THE UNIVERSITY OF MICHIGAN
COLLEGE OF ENGINEERING
Department of Naval Architecture and Marine Engineering

Final Report

EVALUATION OF WAVE-RESISTANCE COEFFICIENTS FOR POLYNOMIAL CENTERPLANE SINGULARITY DISTRIBUTIONS

Finn C. Michelsen
Hun Chol Kim

ORA Project 04534

under contract with:

DEPARTMENT OF THE NAVY
DAVID TAYLOR MODEL BASIN
CONTRACT NO. Nonr-1224(40)
WASHINGTON, D.C.

administered through:

OFFICE OF RESEARCH ADMINISTRATION ANN ARBOR

March 1966

This research was sponsored by the Bureau of Ships Fundamental Hydromechanics Research Program administered by the David Taylor Model Basin, Contract No. Nonr-1224(40).

ABSTRACT

Wave-resistance coefficients for "thin ships" are obtained from the Michell integral by the application of the Birkoff-Kotik transformation. Mathematical expressions derived for the case of polynomial centerplane singularity distributions are shown to be of rather simple form and definable in terms of integral functions possessing nonsingular integrands.

A computer program for the calculation of wave-resistance coefficients has been prepared and computed results for a simple mathematical hull form are given. These are similar to but differ noticeably from results obtained by means of asymptotic expansions. It is expected that the values of the wave-resistance coefficients reported here are closer to being exact because no approximations are made in the derivation of expressions from which these are calculated beyond the approximations made in the linear wave-resistance theory. Furthermore, all functions involved in the computation are well behaved.

TABLE OF CONTENTS

LIST OF FIGURES

NOMENCLATURE

$f(x,z)$	Singularity distribution function
$g(x,z)$	Equation of the hull surface
$h(h,w)$	Nondimensional singularity distribution function
u,w	Nondimentional coordinates of center plan of hull
x,y,z	Coordinate system fixed in ship
$\overline{x},\overline{y}$	Function variables
$A^{I}_{\alpha\beta}$, $A^{II}_{\alpha\beta}$	Coefficients of hull function polynomial
B	Half-breadth of ship
$C(x,y)$	Michell function
C_w	Wave-resistance coefficient
$[\Delta^{\alpha\beta}C_w]_I$, $[\Delta^{\alpha\beta}C_w]_{II}$	Wave-resistance coefficient related to the α-β term of the hull function
D	Depth of singularity distribution
F	Inverse square of Froude No ($= \frac{gL}{V^2}$)
$H(\xi,\zeta)$	Hull function
L	Length of ship
M,N	Maximum power of the variables of the hull function polynomial
$M^{I}_{\alpha\beta}(\overline{x},\overline{y})$, $M^{II}_{\alpha\beta}(\overline{x},\overline{y})$	Special functions as defined in text

NOMENCLATURE (Concluded)

$P_{2n-1}(\overline{x})$, $P_{2n}(\overline{x})$	Havelock P-function
$\overline{P}_{2n-1}(\overline{x},\overline{y})$, $\overline{P}_{2n}(\overline{x},\overline{y})$	Generalized Havelock P-function
R_w	Wave-resistance, lb
V	Speed of ship, fps
α,β	Non-negative integer powers of the hull function polynomial
$(-\alpha)_k$	Factorial function $(=(-\alpha)(-\alpha+1)\ldots(-\alpha+k-1))$
$\delta(\overline{x})$	Dirac delta function
ξ,ζ	Variables of Birkhoff-Kotik transformation
$\Psi_{\alpha\beta}(\overline{x},\overline{y})$	Special function as defined in text.

INTRODUCTION

The analytical representation of the wave-resistance of a ship published by J. H. Michell[1] in 1898 forms in spite of many recent developments the basis for a major portion of the numerical and theoretical studies being conducted today in the field of wave-resistance theory. The work described in this report is in this respect no exception. What makes the "thin ship" linear theory of Michell particularly attractive is the simplicity of the surface on which we consider the singularities representing the ship's hull to be distributed, this surface being in the case of the Michell theory the centerplane of the ship. The authors believe that the "thin ship" theory provides the greatest flexibility for many studies of fundamental character, such as the problem of minimum wave-resistance, and therefore, that developments related to this theory, will be centered interest if they facilitate computational work, increase accuracy and, perhaps more significantly, provide us with additional analytic properties of Michell's integral.

Within limitations of the linear theory the evaluation of Michell's integral may be expected to give pertinent information.about the wave-resistance of "thin ships" of general shape. It is a quintuple integral, however, and its numerical evaluation is a formidable task. Furthermore, the integral is improper, requiring by most methods of evaluation the application of numerical integration techniques. This will introduce numerical approximations which may prove to be of rather serious magnitude. Several methods of evaluation of Michell's integral have been proposed in the past. One of the most noteworthy is due to Weinblum[2] who developed a set of functions suitable for the case of affine hull forms. Because of this geometric feature his method was severely limited in scope. Other researchers made use of assymptotic expressions for the Michell integral. Inui[3] in particular has followed such a procedure.

A fresh approach to the problem of formulating the wave-resistance of thin ships was taken by Birkhoff, Korvin-Kroukovsky, and Kotik[4] in their joint papers presented before the Society of Naval Architects and Marine Engineers in 1954. Based directly on the Michell integral two transformations were proposed, which would lead to a suitable separation of the functional parameters. The first of these transformations revealed a separation of the functional integrand into two functions, one depending only upon hull form parameters and the other upon ship speed and draft. As a consequence it offered the prospect of providing a better mathematical definition of the relationships between hull form and resistance. Furthermore, it appeared that the Birkhoff-Kotik transformation would lead to simplifications in the numerical work. Especially the idea of being able to tabulate wave-resistance coefficients as functions of hull parameters was attractive at a time when the full impact of the high-speed computers had yet to be felt.

The wave-resistance integral resulting from the Birkhoff-Kotik transformation was left by its proponents in a form which required a double numerical integration over a finite rectangular domain. That operation was complicated by the existance of a singularity located on the boundary of this domain. In an attempt to circumvent the numerical difficulties posed by the singularity, Michelsen[5] assumed the general singularity distribution to be given by a double finite series. Combining this procedure with certain mathematical developments he was able to formally integrate the wave-resistance integral and write the wave-resistance coefficient for each term of the singularity distribution function in terms of an infinite series. These results led to a contract in 1961 with the Bureau of Ships administered by the D.T.M.B. for a research project with the objectives of making a more detailed study of functional properties, and to prepare a computer program needed for the calculation of wave-resistance coefficients which could subsequently be tabulated. In the following are the results of that project.

Because a relatively long time has elapsed following commencement of the study, it may be appropriate to review its history, the reason being that this in itself offers a lesson to be learned. The equations requiring programming were far from simple and did indeed demand the services of an expert programmer. A number of functions had to be calculated from subroutines not available in the computer library. Some of these functions, such as the Confluent Hypergeometric Functions of the second kind, presented very interesting studies in themselves. After about a year of efforts expanded on the writing of suitable subroutines the total program was ready for trial.

Results indicated complete failure. Wave-resistance was for a representative case given as approximately 10^3 times the expected value, and for a different set of conditions the computer dismayingly recorded it as being negative. A careful analysis of each part of the terms entering into the expression for the wave-resistance calculations revealed that the formulation was absolutely impractical for numerical work. No matter what speed was considered one term or another would become very large, and terms of the alternating series would first increase in magnitude making convergence extremely slow. The main problem was simply computer overflow, however. Some terms reached magnitudes of more than 10^{38}.

Several changes took place at this time. Firstly the programmer left the project for the reason of more profitable employment. Secondly, and on the other side of the ledger, a faster and larger computer was being installed. The project was not of very high priority, however, working from the very beginning on a curtailed budget. Application for additional funding was not approved and research results obtained at this point could hardly warrant that any different action be taken. The nature of the project was such that partial success could not be achieved. Either the objectives were fully realized or nothing at all would be gained from our efforts.

Optimism again prevailed when it was found that the mathematical formulation could be modified to avoid some of the difficulties referred to above. Details of these modifications were described by Michelsen[6] in a paper read at the International Seminar on Theoretical Wave-Resistance in 1963. The second programmer did now leave the project, however, and with all funds expended it became a matter of the authors completing the research in their spare time.

During the ensuing work it became apparent that the formulas given by Michelsen in 1963 were not fully satisfactory. The expression for the wave-resistance coefficients related to the individual terms of the singularity distribution function had an assymptotic series which appeared to lead to some erratic behavior of the total wave-resistance coefficient of a simple singularity distribution.

To the researcher it is always amazing how painful it is to find the most logical path to the solution of his problem. The mathematical derivation of the next section is a case in point. As far as the authors can judge it avoids all the pitfalls of previous work and leads to a simple concise formula for the wave-resistance. Accuracy can easily be checked because the functions involved possess several recurrence relations. More important may be the fact that computer time needed is relatively short so that it should prove useful in the determination of optimum hull forms.

DERIVATION OF EXPRESSIONS FOR THE WAVE-RESISTANCE COEFFICIENTS

The Michell integral for the wave-resistance coefficient for a "thin ship" can, upon the application of the Birkhoff-Kotik transformation,[6] be written as

$$C_w = \frac{16F^2}{\pi} \left(\frac{D}{L}\right)^2 \int_0^1 d\xi \int_0^2 d\zeta \; H(\xi,\zeta)\; C(F\xi,\, F\frac{D}{L}\zeta) \qquad (1)$$

This coefficient has here been defined by

$$C_w = \frac{R_w}{\frac{1}{2}\rho V^2 B^2}$$

The hull function $H(\xi,\zeta)$ is continuous. It is, however, defined by two separate expressions referred to two subregions as follows:

$$\text{Region I:} \quad [H(\xi,\zeta)]_I = \int_{-\frac{1}{2}}^{\frac{1}{2}-\xi} du \int_0^{\zeta} dw \; h(u,w)h(\xi+u,\zeta-w)$$

$$0 \leq \xi \leq 1 \quad ; \quad 0 \leq \zeta \leq 1 \qquad (2a)$$

$$\text{Region II:} \quad [H(\xi,\zeta)]_{II} = \int_{-\frac{1}{2}}^{\frac{1}{2}-\xi} du \int_{\zeta-1}^{1} dw \; h(u,w)h(\xi+u,\zeta-w)$$

$$0 \leq \xi \leq 1 \quad ; \quad 0 < \zeta \leq 2 \qquad (2b)$$

It should be noted that the hull function in this form does not depend upon the uniform depth D of the singularity distribution. For a nondimentional singularity distribution given by

$$h(u,w) = \sum_{m=0}^{M} \sum_{n=0}^{N} C_{mn}\, u^m\, w^n \qquad (3)$$

integrations indicated in Equations (2a, 2b) are readily carried out and the results can be written

$$[H(\xi,\zeta)]_{I} = \sum_{\alpha=0}^{2M+1} \sum_{\beta=1}^{2N+1} A_{\alpha\beta}^{I} \xi^{\alpha} \zeta^{\beta} \tag{4a}$$

$$[H(\xi,\zeta)]_{I} = \sum_{\alpha=0}^{2M+1} \sum_{\beta=0}^{2N+1} A_{\alpha\beta}^{II} \xi^{\alpha} \zeta^{\beta} \tag{4b}$$

Derivation of the coefficients $A_{\alpha\beta}^{I}$ and $A_{\alpha\beta}^{II}$ are given in Appendix A.

To the approximation of the Michell "thin ship" theory

$$h(u,w) = \frac{L}{B} \frac{d}{dx} g(x,z) = \frac{L}{B} f(x,z)$$

at corresponding points: $u = \frac{x}{L}$; $w = \frac{z}{D}$, where $g(x,z)$ is the equation of the hull surface. The function $f(x,z)$ may be taken to be the singularity distribution function of sources and sinks located on the longitudinal center plane of the hull.

Equation (1) can be evaluated for each term of Equations (4a) and (4b). By designating the wave-resistance coefficient resulting from the terms $[\xi^{\alpha}\zeta^{\beta}]_{I,II}$ as $[\Delta^{\alpha\beta}C_w]_{I,II}$, respectively, Equation (1) thus becomes

$$C_w = \sum_{\alpha=0}^{2M+1} \sum_{\beta=1}^{2N+1} A_{\alpha\beta}^{I} M_{\alpha\beta}^{I} (\tfrac{D}{L},F) + \sum_{\alpha=0}^{2M+1} \sum_{\beta=0}^{2N+1} A_{\alpha\beta}^{II} M_{\alpha\beta}^{II} (\tfrac{D}{L},F)$$

$$= \sum_{\alpha=0}^{2M+1} \sum_{\beta=1}^{2N+1} [\Delta^{\alpha\beta}C_w]_{I} + \sum^{2M+1} [\Delta^{\alpha\beta}C_w]_{II} \tag{5}$$

The summation on β in the first term in Equation (5) above starts at $\beta = 1$ because the hull function vanishes linearly as $\zeta \to 0$. This is property of the Birkhoff-Kotik transformation.

DERIVATION OF EXPRESSIONS FOR $M^{I}_{\alpha\beta}$ and $M^{II}_{\alpha\beta}$

In order to show the details of Equation (5) consider any one of the terms of the summation. With the Michell function C(x,y) defined by

$$C(x,z) = \frac{1}{2} e^{-y} \int_{0}^{\infty} e^{-yt} \cos x\sqrt{t+1} \, \frac{\sqrt{t+1}}{t^{1/2}} \, dt \qquad (6)$$

we obtain from Equation (1)

$$M^{I} (\tfrac{D}{L},F) = \frac{16F^{2}}{\pi} (\tfrac{D}{L})^{2} \int_{0}^{1}\int_{0}^{1} \xi^{\alpha} \zeta^{\beta} \, d\xi \, d\zeta$$

$$\cdot \frac{1}{2} \int_{0}^{\infty} e^{-\frac{FD}{L}\zeta(1+t)} \cos (F\xi\sqrt{t+1}) \, \frac{\sqrt{t+1}}{t^{1/2}} \, dt \qquad (7)$$

Integration by parts with respect to ζ a number of times equal to β Equation (7) becomes:

$$M^{I}_{\alpha\beta}(\tfrac{D}{L},F) = \frac{16F^{2}}{\pi} (\tfrac{D}{L})^{2} \int_{0}^{1} \xi^{\alpha}(d\xi) \, \frac{1}{2} \int_{0}^{\infty} [\beta! \, (\tfrac{L}{FD})^{\beta+1} (1+t)^{-(\beta+1)}$$

$$- \sum_{p=0}^{\beta} (-\beta)_{p} \, (-1)^{p} \, (\tfrac{L}{FD})^{p+1} (1+t)^{-(p+1)} e^{-\frac{FD}{L}(1+t)}]$$

$$\cdot \cos (F \sqrt{t+1}) \, \frac{\sqrt{t+1}}{t^{1/2}} \, dt \qquad (8)$$

The integral

$$I_{1} = \int_{0}^{1} \xi^{\alpha} \, d\xi \int_{0}^{\infty} \cos (F\xi \sqrt{t+1}) \, \frac{dt}{t^{1/2}(1+t)^{\beta+1/2}} \qquad (9)$$

can be expressed in terms of Havelock P-functions. This is readily shown by integrating by parts with respect to ξ a number of times equal to α.

