DOT HS-803 306

STEERING CONTROLLABILITY CHARACTERISTICS

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Contract No. DOT HS-6-01409
Contract Amt. $107,775

AUGUST 1977

FINAL REPORT

Prepared For
U.S. DEPARTMENT OF TRANSPORTATION
National Highway Traffic Safety Administration
Washington, D.C. 20590

Technical Report Documentation Page

1. Report No. DOT HS-803 306	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle STEERING CONTROLLABILITY CHARACTERISTICS		5. Report Date 8/31/77
		6. Performing Organization Code
7. Author(s) P.S. Fancher, C.B. Winkler, C. C. MacAdam L. Segel, C. Mallikarjunarao		8. Performing Organization Report No. UM-HSRI-77-50
9. Performing Organization Name and Address Highway Safety Research Institute The University of Michigan Huron Parkway & Baxter Road Ann Arbor, Michigan 48109		10. Work Unit No. (TRAIS)
		11. Contract or Grant No. DOT-HS-6-01409
12. Sponsoring Agency Name and Address National Highway Traffic Safety Administration U. S. Department of Transportation Washington, D.C. 20590		13. Type of Report and Period Covered Final 8/1/76 - 9/1/77
		14. Sponsoring Agency Code
15. Supplementary Notes		

16. Abstract

The relationship of the steering controllability characteristics of domestically-produced subcompact, compact, and intermediate size automobiles to specifications on yaw-rate gain and response time as developed in NHTSA research programs is examined through (1) an analytical survey of the yaw response of 1977 vehicles and (2) open- and closed-loop testing of 3 selected vehicles in modified and unmodified conditions. It is found that, with the exception of some vehicle models equipped with manual steering, the directional performance characteristics of currently produced subcompact, compact, and intermediate size cars lie within the "optimum" region that has been defined through NHTSA research. It is recommended that further effort in defining safety-related handling requirements is needed before implementing vehicle performance specifications.

17. Key Words Steering Modification, Driver/Vehicle Control, Yaw-Rate Gain, Yaw-Rate Response Time, Closed-Loop Testing		18. Distribution Statement Document is available to the U.S. public through the National Technical Information Service, Springfield Virginia 22161	
19. Security Classif. (of this report) NONE	20. Security Classif. (of this page) NONE	21. No. of Pages 198	22. Price

Form DOT F 1700.7 (8-72) Reproduction of completed page authorized

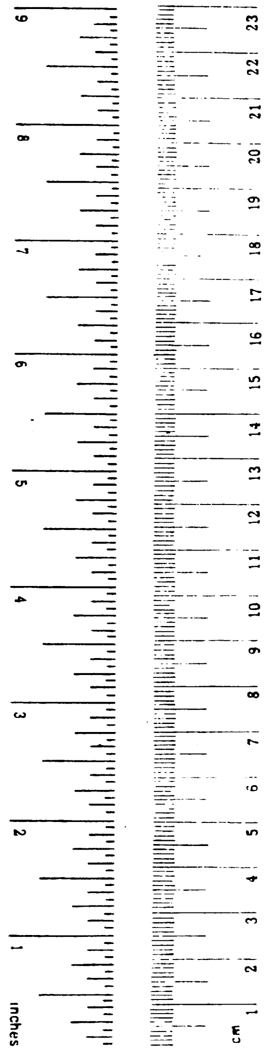
☆ U. S. GOVERNMENT PRINTING OFFICE : 1973 725-504/328

METRIC CONVERSION FACTORS

Approximate Conversions to Metric Measures

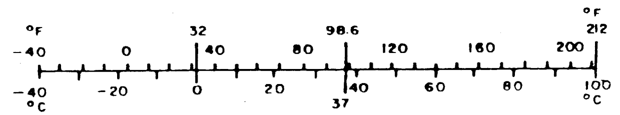
Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
		VOLUME		
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
		TEMPERATURE (exact)		
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

*1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price $2.25, SD Catalog No. C13.10:286.



Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
		AREA		
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
		MASS (weight)		
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
		VOLUME		
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
		TEMPERATURE (exact)		
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



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Summary Report

STEERING CONTROLLABILITY CHARACTERISTICS

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PREFACE

The Final Report for this project is divided into two parts, a Summary Report and a Technical Report. This volume contains the Summary Report which is a condensation of the Technical Report. The conclusions and recommendations developed in the Technical Report are repeated herein.

The ability to apply the latest research findings in this project would not have been possible without the assistance and cooperation of Messrs. Weir and Klein of Systems Technology, Inc. The Contract Technical Manager, Mr. Francis DiLorenzo, was instrumental in arranging this cooperation.

As part of this research program, visits were made to the General Motors Corporation, the Chrysler Corporation, and the Ford Motor Company to consult with industry experts on vehicle modification. The help received from industry personnel is gratefully acknowledged.

1.0 INTRODUCTION

This report presents the conclusions and recommendations obtained in a project entitled "Steering Controllability Characteristics" performed by the Highway Safety Research Institute (HSRI) on behalf of the National Highway Traffic Safety Administration (NHTSA). The following quotation from NHTSA's Request for Proposal indicates the basic intention of this study:

> "It is essential in the progressing studies of driver-vehicle controllability to have "bench-marks" established that firmly relate the research findings to the practical problems of implementing those findings into production vehicles. If such bench-marks are not established, it is conceivable that a serious gap could develop between practical design and that design necessary for compliance to research findings."

Accordingly, this study has addressed practical considerations connected with the application of research findings (obtained in NHTSA programs [1, 2]) to the steering controllability of domestically produced subcompact, compact, and intermediate size automobiles. The methodology used in the investigation described herein consisted of (1) an analytical study of the influence of changes in vehicle parameters on the yaw rate response to steering-wheel control inputs, (2) a survey of the response characteristics of motor cars to select specific subcompact, compact, and intermediate size vehicles for experimental work, and (3) open- and closed-loop tests of modified and unmodified versions of a 1977 Ford Pinto (subcompact), a 1977 Buick Skylark (compact), and a 1977 Plymouth Fury (intermediate).

The handling performance specifications used in this program are stated in terms of the yaw rate exhibited by a given vehicle in response to a steering-wheel displacement input. In particular, an "optimum" region of a "performance space" defined by (1) steady-state gain and (2) a measure of transient response-time was developed in a concurrent (and separate) project [2]. These performance specifications are discussed in the following section (2.0) of this report.

The findings of the analytical study and the survey of vehicle performance are summarized in Section 3.0 The results obtained in the vehicle and driver-vehicle tests are described in Section 4.0. Conclusions and recommendations are presented in Section 5.0.

At the outset, it should be emphasized that the study reported herein indicates that a somewhat unanticipated "bench-mark" has been established. Specifically, the findings show that, with the exception of some vehicle models equipped with manual steering systems, the directional performance characteristics of currently produced subcompact, compact, and intermediate size automobiles lie within the "optimum" region that has been defined by research performed under the auspices of NHTSA. Accordingly, a principal conclusion of this study is that a serious gap does not exist between current design and that design necessary for compliance with recent research findings.

Although the prior research findings (applied in this study) do contribute to the development of an understanding of the relationships between directional performance measures and driver control, this study suggests that there are factors, in addition to specifications on yaw rate response, which should be evaluated to assess the handling qualities of a particular vehicle. For example, steering-wheel torque, roll response to lateral acceleration, lateral acceleration response time, yaw response over a range of forward velocities, and emergency maneuvering performance are some of the factors which appear to be important. Section 5.0 recommends that these factors be addressed in further studies.

2.0 DESCRIPTION OF THE VEHICLE HANDLING SPECIFICATIONS DEVELOPED IN NHTSA RESEARCH STUDIES

Passenger car handling and the control of road vehicles have been the subject of many research studies and technical papers over the last 25 years. Reviews and evaluations of a good sample of pertinent efforts are given in References [3, 4, 5]. In this section, the open-loop vehicle response measures developed in two NHTSA-sponsored studies [1, 2] will be described because those vehicle specifications have been used in this program.

A tentative optimum range of vehicle dynamics for directional control was presented in 1975 in the final report for a study entitled "Automobile Controllability-Driver/Vehicle Response for Steering Control," Contract Number DOT-HS-359-3-762 [1]. In that project vehicles with differing dynamic properties were driven by 16 typical drivers and an expert driver in regulation tasks and transient maneuvers. Much of the experimental work was done with a "laboratory" vehicle which had a servomechanism installed to provide front wheel motions in addition to those steer motions commanded by the driver through the use of the steering wheel. By using transduced vehicle response variables as inputs to the servomechanism, a wide variety of vehicle dynamics characteristics were obtained. In addition, the servomechanism was used to apply known "disturbances" to the front wheels, thereby providing the inputs for sophisticated studies of driver regulation of vehicle path.

The laboratory vehicle used power steering (which helped to isolate the driver from the torques applied by the servomechanism). The laboratory vehicle was "stiffened" in roll to remove the possibly confusing issue of roll dynamics from the initial research efforts and to allow the use of a simple two-degree-of-freedom model in analyzing vehicle test results.

The form of the transfer function used in [1] to describe the yaw rate response to steering-wheel inputs is as follows:

$$\frac{r}{\delta_{sw}} = \frac{(r/\delta_{sw}|_{ss})\ (T_r s + 1)\ (\omega_n^2)}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad (1)$$

where

r = yaw rate

δ_{sw} = steering-wheel angle

$r/\delta_{sw}|_{ss}$ = steady-state gain, yaw velocity to steering-wheel angle

T_r = the yaw velocity numerator time constant

s = Laplace transform variable

ω_n = natural frequency

ζ = damping ratio

Clearly, the transfer function approach implies linear analysis and a linear system.

Nevertheless, in [1], some results for transient maneuvers substantially exceeding the linear range of vehicle performance were studied. In those cases, driver opinion, lane excedances, and steering activity were correlated with vehicle response parameters as defined in the linear analysis.

The initial effort [1] was followed by a subsequent study [2] entitled "Evaluation and Correlation of Driver/Vehicle Data," Contract Number DOT-HS-5-01200. The results from the initial program, along with data from other studies [6, 7], were evaluated. The vehicle rating data for 16 typical drivers from [1] were analyzed to define an optimum region (see Figure 1).

The resulting optimum region for typical drivers is specified in a performance space consisting of steady-state yaw rate gain, r/δ_{sw}, and an effective time constant, T_e. The quantity $1/T_e$ corresponds to the frequency at which there is a 45° phase lag between the steering-wheel angle input and the yaw rate output. The

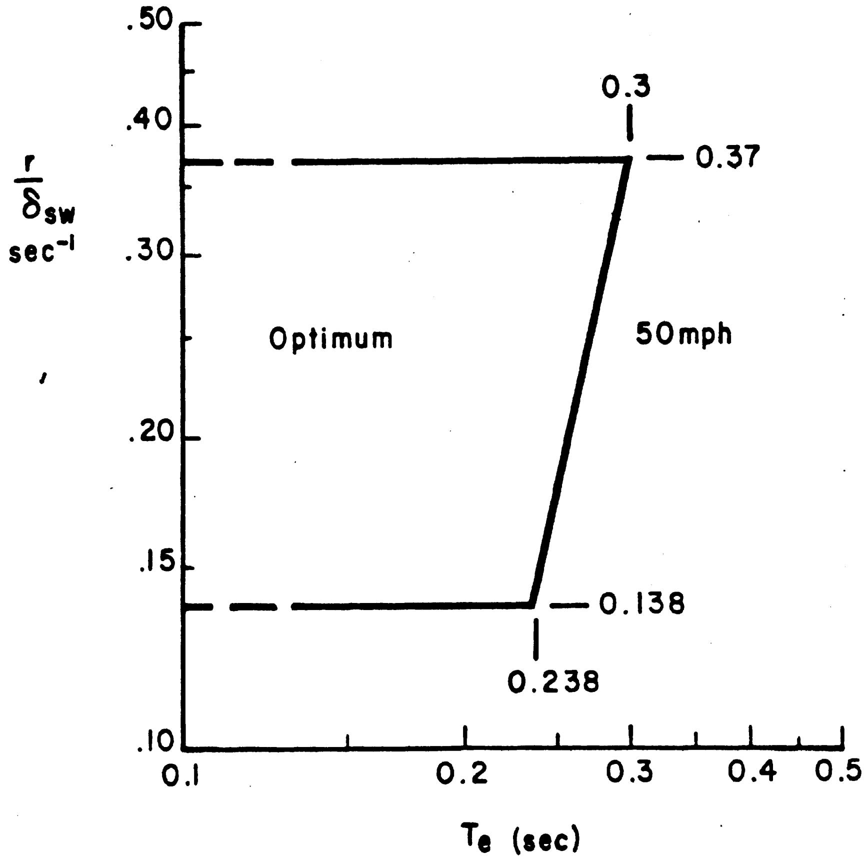

Figure 1. The Specified Performance Space (SPS) typical driver boundaries - 50 mph.

effective time constant, T_e, is used instead of T_r (T_r was used in the original project) because T_e is a more representative measure of vehicle phase lag than T_r for vehicle configurations in which $1/T_r$ is substantially less than the natural frequency, ω_n. (See Equation (1).)

In summary, the specified performance space (SPS), used as a goal for vehicle modification in this project, consists of the optimum region shown in Figure 1 and the additional requirements (included in [1]) that (1) the damping ratio, ζ, exceed 0.5 and (2) the natural frequency, ω_n, exceed 3.0 rad/sec. The notation $r/\delta_{sw}\big|_{50}$, $T_e\big|_{50}$, $\zeta\big|_{50}$, and $\omega_n\big|_{50}$ is used herein to emphasize that the specified performance requirements are all evaluated at 50 mph.

3.0 ANALYSIS OF THE INFLUENCE OF VEHICLE PARAMETERS ON PERFORMANCE RELATIVE TO THE SPECIFIED PERFORMANCE SPACE

The Technical Report presents a detailed discussion of the techniques used in obtaining the analytical results summarized hereinafter. The purpose of this section of the Summary Report is to provide a concise review of the results of an analytical survey of vehicle characteristics performed in this project.

3.1 Factors Influencing Steady-State Yaw Rate Gain

The well-known basic equation describing the steady-state steering of an automobile while turning at less than approximately 0.3 g is as follows:

$$\frac{\delta_{SW}}{N_G} = \frac{\ell 57.3}{R} + K\frac{ur}{g57.3} \qquad (2)$$

where

δ_{SW} = steering-wheel angle, degrees

N_G = total steering ratio

ℓ = wheelbase, feet

R = radius of the turn, feet

K = understeer factor, degrees/g (understeer/oversteer gradient)

u = forward velocity, ft/sec

r = yaw rate, deg/sec

g = gravitational constant, 32.2 ft/sec^2

For uniform velocity on a circular path, the radius, velocity, and yaw rate are related by the following equation:

$$\frac{rR}{57.3} = u \qquad (3)$$

In addition, the lateral acceleration, A_y, is given by

$$A_y = \left(\frac{ur}{57.3}\right) = u^2(1/R) \qquad (4)$$

where 1/R is the path curvature. The steady-state gains (yaw rate, path curvature, or lateral acceleration) can be obtained by combining Equations (2), (3), and (4). Specifically, the expression for yaw rate gain, G, is

$$G = \left.\frac{r}{\delta_{sw}}\right|_{ss} = \left(\frac{1}{N_G}\right)\left(\frac{(u/\ell)}{1 + \frac{u^2K}{g\ell 57.3}}\right), \ \sec^{-1} \qquad (5)$$

As indicated by Equation (5), there are two factors, N_G and K, which can be varied independently to achieve a required yaw rate gain for a vehicle of given wheelbase at a speed of 73.3 ft/sec (50 mph). Clearly, it is obvious that the yaw rate gain can be changed directly by altering steering ratio. Even though it is patently clear, it should be emphasized that changing steering ratio has the advantages that it does not change (1) the understeer/oversteer gradient, (2) steady-state roll response in degrees per unit lateral acceleration, or (3) the form of the transient response.

(Depending upon the type of steering system employed on a particular vehicle, the changing of steering ratio may cause a significant change in the torque the driver feels at the steering wheel. At present, boundaries on steering-wheel torque have not been specified.)

The understeer/oversteer gradient, K, in Equation (5) can also be varied to change yaw rate gain. However, K is dependent upon many vehicle parameters and changes in vehicle parameters which influence K also affect transient response.

To examine the influence of vehicle parameters on understeer, the total expression for understeer has been divided into four contributing factors*, specifically,

$$K = K_1 - K_2 + K_3 + K_4 \tag{6}$$

where

K_1 is the contribution to understeer due to (1) front tire cornering stiffness, (2) the load on the front wheels, and (3) the influence of steering system stiffness, mechanical, trail, and aligning torque in changing front wheel angle

K_2 is the contribution due to rear tire cornering stiffness and the load on the rear wheels

K_3 is the contribution due to roll effects including roll stiffness, camber forces, roll steer, suspension roll center heights, and the height of the center of gravity of the sprung mass

K_4 is the contribution due to rigid body aligning torque.

The largest contributor to understeer comes from K_1. The influence of steering system compliance is to increase K_1. In effect, this influence may be thought of as an increase in the value of front cornering compliance. The results of the survey of vehicle characteristics indicate that for subcompact, compact, and intermediate size automobiles, typical values of K_1 range from 8 to 11 deg/g.

It should be noted that the value of steering system compliance is not included in currently available vehicle specification data. Data on the steering compliance of typical manual steering systems

*Analytical expressions for these factors are given in Section 3.0 of the Technical Report.

is practically non-existent. Nevertheless, based on limited information [8], it is estimated that changing from power steering to a conventional manual steering system may increase understeer by as much as 2 deg/g.

The quantity K_2 is closely related to the numerator time constant, T_r, defined by STI in [1]*, viz.

$$T_r = \frac{K_2 u}{g57.3} \tag{7}$$

where u is the forward velocity and g is the gravitational constant. Typically, K_2 ranges from approximately 5.8 to 8.0 deg/g, corresponding to T_r values between 0.23 and 0.32 sec at 50 mph for the vehicle sizes addressed in this project.

Since K_1 and K_2 are roughly equal for typical automobiles, the quantity K_3 can be an important factor in determining understeer. The roll related terms are estimated to add approximately 1.2 to 2.6 deg/g to the understeer, depending upon (1) the camber stiffnesses of the tires employed, (2) whether the vehicle has a front anti-roll bar, and (3) the roll steer properties of the front and rear suspensions. The parametric data needed to make accurate estimates of K_3 are not generally published, particularly the roll steer properties and the roll center heights of the suspensions are not usually available.

The rigid body aligning torque effects that constitute K_4 total about 0.5 deg/g of understeer. This is a small effect but it is fairly constant from car to car. Also, ±0.5 deg/g is about the best that is currently done in measuring understeer using full-scale vehicle testing.

The overall findings from the analysis of vehicles, including both gain and response time effects, will be presented in Section 3.3

*For a simple non-rolling vehicle model, K_2 is the "rear cornering compliance" introduced by Bundorf and Leffert [9]. Recognizing this, much of the work persented here and in [1] can be related to [9].

after expressions pertaining to transient response have been presented in the next section.

3.2 Relationships Between Vehicle Characteristics and Transient Response

The yaw moment of inertia is the primary vehicle parameter which (1) has an influence on transient response and (2) does not influence steady-state response. However, the yaw moment of inertia can be estimated knowing the loads on the front and rear wheels [9]. Accordingly, transient response parameters (ζ, ω_n, and T_e) can be estimated from the steering ratio, wheelbase, vehicle speed, steady-state yaw rate gain, and the factors K_1 and K_2 which contribute to understeer.

In the Technical Report, equations were developed for estimating transient response properties, viz.,

$$\zeta^2 \doteq \frac{\ell \, N_G (r/\delta_{sw}|_{ss})}{u} \qquad (8)$$

$$1/\omega_n^2 \doteq \zeta^2 \left(\frac{u}{g57.3}\right)^2 K_1 K_2 \qquad (9)$$

$$T_r = \frac{K_2 u}{g57.3} \qquad \text{(repeat of (7))}$$

and

$$(1/T_e)^3 + (1/T_r - 2\zeta\omega_n)(1/T_e)^2 + (2\zeta\omega_n/T_r - \omega_n^2)(1/T_e) - \omega_n^2/T_r = 0 \qquad (10)$$

Once Equations (7), (8), and (9) have been evaluated for ζ, ω_n, and T_r, then Equation (10) can be solved numerically for T_e. The results presented in the next section are based on using Equation (10) to evaluate T_e.

3.3 Summary of the Analytical Survey of Vehicle Characteristics

Parametric data from References [10,11,12] (and, where necessary, estimated parametric values) were used in this project to calculate the steady-state yaw rate gain, damping ratio, natural frequency, and effective time constant for domestically-produced 1977 subcompact, compact, and intermediate size automobiles. For all the vehicles surveyed, the damping ratio, ζ, was greater than 0.5 and the natural frequency, ω_n, was greater than 3.0 rad/sec. The relationship between the properties of the vehicles surveyed and the specific performance space (SPS) is illustrated in Figure 2.

Even though these results were obtained by analytical procedures using parametric data available in the literature, a number of important trends can be derived from the results presented in Figure 2. Specifically, the findings are as follows:

1) The response times (T_e) for subcompact, compact, and intermediate size automobiles are within the SPS. (Possibly, larger vehicles might have response times which would fall outside the SPS.)

2) With the exception of cars equipped with manual steering, the yaw rate gain of typical cars in a nominal condition (two passengers in the front seat) will be large enough to lie within the SPS.

3) The obvious modification to bring manual steering cars into the SPS is to equip them with power steering. Another approach would be to decrease the steering ratio but this might increase the steering-wheel torque to an objectionable level.

Additional calculations were made to assess the influence of service factors on vehicle characteristics. Changes in the loading or the condition of the rear tires were taken to be important. Table 1 presents a summary of pertinent results. The results given in

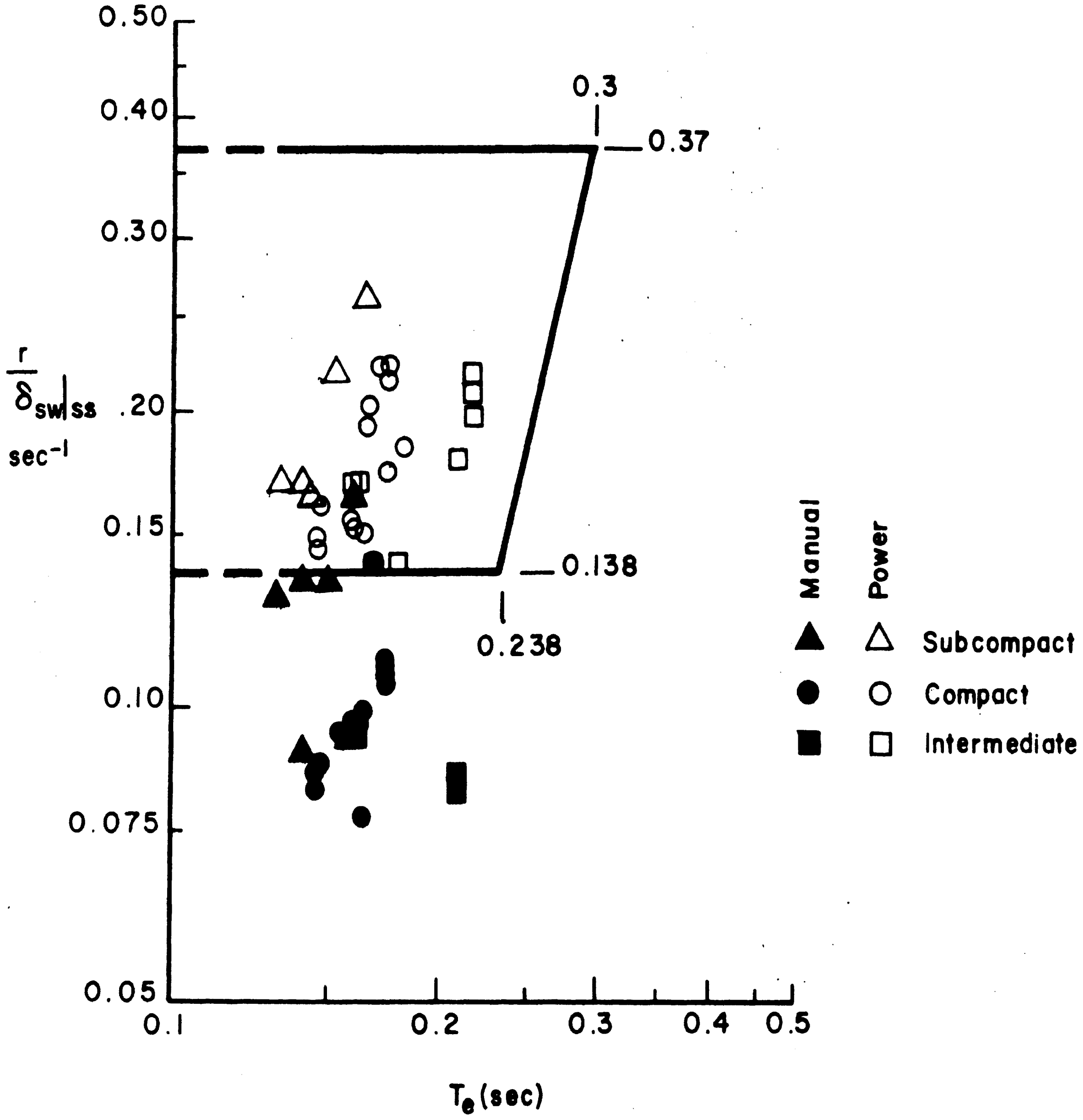

Figure 2. Relationship of estimates of 1977 vehicle performances to the SPS.

Table 1	
Service Factor Variation	Approximate Change in K_2
Change from belted-bias tires to radial tires on the rear axle	1.5 deg/g reduction
3 rear seat passengers or an additional 400 lbs on the rear wheels	1.3 deg/g increase
±3 psi change in inflation pressure from nominal value	±0.5 deg/g

Table 1 indicated that service factors must be controlled fairly closely if vehicle handling characteristics are to be maintained reasonably constant in use.

4.0 VEHICLE TESTS

Two series of vehicle tests were conducted for this research program. A preliminary test series was conducted on vehicles tentatively chosen as subjects for the modification program. These tests confirmed the position of the vehicles relative to the SPS. Later, confirmation testing was conducted on the chosen vehicles in both modified and unmodified conditions.

Both test series included open-loop testing designed to measure handling qualities which are germane to the SPS. Open-loop testing included (1) "step" steer tests and (2) "pulse" steer tests. The step steer tests are characterized by a rapid quasi-step steering input, initiated with the vehicle traveling essentially straight ahead at 50 mph. Both yaw rate response time and yaw rate gain data were obtained from this test.

"Pulse" steer tests are characterized by a series of "random" steering pulses input freely by the driver. An attempt is made to input steering-wheel motion with wide ranging frequency content. This test is used to gather yaw rate response time data.

The preliminary test series indicated that each of the three vehicles (subompact: Pinto; compact: Skylark; and intermediate: Fury) possessed yaw rate time response characteristics ($T_e|_{50}$) which were satisfactory relative to the SPS. The Pinto and Fury were found to have steady-state yaw rate gains which were too low. To modify these vehicles, steering ratio was altered using the variable ratio steering wheel limiter shown in Figure 3. Ratios were chosen that increased the gain to acceptable levels. Since the Skylark fell within the SPS range in its unmodified condition, a ratio was chosen for it which resulted in an unacceptably low yaw rate gain. Figure 4 presents steady-state gain and response time data for each of the vehicles as well as the boundaries of the SPS.

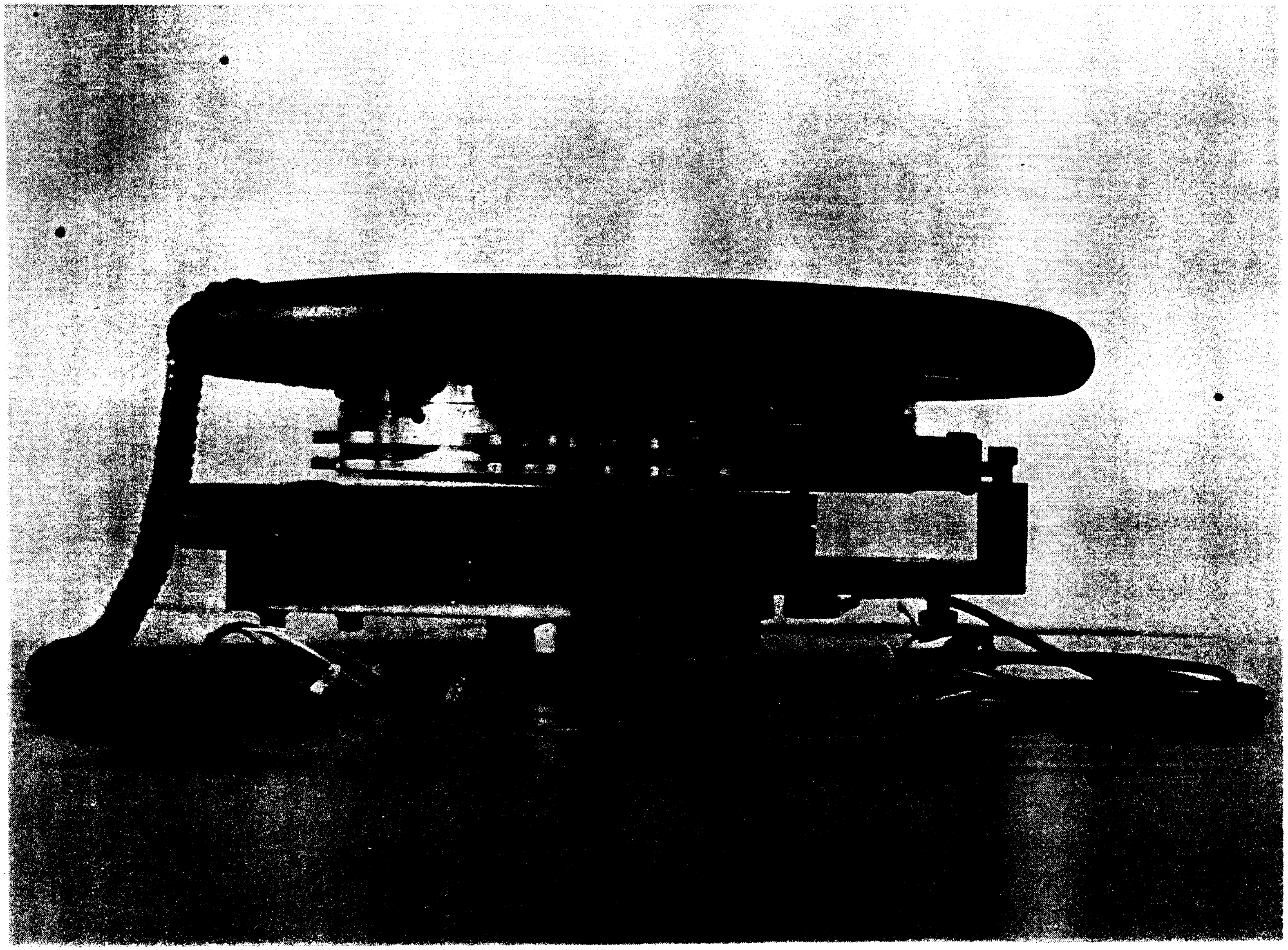

Figure 3. Variable ratio steering wheel limiter.

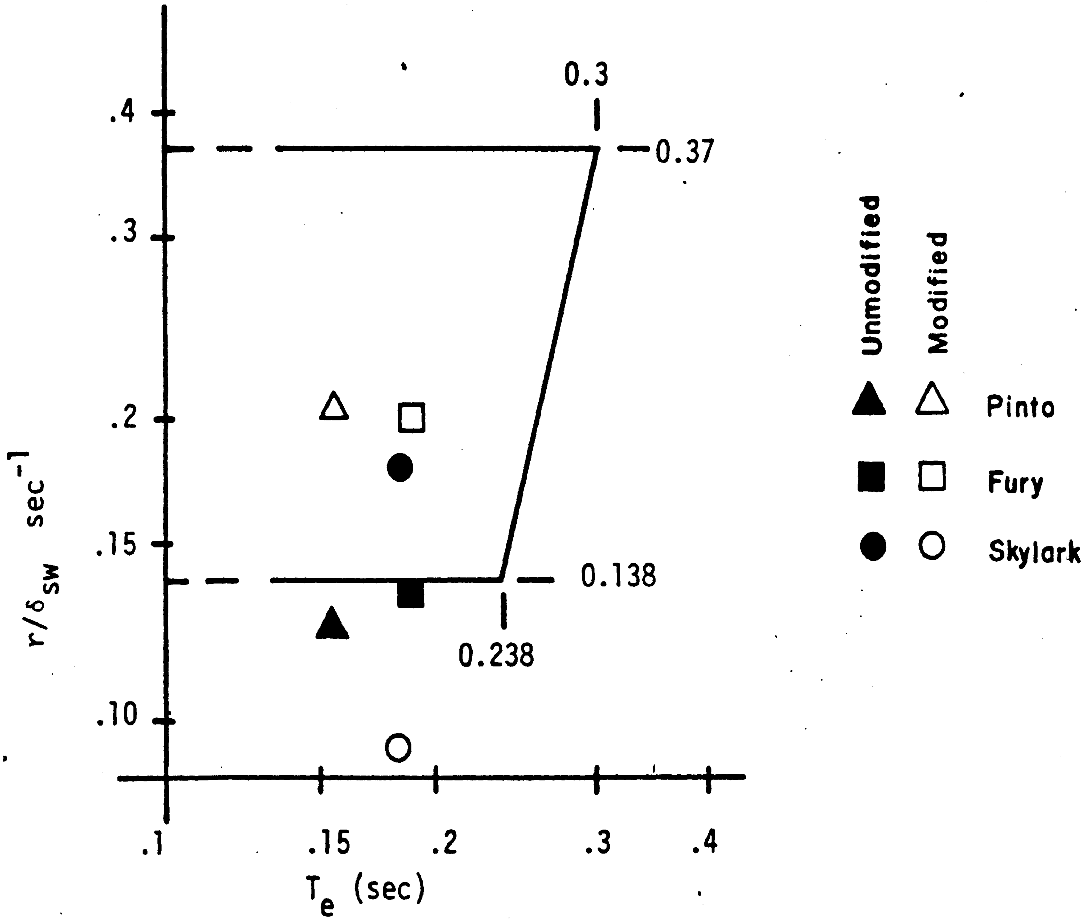

Figure 4 . Steady-state yaw rate gain and effective time constant of modified and unmodified test vehicles at 50 mph.

The verification test series also included some closed-loop vehicle testing. These tests were conducted to confirm that the modified vehicles were acceptable highway vehicles and to tentatively judge the handling value of the modifications.

Four experienced drivers and one expert driver were used as drivers in the closed-loop portion of the vehicle testing. In each case, the driver was given twenty minutes of in-town driving and twenty minutes of highway driving to familiarize himself with the test vehicle. Then the driver was asked to guide the vehicle through a tightly restricted cone course. Data was taken on cone strikes in negotiating the course, and a driver opinion of handling quality in both general driving and course driving. Both modified and unmodified vehicles were tested.

The general layout of the cone course used is shown in Figure 5. The course was designed to be run at a speed of 50 mph. At this speed, the curved portion defines a constant radius turn which, at steady-state, produces a lateral acceleration of .25 g. The "lane changes" (avoidance maneuvers) at either end also produced approximately .25 g lateral acceleration. The lane changes are sufficiently short as to elicit relatively abrupt steering inputs and the step transitions from straight to curve and back to straight require step-like steering changes which have relatively strong frequency content at the levels of interest. A "short" course was also used in which the lane changes were eliminated.

The width of the course lanes was determined empirically for each vehicle in order to obtain a level of difficulty which was challenging but not overwhelming. Lane widths were set such that nominal clearance with respect to vehicle width ranged from six to nine and one-half inches.

The degree of difficulty in negotiating the course was found to be sensitive to small (1 1/2 in) changes in lane width. Clearances used were so small as to make vehicle sideslip angle a significant

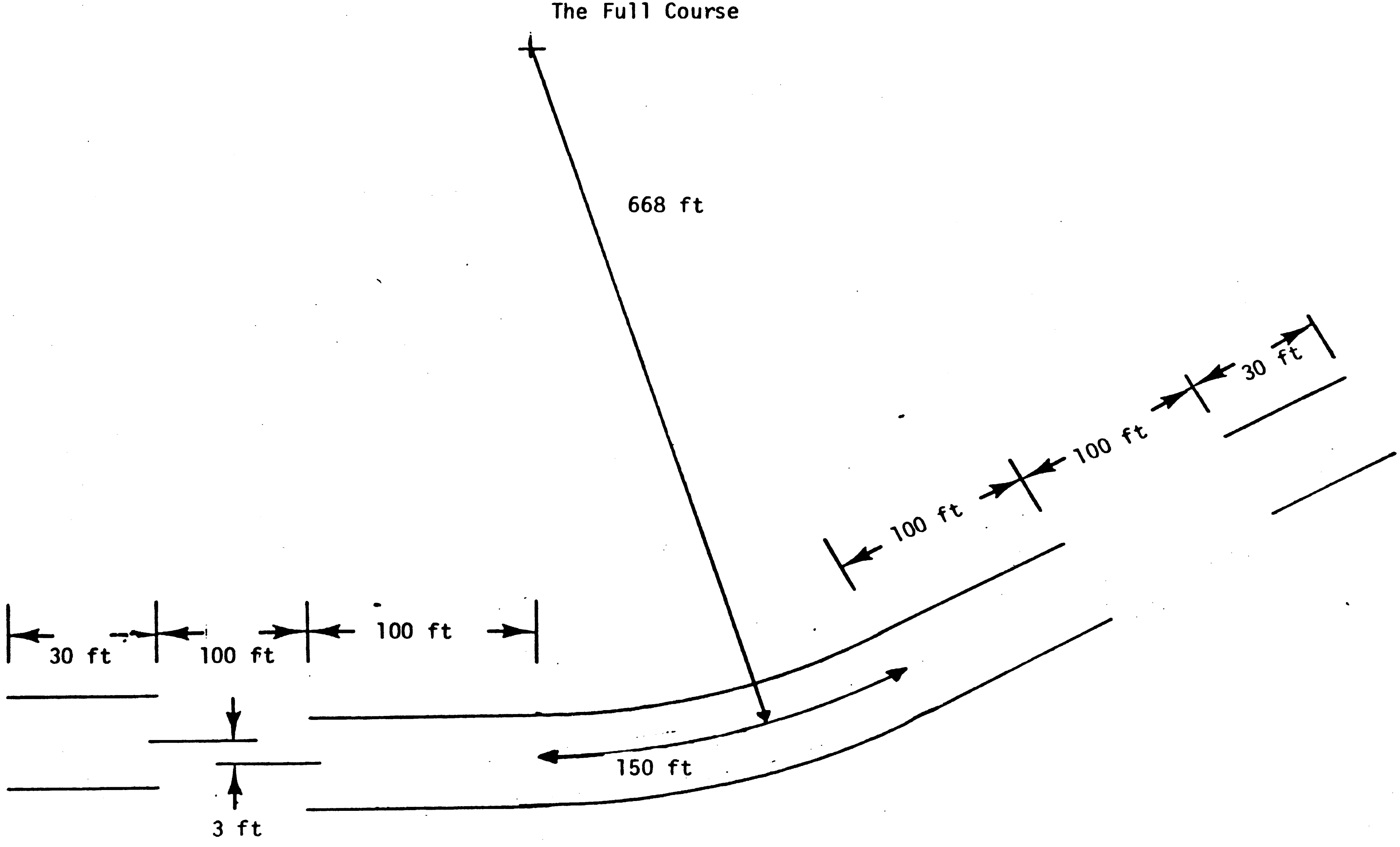

Figure 5. Layout of the cone course used in closed-loop driver-vehicle system testing.

influence on the degree of difficulty. This means that reasonable comparisons of test data can only be made between modified and unmodified vehicles* and not between different vehicle types.

