 T(I,J)=T(I,J)
0058 14 N3=I
0059 N4=1
0060 P1=PRESS
0061 102 NT=NT+1
0062 TSATK=TSATK/1.8
0063 CALL ATSM(TSATK,ZIT,FHFG,12,1,IT,HHFG,NDIM)
0064 CALL ALT(TSATK,TT,HHFG,HFGJ,NDIM,0.1,IER)
0065 HFG=0.215*HFGJ
0066 RP=PRESS/PO
0067 J=(P+M)/2.*1
0068 BEITAG = 1.0/(TINIT+T(I,J)/CONST1)
0069 M4=(GM-1.)/GM
0070 CT=1.
0071 DX=X/M
0072 DX2=DX*DX
0073 DZ=(1.0-X)/(P-M)
0074 DZX=DZ/DX
0075 DZX=DZX+1.
0076 DZX2=-5*DX*DZ*DZ
0077 DZXH=2./(DZ+DX)
0078 DZ2=DZ*DZ
0079 DRZ=ER2/DZ2
0080 F3=DRZ**Y
0081 BJ=2.*F3
0082 DXR=DX2/ER2
0083 DRX=ER2/ER2
0084 F1=DRX**Y
0085 F2=DXR**Y
0086 DZ2=DZ/D2.
0087 ZOVRZ=Z/2.0
0088 BZ=2.*F1
0089 BO=2.*F1+F1
0090 EJ1(I)=0.
0091 EJ1(I)=0.
0092 DO 15 J=2,K2

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0093 A2(J) = DX2*ER2*(J-1,J)*Y/Y
0094 A3(J) = DX2*YY/Y/(J-1,J)*ER2/2.
0095 D1(J) = BJ+CZ(J)*EJ(J-1)
0096 R2(J) = IZ*(J,J) + OR*DX
0097 D2(J) = BI+CZ(J)*EJ(J-1)
0098 EJ(J) = (I-1,J)/(2.0*(J-1,J))/D2(J)
0099 R4(J) = Y*Y/(J-1,J)/(J-1,J)*ER2/2*DX2/2.
0100 R5(J) = Y*Y/(J-1,J)/(J-1,J)*ER2/2*DX2/2.
0101 R6(J) = IZ*(J,J)/D2(J)
0102 15 EJ(J) = (J-1,J)/(J-1,J)/D1(J)
0103 IF(CODE.NE.1) GO TO 107
0104 DO 12 J=2,N
0105 16 SF(M+1,J) = 0.
0106 DO 10 I=2,M
0107 DO 17 J=2,N
0108 17 WI(J) = (SF(I+1,J)-2.0*SF(I,J)+SF(I-1,J))*/Y/Y/2-2*SF(I,J)+SF(I-1,J)
0109 DO 18 I=2,M
0110 18 WI(I,N+1) = (8.0*SF(I,N)-SF(I,N-1)-7.0*SF(I,N+1))/DYY
0111 DO 19 I=1,K1
0112 DO 19 J=1,K2
0113 19 R0(I,J) = CON8*PREST**14.0/RSAS/(I(I,J)+M5)
0114 DO 20 I=K4,J
0115 20 WI(I,N+1) = (8.0*SF(I,N)-SF(I,N-1)-7.0*SF(I,N+1))/DYY/R0(I,N+1)
0116 UG(U) = (ROL/R0(I,N+1))**1.1-1.0
0117 DO 21 I=2,K2
0118 21 SF(M+1,I) = R0(M+1,I)*UG*ER2*(J-1,0)*(J-1,0)/D2.0
0119 DO 22 I=K1,J
0120 U1(I,N) = (3.0*SF(I,N)-6.0*SF(I,N-1)-SF(I,N-2))/R1(N)/R0(I,N)
0121 U1(I,1) = SF(I,2)/ER6/R0(I,1)
0122 DO 22 I=2,K1,J
0123 DO 23 I=K1,J
0124 23 U1(I,J) = (SF(I,J-2)-8.0*SF(I,J-1)+8.0*SF(I,J+1)-SF(I,J-2))/R1(J)
0125 1 R0(I,J)
0126 DO 24 J=2,N
0127 24 VM+2(J) = (3.0*SF(M+2,J)+SF(M+4,J)-6.0*SF(M+3,J)+2.0*SF(M+1,J)
0128 1 R6(J)/R0(M+2,J)
0129 24 VM(J) = (3.0*SF(M+3,J)-8.0*SF(M+2,J)+7.0*SF(M+1,J))/R6(J)/R0(M+1,J)
0130 DO 25 J=2,N
0131 25 VM(P,J) = (6.0*SF(P-1,J)-3.0*SF(P,J)-SF(P+2,J))/R6(J)/R0(P,J)
0132 DO 25 J=2,N
0133 25 VM(I,J) = (SF(I+2,J)-8.0*SF(I+1,J)+8.0*SF(I-1,J)-SF(I-2,J))/R1(J)/R0(I,J)
0134 DO 26 J=2,N
0135 26 WI(I,J) = (SF(I+1,J)-2.0*SF(I,J)+SF(I-1,J))/D2*(SF(I+1,J)-
0136 1 SF(I,J-1))/ER2/(J-1,J)/2.0*(SF(I,J-1)-2.0*SF(I,J)+SF(I,J-1))/ER2/2.
0137 DO 26 J=2,N