Thus

$$I_1 = 2(-1)^{\beta} F^{-(\alpha+1)} \Bigg[\Big\{ \sum_{k=0}^{\alpha} F^{\alpha-k} (-\alpha)_k P_{2\beta+k}(F) \Big\}$$

$$+ \alpha! \, P_{2\beta+\alpha}(0) \quad \delta(1 + (-1)^{\alpha}) \Bigg] \tag{10}$$

where $\delta(x)$ is the Dirac delta function. The Havelock P-functions are defined as follows

$$P_{2n-1}(x) = \frac{1}{2}(-1)^n \int_0^{\infty} \cos(x\sqrt{t+1}) \frac{dt}{t^{1/2}(1+t)^{n+1/2}} \tag{11}$$

$$P_{2n}(x) = \frac{1}{2}(-1)^n \int_0^{\infty} \sin(x\sqrt{t+1}) \frac{dt}{t^{1/2}(1+t)^{n+1}} \tag{11b}$$

The integral

$$I_2 = \int_0^1 \xi^{\alpha} d\xi \int_0^{\infty} e^{-\frac{FD}{L}(1+t)} \cos(F\sqrt{t+1}) \frac{dt}{t^{1/2}(1+t)} \tag{12}$$

can similarly be shown to be given by

$$I_2 = 2(-1)^{\beta} F^{-(\alpha+1} \Bigg[\Big\{ \sum_{k=0}^{\alpha} F^{\alpha-k} (-\alpha)_k \bar{P}_{2\beta+k}(F, \frac{FD}{L}) \Big\}$$

$$+ \alpha! \, \bar{P}_{2\beta+\alpha}(0) \, \delta(1+(-1)^{\alpha}) \Bigg] \tag{13}$$

where the $\bar{P}$-functions are given by

$$\bar{P}_{2n-1}(x,y) = \frac{1}{2}(-1)^n \int_0^{\infty} e^{-yt} \cos(x\sqrt{t+1}) \frac{dt}{t^{1/2}(1+1)^{n+1/2}}$$

$$= (-1)^n \int_0^{\frac{\pi}{2}} e^{-y \tan^2\theta} \cos^{2n-1}\theta \cos(x \sec\theta) d\theta \tag{14a}$$

$$\bar{P}_{2n}(x,y) = \frac{1}{2}(-1)^n \int_0^\infty e^{-yt} \sin(x\sqrt{t+1}) \frac{dt}{t^{1/2}(t+1)^{n+1}}$$

$$= (-1)^n \int_0^{\frac{\pi}{2}} e^{-y \tan^2 \theta} \cos^{2n}\theta \sin(x \sec \theta)\, d\theta \qquad (14b)$$

If I_2 is written as $\Psi_{\alpha\beta}(F, \frac{FD}{L})$ we note the integral I_1 will be given by $\Psi_{\alpha\beta}(F.0)$. With this notation the expression for the wave-resistance coefficient becomes very simple in form. Substitution into Equation (8) gives

$$M^{I}_{\alpha\beta}(\frac{D}{L},F) = \frac{8}{\pi}(\frac{FD}{L})^{1-\beta}\Big[\beta!\ \Psi_{\alpha\beta}(F,0) - e^{-\frac{FD}{L}} \sum_{p=0}^{\beta} (-\beta)_p (-1)^p (\frac{FD}{L})^{\beta-p}\ \Psi_{\alpha p}(F, \frac{FD}{L})\Big] \qquad (15)$$

From these results and Equation (1) it follows that for the region $0 \leq \xi \leq 1$, $1 \leq \zeta \leq 2$ we can immediately write

$$M^{II}_{\alpha\beta}(\frac{D}{L}, F) = \frac{8}{\pi}(\frac{FD}{L})^{1-\beta}\Big\{\sum_{p=0}^{\beta}(-\beta)_p(-1)^p(\frac{FD}{L})^{\beta-p} \times \Big[e^{-\frac{FD}{L}}\ \Psi_{\alpha p}(F, \frac{FD}{L}) - (2)^{\beta-p} e^{-\frac{2FD}{L}}\ \Psi_{\alpha p}(F, \frac{2FD}{L})\Big]\Big\} \qquad (16)$$

Equations (15) and (16) are the complete expressions for the wave-resistance coefficients of a "thin ship" based on the Birkhoff-Kotik transformation of the Michell integral. It should be pointed out that all functions involved are well behaved and easy to compute. Details of the computer programs are given in Appendix B.

NUMERICAL EXAMPLE

To check on calculations throughout the development of our mathematical formulations and the associated computer programs it was decided to consider a simple hull form for which the wave-resistance had been previously computed from the "thin ship" theory. An easy choice was the hull form of perabolic W.L. used by Inui and others. Results for this case are readily available in the literature.[3]

Choosing $$\frac{D}{L} = .10$$

and $$h(u,w) = -8u \qquad (17)$$

we obtain from Equations (2a) and (2b)

Region I

$$A^{I}_{01} = \frac{16}{3} \quad ; \quad A^{II}_{11} = -16 \quad ; \quad A^{I}_{31} = \frac{32}{3}$$

Region II

$$A^{I}_{01} = -\frac{16}{3} \quad ; \quad A^{I}_{11} = 16 \quad ; \quad A^{I}_{31} = -\frac{32}{3}$$

$$A^{I}_{00} = \frac{32}{2} \quad ; \quad A^{II}_{10} = -32 \quad ; \quad A^{II}_{30} = \frac{64}{3}$$

With these values for the coefficients of the hull function polynomial as input, the computer provides us with the wave-resistance coefficient. The graph of this coefficient as a function of Froude number is shown in Figure 1. It is noteworthy that results differ significantly from those obtained by means of Inui's asymptotic approximation.

The contributions made to the wave-resistance coefficient by the individual terms of the hull function are given in Figures 2 through 10. A somewhat peculiar behavior was noted in the coefficients for region II in the range of Froude number $\simeq .30$ whenever $\beta = 1$. Accuracy of calculations have been verified several times and it has been concluded that the values of the coefficients are indeed as shown in the figures.

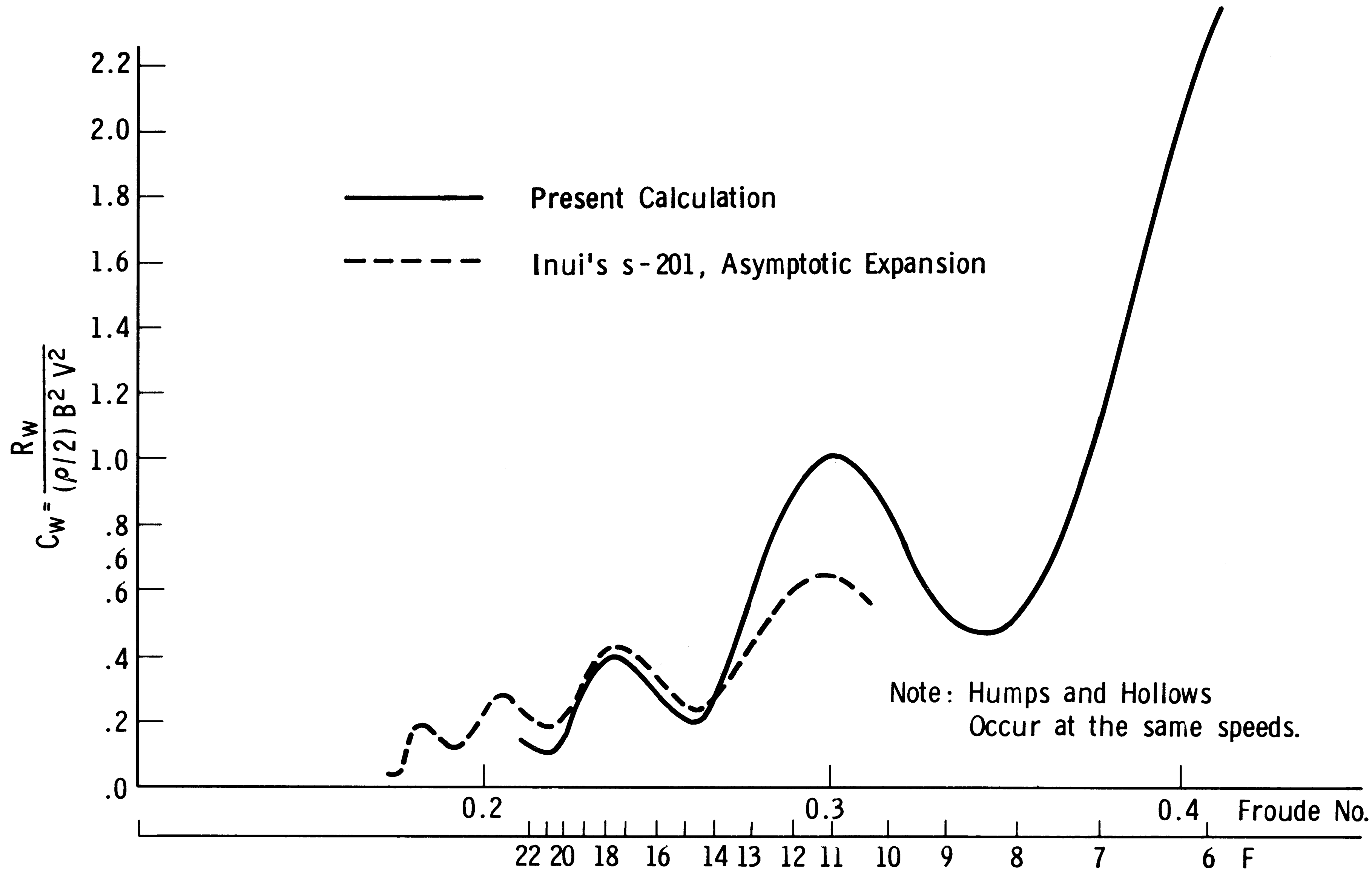

Figure 1. Wave-resistance coefficient for h(u,w)=-8u, D/L=.10

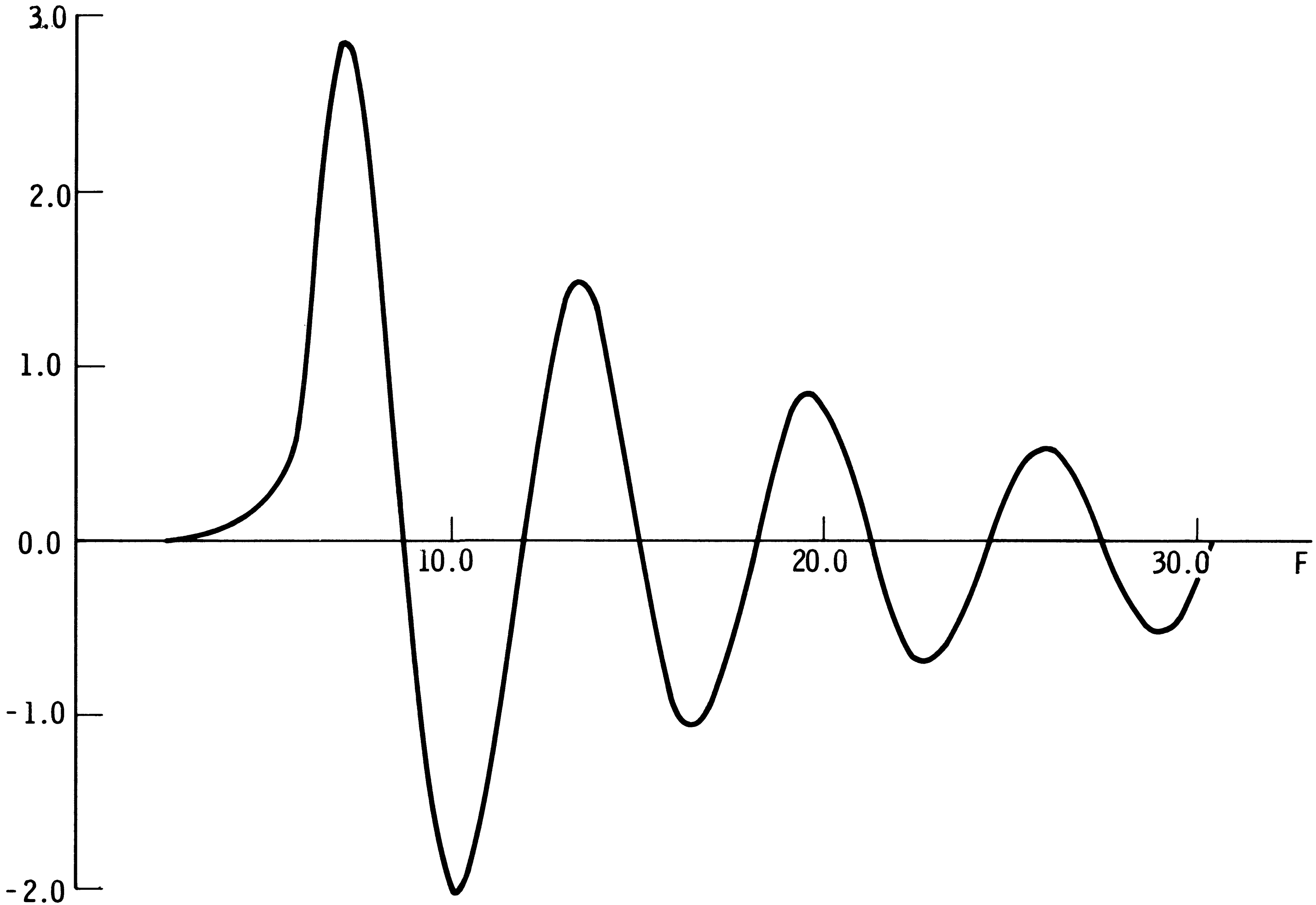

Figure 2. $M_{01}^{I} \times 10$ versus F ($\frac{D}{L}$ = .1 ; h(u,w) =-8u)