Cone strike and driver opinion data are shown in normalized form in Figure 6. For each measure, performance with the unmodified versions of the car is used as the normalizer. Thus, all normalized measures are unity for the unmodified cars. For the modified cars, values larger than unity imply "better" performance (less cone strikes or higher subjective ratings) and values smaller than unity imply worse performance.

Figure 6 shows that results of the closed-loop testing were generally inconclusive. Driver opinion of handling quality in the course showed no trends, although the drivers, with only one exception, preferred the unmodified vehicles in general driving. Cone strike data showed that the modified Pinto performed better than the unmodified Pinto on the full length course. Other cone strike data was mixed.

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*Even this comparison is reasonable only because of the specific modification method used in this study.

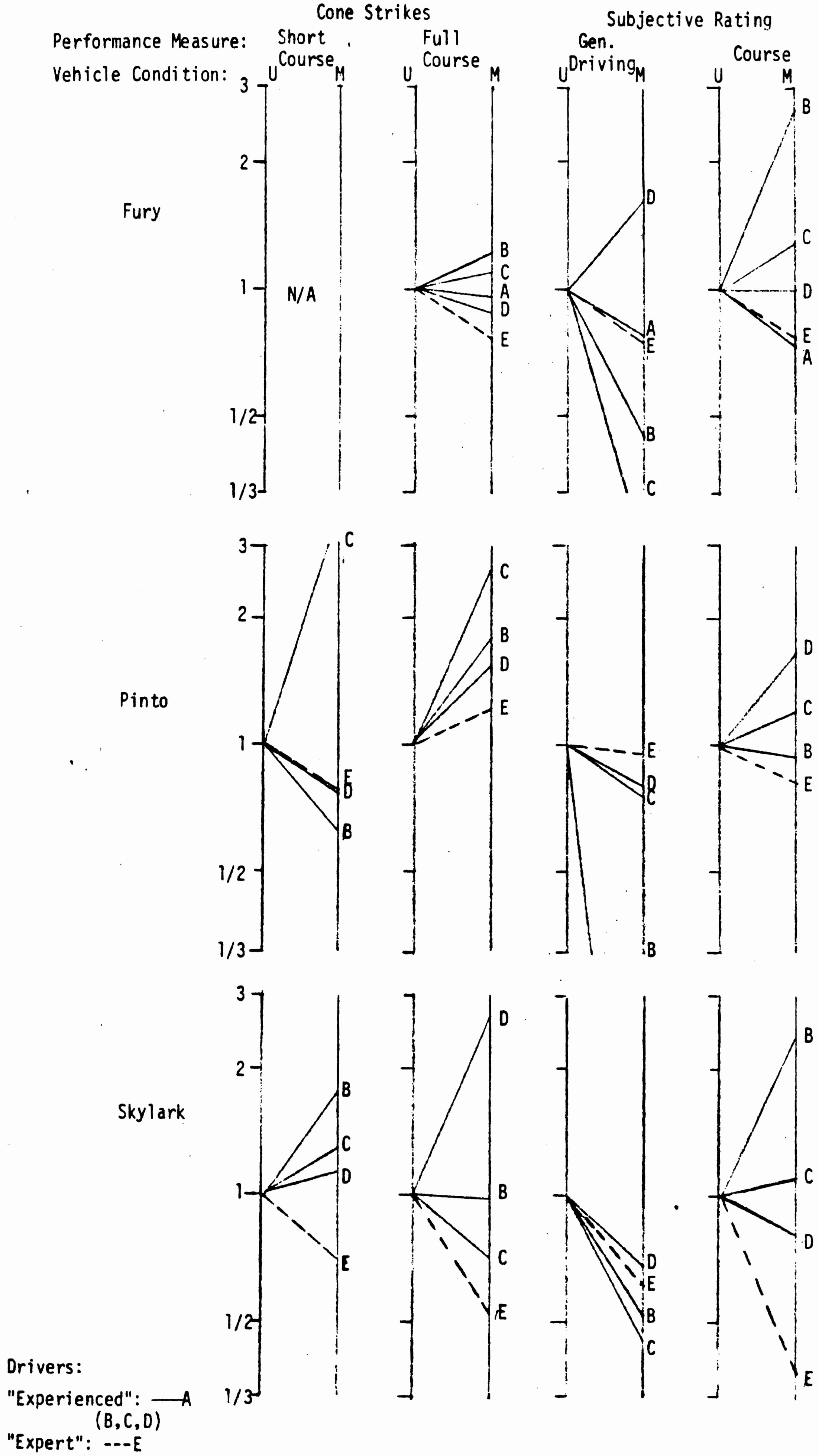

Figure 6. Normalized test data.

5.0 CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

This research project has endeavored to address practical considerations associated with applying the research findings obtained in NHTSA programs [1, 2] to the steering controllability of domestically-produced subcompact, compact, and intermediate size automobiles. It was the purpose herein to identify reasonable techniques whereby such vehicles could be made to fall within the boundaries of the Specified Performance Space (SPS), as defined by those studies.

The work which was undertaken has led to a series of conclusions which lend themselves to organization under the following two major classifications:

1) Conclusions regarding individual vehicles and/or the vehicle population (specifically, subcompact, compact, and intermediate size vehicles) and their relationship with the SPS.

2) Conclusions regarding the SPS specifically and its place in the developing art of the objective characterization of vehicle handling quality.

Even though the expressed objectives of this research project were directed toward obtaining conclusions which would fall within the first category, conclusions of the second type were a natural result of this effort.

Conclusions drawn from this project are listed below according to these two classifications.

1. Conclusions regarding the relationship of vehicles to the SPS.

   a) Most domestically-produced passenger cars of the subcompact, compact, and intermediate sizes fall within the SPS.

b) Vehicles of these sizes show very little variance in $T_e|_{50}$, one from another, and generally fall comfortably within the specified range for $T_e|_{50}$.

c) These size vehicles, when equipped with manual steering, often fall outside of the SPS because they tend toward unacceptably low values of $r/\delta_{sw}|_{50}$. The large value of overall steering ratio which is necessary because of steering effort considerations, is the cause of this low gain.

d) Some intermediate vehicles equipped with power steering fall outside of the SPS, again because $r/\delta_{sw}|_{50}$ is too low.

e) A reasonable modification to move vehicles which are outside of the SPS (because of low yaw rate gain) into the space is to lower the steering ratio. In the case of manual steering cars, this reduction is easily accomplished by changing to a power steering gear.

f) Steering ratio is virtually the only vehicle parameter which can effect a change in steady-state yaw rate gain without causing changes in transient yaw response.

g) In-use factors, especially those deriving from loading and tires, have an effect on vehicle performance in the dimensions of the SPS. Changes in in-use factors can cause individual vehicles to cross the boundaries of the space. Thus it would be necessary to constrain the variation in in-use factors if a vehicle's position relative to the SPS is to be maintained over time.

2. Conclusions regarding the SPS specifically.

a) The SPS, and especially the research efforts which led to its development, make a significant contribution to the developing art of the objective definition of vehicle handling qualities. However, the SPS (as well as this art in general) does not appear to be sufficiently developed as to be adequate for the general identification of handling quality.

b) It is possible for vehicles, whose yaw response properties lie within the SPS, to have widely varying, and possibly unacceptable levels of understeer (oversteer).

c) Vehicles which have similar yaw response properties at 50 mph, as defined by the SPS, may have widely varying yaw response properties at other velocities.

d) Many vehicle response properties (other than the yaw response properties which constitute the SPS) need consideration in evaluating vehicle handling quality.

In a separate area which does not fit conveniently into the above classifications, this program has found that

> The use of the precision cone course and resulting cone strike data for the evaluation of driver-vehicle system handling performance within the linear range is of questionable validity.

5.2 Recommendations

·Recommendation:

Efforts should be made to establish the safety significance of the SPS through a study of the accident record.

Discussion:

The preceding "Conclusions" section points out that, in large part, vehicles presently being manufactured conform to the performance specifications developed in NHTSA research studies [1, 2]. Given that subjective ratings by drivers have played a major role in establishing both the performance space specified by that research and the current design practices of vehicle manufacturers, this finding is not particularly surprising. It is valuable to note, however, that this study has identified vehicles which fall within the SPS and other vehicles that fall without. It would then appear that there is a potential mechanism for evaluating the safety significance of the SPS through the comparative evaluation of the accident records of these two sets of vehicles, even though problems related to the significance of the results due to the influence of other, uncontrolled variables could be very large.

An approach, which would eliminate some of the uncontrolled differences in vehicles would be to compare vehicles of the same basic model but differing in certain mechanical features such as power versus manual steering, radial versus bias tires, and/or a front anti-roll bar versus no anti-roll bar. Clearly, it would be necessary to (1) test the various versions of the vehicle models to be studied to establish their relationship to the SPS, (2) maintain the vehicles so that their relationship to the SPS did not change significantly in use, and (3) study a sizeable sample of these vehicles to attempt to remove the influences introduced by the varying characteristics of the drivers involved.

Recommendation:

As a preliminary step to further handling research, closed-loop, driver-vehicle handling test methodologies which (1) yield objective measures of driver-vehicle system performance and (2) provide an assessment of the control difficulties associated with particular vehicles should be developed.

Discussion:

It is of interest to note that the boundaries of the SPS have been set largely through the subjective ratings of drivers. Presumably, then, the SPS defines a set of vehicle handling properties which drivers like. It is not yet well established, however, that this same set of vehicle properties are necessary for safe handling characteristics. Indeed, the fact that drivers are adaptable to the characteristics of the vehicle they are driving and can compensate for broad differences in vehicle characteristics might suggest that "safe" handling characteristics may cover a significantly broader range than "likeable" characteristics. Accordingly, future research efforts might search for an outer region in which control properties become unacceptable, rather than for an inner region in which they are "optimum."

One manner in which the safety significance of the SPS might be examined through accident data analysis has been suggested above. For purposes of safety-related vehicle handling research, the development of closed-loop vehicle handling test methodologies which would yield objective measures of safety-related driver-vehicle handling performance quality would also be desirable. It is recognized that the safety-relevance of objective measures obtained on the test track is extremely difficult to establish. Nonetheless, the state-of-the-art of driver-vehicle performance testing appears severely wanting relative to the need for objective data.

Recommendation:

Investigations should be undertaken to examine the significance of the many other vehicle factors which might contribute to linear-regime handling quality. The SPS concept should be appropriately expanded according to the results of such investigations.

Discussion:

In connection with further research into the control quality of vehicles in the normal driving range, it would appear that more broadly based investigations are called for. The subject under consideration appears to be most complex with a large number of interrelated properties contributing to overall handling quality. In the future, it will be necessary to consider at least (1) both lateral acceleration and yaw rate response times, (2) the influence of the amount and timing of roll-related properties, and (3) the importance of the level and nature of steering torque or "feel" prior to the establishment of firmly based handling performance specifications. Even though the

SPS, as currently defined, represents a useful, interesting approach, its definition should be expanded and/or revised according to the findings of future research into these areas. Furthermore, vehicle performance over a range of velocities should be examined and specified.

•Recommendation:

Research on vehicle handling as it is affected by the transition from the linear range through the nonlinear range to the limit of turning performance should be undertaken. The findings derived therefrom should be incorporated in any evolving vehicle specification scenario.

Discussion:

Overall handling quality, specifically safety-related quality, would certainly appear to involve more than the normal driving regime. In addition to closed-loop control in normal driving, NHTSA has in the past sponsored research studies addressing the limit performance of passenger cars [12, 14]. In those studies, vehicle handling test procedures (VHTP) were developed and used to examine open-loop performance with the idea of seeing if vehicles possess response characteristics which are uncontrollable in extreme maneuvers. That is, do vehicles reach a limit response beyond which driver skill and experience is of little avail?

Two maneuvers which only involve turning, viz., a rapid turn (called "trapezoidal steer") and a reverse steer or lane-change-like maneuver (called "sinusoidal steer") have been included in the developed test procedures. Of these two turning maneuvers, the lane change is believed to be more realistic since it can be related to driver-vehicle control situations on typical roads.

Directional response in emergency turning maneuvers is partially dependent upon the vehicle characteristics (including tire characteristics) used in the definition of the SPS. The yaw rate conditions prevailing during the initial phase of a drastic steering maneuver are determined by the linear-range yaw-response properties (that is, the SPS) of the vehicle. At the start of a steering maneuver the front wheels generate a side force causing the vehicle to yaw. As the vehicle yaws and starts to sideslip, the rear tires generate forces providing additional acceleration in the direction of the turn. For a controlled turn to develop, the front and rear tire forces must produce a yaw moment balance appropriate for the desired turn. How the transition from rapid yaw acceleration into yaw moment balance occurs is crucial in establishing a good turn.

Further research on the transition from the linear range through the nonlinear range to the limit of turning performance appears valuable in order to gain an understanding of the events which can lead to loss of control in attempted turning maneuvers. Previous closed-loop studies [6] have shown that drivers are capable of applying inputs which will lead to loss of control at the limit for particular vehicles. But means for assessing the performance capabilities of the driver-vehicle system in extreme maneuvers have not been established [4]. Both open- and closed-loop results for evasive performance tests (lane-change maneuvers) are needed to illuminate meaningful, objective measures of vehicle dynamics characteristics which are pertinent to vehicle control in accident avoidance maneuvers. Until the interaction between driver control and vehicle dynamics characteristics in evasive maneuvers is well understood, our knowledge of steering controllability will be incomplete with respect to safety-related accident avoidance considerations.

Specifically, further study of driver-vehicle system performance in evasive, lane-change maneuvers is suggested. Even though the lane-change maneuver has been used in many studies with only limited success [15], it still appears to be a promising maneuver to investigate in the future. With regard to the type of maneuver involved, the results of this study indicate that a precision, tightly-constrained lane change course is not appropriate for comparing different vehicle types or models. Accordingly, a course arranged to challenge the responsiveness of the vehicle in avoiding an obstacle while allowing a fairly reasonable space laterally (such as a lane width) for recovering the original direction of travel appears to be a good candidate for further study.

•Concluding Recommendation:

Implementation of vehicle handling performance specifications is not recommended at this time, pending further development as amplified above.

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UM-HSRI-77-50-2

Final Technical Report

STEERING CONTROLLABILITY CHARACTERISTICS

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Contract Number DOT-HS-6-01409
Contract Amount: $107,775.00

August, 1977

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The University of Michigan

Prepared for:

National Highway Traffic Safety Administration
U. S. Department of Transportation

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PREFACE

The work performed in this study examines the relationship of steering controllability specifications developed in research programs sponsored by the National Highway Traffic Safety Administration (NHTSA) to the performance characteristics of domestically-produced subcompact, compact, and intermediate size automobiles. The NHTSA Contract Technical Manager, Mr. Francis DiLorenzo, was instrumental in establishing contact with technical personnel (from Systems Technology, Incorporated (STI)) who were developing steering controllability specifications for NHTSA in a separate research investigation (Contract No. DOT-HS-5-01200).

The assistance of Messrs. Weir, Klein, and McRuer of STI in explaining and keeping the authors up to date on the progress of STI's work is gratefully acknowledged. The ability to apply the latest research results would not have been possible without their cooperation. The effort expended by STI and the CTM in keeping HSRI informed is very much appreciated.

As part of this research program, visits were made to consult on vehicle modifications with technical personnel from General Motors, Ford, and Chrysler. The arrangements for those meetings were made through Mr. R. Humphries of General Motors, Mr. C. Kennedy of Chrysler, and Mr. M. Webb of Ford. The discussions at the meetings were helpful in formulating an understanding of the wide range of considerations applicable to the development of suitable handling qualities for production vehicles with various options. In this regard, the assistance of the following technical personnel should be noted:

From General Motors

R. Rasmussen

T. Bundorf

S. Anderson

R. Riefe

From Chrysler

E. Kramer

M. Agar

From Ford

L. Forbes

E. Farber

W. Bergman

T. Richardson

P. Schuba

R. Freeman

A. Anderson

Mr. David Finch was the expert driver employed in this study. His willingness to participate as needed was helpful in completing the test program efficiently.

Finally, a demonstration exercise, including both modified and unmodified versions of the example vehicles in this project, was conducted at the end of this program to provide a "hands-on" assessment to supplement the final report. Thanks are extended to those who participated in the demonstration.

1.0 INTRODUCTION

This report presents the results obtained in a project entitled "Steering Controllability Characteristics" performed by the Highway Safety Research Institute (HSRI) on behalf of the National Highway Traffic Safety Administration (NHTSA). The following quotation from NHTSA's Request for Proposal indicates the basic intention of this study:

> "It is essential in the progressing studies of driver-vehicle controllability to have "bench-marks" established that firmly relate the research findings to the practical problems of implementing those findings into production vehicles. If such bench-marks are not established, it is conceivable that a serious gap could develop between practical design and that design necessary for compliance to research findings."

Accordingly, this study has addressed practical considerations connected with the application of research findings (obtained in NHTSA programs [1, 2]) to the steering controllability of domestically-produced subcompact, compact, and intermediate size automobiles. The methodology used in the investigation described herein consisted of (1) an analytical study of the influence of changes in vehicle parameters on the yaw rate response to steering-wheel control inputs, (2) a survey of the response characteristics of motor cars to select specific subcompact, compact, and intermediate size vehicles for experimental work, and (3) open- and closed-loop tests of modified and unmodified versions of a 1977 Ford Pinto (subcompact), a 1977 Buick Skylark (compact), and a 1977 Plymouth Fury (intermediate).

The handling performance specifications used in this program are stated in terms of the yaw rate exhibited by a given vehicle in response to a steering-wheel displacement input. In particular, an "optimum" region of a "performance space" defined by (1) steady-state gain and (2) a measure of transient response-time was developed in a concurrent (and separate) project [2]. These performance specifications are discussed in the following section (2.0) of this report. The findings of the analytical study and the survey of vehicle performance are summarized in Section 3.0. The results obtained in the vehicle and driver-vehicle tests are described in Section 4.0. Conclusions and recommendations are presented in Section 5.0.

At the outset, it should be emphasized that the study reported herein indicates that a somewhat unanticipated "bench-mark" has been established. Specifically, the findings show that, with the exception of some vehicle models equipped with manual steering systems, the directional performance characteristics of currently produced subcompact, compact, and intermediate size automobiles lie within the "optimum" region that has been defined by research performed under the auspices of NHTSA. Accordingly, a principal conclusion of this study is that a serious gap does not exist between current design and that design necessary for compliance with recent research findings.

Although the prior research findings (applied in this study) do contribute to the development of an understanding of the relationships between directional performance measures and driver control, this study suggests that there are factors, in addition to specifications on yaw rate response, which should be evaluated to assess the handling qualities of a particular vehicle. For example, steering wheel torque, roll response to lateral acceleration, lateral acceleration response time, yaw response over a range of forward velocities, and emergency maneuvering performance are some of the factors which appear to be important. Section 5.0 recommends that these factors be addressed in further studies.

In summary, an overall conclusion of this study is that a design serving to satisfy a set of specifications on yaw rate response, as developed earlier in a separate study, is not sufficient to insure a good handling automobile. This conclusion does not come as a surprise in view of the total absence of any consensus relative to the art of defining and/or identifying good handling characteristics.

The body of this report presents evidence supporting the two main conclusions stated above.

2.0 DISCUSSION OF VEHICLE HANDLING SPECIFICATIONS

Passenger car handling and the control of road vehicles have been the subject of many research studies and technical papers over the last 25 years. Reviews and evaluations of a good sample of pertinent efforts are given in References [3, 4, 5]. In this section, the open-loop vehicle response measures developed in two NHTSA-sponsored studies [1, 2] will be described because those vehicle specifications have been used in this program.

Both what the specifications are and what they are not will be discussed. The discussion of factors not specified is intended to provide an indication of directions which progressing studies of driver-vehicle control might take in the future.

2.1 Review of the Preceding and Concurrent Projects

A tentative optimum range of vehicle dynamics for directional control was presented in 1975 in the final report for a study entitled "Automobile Controllability-Driver/Vehicle Response for Steering Control," Contract Number DOT-HS-359-3-762 [1]. In that project vehicles with differing dynamic properties were driven by 16 typical drivers and an expert driver in regulation tasks and transient maneuvers. Much of the experimental work was done with a "laboratory" vehicle which had a servomechanism installed to provide front wheel motions in addition to those steer motions commanded by the driver through the use of the steering wheel. By using transduced vehicle response variables as inputs to the servomechanism, a wide variety of vehicle dynamics characteristics were obtained. In addition, the servomechanism was used to apply known "disturbances" to the front wheels, thereby providing the inputs for sophisticated studies of driver regulation of vehicle path.

The laboratory vehicle used power steering (which helped to isolate the driver from the torques applied by the servomechanism).

The laboratory vehicle was "stiffened" in roll to remove the possibly confusing issue of roll dynamics from the initial research efforts and to allow the use of a simple two-degree-of-freedom model in analyzing vehicle test results.

The form of the transfer function used in [1] to describe the yaw rate response to steering wheel inputs is as follows:

$$\frac{r}{\delta_{sw}} = \frac{(r/\delta_{sw}|_{ss})\ (T_r S + 1)\ (\omega_n^2)}{S^2 + 2\zeta\omega_n S + \omega_n^2} \qquad (1)$$

where

r = yaw rate

δ_{sw} = steering-wheel angle

$r/\delta_{sw}|_{ss}$ = steady-state yaw velocity to steering gain

T_r = the yaw velocity numerator time constant

S = Laplace transform variable

ω_n = natural frequency

ζ = damping ratio

Clearly, the transfer function approach implies linear analysis and a linear system. Nevertheless, in [1], some results for transient maneuvers substantially exceeding the linear range of vehicle performance were studied. In those cases, driver opinion, lane exceedances, and steering activity were correlated with vehicle response parameters, as defined in the linear analysis.

The results of the initial study may be summarized as follows:

1) $r/\delta_{sw}|_{ss}$ and T_r appear to be the most important vehicle characteristics in terms of driver rating;

2) At 50 mph, tentative boundaries on $r/\delta_{sw}|_{ss}$ and T_r for a region of optimum vehicle response for an experienced test driver are those given in Figure 1; and

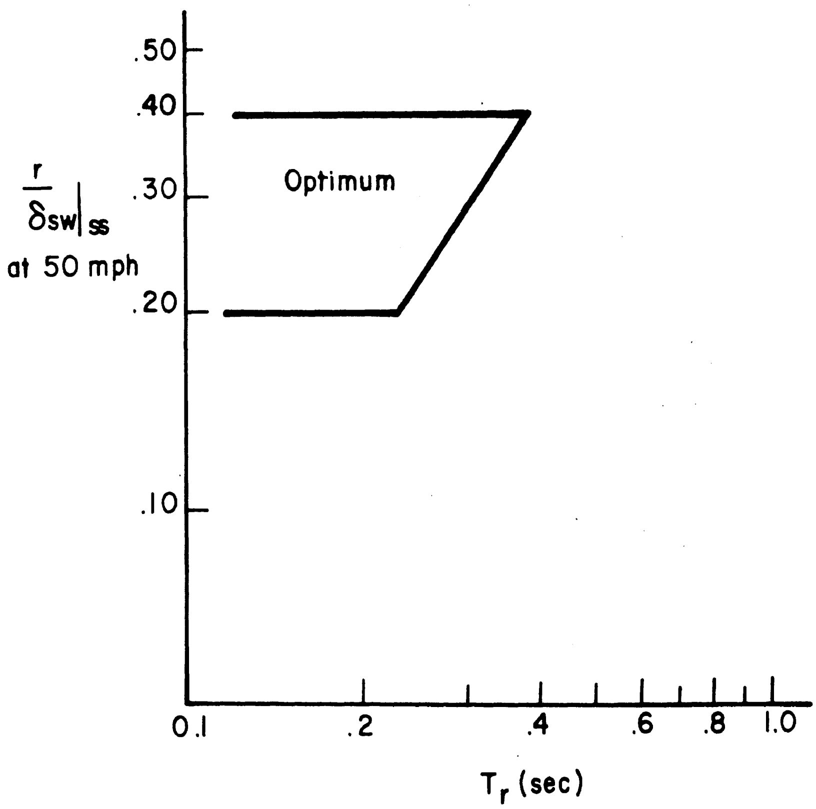

Figure 1. Tentative boundaries of optimum vehicle response at 50 mph as determined from subjective ratings of experienced test driver [1].

3) The lower bounds on $\omega_n\big|_{50}$ and $\zeta\big|_{50}$ are approximately 3.0 rad/sec and 0.5, respectively (where the notation $(\big|_{50})$ indicates that the parameter is evaluated at a vehicle velocity of 50 mph).

These specified restrictions on vehicle performance are all for a forward speed of 50 mph. The use of a particular speed is important to observe because all of the vehicle performance parameters used in Equation (1) are functions of forward velocity.

Although the initial optimum space was modified significantly in a second study [2], it is of interest to note that many domestic passenger cars do not have large enough steady-state yaw rate gains to fall into the initial optimum region. Typically, gain increases of approximately 1.3 times would be needed to cause conventional passenger cars to meet the specifications given by Figure 1.

The initial effort [1] was followed by a subsequent study [2] entitled "Evaluation and Correlation of Driver/Vehicle Data," Contract Number DOT-HS-5-01200. The results from the initial program along with data from other studies [6, 7] were evaluated using revised procedures. The vehicle rating data for 16 typical drivers from [1] were analyzed to define a new optimum region (see Figure 2).

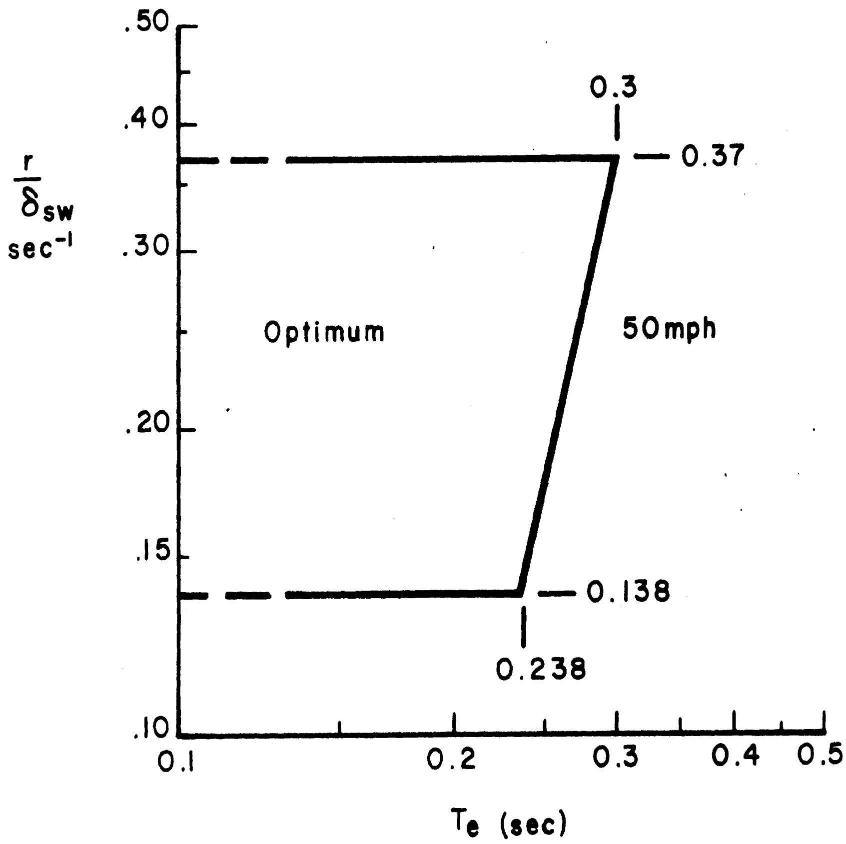

Figure 2. Typical driver boundaries - 50 mph.

The new optimum region is specified in a performance space consisting of steady-state yaw rate gain (at 50 mph) and an effective time constant, $T_e|_{50}$. The quantity $1/T_e$ corresponds to the frequency at which there is a 45° phase lag between the steering-wheel angle input and the yaw rate output. The effective time constant, T_e, is used instead of T_r because T_e is a more representative measure of vehicle phase lag than T_r for vehicle configurations in which $1/T_r$ is substantially less than the natural frequency, ω_n (see Eq. (1)).

Although the use of T_e in Figure 2 in place of T_r, as used in Figure 1, necessitates a frequency domain analysis of the vehicle response data, the most significant difference between Figures 1 and 2 for the study described here is the large change in the lower boundary for steady-state gain between the expert driver boundary and the typical driver boundary. Specifically, the lower boundary for steady-state gain at 50 mph for the expert driver is 0.2 sec^{-1} and the lower boundary for typical drivers is below 0.14 sec^{-1}.

In terms of current vehicle design, the results of this study (presented in Sections 3.0 and 4.0) indicate that the steady-state yaw rate gains for many 1977 vehicle models lie between the lower boundaries for typical and expert drivers. Accordingly, the change from expert driver results to typical driver results means a change from a situation in which the research results imply an increase in gain to a situation in which no vehicle modification is required for many typical production vehicles.

2.2 Factors Not Included in the Specified Performance Space (SPS)

The boundaries shown in Figure 2, along with the restriction that $\zeta|_{50}$ be greater than 0.5 and $\omega_n|_{50}$ be greater than 3.0 rad/sec, define the specified performance space (SPS). However, the following factors not included in the SPS can be important in attempting to develop straightforward, inexpensive methods to bring a vehicle's yaw response into the SPS without degrading the vehicle in some other aspect of performance:

1) steering torque characteristics

2) influence of roll on acceptable levels of yaw rate gain

3) lateral acceleration response time

4) sideslip angle response

5) "spin-out" or "plow-out" at the limit

6) conditions for wheel "lift-off"

7) performance at speeds other than 50 mph.

Comprehensive investigations of each of these items were beyond the scope of this project. Each of these factors could be the subject of a separate research task or project in future studies. The discussions which follow present reasons for considering these factors.

2.2.1 Steering Torque Characteristics. Most automobiles equipped with manual steering have a significantly larger steering ratio than that used on an equivalent model equipped with power steering. The purpose of the larger steering ratio is to keep the torque level required from the driver within acceptable bounds. (It is believed that (1) drivers are sensitive to steering torque and (2) they do not like to apply relatively large or relatively small torques in controlling a vehicle.)

For manual steering automobiles, the desire for limited steering torques conflicts with the requirement for exceeding the specified lower boundary for steady-state yaw rate gain. One obvious solution to the conflict is to use power steering if the cost is acceptable. Another approach would be to conduct research with the goal of establishing acceptable bounds on steering-wheel torque. Also, research could be performed to see if there is an interaction between the level of steering torque required and the directional response properties which are good for driver control. It is possible that steering-wheel torque influences the ranges of yaw rate gain and response time (effective time constant) suitable for good steering controllability.

2.2.2 Influence of Roll on Acceptable Levels of Yaw Rate Gain. Recent research [8] indicates that drivers prefer lower levels of yaw rate gain for vehicles, which have a greater roll response (degrees roll per unit of lateral acceleration), than the levels of gain they desire for "stiffer" vehicles. Since the initial study [1] was conducted primarily with roll-stiffened vehicles, the possibility exists that the results obtained therefrom need to be extended to include the influence of vehicle roll.

In the follow-on study [2], and in this project, the influence of vehicle roll on yaw gain and effective time constant have been considered. But the influence of roll per se (for example, the lateral acceleration felt by the driver due to roll motion) was not studied. Whether roll influences the driver's ability to steer is a candidate subject for further research.

2.2.3 Lateral Acceleration Response Time. During this project, consultation visits were made to three automobile manufacturing firms to speak with persons knowledgeable in practical matters associated with the steering and controllability of passenger cars. On two of these visits, the representatives from industry volunteered that lateral acceleration response time was a more discriminating measure of directional performance than yaw rate response time (or the effective time constant for yaw).

The industry personnel observed from their experience in experimental work that most vehicles of a particular size had very similar yaw rate response times which did not change much with changes in vehicle parameters. However, the changes in vehicle response due to changes in parameters were observable in the lateral acceleration response time. Accordingly, they prefer to work with lateral acceleration response time because they can more readily interpret their test results using this performance measure.

This situation does not appear to be well documented in the technical literature, although lateral acceleration response time is used in several references [9, 10, 11, 12] dealing with the directional response and control of passenger cars. It should be noted that there does not appear to be agreement within the technical community as to suitable measures for quantifying transient response. In this regard, yaw rate is quite often used as the motion variable employed because the steering system's basic function is to turn the vehicle towards desired heading angles. Nevertheless, the avoidance of obstacles at high speed requires rapid lateral deviations which entail short lateral acceleration response times. Interestingly, one researcher [11] has used a measure which is equivalent to the difference between the lateral acceleration and the yaw rate response times.

In summary, researchers have not yet reached agreement on appropriate measures for quantifying transient directional-response properties. Undoubtedly, this subject will continue to receive attention in future studies, attempting to evaluate vehicle control and accident avoidance capability.

2.2.4 Sideslip Angle Response. The sideslip angle, defined as the angle between an axis directly out the front of the vehicle and the velocity vector, is small (usually less than 2°) for normal driving maneuvers. Using commonly available devices, it is difficult to measure sideslip angle accurately enough for use in studying normal driving. However, in severe maneuvers large sideslip angles can develop and measured (derived) sideslip angle data can be used to provide an indication of the type and severity of response in emergency maneuvers approaching the limits of vehicle performance [13].

Nevertheless, the sideslip angle can be important in relatively low-level tracking or precision steering courses because even below a sideslip angle of 2° the effective width of the vehicle is increased by an amount which can be significant to the driver. This phenomenon appeared to be of importance in this study and the implications of sideslip angle with regard to driver-vehicle testing are described in Section 4.2.2.

2.2.5 "Spin-Out" or "Plow-Out" at the Limit. Upon initiating a sudden turn, a vehicle's front tires produce a side force causing the vehicle to accelerate in yaw and lateral motion. Then, the rear tires start to produce lateral forces as the vehicle develops yaw and lateral velocities. The forces from the rear tires reduce the yaw acceleration and increase the lateral acceleration of the vehicle. If the forces from the rear tires cannot arrest the yaw motion, the vehicle will spin out. If the front tire forces saturate before the rear tire forces reach their maximum value, the vehicle will plow out, that is, the driver can increase the steer angle above the level for front tire saturation without causing an increase in path curvature.

In order to increase steady-state yaw-rate gain in the normal or linear driving range, consideration has been given to increasing the cornering stiffnesses of the front tires and decreasing the cornering stiffnesses of the rear tires.

A possibility for accomplishing this difference in tire characteristics is to use a different type of tire on the rear axle than that used on the front axle. Even though this situation occurs fairly frequently for vehicles in use (that is, people often replace two tires at a time), it is considered to be impractical to maintain a specified difference in the types of tires installed on the front and rear axles.

However, differences in tire mechanical characteristics have been achieved by using different inflation pressures between front and rear tires. To obtain substantial changes in gain, extraordinarily low levels of inflation pressure are needed in the rear tires [14]. This situation can lead to a tendency for the vehicle to spin out in emergency maneuvers. During the initial phase of a sudden turn, a large yaw acceleration can be developed. Even if the forces from the front and rear tires tended to saturate at levels which produce a yaw moment balance on the vehicle later in the manuever, they would not be able to reduce the high yaw rate established during the time when the yaw acceleration was large. If the large yaw rate persists, the vehicle will spin out.

There are differences in opinion regarding whether a "spin-out" or a "plow-out" is least safe. In the case of a spin-out, the driver is turning in an uncontrolled manner which he probably cannot correct. In a plow-out, the driver cannot increase the severity of the turn, but he can reduce the severity of the turn and he is oriented somewhat in the direction of the vehicle's velocity vector. From a controllability standpoint, a plow-out appears to have an advantage over a severe spin-out. Nonetheless, the matter of plow-out versus spin-out has not been resolved and the prudent course of action is to try to avoid either one.

Accordingly, large reductions in rear tire inflation pressure to achieve significant changes in yaw rate gain are not recommended even though this modification might move the performance characteristics of a particular vehicle into the SPS.

2.2.6 <u>Conditions for Wheel "Lift-Off</u>." This subject was mentioned by representatives from a vehicle manufacturing firm as one of a number of items a manufacturer might consider in evaluating a vehicle's response in severe maneuvers. The roll stiffnesses of front and rear suspensions, jacking effects, front and rear roll center heights, sprung-mass center of gravity height, and the locations of bump stops are important with regard to the level of maneuver at which wheels start to lift off the ground.

For a vehicle with a substantial front anti-roll bar and a fairly roll-compliant rear suspension, the inside front wheel may be entirely unloaded in a severe turning maneuver. The unloading of one front wheel and the loading of the other front wheel can cause a net reduction in tire side force from the front wheels. This phenomenon can reduce the tendency for a vehicle to spin out (enhance the tendency for plow out) in an abrupt turning maneuver.