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1  *R5(J)/RO(P+1,J)
0138  260  W1(J)=(-SF(3,J)+8.*SF(2,J))*R4(J)
0139  CODE=2
0140  GO TO 103
0141  107  SMAX=0.
0142  DO 27 I=2,K1
0143  DO 27 J=1,N
0144  S=BO*ABS(U(I,J))/DX+ABS(V(I,J))/DR
0145  27  IF(SMAX.LT.S) SMAX=S
0146  IF(PR0.LE.1.) GO TO 28
0147  B00=4.*PR0/ER2+1.*PR0*Y/DZ2
0148  B0=2.*PR0*(Y/DZ2+1./ER2)
0149  GO TO 301
0150  28  B00=4./ER2+2.*Y/DZ2
0151  B0=2.*Y/DZ2+1./ER2
0152  301  DO 29 I=K4,P
0153  DO 29 J=2,N
0154  S=BO*ABS(U(I,J))/DX+ABS(V(I,J))/DR
0155  29  IF(SMAX.LT.S) SMAX=S
0156  DO 30 I=K4,P
0157  S=BO0*ABS(U(I,1))/DZ
0158  30  IF(SMAX.LT.S) SMAX=S
0159  IF((SMAX.DT)/0.8) DT=0.8/SMAX
0160  TIME=TIME+DT
0161  TAU=TIME * CONST
0162  X=X+UU*DT
0163  DX1=DT/DX
0164  DZ1=DT/DZ
0165  DZ3=CT*DT*Y*Y*RALPHA/DZ2
0166  DZ5=2.*DZ3
0167  DZ7 = DT * Y * Y * PR * RNW / DZ2
0168  DZ8=DT*Y*Y*ALPHA/ALPHA/DZ2
0169  DZ8=DT*Y*Y*ALPHA/ALPHA/DZ2
0170  M1=DT+.5*BT*A*G*(A**6)/(ALPHA*ALPHA*NEW*ROW*CPW*DELTA*B)
0171  M2=+ .5*DT*A/(ALPHA*ROW*CPW*DELTA)
0172  DY3=CT*DT*RALPHA/ER2
0173  DY1=2.*DY3
0174  DY7 = DT * RNW * PR / ER2
0175  DY5=4.*DY3
0176  DX3=DT*Y*Y/DX2
0177  ER3=N*N*DT
0178  ER4=N*DT
0179  ER5=4.*ER3
0180  DX5=2.*DX3
0181  ER1=2.*ER3
0182  DX7=DX3*PR
0183  DX8=DT*Y*Y*ALPHA/ALPHA/DX2
0184  ER7=ER3*PR
0185  EI=1.0-DX5-ER5
0186  E2=1.0-UX5-ER1
0187  E3=1.0-2.0*DX7-2.0*ER7
0188  DO 31 J=2,K2
0189  J=J+2
0190  EI=1.57*(J-1.0)
0191  DR1(J)=ER3*(1.0-FI)

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0235 DQWD=Q*KG*6.28*NEW*ALPHA/BEITA/G/A/A/A/ADVERB/ADVERB
0236 RQI=DQWD/ADVERB/6.28/A/A/1/DSRUF/G/1-X
0237 DQILDT = 3.14*DQIL*KL*NEW*ALPHA/BEITA/G/A/A
0238 DQI=DQI*3.14*QI*KG*NEW*ALPHA/BEITA/G/A/A
0239 DME=DT+IU*ROL*(A**3)*3.14/ADVERB
0240 DM=DM+DME
0241 DM=DMB+DME
0242 PRESS+PRESS+RAS*ADVERB/(3.14*144.*(A**3)*(1.-X))*(DQWDT-
0243 DO 47 DQILDT)*DT*A**2/(CV*ALPHA)-DM*GAM*TSAT
0244 1 A=(PRESS-P1)/PRESS
0245 6=(PRESS-P1)/P1
0246 P1=PRESS
0247 BB=(B3*B4*ALOG10(PRESS)+B1)
0248 C=(B2-B3*ALOG10(PRESS)+B1)+B3
0249 D=BB*BB-4.*B4*C
0250 TSAT=(-BB+SQRT(D))/2.*B4
0251 AS=2.*DT*ADVERBG*(A**3)/ALPHA/ALPHA
0252 DO 41 J=1,N
0253 T(I,J)=TO(I,J)*E2+TO(I-1,J)*DX5*TO(I,J-1)*ER5
0254 DO 43 I=2,M
0255 IFIU(I,J)-E0.0. GO TO 42
0256 C=(TO(I-1,J)-TO(I,J))*C1(I,J)
0257 GO TO 42
0258 42 C=(C1(I,J)-TO(I-1,J))*C1(I,J)
0259 43 T(I,J)=T(I,J)+ER5*(TU(I-1,J)+TO(I,J-1))*DX3+C
0260 DO 44 J=2,N
0261 44 T(I,J)=TO(I,J)+E2+TO(I-2,J)*DX5*TO(I,J-1)*DR1(J)
0262 1+TO(I,J-1)*DR2(J)
0263 DO 47 J=2,N
0264 DO 41 I=2,M
0265 IFIU(I,J)-E0.0. GO TO 45
0266 C=(TO(I-1,J)-TO(I,J))*C1(I,J)
0267 GO TO 303
0268 45 C=(C1(I,J)-TO(I,J-1))*C1(I,J)
0269 303 IFIV(I,J)-E0.0. GO TO 46
0270 D=(TO(I,J-1)-TO(I,J))*C3(I,J)
0271 GO TO 47
0272 47 D=(TO(I,J)-TO(I,J-1))*C3(I,J)
0273 DO 50 I=2,M
0274 DO 50 J=2,N
0275 IF=K3(I,J)*(T(I,J)+T(I,J-1)-T(I,J-1))
0276 IFIU(I,J)-E0.0. GO TO 48
0277 C=(WO(I-1,J)-WO(I,J))*C1(I,J)
0278 GO TO 305
0279 48 C=(WO(I,J)-WO(I+1,J))*C1(I,J)
0280 305 IFIV(I,J)-E0.0. GO TO 49
0281 D=(WO(I,J-1)-WO(I,J))*C3(I,J)
0282 GO TO 50
0283 49 D=(WO(I,J)-WO(I,J-1))*C3(I,J)
0284 50 W(I,J)=WO(I,J)+E3*(WO(I+1,J)+WO(I-1,J))*DX7+C+D+
0285 1*DR3(J)+WO(T,J+1)*DR4(J)*FI
0286 DO 51 J=2,K2