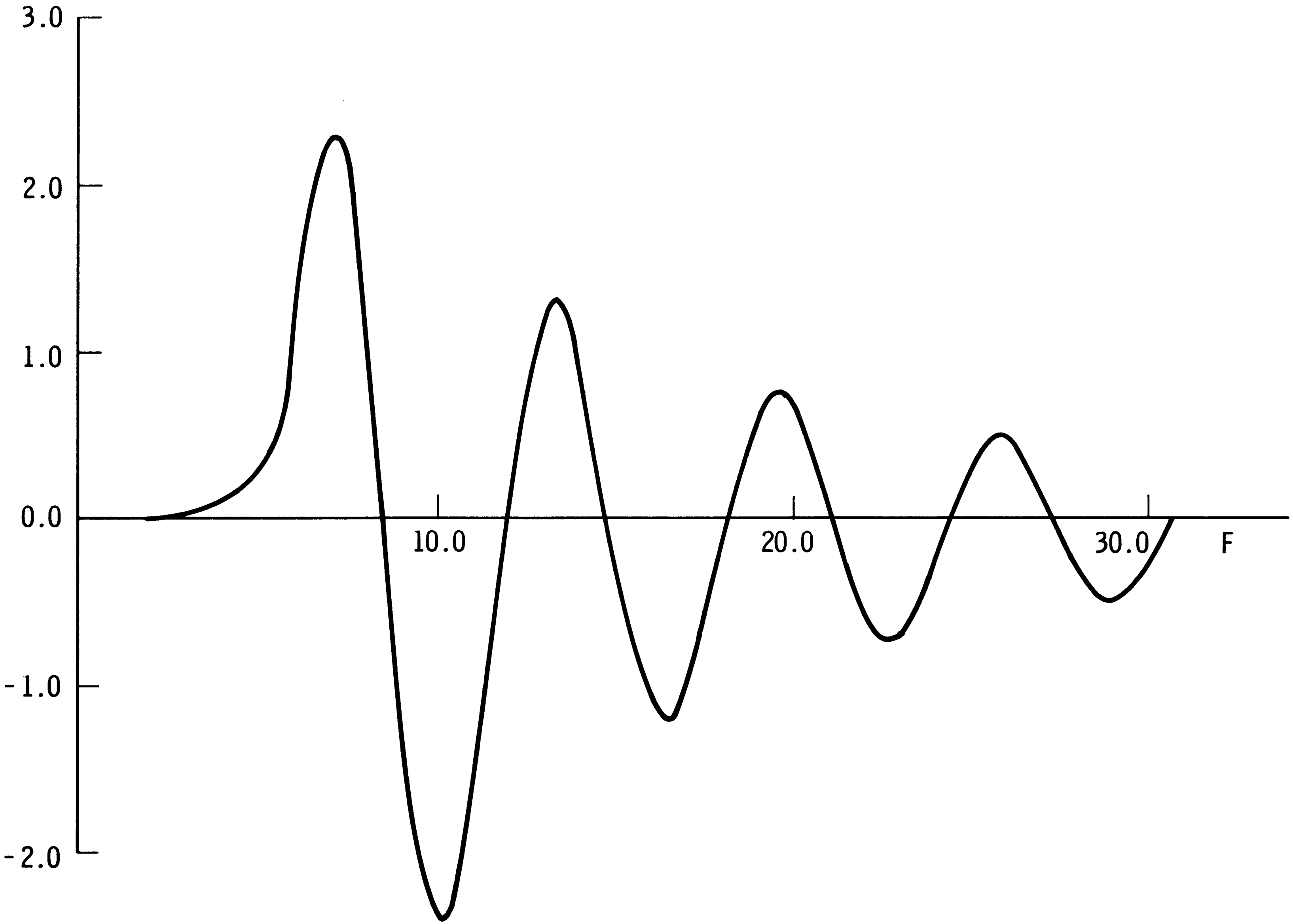

Figure 3. M_{11}^{I} x 10 versus F ($\frac{D}{L}$ = .1 ; h(u,w) =-8u)

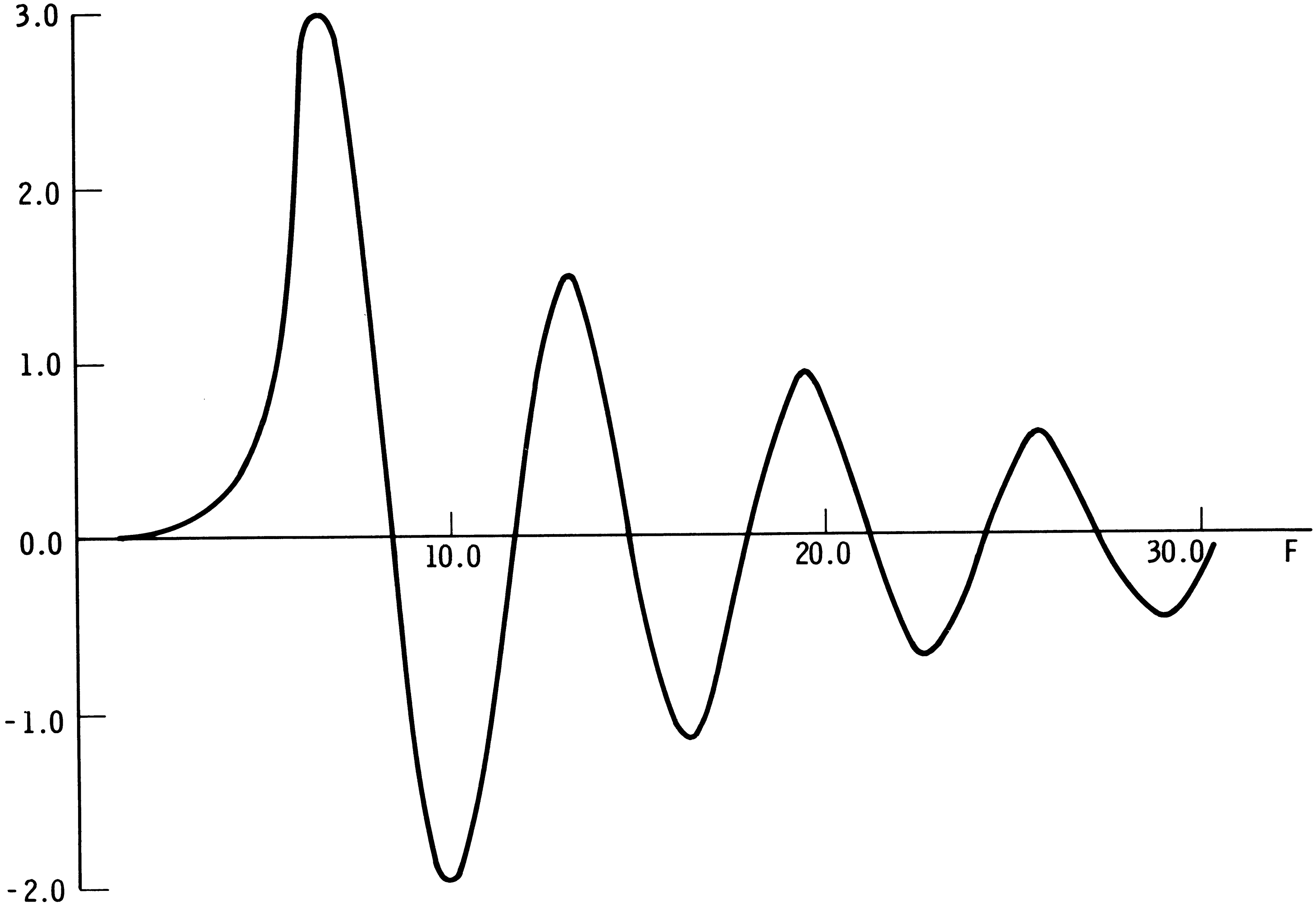

Figure 4. M^{I}_{31} x 10 versus F ($\frac{D}{L}$ = .1 ; h(u,w) =-8u)

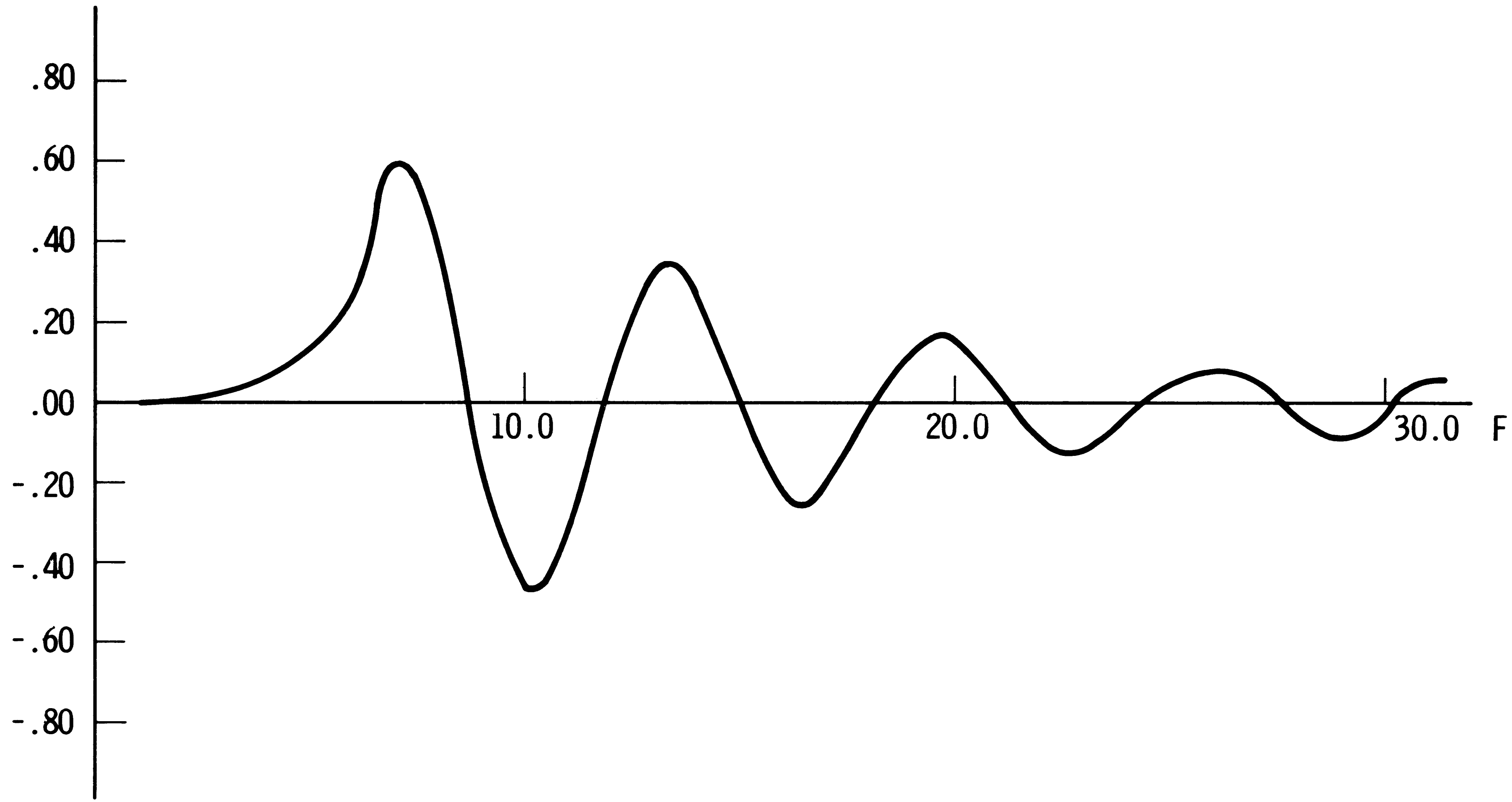

Figure 5. M_{00}^{II} x 10 versus F ($\frac{D}{L}$ = .1 ; h(u,w) =-8u)

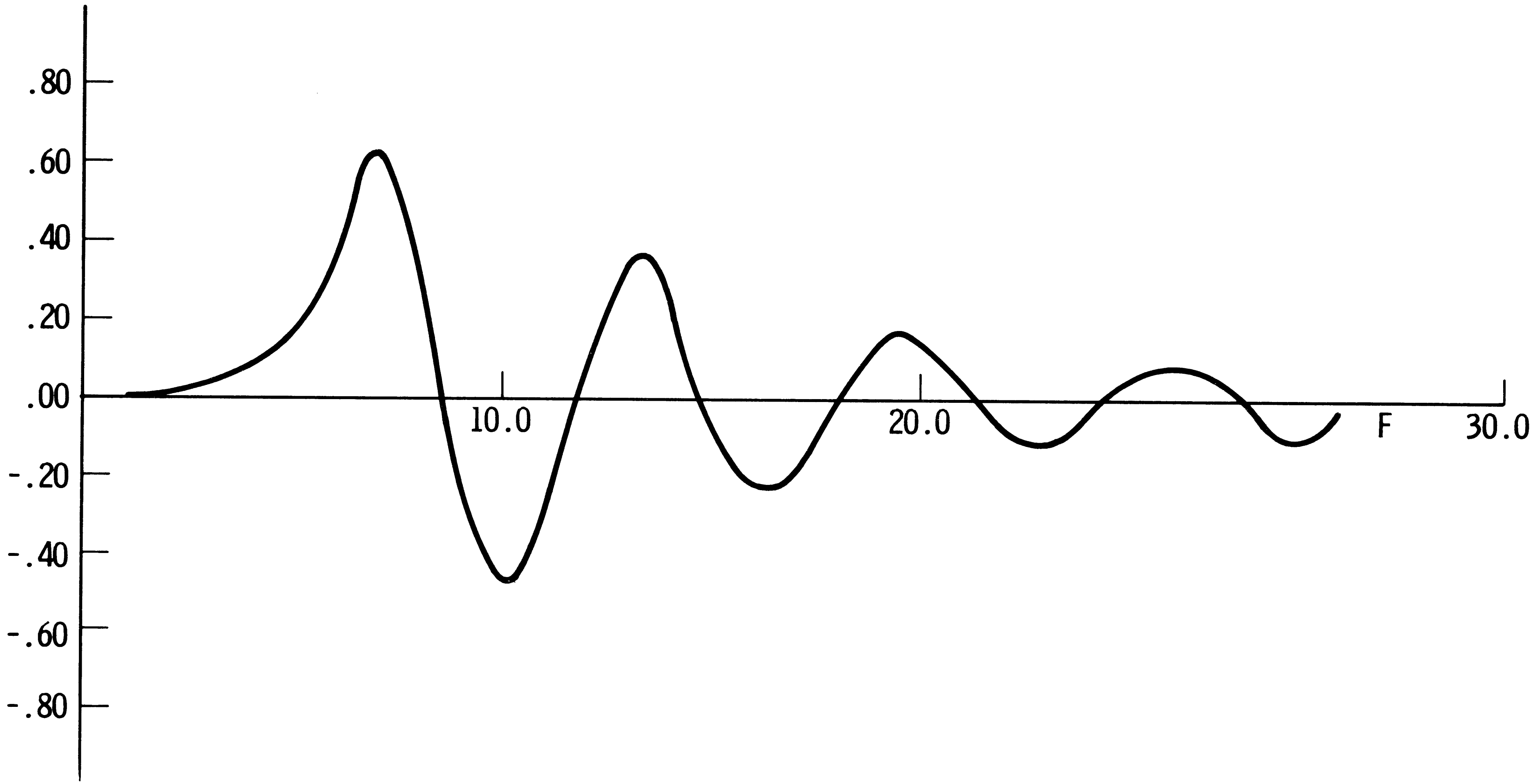

Figure 6. M_{10}^{II} x 10 versus F ($\frac{D}{L}$ = .1 ; h(u,w) =-8u)

