Snubbing and Bundling Effects on Multiple Crack Spacing of Discontinuous Random Fiber-Reinforced Brittle Matrix Composites

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Multiple cracking due to pseudo strain–hardening of fiber-reinforced brittle matrix composites contributes significantly to enhanced composite strain capacity as well as toughness. This unique phenomenon is true for both continuous aligned and discontinuous random fiber composites. Theoretical formulation of multiple crack spacing is reviewed in this paper. In particular, a snubbing effect of fiber pullout and a bundling effect of fiber strands on crack spacing are discussed. Taking account of these effects, expressed via the snubbing coefficient, f, and effective bond strength, τ_{eff}, a better spacing prediction has been obtained for a discontinuous random glass-strand-reinforced cement composite.

I. Introduction

FIBER reinforcement is an effective way of improving strength as well as toughness of brittle materials. This is generally attributed to microcrack suppression and fiber pullout. In addition, *adequate* fiber reinforcement can offer much enhanced composite strain capacity due to pseudo strain–hardening.[1,2] This means that, beyond the limit of proportionality, the composite behaves more like ductile metals than like brittle materials. The classical work by Aveston *et al.*[3] and later extensions by Marshall *et al.*[4,5] laid the foundations for micromechanical modeling of the pseudo strain–hardening phenomenon for continuous aligned fiber-reinforced brittle matrix composites (designated as CA composites). Further extension to discontinuous random fiber-reinforced composites (designated as DR composites), which possess low production cost and processing ease, is proposed by Li and Leung[6] and Li and Wu.[2] These models provide linkage between micromechanical parameters such as fiber, matrix, and interface properties, and the composite properties.

The pseudo strain–hardening process is made possible by the stress transfer capability of bridging fibers. This phenomenon is associated with the multiple cracking of the brittle matrix. When the composite is loaded up to its first cracking strength under uniaxial tensile loading, a first macroscopic crack is formed in the composite. The composite load will then be shared by the bridging fibers. These fibers then transfer the load via their interface back into the matrix. If enough load is transferred, the matrix may crack again and the process repeats until the matrix is broken by a series of subparallel cracks of approximately equal crack spacing.[2]

II. Theory

(1) Snubbing Effect

The physical interpretation of multiple cracking mentioned above is applicable to both CA and DR composites. Aveston *et al.*[3] have determined the multiple crack spacing of CA composites between x' and $2x'$, and x' is defined as

$$x' = \frac{E_m V_m}{V_f}\frac{\varepsilon_{mu} r_f}{2\tau_{eff}} = \frac{V_m}{V_f}\frac{\sigma_{mu} r_f}{2\tau_{eff}} \tag{1}$$

where E_m is the matrix modulus, V_f, V_m are the volume fraction of fiber and matrix, respectively, ε_{mu}, σ_{mu} are the matrix failure strain/stress, r_f is the fiber radius, and τ_{eff} is the effective interfacial bond strength.

In DR composites, Aveston *et al.*[7] considered the stress transfer via inclined short fibers to the cracked plane for 2-D composites, which leads to

$$x_2' = \frac{L_f - \sqrt{L_f^2 - 2\pi L_f x'}}{2} \tag{2}$$

where L_f is the discontinuous fiber length; x' is defined in Eq. (1).

As shown in Eq. (2), x_2' (subscript 2 here indicates 2-D randomness in fiber orientation) is less than $L_f/2$, which is the average embedment length in either side of the cracked plane. In other words, bridging fibers should be long enough to transfer stress back into the matrix for subsequent matrix cracking. In deriving Eq. (2), the inclined fibers are treated as bending over a frictionless pulley at their exit points from the cracked plane.[7] The pulley serves only to change the loading direction. More recently, a generalized formulation of load transfer by bridging fibers accounting for general randomness (2-D or 3-D) of fiber orientation and location has been presented.[8] In the special case of 2-D uniform random fiber distribution with the frictionless pulley assumption, this formulation leads to the crack spacing given in Eq. (2) (see Eq. (4) below with $f = 0$).

Beyond the frictionless pulley assumption, Li *et al.*[9] reported a snubbing effect which results in better agreement with experimental data of inclined fiber pullout tests. This can be envisioned as a flexible rope passing over a frictional pulley (see Fig. 1). The snubbing effect amplifies the bridging force due to fiber/matrix snubbing when a fiber is pulled out at an angle to the loading axis, expressed via the snubbing coefficient, f, where f is found to be 0.994 for nylon fiber and 0.702 for polypropylene fiber.[9] When the snubbing effect is inactive, f equals zero and coincides with the frictionless pulley assumption.