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0286        51       SF(M+1,J)=0.
0287        52       NEI=0
0288        52       NEI=NEI+1
0289   IF(NEI.GT.NE) GO TO 104
0290     DO 53 I=1,K1
0291     DO 53 J=1,K2
0292   C3 (I,J)=SF(I,J)
0293        54       I=I+1
0294        54       F(I)=0.
0295     DO 55 J=2,NR
0296   D(I,J)=(C3(I,J)+C3(I-1,J))*F1-W(I,J)*A1(J)
0297   F(J)=(D1(J)*D2(J))/D1(J)
0298   DO 56 J=1,K5
0299   H=N+1-J
0300   SF(I,H)=EJ(H)*SF(I,H+1)+F(H)
0301   IF(I.LT.M) GO TO 54
0302       57       RMAX=C.
0303   DO 57 I=2,M,NR
0304   RR3=ABS((SF(I,J)-C3(I,J))/C3(I,J)+1.0E-20)
0305   IF(RMAX.GT.RR3) RMAX=RR3
0306   IF(RMAX.GT.EPSILON) GO TO 52
0307     DO 58 I=1,K1
0308   IF(T(I,N+1).GT.(T(M+1,N)+DTS*CONST1)) T(I,N+1)=T(M+1,N)+DTS*CONST1
0309     DO 58 J=1,K2
0310       59       DQS = 0.
0311       60       DQL = 0.
0312       60       DMB=0.
0313   CT2=6.28*A*A*DX*ROW*CPW*DELTA/ADVERB/2.0/CONST1
0314     DO 59 J=1,M
0315   C=C(T(I,N+1)+T(I+1,N+1))
0316   DQS=DQS+C*C4(I,N+1)+CT2
0317     DO 60 J=1,K2
0318   DRI(J)=3.14*ROL*CP*(A**3)*DX*ER2*(2.0*J-1.0)/ADVERB
0319   DQI=6.28*A*A*QSURF/ADVERB+DQILD*DT*CONST
0320     DO 61 J=1,M
0321   C=(T(I,J)+T(I,J+1)+T(I+1,J)+T(I+1,J+1))/4.0/CONST1
0322     DO 61 J=1,N
0323     DQI=DQI+C+C4(I+1,J)
0324     DQL=DQL+DQI*DR1(I,J)
0325   DQI=(DQI-DQL-DQS)/HFG
0326   IF(DMB.LT.0.) DMB=0.
0327   DMB=DMB+DMB
0328   DMT=DMT+DMT
0329   UU=UU-ADVERB*DMB/3.14/(A**3)*ROL/OT
0330   DWDT=-5.81*PRESS*AUPH*UU/ADVERB
0331   UG=-UU*ROL/RD(M+1,1)+1.0
0332   E1=1-DZ5-DY5
0333   E2=1-DZ5-DY5
0334   E3=1-2*DZ7-2*DY7
0335     DO 62 I=K4,P
0336     DO 62 J=1,N
0337       63       C1(I,J)=DZ1*U(I,J)
0341  62  C3(I,J)=V(I,J)*ER4
0342        T(P+1,I)=T0(P+1,I)*E1+T0(P,1)*DZ5+T0(P+1,2)*DY5
          +M4*6*[(M5+T0(P+1,1))
0343  63  DO 63 J=Z,N
0344        T(P+1,J)=T0(P+1,J)*E2+T0(P,J)*DZ5+T0(P+1,J-1)*DZ5(J)+T0(P+1,J
0345        +1)*DR6(J)+M4*6*[(M5+T0(P+1,1))
0346           DO 65 I=K4,P
0347          IF(V(I,J).LE.0.) GO TO 64
0348        C=(T0(I-1,1)-T0(I,1))*C1(I,J)
0349        GO TO 65
0350        T(I,1)=T0(I,1)*E1+T0(I,2)*DZ5+T0(I+1,1)*DZ5 +C
          +M4*6*[(M5+T0(I,1))
0351        DO 68 I=K4,P
0352        DO 68 J=Z,N
0353          IF(V(I,J).LE.0.) GO TO 66
0354        C=(T0(I-1,J)-T0(I,J))*C1(I,J)
0355        GO TO 308
0356        T(I,J)=T0(I,J)-T0(I,J)*C1(I,J)
0357        DO 67 IF(V(I,J).LE.0.) GO TO 67
0358        D=(T0(I,J)-T0(I,J))*C3(I,J)
0359        GO TO 68
0360        T(I,J)=T0(I,J)-T0(I,J)*C3(I,J)
0361        IF(V(I,J).LE.0.) GO TO 69
0362        F1=A4(J)*(-(*2*(U(I,J)-U0(I,J)))*C1(I,J)*U(I,J)-U(I,J))
0363        +C3(I,J)*U(I,J)-U(I,J)*U(I,J)-U(I,J)*RO(I,J)-RO(I,J)
          +ADVBR2*(2,
0364        +*V(I,J)-V0(I,J))*C1(I,J)*V(I,J)-V(I,J)
          +C3(I,J)*V(I,J)
0365        DO 308 IF(V(I,J).LE.0.) GO TO 69
0366        C=(w0(I,J)-w0(I,J))*C1(I,J)
0367        GO TO 310
0368        69  C=(w0(I,J)-w0(I,J))*C1(I,J)
0369        IF(V(I,J).LE.0.) GO TO 70
0370        D=(w0(I,J)-w0(I,J))*C3(I,J)
0371        GO TO 71
0372        70  D=(w0(I,J)-w0(I,J))*C3(I,J)
0373        71  W(I,J)=w0(I,J)*E3+(w0(I,J)+w0(I,J))*DZ7 +C+) +
          W0(I,J-1)*DR7(J)*W0(I,J+1)*UR8(J)+F1
0374        DO 72 J=Z,N
0375          SF(M+1,J)=RO(M+1,J)*UG*ER2*(J-1.0)*(J-1.0)2.0
0376          SF(J)=SF(M+1,J)
0377          DO 73 I=K4,P
0378          SF(SF(I,N)+SF(M+1,N)-1)
0379        IF(N2.EQ.0)
0380          NE2=1
0381          IF(N2.GT.NE) GO TO 104
0382          DO 75 I=K1,K3
0383        DO 75 J=K2
0384        C3(I,J)=SF(I,J)
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0385 I=M+1
0386 76 I=I+1
0387 DO 77 J=2,N
0388 DI(J)=C3(I+1,J)+C3(I-1,J)*F3-(W(I,J)*RO(I,J)+A4(J)*(U(I,J)*
0389 1 (RO(I,J+1)-RO(I,J-1))/D2R-V(I,J)*(RO(I+1,J)-RO(I-1,J))/D2I)*
0389 2 A1(I,J)
0390 77 F(I,J)=(DI(J)+F(J-1)*C2(J))/D2I)
0390 DO 78 J=2,N
0391 K=N+2-J
0392 SF(I,K)=EJ1(K)*SF(I,K+1)+F(K)
0393 IF(I.LT.P) GO TO 76
0394 IF(N.EQ.1.AND.NE2.EQ.1) GO TO 74
0395 RMAXX=0.
0396 DO 79 I=K1,P,NR
0397 DO 79 J=2,N,ND
0398 RR3=ABS((SF(J,J-C3(I,J))/C3(I,J)+1.0E-20))
0399 79 IF(RMAXX.LT.RR3) RMAXX=RR3
0400 IF(RMAXX.GT.EPSLON) GO TO 74
0401 DO 80 I=2,N
0402 W(I,N+1)=(W*SF(I,N)-SF(I,N-1)-7.*SF(I,N+1))/DYY
0403 DO 81 I=K4,P
0404 W(I,N+1)=(W*SF(I,N)-SF(I,N-1)-7.*SF(I,N+1))/DYY/R0(I,N+1)
0405 DO 82 J=2,N
0406 W(P+1,J)=(-SF(P-1,J)+8.*SF(P,J)-7.*SF(P+1,J))*R5(J)/R0(P+1,J)
0407 82 W(I,J)=(-SF(3,J)+8.*SF(2,J))*R4(J)
0408 103 DO 83 I=K1,K3
0409 DO 83 J=1,K2
0410 V0(I,J)=V(I,J)
0411 83 U0(I,J)=U(I,J)
0412 DO 84 I=2,M
0413 U(I,N)=2.*3.*(SF(I,N)-6.*SF(I,N-1)+SF(I,N-2))/R1(N)
0414 U(I,2)=2.*3.*(SF(I,3)-3.*SF(I,2)+SF(I,4))/R1(2)
0415 84 U(I,2)=2.*3.*(SF(I,3)-3.*SF(I,2)+SF(I,4))/R1(2)
0416 DO 85 I=K1,P
0417 U(I,N)=2.*3.*(SF(I,N)-6.*SF(I,N-1)+SF(I,N-2))/R1(N)/R0(I,N)
0418 85 U(I,2)=2.*3.*(SF(I,3)-3.*SF(I,2)+SF(I,4))/R1(2)/R0(I,2)
0419 86 I=2,M
0420 DO 86 J=3,K5
0421 DO 86 J=3,K5
0422 U(I,J)=(SF(I,J-2)-8.*SF(I,J-1)+8.*SF(I,J+1)-SF(I,J+2))/R1(J)
0423 DO 89 I=K1,P
0424 DO 89 J=3,K5
0425 89 U(I,J)=(SF(I,J-2)+8.*SF(I,J-1)+8.*SF(I,J+1)-SF(I,J+2))/R1(J)
0426 DO 90 J=2,N
0427 V(M+2,J)=2.*3.*(SF(M+2,J)-6.*SF(M+3,J)+2.*SF(M+1,J))/R6(J)/R0(M+2,J)
0428 V(P,J)=2.*3.*(SF(P-1,J)-3.*SF(P,J)-SF(P-2,J))/R6(J)/R0(P,J)
0429 90 V(G,J)=2.*3.*(SF(M+3,J)-6.*SF(M+2,J)+7.*SF(M+1,J))/R6(J)/R0(M+1,J)
0430 DO 91 J=2,K2
0431 DO 91 J=2,N
0432 91 U(I,J)=(SF(I,J-2)-8.*SF(I,J-1)+8.*SF(I,J+1)-SF(I,J+2))/R6(J)/R0(I,2)