For vehicles without a front anti-roll bar, the addition of a front anti-roll bar can be used to increase the yaw rate gain in normal driving. (See Section 3.1.) Thus this modification could be used to bring a vehicle whose gain is slightly below the lower boundary of the SPS into the SPS. Whether the additional front roll stiffness will cause an unfavorable wheel lift-off situation or an inappropriate loss in net side force from the front wheels in maneuvers in the nonlinear range of vehicle performance has not been established, and any general results of this nature are beyond the scope of this project and the requirements defining the SPS.

2.2.7 Performance at Speeds Different from 50 mph. The SPS is based on parameters which are all determined at a speed of 50 mph. As discussed in the next section, reasonable values of gain and response time at 50 mph do not insure reasonable values of gain and response time at speeds slightly removed from 50 mph.

If overall steering ratio were to be constrained to a narrow range, then gain and response time measures at 50 mph would adequately define performance at other speeds. But overall steering ratio is not constrained to a narrow range either in practice or by the SPS. Thus, specifications at 50 mph are not sufficient to insure good handling qualities throughout the entire range of typical forward velocities.

There are vehicle performance parameters which are not speed dependent. For example, the understeer/oversteer gradient, a traditional open-loop handling measure, does not depend upon speed in the normal driving range. Also, the front and rear cornering compliances [9] are independent of speed.

The next section examines what the requirements of the SPS imply about (1) understeer/oversteer gradient and (2) handling properties at speeds different from 50 mph.

2.3 Limitations of the SPS in Determining the Yaw Response of a Vehicle

The following discussion will demonstrate, using very simple analytical models, that

1) It is possible for vehicles which fall within the boundaries of the SPS to have understeer/oversteer gradient values which would generally be deemed unacceptable. These values could range at least as high as 9.5°/g understeer to 1°/g oversteer.

2) Vehicles which have similar properties or "acceptable" properties (according to the SPS requirement) at 50 mph may have widely varying or "unacceptable" properties at velocities other than 50 mph.

To examine the manner in which the SPS serves to determine a vehicle's handling properties, expressions derived from the well-known two-degree-of-freedom, linear range, "bicycle" model of the passenger car will be used. The basic equation defining steady-state steering, as derives from such a model, is

$$\frac{\delta_{SW}}{N_G} = \frac{\ell 57.3}{R} + K\frac{ur}{g57.3} \qquad (2)$$

where

δ_{SW} = steering wheel angle, degrees

N_G = overall steering ratio

ℓ = wheelbase, feet

R = turn radius, feet

K = understeer/oversteer gradient, degrees per g

u = forward velocity, ft/sec

r = yaw rate, degrees/sec

g = gravitational constant, 32.2 ft/sec^2

Further, under steady-state conditions

$$\frac{rR}{57.3} = u \tag{3}$$

From these expressions, the steady-state yaw rate gain (G) may be expressed as

$$G = \left.\frac{r}{\delta_{sw}}\right|_{ss} = \left(\frac{1}{N_G}\right) \frac{u/\ell}{1 + \frac{u^2 K}{g\ell 57.3}}, \ \sec^{-1} \tag{4}$$

In the earlier work [1], the transfer function for the non-rolling vehicle model was shown to be

$$\frac{r}{\delta_{sw}}(S) = \frac{T_r S + 1}{\frac{1}{\omega_n^2} S + \frac{2\zeta}{\omega_n} S + 1} \left(\frac{r}{\delta_{sw}}\right)_{ss} \tag{5}$$

The phase shift (ϕ) between input (δ_{sw}) and output (r) of the system defined by Equation (5) may be expressed as follows:

$$\phi = \tan^{-1}(T_r\omega) - \tan^{-1}\left(\frac{2\zeta\ \omega/\omega_n}{1 - (\omega/\omega_n)^2}\right) \tag{6}$$

where ω is the frequency of system excitation.

From (6),

$$\tan\phi = \tan\left[\tan^{-1}(T_r\omega) - \tan^{-1}\left(\frac{2\zeta\ \omega/\omega_n}{1 - (\omega/\omega_n)^2}\right)\right] \tag{7}$$

and using the identity

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y} \tag{8}$$

it can be shown that

$$\tan \phi = \left[\frac{T_r\omega - \dfrac{2\zeta\ \omega/\omega_n}{1 - (\omega/\omega_n)^2}}{1 + T_r\omega \dfrac{2\zeta\ \omega/\omega_n}{1 - (\omega/\omega_n)^2}} \right] \tag{9}$$

The equivalent time constant (T_e), which is one dimension of the SPS, is defined such that, when

$$\omega = 1/T_e \tag{10a}$$

then

$$\phi = -45° \tag{10b}$$

Substituting Equation (10) into Equation (9) and rearranging, the following cubic equation derives*

$$(1/T_e)^3 + (1/T_r - 2\zeta\ \omega_n)(1/T_e)^2 + (2\zeta\ \omega_n/T_r - \omega_n^2)(1/T_e) - \omega_n^2/T_r = 0 \tag{11}$$

Finally, Bundorf and Leffert [9], using the cornering compliance concept, have shown that damping ratio and natural frequency may be expressed, respectively, as:

$$\zeta^2 = \frac{1}{[1 + \frac{u^2K}{\ell 57.3g}]} \tag{12}$$

and

$$\omega_n^2 = \frac{(57.3g)^2}{D_f D_r u^2}[1 + \frac{u^2K}{\ell 57.3g}] \tag{13}$$

---

*A closed-form solution for $(1/T_e)$ can be obtained from this equation, but it is very complex. This "solution," i.e., Equation (11), will be sufficient for purposes herein. Later in this discussion, example values of T_e will be given which are obtained through numerical solution of Equation (11).

where

D_f = front cornering compliance, deg/g

D_r = rear cornering compliance, deg/g

Equations (12) and (13) are actually approximations which require the conditions

$$k^2 \doteq a\ b \tag{14}$$

and

$$1/2D_r \leq D_f \leq 2D_r \tag{15}$$

where

k = radius of gyration of the vehicle in yaw, ft

a = longitudinal distance from the center of gravity to the front axle, ft

b = longitudinal distance from the center of gravity to the rear axle, ft

(Equation (13) is exactly true if Equation (14) holds. According to [9], it is generally true that $0.9 < k^2/ab < 1.2$ such that Equation (14) is sufficiently accurate for purposes herein.)

For the simplest vehicle models, wherein only tire cornering stiffness and weight distribution are considered, cornering compliances may be expressed as:

$$D_i = \frac{W_i}{C_{\alpha_i}} \qquad i = F \text{ or } R \tag{16}$$

where W_i = weight on the axle, lb

C_{α_i} = total tire cornering stiffness on the axle, lb/deg

For more complex models, cornering compliances may also include the effects of roll steer and the several compliance steer effects.

It should be noted that

$$K = D_f - D_r \tag{17}$$

and that

$$T_r = \frac{W_r}{C_{\alpha_r}} \frac{u}{g57.3} \approx D_r \frac{u}{g57.3} \tag{18}$$

The above equations may be used to illuminate the significance of the performance parameter constraints represented by the SPS. Equation (4) indicates that there are two parameters available which can be altered in order to obtain a desired steady-state, 50 mph, yaw rate gain. These are the understeer coefficient (K) and the overall steering ratio (N_G) (assuming the wheelbase is a fixed value for a given vehicle). Equation (12), however, points out that the restriction on damping ratio, i.e., $\zeta\big|_{50} > .5$, essentially places an upper bound on the value of K. (Here again, ℓ is assumed fixed, and it is also assumed that the inertial properties of the vehicle are not altered significantly from those implied by Equation (14).) The remaining restriction on natural frequency ($\omega_n\big|_{50} > 3$ rad/sec) and on $T_e\big|_{50}$ (see Fig. 2) do not yield to such straightforward interpretation. In effect, these restrictions, through Equations (11), (13), and (18) put a lower bound on K and limit the total cornering compliance on the two axles of the vehicle.

Now consider an example based on a real vehicle. Reference [10] presents cornering compliance data* for a compact car which resulted from a variety of load and tire condition variations (including wear, construction type, and inflation pressure variations). Table 1 displays the front and rear cornering compliances obtained in that

*The original cornering compliance definitions given in [9] have been extended in [10] to include vehicle factors not considered in the earlier work.

Table 1. Cornering Compliances Obtained for a Compact Car Through Load and Tire Factor Variations.

Vehicle Configuration	Front Cornering Compliance, °/g	Rear Cornering Compliance, °/g
1	10.5	6.1
2	10.3	5.1
3	10.6	5.5
4	10.5	5.5
5	10.6	6.1
6	8.3	7.7
7	6.0	6.1
8	7.3	4.2
9	6.0	4.9
10	10.5	7.2
11	10.5	7.1
12	5.1	7.2

work. The vehicle from which these data were obtained was a compact car very much like the Buick Skylark used in this project. Using the wheelbase of the Skylark (9.25 ft) and the compliance data of Table 1, the values of K, $\omega_n|_{50}$, $\zeta|_{50}$, and $T_e|_{50}$ can be calculated for each of the 12 vehicle configurations of Table 1. The results appear in Table 2. Table 2 shows that, of the 12 configurations, only number 12 is necessarily outside of the SPS. This configuration fails the requirements on two counts, viz., (1) its 50-mph natural frequency is less than 3 rad/sec and (2) its 50-mph effective time constant is greater than 0.3 sec. All other configurations can be made to fall in the SPS if a steering ratio (N_G) is chosen which results in an appropriate 50-mph yaw rate gain (see Eq. (4)). (For most of the configurations any gain from 0.138 sec^{-1} to 0.37 sec^{-1} is acceptable. Configuration numbers 6 and 7 require more specific control, for their time constants fall in the 0.238-0.3 sec range, i.e., near the sloping right-hand boundary seen in Figure 2.)

Table 2. Performance Parameters for the Vehicle Configurations of Table 1.

Vehicle Configuration	K	$\zeta\|_{50}$	$\omega_n\|_{50}$	$T_e\|_{50}$
1	4.4	.647	4.86	0.191
2	5.2	.616	5.64	.167
3	5.1	.619	5.32	.176
4	5.0	.623	5.31	.176
5	4.5	.643	4.87	.190
6	0.6	.917	3.43	.280
7	-0.1	1.016	4.09	.246
8	3.1	.711	6.39	.151
9	1.1	.862	5.38	.179
10	3.3	.700	4.13	.221
11	3.4	.695	4.19	.218
12	-2.1	1.719	2.41	0.943

The data of Tables 1 and 2 demonstrate that a rather broad range of vehicles, in terms of their understeer/oversteer character, can fall within the SPS. The tables show that as understeer becomes large, damping ratio is the limiting criterion. (This relationship was expressed precisely in Equation (12).) In fact, for the vehicle used here (9.25 ft wheelbase), Equation (12) shows that the upper bound on K is 9.5 °/g. As the vehicle tends toward oversteer, both $\omega_n|_{50}$ and $T_e|_{50}$ become the limiting factors. The relationship between K and these limits is not so straightforward, however, and depends on the total amount of cornering compliance in the vehicle. That is, as the cornering compliance at both ends of the vehicle decreases, more oversteer is acceptable. To demonstrate this point, consider the three vehicle configurations of Table 3. These hypothetical configurations use front and rear compliances which are generally low, but not unreasonably so. Note that all three vehicle configurations (including one with 1 °/g oversteer) could fall within

Table 3

Vehicle Configuration	13	14	15
D_f	5.1	5.1	4.5
D_r	5.1	5.6	5.5
K	0.0	-0.5	-1.0
$\zeta\|_{50}$	1.00	1.09	1.21
$\omega_n\|_{50}$	4.93	4.32	4.18
$T_e\|_{50}$	.203	.242	.273

the SPS with the proper choice of steering ratios. It is because the compliances are generally low that the allowable value of K is also low.

The preceding discussion has highlighted the relationship between the parameters $G|_{50}$, $T_e|_{50}$, $\omega_n|_{50}$, and $\zeta|_{50}$ and the traditional vehicle parameter, K, as well as the front and rear cornering compliances (which, when combined, determine the value of K). It is of interest to observe the behavior of G, T_e, ω_n, and ζ at velocities other than 50 mph. Figures 3 through 6 present plots of G, T_e, ω_n, and ζ, respectively, as a function of velocity for several selected vehicle configurations. (All the vehicles have a wheelbase of 9.25 feet. Their understeer gradient, K, varies from 9.5 to -1.0, a range which the preceding discussion has shown to be acceptable, according to the SPS requirements. Where the specific values of D_f and D_r are significant, i.e., in plots of T_e and ω_n, the vehicle configurations have been selected from Tables 1 and 3.)

Figure 3 presents yaw rate gain as a function of velocity. Plots are shown for vehicles of varying levels of K from 9.5 °/g to -1.0 °/g. In each case, overall steering ratio, N_G, has been chosen such that $G|_{50} = .2\ sec^{-1}$, a value which is comfortably within the gain range of the SPS. The figure clearly indicates that, although all the vehicles have identical gains at 50 mph, at other velocities their gains can vary widely, and the rate of change in gain with

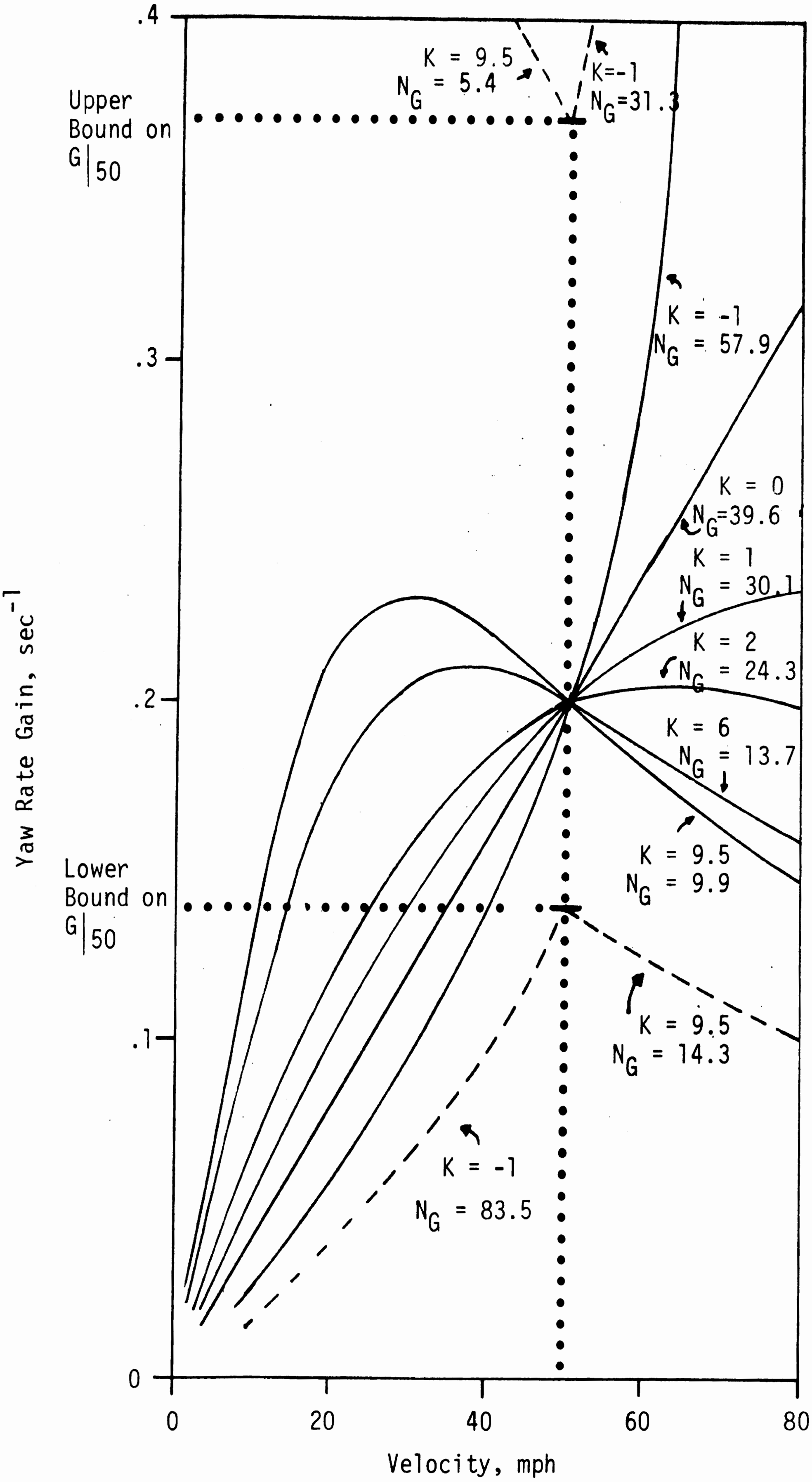

Figure 3. Yaw rate gain as a function of velocity for vehicles with various levels of understeer.

respect to velocity can be very high at 50 mph. Also shown in dashed lines on the figure is the envelope of performance of all possible "optimum" vehicles obtained by choosing steering ratios which would lead to $G|_{50}$ = .37 sec^{-1} or $G|_{50}$ = .138 sec^{-1}, the boundaries of the SPS. This envelope serves to illustrate that a vehicle could have a yaw rate gain within these bounds at 50 mph (thus being inside the "optimum space"), but at a velocity of only a few mph difference, the same vehicle could have a yaw rate substantially removed from these bounds.

Figure 4 shows the sensitivity of T_e to velocity for several vehicle configurations chosen from Tables 1 and 3. This figure illustrates again that vehicles which have similar parametric values at 50 mph, in this case, $T_e|_{50}$, may have wide variance in the same parameter at other velocities. In this case, for vehicles with lower values of K, T_e increases rapidly at velocities above 50 mph.

Figures 5 and 6 present the sensitivity of ω_n and ζ to velocity. In Figure 5, several vehicle configurations from Tables 1 and 3 are used; in Figure 6, vehicles ranging in K from 9.5 to -1.0 are displayed. These two figures show that there is some tendency for ω_n or ζ to stray out of the SPS range at velocities slightly removed from 50 mph, but this tendency is not severe.

This discussion has served to indicate the limitations of the SPS in determining yaw response of a vehicle. It has been shown that, while the SPS may identify good yaw response qualities at 50 mph, it may also encompass vehicles with an understeer (oversteer) level conventionally judged unacceptable for highway vehicles. These same vehicles, then, may have widely varying yaw response properties at speeds other than 50 mph.

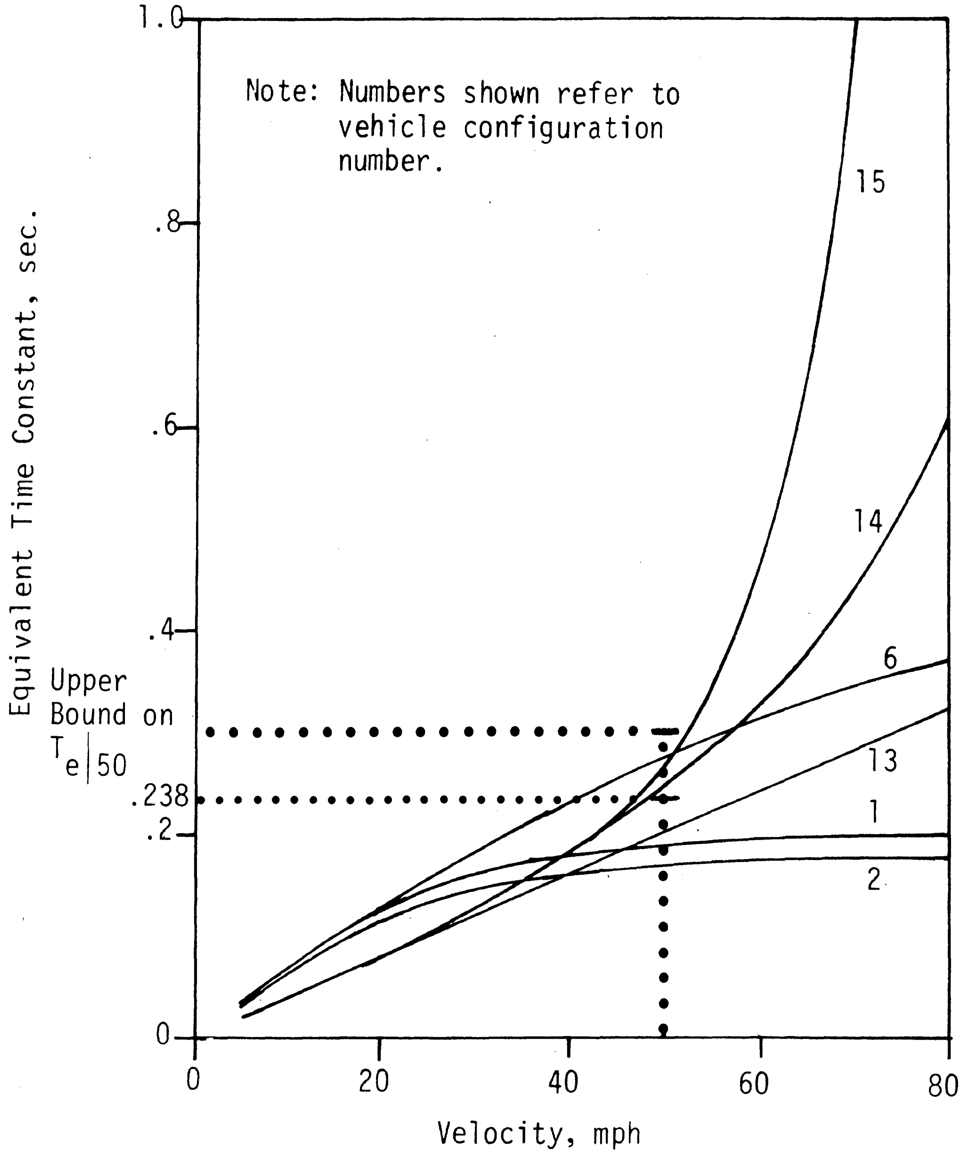

Figure 4. Equivalent time constant as a function of velocity for selected vehicle configurations from Tables 1 and 3.

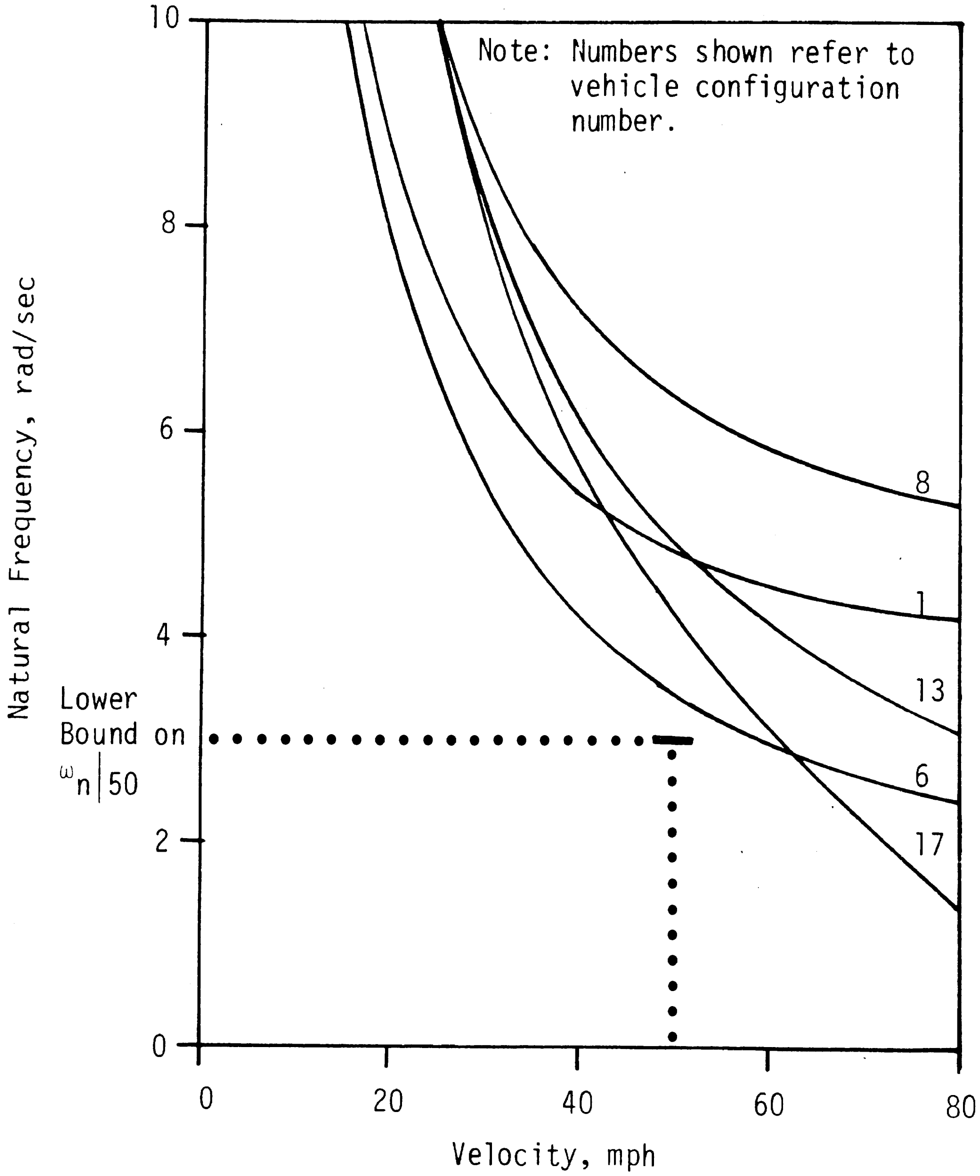

Figure 5. Natural frequency as a function of velocity for selected vehicle configurations from Tables 1 and 3.

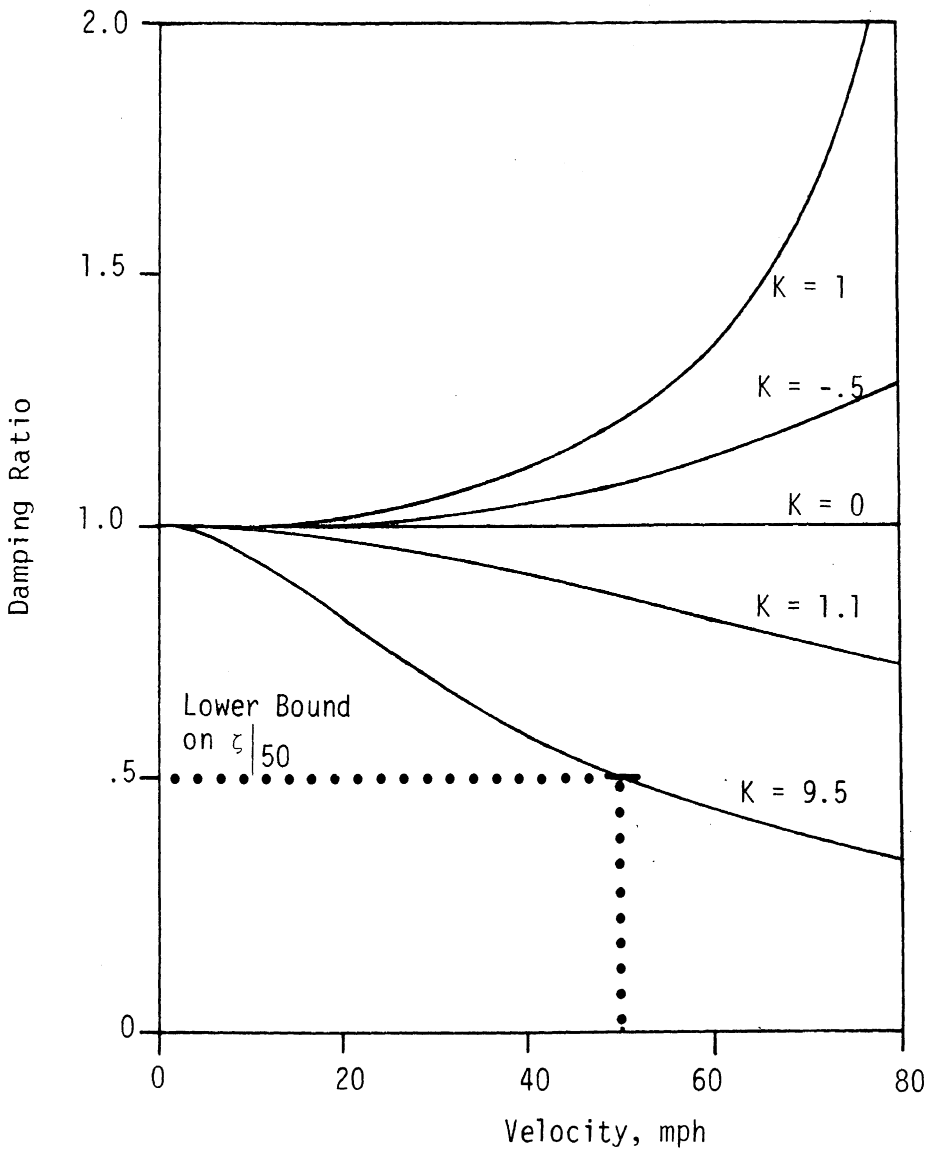

Figure 6. Damping ratio as a function of velocity for vehicles with various levels of understeer.

3.0 ANALYSIS OF THE INFLUENCE OF VEHICLE PARAMETERS ON PERFORMANCE

Steady-state gain is discussed first in this section because almost all vehicle parameters which influence transient response in normal driving affect steady-state response. In fact, the yaw moment of inertia and the damping in the suspensions are the only vehicle parameters which (1) have an influence on transient response and (2) do not influence steady-state response. Accordingly, the terms and definitions used to describe steady-state response will be employed later on in discussing the relationships between vehicle characteristics and transient response.

3.1 Factors Influencing Steady-State Yaw Rate Gain

As previously discussed in Section 2.0 and as indicated by Equation (4), there are two factors, N_G and K, which can be varied independently to achieve a required yaw rate gain for a vehicle of given wheelbase at a speed of 73.3 ft/sec (50 mph). Clearly, it is obvious that the yaw rate gain can be changed directly by altering steering ratio. Furthermore, small percentage changes in gear ratio will produce corresponding percentage changes in yaw rate gain, viz.,

$$\frac{\Delta G}{G} \doteq - \frac{\Delta N_G}{N_G} \tag{19}$$

where ΔN_G is a small change in overall steering ratio and ΔG is the corresponding change in yaw rate gain from a nominal yaw rate gain, G, corresponding to the nominal steering ratio, N_G.

Even though it is patently clear, it should be emphasized that changing steering ratio has the advantages that it does not change (1) the understeer/oversteer gradient, (2) steady-state roll response in degrees per unit lateral acceleration, or (3) the form of the transient response.

(Depending upon the type of steering system employed on a particular vehicle, the changing of steering ratio may cause a significant change in the torque the driver feels at the steering wheel. At present, boundaries on steering-wheel torque have not been specified.)

The understeer/oversteer gradient, K, in Equation (4) can also be varied to change yaw rate gain. However, K is dependent upon many vehicle parameters and changes in vehicle parameters which influence K also affect transient response.

A general impression of the influence of K in degrees/g on the steady-state yaw rate response, r_{ss}, to a reference front-wheel angle (defined by δ_{sw}/N_G) may be derived from Figure 7 (in which $G_r = r_{ss}/(\delta_{sw}/N_G)$. The dashed horizontal lines on Figure 7 indicate that, for subcompact, compact, and intermediate cars with an overall steering ratio of approximately 19, the understeer factor may vary from approximately 6.5 degrees/g to nearly neutral steer (0.0 degrees/g) without violating the specified range of yaw rate gain at 50 mph. The vertical dashed line represents a typical example of a domestic car with an understeer of approximately 5 deg/g corresponding to an $N_G G$ product (that is, G_r) in the neighborhood of 3.2 sec^{-1}, which is above the lower boundary of the SPS for $N_G = 19$.

As illustrated in Figure 7, the sensitivity of $G_r\big|_{50}$ to changes in K increases as K decreases. Thus the yaw rate gain becomes more sensitive to changes in service factors (loading, replacement tire characteristics, etc.) at low values of understeer than it is at moderate values of understeer (approximately 5 deg/g).

Using Equation (4) to develop an expression for estimating the influence of small changes in K on the yaw rate gain yields the following equation:

$$\frac{\Delta G}{G} = \frac{B\Delta K}{1+BK} \tag{20}$$

where

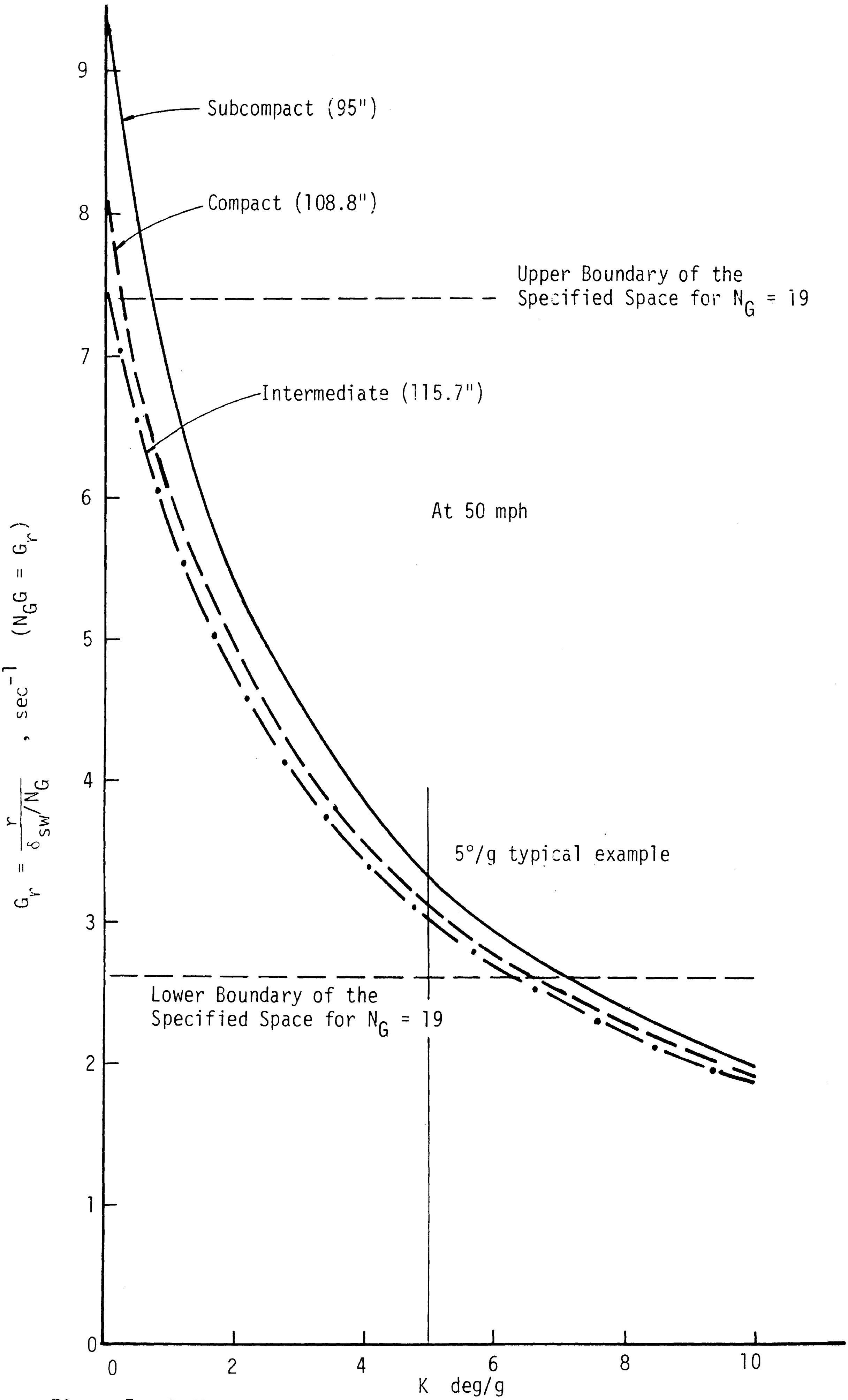

Figure 7. Influence of understeer/oversteer gradient, K, on yaw rate gain.

$$B = (u^2/\ell g \; 57.3)$$

For example, for a typical compact car with a nominal value of K equal to 5 deg/g, the fractional change in gain at 50 mph is given numerically by:

$$\frac{\Delta G}{G} = -\,0.123\,\Delta K \quad (\text{for } B = 0.3214)$$

That is, a 0.5 deg/g decrease in K (i.e., a 10% change at 5 deg/g) will produce approximately a 6% increase in yaw rate gain. The numbers in this example have some practical significance in that current practice is challenged to measure or estimate K any more accurately than ±0.5 deg/g, corresponding to approximately a ±6% uncertainty in yaw rate gain for $K \doteq 5$ deg/g.

3.1.1 The Influence of Vehicle Parameters on Understeer. In the preceding discussion, the basic influences of K and N_G on yaw rate gain were treated in elementary terms. The following material discusses the relationship between vehicle parameters and understeer, K, thereby providing a connection between vehicle design (modification) parameters and yaw rate gain.

The derivation of the expression used hereinafter for evaluating K involves a number of vehicle dynamics considerations, lengthy algebraic manipulations, and many parameters. Nonetheless, this subject has been treated by numerous research investigators [e.g., see 10, 14, 15, 16] and their work can be compared with the equations presented hereinafter. (The derivation outlined in Appendix C provides a means for relating steady-state gain to the physical processes involved.) Accordingly, this discussion will start from a basic expression for understeer in terms of vehicle parameters and proceed to address the implications of modifying vehicle parameters to obtain a required level of understeer.