In considering the snubbing effect, the multiple crack spacing of discontinuous random fiber-reinforced composites can be derived:[10]

$$x_1' = \frac{L_f - \sqrt{L_f^2 - 2\pi L_f \alpha x'}}{2} \quad \text{for 1-D} \tag{3}$$

where

$$\alpha = \frac{2}{\pi}$$

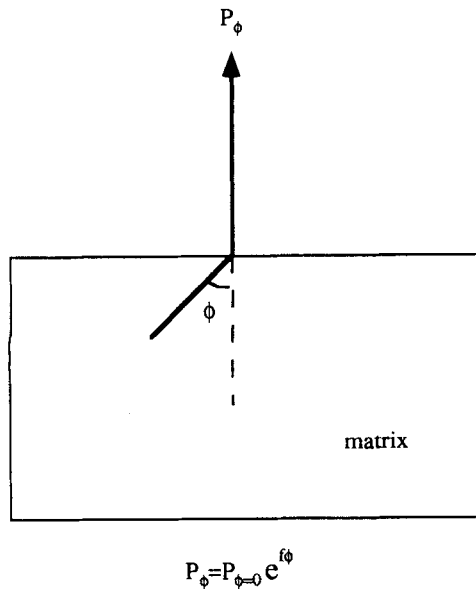**Fig. 1.** Fiber pullout at an angle from the matrix, showing snubbing effect.

$$x'_2 = \frac{L_f - \sqrt{L_f^2 - 2\pi L_f \kappa x'}}{2} \quad \text{for 2-D} \tag{4}$$

where

$$\kappa = \frac{1 + f^2}{e^{\pi f/2} - f}$$

and

$$x'_3 = \frac{L_f - \sqrt{L_f^2 - 2\pi L_f \lambda x'}}{2} \quad \text{for 3-D} \tag{5}$$

where

$$\lambda = \frac{2}{\pi} \frac{4 + f^2}{e^{\pi f/2} + 1} = \frac{4}{\pi g}$$

g is snubbing factor $= 2\dfrac{e^{\pi f/2} + 1}{4 + f^2}$

The dependency of snubbing coefficient on the crack spacing reduction parameters is plotted in Fig. 2 for 1-D, 2-D, and 3-D fiber randomness. It is most efficient for stress transfer when fibers are aligned with the loading direction. Therefore, as suggested in Fig. 2 and Eqs. (3)–(5), the crack spacing of 1-D, 2-D and 3-D fiber composites is in increasing order. However, the
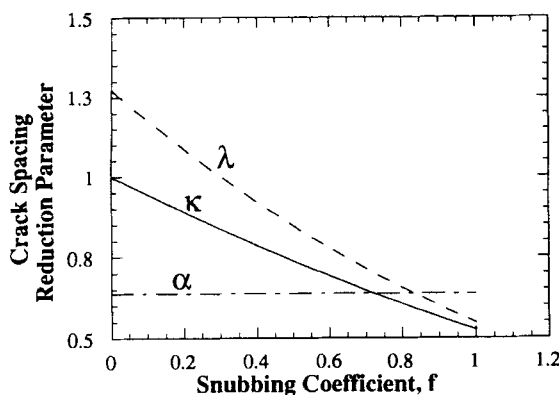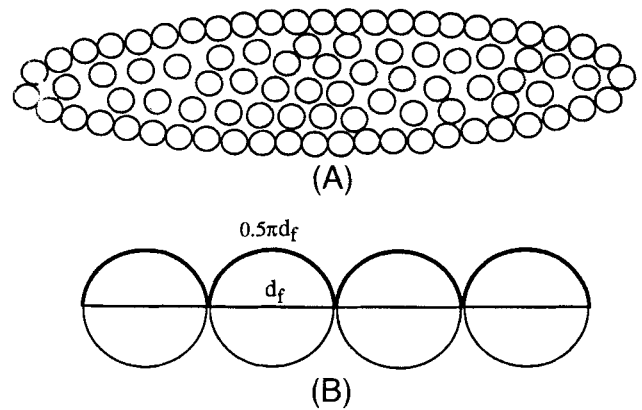

Fig. 3. (A) Schematic cross section of a glass fiber strand. (B) Surface roughness along circumference in (A).

snubbing effect is in favor of short crack spacing. In 1-D composites, the snubbing effect does not exist. Hence, for 2-D or 3-D composites with high snubbing coefficients, the snubbing effect on crack spacing can dominate over the fiber orientation effect and result in smaller crack spacing than 1-D composites.

(2) Bundling Effect

In practice, many reinforcing fibers such as SiC or glass are frequently used in discontinuous chopped lengths, normally in bundles or strands containing 100–200 individual filaments. The shape of the cross section of such fiber strands in the composites is commonly observed as an ellipse with varied ellipticity (Fig. 3(A)).[11,12] Bond strength, τ, is typically measured on a single-fiber basis. Hence, an effective τ should be employed when fiber bundles instead of discrete fibers are used in load transfer models. τ_{eff} may be determined from a geometric consideration as follows.