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0434  92  SF(I+1,J)=0.
0435  93  DO 93 J=2,N
0436  94  V(2,J)=2.0*(SF(4,J)-6.0*SF(3,J)+3.0*SF(2,J))/R2(J)
0437  95  V(M,J)=2.0*(6.0*SF(M-1,J)-3.0*SF(M,J)-SF(M-2,J))/R2(J)
0438  96  IF(INT.LT.N1) GO TO 104
0439  97  IF(NP1.GE.1.1) GO TO 104
0440  98  DO 98 I=3,K8
0441  99  V(I,J)=(SF(I+2,J)-8*SF(I+1,J)+8*SF(I-1,J)-SF(I-2,J))/R2(J)
0442  100  IF(NP1.GE.1.1) GO TO 104
0443  101  IF(INT.NE.NMAX) GO TO 95
0444  102  PUNCH 204, A, PRESS, PO, X, N, TIME, U, DMBT, DMTT, (T(I,J), J=1,K2),
0445  103  I=1,K3, (SF(I,J), J=1,K2), I=1,K3
0446  104  RRI=0
0447  105  IF(NP1.GE.1.1) GO TO 104
0448  106  DO 106 I=2,M
0449  107  RR2=ABS((T(I,J)-TO(I,J))/TO(I,J)+1.E-20)
0450  108  IF(RR1.LT.RR2) RR1=RR2
0451  109  DO 109 I=1,K3
0452  110  HL(I)=ABS(KL*OTDRW(I)/A/(T(I,N+1)-T(I,1))
0453  111  RAL=RAL+DX*(T(I,N+1)
0454  112  NUG=NUG+DX*HL(I)*A*X/AOVB/RKL
0455  113  RAL=RAL+X**3/(AOVBR**3)
0456  114  DO 114 I=1,K3
0457  115  RAG=0.
0458  116  DO 116 I=1,K1
0459  117  J=I-M
0460  118  HG(J)=ABS(KG*OTDRW(I)/A/(T(I,N+1)-T(I,1))
0461  119  RAG=RAG+DX*(T(I,N+1)
0462  120  RAG=RAG+X**3/(AOVBR**3)
0463  121  CL=NUL/(RAG**0.25)
0464  122  CG=NUG/(RAG**0.25)
0465  123  UR=0.
0466  124  UR=(UR+H(U1,J)*R1(J))/12.0
0467  125  DO 125 J=1,K3
0468  126  DO 126 I=1,K2
0469  127  RT(I,J)=CCNST3(T(I,J)
0470  128  WRITE(6,210) NT,TIME, TA(U,DT,UR, PRESS, Z, X, DX, DME, DMT, DMB, DMBT,
0471  129  IOM, DMTT, RQI, DQI, DQDT, DQMDT, HFG, TSAT, TINIT
0472  130  210 FORMAT(1H1, NT=I13, TIME=E13.6,
0473  131  TAU=F13.6, DT=F13.6/
0474  132  21X, UR=E13.6, /
0475  133  21X, PRESS=F13.6, /
0476  134  21X, X=F13.6, /
0477  135  21X, DME=F13.6, /
0478  136  21X, DMT=F13.6, /
0479  137  21X, DMB=F13.6, /
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</thead>
<tbody>
<tr>
<td>7</td>
<td>DM=' E13.6,'</td>
<td>DMTT=' E13.6//'</td>
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<td>RQIN=' E13.6,'</td>
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<td>320X</td>
<td>AXIAL VELOCITY COMPONENT---U(I,J)///</td>
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<td>0480</td>
<td>DO 601 I=1,K3</td>
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<tr>
<td>0481</td>
<td>WRITE(6,211) I,UL(I,J),J=1,K2</td>
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<td>0482</td>
<td>FORMAT(1H0,13,T(5X,E13.6,1/4X,7(5X,E13.6))</td>
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<td>0483</td>
<td>WRITE(6,212)</td>
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<td>0484</td>
<td>FORMAT(1H0,20X,&quot;RADIAL VELOCITY COMPONENT---V(I,J)///)</td>
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<tr>
<td>0485</td>
<td>DO 302 I=1,K3</td>
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<tr>
<td>0486</td>
<td>WRITE(6,211) I,(V(I,J),J=1,K2)</td>
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<td>0487</td>
<td>WRITE(6,213)</td>
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<td>0488</td>
<td>FORMAT(1H0,20X,&quot;NON DIMENSIONAL TEMPERATURE RISE---T(I,J)///)</td>
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<td>0489</td>
<td>DO 603 I=1,K3</td>
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<tr>
<td>0490</td>
<td>WRITE(6,211) I,(T(I,J),J=1,K2)</td>
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<td>0491</td>
<td>WRITE(6,221)</td>
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<td>FORMAT(1H0,20X,&quot;REAL TEMPERATURE RISE---RT(I,J)///)</td>
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<td>0493</td>
<td>DO 608 I=1,K3</td>
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<tr>
<td>0494</td>
<td>WRITE(6,211) I,(RT(I,J),J=1,K2)</td>
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<td>0495</td>
<td>WRITE(6,214)</td>
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<td>0496</td>
<td>FORMAT(1H0,20X,&quot;STREAM FUNCTION---SF(I,J)///)</td>
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<td>0497</td>
<td>DO 304 I=1,K3</td>
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<tr>
<td>0498</td>
<td>WRITE(6,211) I,(SF(I,J),J=1,K2)</td>
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<td>0499</td>
<td>WRITE(6,215)</td>
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<td>0500</td>
<td>FORMAT(1H0,20X,&quot;RADIAL VAPOR VELOCITY COMPONENT AT INTERFACE---IVG(J)///)</td>
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<td>0501</td>
<td>WRITE(6,216) (VG(J),J=1,K2)</td>
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<td>0502</td>
<td>FORMAT(4X,7(5X,E13.6,1/4X,7(5X,E13.6))</td>
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<td>0503</td>
<td>WRITE(6,217)</td>
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<td>0504</td>
<td>FORMAT(1H0,20X,&quot;INTERFACE VELOCITY DUE TO INTERFACE CHANGE---UL(I,J)///)</td>
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<td>0505</td>
<td>WRITE(6,216) (UL(I,J),J=1,K2)</td>
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<td>0506</td>
<td>WRITE(6,218)</td>
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<td>0507</td>
<td>FORMAT(1H0,20X,&quot;VAPOR STREAM FUNCTION AT INTERFACE---SFI(J)///)</td>
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<td>0508</td>
<td>WRITE(6,216) (SFI(J),J=1,K2)</td>
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<td>0509</td>
<td>WRITE(6,219)</td>
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<td>0510</td>
<td>FORMAT(1H0,20X,&quot;HEAT TRANSFER COEFFICIENT OF LIQUID---HL(J)///)</td>
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<tr>
<td>0511</td>
<td>WRITE(6,216) (HL(J),J=1,K1)</td>
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<td>0512</td>
<td>WRITE(6,220)</td>
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<td>0513</td>
<td>FORMAT(1H0,20X,&quot;HEAT TRANSFER COEFFICIENT OF VAPOR---HG(J)///)</td>
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<td></td>
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<tr>
<td>0516</td>
<td>WRITE(6,221) NUL,NUG,RR1,NE1,NE2,RAL,RAG,CL,CG,RP,DQILD,</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>0517</td>
<td>DQI,DQS,DQL</td>
<td></td>
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<tr>
<td>0518</td>
<td>N3N3+1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
FORTRAN IV G COMPILER