The total expression for understeer has been divided into four contributing factors, specifically,

$$K = K_1 - K_2 + K_3 + K_4 \tag{21}$$

where

K_1 is the contribution to understeer due to front cornering coefficient and the influence of steering system compliance

K_2 is the contribution to understeer due to rear cornering coefficient

K_3 is the contribution to understeer due to roll effects

K_4 is the contribution to understeer due to rigid body aligning torque

The equations used for each of these factors are:

$$K_1 = \frac{W_f}{C_f}(1 + A_1) = \frac{W_f}{C_{fe}} = \frac{T_{fe}\ g57.3}{u} \tag{22}$$

where $$A_1 = \frac{C_f(X_{pf} + X_m)}{K_{ss}}$$

and $$C_{fe} = \left(\frac{C_f}{1+A_1}\right)$$

$$K_2 = \frac{W_r}{C_r} = \frac{T_r\ g57.3}{u} \tag{23}$$

$$K_3 = K_\phi\left(K_{frs} + \frac{C_\gamma K_{\gamma\phi}}{C_{fe}} - K_{rrs}\right) = K_\phi(K_{rs} + K_{r\gamma}) \tag{24}$$

$$K_4 = \frac{W}{\ell}\left(\frac{X_{pr}}{C_{fe}} + \frac{X_{pf}}{C_r}\right) \tag{25}$$

where

a = distance from total vehicle c.g. to front axle

b = distance from total vehicle c.g. to rear axle

W = total vehicle weight

ℓ = wheelbase (a+b)

W_f = weight on the front wheels ($W_f = bW/\ell$)

W_r = weight on the rear wheels ($W_r = aW/\ell$)

C_f = twice the cornering stiffness of one front tire

C_r = twice the cornering stiffness of one rear tire

X_{pf} = front pneumatic trail, AT_f/C_f (see Note 2)

X_{pr} = rear pneumatic trail, AT_r/C_r (see Note 2)

X_m = mechanical trail

C_γ = twice the camber stiffness of one front tire

K_{ss} = steering system stiffness

K_ϕ = roll angle per g of lateral acceleration

K_{frs} = front roll steer coefficient (positive for a positive roll angle (roll to the right) producing a positive steer angle (to the right))

K_{rrs} = rear roll steer coefficient (positive for a positive roll angle producing a positive steer angle)

$K_{\gamma\phi}$ = average front wheel camber angle per degree of roll angle

T_r = numerator time constant = $m\ a\ u/\ell C_r 57.3$

u = forward velocity

T_{fe} = effective front time constant

K_{rs} = roll steer effect ($K_{rs} = K_{frs} - K_{rrs}$)

$K_{r\gamma}$ = camber effect ($K_{r\gamma} = C_\gamma K_{\gamma\phi}/C_{fe}$)

(Notes: 1) for roll equilibrium

$$-K_\gamma \phi - W_s h \frac{ur}{g} = 0$$

where W_s = sprung weight

h = height of sprung mass c.g. above the roll axis

$$K_r = K_{rf} + K_{rr} - W_s h$$

where K_{rf} = roll stiffness of front suspension

K_{rr} = roll stiffness of rear suspension

then $$\phi = - \frac{W_s h}{K_r} \left(\frac{ur}{g}\right)$$

and $$K_\phi = \frac{W_s h}{K_r} \text{ degs/g}$$

2) The aligning stiffness of the front tires is given by

$$AT_f = C_f X_{pf}$$

for the rear tires

$$AT_r = C_r X_{pr} \,)$$

The largest contributor to understeer comes from K_1. The influence of steering system compliance is to increase K_1 or, effectively, to increase the value of front cornering compliance. Typical values of K_1 range from 8 to 11 degs/g (with A_1 representing the influence of steering compliance, caster, and tire torques estimated to equal approximately 0.25 for cars with power steering or rack-and-pinion manual steering.)*

The quantity K_2 represents the influence of the rear tires and the load on the rear wheels in determining the value of a vehicle's understeer/oversteer gradient. The quantity K_2 and the numerator

*Estimates of vehicle performance parameters for many 1977 subcompact, compact, and intermediate size vehicles are presented in Appendix D.

time constant, T_r, defined by STI in Reference [1], are closely related*, viz.,

$$T_r = \frac{a\ m\ u}{\ell\ C_r(57.3)} = \frac{W_r}{gC_r}\frac{u}{(57.3)} = \frac{K_2 u}{g57.3} \qquad (27)$$

Typically, K_2 ranges from approximately 5.8 to 8.0 deg/g, corresponding to T_r values between 0.23 and 0.32 seconds at 50 mph.

The quantity K_3 indicates the influence on understeer of the most significant roll-related factors. For the cars surveyed in this study, the estimated roll angle per g lateral acceleration (K_ϕ) varied from 5 to 11 deg/g.

The roll steer quantities, K_{frs} and K_{rrs}, are not usually given in published specifications and they often change with vehicle loading. For many cars the influences of front and rear roll steer tend to cancel one another. Accordingly, the influence of roll steer may not be known accurately, but it may not have a large effect on understeer [10].

Usually, the most significant contribution to K_3 comes from the camber stiffnesses of the front tires if these tires are of bias or bias-belted construction. In this regard, radial tires tend to have much less camber stiffness than bias or bias-belted tires [14].

The roll-related terms in Equation (7) add approximately 1.2 to 2.6 deg/g to the understeer. The influence of the roll-related terms is reduced by increasing the roll stiffness of the vehicle.

The rigid body aligning torque terms that make up K_4 total about 0.5 deg/g of understeer. This is a small effect but it is fairly constant from car to car and always acts to increase understeer.

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*For a simple non-rolling vehicle model, the quantity W_r/C_r is the "rear cornering compliance" introduced by Bundorf and Leffert [9]. Recognizing this, much of the work presented in [1] can be related to that of [9]. Furthermore, the work presented herein can be readily related to the original definitions used in the cornering compliance concept.

3.1.2 Changes in Understeer Due to Changes in Vehicle or Tire Factors. The quantity K_1 depends upon the load on the front wheels, the cornering stiffness of the front tires, and the factors contributing to a torque balance in the steering system. In estimating the performance of particular vehicles, knowledge of the steering system stiffness is not generally available. An estimate of the influence of steering system stiffness on understeer can be obtained using representative numbers. For example, if

$$K_{ss} = 200 \text{ ft-lbs/deg}$$

$$AT_f + C_\alpha X_m = 50 \text{ ft-lbs/deg}$$

and $$W_f/C_f = 8 \text{ deg/g}$$

then $$A_1 = 0.25 \text{ and } K_1 = 10 \text{ deg/g}$$

However, if K_{ss} were actually twice as large as originally estimated, that is, 400 ft-lbs/deg (which is not an impossible error given the data available in the literature), then, all else being equal,

$$A_1 = 0.125 \quad \text{and} \quad K_1 = 9 \text{ deg/g.}$$

Roughly a factor of two increase in K_{ss} corresponds to a 1 deg/g change in understeer.

It is believed that vehicles with rack-and-pinion manual steering systems have steering system stiffnesses nearly equal to those of typical power steering systems. However, data presented in [17] indicate that conventional manual steering systems can be at least twice as compliant as power steering systems. Going back to the previous example with $K_{ss} \doteq 100$ ft-lbs/deg (to represent a manual steering system) yields $A_1 = 0.5$ and $K_1 = 12$ deg/g, which indicates an estimated 2 deg/g increase in understeer in changing from power steering to manual steering.

The cornering stiffness of typical passenger car tires depends upon vertical load. For most passenger car tires C_α increases as vertical load increases up to the rated load of the tire at the specified inflation pressure. This characteristic of passenger car tires tends to moderate the influence of changes in vertical load on K_2. Nevertheless, changes in vehicle loading can significantly influence K_2. Currently published vehicle specification data [18] indicate that front seat load is nearly equally distributed to the front and rear tires and approximately 80 to 85% of the rear seat load is reacted at the rear tires, or for three large, rear-seat passengers the vertical load on the rear tires could be increased by about 400 lbs over the load with a nominal condition of two front-seat passengers.

For example, the load on the rear wheels of a compact car could change from 1800 lbs to 2200 lbs with the addition of a load equivalent to three passengers in the rear seat. Using data for an E78-14 tire (tire number 063 in [19]) yields:

$$K_2 = \frac{1}{0.144} = 6.9 \text{ deg/g at 1800 lbs (900 lbs/tire)}$$

and

$$K_2 = \frac{1}{0.122} = 8.3 \text{ deg/g at 2200 lbs (1100 lbs/tire)}$$

These results indicate that a 25% increase in load on the rear wheels will cause approximately a 19% increase in K_2 corresponding to a 1.3 deg/g reduction in understeer.

Without the influence of C_α varying with vertical load, a 25% increase in vertical load on the rear tires in the previous example would have caused a 25% increase in K_2, yielding a 1.7 deg/g reduction in understeer. The curvature of the relationship of cornering stiffness to vertical load and the values of vertical load involved for a particular vehicle determine the value of the change in K_2.

It should be emphasized that (1) tire cornering stiffnesses are the most significant factors determining K_1 and K_2 and (2) tire replacement practices, especially if either the front or rear set of tires are replaced with a different type of tire, can have an important influence on K [10, 14]. For example, if E78-14 bias-belted tires (similar to tire no. 063 in [19]) are replaced by ER78-14 radial tires (similar to tire no. 032 in [19]), K_2 changes from 6.9 deg/g to 5.4 deg/g at approximately 1800 lbs load on the vehicle's rear wheels.

Tire inflation pressure also affects cornering stiffness and there is interaction between the influences of load and inflation pressure on cornering stiffness [14]. Generally, for moderate tire loads, the cornering stiffness does not start to fall off drastically until inflation pressure drops well below 20 psi. Example test results for a G78-14 tire (tire no. 460 in [19]) are as follows:

Cold Inflation Pressure	at 1032 lbs
psi	C_α (lbs/deg)
18	154
25	167
32	178
	at 1374 lbs
18	145
25	167
32	196

These data can be used to evaluate W/C_α as a function of psi and load, viz.,

at 1032 lbs

(Cold Inflation Pressure) psi	W/C_α deg/g
18	6.7
25	6.2
32	5.8

at 1374 lbs

(Cold Inflation Pressure) psi	W/C_α deg/g
18	9.5
25	8.3
32	7.0

Inspection of these results indicates that for a nominal inflation pressure of 25 psi holding inflation pressure within ±6 psi will maintain K_2 within ±0.5 deg/g at 1032 lbs load per wheel while at 1374 lbs load per wheel the inflation pressure would have to be held within ±3 psi to keep K_2 within approximately ±0.5 deg/g.

At present we do not know of a large set of tire data which can be used to evaluate the generality of the results just presented. However, it is straightforward to derive the following expression which can be used to estimate the influence of inflation pressure on K_2.

$$\Delta K_2 \doteq \frac{\partial K_2}{\partial C_\alpha} \frac{\partial C_\alpha}{\partial psi} \Delta psi \tag{28}$$

where $\dfrac{\partial K_2}{\partial C_\alpha} = \dfrac{K_2}{C_\alpha}$,

$\dfrac{\partial C_\alpha}{\partial psi}$ is the influence of inflation pressure on cornering stiffness,

C_α is the cornering compliance for a single tire,

and ΔK_2 is the change in K_2 due to a change in pressure equal to Δpsi.

Or, by rearranging (28),

$$\Delta psi = \left| \frac{C_\alpha}{K_2} (\partial C_\alpha / \partial psi) \right| \cdot |\Delta K_2| \tag{29}$$

where $\pm\Delta psi$ is the tolerance in pressure determined by a selected tolerance in K_2 (that is, $|\Delta K_2|$), and C_α and K_2 are nominal values for a particular vehicle.

Vehicle parameters for estimating K_ϕ are generally available [18] with the exception of h, the distance between the roll axis and the sprung mass c.g. The roll steer quantities are not generally available, particularly the front roll steer parameter, K_{frs}. Example values for the rear roll steer are given in [20]. It is estimated that K_{rs} is usually less than 0.1, meaning that for K_ϕ less than 10 deg/g the roll steer effect is usually less than 1.0 deg/g. (This effect can be small, almost 0.0 deg/g, if K_{frs} and K_{rrs} tend to cancel each other [10].)

The quantity $K_{r\gamma}$ in Equation (24) depends upon tire properties and $K_{\gamma\phi}$ which is the side-to-side average change in camber angle per unit change in roll angle. The quantity $K_{\gamma\phi}$ varies from vehicle to vehicle from approximately 0.7 to 1.0 with an average value around 0.88. For bias and belted-bias passenger car tires the ratio of camber stiffness to cornering stiffness is approximately 1/6. Accordingly, $K_{r\gamma} \doteq 0.18$ for $A_1 = 0.25$. If $K_\phi = 10$ deg/g, for example, then the camber term may contribute approximately 1.8 deg/g to a vehicle's understeer.

Examination of vehicle specifications indicates that for some vehicles equipped with radial tires large caster angles on the order of 3 or 4 degrees are used. This practice compensates for the loss of understeer due to the relatively low camber stiffness of radial tires. The significance of caster angle, which determines the mechanical trail, X_m, depends upon the steering system stiffness.

The addition of an anti-roll bar to a suspension adds an additional roll stiffness approximately equal to the roll stiffness due to the suspension springs [20]. For typical vehicles the front roll stiffness (for a vehicle without an anti-roll bar) may be approximately twice the rear roll stiffness and the addition of a front anti-roll bar will increase the total roll stiffness by about 60%. For example, the addition of a front anti-roll bar to a vehicle with K_ϕ = 10 deg/g could result in K_ϕ being reduced to about 6 deg/g. If the original vehicle had 2 deg/g of understeer due to roll-related factors, then the roll stiffened vehicle (in this example) would have 1.2 deg/g of understeer from roll. A penalty for this additional roll stiffening accrues from the rough ride experienced when one front wheel hits a bump or chuckhole and the other front wheel is on smooth road. Nonetheless, for typical vehicles, the addition of an anti-roll bar can lead to about a 10% increase in steady-state gain resulting from a 16% decrease in understeer.

This section has presented numerous examples to illustrate the influence of various vehicle parameters on understeer. Usually a number of design factors contribute to the overall understeer of a particular vehicle, and, accordingly, any of a number of design factors could be modified to change the yaw rate gain of a selected vehicle. Nonetheless, changing steering ratio is conceptually the simplest way of changing yaw rate gain. Furthermore, N_G is virtually the only design parameter that can be used to alter yaw rate gain without having any effect on vehicle transient response characteristics.

3.2 Relationships Between Vehicle Characteristics and the Transient Response Specifications

A theoretical foundation for the vehicle dynamics requirements to be used in this project was developed in [1] using a two-degree-of-freedom model for the non-rolling vehicle. The vehicle model presented in [1] can easily be related to the analysis presented in References [9 and 21], in which the cornering compliance concept was used to provide an understanding of the basic vehicle factors influencing transient response. Nevertheless, experience in this project at attempting to match vehicle test data with a two-degree-of-freedom model indicated difficulties in matching the initial yaw acceleration and the time at which the maximum yaw rate response to a rapid, ramp-step steering input occurs.

The difficulties were associated with the treatment of the roll degree of freedom which is either neglected or handled quasi-statically in the two-degree-of-freedom model.

By using a simplified three-degree-of-freedom model, the roll mode was seen to have a significant influence on the time and the magnitude of the maximum yaw rate response. In a dynamic maneuver, like a ramp-step steering maneuver, the timing of the understeer properties due to vehicle roll (that is, the camber forces and the roll steer effects) determine the time and magnitude of the peak yaw rate response.

Nevertheless, a two-degree-of-freedom model which includes roll effects in an approximate manner will be presented hereinafter to provide an analytical understanding of pertinent factors influencing transient yaw response. The simplified three-degree-of-freedom model used for data matching experiments is described in Appendix E, and complete descriptions of three-degree of freedom models can be found in the fundamental work of Segel [15, 22] and in Weir, Shortwell, and Johnson [16].

Similar to the situation presented in Section 3.1.1 (pertaining to steady-state yaw rate gain), numerous vehicle dynamics considerations and lengthy algebraic manipulations are required to develop the differential equations describing vehicle response in transient (dynamic) maneuvers. However, only the basic equations and assumptions will be given here before presenting the final results. The intent is to concentrate on the implications of these results with respect to the requirements of this steering controllability project rather than to emphasize their derivation.

The equations of motion used in the simplified two-degree-of-freedom model employed herein are:

$$m(\dot{v} + ur) = F_f + F_r \qquad (30)$$

$$I\dot{r} = a'F_f - b'F_r \qquad (31)$$

where m is the mass of the vehicle

v is the lateral velocity

u is the forward velocity

r is the yaw rate

F_f is the lateral force from the front tires

F_r is the lateral force from the rear tires

I is the yaw moment of inertia

$a' = a - X_{pf}$

$b' = b + X_{pr}$

The forces are given by:

$$F_f = -C_f\alpha_f + C_\gamma K_{\gamma\phi}\phi$$

$$F_r = -C_r\alpha_r$$

where α_f is the slip angle of the front wheels

α_r is the slip angle of the rear wheels

ϕ is the roll angle of the sprung mass

The steering system is treated "quasi-statically" using the following equation for a moment balance in the steering system:

$$0 = K_{ss}\left(\frac{\delta_{sw}}{N_G} - \delta_f\right) - C_\gamma K_{\gamma\phi}\phi X_m - \left(-C_f\alpha_f(X_m + X_{pf})\right) \tag{32}$$

Or, by rearranging (32)

$$\delta_f = \frac{\delta_{sw}}{N_G} - \frac{C_\gamma K_{\gamma\phi}\phi X_m}{K_{ss}} + \frac{C_f\alpha_f(X_{pf} + X_m)}{K_{ss}} \tag{33}$$

A major assumption of this model is that roll motion can be treated quasi-statically using the following equation:

$$\phi \doteq \frac{-W_s h}{K_r} \left(\frac{ur}{g}\right) \qquad (34)$$

$$\left(\text{let } K_{\phi 1} = \frac{W_s h}{K_r}, \text{ radians}\right)$$

Equation (34) is the same expression as that employed in the steady-state analysis. Comparison of transient responses calculated with a two-degree-of-freedom model employing (34) with calulations from a three-degree-of-freedom model indicates that the rolling vehicle will respond a little more quickly in yaw than predicted by the two-degree-of-freedom model. Example calculations show that the peak yaw rate response to a ramp-step input is obtained sooner in calculations including roll dynamics than in calculations employing the quasi-static assumption. Nonetheless, the frequency response of the two-degree-of-freedom model is expected to match approximately the frequency response of the three-degree-of-freedom model throughout the mid-frequency range [2] while only failing to match at the higher frequencies excited by rapid steering inputs.

(The front and rear slip angles, α_f and α_r, needed for computing tire forces are defined in the steady-state analysis presented in Appendix C.)

Based on the considerations and assumptions just presented and neglecting small terms resulting from rigid body aligning torque effects, the following expression for the yaw rate transfer function is obtained.

$$\frac{r(s)}{\delta_{sw}(s)} = \left[\frac{\left.\frac{r}{\delta_{sw}}\right|_{ss} (T_r s + 1)}{\left[G_r \left(\frac{mIu}{C_{fe} C_{r\ell}}\right)s^2 + G_r\left(\frac{m(a^2 C_{fe} + b^2 C_r) + I(C_{fe} + C_r) - u^2 A_5 K_{\phi 1} m}{C_{fe} C_{r\ell}}\right)s + 1\right]}\right] \qquad (35)$$

where $\left.\frac{r}{\delta_{sw}}\right|_{ss}$ is given by Equation (4)

$$G_r = \left.\frac{r}{\delta_{sw}}\right|_{ss} N_G \quad \text{(the steady-state gain to a "reference" front wheel angle)}$$

$$A_5 = (aA_2 - bA_3)/g \text{ in which}$$

$$A_2 = C_{fe}K_{frs} - \frac{C_{fe}C_\gamma K_{\gamma\phi}X_m}{K_{ss}} + C_\gamma K_{\gamma\phi}$$

$$A_3 = C_r K_{rrs}$$

The numerator time constant is T_r, as defined in the steady-state analysis (even though roll effects have been included). The terms in the denominator of (35) can be used to express the natural frequency and the damping ratio in terms of vehicle parameters, viz.,

$$\frac{1}{\omega_n^2} = G_r \frac{mIu}{C_{fe}C_r\ell} \tag{36}$$

$$\frac{2\zeta}{\omega_n} = \frac{G_r m}{C_{fe}C_r\ell}\left[(a^2C_{fe}+b^2C_r) + \frac{I}{m}(C_{fe}+C_r) - u^2 A_5K_{\phi 1}\right] \tag{37}$$

Fortunately, earlier work by Bundorf and Leffert [9] indicates useful means for interpreting Equations (36) and (37). First, considering natural frequency, let

$$I = abm(1 + k')$$

where $-0.1 < k' < 0.2$ for typical cars [9]. Then (36) can be expressed as

$$\frac{1}{\omega_n^2} = \frac{G_r\ell}{u}\left(\frac{u}{g57.3}\right)^2 K_1K_2(1+k') \tag{38}$$

Furthermore, for $k' \doteq 0$, $\frac{K_2}{2} \leqq K_1 \leqq 2K_2$, and

$$|(K_1+K_2)| >> |\frac{K_{\phi 1} u^2}{\ell g} (K_2 \frac{A_2}{C_{fe}} + K_1 K_{rrs})| ,$$

$$\zeta^2 \doteq (\frac{\ell G_r}{u}) = \frac{N_G \left.\frac{r}{\delta_{sw}}\right|_{ss}}{u} \qquad (39)$$

Equation (39) allows one to estimate the damping ratio, ζ, from steady-state gain, steering ratio, wheelbase, and velocity. This estimate for ζ could possibly be 30% low in an extreme case in which the roll-related factors are large. However, damping ratio is not a crucial factor in this project, that is, vehicle design is not tightly constrained by the requirement that $\zeta|_{50}$ (i.e., ζ at a vehicle velocity of 50 mph) be greater than 0.5 as specified in [1].

Using Equation (39), a "worst case" estimate of damping ratio can be made for passenger cars with yaw rate gains meeting the requirements of the specified performance space. The lower boundary of the required performance space is at $r/\delta_{sw}|_{ss} \doteq 0.14$. For a subcompact car with a wheelbase of approximately eight feet and a steering ratio of 19, Equation (39) yields a worst case damping ratio (at 50 mph) equal to 0.54. Accordingly, it is estimated that for typical vehicles (of subcompact, compact, or intermediate size) the damping ratio will be greater than 0.5 (at 50 mph) if the yaw rate gain falls within the specified performance space.

Equation (39) is in the most convenient form for working with measured values of steady-state gain. However, it can be restated in terms of understeer as follows:

$$\zeta^2 \doteq \left[\frac{1}{\left(1 + \frac{u^2 K}{g\ell 57.3}\right)}\right] \qquad (40)$$

(Note that damping ratio is independent of steering ratio as should be expected.)

For 50 mph, $\zeta = 0.5$, and $\ell = 8$ ft (a subcompact car), Equation (40) is satisfied if K = 8.3 deg/g. For passenger cars with wheelbases greater than 8 feet and values of K less than 8.3 deg/g, the damping ratio will be greater than 0.5. For a neutral steer vehicle K = 0 and $\zeta \equiv 1$, as indicated by Equation (40). Thus, it is estimated that at 50 mph over the range of damping ratio between 0.5 and 1.0 ($0.5 < \zeta < 1.0$), vehicles can vary from highly understeer (K = 8.2 deg/g) to neutral steer (K = 0.0 deg/g).

The given constraint on natural frequency is that ω_n at 50 mph be greater than 3.0 rad/sec. For examining the relationship between vehicle parameters and ω_n it is convenient to re-express (38) in the following form.

$$\omega_n = \left[\frac{1 + K\left(\frac{u^2}{g\ell 57.3}\right)}{T_r T_{fe}(1+k')}\right]^{1/2} = \left[\frac{1 + K\frac{u^2}{\ell g 57.3}}{K_1 K_2\left(\frac{u}{g57.3}\right)^2(1+k')}\right]^{1/2} \tag{41}$$

By examining the results of the survey of typical cars presented in Appendix D, it is seen that the following conditions are almost always satisfied: K > 3 deg/g and $K_1K_2 < 80\ (\text{deg/g})^2$. Using these numbers to calculate natural frequency for a vehicle with $\ell = 10$ ft, $k' \doteq 0$, and u = 50 mph yields a very conservative lower bound of $\omega_n\big|_{50\text{ mph}} \geqq 3.8$ rad/sec for currently-produced subcompact, compact, and intermediate size automobiles.

Theoretically, the effective time constant, T_e, now used to define the boundary of the optimum region can be evaluated in a straightforward manner, but this requires measuring or evaluating numerous vehicle parameters. Consequently, the quantity T_e is easier to determine directly from full-scale vehicle experiments than from an analysis based on measured or calculated parametric data.

Nevertheless, an understanding of the factors influencing T_e can be obtained through consideration of the following form of the yaw rate transfer function.

$$\frac{r(j\omega)}{\delta_{sw}(j\omega)} = \left[\frac{\left.\frac{r}{\delta_{sw}}\right|_{ss}(T_r j\omega + 1)}{(\frac{j\omega}{\omega_n})^2 + (\frac{2\zeta}{\omega_n})j\omega + 1}\right] \tag{42}$$

where

ω is frequency

j is the complex number $\sqrt{-1}$, and

$\left.\frac{r}{\delta_{sw}}\right|_{ss}$, T_r, ω_n, and ζ are as defined previously.

(It should be emphasized that a vehicle speed of 50 mph (73.33 ft/sec) applies throughout this discussion.) The phase angle, ϕ_y, of the yaw rate transfer function (42) is given by:

$$\phi_y = \tan^{-1}(T_r\omega) - \tan^{-1}\left(\frac{2\zeta\omega}{\omega_n\left(1 - (\frac{\omega}{\omega_n})^2\right)}\right) \tag{43}$$

For $\phi_y = -45°$, that is, for $\omega = \omega_e = 1/T_e$, the properties of the tangent function can be used to deduce the following equation from Equation (43):

$$\omega_e^3 + (\frac{1}{T_r} - 2\zeta\omega_n)\omega_e^2 + (\frac{2\zeta\omega_n}{T_r} - \omega_n^2)\omega_e - \frac{\omega_n^2}{T_r} = 0 \tag{44}$$

Equation (43) can be solved numerically for ω_e if T_r, ζ, and ω_n are known. Nevertheless, it provides insight to consider solving for T_e graphically. Figure 8 shows the phase lead obtained from the numerator term in (42) and Figure 9 shows the phase lag determined by the denominator of (42). For given values of T_r, ζ, and ω_n the

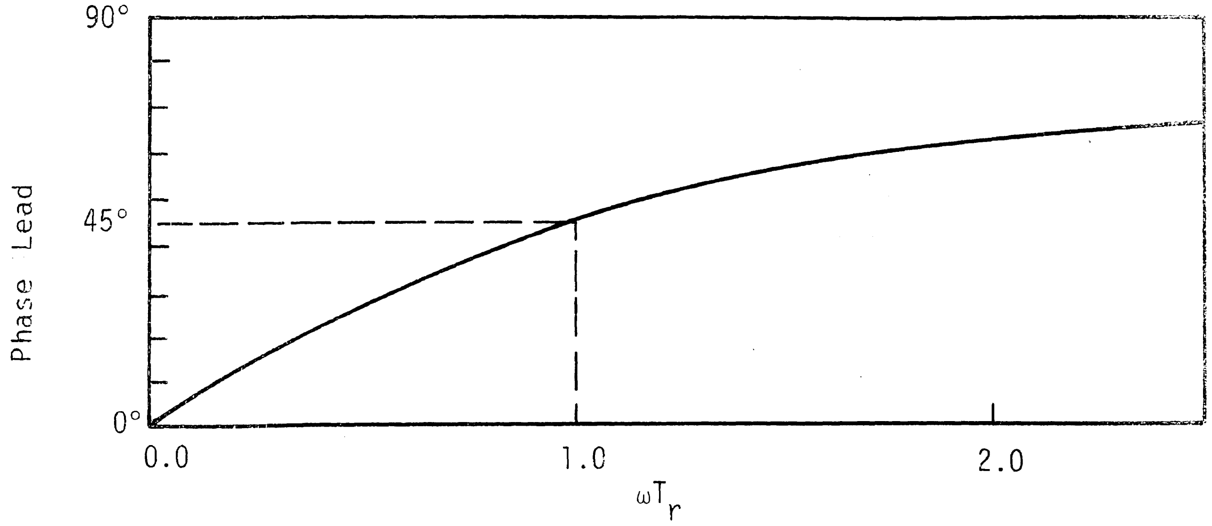

Figure 8. Phase shift due to r/δ_{sw} numerator.

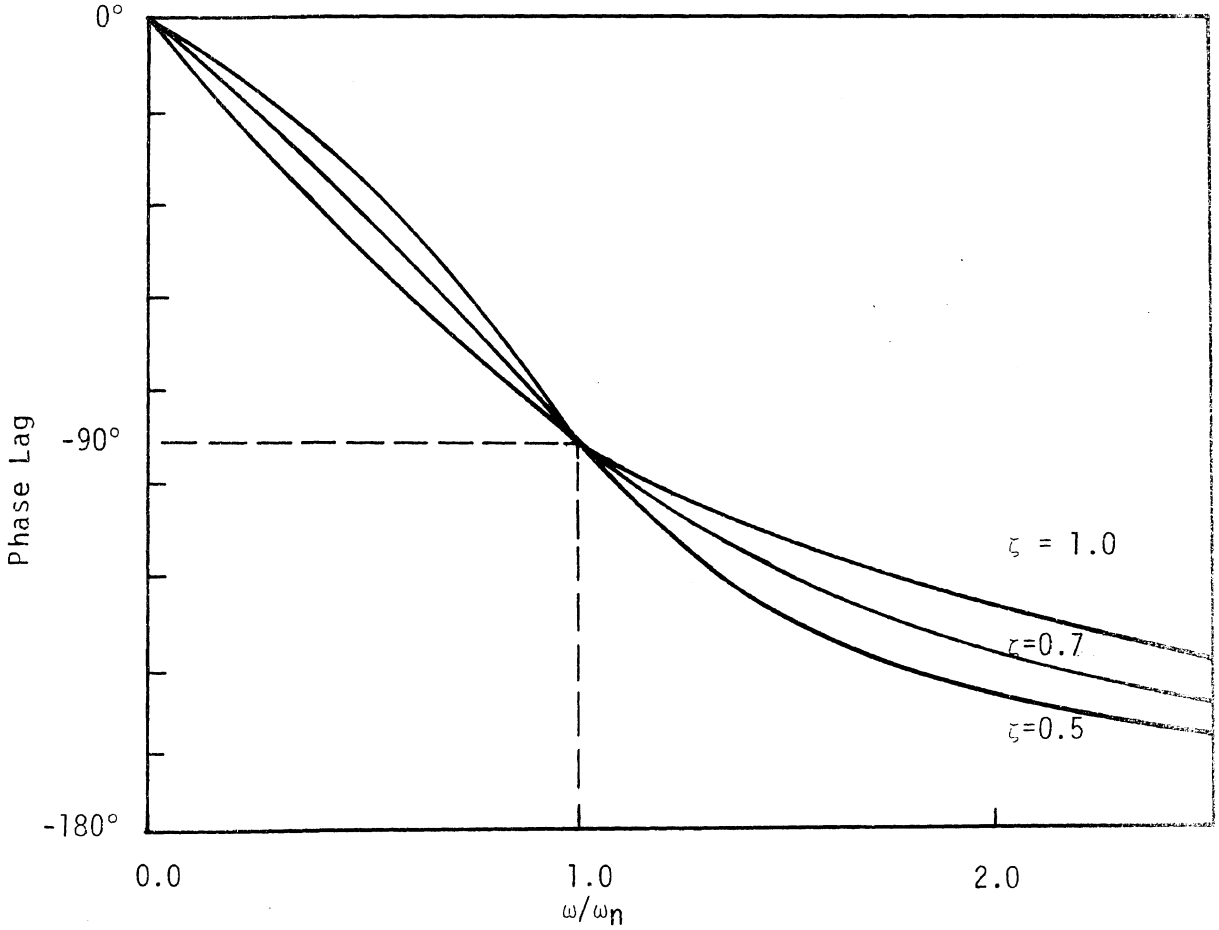

Figure 9. Phase shift due to r/δ_{sw} denominator.

frequency at which -45° phase shift occurs can be found using Figures 8 and 9 and a trial-and-error approach.

Inspection of Figures 8 and 9 indicates that if $1/T_r < \omega_n$, then $\omega_e > \omega_n$. This result can be shown readily from the following simple analysis. At $\omega = \omega_n$ the denominator of (42) yields 90° phase lag for all values of damping ratio. If $1/T_r$ is less than ω_n, then the numerator of (42) yields more than 45° phase lead at $\omega = \omega_n$. Consequently, the value of ω at which 45° phase lag occurs must be greater than ω_n.

The foregoing discussion can be used to develop an estimate of vehicle characteristics such that T_e will be within the boundary of the SPS. Examination of the SPS indicates that if T_e (for a particular vehicle) is less than 0.238, then the effective time constant will be small enough. (The inequality $T_e < 0.238$ sec^{-1} is equivalent to the inequality $\omega_e > 4.2$ rad/sec where $\omega_e = 1/T_e$.) Accordingly, if

$$1/T_r < \omega_n \tag{45}$$

and

$$\omega_n > 4.2 \tag{46}$$

then it follows that $T_e < 0.238$.

For current domestic vehicles $1/T_r$ is less than ω_n. Even at neutral steer (K=0) with $K_2 \doteq K_1$ and $k' = 0$, Equation (41) indicates that $\omega_n \doteq 1/T_r$. For values of $K > 0$, ω_n will be greater than $1/T_r$ if the difference between K_2 and K_1 is not too large. To quantify the magnitude of the allowable difference, let $F_1 K_2 = K_1$. Then from (41)

$$\omega_n = \frac{1}{T_r}\left[\frac{1 + K\,u^2/g\ell 57.3}{F_1(1+k')}\right]^{1/2}$$

Thus, for $k' \doteq 0$, ω_n is greater than $1/T_r$ if

$$\left(\frac{1 + K\ u^2/g\ell 57.3}{F_1}\right) > 1$$

or if

$$(1 + K\ u^2/g\ell 57.3) > (K_1/K_2) \tag{47}$$

Combining the approximate result for ζ^2 from (40) with (47) yields the restriction

$$\left(\frac{1}{\zeta^2}\right) K_2 > K_1 \tag{48}$$

for $1/T_r < \omega_n$ (at 50 mph) for ζ evaluated at 50 mph.

Calculations based on the estimates made in the vehicle survey (Appendix D) indicate that the inequalities $\omega_n\big|_{50\text{ mph}} > 4.2$ and $\omega_n\big|_{50\text{ mph}} > 1/T_r$ will nearly always be satisfied for typical subcompact, compact, and intermediate cars. In general, if K_1 is considerably larger than K_2, the value of understeer, K, will be large, implying that the damping ratio at 50 mph will be relatively small (but larger than 0.5). For the values of cornering compliances, K_1 and K_2, applicable to typical cars, the natural frequency at 50 mph will be greater than both 4.2 rad/sec and $1/T_r$.

In summary, it appears from this analytical consideration of vehicle dynamics that currently produced subcompact, compact, and intermediate cars have transient response properties at 50 mph which satisfy the specified performance requirements on ζ, ω_n, and T_e.

4.0 VEHICLE TESTS

Two series of vehicle tests were conducted for this research study. The first series, referred to as the preliminary tests, were conducted on the vehicles tentatively chosen as subjects for the modification program. Generally, vehicles were chosen because the analytical work conducted during the vehicle survey indicated that they were probably outside or near the boundaries of the optimum space. Preliminary tests were conducted to confirm these analytical results and to precisely identify the dynamic characteristic of the test vehicles. Later in the study a verification test program was conducted on the test vehicles which had been selected and then modified to conform to the SPS. These tests were conducted in order to verify that the accomplished modifications did indeed result in vehicles whose dynamic properties placed them near the center of the SPS. Also, cursory closed-loop handling tests, using both modified and unmodified vehicles, were conducted to confirm that modified vehicles were acceptable highway vehicles and to tentatively judge the handling value of the modifications.

The preliminary test program consisted totally of open-loop testing designed to measure the handling qualities germane to the SPS. The verification test series consisted of these same open-loop tests plus a series of closed-loop tests. In the following subsections, these test programs will be described under the headings:

Test Equipment and Instrumentation

Test Procedures and Results

- Open-Loop Tests
- Closed-Loop Tests

4.1 Test Equipment and Instrumentation

The instrumentation package used in the vehicle testing measured five operating variables, viz.:

1. Steer angle (measured at the input end of the steering column), δ_{sc}
2. Steering wheel torque*, T_{sw}
3. Yaw rate, r
4. Roll rate*, p
5. Velocity, V

In addition, on-board analog computations were performed to make the following signals available:

1. Yaw acceleration, $\dot{r}$
2. Steady-state lateral acceleration, rV
3. Steady-state turn radius, V/r

Data signals were recorded on magnetic tape using an on-board FM tape recorder. A light beam oscillograph was also installed in the test vehicle so that recorded data could be played back, on line, providing a visual check on data system integrity. Later, the tapes were also played back into a hybrid computer where the data was ultimately reduced via an automated process. In the field, an additional "control voltage" level was also recorded on one channel of tape. This signal is used to indicate the mode of system operation (calibrate, standby, test, etc.) to the computer reduction program. The control signal level could be selected by hand or could automatically be switched from standby to test via a "drag switch" arrangement installed on the test vehicle. This automated system was used primarily to lessen driver burden during closed-loop testing. A second advantage was to precisely locate the vehicle on

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*Steering wheel torque and roll rate transducers were employed only in the verification test program.

the test course for purposes of data evaluation.

Both steering-wheel torque and steer angle transducers were incorporated into a "variable ratio steering-wheel limiter" (VRSWL) fitted to the vehicle steering column. The device is shown mounted in a test vehicle in Figure 10. In addition to the two transducer functions, this device also allows for simple and quick variations in the overall steering ratio and provides a steering-wheel stop mechanism for open-loop, step-steer tests. By selecting the appropriate set of change gears for installation in the device, gear ratios between steering wheel and steering column may be varied from 4:1 to 1:4.

A schematic diagram of the instrumentation system appears on Figure 11. A more detailed description of test vehicle equipment and instrumentation appears in Appendix B.

4.2 Test Procedures and Results

In the following two subsections, specific procedures for the open- and closed-loop test series will be described. In the case of both test series it was necessary to carefully control both tire condition and vehicle loading condition since both of these variables are known to influence the vehicle handling properties of interest.

For all vehicles tested, loading consisted of driver plus instrumentation. The instrument package weighed a total of 280 lb, and the majority of this mass was located in the front passenger seat. (See Appendix B for specific details.) Tests were conducted with gas tanks in a 1/2 to full condition.