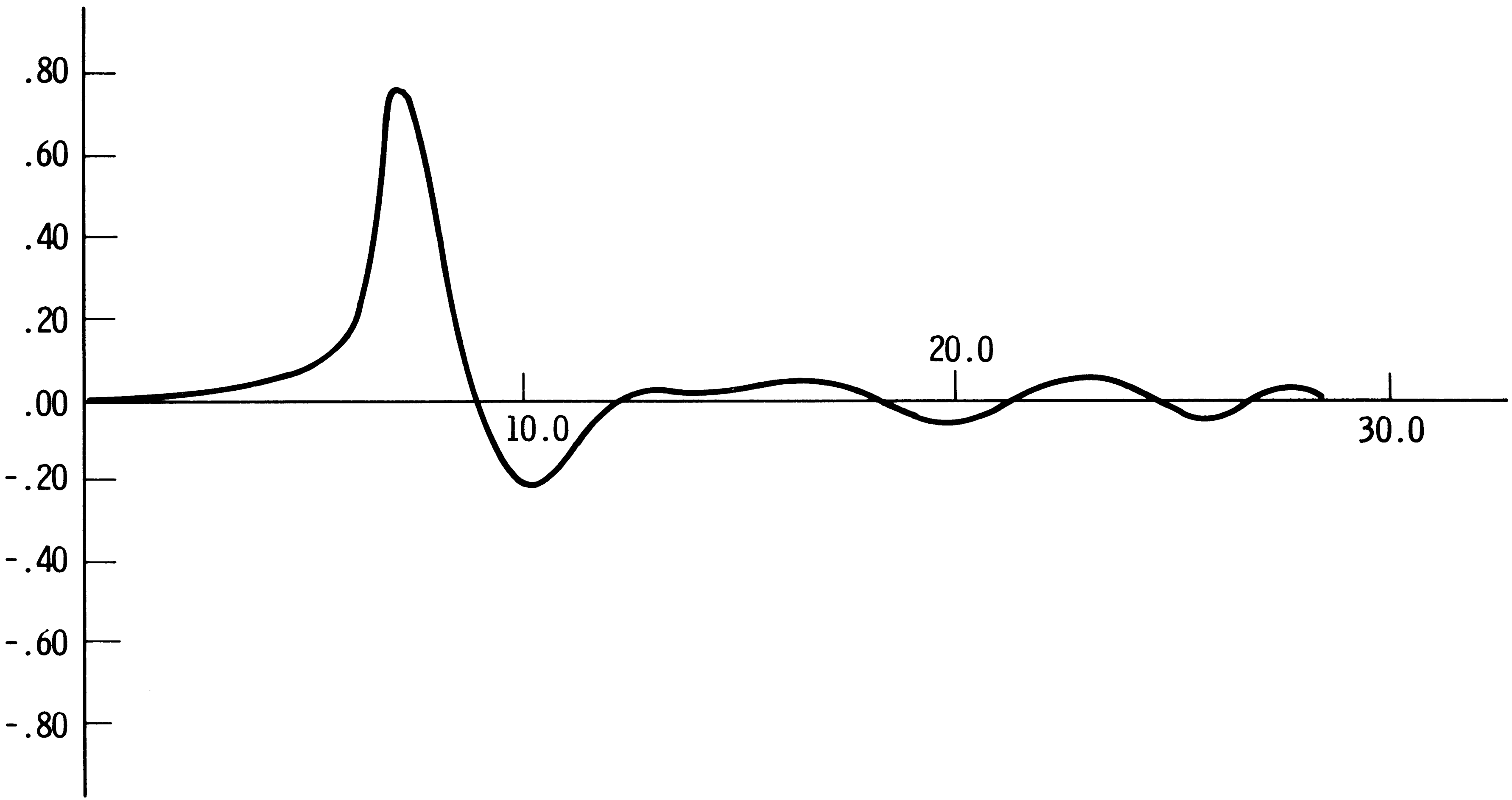

Figure 7. $M_{01}^{II} \times 10$ versus F ($\frac{D}{L}$ = .1 ; h(u,w) =-8u)

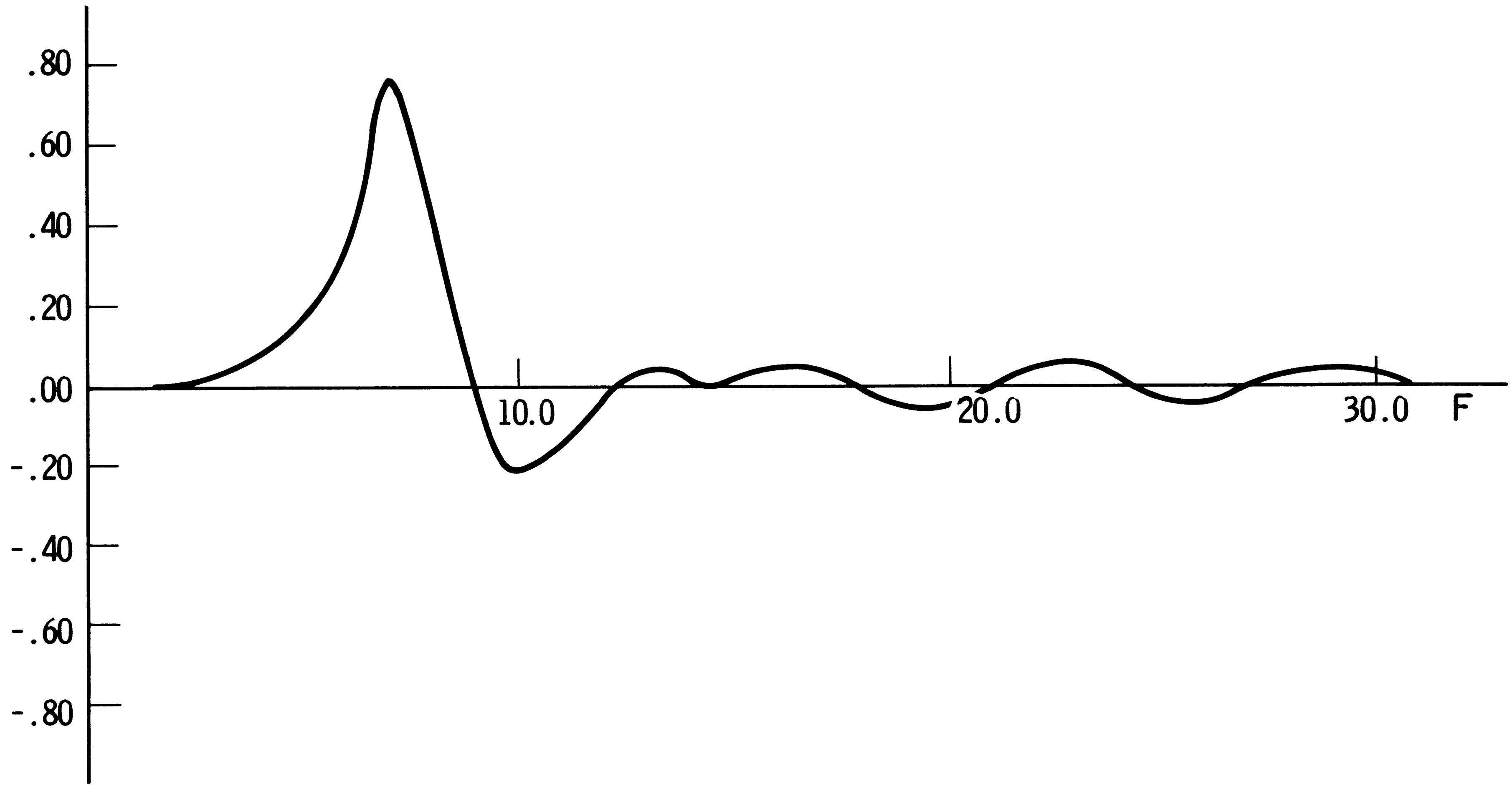

Figure 8. M_{11}^{II} x 10 versus F ($\frac{D}{L}$ = .1 ; h(u,w) =-8u)

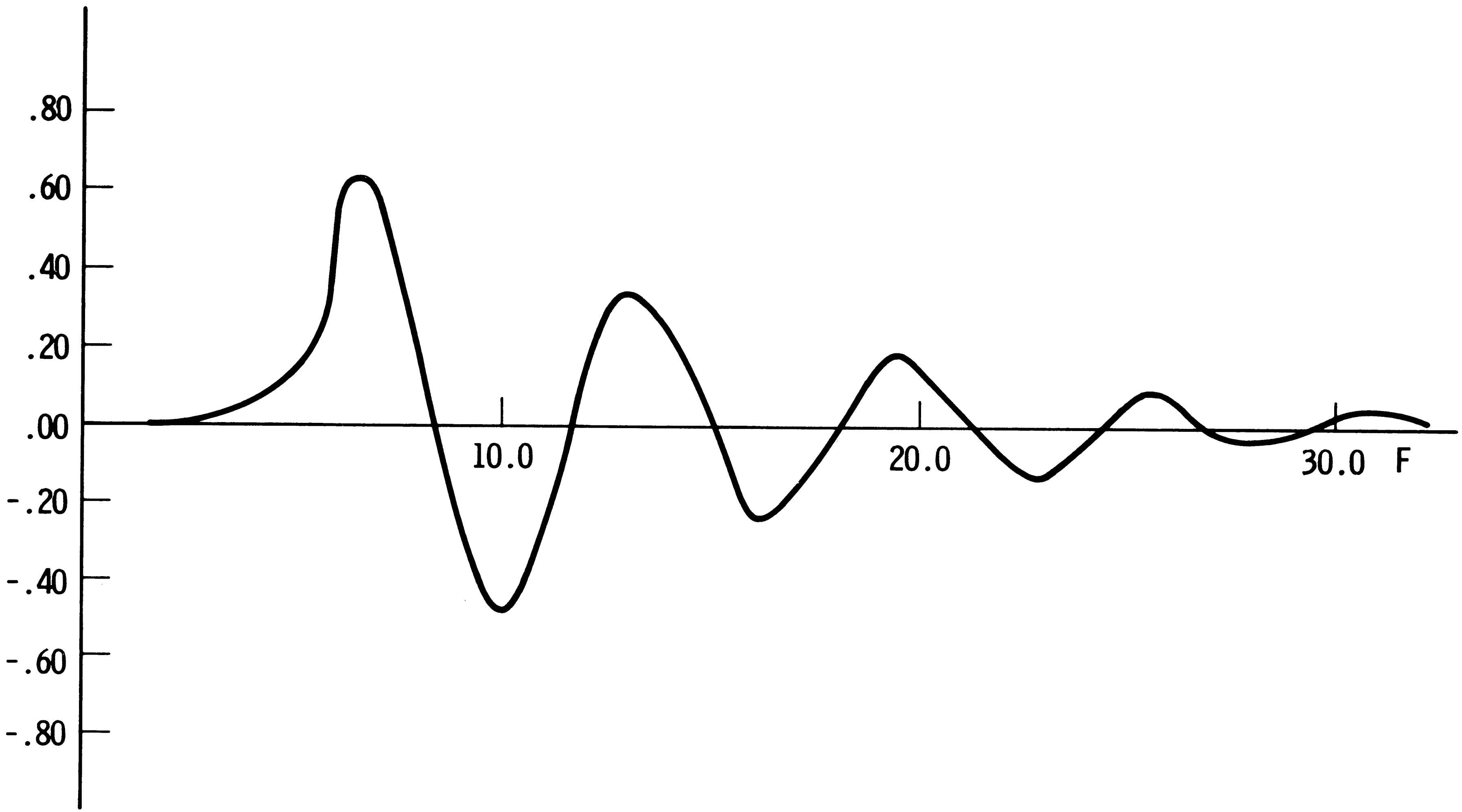

Figure 9. M_{30}^{II} x 10 versus F ($\frac{D}{L}$ = .1 ; h(u,w) =-8u)

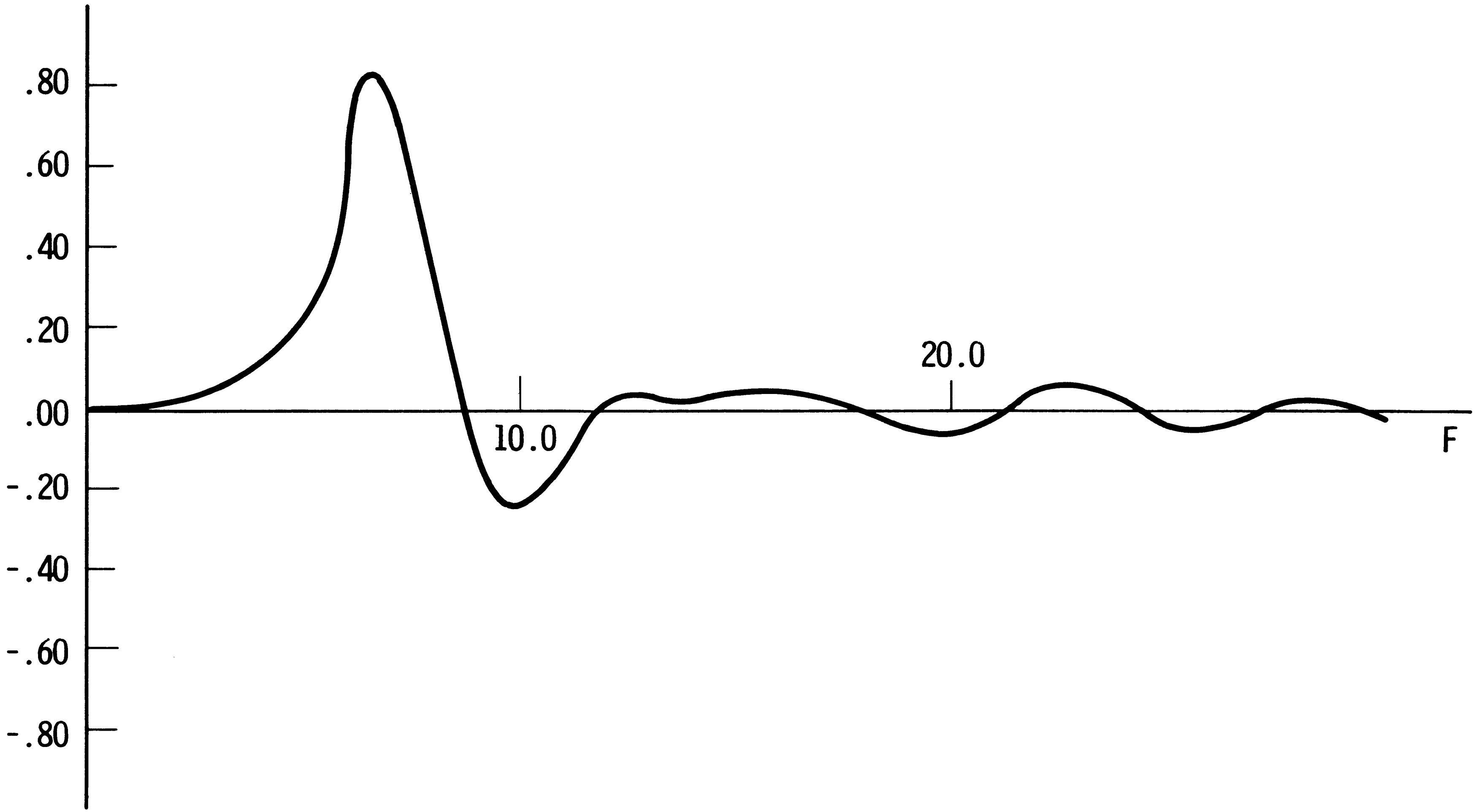

Figure 10. $M_{31}^{II} \times 10$ versus F ($\frac{D}{L}$ = .1 ; h(u,w) =-8u)

CONCLUSIONS AND RECOMMENDATIONS

It is believed that the method of evaluation of the Michell integral presented in this report may well prove to be the most general, fastest, and most accurate in existance. Low and high speeds pose no special problems. In fact the middle speed range is where computation time is the longest. Even so it takes only a few seconds to calculate the wave-resistance for the sample hull form considered in the report. At this time we have not taken full advantage of all recurrence relationships of the generalized Havelock $\bar{P}$-functions and other features which can further reduce computing time. It is therefore recommended that the computer program now available be modified to take advantage of all possible short cuts that can be incorporated.

Of equal importance to the prepared computer programs in the mathematical formulation of the wave-resistance coefficients on which they are based. It is noted that the finite sums of the expression for C_w form a linear system in term of the coefficients $A^{I}_{\alpha\beta}$ and $A^{II}_{\alpha\beta}$ of the hull function polynomial. These functions are in turn linearly dependent upon the coefficients C_{mn} of the singularity distribution function. The formulation of a minimum wave-resistance problem with suitable constraints is therefore fairly straightforward and should be pursued as soon as possible.

Another extension to the present work worthy of consideration would be the inclusion in the theory of concentrated sources and sinks, as well as singularities of higher orders. Such developments would be necessary to make the formulation of the wave-resistance of the "thin ship" as described in this report as flexible and complete as possible.

ACKNOWLEDGMENTS

The authors would like to thank Messrs. James W. Thatcher and Charles M. Greene, research assistants and programmers, for their aid at the beginning stages of this research project. Although their programs are not now being used, preliminary computer results obtained with these programs where extremely helpful in arriving at a workable solution.