0519  NP2 = TAU/(N4*TAU2)
0520  IF(NP2.GE.1.) N4 = N4 + 1
0521  IF(NP2.GE.1.) WRITE(7,204) A, PRESS, PO, X, NTIME, UU, DMBT, DMTT,
0522     1((T(I,J),J=1,K2),I=1,K3),((SF(I,J),J=1,K2),I=1,K3)
0523  IF(NE1.GT.NE) GO TO 500
0524   500 CALL SYSTEM
0525   500 CALL SYSTEM
0526   END
APPENDIX C

\( \text{LH}_2 \) PROGRAM NOMENCLATURE
Symbols preceding the expression indicate the following:

* - Input data
** - Printout

Symbols following the expression indicate the following:

[0] - Integer
[1] - Dimensionless
[] - Units
() - Statement number in the source program of Appendix B

A

<table>
<thead>
<tr>
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<th>Description</th>
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<tbody>
<tr>
<td>*A</td>
<td>[ft]</td>
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<tr>
<td>Aoverall</td>
<td>[1]</td>
<td>A/B</td>
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<tr>
<td>Aoverall2</td>
<td>[1]</td>
<td>(A/B)^2</td>
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<tr>
<td>A5</td>
<td>[1]</td>
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<td>A1(J)</td>
<td>[1]</td>
<td>(243)</td>
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<tr>
<td>A2(J)</td>
<td>[1]</td>
<td>(48)</td>
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<tr>
<td>A3(J)</td>
<td>[1]</td>
<td>(95)</td>
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<td>A4(J)</td>
<td>[1]</td>
<td>(94)</td>
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<td>A5(J)</td>
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<td>(50)</td>
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<tr>
<td>*ALPHA</td>
<td>[ft^2/sec]</td>
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</tr>
<tr>
<td>*ALPHAaw</td>
<td>[ft^2/sec]</td>
<td>-</td>
</tr>
<tr>
<td>*ALPHAav</td>
<td>[ft^2/sec]</td>
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B

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<tr>
<td>B</td>
<td>[ft]</td>
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<tr>
<td>BJ1</td>
<td>[1]</td>
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<td>BO</td>
<td>[1]</td>
<td>(89,118,151)</td>
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<tr>
<td>BOO</td>
<td>[1]</td>
<td>(147,150)</td>
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<tr>
<td>*BEITA</td>
<td>[°F^-1]</td>
<td>-</td>
</tr>
<tr>
<td>BEITAG</td>
<td>[°F^-1]</td>
<td>(68)</td>
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</table>

C

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<thead>
<tr>
<th>Symbol</th>
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<tr>
<td>C1(J)</td>
<td>[1]</td>
<td>(202,340)</td>
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<tr>
<td>C2(J)</td>
<td>[1]</td>
<td>(49)</td>
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<tr>
<td>C3(J)</td>
<td>[1]</td>
<td>(203,341,384)</td>
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<tr>
<td>C4(J)</td>
<td>[1]</td>
<td>(319,526)</td>
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<tr>
<td>**CL</td>
<td>[1]</td>
<td>(470)</td>
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<tr>
<td>**CG</td>
<td>[1]</td>
<td>(471)</td>
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<table>
<thead>
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<th>Symbol</th>
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<th>Description</th>
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</thead>
<tbody>
<tr>
<td>*CP</td>
<td>[Btu/lbm^-°F]</td>
<td>-</td>
</tr>
<tr>
<td>*CPV</td>
<td>[Btu/lbm^-°F]</td>
<td>-</td>
</tr>
<tr>
<td>*CV</td>
<td>[Btu/lbm^-°F]</td>
<td>-</td>
</tr>
<tr>
<td>*CPW</td>
<td>[Btu/lbm^-°F]</td>
<td>-</td>
</tr>
<tr>
<td>CT</td>
<td>[1]</td>
<td>(70)</td>
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</table>