Tire conditions were also carefully controlled. In the preliminary test series, tires installed on the vehicle by the rental agency were used if (1) they were consistent with the OE tire for that vehicle in size and construction type, (2) the same tire was installed on all wheels, (3) the tire was judged to be in good condition for testing. Otherwise, tires were replaced to meet these

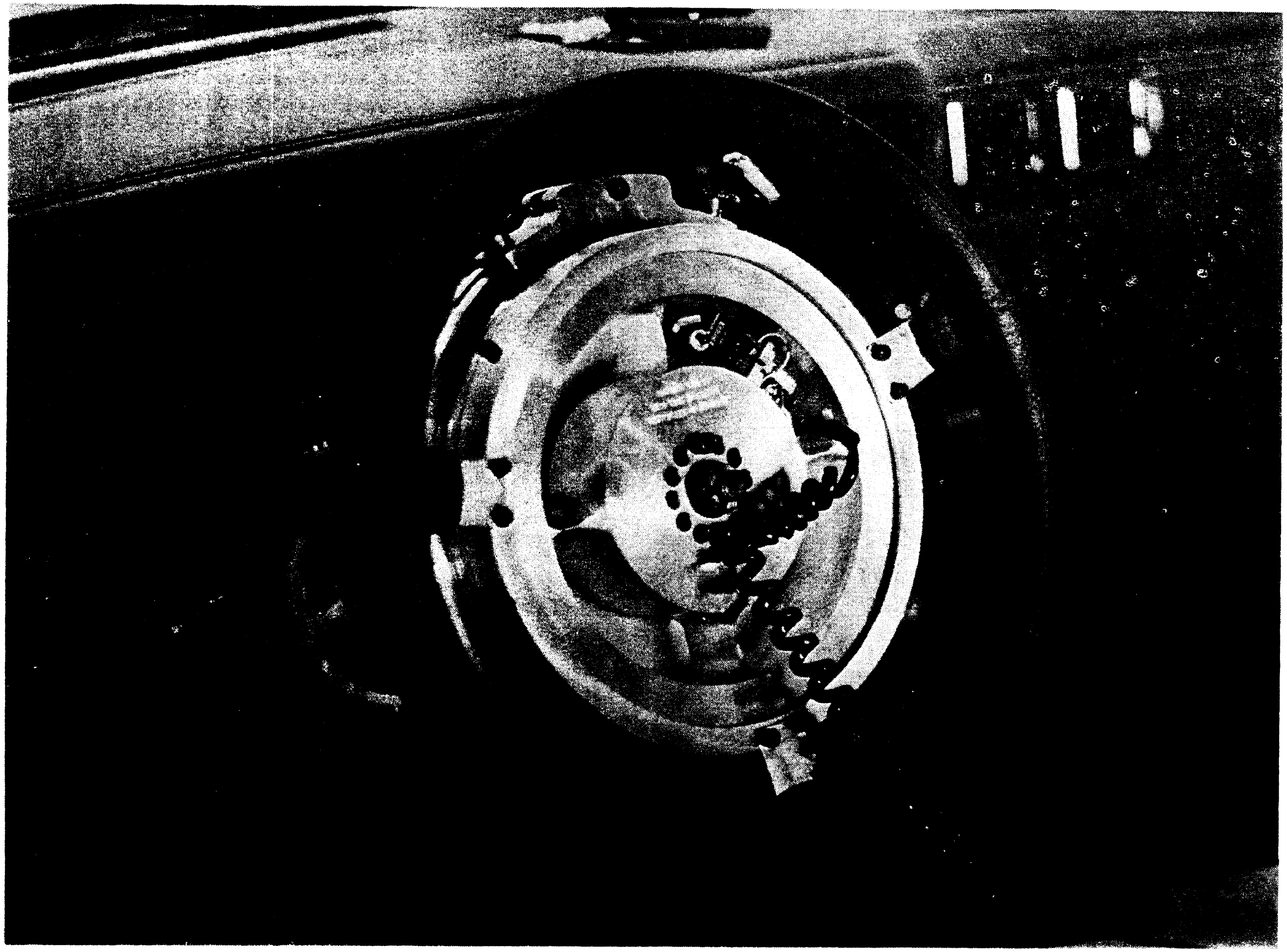


Figure 10. The Variable Ratio Steering Wheel Limiter

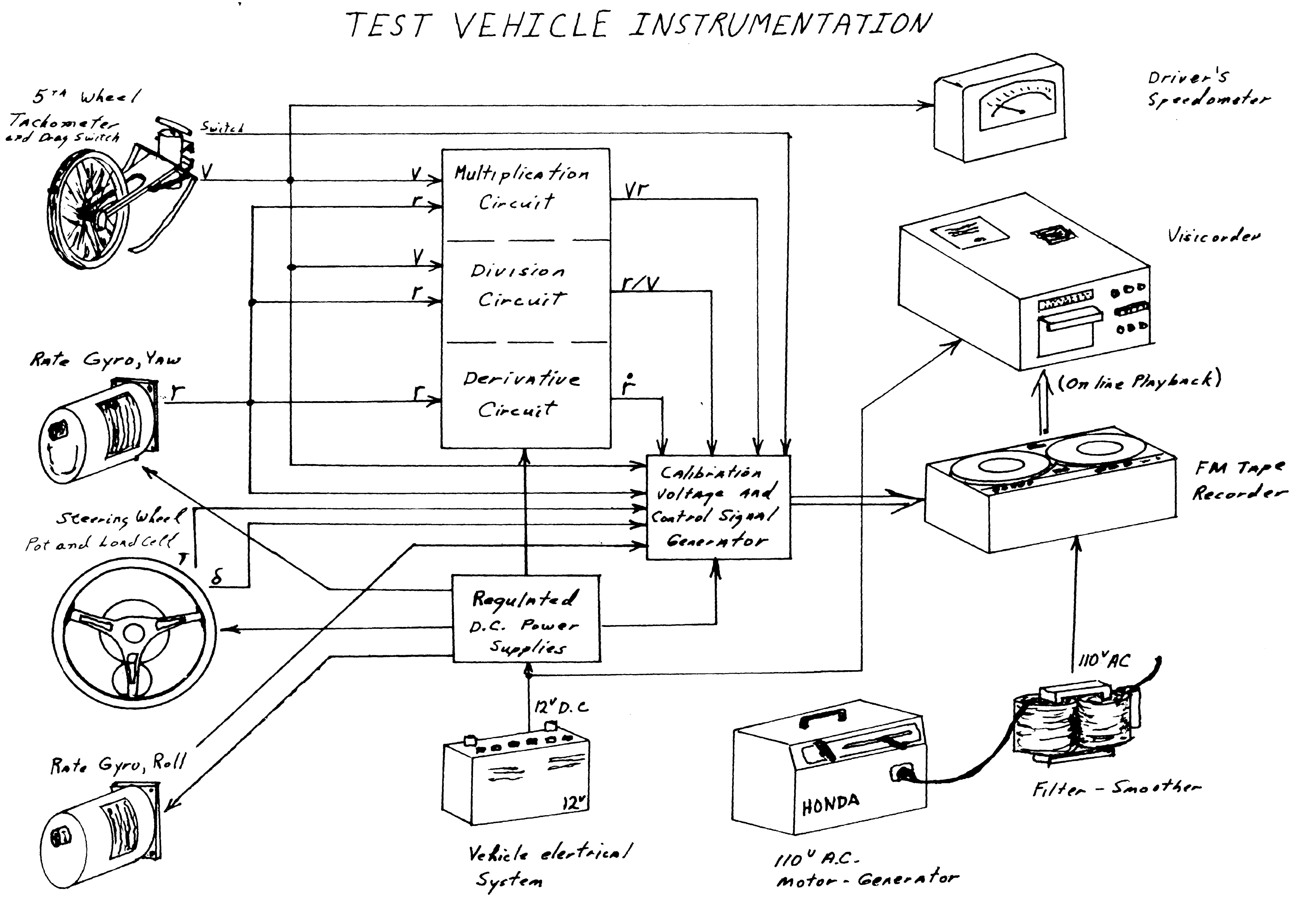


Figure 11. Test vehicle instrumentation

conditions. For the validation testing program, in which a limited number of vehicles were tested, new OE tires were installed. Tires were inflated to the vehicle manufacturer recommended cold pressures. The resulting hot pressures were determined and inflation pressures maintained at this level throughout testing.

4.2.1 Open-Loop Test Series. The purpose of the open-loop test series was to provide data for the determination of handling properties of the subject vehicles, be it a modified or unmodified vehicle. Of most immediate interest was the measurement of the two properties, yaw rate gain, r/δ_{SW}, and the equivalent yaw rate time constant, T_e,* where both values are determined for 50 mph vehicle velocity and for linear regime performance.

Data was obtained from two test procedures, viz.:

1) "Step" steer tests

2) "Pulse" steer tests

Procedures for these tests were as follows:

1) Step steer tests. With the vehicle initially traveling in a straight line at a steady speed of 50 mph, the driver displaces the steering wheel as rapidly as possible up to the level predetermined by the setting of the steering-wheel stop mechanism. This level of steer input and the 50-mph speed level are maintained until the vehicle has established and maintained a steady-state turn for several seconds. This procedure is repeated twice each, for steer inputs corresponding to $\pm.8\delta_0$, $\pm\delta_0$, $\pm1.2\delta_0$, and $\pm1.4\delta_0$ where δ_0 is predetermined as follows.

*T_e is defined as the inverse of the frequency at which 45° phase shift occurs for the vehicle's yaw rate transfer function. Early in the program, the numerator time constant, T_r, was used. However, both time constants are derivable from similar vehicle data.

A "cone course" consisting of a narrowly confining lane of a constant, 668-foot radius curve of sufficient length to provide several seconds of steady-state turning at a speed of 50 mph, is established. (This arc, traversed at 50 mph, produces .25 g lateral acceleration.) A skilled driver negotiates this course at a steady velocity of 50 mph, repeating the procedure several times for turns of both directions. During these runs, the driver observes (via the index steering wheel described in Appendix B) the level of steady-state steer angle required to negotiate the course. (Steering level may also be observed via recorded data, although the precision obtainable in the field using this method may not be satisfactory.) The observed steer angle is defined as δ_0.

The steady-state response of the vehicle to this test series is used to obtain a measure of the vehicle's yaw rate gain. The transient response of the vehicle to the step-like steer inputs is transformed to the frequency domain and is used in evaluating the vehicle time constants.

2) "Pulse" steer test. With the vehicle initially traveling in a straight line at a steady speed of 50 mph a series of both right- and left-hand steering pulses are input to the vehicle. Magnitude of the pulses are in the range of steering inputs used in the step steer tests of the same vehicle. Rate and spacing of the pulses are varied by the driver over a wide range in an attempt to obtain strong frequency content throughout the 1 to 10 rad/sec range. The procedure is repeated four times.

   Steering input and yaw rate response data gathered from these tests are transformed to the frequency domain and are used in obtaining T_e for the vehicle.

Data from the open-loop step steer and pulse steer test procedures discussed above were digitally processed by Fourier transform methods to obtain tabulated yaw rate to steering wheel angle vehicle transfer functions. The transfer function information consisted of yaw rate gain and phase angle tabulations versus frequency. The effective time constant, T_e, for each test was obtained by noting the frequency, ω_e, at which 45° of phase shift occurs. The inverse of this value is, by definition, T_e.

Appendix A discusses in greater detail the methods and programs used for analyzing the two test responses. The step responses were processed by a Fourier technique used by Samulon [23] as derived from earlier work by Bedford and Fredendall [24]. The pulse steer inputs were processed by a standard Fourier series transform. The T_e values obtained for a given vehicle from the two separate test procedures and analyses were in close agreement. Table 4 summarizes the T_e values obtained from both step response and random steer input tests for several vehicles.

Table 4. Yaw Rate Effective Time Constant Results of Preliminary Test Series.

Vehicle	Effective Time Constant T_e (sec) Pulse Steer	Step Steer
Ford Pinto:		
Manual Steering		.14
Power Steering	.16	.15
Plymouth Fury:		
Manual Steering	.17	.19
Power Steering	.18	.19
Chevrolet Nova:	.16	.17
Buick Skylark:	.18	.15

In addition to the T_e calculations for each vehicle, steady-state gain values were obtained from the step steer tests. Changes in the steady-state yaw rate were divided by the corresponding changes in steering angle for different levels of steer and then averaged. Table 5 shows the steady-state gains as evaluated for each vehicle.

Table 5. Steady-State Yaw Rate Gain Results of Preliminary Test Series

Vehicle	Yaw Rate Gain $(sec)^{-1}$
Ford Pinto:	
Manual Steering	.12
Power Steering	.16
Plymouth Fury:	
Manual Steering	.09
Power Steering	.13
Chevrolet Nova:	.15
Buick Skylark:	.18
Ford LTD II:	.18
Plymouth Aspen	.18

During the course of the testing, a modification of the standard step steer maneuver helped facilitate the yaw rate gain calculation. The modification was to initiate the step steer not from a straight course or zero steer level, but instead from a small steer angle level. The purpose of the initial small steer angle was to take up any play in the steering system and thus provide a more definite initial reference condition. A substantial improvement in consistency was noted in the run-to-run yaw rate gain calculations using this test procedure.

The yaw rate gains and the effective time constants measured for the modified and unmodified Pinto, Skylark, and Fury used in the closed-loop test program are plotted on a graph showing the boundaries of the optimum space in Figure 12.

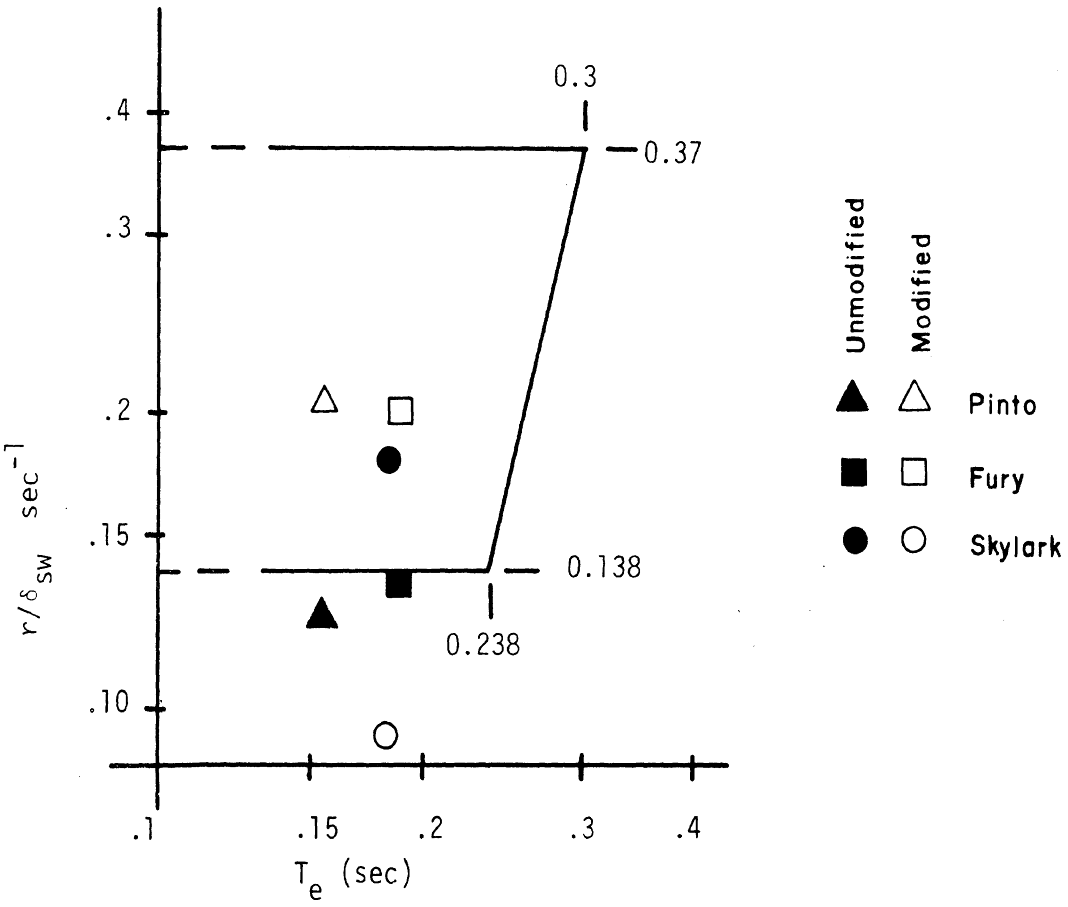


Figure 12. Steady-state yaw rate gain and effective time constant of modified and unmodified test vehicles at 50 mph.

4.2.2 Closed-Loop Test Procedures and Results. The plan for this project allowed for a "cursory" closed-loop vehicle test program only. Thus, relatively simplistic test methods were necessary. A procedure in which the driver was requested to guide the vehicle through a tightly restricted cone course was chosen as the primary closed-loop test method.

The course was designed to elicit both transient and steady-state vehicle response within the linear (up to ~.3 g) performance regime. The attempt was made to arrange the course such that the driver would input steering time histories with strong frequency content up to and beyond the $1/T_e$ range (4~6 rad/sec). Further, the course was designed to represent a significant, but not overwhelming, challenge to the driver in terms of cone strikes such that this parameter could be used as an objective measure in comparing performance attained with the modified and unmodified cars.

The general layout of the course which was chosen is shown in Figure 13. The course was designed to be run at a vehicle speed of 50 mph. The curved portion of the course is a constant radius (670 ft) turn which, at steady-state, produces lateral acceleration of .25 g at 50 mph. The lane changes also produce approximately .25 g lateral acceleration when taken at 50 mph. Although they do not require high acceleration, the lane changes are sufficiently short as to elicit relatively abrupt steering inputs. The step transitions from straight section to curve and then back to straight also require step-like steering changes which have relatively strong frequency content at the levels of interest.

The width of the lane used in the course was determined empirically. Prior to actual testing with each particular vehicle, two test drivers (not those used in actual testing) drove the course repeatedly, adjusting lane width until the desired degree of difficulty was attained. In testing, the lane width was maintained the same for both the modified and ummodified versions of a particular car, but was varied for each of the three test cars.

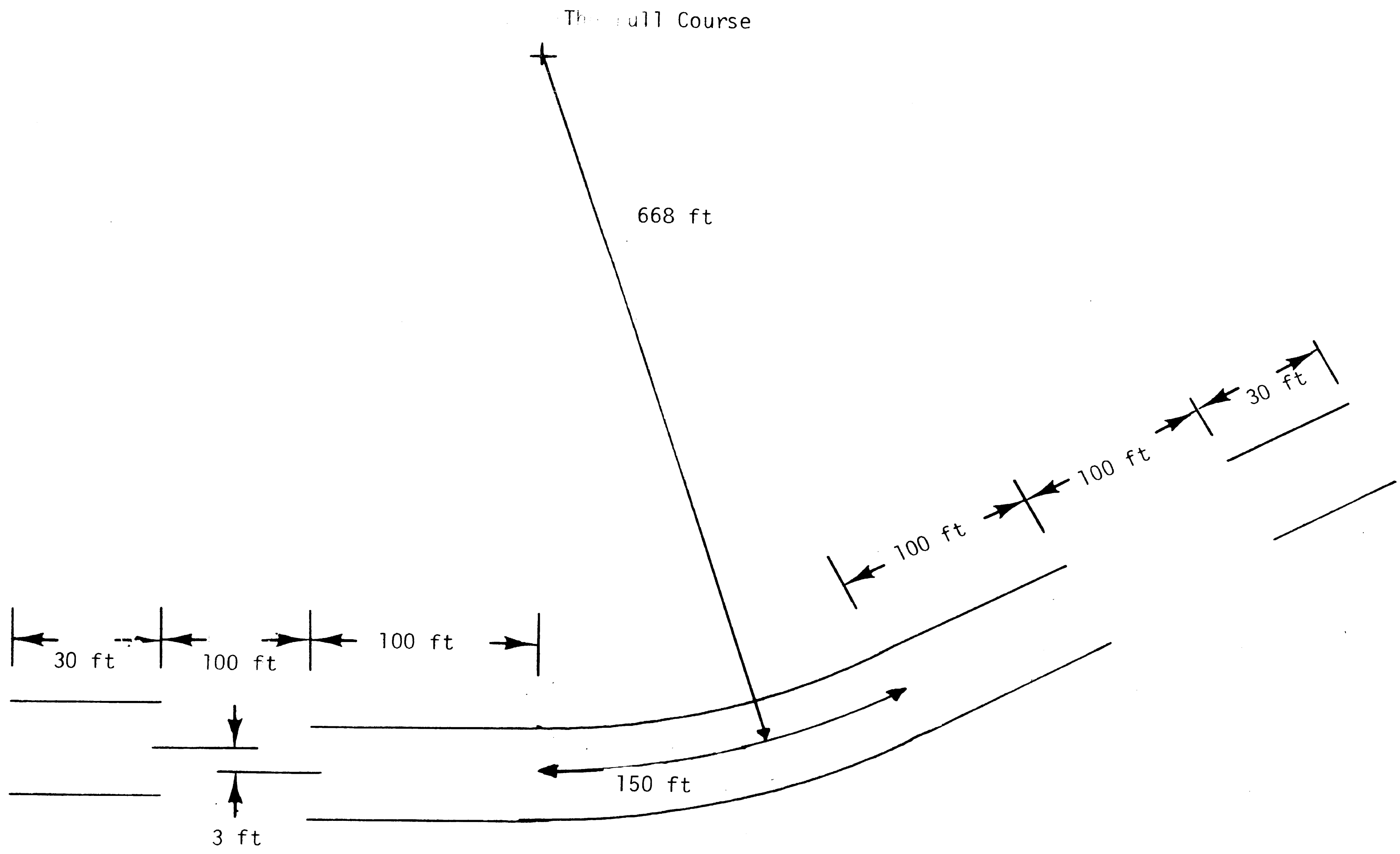


Figure 13. Layout of the cone course used in closed-loop driver-vehicle system testing.

Referencing lane width to the overall track width of the vehicle (see Figure 14), the lane clearance for the cars tested was: (1) Plymouth Fury, 9 1/2 inches; (2) Buick Skylark, 8 1/2 inches; (3) Ford Pinto, 6 inches.

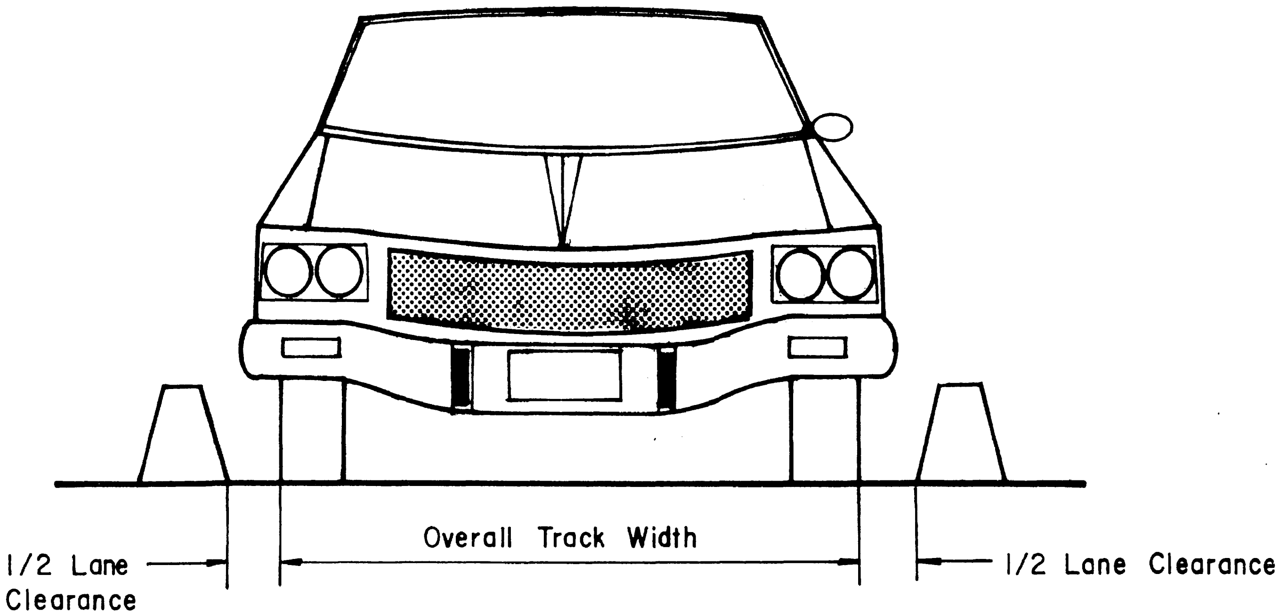


Figure 14. Cone course lane clearance.

Several observations with regard to these clearances should be noted. First, it seems somewhat surprising that the lane clearance resulting in a challenging, but negotiable, course is so small. The course was constructed using 84 cones, yet with the small clearances indicated above, the average number of cone strikes per run for all drivers and all cars was just 6.1. In addition, the test drivers who determined the course width found that small deviations from the values given above produced substantial changes in difficulty. Increasing or decreasing total lane width by 1 1/2 inches was found to significantly alter the difficulty factor. Increasing the lane width by three inches was found to result in a course which was easily negotiated with no cone strikes, while decreasing the width by three inches increased the number of cone strikes many times over.

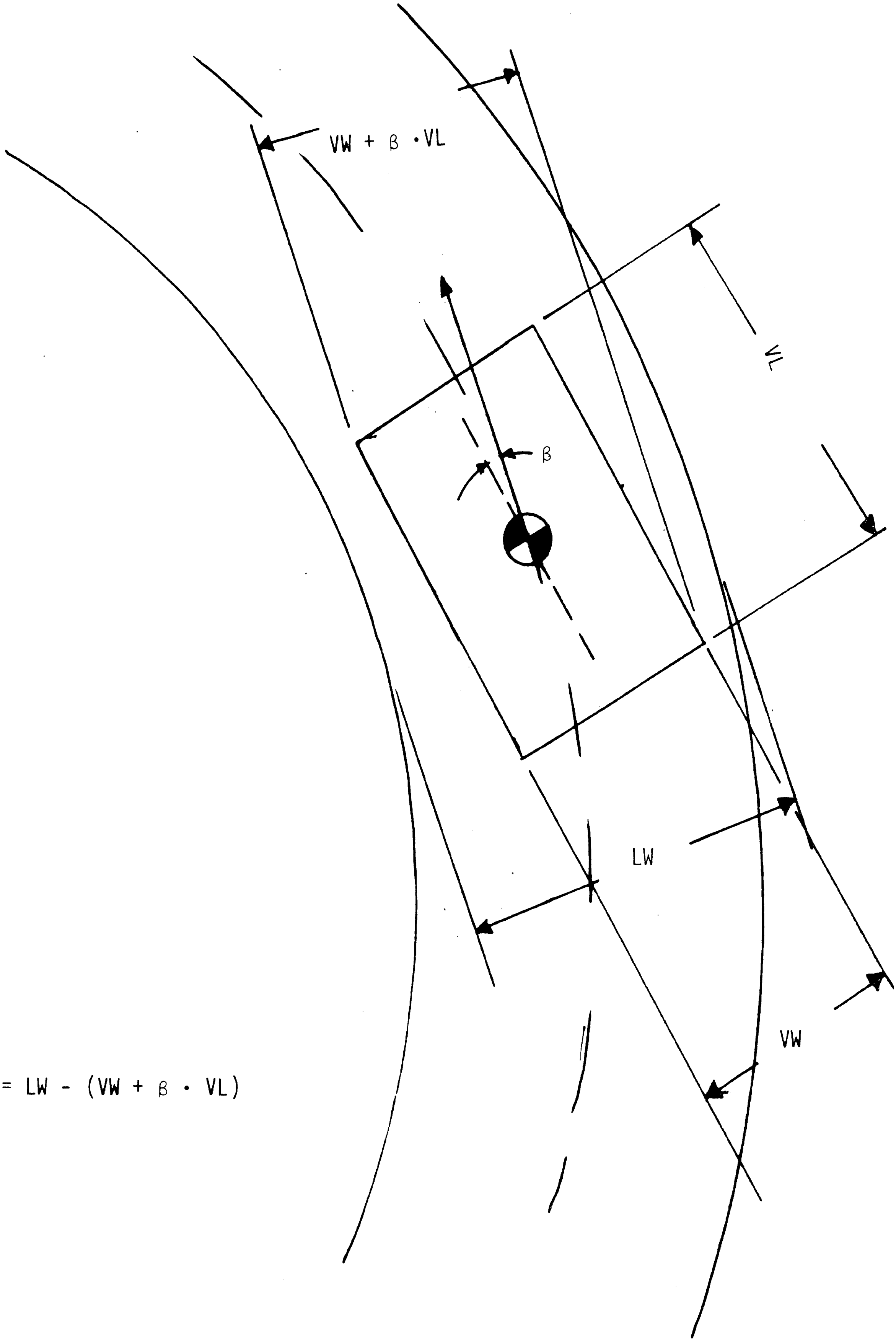
It was not possible to examine this sensitivity in any depth in this program, but it should be noted that if the initial indications of such high sensitivity hold true, there may be important ramifications upon the usefulness of this testing method. First, high sensitivity would require very accurate cone placement if the results are to be dependent on driver-vehicle performance rather than cone placement accuracy. A more subtle point is that this sensitivity puts in serious question the ability to make meaningful comparisons between cars. Consider Figure 15 which represents a vehicle passing through the curve portion of the cone course at speed. Note that in the figure, as is generally true, the vehicle has assumed some non-zero sideslip angle (β). Thus, it is clear that the actual clearance which the vehicle has in the lane is a function not only of vehicle width and lane width, but also of vehicle length and sideslip angle. In fact, for small β

$$LC = LW - (VW + \beta \cdot VL) \tag{49}$$

where

- LC is lane clearance
- LW is lane width
- VW is vehicle width
- VL is vehicle length

Even for small vehicles, VL may be on the order of 100 inches and β on the order of 1°. Thus, $\beta \cdot VL$ may have an effect on lane clearance on the order of two inches, and as noted above, two-inch changes in lane clearance have been found to be of significant magnitude. It would seem that if vehicle-driver performance is to be compared across different vehicles, with number of cone strikes as the criterion, then lane clearance should be equal between vehicles. Figure 15 implies that this requires considering vehicle length and sideslip angle as well as vehicle width in choosing lane width.



For small β

Lane Clearance = LW - (VW + β · VL)

Figure 15. The effect of vehicle sideslip on lane clearance.

Using the very simple "bicycle" model of a vehicle, it may be shown that for steady-state turning,

$$\beta = \frac{57.3b}{R} - \frac{a\,W}{\ell C_{\alpha R}} \frac{u^2}{gR} \tag{50}$$

for small β and where $C_{\alpha R}$ is the cornering stiffness of the rear tire and u is the longitudinal velocity. Or, using the cornering compliance concept,

$$\beta = \frac{57.3b}{R} - D_r \frac{u^2}{gR} \tag{51}$$

where D_r is the rear cornering compliance of the vehicle.

From these two equations it is seen that β is dependent on turn radius, vehicle geometry, velocity, and rear cornering compliance, where, in the real vehicle, this last factor is dependent on vehicle weight, longitudinal c.g. position, rear tire and rear suspension properties, as well as vehicle roll stiffness. All these factors affect steady-state sideslip angle. These, and others, affect transient β performance. It would seem, then, that the term $\beta \cdot VL$ of Equation (49) is, in fact, most difficult to evaluate with sufficient accuracy for comparative test purposes.

It is of further interest to note that, as a consequence of Equation (51), each individual vehicle possesses one speed at which steady-state β becomes zero, regardless of turn radius. That is, from (51)

$$u\Big|_{\beta=0} = \left[\frac{57.3g\ b}{D_r}\right]^{1/2} \tag{52}$$

Figure 16 illustrates this point through a plot of β versus u for a variety of R values. Thus, it would seem that a cone course test could be "tuned" to favor a particular vehicle through the choice of velocity.

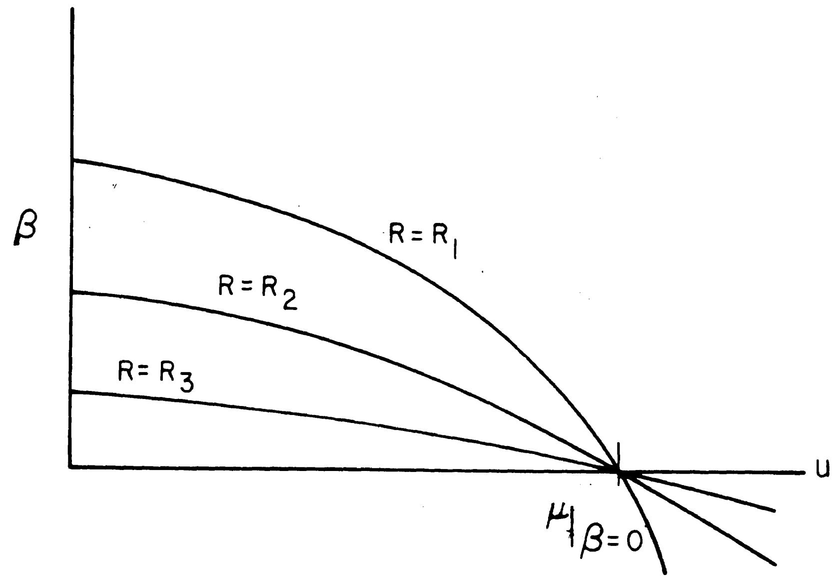


Figure 16. Steady-state sideslip angle as a function of velocity for several turn radii.

All these facets should be considered (and explored further) in comparative testing of driver-vehicle systems. In this program, however, comparisons are made only between unmodified and modified versions of the same vehicle—not between different vehicles. Further, the modifications were made to steering ratio only, one of the few vehicle handling properties which can seemingly have no effect on the $\beta \cdot VL$ term of Equation (49).

Four experienced drivers and one expert driver were used as drivers in the closed-loop portions of the vehicle testing activity. The experienced drivers were taken from the HSRI staff, and although they were all experienced drivers in the normal sense, none were experienced as "test drivers." The expert driver was an experienced and successful road racing driver.

The following procedure was used with each driver* in evaluating each of the three vehicles chosen for modification.

A test day began at approximately 9 a.m. with the subject driver and an observer leaving the HSRI facility in the instrumented vehicle. At this time, the VRSWL installed in the test vehicle would be fitted either with a 1:1 ratio, thus making the vehicle "unmodified," or with the ratio, chosen as a result of preliminary testing, which resulted in the desired yaw rate gain for the modified vehicle. The first 20 minutes were spent in ordinary in-town driving, generally in residential areas. Following this period, approximately 20 minutes were spent in freeway driving on the way to the test facility. Immediately upon reaching the facility, the driver was requested to rate the vehicle using the rating sheet shown in Figure 17.

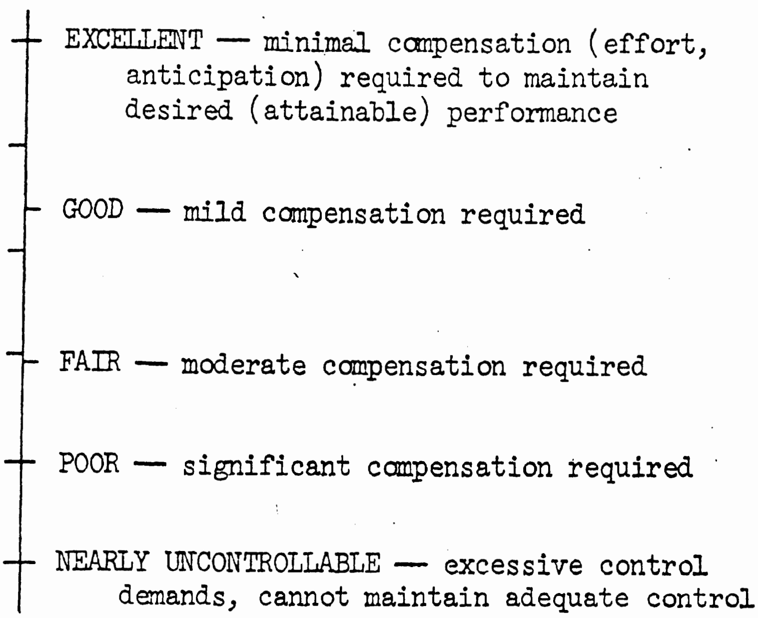


Figure 17. Rating sheet for general driving.

*All four experienced drivers were used in testing the Fury, while three were used in tests on the Pinto and Skylark.

The driver and observer then proceeded to the handling area where a cone course had been laid out previous to their arrival.

Two cone courses were involved in the vehicle tests. One was the "full course," discussed above and shown in Figure 13. A "short course" was attained by eliminating the lane changes.* This was done by changing the 3-foot offsets, shown in Figure 13, to zero rather than by removing cones. This course produced simplier and more easily interpreted tape recorded data with well established initial and final conditions of zero yaw velocity.

Initially, the test observer drove the vehicle through the course with the test driver on board to introduce him (her) to the course and the test area in general. Then, beginning with the short course, the test driver was allowed to practice driving the course in both directions until he (she) satisfied himself (herself) that he (she) was ready.

At this time, six runs (three in each direction, alternating direction from run to run) were made through the short course. The number of cone strikes were recorded for each run, and data was taken on magnetic tape throughout. The entire practice and test procedure was then repeated using the full course. This time, however, ten test runs of alternating directions were made.

Upon completing the course driving, the driver was again asked to evaluate the car, this time using the rating sheet of Figure 18.

Following a break for lunch, this entire procedure, starting with 20 minutes of in-town driving, was then repeated with the second chosen gear ratio installed in the VRSWL.

Results of the closed-loop, driver-vehicle test series consist of cone strike data, an objective measure, and the subjective ratings which the drivers made with respect to both general driving and cone course driving.

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*This short course was not introduced into the program until the second test vehicle. Thus, it was not used during testing of the Plymouth Fury.