REFERENCES

1. Michell, J. H. "The Wave Resistance of a Ship," Philosophical Magazine, 45 (1898), pp. 106-122.

2. Weinblum, G. P. "A Systematic Evaluation of Michell's Integral," Taylor Model Basin Report 886, June, 1955.

3. Inui, T. "Study on Wave-Making Resistance of Ships," The Society of Naval Architects of Japan, 60th Anniversary Series, II (1957), pp. 173-355.

4. Birkhoff, G., Korvin-Kroukovsky, B. V. and Kotik, J. "Theory of the Wave-Resistance of Ships," Transactions of the Society of Naval Architects and Marine Engineers, 62 (1954), pp. 359-385.

5. Michelsen, F. C. Wave Resistance Solution of Michell's Integral for Polynomial Ship Forms, Doctoral Dissertation, The University of Michigan, 1960.

6. Michelsen, F. C. "Expressions for the Evaluation of Wave-Resistance for Polynomial Certerplane Singularity Distributions," Proceedings of International Seminar on Theoritical Wave Resistance, Ann Arbor, Michigan August 1963, pp. 859-892.

APPENDIX A

COEFFICIENTS OF THE HULL FUNCTION POLYNOMIALS

From Equations (2), (3), and (4) it follows that the hull function can be written

$$[H(\xi,\zeta)]_I = \sum_{\alpha=0}^{2M+1} \sum_{\beta=0}^{2N+1} A^I_{\alpha\beta}\, \xi^\alpha \zeta^\beta \tag{19}$$

$$= \int_{-\frac{1}{2}}^{\frac{1}{2}-\xi} du \int_0^1 dw \left[\sum_{i=0}^{M} \sum_{j=0}^{N} C_{ij}\, u^i w^j\right]$$

$$\left[\sum_{m=0}^{M} \sum_{n=0}^{N} C_{m,n} (u+\xi)^m (\zeta - w)^n\right]$$

$$= \sum_i^M \sum_j^N \sum_m^M \sum_n^N C_{ij}\, C_{mn} \int_{-\frac{1}{2}}^{\frac{1}{2}-\xi} u^i (u+\xi)^m\, du \int_0^1 w\, (\zeta - w)^n\, dw$$

Considering the integral

$$[I_1]_I = \int_{-\frac{1}{2}}^{\frac{1}{2}-\xi} u^i\, (u+\xi)^m\, du \tag{20}$$

By the binomial theorem we have that

$$[I_1]_I = \sum_{k=0}^{m} \binom{m}{k} \xi^{m-k} \int_{-\frac{1}{2}}^{\frac{1}{2}-\xi} u^{i+k}\, du$$

$$= \sum_{k=0}^{m} \binom{m}{k} \xi^{m-k} (i+k+1)^{-1}\; u^{i+k+1} \Bigg|_{-\frac{1}{2}}^{\frac{1}{2}-\xi}$$

$$= \sum_{k=0}^{m} (-1)^{i+k+1} \binom{m}{k} (i+k+1)^{-1} \xi^{m-k}$$

$$\left[\sum_{\ell=0}^{i+k+1} \binom{i+k+1}{\ell} (-1)^{\ell} \left(\tfrac{1}{2}\right)^{\ell} \xi^{i+k+1-\ell} - \left(\tfrac{1}{2}\right)^{i+k+1} \right] \quad (21)$$

where $\binom{m}{k} = \dfrac{m!}{k!\,(m-k)!}$

Similarly we find that

$$[I_2]_I = \int_0^1 w^j \, (\zeta - w)^n \, dw$$

$$= \sum_{p=0}^{n} \binom{n}{p} \zeta^{n-p} (-1)^p (j+p+1)^{-1} \quad (22)$$

Substitution of (21) and (22) into (19) then gives

$$H(\xi,\zeta)]_I = \sum_i^M \sum_j^N \sum_m^M \sum_n^N \sum_{k=0}^{m} \sum_{p=0}^{n} (-1)^{i+k+1+p} \binom{m}{k} \binom{n}{p}$$

$$\frac{C_{ij} \quad C_{mn}}{(i+k+1)(j+p+1)} \xi^{m-k} \zeta^{n-p} \left[\sum_{\ell=0}^{i+k+1} \binom{i+k+1}{\ell} (-1)^{\ell} \left(\tfrac{1}{2}\right)^{\ell} \right.$$

$$\left. \xi^{i+k+1-\ell} - \left(\tfrac{1}{2}\right)^{i+k+1} \right]$$

$$= \sum_i^M \sum_j^N \sum_m^M \sum_n^N \sum_k^m \sum_p^n \sum_{\ell}^{i+k+1} (-1)^{i+k+p+\ell+1} \frac{\left(\tfrac{1}{2}\right)^{\ell}}{(i+k+1)(j+p+1)}$$

$$\binom{m}{k}\binom{n}{p}\binom{i+k+1}{\ell}\ \xi^{m+i+1-\ell}\ \zeta^{n-p}\ C_{ij}\ C_{mn}$$

$$+\sum_{i}\sum_{j}\sum_{m}\sum_{n}\sum_{k}\sum_{p}(-1)^{i+k+p}\ \frac{(\frac{1}{2})^{i+k+1}}{(i+k+1)(j+p+1)} \tag{23}$$

$$\binom{m}{k}\binom{n}{p}\ \xi^{m-k}\ \zeta^{n-p}\ C_{ij}C_{mn}$$

Following an identical procedure it can be shown that

$$[H(\xi,\zeta)]_{II} = \sum_{i,j,m,n,k,p,\ell} C_{ij}\ C_{mn}(-1)^{i+k+1+p+\ell}\binom{m}{k}\binom{n}{p}$$

$$\binom{i+k+1}{\ell}\left(\frac{1}{2}\right)^{\ell}\ \frac{1}{(i+k+1)(j+p+1)}\ \xi^{m+i+1-\ell}\ \zeta^{n-p}$$

$$+\sum_{i,j,m,n,k,p,\ell}\ \sum_{q=0}^{j+p+1} C_{ij}\ C_{mn}(-1)^{i+k+p+q+\ell}\binom{m}{k}\binom{n}{p}\binom{i+k+1}{\ell}$$

$$\binom{j+p+1}{q}\left(\frac{1}{2}\right)^{\ell}\ \frac{1}{(i+k+1)(j+p+1)}\ \xi^{m+i+1-\ell}\ \zeta^{n+j+1-q}$$

$$+\sum_{i,j,m,n,k,p} C_{ij}\ C_{mn}(-1)^{i+k+p}\binom{m}{k}\binom{n}{p}\left(\frac{1}{2}\right)^{i+k+1}\frac{1}{(i+k+1)(j+p+1)}$$

$$\xi^{m-k}\ \zeta^{n-p}$$

$$+\sum_{i,j,m,n,k,p,q} C_{ij}\, C_{mn} (-1)^{i+k+p+q+1} \binom{m}{k} \binom{n}{p} \binom{j+p+1}{q}$$

$$\left(\frac{1}{2}\right)^{i+k+1} \frac{1}{(i+k+1)(j+p+1)}\, \xi^{m-k}\, \zeta^{n+j+1-q} \qquad (24)$$

The method used in determining the coefficients $A^{I}_{\alpha\beta}$ and $A^{II}_{\alpha\beta}$ is to calculate and collect the coefficients of equal powers in ξ and ζ. A computer program written in MAD language that will perform this task is shown in Appendix "B."

APPENDIX B

COMPUTER PROGRAMS

Because programmers at The University of Michigan are generally more familiar with MAD, this language has been used throughout. The programs can readily be translated into other languages, however.

WAVE-RESISTANCE COEFFICIENT

Wave-resistance is calculated from Equations (5), (15), (16) and from the coefficients of Equations (23) and (24). The flow diagram is shown in Figure 11 below followed by the associated program.