- Tank radius
- Thermal diffusivity of liquid
- Thermal diffusivity of wall
- Thermal diffusivity of vapor
- Temporary storage
- Total tank height
- Volumetric coefficient of expansion-liquid
- Volumetric coefficient of expansion-vapor
- Pressure temperature relation
- Coefficient of Rayleigh number correlation-liquid
- Coefficient of Rayleigh number correlation-vapor
- Specific heat-liquid
- Specific heat-vapor-constant pressure
- Specific heat-vapor-constant volume
- Specific heat-wall
- Property term for vapor (=1 for ideal gas)
CT2 * - [Btu] - (315) - Property term for wall
*CODE - [0] - (52,103,139) - = 2 for a new run
           = 1 for continuation
           of prior run via READ FORMAT INPUT
**CONST - [sec] - (39) - Conversion constant for dimensionless
time
**CONST1 - ["F"⁻¹] - (36) - Conversion constant for dimensionless
temperature
**CONST2 - [sec⁻¹] - (37) - = 1/CONST
**CONST3 - ["F"] - (38) - = 1/CONST1

D

D - [1] - Temporary storage
D1(J) - [1] - (99)
D2(J) - [1] - (97)
D1(J) - [1] - (388)
D1(J) - [1] - (191,321)
D2(J) - [1] - (192)
D3(J) - [1] - (193)
D4(J) - [1] - (194)
D5(J) - [1] - (196)
D6(J) - [1] - (197)
D7(J) - [1] - (198)
D8(J) - [1] - (199)
D2X - [1] - (74)
D2X1 - [1] - (75)
D2X2 - [1] - (76)
D2XH - [1] - (77)
DR - [1] - (24) - Radial grid space
D2R - [1] - (45) - DR/2
D2Z - [1] - (79) - (DR/DZ)²
**DX - [1] - (71) - Axial grid space-liquid
DX1 - [1] - (163) - DT/DX
DX2 - [1] - (72) - DX²
DX3 - [1] - (176)
DX5 - [1] - (180)
DX7 - [1] - (182)
DX8 - [1] - (183)
DXR - [1] - (82) - (DX/DR)²
DRX - [1] - (83) - 1/DXR
DAY - [1] - (27) - 2(DR)²
DY1 - [1] - (173)
DY3 - [1] - (172)
DY5 - [1] - (175)
DY7 - [1] - (174)
DZ - [1] - (73) - Axial grid space-vapor
D2L - [1] - (164) - DT/DZ
D2Z - [1] - (78) - DZ²
D2Z2 - [1] - (86)
D2Z3 - [1] - (165)
D2Z5 - [1] - (166)
D2Z7 - [1] - (167)
D2Z8 - [1] - (168)
D2Z88 - [1] - (169)
**DM - [lbm] - (241) - Net phase change in time DT (minus sign
           for evaporation)
**DMB - [lbm] - (314,328) - Phase change in time DT from energy
           balance on tank wall and liquid system
           (plus sign for evaporation)
**DME** - [lbm] - (239)
**DMT** - [lbm] - (240)
**DMBT** - [lbm] - (330)
**DMTT** - [lbm] - (331)
*DT* - [1] -
*DTS* - [°F] - (311)
*DTW* - [°F] - (310)
**DIDRW(I)** - [1] - (226,229)
**DIDXG(J)** - [1] - (220)
**DIDXL(J)** - [1] - (219)
**DQIL** - [1] - (231,233)
**DQILDT** - [Btu/sec] - (237)
**DQIDT** - [Btu/sec] - (238)
**DQMDT** - [Btu/sec] - (239)
**DQMDT** - [Btu/sec] - (333)
**DQWT** - [Btu/sec] - (335)
**DQI** - [Btu] - (327)
**DQL** - [Btu] - (313,325)
**DQS** - [Btu] - (312,318)
*DELTA* - [ft] -

**EJ(J)** - [1] - (90,102)
**EJ1(J)** - [1] - (91,98)
*EPSILON* - [1] - (508,400)

**E1** - [1] - (185)
**E2** - [1] - (186)
**E3** - [1] - (187)
**E4** - [1] - (190)
**ER1** - [1] - (181)
**ER2** - [1] - (25)
**ER3** - [1] - (177)
**ER4** - [1] - (178)
**ER5** - [1] - (179)
**ER6** - [1] - (28)
**ER7** - [1] - (184)

**F**

**F(J)** - [1] - (295,298,389)
**FI** - [1] - (189,364)

- Phase change at L-V interface in time DT (minus sign for evaporation)
- Cumulative total of DME (minus sign for evaporation)
- Cumulative total of DMB
- Cumulative total of DM (minus sign for evaporation)
- Time step interval
- Max. permissible liquid superheat at node adjacent to wall
- Max. permissible superheat of wall in contact with liquid
- Temperature gradient in fluid at wall
- Temperature gradient in vapor at liquid-vapor interface
- Temperature gradient in liquid at liquid-vapor interface
- Mean temperature gradient in liquid at L-V interface
- Heat transfer rate in liquid at L-V interface (plus for heat transfer to liquid)
- Heat transfer rate in vapor at L-V interface (plus for heat transfer out of vapor)
- Heat transfer rate to vapor from wall (plus for heat transfer to vapor)
- Work rate by vapor control volume
- Sum of heat transfer to wall in contact with liquid and to liquid at L-V interface in DT (plus for heat transfer in)
- Enthalpy rise of liquid in DT
- Enthalpy rise of wall in contact with liquid in DT
- Wall thickness