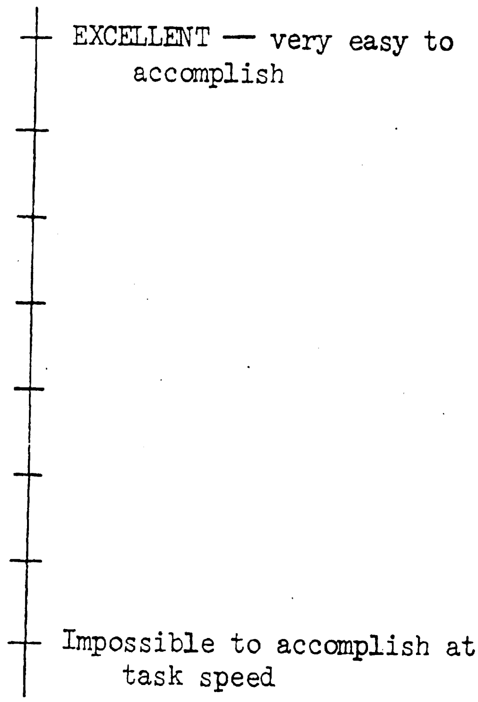

Figure 18. Rating sheet for cone course.

Cone strike data is presented in Table 6, and subjective data in Table 7. The subjective data results from converting the subjective rating scales, shown previously in Figures 17 and 18, to linear scales ranging from 0 to 10 (zero being "good," and 10 "bad"). It should be reiterated that these data, especially cone strike data, should only be used to compare modified and unmodified versions of the same vehicle. Further, note that the scatter in number of cone strikes from run to run, as indicated by the standard deviation values given in Table 6, is substantial, and detracts from the significance of this data.

The mean data from these tables have been reduced and presented in a graphical form in Figures 19 and 20. These figures present the data in a normalized form. For each driver-vehicle-measure

Table 6. Cone Strike Data: Mean and Standard Deviations of the Number of Cones Struck Per Run.

		Shortened Course						Full Course					
		Unmodified			Modified			Unmodified			Modified		
Vehicle	Driver*	Left Turn	Right Turn	All	Left Turn	Right Turn	All	Left Turn	Right Turn	All	Left Turn	Right Turn	All
Ford Pinto	A	3.0/2.64	10/2.0	6.5/4.37	0.0/0.0	3.67/2.89	1.83/2.71	4.6/4.5	3.6/3.05	4.1/3.66	1.2/1.30	2.0/1.414	1.6/1.35
	B	6.33/7.57	7.0/5.0	6.66/5.75	6.33/3.21	15.0/7.55	10.67/7.03	12.6/9.18	11.0/5.83	11.8/7.30	5.2/3.42	7.8/3.76	6.5/3.66
	D	2.67/2.52	3.33/3.51	3.0/2.76	2.33/1.53	5.33/4.73	3.83/3.54	3.0/2.12	5.8/2.39	4.4/2.59	1.2/1.64	4.4/2.51	2.8/2.62
	E	2.0/2.65	1.67/1.15	1.83/1.835	1.67/1.52	3.0/3.46	2.33/2.5	1.0/1.41	4.4/4.56	2.7/3.65	2.2/2.28	2.4/1.82	2.3/1.95
	Non-Expert Avg.			5.39			5.44			6.77			3.63
	All Drivers Avg.			4.5			4.66			5.75			3.30
Buick Skylark	B	8.0/5.1	7.3/1.2	7.6/3.7	4.3/2.5	4.3/3.3	4.3/2.9	12.8/5.6	1.4/1.9	7.1/7.1	10.2/7.8	3.6/2.3	7.2/6.5
	C	3.7/.5	3.3/2.5	3.5/1.8	1.0/0	4.3/2.5	2.7/2.4	6.6/4.6	2.2/1.6	4.4/4.1	7.0/4.8	5.4/2.7	6.2/4.0
	D	2.3/2.1	6.3/1.2	4.3/2.6	3.0/3.6	4.7/0.5	3.8/2.7	4.6/3.1	4.0/2.0	3.8/3.0	0.8/0.7	2.0/1.1	1.4/1.1
	E	1.3/1.2	1.0/0.8	1.2/1.1	1.0/.8	2.3/1.7	1.7/1.5	0.4/0.5	2.0/1.7	1.2/1.5	3.0/2.4	1.6/1.0	2.3/2.0
	Non-Expert Avg.			5.1			3.6			5.1			4.9
	All Drivers Avg.			4.2			3.1			4.1			4.3
Plymouth Fury	A							7.4/3.9	7.6/3.4	7.5/3.7	7.2/5.8	8.4/6.2	7.8/6.0
	B							14.4/2.7	13/2.6	13.7/2.8	13.4/4.7	9.0/5.8	11.2/5.7
	C							13.6/8.5	2.0/1.6	7.3/8.2	12.6/4.7	2.6/2.1	7.6/6.2
	D							13.6/3.8	8.0/1.3	10.8/4.0	12.6/2.1	11.8/4.5	12.3/3.5
	E							5.4/4.4	4.0/2.9	4.7/3.8	5.8/4.6	6.4/3.7	6.1/4.2
	Non-Expert Avg.									9.8			9.7
	All Drivers Avg.									8.8			9.0

*Driver E was the "expert driver."

Table 7. Subjective Rating Data*

Vehicle	Driver**	General Driving Rating Unmodified	General Driving Rating Modified	Cone Course Rating Unmodified	Cone Course Rating Modified
Plymouth Fury	A	3.1	4.0	5.0	6.8
	B	1.4	3.1	7.7	2.9
	C	1.5	6.2	2.6	2.0
	D	7.1	5.2	7.4	7.4
	E	2.7	3.6	3.6	4.7
Ford Pinto	A	3.6	5.8	1.1	0.9
	B	0.0	7.5	2.7	2.9
	D	3.0	3.8	1.5	0.9
	E	2.7	2.8	2.0	2.4
Buick Skylark	B	3.0	5.8	4.3	1.8
	C	2.5	5.5	5.0	4.5
	D	4.0	5.8	1.4	1.7
	E	4.7	7.6	2.3	5.9

*Small values imply a good rating; large numbers imply a bad rating

**Driver E was the "expert driver."

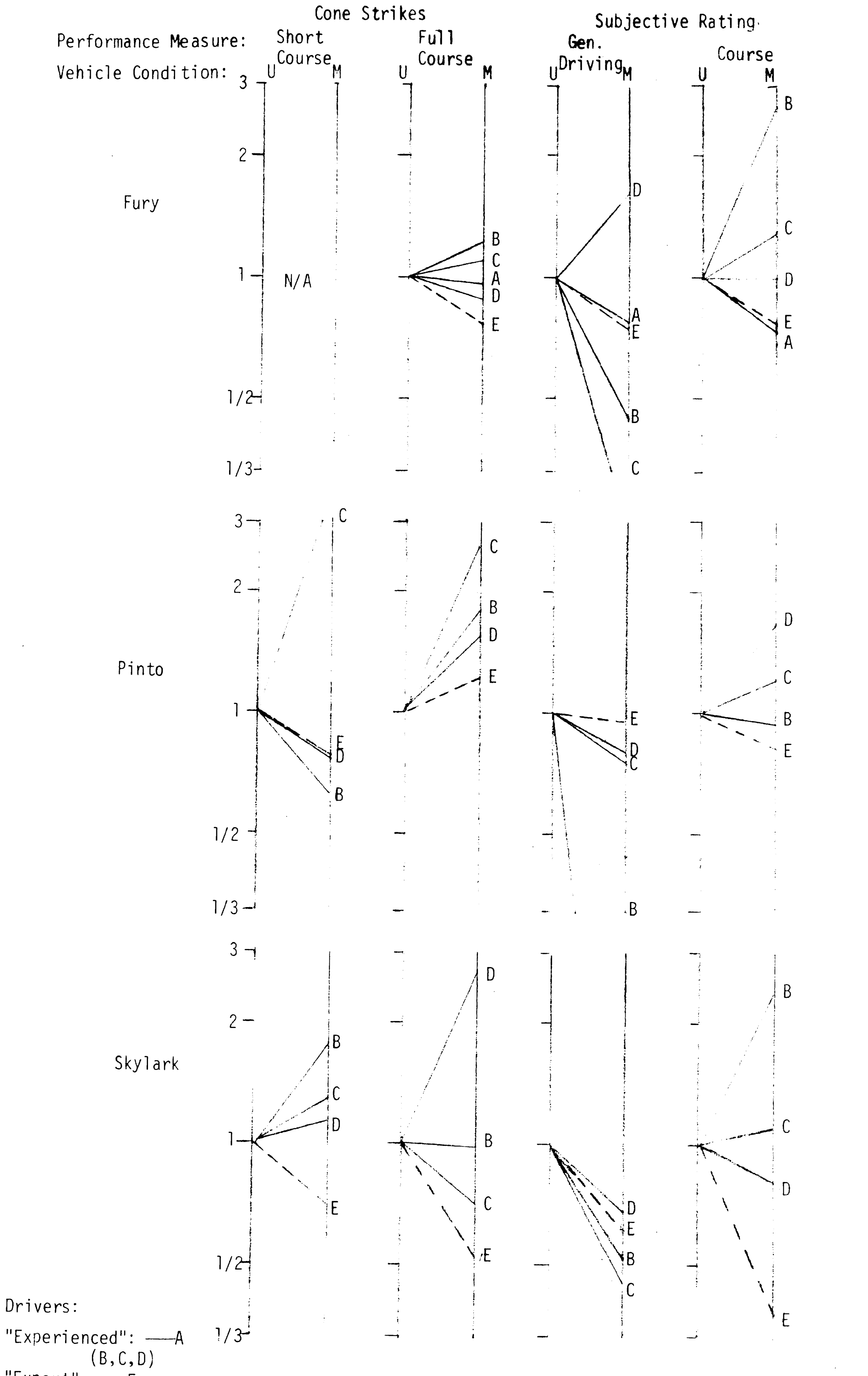

Figure 19. Normalized Closed-Loop Test Results: (Measure by Measure)

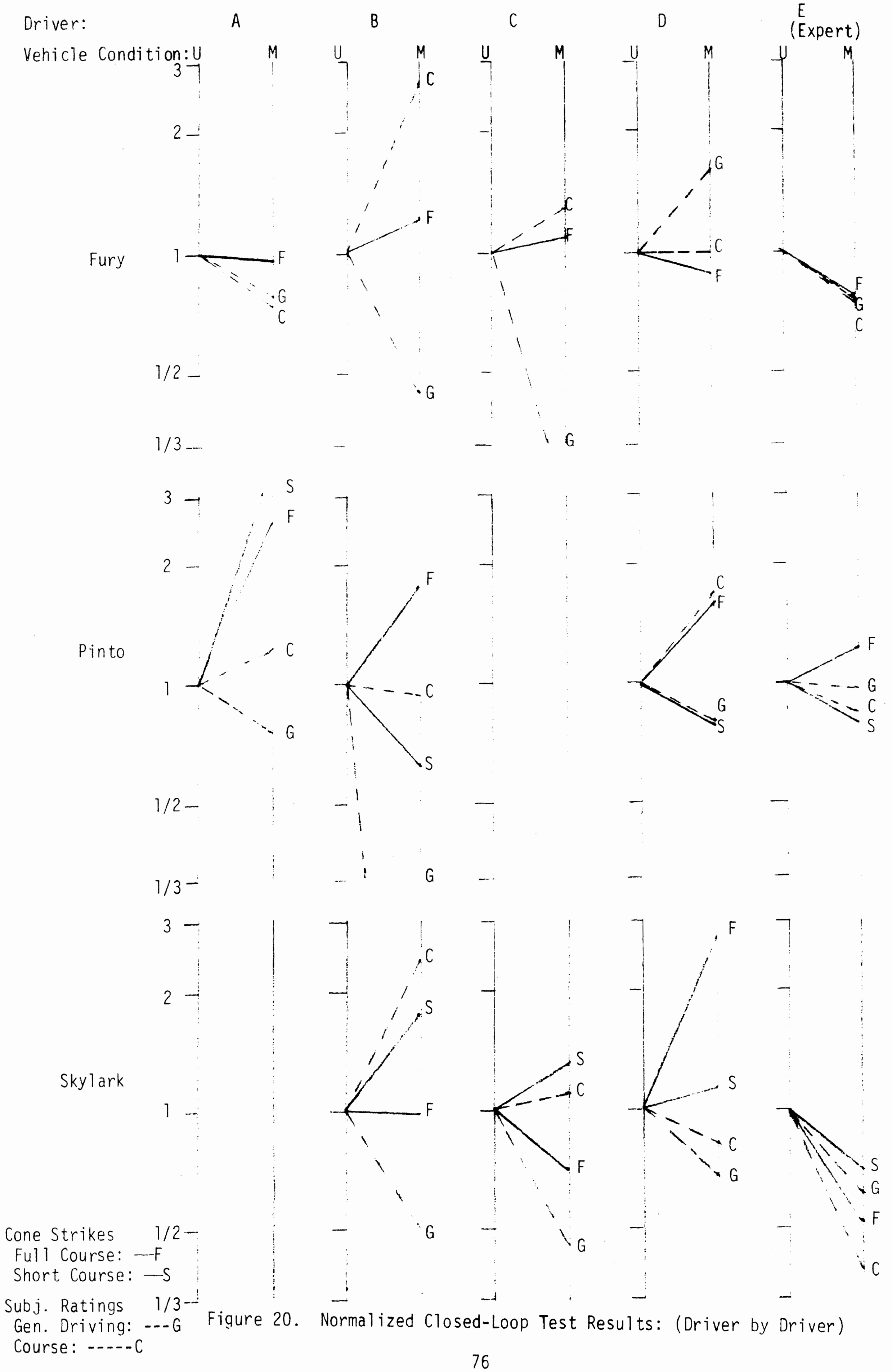

Figure 20. Normalized Closed-Loop Test Results: (Driver by Driver)

combination, the measure attained in the unmodified vehicle is used as a normalizer for both unmodified and modified vehicle results. Thus, unmodified vehicle measures are always normalized to a value of 1 and become the reference for comparison. When the normalized value for modified vehicles is greater than one, then the performance measure of the modified vehicle was "better" than the same performance measure for the same unmodified vehicle. If the normalized value is less than one, then, of course, the performance measure was "worse." (This polarity requires that the normalizing procedure be the inverse of the usual procedure, as shown in the example below.) The resulting normalized data is displayed on a log scale. (This scale produces a visual effect which makes a X2 or X 1/2 change, for example, appear as the same magnitude.) As an example of the normalizing procedure, consider cone strike data for driver A in the full course driving the Pinto. The average cone strikes per run are

Unmodified Vehicle:	4.1
Modified Vehicle:	1.6

The resulting normalized values are:

Unmodified Vehicle:	4.1/4.1 = 1.0
Modified Vehicle:	4.1/1.6 = 2.6

In Figures 19 and 20, the normalized data is presented in two forms. Figure 19 groups the data by test vehicle and performance measure, each group having one line plotted for each driver. This figure readily demonstrates the effect which the modification had on each performance measure for each vehicle. Figure 20, on the other hand, groups the data by test vehicle and driver with a line plotted in each group for each performance measure. This figure would indicate the effect which the vehicle modification had on each driver-vehicle system as interpreted by the various performance measures.

From Figure 19, it can be seen that the modification made to the Plymouth Fury did not appear to have consistent effect across drivers.

In fact, the distribution in cone strike results appear remarkably even. Only the subjective evaluation of general driving shows a definite downward trend. Interpretation of the drivers' verbal remarks would indicate that their disapproval of the modification, as indicated by this measure, resulted from increased steering torque requirements.

The Pinto suffered in the general driving rating due to modification, also. And, again, the drivers indicated that this was a result of the increase in required torque. However, cone strike data, at least for the full course, would indicate an improvement due to the modification. Other results for the Pinto are mixed, although there appears to be a downward trend for cone strike data for the shortened course.

Once again, the drivers' disapproved of the modification made to the Skylark. This time, steering effort was considered too light and steering gain, particularly for in-town, low speed maneuvers, was considered to be too small. Other measures for this vehicle are quite scattered.

Figure 19 serves to illustrate an interesting point, namely, that a given driver-vehicle system does not necessarily yield consistent results across the several objective and subjective measures used here. Only the expert driver tended to produce consistent results across the various measures. For the other drivers, opinion and cone strike performance did not generally show agreement in trends.

5.0 CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

This research project has endeavored to address practical considerations associated with applying the research findings obtained in NHTSA programs [1, 2] to the steering controllability of domestically-produced subcompact, compact, and intermediate size automobiles. It was the purpose herein to identify reasonable techniques whereby such vehicles could be made to fall within the boundaries of the Specified Performance Space (SPS), as defined by those studies.

The work which was undertaken has led to a series of conclusions which lend themselves to organization under the following two major classifications:

1) Conclusions regarding individual vehicles and/or the vehicle population (specifically, subcompact, compact, and intermediate size vehicles) and their relationship with the SPS.

2) Conclusions regarding the SPS specifically and its place in the developing art of the objective characterization of vehicle handling quality.

Even though the expressed objectives of this research project were directed toward obtaining conclusions which would fall within the first category, conclusions of the second type were a natural result of this effort.

Conclusions drawn from this project are listed below according to these two classifications.

1. Conclusions regarding the relationship of vehicles to the SPS.

   a) Most domestically-produced passenger cars of the subcompact, compact, and intermediate sizes fall within the SPS.

b) Vehicles of these sizes show very little variance in $T_e|_{50}$, one from another, and generally fall comfortably within the specified range for $T_e|_{50}$.

c) These size vehicles, when equipped with manual steering, often fall outside of the SPS because they tend toward unacceptably low values of $r/\delta_{sw}|_{50}$. The large value of overall steering ratio which is necessary because of steering effort considerations, is the cause of this low gain.

d) Some intermediate vehicles equipped with power steering fall outside of the SPS, again because $r/\delta_{sw}|_{50}$ is too low.

e) A reasonable modification to move vehicles which are outside of the SPS (because of low yaw rate gain) into the space is to lower the steering ratio. In the case of manual steering cars, this reduction is easily accomplished by changing to a power steering gear.

f) Steering ratio is virtually the only vehicle parameter which can effect a change in steady-state yaw rate gain without causing changes in transient yaw response.

g) In-use factors, especially those deriving from loading and tires, have an effect on vehicle performance in the dimensions of the SPS. Changes in in-use factors can cause individual vehicles to cross the boundaries of the space. Thus it would be necessary to constrain the in-use factors if a vehicle's position relative to the SPS is to be maintained over time.

2. Conclusions regarding the SPS specifically.

a) The SPS, and especially the research efforts which led to its development, make a significant contribution to the developing art of the objective definition of vehicle handling qualities. However, the SPS (as well as this art in general) does not appear to be sufficiently developed as to be adequate for the general identification of handling quality.

b) It is possible for vehicles, whose yaw response properties lie within the SPS, to have widely varying, and possibly unacceptable, levels of understeer (oversteer).

c) Vehicles which have similar yaw response properties at 50 mph, as defined by the SPS, may have widely varying yaw response properties at other velocities.

d) Many vehicle response properties (other than the yaw response properties which constitute the SPS) need consideration in evaluating vehicle handling quality.

In a separate area which does not fit conveniently into the above classifications, this program has found that

> The use of the precision cone course and resulting cone strike data for the evaluation of driver-vehicle system handling performance within the linear range is of questionable validity.

5.2 Recommendations

Recommendation:

Efforts should be made to establish the safety significance of the SPS through a study of the accident record.

Discussion:

The preceding "Conclusions" section points out that, in large part, vehicles presently being manufactured conform to the performance specifications developed in NHTSA research studies [1, 2]. Given that subjective ratings by drivers have played a major role in establishing both the performance space specified by that research and the current design practices of vehicle manufacturers, this finding is not particularly surprising. It is valuable to note, however, that this study has identified vehicles which fall within the SPS and other vehicles that fall without. It would then appear that there is a potential mechanism for evaluating the safety significance of the SPS through the comparative evaluation of the accident records of these two sets of vehicles, even though problems related to the significance of the results due to the influence of other, uncontrolled variables could be very large.

An approach which would eliminate some of the uncontrolled differences in vehicles would be to compare vehicles of the same basic model but differing in certain mechanical features such as power versus manual steering, radial versus bias tires, and/or a front anti-roll bar versus no anti-roll bar. Clearly, it would be necessary to (1) test the various versions of the vehicle models to be studied to establish their relationship to the SPS, (2) maintain the vehicles so that their relationship to the SPS did not change significantly in use, and (3) study a sizeable sample of these vehicles to attempt to remove the influences introduced by the varying characteristics of the drivers involved.

•Recommendation:

As a preliminary step to further handling research, closed-loop, driver-vehicle handling test methodologies which (1) yield objective measures of driver-vehicle system performance and (2) provide an assessment of the control difficulties associated with particular vehicles should be developed.

Discussion:

It is of interest to note that the boundaries of the SPS have been set largely through the subjective ratings of drivers. Presumably, then, the SPS defines a set of vehicle handling properties which drivers like. It is not yet well established, however, that this same set of vehicle properties are necessary for safe handling characteristics. Indeed, the fact that drivers are adaptable to the characteristics of the vehicle they are driving and can compensate for broad differences in vehicle characteristics might suggest that "safe" handling characteristics may cover a significantly broader range than "likeable" characteristics. Accordingly, future research efforts might search for an outer region in which control properties become unacceptable, rather than for an inner region in which they are "optimum."

One manner in which the safety significance of the SPS might be examined through accident data analysis has been suggested above. For purposes of safety-related vehicle handling research, the development of closed-loop vehicle handling test methodologies which would yield objective measures of safety-related driver-vehicle handling performance quality would also be desirable. It is recognized that the safety-relevance of objective measures obtained on the test track is extremely difficult to establish. Nonetheless, the state-of-the-art of driver-vehicle performance testing appears severely wanting relative to the need for objective data.

Recommendation:

Investigations should be undertaken to examine the significance of the many other vehicle factors which might contribute to linear-regime handling quality. The SPS concept should be appropriately expanded according to the results of such investigations.

Discussion:

In connection with further research into the control quality of vehicles in the normal driving range, it would appear that more broadly based investigations are called for. The subject under consideration appears to be most complex with a large number of inter-related properties contributing to overall handling quality. In the future, it will be necessary to consider at least (1) both lateral acceleration and yaw rate response times, (2) the influence of the amount and timing of roll-related properties, and (3) the importance of the level and nature of steering torque or "feel" prior to the establishment of firmly based handling performance specifications. Even though the SPS, as currently defined, represents a useful, interesting approach, its definition should be expanded and/or revised according to the findings of future research into these areas. Furthermore, vehicle performance over a range of velocities should be examined and specified.

Recommendation:

Research on vehicle handling as it is affected by the transition from the linear range through the nonlinear range to the limit of turning performance should be undertaken. The findings derived therefrom should be incorporated in any evolving vehicle specification scenario.

Discussion:

Overall handling quality, specifically safety-related quality, would certainly appear to involve more than the normal driving regime. In addition to closed-loop control in normal driving, NHTSA has in the past sponsored research studies addressing the limit performance of passenger cars [13, 25]. In those studies, vehicle handling test procedures (VHTP) were developed and used to examine open-loop performance with the idea of seeing if vehicles possess response characteristics which are uncontrollable in extreme maneuvers. That is, do vehicles reach a limit response beyond which driver skill and experience is of little avail?

Two maneuvers which only involve turning, viz., a rapid turn (called "trapezoidal steer") and a reverse steer or lane-change maneuver (called "sinusoidal steer") have been included in the developed test procedures. Of these two turning maneuvers, the lane change is believed to be more realistic since it can be related to driver-vehicle control situations on typical roads.

Directional response in emergency turning maneuvers is partially dependent upon the vehicle characteristics (including tire characteristics) used in the definition of the SPS. The yaw rate conditions prevailing during the initial phase of a drastic steering maneuver are determined by the linear-range yaw-response properties (that is, the SPS) of the vehicle. At the start of a steering maneuver the front wheels generate a side force causing the vehicle to yaw. As the vehicle yaws and starts to sideslip, the rear tires generate forces providing additional acceleration in the direction of the turn. For a controlled turn to develop, the front and rear tire forces must produce a yaw moment balance appropriate for the desired turn. How the transition from rapid yaw acceleration into yaw moment balance occurs is crucial in establishing a good turn.

Further research on the transition from the linear range through the nonlinear range to the limit of turning performance appears valuable in order to gain an understanding of the events which can lead to loss of control in attempted turning maneuvers. Previous closed-loop studies [6] have shown that drivers are capable of applying inputs which will lead to loss of control at the limit for particular vehicles. But means for assessing the performance capabilities of the driver-vehicle system in extreme maneuvers have not been established [4]. Both open- and closed-loop results for evasive performance tests (lane-change maneuvers) are needed to illuminate meaningful, objective measures of vehicle dynamics characteristics which are pertinent to vehicle control in accident-avoidance maneuvers. Until the interaction between driver control and vehicle dynamics characteristics in evasive maneuvers is well understood, our knowledge of steering controllability will be incomplete with respect to safety-related accident-avoidance considerations.

Specifically, further study of driver-vehicle system performance in evasive, lane-changing maneuvers is suggested. Even though the lane-change maneuver has been used in many studies with only limited success [26], it still appears to be a promising maneuver to investigate in the future. With regard to the type of maneuver involved, the results of this study indicate that a precision, tightly-constrained lane-change course is not appropriate for comparing different vehicle types or models. Accordingly, a course arranged to challenge the responsiveness of the vehicle in avoiding an obstacle while allowing a fairly reasonable space laterally (such as a lane width) for recovering the original direction of travel appears to be a good candidate for further study.

•Concluding Recommendation:

Implementation of vehicle handling performance specifications is not recommended at this time, pending further development as amplified above.

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APPENDIX A
DATA ANALYSIS PROGRAMS

This appendix contains descriptions of the data analysis methods used for determining the effective time constant, T_e, from the step-steer and pulse-steer yaw rate responses.

A.1 Step Steer

The method used for obtaining the Fourier transforms of the yaw rate step response was based on that used by Samulon [23], and earlier by Bedford and Fredendall [24]. Samulon's technique approximates the given response by a sum of (sin x/x) functions. The Fourier transforms of the individual (sin x/x) functions are then computed and summed to obtain the total Fourier transform. Bedford and Fredendall used the same approach but approximated the response by a sum of step functions instead.

Using Samulon's notation, if F(t) is the measured yaw rate response, a discrete approximation is given by

$$F(t) = \sum_{n=0}^{\infty} A_n \frac{\sin f_c(t-n\tau)2\pi}{f_c(t-n\tau)2\pi} \tag{A.1}$$

where

τ is the sampling interval

f_c is the highest frequency component in F(t), (Hz)

A_n are the amplitudes of F(t) at the sampling points.

It can be shown that the complete Fourier spectrum of the given F(t) expression (A.1), summing over all terms, is given by

$$\Phi(\omega) = \sum_{n=0}^{\infty} A_n \frac{e^{-jn\omega\tau}}{2f_c} \quad ; \quad \tau = \frac{1}{2f_c} \tag{A.2}$$

where ω is frequency in radians/second, ($\omega = 2\pi f$).

By expressing the same transform in terms of first differences of A_n, $B_n = A_n - A_{n-1}$, Equation (A.2) becomes

$$\Phi(f) = \frac{1}{j4f_c \sin[\frac{\pi}{2} \frac{f}{f_c}]} \sum_{n=0}^{\infty} B_n e^{-j(n+1/2)\pi \frac{f}{f_c}} \qquad (A.3)$$

In response to an ideal step function, (A.3) is divided by $1/j\omega$ and becomes the transfer function,

$$H(f) = \frac{\frac{\pi}{2} \cdot \frac{f}{f_c}}{\sin(\frac{\pi}{2} \cdot \frac{f}{f_c})} \sum_{n=0}^{\infty} B_n e^{-j[n+1/2]\pi \frac{f}{f_c}} \qquad (A.4)$$

This is the same result obtained by Bedford and Fredendall with the exception of the gain correction term,

$$\frac{\frac{\pi}{2} \cdot \frac{f}{f_c}}{\sin(\frac{\pi}{2} \frac{f}{f_c})} \quad , \text{(i.e.,} \frac{x}{\sin x} \quad , \quad x = \frac{\pi}{2} \frac{f}{f_c}) \; ,$$

and the constant time delay correction term, $e^{-j\,\pi/2\,f/f_c}$. Equation (A.4) represents the exact Fourier transform of the ideal step response provided the response contains no frequency components greater than f_c. Bedford and Fredendall's result (without the gain and phase correction terms) is a good approximately to Equation (A.4) in the lower range of the frequency band (0, f_c).

HSRI used Equation (A.4) to calculate the Fourier transforms of the yaw-rate step response with a correction procedure added to account for an actual ramp-step input instead of the ideal step input assumed by Equation (A.4). The correction procedure principally affects the phase angle calculations and appears as a phase lead term given by

$$\Delta\psi = \tan^{-1} \frac{1 - \cos \omega k}{\sin \omega k} = \frac{\omega k}{2} \qquad (A.5)$$

where k is the delay between two ramp functions, the sum of which yield a ramp-step function (see Figure A.1).

The correction term of Equation (A.5) is derived from the Fourier transform of the ramp-delayed ramp sum shown in Figure A.1.

A more fundamental approach is to view the response to a ramp-step input as the response to a pure step input starting mid-way through the ramp portion of the ramp-step input as shown in Figure A.2. In most cases, this method gave nearly identical numerical results as the above procedure and Equation (A.5).

Interestingly, the result of Equation (A.5) suggests exactly this procedure. That is, a pure step input advanced k/2 seconds would produce a phase lead term of $\omega k/2$.

A.2 Pulse Steer

The pulse steer responses (for example, see Figure A.3 in Section A.4) were processed by a conventional Fourier transform method. The finite Fourier series for both input and output was calculated and tabulated in terms of gain and phase as a function of frequency. The yaw rate to steering angle transfer function was then obtained by dividing the output gain by input gain and subtracting the input phase angle from output phase angle. The resulting transfer function gain and phase was then curve fit by a weighted third-order least squares procedure.

The Fourier series expression for a function, F(t), assumed periodic over an interval, T, and represented by N equally-spaced samples ($F(t_p)$, p=0,1,...N-1) is given by

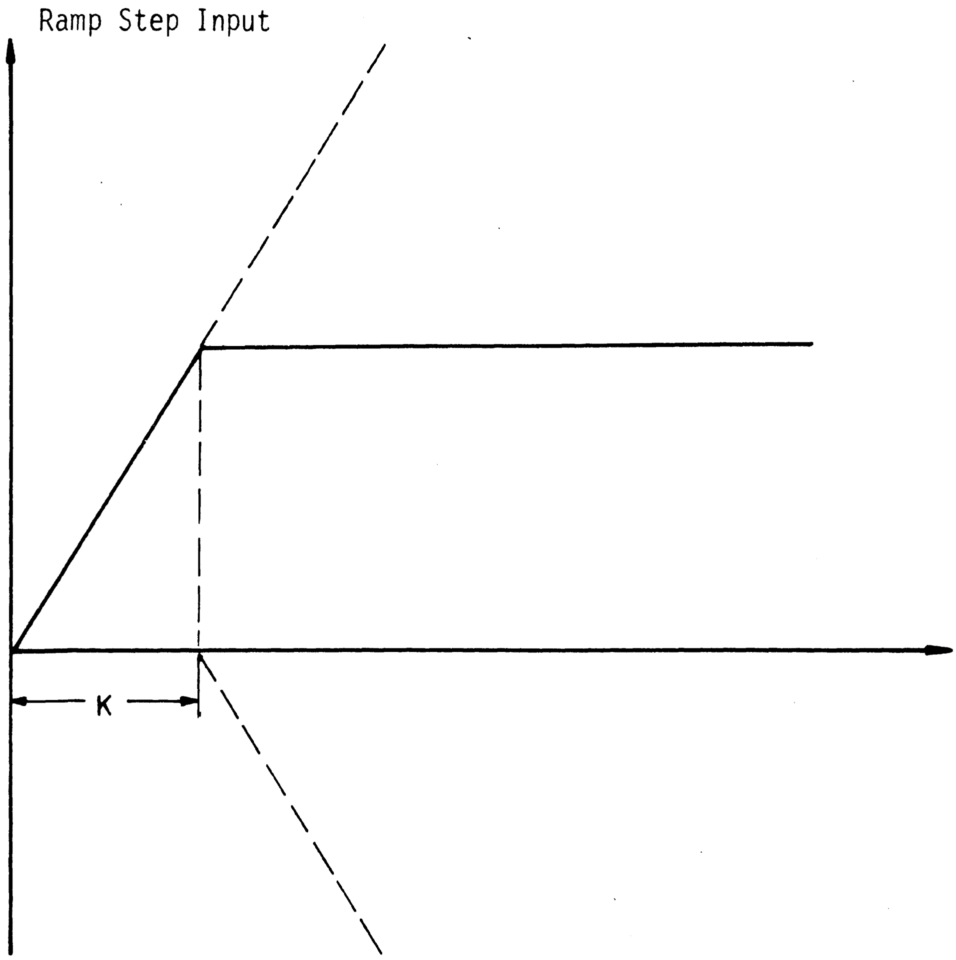

Figure A.1. Ramp-step input.

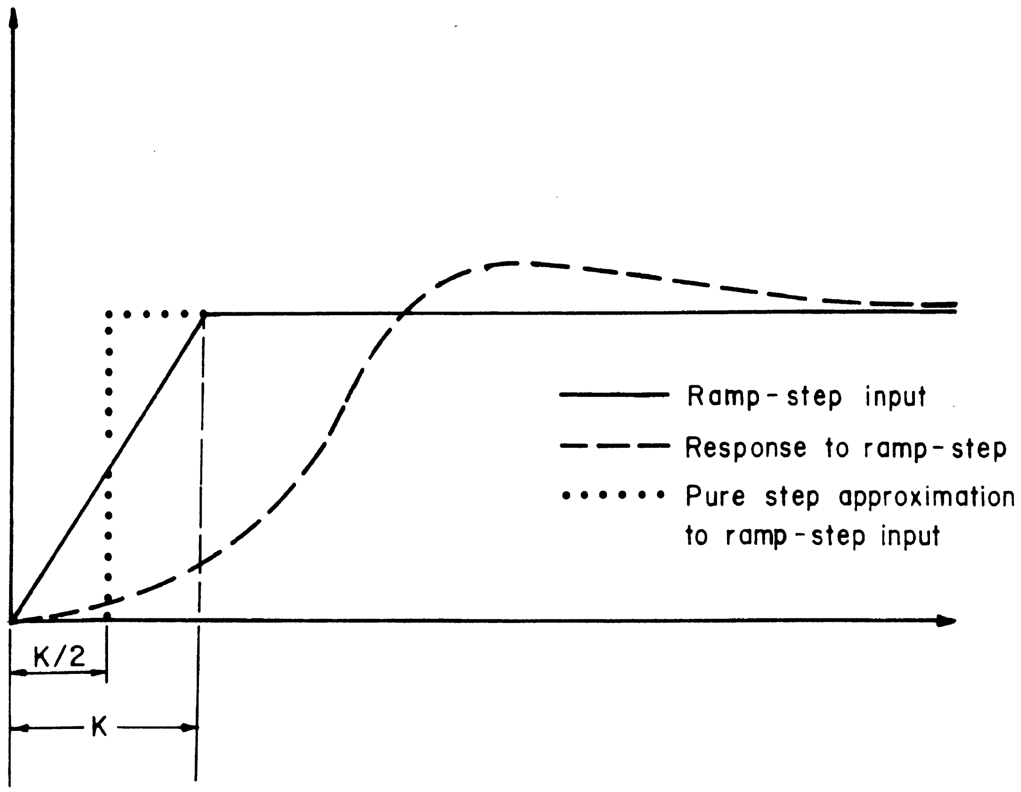
Ramp-step input
Response to ramp-step
Pure step approximation to ramp-step input
K/2
K

Figure A.2

$$F(t) = \frac{A_o}{2} + \sum_{k=1}^{N-1} (A_k \cos \frac{2\pi}{T} kt + B_k \sin \frac{2\pi}{T} kt) + \frac{A_N}{2} \cos \frac{2\pi}{T} Nt \tag{A.6}$$

with the Fourier coefficients A_k, B_k defined by

$$A_k = \frac{1}{N} \sum_{p=0}^{N-1} F(t_p) \cos \frac{2\pi}{T} k\, t_p \quad ; \quad k=0,1,\ldots,N \tag{A.7}$$

$$B_k = \frac{1}{N} \sum_{p=0}^{N-1} F(t_p) \sin \frac{2\pi}{T} k\, t_p \quad ; \quad k=1,2,\ldots,N-1 \tag{A.8}$$

The gain, G, and phase angle, ψ, for $F(t_p)$ at a frequency k is provided by the Fourier coefficients A_k, B_k:

$$G = \sqrt{A_k^2 + B_k^2} \tag{A.9}$$

$$\psi = \tan^{-1}(\frac{B_k}{A_k}) \tag{A.10}$$

The power, P, at a frequency, k, is given by

$$P = A_k^2 + B_k^2 = G^2 \tag{A.11}$$

Applying this method to the pulse steer yaw rate response and corresponding steering input, the transfer function for yaw rate to steer, $Y_{r\delta}$, is given by

$$Y_{r\delta} = \frac{G_r}{G_\delta} \angle \psi_r - \psi_\delta \tag{A.12}$$

where

G_r = yaw rate gain

G_δ = steering angle gain

ψ_r = yaw rate phase angle

ψ_δ = steering angle phase

Equation (A.12) was used by HSRI to calculate the yaw rate/steer angle transfer function. Depending on the nature of the input provided by the driver, various input power spectrums occur over the frequency band of interest. In order to improve the estimate of the transfer function, $Y_{r\delta}$, a third-order weighted least squares curve fit was applied to Equation (A.12). The least square weights were selected in proportion to G_δ, or the square root of the input power.

A.3 Advantages/Disadvantages of the Step and Pulse Steer Methods

1) The step-steer method yields better steady-state (d.c.) and low frequency information. The pulse steer is less reliable in the low frequency range for two reasons: (a) lash or play in the steering system prevents accurate calculation of the d.c. and low frequency components of the yaw rate; and (b) relatively low level turns are necessary in combination with the pulse inputs to guarantee a vehicle response within the linear range while also providing yaw rate and steering input signals large enough with which to calculate (ratio) the steady-state gain.

2) The pulse-steer method provides more reliable mid- and high-frequency information since the input is designed to contain a more evenly distributed power spectrum. This method is also less susceptible to external disturbances (wind, road) unrelated to the steering input, which are reflected in the output. While these external disturbances are often small, their frequency content

is generally spread over a wide frequency range, thereby capable of contributing far greater error to the total input spectrum of a step + external disturbance than of a pulse steer + external disturbance.