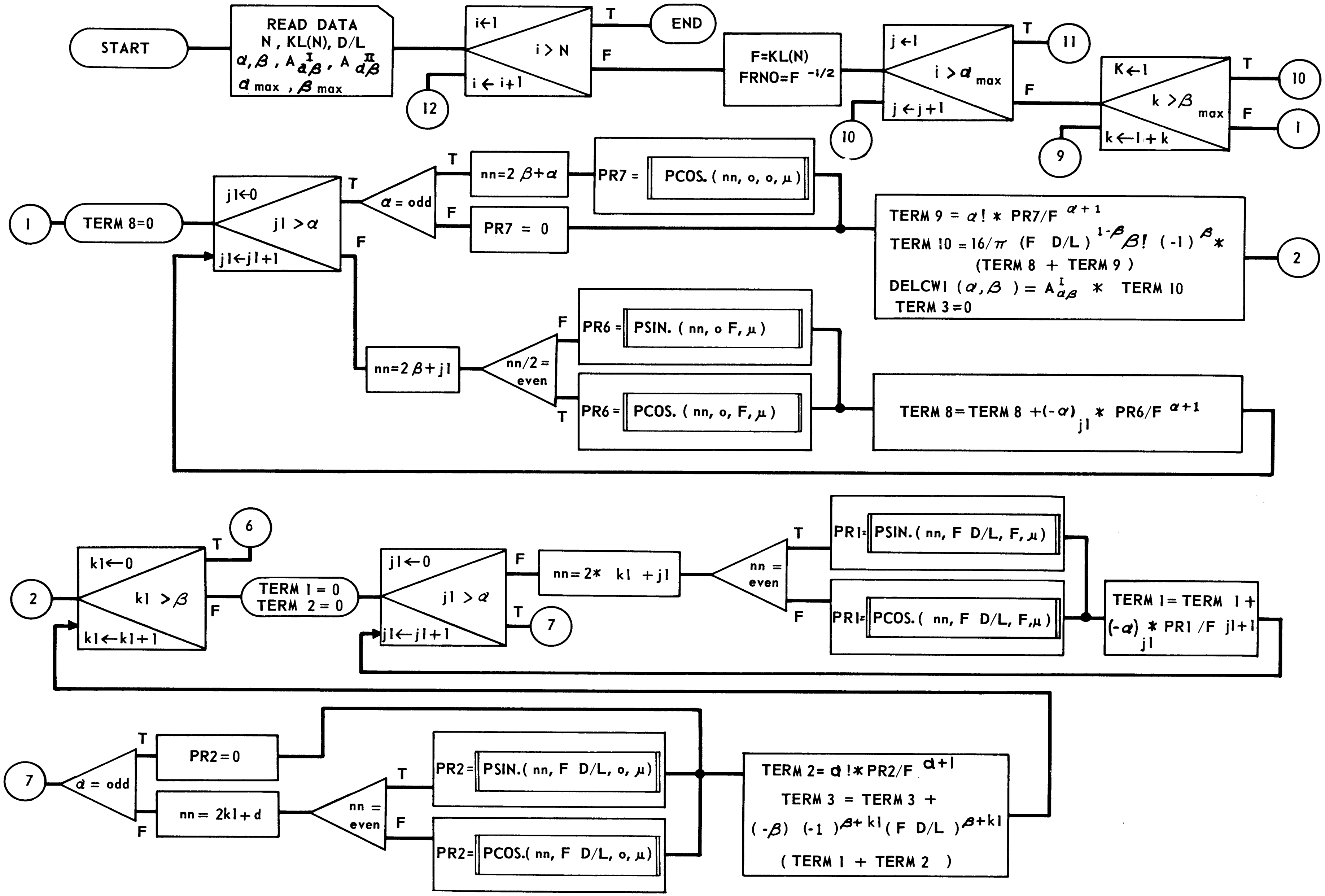

Figure 11. Flow Diagram for Wave-Resistance Calculation

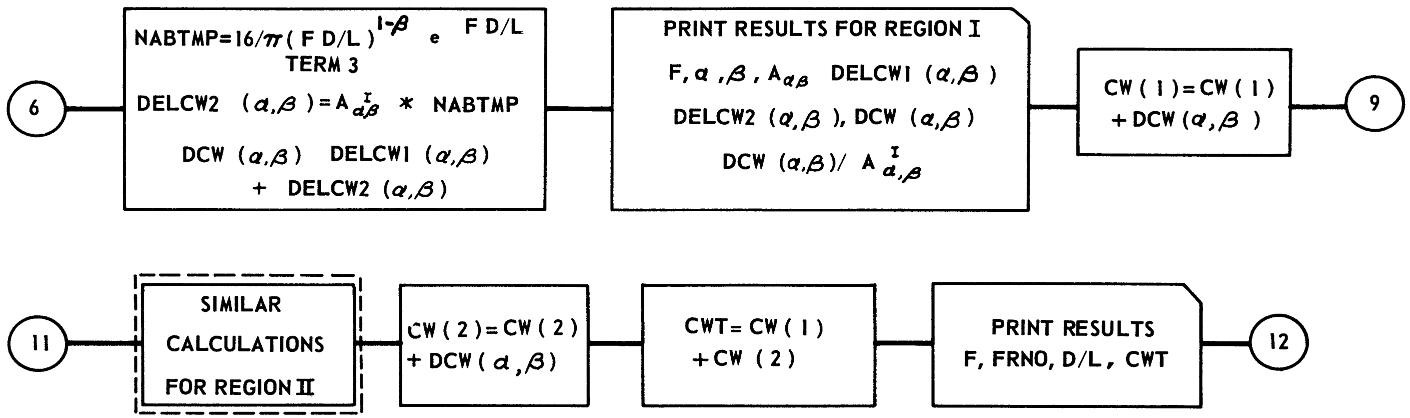

Figure 11 (Concluded)

```
$COMPILE MAD,EXECUTE,DUMP,PRINT OBJECT,I/O DUMP,PUNCH OBJECT    CWK                018659   08/19/65      6 56 59.0 PM

MAD (09 AUG 1965 VERSION) PROGRAM LISTING ... ... ...

           MICHELL INTEGRAL                    F.C. MICHELSEN - H. KIM

                          FACT.(X,N)
                          PSIN.(N,FDL,F,MU)
                          PCOS.(N,FDL,F,MU)
                          INTERNAL FUNCTION Z.(KK)=FACT.(1,KK)                      *001

            EXECUTE  FTRAP.                                                         *002
            PRINT COMMENT $1$                                                       *003
            PRINT COMMENT $4                      MICHEL INTEGRAL$                  *004
            PRINT COMMENT $4                                                        *005
          1   FINN C. MICHELSEN AND HUN CHOL KIM$                                   *005
            PRINT COMMENT $1$                                                       *006
START       READ AND PRINT DATA N                                                   *007
            READ FORMAT KLERM,  KL(1)...KL(N)                                       *008
            PRINT COMMENT $0  REGION I  INPUT DATA$                                 *009
START 1     READ AND PRINT DATA   MAX A, MAX B,DOVERL                               *010
            READ FORMAT ABIN, ALPHA(1)...ALPHA(MAXA)                                *011
            VECTOR VALUES ABIN=$16I5*$                                              *012
            READ FORMAT ABIN, BETA(1)...BETA(MAXB)                                  *013
            MDIM(1)=MAXB+2                                                          *014
            MDIM(2)=MAX B                                                           *015
            DIM1(1)=BETA(MAXB)+2                                                    *016
            DIM1(2)=BETA(MAXB)                                                      *017
            READ AND PRINT DATA  AAB(ALPHA, BETA)                                   *018
            PRINT COMMENT $0  REGION II INPUT DATA$                                 *019
            READ AND PRINT DATA  MAXA1, MAXB1, ALP MAX                              *020
            READ FORMAT ABIN, ALPHA1(1)...ALPHA1(MAXA)                              *021
            READ FORMAT ABIN,BETA1(1)...BETA1(MAXB)                                 *022
            MDIM1(1)=MAXB1+2                                                        *023
            MDIM1(2)=MAX B1                                                         *024
            DIM2(1)=BETA1(MAXB1)+2                                                  *025
            DIM2(2)=BETA1(MAXB1)                                                    *026
            READ AND PRINT DATA  AAB1(ALPHA1, BETA1)                                *027
            PRINT COMMENT $8 SPEED INPUT$                                           *028
            PRINT RESULTS KL(1)...KL(N)                                             *029
            THROUGH L1, FOR I=1,1,I.G. N                                            *030
                        PRINT COMMENT$1$                                            *031         01
            F=KL(I)                                                                 *032         01
            FDL=F*D OVER L                                                          *033         01
            F? NO=F .P.-0.5                                                         *034         01
            CW(1)=0.                                                                *035         01
            NN=ALPHA(MAX A)+ 2*BETA(MAX B)                                          *036         01
            WHENEVER NN .L. ALPHA1(MAXA1)+2*BETA1(MAXB1)                            *037         01
                     NN= ALPHA1(MAXA1)+2*BETA1(MAXB1)                               *038    01   01
            END OF CONDITIONAL                                                      *039    01   01
                 WHENEVER SW(2), PRINT RESULTS AAB(1,1),AAB1(1,1)                   *040         01
            THROUGH LOOP, FOR J=0,1,J.G. NN                                         *041         01
            PSNSW(J)=0B                                                             *042         02
            PCNSW(J)=0B                                                             *043         02
            PSNSW1(J)=0B                                                            *044         02
            PCNSW1(J)=0B                                                            *045         02
```

```
          PSNSW6(J)=0B                                              *046          02
          PCNSW6(J)=0B                                              *047          02
          PCNSW7(J)=0B                                              *048          02
          PSNSW7(J)=0B                                              *049          02
          PSNSW4(J)=0B                                              *050          02
          PCNSW4(J)=0B                                              *051          02
          PSNSW5(J)=0B                                              *052          02
LOOP      PCNSW5(J)=0B                                              *053          02
                  WHENEVER SW(1)                                    *054          01
          PRINT COMMENT$0          RESULTS FOR REGION 1$            *055    01    01
          END OF CONDITIONAL                                        *056    01    01
                                                                    *057          01
          THROUGH L2, FOR J=1,1,J.G. MAX A                          *058          01
          ALP=ALPHA(J)                                              *059          02
          THROUGH L2, FOR K=1,1,K.G. MAX B                          *060          02
          BET=BETA(K)                                               *061          03
                                                                    *062          03
                                                                    *063          03
          EXECUTE ZERO.(TERM 8)                                     *064          03
          THROUGH L8, FOR J1=0,1,J1.G.ALP                           *065          03
          NN=2*BET+J1                                               *066          04
          NNH=NN/2                                                  *067          04
          WHENEVER NNH*2.E.NN                                       *068          04
                WHENEVER PSNSW6(NN)                                 *069    01    04
                PR6=PSNS6(NN)                                       *070    02    04
                OTHERWISE                                           *071    02    04
                PR6=PSIN.(NN,0.,F,MU)                               *072    02    04
                    WHENEVER SW(2), PRINT COMMENT$ PSIN$            *073    02    04
                    WHENEVER SW(2), PRINT RESULTS NN, BET, J1, PR6  *074    02    04
                PSNSW6(NN)=1B                                       *075    02    04
                PSNS6(NN)=PR6                                       *076    02    04
                END OF CONDITIONAL                                  *077    02    04
          OTHERWISE                                                 *078    01    04
                WHENEVER PCNSW6(NN)                                 *079    01    04
                PR6=PCNS6(NN)                                       *080    02    04
                OTHERWISE                                           *081    02    04
                PR6=PCOS.(NN,0.,F,MU)                               *082    02    04
                    WHENEVER SW(2), PRINT COMMENT$ PCOS$            *083    02    04
                    WHENEVER SW(2), PRINT RESULTS NN, BET, J1, PR6  *084    02    04
                PCNSW6(NN)=1B                                       *085    02    04
                PCNS6(NN)=PR6                                       *086    02    04
                END OF CONDITIONAL                                  *087    02    04
          END OF CONDITIONAL                                        *088    01    04
L8        TERM 8=TERM 8+FACT.(-ALP,J1)*PR6/F.P.(J1+1)               *089          04
          ALPH=ALP/2                                                *090          03
          WHENEVER ALPH*2.NE.ALP                                    *091          03
          NN=2*BET+ALP                                              *092    01    03
                NNH=NN/2                                            *093    01    03
                WHENEVER PCNSW7(NN)                                 *094    01    03
                PR7=PCNS7(NN)                                       *095    02    03
                OTHERWISE                                           *096    02    03
                PR7=PCOS.(NN,0.,0.,MU)                              *097    02    03
                    WHENEVER SW(2), PRINT COMMENT$ PCOS$            *098    02    03
                    WHENEVER SW(2), PRINT RESULTS NN, BET,ALP, PR7  *099    02    03
                  WHENEVER SW(2), PRINT RESULTS PR7, NN             *100    02    03
                PCNSW7(NN)=1B                                       *101    02    03
                PCNS7(NN)=PR7                                       *102    02    03
                END OF CONDITIONAL                                  *103    02    03
          OTHERWISE                                                 *104    01    03
          PR7=0.                                                    *105    01    03
```

```
      END OF CONDITIONAL                                                  *106   01   03
      TERM 9=Z.(ALP)*PR7/F.P.(ALP+1)                                      *107        03
      TERM 10=16./(3.14159*FDL.P.(BET-1))*Z.(BET)*(-1).P.(BET)*           *108        03
    1  (TERM 8+TERM 9)                                                    *108
      DELCW1(ALP,BET)=AAB(ALP,BET)*TERM 10                                *109        03
      EXECUTE ZERO. (TERM 3)                                              *110        03
      THROUGH L3, FOR K1=0,1,K1.G. BET                                    *111        03
      EXECUTE ZERO. (TERM1,TERM2)                                         *112        04
      THROUGH L6, FOR J1=0,1,J1.G. ALP                                    *113        04
      NN=2*K1+J1                                                          *114        05
      WHENEVER (NN/2)*2 .E. NN                                            *115        05
            WHENEVER PSNSW(NN)                                            *116   01   05
            PR1=PSNS1(NN)                                                 *117   02   05
            OTHERWISE                                                     *118   02   05
      PR1=PSIN.(NN,FDL,F,MU)                                              *119   02   05
                WHENEVER SW(2), PRINT COMMENT$ PSIN$                      *120   02   05
                WHENEVER SW(2), PRINT RESULTS NN,K1,J1,PR1                *121   02   05
            PSNSW(NN)=1B                                                  *122   02   05
            PSNS1(NN)=PR1                                                 *123   02   05
              END OF CONDITIONAL                                          *124   02   05
      OTHERWISE                                                           *125   01   05
            WHENEVER PCNSW(NN)                                            *126   01   05
            PR1=PCNS1(NN)                                                 *127   02   05
            OTHERWISE                                                     *128   02   05
      PR1=PCOS.(NN,FDL,F,MU)                                              *129   02   05
                WHENEVER SW(2), PRINT COMMENT$ PCOS$                      *130   02   05
                WHENEVER SW(2), PRINT RESULTS NN,K1,J1,PR1                *131   02   05
            PCNSW(NN)=1B                                                  *132   02   05
            PCNS1(NN)=PR1                                                 *133   02   05
              END OF CONDITIONAL                                          *134   02   05
              END OF CONDITIONAL                                          *135   01   05
L6    TERM1=TERM1+FACT.(-ALP,J1)*PR1/F.P.(J1+1)                           *136        05
              ALPH=ALP/2                                                  *137        04
      WHENEVER ALPH*2 .NE. ALP                                            *138        04
      NN=2*K1+ALP                                                         *139   01   04
      WHENEVER (NN/2)*2 .E. NN                                            *140   01   04
            WHENEVER PSNSWO(NN)                                           *141   02   04
            PR2=PSNSO(NN)                                                 *142   03   04
            OTHERWISE                                                     *143   03   04
      PR2=PSIN.(NN,FDL,0.,MU)                                             *144   03   04
                WHENEVER SW(2), PRINT COMMENT$ PSIN$                      *145   03   04
                WHENEVER SW(2), PRINT RESULTS NN,K1,ALP,PR2               *146   03   04
            PSNSWO(NN)=1B                                                 *147   03   04
            PSNSO(NN)=PR2                                                 *148   03   04
            END OF CONDITIONAL                                            *149   03   04
      OTHERWISE                                                           *150   02   04
            WHENEVER PCNSWO(NN)                                           *151   02   04
            PR2=PCNSO(NN)                                                 *152   03   04
            OTHERWISE                                                     *153   03   04
      PR2=PCOS.(NN,FDL,0.,MU)                                             *154   03   04
                WHENEVER SW(2), PRINT COMMENT$ PCOS$                      *155   03   04
                WHENEVER SW(2), PRINT RESULTS NN,K1,ALP,PR2               *156   03   04
            PCNSWO(NN)=1B                                                 *157   03   04
            PCNSO(NN)=PR2                                                 *158   03   04
            END OF CONDITIONAL                                            *159   03   04
            END OF CONDITIONAL                                            *160   02   04
              OTHERWISE                                                   *161   01   04
              PR2=0.                                                      *162   01   04
              END OF CONDITIONAL                                          *163   01   04
      TERM 2=Z.(ALP)*PR2/F.P.(ALP+1)                                      *164        04
```

```
L3      TERM 3=TERM 3+FACT.(-BET,K1)*(-1.).P.(BET+K1)*FDL.P.(BET-K1)            *165          04
      1 *(TERM 1+ TERM 2)                                                        *165
        NABTMP=-16.*TERM 3/(3.14159*FDL.P.(BET-1)*NAPE.P.FDL)                    *166          03
        DELCW2(ALP,BET)=AAB(ALP,BET)*NABTMP                                      *167          03
        DCW(ALP,BET)=DELCW1(ALP,BET)+DELCW2(ALP,BET)                             *168          03
                 WHENEVER SW(1)                                                  *169          03
        PRINT COMMENT$0              F  ALPHA BETA     AAB          FIRS         *170     01   03
      1 T         SECOND             SUM             COEFF$                      *170
        PRINT FORMAT OUTPUT,F,ALP,BET,AAB(ALP,BET),DELCW1(ALP,BET),              *171     01   03
      1 DELCW2(ALP,BET), DCW(ALP,BET),DCW(ALP,BET)/AAB(ALP,BET)                  *171
                 END OF CONDITIONAL                                              *172     01   03
        WHENEVER SW(5)                                                           *173          03
        PUNCH FORMAT MK, DOVERL,F,KKK,DCW(ALP,BET)/AAB(ALP,BET)                  *174     01   03
        VECTOR VALUES MK=$2F10.5,I5,S5, E25.9*$                                  *175     01   03
        END OF CONDITIONAL                                                       *176     01   03
L2      CW(1) = CW(1) + DCW(ALPHA(J), BETA(K))                                   *177          03
                 WHENEVER SW(1)                                                  *178          01
        PRINT COMMENT $0            RESULTS FOR REGION 2$                        *179     01   01
                 END OF CONDITIONAL                                              *180     01   01
        CW(2)=0.                                                                 *181          01
        THROUGH L4, FOR J=1,1,J.G. MAX A1                                        *182          01
        ALP=ALPHA1(J)                                                            *183          02
        THROUGH L4, FOR K=1,1, K.G. MAXB1                                        *184          02
        BET=BETA1(K)                                                             *185          03
        KKK=KKK+1                                                                *186          03
        AAB1(ALP,BET)=1.0                                                        *187          03
        EXECUTE ZERO. (TERM3,TERM6)                                              *188          03
        THROUGH L5, FOR K1=0,1,K1.G. BET                                         *189          03
        EXECUTE ZERO. (TERM1,TERM2,TERM4,TERM5)                                  *190          04
        THROUGH L7, FOR J1=0,1,J1.G. ALP                                         *191          04
        NN=2*K1+J1                                                               *192          05
         NNH=NN/2                                                                *193          05
         WHENEVER NNH*2 .E. NN                                                   *194          05
               WHENEVER PSNSW(NN)                                                *195     01   05
               PR1=PSNS1(NN)                                                     *196     02   05
                 OTHERWISE                                                       *197     02   05
        PR1=PSIN.(NN,FDL,F,MU)                                                   *198     02   05
                   WHENEVER SW(2), PRINT COMMENT$ PSIN$                          *199     02   05
                   WHENEVER SW(2), PRINT RESULTS NN,K1,J1,PR1                    *200     02   05
               PSNSW(NN)=1B                                                      *201     02   05
               PSNS1(NN)=PR1                                                     *202     02   05
               END OF CONDITIONAL                                                *203     02   05
               WHENEVER PSNSW4(NN)                                               *204     01   05
               PR4=PSNS4(NN)                                                     *205     02   05
                 OTHERWISE                                                       *206     02   05
               PR4=PSIN.(NN,2.*FDL,F,MU)                                         *207     02   05
                   WHENEVER SW(2), PRINT RESULTS NN,K1,J1,PR4                    *208     02   05
               PSNSW4(NN)=1B                                                     *209     02   05
               PSNS4(NN)=PR4                                                     *210     02   05
               END OF CONDITIONAL                                                *211     02   05
        OTHERWISE                                                                *212     01   05
               WHENEVER PCNSW(NN)                                                *213     01   05
               PR1=PCNS1(NN)                                                     *214     02   05
               OTHERWISE                                                         *215     02   05
        PR1=PCOS.(NN,FDL,F,MU)                                                   *216     02   05
                   WHENEVER SW(2), PRINT RESULTS NN,K1,J1,PR1                    *217     02   05
               PCNSW(NN)=1B                                                      *218     02   05
               PCNS1(NN)=PR1                                                     *219     02   05
               END OF CONDITIONAL                                                *220     02   05
               WHENEVER PCNSW4(NN)                                               *221     01   05
```

```
             PR4=PCNS4(NN)                                                        *222     02   05
             OTHERWISE                                                            *223     02   05
             PR4=PCOS.(NN,2.*FDL,F,MU)                                            *224     02   05
                 WHENEVER SW(2), PRINT RESULTS NN,K1,J1,PR4                       *225     02   05
             PCNSW4(NN)=1B                                                        *226     02   05
             PCNS4(NN)=PR4                                                        *227     02   05
             END OF CONDITIONAL                                                   *228     02   05
               END OF CONDITIONAL                                                 *229     01   05
       WHENEVER SW(3) , PRINT RESULTS K1,J1,PR1,PR4                               *230          05
       TERM1=TERM1+FACT.(-ALP,J1)*PR1/(F.P.J1*NAPE.P.FDL)                         *231          05
L7     TERM 4=TERM 4+FACT.(-ALP,J1)*PR4/(F.P.J1*NAPE.P.(2.*FDL))                  *232          05
       ALPH=ALP/2                                                                 *233          04
       WHENEVER ALPH*2.NE. ALP                                                    *234          04
       NN=2*K1+ALP                                                                *235     01   04
             WHENEVER PCNSWO(NN)                                                  *236     01   04
             PR2=PCNSO(NN)                                                        *237     02   04
             OTHERWISE                                                            *238     02   04
       PR2=PCOS.(NN,FDL,O.,MU)                                                    *239     02   04
                 WHENEVER SW(2), PRINT RESULTS NN,K1,ALP,PR2                      *240     02   04
             PCNSWO(NN)=1B                                                        *241     02   04
             PCNSO(NN)=PR2                                                        *242     02   04
             END OF CONDITIONAL                                                   *243     02   04
             WHENEVER PCNSW5(NN)                                                  *244     01   04
             PR5=PCNS5(NN)                                                        *245     02   04
             OTHERWISE                                                            *246     02   04
             PR5=PCOS.(NN,2.*FDL,O.,MU)                                           *247     02   04
                 WHENEVER SW(2), PRINT RESULTS NN,K1,ALP,PR5                      *248     02   04
             PCNSW5(NN)=1B                                                        *249     02   04
             PCNS5(NN)=PR5                                                        *250     02   04
             END OF CONDITIONAL                                                   *251     02   04
           OTHERWISE                                                              *252     01   04
           PR5=0.                                                                 *253     01   04
           PR2=0.                                                                 *254     01   04
       END OF CONDITIONAL                                                         *255     01   04
       WHENEVER SW(3), PRINT RESULTS K1,J1,PR2,PR5                                *256          04
       TERM 2=Z.(ALP)*PR2/(NAPE.P.FDL*F.P.ALP)                                    *257          04
       TERM 5=Z.(ALP)*PR5/(NAPE.P.(2.*FDL)*F.P.ALP)                               *258          04
        WHENEVER SW(3),PRINT RESULTS TERM1,TERM2,TERM1+TERM2                      *259          04
       WHENEVER SW(3), PRINT RESULTS TERM4, TERM5, TERM4+TERM5                    *260          04
       TERM 3=TERM 3+FACT.(-BET,K1)*FDL.P.(BET-K1)*(TERM1+TERM 2)*                *261          04
     1 (-1.).P.(BET+K1)                                                           *261
L5     TERM 6=TERM 6+FACT.(-BET,K1)*(2.*FDL).P.(BET-K1)*(TERM4+                   *262          04
     1 TERM 5)*(-1.).P.(BET+K1)                                                   *262
       TMP1=16.*TERM3/(3.14159*FDL.P.(BET-1))/F                                   *263          03
       TMP2=16.*TERM6/(3.14159*FDL.P.(BET-1))/F                                   *264          03
       DCW1(ALP,BET)=AAB1(ALP,BET)*TMP1                                           *265          03
       DCW2(ALP,BET)=AAB1(ALP,BET)*TMP2*(-1.)                                     *266          03
       DCW3(ALP,BET)=DCW1(ALP,BET)+DCW2(ALP,BET)                                  *267          03
               WHENEVER SW(1)                                                     *268          03
       PRINT COMMENT$0             F  ALPHA BETA    AAB1          THIR            *269     01   03
     1 D          FOURTH           SUM            COEFF$                          *269
       PRINT FORMAT OUTPUT,F,ALP,BET,AAB1(ALP,BET),DCW1(ALP,BET),                 *270     01   03
     1 DCW2(ALP,BET),DCW3(ALP,BET), DCW3(ALP,BET)/AAB1(ALP,BET)                   *270
               END OF CONDITIONAL                                                 *271     01   03
                                                                                  *272          03
                                                                                  *273     01   03
                                                                                  *274     01   03
L4     CW(2) = CW(2) + DCW3(ALP,BET)                                              *275          03
       CWT=CW(1) + CW(2)                                                          *276          01
       PRINT COMMENT $8$                                                          *277          01
```

```
L1        PRINT RESULTS F,ERNO,DOVERL,  CWT,CW(1),CW(2)                    *278
          PRINT COMMENT$1$                                                 *279
          TRANSFER TO START                                                *280

          VECTOR VALUES KLFRM=$16F5.1*$                                    *281
          VECTOR VALUES MDIM=2,1,10                                        *282
          VECTOR VALUES MDIM1=2,1,10                                       *283
          VECTOR VALUES OUTPUT=$S10,F5.1,2I4,5E15.7*$                      *284
          VECTOR VALUES SW=0B,0B,0B                                        *285
          VECTOR VALUES NAPE=2.7321828182 8                                *286
          INTEGER KK,NNH , KKK                                             *287
          INTEGER N, MAXA,MAXA1,MAXB,MAXB1,I,J,K,J1,K1,ALPHA,ALPHA1,       *288
        1  BETA,BETA1,NN ,ALP, BET, ALPH                                   *288
          VECTOR VALUES DIM1=2,1,10                                        *289
          VECTOR VALUES DIM2=2,1,10                                        *290
          DIMENSION  AAB(1000,MDIM),AAB1(1000,MDIM1), I2AB(1000,DIM1),     *291
        1  DELCW1(1000,DIM1), DELCW2(1000,DIM1), DCW(1000,DIM1),           *291
        2  DCW1(1000,DIM2), CW(2), ALPHA(50), ALPHA1(50), BETA(50),        *291
        3  BETA1(50), KL(100), SW(5)                                       *291
          DIMENSION DCW2(1000,DIM2),DCW3(1000,DIM2)                        *292
          DIMENSION PSNS1(100),PCNS1(100),PSNS0(100),PCNS0(100),           *293
        1 PSNS4(100),PCNS4(100),PSNS5(100),PCNS5(100),                     *293
        2 PSNSW(100),PCNSW(100),PSNSW0(100),PCNSW0(100),PSNSW4(100),       *293
        3 PCNSW4(100),PSNSW5(100),PCNSW5(100)                              *293
          DIMENSION PSNSW6(100), PCNSW6(100), PSNSW7(100), PCNSW7(100),    *294
        1  PSNS6(100), PCNS6(100),PSNS7(100), PCNS7(100)                   *294
          BOOLEAN SW,PSNSW,PCNSW,PSNSW0,PCNSW0,PSNSW4,PCNSW4,PSNSW5,       *295
        1  PCNSW5                                                          *295
          BOOLEAN PSNSW6, PCNSW6, PSNSW7, PCNSW7                           *296
          END OF PROGRAM                                                   *297

THE FOLLOWING NAMES HAVE OCCURRED ONLY ONCE IN THIS PROGRAM.
COMPILATION WILL CONTINUE.

     START1        *010
```