- Maximum fractional change in stream function between iterations before iteration is terminated
- Temporary storage
- Temporary storage
- Temporary storage
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<td>$F_1$</td>
<td>- [1] - (84)</td>
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<tr>
<td>$F_2$</td>
<td>- [1] - (85)</td>
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<tr>
<td>$F_3$</td>
<td>- [1] - (80)</td>
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<tr>
<td>$F_HF$</td>
<td>[Joule/gram mole] - (62)</td>
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<tr>
<td>$G$</td>
<td>[ft/sec$^2$] -</td>
</tr>
<tr>
<td>$GAM$</td>
<td>[1] - (26)</td>
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<tr>
<td>$H$</td>
<td>[0] - (300)</td>
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<td>$H_{HF}$</td>
<td>[Btu/lbm] - (64)</td>
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<tr>
<td>$H_{GFJ}$</td>
<td>[Joule/gram mole] - (63,64)</td>
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<td>$H_{LG(I)}$</td>
<td>[Btu/sec-ft$^2$-$^\circ$F] - (466)</td>
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<tr>
<td>$H_{LI(I)}$</td>
<td>[Btu/sec-ft$^2$-$^\circ$F] - (460)</td>
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<tr>
<td>$H_{HF}$</td>
<td>[Joule/gram mole] - (62,63)</td>
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<td>[0] -</td>
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<tr>
<td>$I_{ER}$</td>
<td>[0] - (63)</td>
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<tr>
<td>$J$</td>
<td>[0] -</td>
</tr>
<tr>
<td>$K$</td>
<td>[Btu/sec-ft-$^\circ$F] -</td>
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<tr>
<td>$K_{L}$</td>
<td>[Btu/sec-ft-$^\circ$F] -</td>
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<td>$K_8$</td>
<td>- (15)</td>
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<tr>
<td>$M$</td>
<td>[0] -</td>
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<td>[1] - (40)</td>
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<td>$M_{3}$</td>
<td>[1] - (42)</td>
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<td>$M_4$</td>
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<td>$M_{11}$</td>
<td>[1] - (170)</td>
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<tr>
<td>$M_{23}$</td>
<td>[1] - (171)</td>
</tr>
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</table>

- Input function value of HFG in the subroutine ATSM
- Acceleration corresponding to effective body force acting on container
- Ratio of specific heats-vapor
- Index
- Latent heat of vaporization corresponding to TSAT
- HFG in Joule/gram mole corresponding to TSAT obtained by subroutine ATSM
- Local heat transfer coefficient of vapor based on $\Delta T$ between wall and centerline
- Local heat transfer coefficient of liquid based on $\Delta T$ between wall and centerline
- Function value of HFG in given table-HFG vs. TSAT appearing subroutine ALI and ATSM
- Axial nodal index number
- Mid-vertical height of liquid
- Error parameter for subroutine ALI
- Radial nodal index number
- Thermal conductivity-vapor
- Thermal conductivity-liquid
- Number of vertical divisions in liquid $(M + 1 = L-V$ interface)$
- Liquid-wall property
- Liquid-wall property
- Vapor-wall property
- Vapor property
- Initial temperature
- Pressure rise ratio in DT
- Vapor-wall property
| N   | [0] | - | Number of radial divisions (N + 1 = wall) |
| N1  | [1] | (442) | Minimum number of time steps before punch is permitted |
| N3  | [0] | (57,518) | Control number for establishing time steps between printouts |
| N4  | [0] | (58,520) | Control number for establishing time steps between punchouts |
| NC  | [0] | (449) | Minimum number of time steps NT before fractional temperature changes between time steps is computed |
| NE  | [0] | (381) | Maximum number of iterations permitted in computations of stream function |
| NE1 | [0] | (287,288,289) | Counter on iterations on stream function-liquid |
| NE2 | [0] | (379,380,381) | Counter on iterations on stream function-vapor |
| N1P | [1] | (446,447) | Control for printout instruction |
| N2P | [1] | (519) | |
| NR  | [0] | (396,397) | Index steps of grid spaces for which fractional change in stream function with iterations is computed |
| NT  | [0] | (60,523) | Time step number |
| NMAX | - | (523) | Maximum number of time steps NT permitted before program is terminated |
| NEW | [ft²/sec] | - | Kinematic viscosity-liquid |
| NEWV | [ft²/sec] | - | Kinematic viscosity-vapor |
| NUG | [1] | (459,468) | Mean Nusselt number-vapor |
| NUL | [1] | (456,462) | Mean Nusselt number-liquid |
| NDIM | [0] | (62,63) | Number of points which must be selected out of given table (ZTI,FHHFG) |

| P | [0] | - | Total number of vertical divisions |
| PO | [lbf/in.²] | - | Initial system pressure |
| P1 | [lbf/in.²] | (59,245) | Pressure at previous time step |
| PRESS | [lbf/in.²] | (29,242) | Current system pressure |
| PR | [1] | (20) | Prandtl number-liquid |
| PRV | [1] | (21) | Prandtl number-vapor |

| Q | [1] | (230,234) | Mean temperature gradient in vapor at L-V interface |
| OW | [1] | (227,229) | Mean temperature gradient in vapor at wall |
| QSURG | [Btu/sec-ft²] | - | Imposed heat flux on container exterior on vapor portion |
| QSURFL | [Btu/sec-ft²] | - | Imposed heat flux on container exterior on liquid portion |

<p>| R | [1] | (450,454) | Maximum value of R2 |
| RR2 | [1] | (453,454) | Fractional change in temperature between time steps |
| RR3 | [1] | (306,307,398,399) | Fractional change in stream function with iteration |
| R1(J) | [1] | (51) | |
| R2(J) | [1] | (96) | |</p>
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
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<td>R3(J)</td>
<td>- [l]</td>
<td>- (195)</td>
</tr>
<tr>
<td>R4(J)</td>
<td>- [l]</td>
<td>- (99)</td>
</tr>
<tr>
<td>R5(J)</td>
<td>- [l]</td>
<td>- (100)</td>
</tr>
<tr>
<td>R6(J)</td>
<td>- [l]</td>
<td>- (101)</td>
</tr>
<tr>
<td>RO(I,J)</td>
<td>- [lbm/ft³]</td>
<td>- (113,217)</td>
</tr>
<tr>
<td>*ROL</td>
<td>- [lbm/ft³]</td>
<td>- (Liquid density)</td>
</tr>
<tr>
<td>*ROW</td>
<td>- [lbm/ft³]</td>
<td>- (Wall density)</td>
</tr>
<tr>
<td>**RP</td>
<td>- [l]</td>
<td>- (65)</td>
</tr>
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<td>**RMAX</td>
<td>- [l]</td>
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<td>RMAXX</td>
<td>- [l]</td>
<td>- (395,399,400)</td>
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<tr>
<td>**RAG</td>
<td>- [l]</td>
<td>- (458,467,469)</td>
</tr>
<tr>
<td>**RAL</td>
<td>- [l]</td>
<td>- (457,461,463)</td>
</tr>
<tr>
<td>*RGAS</td>
<td>- [ft-lbf/lbm-°R]</td>
<td>- (Gas constant-vapor)</td>
</tr>
<tr>
<td>**RQIN</td>
<td>- [l]</td>
<td>- (236)</td>
</tr>
<tr>
<td>RALPHA</td>
<td>- [l]</td>
<td>- (23)</td>
</tr>
<tr>
<td>RNEW</td>
<td>- [l]</td>
<td>- (22)</td>
</tr>
<tr>
<td>RT(I,J)</td>
<td>- [°F or °R]</td>
<td>- (477)</td>
</tr>
</tbody>
</table>