3) The step steer method requires a higher digitizing rate to minimize the error associated with not knowing the exact starting point of the input because of its discrete representation. Depending on whether sample i or sample i+1 is considered the start of the input, a maximum phase angle shift of $\pm \omega\tau/2$ can occur. This phase shift can be reduced by increasing the sampling frequency during digitizing. A sampling frequency of 60 Hz was used for most of the step steer data processed during the course of this project. In contrast, the pulse steer tests were digitized at 20 Hz primarily to avoid any potential aliasing effects arising from wheel hop frequencies in the 7-10 Hz band. However, this sampling rate can be lowered by effective analog filtering prior to digitizing. If the frequencies of interest are less than $\overline{f}$, the analog filters can be set at $\overline{f}$ and the data digitized at $2\overline{f}$ or more to avoid aliasing.

4) Even though the step steer requires a higher digitizing rate for data processing, it requires less test time and space to conduct. While 2 or 3 seconds is adequate for the step steer, at least 10 seconds of test time is desirable for the pulse steer in order to achieve adequate resolution in the frequency domain.

5) The digital storage requirements for the two methods are comparable. While the step steer requires less time per test, it does require a greater sampling rate thereby generating an equivalent amount of data per test as the pulse steer method.

6) The degree of scatter in the transform results for the pulse steer test can be traced directly to the quality of the steering input power spectrum. A curve fit routine which accounted for power differences of the input signal greatly enhanced the appearance and consistency of the raw transform results. The main source of scatter for the step steer results appeared as external wind and road disturbances.

A.4 Pulse and Step Steer Examples

Figures A.3 and A.4 show the steering angle (δ_{sw}) and yaw rate (r) analog signals for a representative pulse steer and step steer test with the Chevrolet Nova. These signals were digitized at 20 and 60 Hz, respectively, and stored on magnetic tape. A digital computer program processed the digital versions of these signals using the aforementioned methods. The corresponding Fourier transform output of the digital program is shown in Figures A.5 and A.6.

The top portion of Figure A.5 shows the raw transform (periodogram) prior to a third-order least squares curve fit. The corresponding least square weights (constrained between 0.2 and 2.0) are shown in the rightmost column. A weight value of 1.0 corresponds to a root input power equal to the average root power over the frequency interval. The lower portion of Figure A.5 shows the smoothed or curve fit results. In this example, 45° of phase shift occurs at 6.3 rad/sec yielding a T_e of 1/6.3 or 0.16 seconds.

Figure A.6 shows the digital output from processing the step steer test of Figure A.4. The top portion of Figure A.6 shows a segment of the digitized steering and yaw rate time histories at the time of the step steer input. The lower portion of Figure A.6 shows the transform output with 45° phase lag at 6 rad/sec or a T_e of 0.166.

The arrows marked "1" and "2" in the upper portion of Figure A.6 show two alternate starting points for the transform calculation. Arrow "1" shows the starting point of the steering ramp-step input. If this point was used as the starting point a k value of 0.16 seconds would be used in the transform calculation, as discussed above. If instead, the mid-point of the ramp was used as the starting point (arrow "2"), a k value of 0.0 would be entered (equivalent pure step). Both methods yield the same transform result.

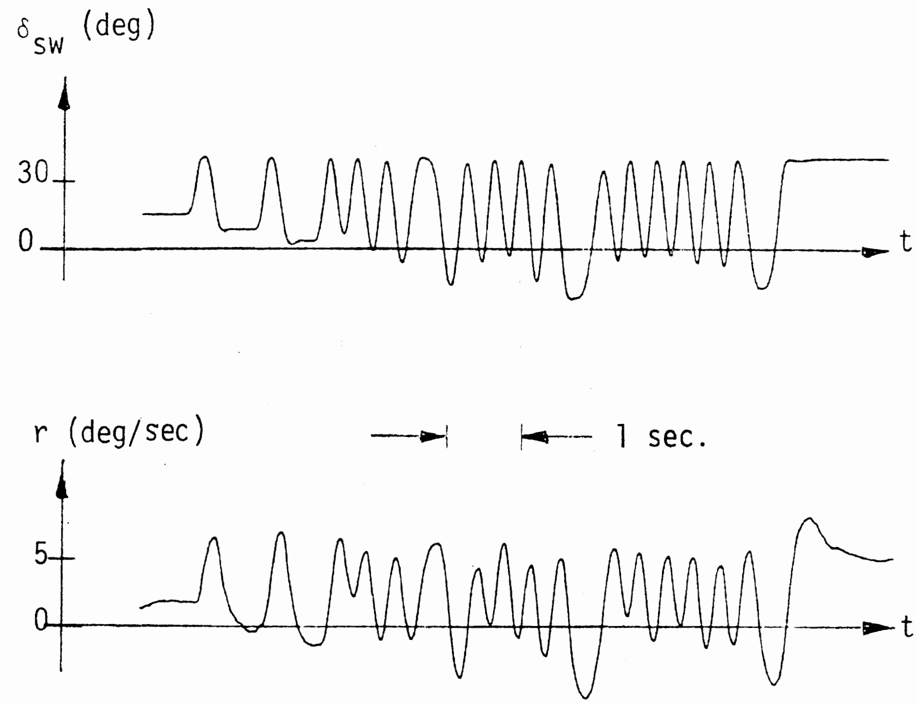

Figure A.3. Pulse-steer test - Chevrolet Nova.

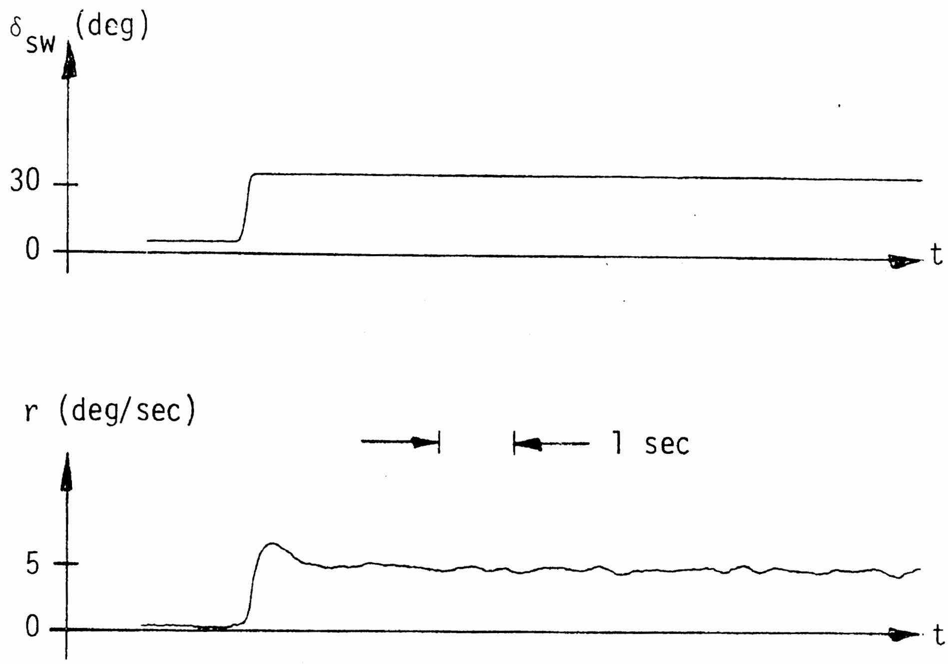

Figure A.4. Step-steer test - Chevrolet Nova.

GAIN	PHASE	FREQ	WEIGHTS	(PERIODOGRAM)
0.79326	0.0	0.0	2.00000	
0.88722	-3.64378	0.65625	2.00000	
0.87557	0.98712	1.31250	1.61288	
0.91094	-4.23595	1.96875	1.78293	
0.97271	-4.55323	2.62500	1.84607	
1.38071	0.30224	3.28125	1.06796	
1.13240	4.64039	3.93750	0.50148	
1.41148	-23.40631	4.59375	1.09522	
1.16916	-44.72310	5.25000	1.40570	
1.51240	-56.30182	5.90625	0.20004	
2.19499	-39.92473	6.56250	0.52932	
1.51639	-54.52803	7.21875	1.67105	
1.47164	-65.59654	7.87500	1.43905	
0.99045	-80.35477	8.53125	0.23756	
1.29074	-110.95860	9.18750	0.41533	
1.04040	-94.20693	9.84375	0.96545	
0.87717	-87.27136	10.50000	1.46834	
0.82059	-97.63396	11.15625	0.42534	
0.97895	-88.02460	11.81250	1.41456	
1.43417	-42.47415	12.46875	0.20000	
0.74968	-110.16821	13.12500	0.99130	
1.01402	-90.61501	13.78125	0.55426	
0.80371	-110.40866	14.43750	1.37704	
1.48337	-111.37447	15.09375	0.24220	
1.02785	-130.85226	15.75000	0.59698	
0.97627	-125.20904	16.40625	0.64771	

GAIN	PHASE	FREQ	(SMOOTHED)
0.64067	0.0	0.0	
0.90378	0.04723	1.00000	
1.09899	-5.84616	2.00000	
1.23173	-13.29440	3.00000	
1.31584	-21.99975	4.00000	
1.35279	-31.66060	5.00000	
1.35211	-41.98126	6.00000	
1.32136	-52.66212	7.00000	
1.26810	-63.40446	8.00000	
1.19966	-73.90964	9.00000	
1.12424	-81.87897	10.00000	

Figure A.5. Pulse-steer results.

r	δ_{sw}	
46.33	167.00	
46.33	167.00	
46.33	166.33	
52.33	167.67	
61.00	168.33	
65.00	169.67	
69.00	171.00	
76.33	173.00	
93.00	176.33	
105.00	183.67	
104.33	199.67	
96.33	228.67	1
96.33	275.33	
109.00	340.67	
127.00	426.33	
137.67	531.00	
143.00	651.67	2
150.33	785.67	
171.00	926.33	
212.00	1057.00	
273.33	1147.00	
354.67	1191.33	
473.00	1206.67	
613.67	1212.00	
757.67	1214.33	
875.00	1215.00	
970.33	1215.67	
1055.00	1214.33	
1131.00	1214.33	
1193.67	1214.33	
1240.00	1215.00	

ramp,
10 sample @
.016 sec
= .16 sec.

GAIN	PHASE	FREQ
853.33325	0.0	0.0
966.08612	3.54624	1.00000
1056.44678	-2.96250	2.00000
1139.35008	-4.27421	3.00000
1422.6[illegible]	-14.2[illegible]	4.00000
1434.74[illegible]	-34.47127	5.00000
1271.7949[illegible]	-44.457[illegible]	6.00000
1221.0[illegible]	-52.9[illegible]	7.00000
1133.08985	-59.54912	8.00000
1211.36578	-67.72925	9.00000
1041.5[illegible]	-84.8[illegible]	10.00000

Figure A.6. Step-steer result.

APPENDIX B

VEHICLE EQUIPMENT AND INSTRUMENTATION

An instrumentation and equipment package, designed to be quickly and easily installed in each test vehicle, was developed for use in this project. A schematic diagram of the system is presented in Figure B.1. As seen in the figure, input variables measured were steer angle (δ_{SS}) and steering-wheel torque (T_{SW}); response variables measured were vehicle velocity (V), yaw rate (r), and roll rate (p). Additionally, on-board analog computation was used to generate steady-state lateral velocity (V r), path curvature (r/ V) and yaw acceleration ($\dot{r}$) signals. These signals (a maximum of 6 at one time) were recorded in the vehicle using an FM magnetic tape recorder. The recorded data was also played back on line and displayed to the driver via a light beam oscillograph, thus providing an immediate visual check on total data system integrity.

In addition to 6 data signals, a "control signal" consisting of selectable D.C. voltage levels was also recorded on one channel of tape. Later, when taped data was reduced using a hybrid computer facility, this signal was used to control the automatic processing routine. Control signal voltage levels generally were determined by driver-operated switches. Additionally, a "drag switch" mounted to the fifth wheel assembly was used to switch between "standby" and "test" modes. The "switch," seen in Figure B.2, consisted of two spring-steel straps dragging on the ground and a piece of aluminum sheet fixed to the ground. When the vehicle passes over the sheet, the switch is closed momentarily and the resulting pulse can be used to trigger a switching circuit. During closed-loop, driver-vehicle experiments, aluminum sheets were placed at the entrance and exit of the cone course. Use of the switch frees the driver from having to perform this task and further provided precise information on the data tape regarding the position of the test vehicle on the course.

TEST VEHICLE INSTRUMENTATION

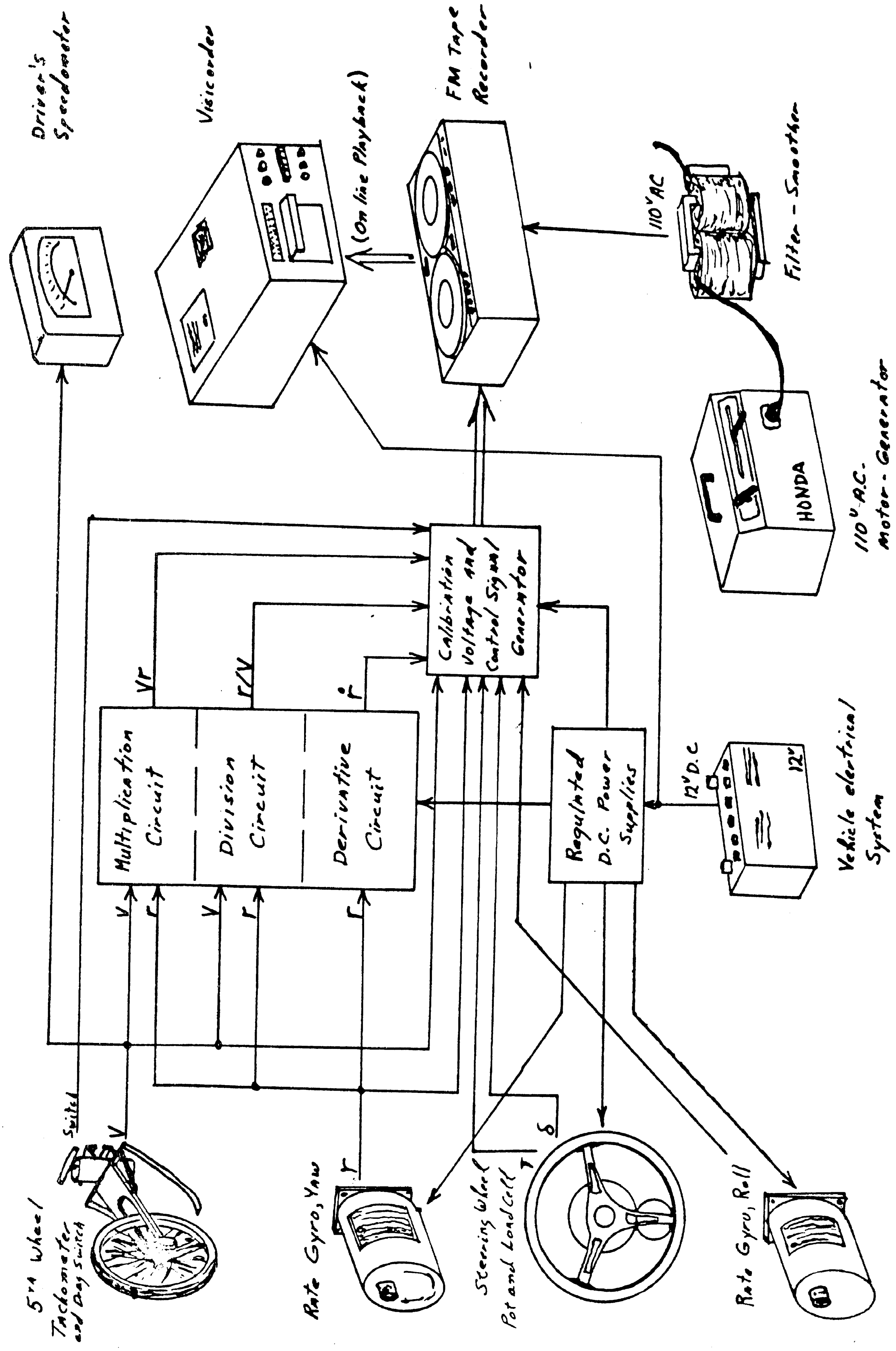

Figure B.1

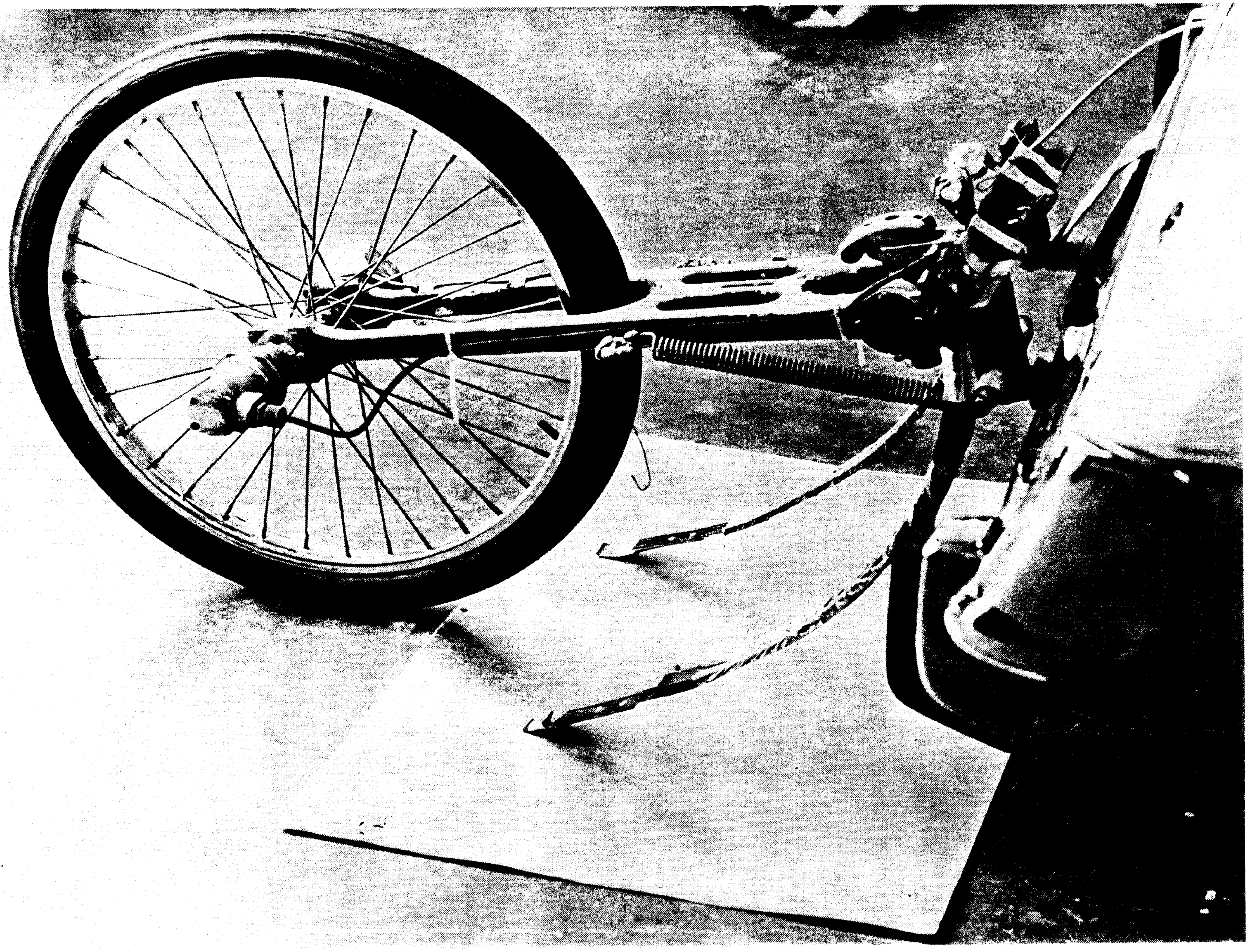

Figure B.2. The fifth wheel and "drag switch."

In addition to the fifth wheel/switch element, the instrumentation system was organized into four major physical units, viz.:

1) Recorder Unit
2) Gyro/Electronic Unit
3) Motor Generator
4) Variable Ratio Steering Wheel Limiter (VRSWL)

Installation of each unit was sufficiently simple that the entire package could be installed in a rented vehicle in a few hours with virtually no restoration required upon removal of the package.

Both recorders were mounted as a unit in the front passenger seat and anchored with seatbelting. The gyro/electronics package was mounted as a heavy steel plate with adjustable legs which were ground to a sharp point. For the linear regime maneuver used in this program, the weight and sharp feet provided more than sufficient anchorage when the package was placed on a carpeted floor area. The motor generator was mounted on a standard roof rack. Photographs of the various elements of instrumentation appear in Figures B.3 through B.6.

The VRSWL performs four functions, viz.:

1) Provides an easily altered gear ratio between the steering wheel and the input (steering wheel) end of the steering shaft.

2) Provides an adjustable limit stop mechanism for steering-wheel motion.

3) Provides for the measurement of the angular displacement of the input end of the steering shaft.

4) Provides for the measurement of steering-wheel torque when equipped with the steering torque transducer described below.

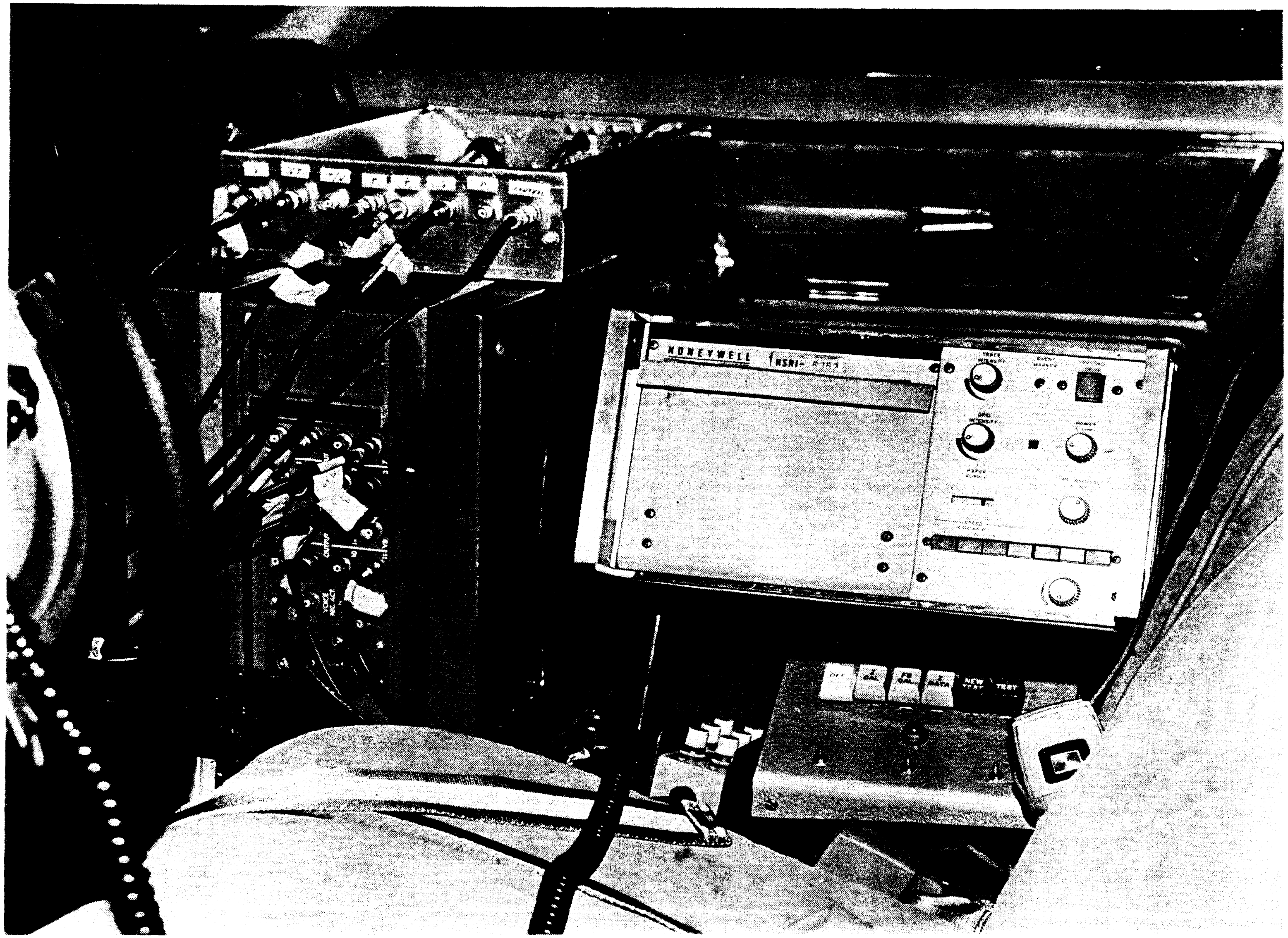

Figure B.3. Tape recorder, oscillograph, and signal conditioning unit in front passenger seat.

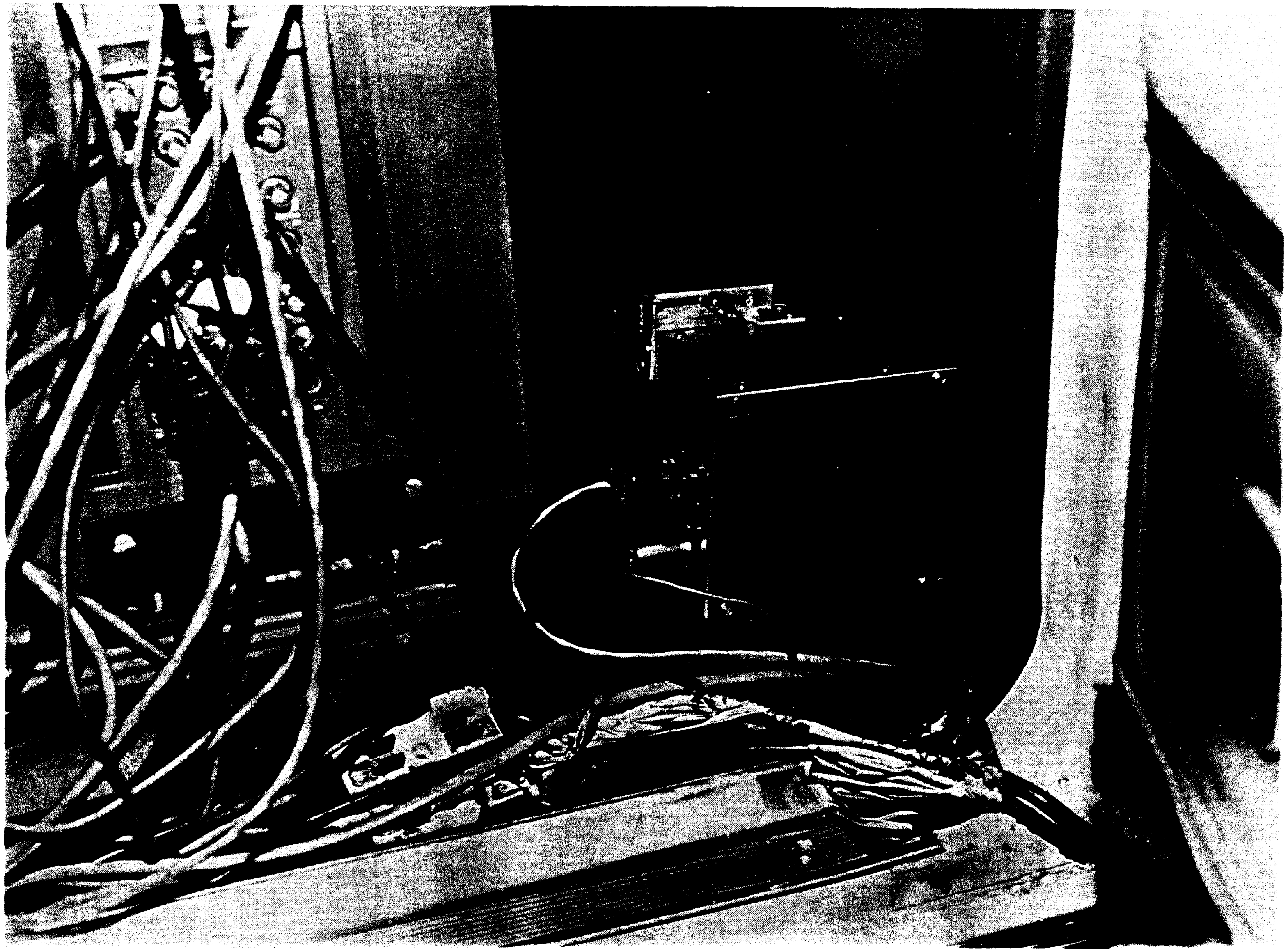

Figure B.4. Gyroscopes in front passenger foot well.

Figure B.5. Motor generator on roof rack.

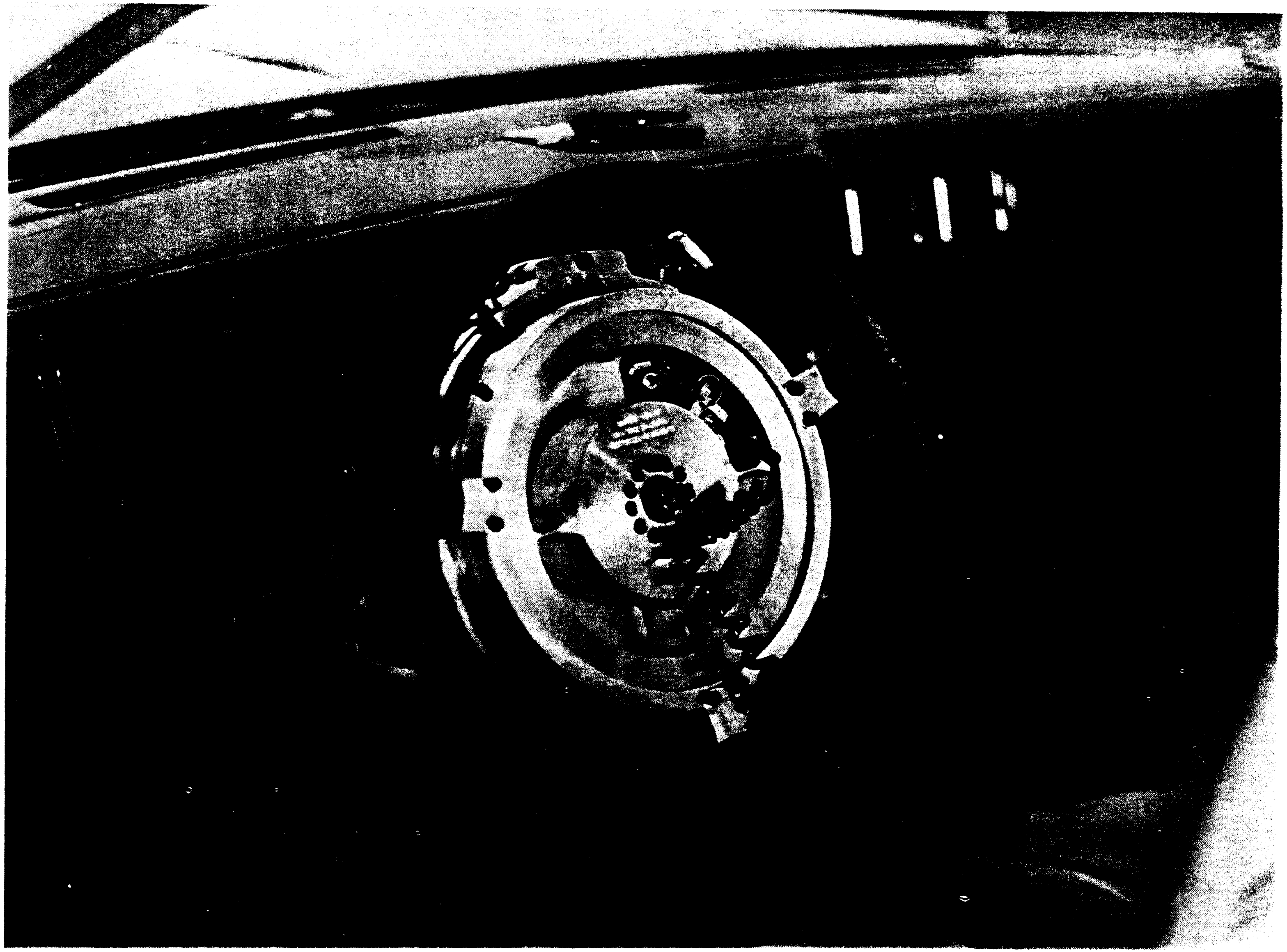

Figure B.6. Variable ratio steering wheel limiter with steering torque load cell.

Figure B.6 illustrates the VRSWL mounted in a test vehicle. Figure B.7 shows a side view of the device. To install the device in a test vehicle requires the fabrication of two simple parts. A steering shaft adaptor for mounting the VRSWL to the steering shaft is fabricated from the hub of an OE steering wheel. A reaction frame bracket which anchors the reaction frame to the drive shaft tunnel (see Figure B.6) or the "A-pillar" is also required.

The gear ratio function is attained using two pairs of change gears.* Referring to Figure B.7, Gear A is keyed to the steering shaft and drives Gear B which is keyed to Gear C. Gear C then drives Gear D which is keyed to the steering wheel but is free to rotate on the center shaft. By proper selection of the gear sizes, hundreds of ratios between at least 4:1 and 1:4 are available. In the field, ratios may easily be changed within a few minutes.

An adjustable stop mechanism provides a steer angle limiter. The stop is easily adjustable by the driver and may provide a limit to steering wheel travel within approximately a ±720 degree range.

Steering angle is transduced at the input end of the steering shaft using a potentiometer geared (using anti-backlash instrumentation gears) to rotate at twice the shaft rate. Shaft angle, rather than actual steering wheel angle is transduced in order that gear ratio changes made to the VRSWL do not affect calibration and/or displacement range of the transducer (for a given vehicle maneuver). Since the gear train of the VRSWL is very rigid and since provisions have been made for eliminating virtually all lash from this train, input shaft angle and steering wheel are simply proportional according to the VRSWL ratio chosen.

A special steering-wheel torque transducer (Figure B.6) was designed and constructed for use under this project. The transducer was specifically designed for use with the VRSWL.

The transducer, itself, is cylindrical in shape and consists of a rigid inner core and concentric outer ring connected by four radial

*Boston Gear Series GB or equivalent.

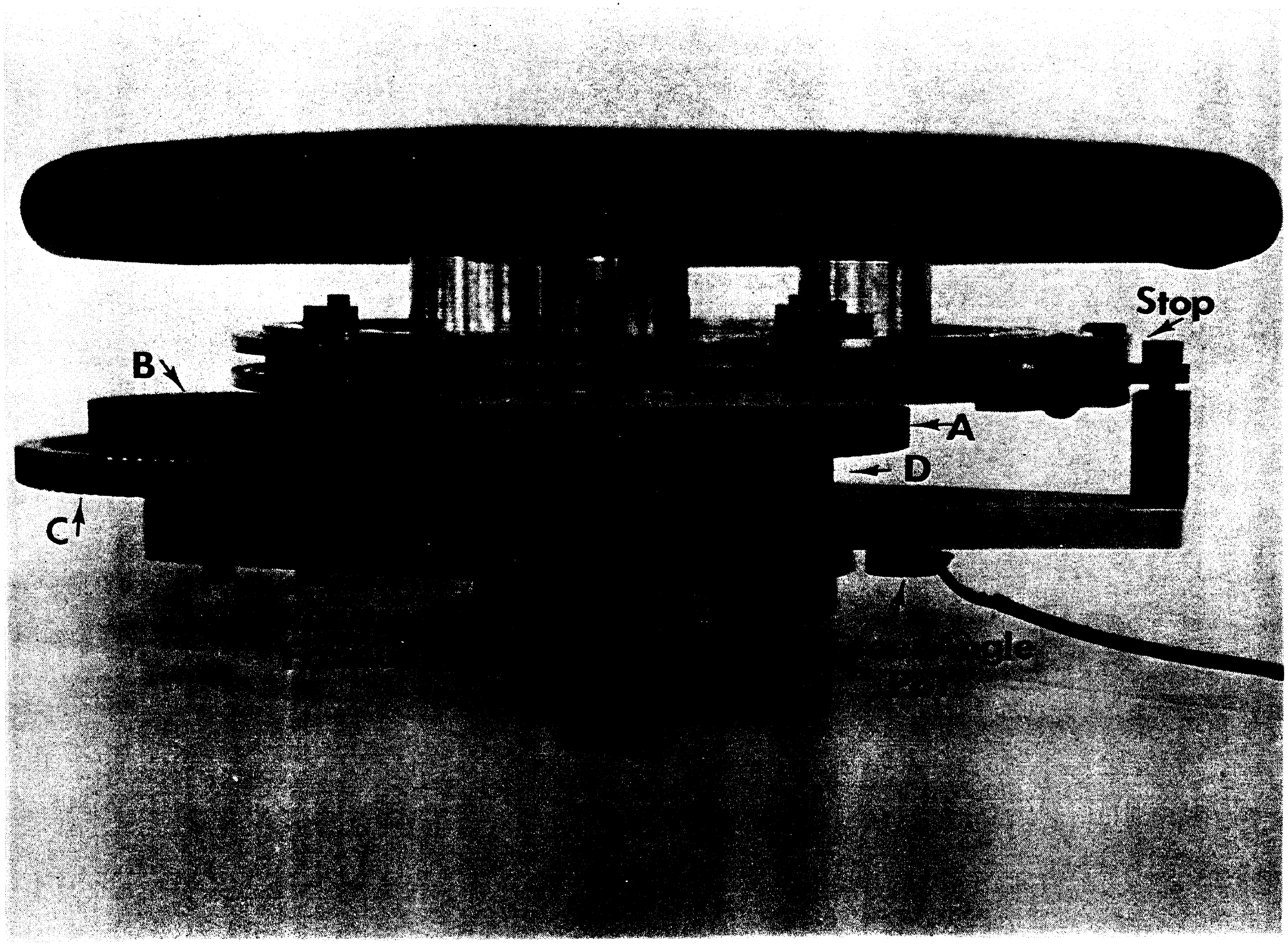

Figure B.7. Side view of VRSWL.

flexture beams. Two of these beams are strain gauged to provide a four-active-arm bridge sensitive to torques about the longitudinal axis of the transducer. A modified steering wheel attaches to the outer ring as does the steering-wheel limiter mechanism. The input gear of the VRSWL is keyed to the inner core. Thus, the transduced torque consists only of torque which is input to the steering system. (That is, driver-applied torque which is reacted by the limiter mechanism is not included in the measurement.)