For a fiber strand of unit thickness, the effective bond strength, τ_{eff}, can be related to the bond strength of a single fiber, τ, as[9]

$$\tau_{eff} = \tau \frac{A_{exposed}}{A_{total}} \tag{6}$$

where $A_{exposed}$ is the exposed surface area of an elliptical fiber strand along its circumference, and A_{total} is the total fiber surface area per strand, expressed as

$$A_{total} = N(\pi d_f) \tag{7}$$

and N is the total numbers of fibers per strand, and d_f is the fiber diameter.

Since the exposed surface of the fiber bundle is approximated as semicircular cylinders superimposed on an ellipse (see Fig.

Fig. 2. Effect of snubbing coefficient on crack spacing reduction for 1-D (α), 2-D (κ), and 3-D (λ) fiber composites (see Eqs. 3–5).
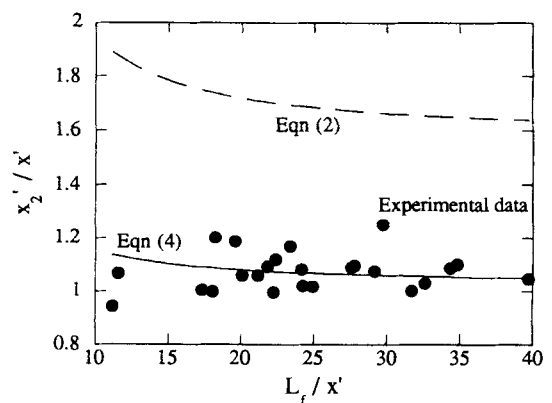

Fig. 4. Normalized crack spacing versus fiber length of glass-strand-reinforced cement (after Oakley and Proctor[11]).

3(B)), the ratio of the circumference of a semicircle to its diameter represents the bundle surface roughness. Hence

$$A_{exposed} = \left(2\pi \sqrt{\frac{a^2 + b^2}{2}}\right) \frac{0.5\pi d_f}{d_f} \tag{8}$$

where the value inside the parentheses is the circumference of an ellipse with axes a and b.

Therefore, using Eqs. (7) and (8) in Eq. (6),

$$\tau_{eff} = \tau \frac{\sqrt{2(a^2 + b^2)}}{Nd_f}(0.5\pi) \tag{9}$$

Equation (9) should be used together with Eqs. (1)–(5) to predict crack spacing whenever fiber bundles of an elliptical geometry are utilized in the composites.

As an example, per Oakley and Proctor,[11] the glass used in their reinforced cement composites was in the form of 38-mm-long chopped strands, and the strands consisted of 200 fibers, each 12.5 μm in diameter, in a flat bundle held together by a size coating. The strand cross section may be approximated as an ellipse with long axis a and short axis b (see Fig. 3(A), also Oakley and Proctor[11]), where $a = 0.325$ mm and $b = 0.055$ mm according to Oakley and Proctor from their experimental measurements of the cross-section area of the glass strands in the cement composites. When these values are used in Eq. (9), τ_{eff} is found to be 0.29τ.

III. Results

The available data on the multiple crack spacing of discontinuous random fiber composites are very limited in the literature. Oakley and Proctor[11] reported a series of tensile tests on glass fiber (2-D orientation) reinforced cement composites, including crack spacing measurements. Their results are reproduced in Fig. 4, together with the predictions of multiple crack spacing obtained from Eqs. (2) and (4) where τ_{eff} (Eq. (9)) is employed in both cases. In Eq. (9), τ ranging from 0.2 to 1.6 MPa are used. These values are experimentally measured for each individual composite.[11] In Eq. (4), since there is no experimental data of snubbing coefficient for glass fiber in cement, $f = 0.7$ is assumed. Limited data on nylon, polypropylene, and steel fibers show f varying between 0.5 and 1. The spacing and fiber length are all normalized by x'_s as defined in Eq. (1). Good agreements are found when the snubbing effect is accounted for in terms of the snubbing coefficient, f; however, the exact magnitude requires further investigation. The frictionless pulley

theory, Eq. (2), as proposed by Aveston *et al.*,[3,7] gives significant overestimation.

IV. Conclusions

Multiple crack spacing of discontinuous random fiber-strand-reinforced brittle matrix composites is dependent on the snubbing effect, which amplifies the bridging force due to fiber/matrix snubbing, and on the bundling effect, which reduces bond strength and bridging force. Good agreements between theoretical predictions and experimental data of a glass-strand-reinforced cement have been found when these two effects are considered.

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