GENERALIZED HAVELOCK P-FUNCTION

Generalized Havelock P-function in Equations (14a) and (14b) are called PCOS.(KK,FDL,FF, MUMU) and PSIN.(KK,FDL,FF, MUMU) respectively, where

KK: integer order of $\overline{P}$-function i.e., 2n-1 or 2n.

FDL: argument y

FF: argument x

MUMU: a number corresponding to an allowable maximum error in numerical integration.

As θ is increased toward $\frac{\pi}{2}$, the oscillating frequency of the integrand increases but the amplitude decreases making the contribution to the integrand smaller. The method of integration is by Gauss-Legendre 9-point quadrature formulae applied in the intervals between two consecutive zero values of the integrand. Zero values of integrands are found from

$$\begin{matrix}\cos\\ \sin\end{matrix}(x \sec \theta) = 0 .$$

Whenever the area under a half-cycle is smaller than a specified number (MUMU), the integration is automatically terminated. The error is always smaller than that due to the oscillation.

MAD (24 SEP 1964 VERSION) PROGRAM LISTING ... ...

```
            EXTERNAL FUNCTION (KK,FDL,FF,MUMU)
             CALCULATION OF GENERALIZED HAVELOCK P FUNCTION BY NUMERICAL
             INTEGRATION
            ENTRY TO PSIN.
            NAPE=2.718281828
              LOWER=0.
              SUM=0.
              A=FF/3.1415927
              MA=A+0.9999999
            THROUGH LOOP A2, FOR M=MA,1,M.G.600
              UPPER=ARCCOS.(A/M)
              AREA=0.
            THROUGH LOOP A1, FOR K=1,1,K.G.9
              X=((1.+HG(10-K))*UPPER+(1.-HG(10-K))*LOWER)*.5
            COSINE=COS.(X)
            SINE=SIN.(X)
            TANGT=SINE/COSINE
            C=FDL*TANGT*TANGT
                    WHENEVER C.G.30.
                    AREA=0.
                    TRANSFER TO E1
                    END OF CONDITIONAL
            EPC=NAPE .P. C
            COSPR=COSINE .P. KK
            WHENEVER KK.E.0, COSPR=1.0
            INTGRD=COSPR/EPC*SIN.(FF/COSINE)
LOOP A1     AREA=AREA+IG(K)*INTGRD*(UPPER-LOWER)*0.5
E1          LIMIT=AREA
            WHENEVER .ABS. (LIMIT) .LE. MUMU, TRANSFER TO LOOP A3
              LOWER=UPPER
LOOP A2       SUM=SUM+AREA
LOOP A3       SUM=SUM*(-1.).P.(KK/2 )
            FUNCTION RETURN SUM
            ENTRY TO PCOS.
            NAPE=2.718281828
              LOWER=0.
              SUM=0.
              A=FF/3.1415927
              MA=A+0.4999999
            THROUGH LOOP A5, FOR M=MA,1,M.G.600
              UPPER=ARCCOS.(A*2./(2.*M+1.0))
              AREA=0.
            THROUGH LOOP A4, FOR K=1,1,K.G.9
              X=((1.+HG(10-K))*UPPER+(1.-HG(10-K))*LOWER)*.5
            COSINE=COS.(X)
            SINE=SIN.(X)
            TANGT=SINE/COSINE
            C=FDL*TANGT*TANGT
                    WHENEVER C.G.30.
                    AREA=0.
                    TRANSFER TO E4
                    END OF CONDITIONAL
            EPC=NAPE .P. C
```

```
            COSPR=COSINE .P. KK                                             *052
            WHENEVER KK.E.0, COSPR=1.0                                      *053
            INTGRD=COSPR/EPC*COS.(FF/COSINE)                                *054
              INTGRD=COSPR*COS.(FF/COS.(X))                                 *055
LOOP A4     AREA=AREA+IG(K)*INTGRD*(UPPER-LOWER)*0.5                        *056
  E4        LIMIT=AREA                                                      *057
            WHENEVER .ABS. (LIMIT) .LE. MUMU, TRANSFER TO LOOP A6           *058
              LOWER=UPPER                                                   *059
LOOP A5       SUM=SUM+AREA                                                  *060
LOOP A6       SUM=SUM*(-1.).P.((KK+1)/2 )                                   *061
            FUNCTION RETURN SUN                                             *062
            INTEGER KK,MA,M,K                                               *063
            DIMENSION IG(10), HG(10)                                        *064
            VECTOR VALUES HG(1) = .96816024,.83603111,.61337143,.32425342    *065
          1                      ,.0,      -.32425342,-.61337143,           *065
          2                      -.83603111,-.96816024                      *065
            VECTOR VALUES IG(1)=.08127439,  .18064816,  .26061070,          *066
          1                     .31234708,  .33023936,  .31234708,          *066
          2                     .26061070,  .18064816,  .08127439           *066
            END OF FUNCTION                                                 *067
```