S
S - [l] - (144,154,155, 157,158) - Stability criteria
SMAx - [l] - (155,158) - Maximum value of S
**SF(I,J) - [l] - Stream function
**SPI(J) - [l] - (376) - Vapor stream function at L-V interface

T
**T(I,J) - [l] - Current temperature
T0(I,J) - [l] - Temperature of prior time step
**TAU - [sec] - (56) - Current time
*TAU1 - [sec] - (161) - Basic multiple of time for which printout occurs
*TAU2 - [sec] - (446) - Basic multiple of time for which punch-out occurs
**TIME - [l] - (160) - Current time
TINIT - [°R] - (34) - Initial saturation temperature corresponding to initial pressure
TSAT - [°R] - (33,249) - Current saturation temperature corresponding to current pressure
TSATK - [°K] - (61,62,63) - Saturation temperature in °K in subroutine ALI and ASTM
TT - [°K] - (62,63) - Argument value for temperature in the subroutine ALI

U
**U(I,J) - [l] - Axial component of velocity
**UU - [l] - (224,332) - Mean velocity of liquid-vapor interface
UG - [l] - (116,334) - Mean velocity of vapor at liquid-vapor interface
**UL(J) - [l] - (221) - Local L-V interface velocity due to interface phase change
UO(I,J) - [l] - (411) - Temporary storage of U(I,J)
**UR - [l] - (472,474) - Mean axial velocity at mid-point of liquid (serves as check on continuity)
\[ \begin{align*}
V & \quad \text{Radial component of velocity} \\
**V(\cdot, J) & \quad [1] \\
V(\cdot, J) & \quad ([1]) \\
**Vg(\cdot) & \quad [1] \\
W & \quad \text{Vorticity} \\
W(I, J) & \quad [1] \\
WO(I, J) & \quad ([1]) \\
X & \quad \text{Liquid fraction in container} \\
x^X & \quad [1] \\
Y & \quad A/B \\
y^Y & \quad [1] \\
Z & \quad \text{Compressibility factor, } Z = 1 \text{ for ideal gas} \\
**Z & \quad [1] \\
Z0VR2 & \quad ([1]) \\
**ZTT & \quad [^\circ \text{K}] \\
- (410) & \quad \text{Temporary storage of } V(I, J) \\
- (128, 429) & \quad \text{Radial velocity of vapor at L-V at interface} \\
- (213) & \quad \text{Vorticity of prior time step} \\
- (162) & \\
- (18) & \\
- & \\
- (87) & \quad = Z/2, = 0.5 \text{ for ideal gas} \\
- (62) & \quad \text{Input argument value of temperature in the subroutine ATSM}
\end{align*} \]
APPENDIX D

LIH₂ PROGRAM—DATA INPUTS
<table>
<thead>
<tr>
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APPENDIX E

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**NUN DIMENSIONAL TEMPERATURE RISE—TI,J**

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**KADIAL VAPOR VELOCITY COEFFICIENT AT INTERFACE---**

\( \mathbf{v}_{\mathbf{g}}(\mathbf{j}) \)
INTERFACE VELOCITY DUE TO INTERFACE CHANGE---\( \text{UL}(j) \)

\[
\begin{array}{cccccccc}
-0.315152E-01 & -0.324620E-01 & -0.344721E-01 & -0.375375E-01 & -0.418359E-01 & -0.475946E-01 & -0.552976E-01 \\
-0.640650E-01 & -0.767604E-01 & -0.917356E-01 & -1.100300E 00 & -0.131936E 00 & -0.157316E 00 & -0.185205E 00 \\
-0.212333E 00 & -0.235093E 00 & -0.246334E 00 & -0.246334E 00 & -0.248436E 00 & -0.246436E 00 & 0.0 \\
\end{array}
\]

VAPOR STREAM FUNCTION AT INTERFACE---\( \text{SF1}(j) \)

\[
\begin{array}{cccccccc}
0.0 & 0.022756E 00 & 0.089106E 00 & 0.206481E 01 & 0.356410E 01 & 0.556891E 01 & 0.801923E 01 \\
0.107131E 02 & 0.142654E 02 & 0.180433E 02 & 0.227556E 02 & 0.269535E 02 & 0.320769E 02 & 0.376458E 02 \\
0.438062E 02 & 0.501202E 02 & 0.570256E 02 & 0.643766E 02 & 0.721731E 02 & 0.804151E 02 & 0.0 \\
\end{array}
\]

HEAT TRANSFER COEFFICIENT OF LIQUID---\( \text{HL}(j) \)

\[
\begin{array}{cccccccc}
0.102777E-01 & 0.312332E-01 & 0.317558E-01 & 0.320641E-01 & 0.322575E-01 & 0.324098E-01 & 0.325391E-01 \\
0.326349E-01 & 0.327387E-01 & 0.328460E-01 & 0.329434E-01 & 0.330402E-01 & 0.331384E-01 & 0.332403E-01 \\
0.333490E-01 & 0.334666E-01 & 0.336055E-01 & 0.337700E-01 & 0.339705E-01 & 0.341523E-01 & 0.142476E-01 \\
\end{array}
\]

HEAT TRANSFER COEFFICIENT OF VAPOR---\( \text{HV}(j) \)

\[
\begin{array}{cccccccc}
0.188462E-02 & 0.647101E-05 & 0.832106E-05 & 0.777129E-05 & 0.533039E-05 & 0.537205E-05 & 0.391993E-05 \\
0.333437E-05 & 0.233831E-05 & 0.183483E-05 & 0.713526E-05 & & & & \\
\end{array}
\]

\[
\begin{array}{cccccccc}
\text{NUL} = 5158.742188 & \text{NUG} = 1.143.777588 & \text{AMAX} = 0.101767E+03 & \text{RR1} = 0.993756E 00 \\
\text{NE1} = 1 & \text{NE2} = 1 & \text{KAL} = 0.133192E 11 & \text{KAG} = 0.665264E 12 \\
\text{CL} = 15.185343 & \text{CG} = 1.155736 & \text{RP} = 1.447804 & \text{DILDT} = -0.017565 \\
\text{JUL} = 0.623943E 02 & \text{DWS} = 0.171359E 00 & \text{QDL} = 0.510014E 02 \\
\end{array}
\]

EXECUTION TERMINATED
REFERENCES


REFERENCES (Concluded)