HULL FUNCTION

The program below describes the calculation of the coefficients of the polynomials in the variables ξ and ζ given by Equations (23) and (24). The form of these expressions are such that a flow diagram is superfluous.

```
$COMPILE MAD, EXECUTE,DUMP,PRINT OBJECT,PUNCH OBJECT             HULL                 01300З   12/09/65       5 01 48.3 PM

MAD (09 AUG 1965 VERSION) PROGRAM LISTING ... ... ...


             THE HULL FUNCTION POLYNOMIAL
                       MICHELSEN AND KIM

             EXTERNAL FUNCTION     FACT.(N,P)
              INTERNAL FUNCTION F.(KKK)=FACT.(1,KKK)                                    *C01
             INTERNAL FUNCTION FAT.(II,JJ)=F.(II)/(F.(JJ)*F.(II-JJ))                    *C02

        PRINT COMMENT $1$                                                               *C03
        PRINT COMMENT $8                   HULL FUNCTICN$                               *C04
        PRINT COMMENT $8                                                                *C05
      1       FINN C. MICHELSEN AND HUN CHOL KIM$                                       *C05
        PRINT COMMENT $1$                                                               *C06
        PRINT COMMENT $4  GENERAL  INPUT$                                               *C07
START   READ AND PRINT DATA, MM,NN ,SW,SW(1)                                            *C08
        EXECUTE FTRAP.                                                                  *C09
        FV=NN+1                                                                         *C10
                AM=2*MM+1                                                               *C11
                BM=2*NN+1                                                               *C12
                AM1=MM                                                                  *C13
                BM1=NN                                                                  *C14
        DIM(1)=NN+2                                                                     *C15
        DIM(2)=NN +1                                                                    *C16
        DIM1(1)=AM+2                                                                    *C17
        DIM1(2)=AM+1                                                                    *C18
                                                                                        *C19
                                                                                        *C20
                                                                                        *C21          C1
                                                                                        *C22          C2
                                                                                        *C23          C2
        PRINT COMMENT $1$                                                               *C24          C2
        PRINT COMMENT $8          POLYNOMIAL COEFFICIENTS ARE$                          *C25          C2
                                                                                        *C26          C2
         PRINT RESULTS C(0,0)...C(MM,NN)                                                *C27          C2
        THROUGH LOOP 1, FOR A=C,1,A.G.AM                                                *C28          C2
        THROUGH LOOP 1, FOR B=C,1, B.G. BM                                              *C29          C3
        AAB2(A,B)=0.                                                                    *C30          C4
LCCP 1  AAB(A,B)=0.                                                                     *C31          C4
        THROUGH LOOP 2, FOR I=0,1, I.G. AM1                                             *C32          C2
        THROUGH LOOP 2, FOR J=C,1, J.G. BM1                                             *C33          C3
        THROUGH LOOP 2, FOR M=0,1, M.G. AM1                                             *C34          C4
        THROUGH LOOP 2, FOR N=C,1, N.G. BM1                                             *C35          C5
                WHENEVER C(I,J).E.0..OR.C(M,N).E.0., TRANSFER TOENTR2                   *C36          C6
        THROUGH  OOP 2, FOR L=C,1, L.G. (I+1)                                           *C37          C6
        THROUGH  OOP 2, FOR K=C,1, K.G. M                                               *C38          C7
                          TEMP=        C(I,J)*F.(I+1)*(-1.).P.L*                        *C39          C8
      1 .5.P.(I+1-L)/(F.(L)*F.(I+1-L))*C(M,N)*F.(M)*F.(N)/(FACT.(I+1,                   *C39
      2  M+1)*FACT.(J+1,N+1))*FACT.(I+1,K)*.5.P.K/F.(K)                                 *C39
                WHENEVER  SW(1)                                                         *C40          C8
        PRINT FORMAT CHK, F.(I+1), (-1.).P.L, .5.P.(I+1-L), F.(L),                      *C41     C1   C8
      1  F.(I+1-L), F.(M), F.(N), FACT.(I+1,M+1), FACT.(J+1, N+1),                      *C41
      2  FACT.(I+1,K), .5.P.K, F.(K)                                                    *C41
```

```
              VECTOR VALUES CHK=$S2,12E9.3*$                          *C42      C1    C8
              PRINT RESULTS AAB(L-K+M,N+J+1), C(I,J), C(M,N)         *C43      C1    C8
                        END CF CONDITIONAL                            *C44      01    C8
OOP 2         AAB(L-K+M,N+J+1)=AAB(L-K+M,N+J+1)+TEMP                  *C45            C8
 ENTR 2       CONTINUE                                                *C46            C6
 LOOP 2       CONTINUE                                                *C47            C6
              THROUGH LOOP 3, FOR I=C,1,I.G. AM1                      *C48            C2
              THROUGH LOOP 3, FOR J=0,1,J.G. BM1                      *C49            C3
              THROUGH LOOP 3, FOR M=C,1, M.G. AM1                     *C5C            C4
              THROUGH LOOP 3, FOR N=C,1, N.G. BM1                     *C51            C5
                        WHENEVER C(I,J).E.0..OR.C(M,N).E.C.,TRANSFERTCENTR3   *C52    C6
              THROUGH  OOP 3, FOR K=C,1, K.G. M                       *C53            C6
              THROUGH  OOP 3, FOR P=C,1, P.G. K                       *C54            07
                              TEMP=          (-1.).P.I*.5.P.(I+1)*C(I,J)*   *C55      C8
            1 C(M,N)*F.(M)*F.(N)/(FACT.(I+1,M+1)*FACT.(J+1,N+1))*(-1.).P.P*  *C55
            2  FACT.(I+1,K)*.5.P.P/(F.(P)*F.(K-P))                    *C55
GOP3          .AAB(M-P,N+J+1)=AAB(M-P,N+J+1)+TEMP                     *C56            C8
 ENTR 3       CONTINUE                                                *C57            C6
 LOOP 3       CONTINUE                                                *C58            C6

              THROUGH LOOP 4, FOR I=0,1,I.G.AM1                       *C59            C2
              THROUGH LOOP 4, FOR J=C,1,J.G.BM1                       *C6C            C3
              THROUGH LOOP 4, FOR M=C,1,M.G. AM1                      *C61            C4
              THROUGH LOOP 4, FOR N=C,1,N.G. BM1                      *C62            C5
                        WHENEVER C(I,J).E.0..OR.C(M,N).E.C.,TRANSFER TCENTR4  *063    C6
              THROUGH  OOP 4, FOR K=0,1,K.G. M                        *C64            C6
              THROUGH  OOP 4, FOR P=C,1,P.G.N                         *C65            07
              THROUGH  OOP 4, FOR L=C,1,L.G.(I+K+1)                   *C66            C8
                              TEMP=           -C(I,J)*C(M,N)*(-1.).P.  *C67            C9
            1 (I+K+P+L)*FA T.(N,P)*FA T.(M,K)*FA T.(I+K+1,L)*.5.P.L/  *067
            2 ((I+K+1)*(J+P+1))                                       *067
                        WHENEVER SW(1)                                *C68            C9
              PRINT RESULTS -C(I,J),C(M,N),(-1.).P.(I+K+P+L),FA T.(N,P),FA  *069  01    C9
            1 T.(M,K),FA T.(I+K+1,L),.5.P.L/((I+K+1)*(J+P+1))        *069
              END OF CONDITIONAL                                      *C7C      01    C9
COP 4         AAB2(M+I-L+1,N-P)=AAB2(M+I-L+1,N-P)+TEMP                *C71            09
 ENTR 4       CONTINUE                                                *072            C6
 LOOP 4       CONTINUE                                                *073            C6
              THROUGH LCOP 5, FOR I=C,1,I.G.AM1                       *C74            C2
              THROUGH LOOP 5, FOR J=0,1,J.G.BM1                       *C75            03
              THROUGH LOOP 5, FOR M=C,1,M.G. AM1                      *076            C4
              THROUGH LOOP 5, FOR N=C,1,N.G. BM1                      *C77            C5
                        WHENEVER C(I,J).E.0..OR.C(M,N).E.0.,TRANSFERTCENTR5   *C78    C6
              THROUGH  OOP 5, FOR K=C,1,K.G. M                        *C79            C6
              THROUGH  GOP 5, FOR P=C,1,P.G.N                         *08C            07
              THROUGH  OOP 5, FOR L=C,1,L.G.(I+K+1)                   *081            C8
              THROUGH  OOP 5, FOR Q=C,1,Q.G.(J+P+1)                   *C82            C9
                              TEMP=                    C(I,J)*C(M,N)*  *C83            1C
            1 (-1.).P.(I+K+P+Q+L)*FA T.(N,P)*FA T.(M,K)*FA T.(I+K+1,L)*  *C83
            2  FA T.(J+P+1,Q)*.5.P.L/((I+K+1)*(J+P+1))               *083
COP 5         AAB2(M+I+1-L,N+J+1-Q)=AAB2(M+I+1-L,N+J+1-Q)+TEMP        *C84            10
 ENTR 5       CONTINUE                                                *C85            C6
 LCOP 5       CONTINUE                                                *C86            C6
              THROUGH LOOP 6, FOR I=C,1,I.G.AM1                       *C87            C2
              THROUGH LOOP 6, FOR J=C,1,J.G.BM1                       *C88            C3
              THROUGH LOOP 6, FOR M=C,1,M.G. AM1                      *C89            C4
              THROUGH LOOP 6, FOR N=0,1,N.G. BM1                      *C9C            C5
                        WHENEVER C(I,J).E.0..OR.C(M,N).E.C.,TRANSFERTCENTR6   *C91    C6
              THROUGH  COP 6, FOR K=C,1,K.G. M                        *C92            C6
```

```
          THROUGH   OOP 6, FOR P=0,1,P.G.N                                   *C93        C7
                                  TEMP=      C(I,J)*C(M,N)*(-1.).P.(I+K+P)*  *C94        C8
        1 FAT.(N,P)*FAT.(M,K)*.5.P.(I+K+1)/((I+K+1)*(J+P+1))                 *C94
COP 6     AAB2(M-K,N-P)=AAB2(M-K,N-P)+TEMP                                   *C95        C8
 ENTR 6   CONTINUE                                                           *C96        C6
 LCOP 6   CONTINUE                                                           *C97        C6
          THROUGH LOOP 7, FOR I=C,1,I.G.AM1                                  *C98        C2
          THROUGH LOOP 7, FOR J=C,1,J.G.BM1                                  *C99        03
          THROUGH LOOP 7, FOR M=0,1,M.G. AM1                                 *1CC        C4
          THROUGH LOOP 7, FOR N=C,1,N.G. BM1                                 *101        C5
                  WHENEVER C(I,J).E.C..OR.C(M,N).E.O.,TRANSFERTCENTR7        *1C2        C6
          THROUGH  OOP 7, FOR K=C,1,K.G.M                                    *1C3        C6
          THROUGH  OOP 7, FOR P=0,1,P.G.N                                    *104        07
          THROUGH  OOP 7, FOR Q=C,1,Q.G.(J+P+1)                              *105        C8
                                  TEMP=          C(I,J)*C(M,N)*(-1.).P.      *1C6        C9
        1 (I+K+P+1+Q)*FA T.(N,P)*FA T.(M,K)*FA T.(J+P+1,C)*.5.P.             *1C6
        2 (I+K+1)/((I+K+1)*(J+P+1))                                          *1C6
CCP7      AAB2(M-K,N+J-Q+1)=AAB2(M-K,N+J-Q+1)+TEMP                           *1C7        C9
 ENTR 7   CONTINUE                                                           *1C8        C6
 LCOP 7   CONTINUE                                                           *1C9        C6
          PRINT COMMENT $1$                                                  *11C        C2
          PRINT COMMENT$0        RESULT FOR REGION 1$                        *111        C2
                                                                             *112        C2
                                                                             *113        C2
                                                                             *114        C3
                                                                             *115        C4
          WHENEVER SW .AND. AAB(A,B).NE.O.                                   *116        C4
          PUNCH FORMAT OUT,A,B,AAB(A,B),LL,K                                 *117   C1   C4
          END OF CONDITIONAL                                                 *118   C1   C4
          PRINT RESULTS LL,K,AAB(A,B)                                        *119        C4
                                                                             *12C        C4
          PRINT COMMENT$1$                                                   *121        C2
          PRINT COMMENT $0    RESULTS FOR REGICN 2$                          *122        C2
                                                                             *123        C2
                                                                             *124        C3
          WHENEVER SW  .AND. AAB2(A,B).NE.C.                                 *125        C4
          PUNCH FORMAT OUT1,A,B,AAB2(A,B),LL,K                               *126        C4
          END OF CONDITIONAL                                                 *127   C1   C4
          PRINT RESULTS LL,K,AAB2(A,B)                                       *128   C1   C4
                                                                             *129        C4
                                                                             *13C        C4
          TRANSFER TO START                                                  *131        C2
          BOOLEAN SW                                                         *132
          INTEGER MM,NN,FV,K,I,J,M,N,P,Q,L,A,B,AM,BM,AM1,BM1,KKK             *133
          INTEGER II,JJ,LL,PP,QQ                                             *134
          FORMAT VARIABLE FV                                                 *135
          VECTOR VALUES OUT=$S10,4HAAB(,I2,1H,,I2,2H)=,E25.8,2I5*$           *136
          VECTOR VALUES OUT1=$S1C,5HAAB1(,I2,1H,,I2,2H)=E24.8,2I5*$          *137
          VECTOR VALUES DIM=2,0,10                                           *138
          VECTOR VALUES DIM1=2,1,10                                          *139
          VECTOR VALUES SW=OB,OB,OB                                          *14C
          DIMENSION C(10C,DIM),AAB(5CO,DIM1), AAB2(5CC, DIM1)                *141
          DIMENSION SW(10)                                                   *142
          END OF PROGRAM                                                     *143
                                                                             *144
```

DISTRIBUTION LIST

1 Commanding Officer
Office of Naval Research
Branch Office
230 North Michigan Avenue
Chicago 1, Illinois

1 Commanding Officer
Office of Naval Research
Branch Office
207 West 24th Street
New York, New York, 10011

1 Commanding Officer
Office of Naval Research
Branch Officer
1030 East Green Street
Pasadena 1, California

1 Commanding Officer
Office of Naval Research
Branch Office
495 Summer Street
Boston 10, Massachusetts

1 Commanding Officer
Office of Naval Research
Branch Office
1000 Geary Street
San Francisco 9, California

10 Commanding Officer
Office of Naval Research
Branch Office
Navy No. 100, Box 39
Fleet Post Office
New York, New York

1 Director, Hydraulic Laboratory
National Bureau of Standards
Washington 25, D.C.

1 Commandant (OAO)
U. S. Coast Guard
1300 E Street, N. W.
Washington 25,D. C.

2 Maritime Administration
U. S. Department of Commerce
Washington 25, D. C.
Attention: Mr. E. Scott Dillon

1 Commander
U. S. Naval Proving Ground
Dahlgren, Virginia

1 Commander
Mare Island Naval Shipyard
Vallejo, California

1 Commander
Norfolk Naval Shipyard
Portsmough, Virginia

1 Commander
Puget Sound Naval Shipyard
Bremerton, Washington

1 Commander
Pearl Harbor Naval Shipyard
Navy No. 128, Fleet Post Office
San Francisco, California

1 Commander
San Francisco Naval Shipyard
San Francisco, California

1 Commander
Long Beach Naval Shipyard
Long Beach, California

1 Commander
Philadelphia Naval Shipyard
Philadelphia, Pennsylvania

DISTRIBUTION LIST (Continued)

1 Commander
Portsmouth Naval Shipyard
Portsmouth, New Hampshire

1 Commander
Boston Naval Shipyard
Boston 29, Massachusetts

1 Commander
Charleston Naval Shipyard
Charleston, North Carolina

1 Commanding Officer
U. S. Naval Repair Facility
U. S. Naval Station
San Diego 36, California

75 Commanding Officer and Director
David Taylor Model Basin
Washington 7, D. C.

10 Chief, Bureau of Ships
Department of the Navy
Washington 25, D. C.
Attn: Codes, 341B, 345
420, 421, 210L (3)

1 Chief of Naval Research
Code 438
Washington, D. C.

2 Chief, Bureau of Yards and Docks
Code D-400
Department of the Navy
Washington 25, D. C.

1 Commanding Officer and Director
U. S. Naval Civil Engineering Laboratory
Port Hueneme, California

2 Chief, Bureau of Naval Weapons
Department of the Navy
Washington 25, D. C.

1 Commander
U. S. Navy Ordnance Laboratory
Silver Spring 19, Maryland

1 Director
U. S. Naval Engineering Experiment Station
Annapolis, Maryland

1 Hydrographer
U. S. Navy Hydrographic Office
Washington 25, D. C.

20 Commander
Defense Documentation Center
Cameron Station
Alexandria, Virginia

1 Director
U. S. Naval Research Laboratory
Washington 25, D. C.

1 U. S. Naval Underwater Sound Laboratory
Fort Trumbull
New London, Connecticut
Attention: Dr. Hartley Pond

1 Superintendent
U. S. Naval Academy
Annapolis, Maryland

1 General Dynamics Corporation
Electric Boat Division
Groton, Connecticut

2 Hydronautics, Inc
Pindell School Road
Howard County, Laurel, Maryland

2 Technical Research Group, Inc.
Route 110
Melville, New York
Attention: Dr. J. Kotik

DISTRIBUTION LIST (Continued)

1 Commanding Officer
U. S. Army Transportation Command
Fort Eustis, Virginia
Attention: Research Reference Center

2 Oceanics, Inc.
Technical Industrial Park
Plainview, L. I., New York
Attention: Dr. Paul Kaplan

1 Director
Institute for Fluid Dynamics and
Applied Mathematics
University of Maryland
College Park, Maryland

1 Director
Scripps Institution of Oceanography
University of California
La Jolla, California

1 Director
Ordnance Research Laboratory
Pennsylvania State University
University Park, Pennsylvania

2 Webb Institute of Naval Architecture
Crescent Beach Road
Glen Cove, L. I., New York
Attention: Professor L. Ward

2 Department of Naval Architecture and
Marine Engineering
Massachusetts Institute of Technology
Cambridge 39, Massachusetts
Attention: Professor M. Abkowitz

1 Institute of Mathematical Sciences
New York University
25 Waverly Place
New York, New York, 10003
Attention: Professor J. Stoker

1 Department of Mathematics
College of Engineering and
Sciences
Carnegie Institute of Technology
Schenley Park
Pittsburgh 13, Pennsylvania
Attention: Professor Richard
MacCamy

1 Department of Engineering Sciences
University of Notre Dame
Notre Dame, Indiana
Attention: Dr. A. G. Strandhagen

1 Grumman Aircraft Engineering
Corporation
Bethpage, L. I., New York
Attention: Marine Division

1 Professor B. V. Korvin-Kroukovsky
East Randolph
Vermont

1 Director
National Aeronautics and Space
Administration
1512 H Street, N. W.
Washington 25, D. C.

1 Superintendent
U. S. Naval Postgraduate School
Monterey, California

2 University of California
Department of Naval Architecture
Berkeley, California
Attention: Professor J. V.
Wehausen

1 Colorado State University
Engineering Research Division
Fort Collins, Colorado

2 Director
Davidson Laboratory
Stevens Institute of Technology
Hoboken, New Jersey
Attention: Dr. J. P. Breslin

2 Hydrodynamics Laboratory
California Institute of Technology
Pasadena 4, California
Attention: Dr. T. Y. Wu

1 Department of Civil Engineering
University of Illinois
Urbana, Illinois

1 St. Anthony Falls Hydraulic Laboratory
University of Minnesota
Minneapolis 14, Minnesota

2 Iowa Institute of Hydraulic Research
Iowa City, Iowa
Attention: Dr. Louis Landweber

2 Department of Naval Architecture and
Marine Engineering
University of Michigan
Ann Arbor, Michigan
Attention: Professor F. C. Michelsen

1 Capt. S. W. W. Shor, USN
3231 Worthington Street, N. W.
Washington, D. C., 20015