The transducer also incorporates a self-contained electronic unit which can be seen in Figure B.6. This package includes amplification and shunt resistor calibration circuitry.

The four element telephone-type cable, seen in Figure B.6, provides for the input of plus and minus 15 volts, for signal output, and for electrical common. In the vehicle testing done in this project, none of the drivers felt that this cable arrangement was at all bothersome. This was largely because none of the test maneuvers required more than ±90° steering-wheel angle motions. For larger motions, a slip ring arrangement might be preferable. Since signal output is already at an amplified level, slip rings should not significantly affect signal noise level.

Specifications for the transducer, as verified by physical calibrations conducted in June of 1977, are as shown in Table B.1. The reference axis system employed in the table is shown in Figure B.8.

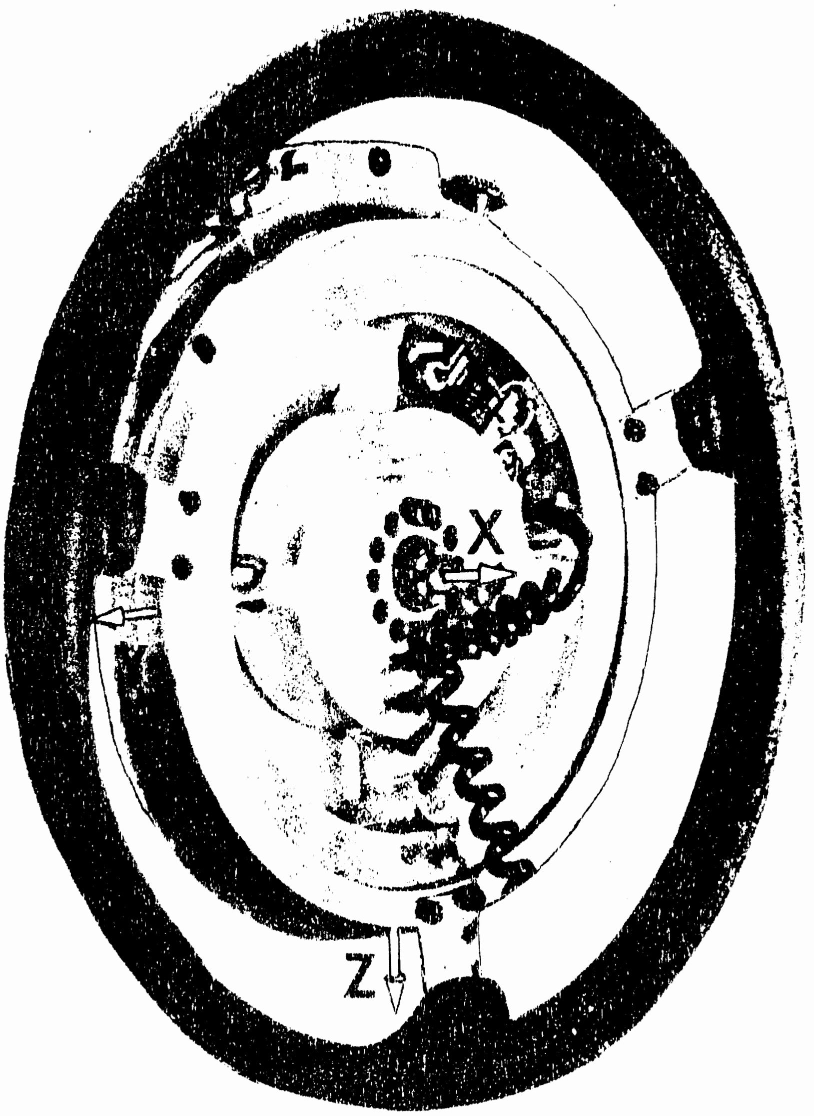

Figure B.8. Steering torque load cell reference axis system.

Table B.1. Steering-Wheel Torque Transducer Calibration Data.

Full Range Input M_x: 600 in-lb

Calibration data for strain gauge bridge only:

Color Code (four element ribbon wire between bridge and electronics):

Red: +Excitation

Black: -Excitation

White: +Signal

Brown: -Signal

Primary Sensitivity*:

M_x: Gain: 4.27 mv/v @ full range

Linearity: ±.25%

Hysteresis: .25%

Cross Sensitivities*:

to F_x: 0.00 mv/v @ 100 lb (0% of full scale)

to F_y: 0.01 mv/v @ 100 lb (.25% of full scale)

to F_z: 0.00 mv/v @ 100 lb (0% of full scale)

to M_y: 0.00 mv/v @ 600 in-lb (0% of full scale)

to M_z: 0.07 mv/v @ 600 in-lb (1.75% of full scale)

System Calibration:

Equivalent output using the installed shunt calibration resistor as obtained by depressing "Calibration" push-button: 304.5 in-lb

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*Instruments used limit the resolution of calibration to 0.01 mv/v or approximately 0.25% of full scale.

APPENDIX C

DERIVATION OF NONDIMENSIONAL PATH-CURVATURE GAIN REFERENCED TO STEERING-WHEEL DISPLACEMENT

There are at least two approaches to deriving the path-curvature gain of an automobile in a steady turn. One approach consists of writing the linear equations of equilibrium in stability derivative form and solving for the gain of interest. A second approach consists of expressing the kinematic relationship existing between the slip angles and the radius of the turn in terms of design variables. This latter approach is adopted below since it better permits the analyst to relate to the physical process that is involved.

The objective of this analysis is to derive the path-curvature gain of a vehicle that is free to roll and is steered by a steering wheel which is connected to the front wheels by a flexible or compliant steering mechanism. Although the aligning stiffness of the front tires will be taken into account in expressing moment equilibrium within the steering mechanism, the influence of aligning moments on yawing moment equilibrium will be neglected. As is usually done, the radius of the curved path will be assumed to be very large relative to the wheelbase and the track width of the motor vehicle. In this event, all angles are sufficiently small that the kinematic condition of turning and the equations of equilibrium can be written as if the vehicle possesses zero width with the right and left tires collapsed into an equivalent tire exhibiting twice the stiffnesses of the single tire.

If the angles between the velocity vectors (at the center of tire contact of the front and rear wheels) and the longitudinal axis are designated, respectively, as β_1 and β_2, the assumptions adopted above lead to the following kinematic result for the steady turn, viz.,

$$\beta_1 - \beta_2 = \ell/R$$

where

ℓ = wheelbase

R = radius of the steady turn

On adopting the SAE sign convention, one can write that the slip angle of the front and rear tires can be expressed as

$$\alpha_1 = \beta_1 - \delta_1$$

$$\alpha_2 = \beta_2 - \delta_2$$

where

α_1, α_2 = slip angle of front and rear tires, respectively

δ_1, δ_2 = steer displacement of the front and rear wheels, respectively

On combining the above expressions, one obtains

$$\alpha_1 + \delta_1 - \alpha_2 - \delta_2 = \ell/R \tag{C.1}$$

Equation (C.1) constitutes the kinematic geometry (see Figure C.1) that must be satisfied in a steady turn.

An expression for path-curvature gain can be obtained by substituting in Equation (C.1) for α_1, δ_1, α_2, and δ_2, respectively. Below, the slip angles are obtained from the requirement that equilibrium be satisfied simultaneously along the y axis and about the roll and yaw axes. The front-wheel steer angle, δ_1, is found from an equation which expresses equilibrium about the steering pivot and the rear-wheel steer angle, δ_2, is found from an equation expressing roll equilibrium.

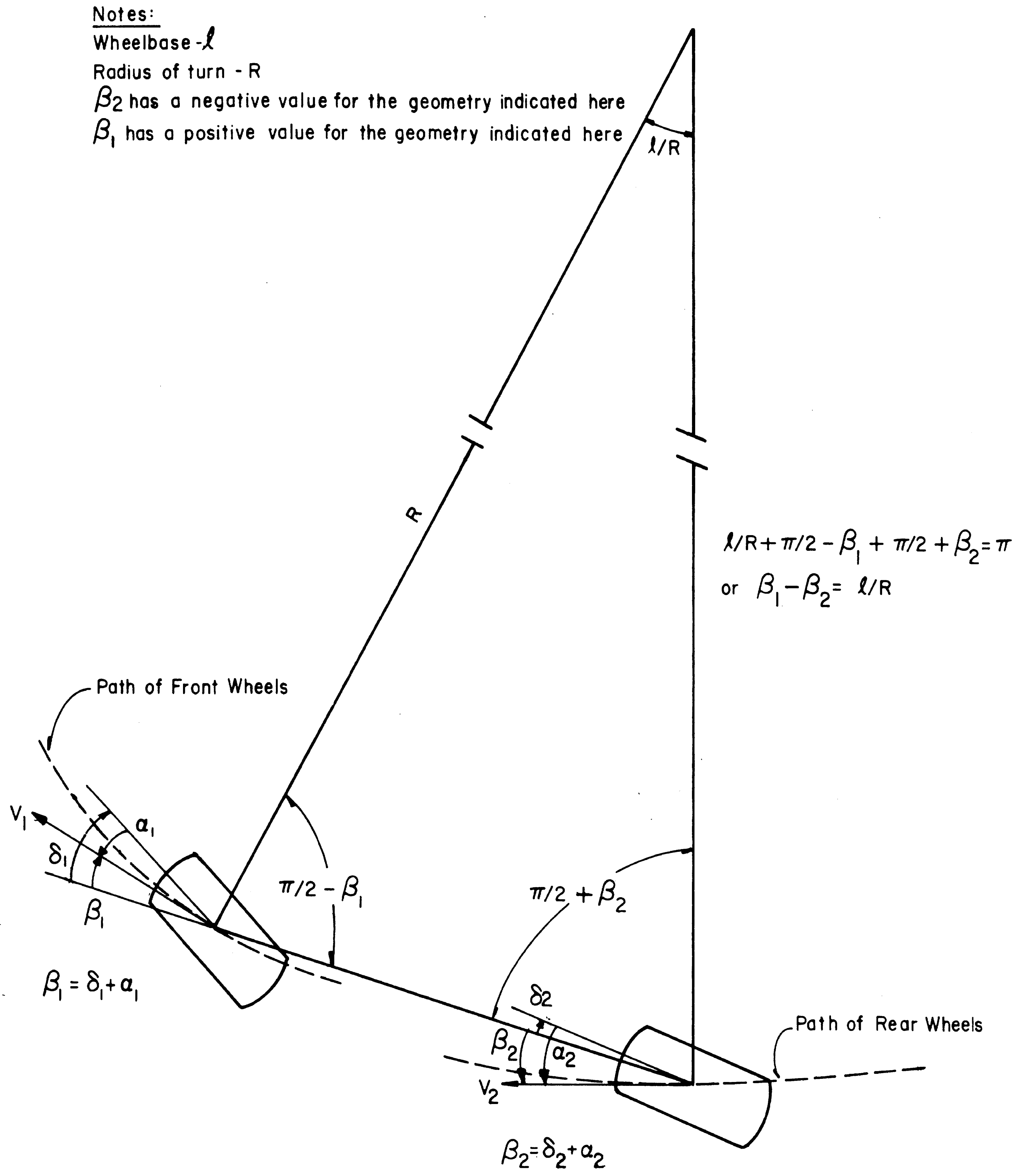

Figure C.1. Kinematic geometry for a high-speed steady turn.

Side-force and yawing-moment equilibrium require that

$$F_{y1} + F_{y2} = mVr$$

$$a F_{y1} - b F_{y2} = 0$$

where

F_{y1}, F_{y2} = lateral force at front and rear tires, respectively

m = mass of the vehicle

V = forward velocity

r = yaw velocity

a,b = distance of the total center of mass from the front and rear axles, respectively

On letting $C_{\alpha 1}$ and $C_{\alpha 2}$ represent the cornering stiffness of the front and rear tires and C_γ represent the inclination stiffness of the front tires (the rear tires will be assumed to be constrained to a nonrolling rear axle), the above equations of equilibrium may be written as

$$\begin{aligned} C_{\alpha 1}\alpha_1 + C_\gamma \frac{\partial\gamma}{\partial\phi}\phi + C_{\alpha 2}\alpha_2 &= mVr \\ a[C_{\alpha 1}\alpha_1 + C_\gamma \frac{\partial\gamma}{\partial\phi}\phi] - b\, C_{\alpha 2}\alpha_2 &= 0 \end{aligned} \qquad (C.2)$$

where

$\frac{\partial\gamma}{\partial\phi}$ = rate of front-wheel inclination per unit roll displacement of the sprung mass

ϕ = roll angle of the sprung mass

In a steady turn, roll equilibrium can be expressed as

$$L_\phi \phi - m_s hVr = 0$$

such that

$$\phi = \frac{m_s hVr}{L_\phi} \tag{C.3}$$

where

m_s = mass of the sprung chassis

h = height of the center of mass of the rolling chassis above the roll axis

L_ϕ = roll moment acting on the sprung mass per unit roll displacement of the chassis

On substituting Equation (C.3) into (C.2), we can solve for the front and rear slip angles, respectively, and obtain

$$\begin{aligned} \alpha_1 &= \frac{mVr}{C_{\alpha_1}} \left[\frac{b}{\ell} + \frac{C_\gamma \dfrac{\partial \gamma}{\partial \phi} \dfrac{m_s}{m} h}{-L_\phi} \right] \\ \alpha_2 &= \frac{a}{\ell} \frac{mVr}{C_{\alpha_2}} \end{aligned} \tag{C.4}$$

(Note that C_{α_1} and C_{α_2} are negative quantities and consequently α_1 and α_2 are negative angles when the vehicle is turning to the right such that it has a positive yaw rate.)

On assuming a solid rear axle that steers when the sprung mass rolls, we can write that

$$\delta_2 = \frac{\partial \delta_2}{\partial \phi} \phi = \frac{\partial \delta_2}{\partial \phi} \frac{m_s hVr}{L_\phi} \tag{C.5}$$

Moment equilibrium about the steering pivot can be expressed as the summation of the torque deriving from the "windup" of the steering mechanism and the torques deriving from the tire-road interface. If front roll steer, due to kinematic interaction between the path of the steering knuckle and the path of the tie-rod end, is defined as $\partial \delta_1 / \partial \phi$, steer moment equilibrium can be expressed as

$$K_{ss}\left[\delta'_{sw} - \left(\delta_1 - \frac{\partial \delta_1}{\partial \phi}\,\phi\right)\right] + \left.\frac{\partial M_z}{\partial \alpha}\right)_1 \alpha_1 + x_c\left[C_{\alpha_1}\alpha_1 + C_\gamma\,\frac{\partial \gamma}{\partial \phi}\,\phi\right] = 0 \tag{C.6}$$

where

K_{ss} = torsional stiffness of the total steering mechanism

δ'_{sw} = steering-wheel displacement divided by the overall gear ratio

$\left.\frac{\partial M_z}{\partial \alpha}\right)_1$ = aligning stiffness of the front tires

x_c = mechanical trail produced by caster angle (positive when the tire contact center is rearward of the steer axis)

On substituting for α_1 and ϕ, respectively, and solving for the front-wheel displacement, δ_1, Equation (C.6) yields

$$\delta_1 = \delta'_{sw} + mVr\left\{\frac{\frac{\partial \delta_1}{\partial \phi}\,\frac{m_s}{m}\,h}{L_\phi} + \frac{1}{K_{ss}}\left[\frac{\left.\frac{\partial M_z}{\partial \alpha}\right)_1}{C_{\alpha_1}}\left(\frac{b}{\ell} + \frac{C_\gamma\,\frac{\partial \gamma}{\partial \phi}\,\frac{m_s}{m}\,h}{-L_\phi}\right) - x_c\,\frac{b}{\ell}\right]\right\} \tag{C.7}$$

On substituting in Equation (C.1) for α_1, α_2, δ_1, and δ_2, as expressed by Equations (C.4), (C.5), and (C.7), respectively, one obtains

$$\ell/R = \delta'_{sw} + mVr\left\{\frac{1}{C_{\alpha_1}}\left[\frac{b}{\ell} + \frac{C_\gamma\,\frac{\partial \gamma}{\partial \phi}\,\frac{m_s}{m}\,h}{-L_\phi}\right] + \frac{\frac{\partial \delta_1}{\partial \phi}\,\frac{m_s}{m}\,h}{L_\phi}\right.$$
$$\left. + \frac{1}{K_{ss}}\left[\frac{\left.\frac{\partial M_z}{\partial \alpha}\right)_1}{C_{\alpha_1}}\left(\frac{b}{\ell} + \frac{C_\gamma\,\frac{\partial \gamma}{\partial \phi}\,\frac{m_s}{m}\,h}{-L_\phi}\right) - x_c\,\frac{b}{\ell}\right] - \frac{a}{\ell}\,\frac{1}{C_{\alpha_2}} - \frac{\partial \delta_2}{\partial \phi}\,\frac{\frac{m_s}{m}\,h}{L_\phi}\right\}$$

Since $r = \frac{V}{R}$, the above equation transforms into the following expression for the nondimensional path-curvature gain, viz.:

$$\frac{\ell/R}{\delta'_{SW}} = \frac{1}{1 + \left(\frac{V}{\sqrt{\ell g}}\right)^2 \left\{ \frac{\frac{a}{\ell} mg}{C_{\alpha_2}} - \frac{\frac{b}{\ell} mg}{C_{\alpha_1}} - \frac{mg}{C_{\alpha_1}} \frac{C_\gamma \frac{\partial \gamma}{\partial \phi} \frac{m_s}{m} h}{-L_\phi} + \frac{mg \frac{\partial \delta_1}{\partial \phi} \frac{m_s}{m} h}{-L_\phi} - \frac{\frac{b}{\ell} mg}{C_{\alpha_1}} \left[\frac{\left.\frac{\partial M_z}{\partial \alpha}\right)_1 - x_c C_{\alpha_1}}{K_{ss}} \right] - mg \frac{\partial \delta_2}{\partial \phi} \frac{\frac{m_s}{m} h}{-L_\phi} - \frac{mg}{C_{\alpha_1}} \frac{\left.\frac{\partial M_z}{\partial \alpha}\right)_1}{K_{ss}} \frac{C_\gamma \frac{\partial \gamma}{\partial \phi} \frac{m_s}{m} h}{-L_\phi} \right\}} \tag{C.8}$$

Note that all terms are nondimensional, with $V/\sqrt{\ell g}$ being the vehicle Froude number and each term within the braces having the dimensions of radians per g. Starting at the left, the first two terms, within the braces, define the understeer contribution deriving from rear- and front-wheel cornering compliances. The third term defines the understeer deriving from the inclination of the front wheels and the fourth term defines the understeer produced by front roll steer. The fifth and seventh terms yield the understeer caused by the finite compliance of the steering mechanism and the sixth term defines the understeer caused by roll steer of the rear axle. If the design variables are such that they produce an understeer effect rather than oversteer effect, all terms within the braces are positive with the exception of the first term.

In order to demonstrate the relative magnitude of the terms within the braces, we shall calculate their numerical values corresponding to the design properties of a representative vehicle, namely, the Ford Pinto. (The relevant design properties of the Ford Pinto are given in Table C.1.) Equation (C.8) yields the following result:

$$\frac{\ell/R}{\delta'_{SW}} = \frac{1}{1 + \left(\frac{V}{\sqrt{\ell g}}\right)^2 \{-.1099 + .1170 + .0274 + .0060 + .0460 + .0142 + .0091\}} \tag{C.9}$$

$$= \frac{1}{1 + \left(\frac{V}{\sqrt{\ell g}}\right)^2 \{.1098\}} \tag{C.10}$$

Table C.1

$mg = 3031$ lb

$a/\ell = .478$

$b/\ell = .522$

$C_{\alpha_1} = -13{,}523$ lb/rad

$C_{\alpha_2} = -13{,}179$ lb/rad

$C_{\gamma} = 3094$ lb/rad

$\partial\gamma/\partial\phi = 0.8$

$m_s/m = .879$

$h = 1.48$ ft.

$-L_{\phi} = 26{,}358$ ft-lb/rad

$\partial M_z/\partial\alpha)_1 = 1585$ ft-lb/rad

$x_c = .0217$ ft.

$K_{ss} = 4775$ ft-lb/rad

$\partial\delta_1/\partial\phi = 0.04$

$\partial\delta_2/\partial\phi = -0.095$

APPENDIX D

STEADY-STATE YAW RATE GAIN AND ROLL/G OF LATERAL ACCELERATION

D.1 Introduction

In this appendix, calculated values of yaw rate gain at 50 mph ($r/\delta_{sw}|_{50}$) and roll/G of lateral acceleration ($K\phi$) for a wide range of U.S. made automobiles are presented in tabular form. In addition, calculated estimates of transient response parameters (ζ, ω_n, and T_e) evaluated at 50 mph are included.

The steady-state yaw rate gain is sensitive to a host of vehicle parameters such as steering system stiffness, steering gear ratio, roll stiffness, and the cornering stiffnesses of the tires, while the presence of a roll stabilizer bar has an appreciable effect on $K\phi$. Calculations were hence made for both, standard models as well as models equipped with such optional equipment as power steering, radial tires, and roll stabilizer bars.

D.2 Vehicle Parameters Used in Calculations

Values of various vehicle parameters used in the calculation of $K\phi$ and $r/\delta_{sw}|_{50}$ were obtained from the specifications published by the Motor Vehicle Manufacturers Association [18] and data reported by Basso, G.L. [20]. Tire properties were obtained from a report published by Calspan Corporation [19]. For certain cars, such vehicle parameters as steering system stiffness and roll steer factor were not available. Estimates based on parametric values of other similarly sized cars were made in such instances.

Clearly, the validity of estimates of vehicle performance are no better than the accuracy of the parametric data available. Hopefully, future government research programs will provide more knowledge of parametric values, especially with regard to steering system stiffnesses and roll-related factors.

D.3 Description of Tabulated Data

Calculated values of $r/\delta_{sw}|_{50}$ and $K\phi$ are presented for a total of 24 cars, of which 8 are of the intermediate category, 11 of the compact category, and 5 of the subcompact category.

Column 1 contains the brand name of the car and a description of the optional equipment used. Column 2 contains the steering gear ratio, while the roll stiffness, $K\phi$, is given in Column 3. Columns 4 through 7 contain quantities which contribute to the understeer factor as described in Section 3 of the report.* Column 8 contains the understeer factor K while the yaw rate gain $r/\delta_{sw}|_{50}$ is given in Column 9. Transient response parameters (ζ, ω_n, T_r, and T_e) evaluated at 50 mph are given in the remaining columns.

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*$K = K_1 - K_2 + K_3 + K_4$

where K_1 is the contribution due to the front cornering coefficient and steering system compliance

K_2 is the contribution due to the rear cornering coefficient,

K_3 is the contribution due to roll effects, and

K_4 is the contribution due to rigid body aligning torque effects.

Subcompact

CAR		N_G	$K\phi$ (deg/G)	K_1 (deg/G)	K_2 (deg/G)	K_3 (deg/G)	K_4 (deg/G)	K (deg/G)	$r/\delta_{sw}\|_{50}$ (sec^{-1})	ζ	$\overline{\omega}_n$	T_r	$1/T_r$	T_e
Vega	Manual steering, front stabilizer standard	22.5	6.5	8.38	6.24	1.47	0.394	4.00	0.165	0.639	5.44	0.248	4.032	0.160
	Power steering, front stabilizer standard	16.5 – 13.5	6.5	7.55	6.24	1.34	0.374	3.02	0.263	0.692	5.30	0.248	4.032	0.164
Astre	Manual steering without front stabilizer	22.5	9.7	8.98	6.17	2.28	0.36	5.45	0.136	0.581	5.82	0.245	4.078	0.149
	Power steering without front stabilizer	16.5 – 13.5	9.7	7.89	6.17	2.04	0.34	4.10	0.222	0.635	5.68	0.245	4.078	0.152
	Manual steering with front stabilizer	22.5	6.04	8.98	6.17	1.42	0.36	4.59	0.152	0.614	5.51	0.245	4.078	0.159
	Power steering with front stabilizer	16.5 – 13.5	6.04	7.89	6.17	1.27	0.34	3.33	0.249	0.674	5.35	0.245	4.078	0.163
Pinto (Rack & pinion steering)	Manual steering without front stabilizer	24.06	9.11	8.07	5.90	2.48	0.36	5.01	0.135	0.592	6.16	0.234	4.265	0.140
	Power steering without front stabilizer	19.04	9.11	8.07	5.90	2.48	0.36	5.01	0.171	0.592	6.16	0.234	4.265	0.140
	Manual steering with front stabilizer	24.06	5.83	8.07	5.90	1.59	0.36	4.12	0.1534	0.629	5.793	0.234	4.265	0.151
	Power steering with front stabilizer	19.04	5.83	8.07	5.90	1.59	0.36	4.12	0.194	0.629	5.793	0.234	4.265	0.151
Bobcat (Rack & pinion steering)	Manual steering without front stabilizer	24.06	9.02	7.718	5.47	2.46	0.35	5.05	0.135	0.590	6.56	0.217	4.599	0.132
	Power steering without front stabilizer	19.04	9.02	7.718	5.47	2.46	0.35	5.05	0.171	0.590	6.56	0.217	4.599	0.132
	Manual steering with front stabilizer	24.06	5.78	7.718	5.47	1.58	0.35	4.17	0.152	0.627	6.17	0.217	4.599	0.142
	Power steering with front stabilizer	19.04	5.78	7.718	5.47	1.58	0.35	4.17	0.192	0.627	6.17	0.217	4.599	0.142
Gremlin	Manual steering without front stabilizer	29.1	9.59	9.59	5.80	2.52	0.39	6.70	0.091	0.539	6.26	0.231	4.338	0.140
	Power steering without front stabilizer	19.4	9.59	8.29	5.80	2.23	0.36	5.08	0.166	0.592	6.13	0.231	4.338	0.142
	Manual steering with front stabilizer	29.1	6.02	9.59	5.80	1.58	0.39	5.76	0.102	0.568	5.94	0.231	4.338	0.149
	Power steering with stabilizer	19.4	6.02	8.29	5.80	1.40	0.36	4.25	0.185	0.626	5.79	0.231	4.338	0.152

Compact (pg. 1 of 2)

CAR		N_G	$K\phi$ (deg/G)	K_1 (deg/G)	K_2 (deg/G)	K_3 (deg/G)	K_4 (deg/G)	K (deg/G)	$r/\delta_{sw}\|_{50}$ (sec^{-1})	ζ	$\overline{\omega_n}$	T_r	$1/T_r$	T_e
Nova Front stabilizer standard	Manual steering bias ply tires	26.41	6.84	10.20	6.67	1.75	0.48	5.76	0.107	0.59	5.11	0.265	3.772	0.173
	Power steering bias ply tires	15.07	6.84	9.22	6.67	1.63	0.45	4.63	0.214	0.638	5.031	0.265	3.772	0.174
	Manual steering radial tires	26.41	6.84	8.19	5.71	0.918	0.37	3.77	0.137	0.676	5.442	0.227	4.406	0.165
	Power steering radial tires	15.07	6.84	7,45	5.71	0.88	0.35	2.97	0.272	0.719	5.367	0.227	4.406	0.167
Ventura Front stabilizer standard	Manual steering bias ply tires	26.40	7.71	10.30	6.56	2.23	0.441	6.41	0.099	0.576	5.317	0.261	3.835	0.165
	Power steering bias ply tires	15.10	7.71	9.15	6.56	2.04	0.417	5.04	0.203	0.622	5.22	0.261	3.835	0.167
Omega Front stabilizer standard	Manual steering bias ply tires	26.16	7.2	9.88	6.66	1.75	0.43	5.38	0.112	0.609	5.09	0.265	3.778	0.173
	Power steering bias ply tires	15.1	7.2	9.01	6.66	1.617	0.412	4.38	0.224	0.648	5.01	0.265	3.778	0.175
Skylark Front stabilizer standard	Manual steering bias ply tires	26.16	7.2	10.04	6.69	1.78	0453	5.58	0.110	0.602	5.099	0.266	3.761	0.173
	Power steering bias ply tires	18.9	7.2	9.10	6.69	1.632	0.432	4.47	0.174	0.644	5.004	0.266	3.761	0.175
	Manual steering radial tires	26.16	7.2	8.36	5.56	0.512	0.37	3.68	0.140	0.6805	5.423	0.221	4.525	0.168
	Power steering radial tires	18.9	7.2	7.5	5.56	0.48	0.35	2.77	0.224	0.731	5.332	0.221	4.525	0.171
Maverick Front stabilizer standard	Manual steering bias ply tires	29.4	7.22	9.33	6.19	2.316	0.377	5.832	0.095	0.592	5.594	0.246	4.065	0.157
	Power steering bias ply tires	21.3	7.22	8.21	6.19	2.11	0.353	4.48	0.154	0.642	5.496	0.246	4.065	0.158
Comet Front stabilizer standard	Manual steering bias ply tires	29.4	7.27	9.34	6.29	2.33	0.38	5.76	0.096	0.594	5.525	0.249	4.00	0.159
	Power steering bias ply tires	21.3	7.27	8.22	6.29	2.13	0.357	4.42	0.156	0.645	5.427	0.249	4.00	0.160
	Manual steering radial tires	29.4	7.27	9.14	6.18	0.97	0.42	4.35	0.114	0.648	5.169	0.246	4.071	0.173
	Power steering radial tires	21.3	7.27	7.92	6.18	0.91	0.39	3.04	0.191	0.713	5.044	0.246	4.071	0.176

Compact (pg. 2 of 2)

CAR		N_G	$K\phi$ (deg/G)	K_1 (deg/G)	K_2 (deg/G)	K_3 (deg/G)	K_4 (deg/G)	K (deg/G)	$r/\delta_{sw}\|_{50}$ (sec^{-1})	ζ	$\overline{\omega_n}$	T_r	$1/T_r$	T_e
Granada & Monarch Front stabilizer standard	Manual steering, radial tires	29.4	7.88	9.66	6.47	1.09	0.47	4.75	0.108	0.631	5.043	0.257	3.889	0.176
	Power steering, radial tires	21.3	7.88	8.29	6.47	1.01	0.43	3.26	0.184	0.701	4.903	0.257	3.889	0.181
Valiant & Dart Bias tires	Manual steering without front stabilizer	27.55	11.4	9.53	6.20	3.65	0.364	7.35	0.087	0.549	5.96	0.246	4.058	0.146
	Power steering without front stabilizer	18.02	11.4	8.56	6.20	3.36	0.346	6.07	0.151	0.586	5.894	0.246	4.058	0.146
	Manual steering with front stabilizer	27.55	6.8	9.53	6.20	2.18	0.364	5.87	0.101	0.592	5.525	0.246	4.058	0.159
	Power steering with front stabilizer	18.02	6.8	8.56	6.20	2.00	0.346	4.71	0.177	0.635	5.443	0.246	4.058	0.161
Aspen & Volare Front stabilizer standard	Manual steering bias tires	28.6	6.4	11.66	6.41	2.30	0.444	7.99	0.078	0.536	5.428	0.255	3.925	0.164
	Power steering bias tires	18.7	6.4	9.70	6.41	1.99	0.405	5.68	0.151	0.602	5.303	0.255	3.925	0.166
	Manual steering radial tires	28.6	6.4	10.1	5.62	0.95	0.41	5.84	0.097	0.596	5.599	0.223	4.477	0.163
	Power steering radial tires	18.7	6.4	8.17	5.62	0.85	0.37	3.77	0.192	0.679	5.469	0.223	4.477	0.165
Hornet Bias tires	Manual steering without front stabilizer	29.1	10.9	9.56	6.02	2.80	0.398	6.74	0.088	0.561	5.916	0.239	4.179	0.148
	Power steering without front stabilizer	19.4	10.9	8.29	6.02	2.46	0.372	5.10	0.158	0.614	5.799	0.239	4.179	0.149
	Manual steering with front stabilizer	29.1	6.5	9.56	6.02	1.67	0.398	5.61	0.099	0.596	5.566	0.239	4.179	0.159
	Power steering with front stabilizer	19.4	6.5	8.29	6.02	1.47	0.372	4.11	0.180	0.655	5.437	0.239	4.179	0.162
Pacer Bias tires	Manual steering without front stabilizer	30.0	11.8	10.12	6.34	3.13	0.478	7.38	0.082	0.528	5.94	0.252	3.969	0.146
	Power steering without front stabilizer	20.85	11.8	8.62	6.34	2.73	0.44	5.45	0.145	0.5866	5.8	0.252	3.969	0.148
	Manual steering with front stabilizer	30.0	7.4	10.12	6.34	1.96	0.478	6.22	0.092	0.561	5.597	0.252	3.969	0.157
	Power steering with front stabilizer	20.85	7.4	8.62	6.34	1.71	0.44	4.43	0.166	0.626	5.434	0.252	3.969	0.148

Intermediate

CAR		N_G	K_ϕ (deg/G)	K_1 (deg/G)	K_2 (deg/G)	K_3 (deg/G)	K_4 (deg/G)	K (deg/G)	$r/\delta_{sw}\|_{50}$ (sec^{-1})	ζ	$\overline{\omega_n}$	T_r	$1/T_r$	T_e
Chevelle 1. Front stabilizer standard 2. Radial tires	Manual steering without rear stabilizer	34.7–32.8	8.1	11.9	7.8	0.93	0.50	5.53	0.082	0.612	4.26	0.310	3.226	0.208
	Power steering without rear stabilizer	17.6–14.2	8.1	9.94	7.8	0.82	0.46	3.42	0.212	0.702	4.07	0.310	3.226	0.217
	Manual steering with rear stabilizer	34.7–32.8	6.4	11.9	7.8	0.74	0.50	5.34	0.084	0.619	4.22	0.310	3.226	0.211
	Power steering with rear stabilizer	17.6–14.2	6.4	9.94	7.8	0.65	0.46	3.25	0.217	0.711	4.02	0.310	3.226	0.221
Lemans 1. Front stabilizer standard 2. Radial tires	Manual steering without rear stabilizer	32.5	8.3	12.04	7.87	0.96	0.52	5.65	0.086	0.608	4.25	0.313	3.197	0.208
	Power steering without rear stabilizer	17.5–14.2	8.3	10.36	7.87	0.86	0.48	3.83	0.201	0.681	4.09	0.313	3.197	0.216
	Manual steering with rear stabilizer	32.5	6.4	12.04	7.87	0.74	0.52	5.43	0.089	0.616	4.20	0.313	3.197	0.211
	Power steering with rear stabilizer	17.5–14.2	6.4	10.36	7.87	0.66	0.48	3.63	0.204	0.691	4.03	0.313	3.197	0.220
Cutlass	Power steering with front stabilizer and radial tires	18.2–16.2	8.4	10.45	7.69	0.88	0.47	4.11	0.180	0.667	4.209	0.306	3.272	0.209
Century 1. Front stabilizer 2. Radial tires	Power steering without rear stabilizer	17.89	8.4	9.62	7.72	0.821	0.397	3.12	0.219	0.718	4.067	0.307	3.259	0.217
	Power steering with rear stabilizer	17.89	6.4	9.62	7.72	0.63	0.397	2.93	0.225	0.729	4.006	0.307	3.259	0.222
LTD II & Cougar	Power steering with front stabilizer std. and radial tires	21.76	6.6	11.61	6.52	1.04	0.65	6.78	0.114	0.576	5.02	0.259	3.859	0.179
Monaco 1. Front stabilizer standard 2. Bias ply tires	Manual steering without rear stabilizer	28.8	5.5	9.52	5.85	1.95	0.37	5.99	0.094	0.599	5.63	0.233	4.301	0.159
	Power steering without rear stabilizer	18.8	5.5	8.36	5.85	1.68	0.34	4.53	0.170	0.652	5.51	0.233	4.301	0.161
	Manual steering with rear stabilizer	28.8	4.7	9.52	5.85	1.67	0.37	5.70	0.096	0.609	5.54	0.233	4.301	0.162
	Power steering with rear stabilizer	18.8	4.7	8.36	5.85	1.44	0.34	4.29	0.175	0.663	5.43	0.233	4.301	0.164

Recalculations

CAR		N_G	K_ϕ (deg/G)	K_1 (deg/G)	K_2 (deg/G)	K_3 (deg/G)	K_4 (deg/G)	K (deg/G)	r/δ_{sw50} (sec^{-1})	ζ	$\overline{\omega}_n$	T_r	$1/T_r$	T_e
LTD II & Cougar	Power steering with front stabilizer std. and radial tires	21.76	6.6	9.80	6.52	0.95	0.59	4.82	0.141	0.642	4.9	0.259	3.859	0.183
Nova	Manual steering with front stabilizer std. and radial tires	26.41	7.7	9.61	5.83	1.12	0.45	5.34	0.112	0.611	5.5	0.232	4.316	0.164
	Power steering with front stabilizer std. and radial tires	15.07	7.7	8.50	5.83	1.05	0.43	4.15	0.228	0.658	5.43	0.232	4.316	0.165
Skylark	Manual steering with front stabilizer std. and radial tires	26.16	7.2	8.60	5.83	0.52	0.40	3.70	0.129	0.681	5.22	0.232	4.316	0.174
	Power steering with front stabilizer std. and radial tires	18.9	7.2	7.67	5.83	0.49	0.38	2.71	0.225	0.736	5.11	0.232	4316	0.172
Volare & Aspen	Manual steering with front stabilizer std. and radial tires	28.6	7.2	8.83	5.63	0.98	0.38	4.60	0.112	0.642	5.56	0.224	4.469	0.162
	Power steering with front stabilizer std. and radial tires	18.7	7.2	7.71	5.63	0.92	0.36	3.36	0.204	0.699	5.46	0.224	4.469	0.165

NOTES

1. LTD II & Cougar recalculated using higher steering system stiffness. $K_{SS} = 400$
2. Nova, Skylark, Volare and Aspen recalculated using measured vehicle weight.

   Nova: $W_f = 2255$ $W_r = 1890$

   Skylark: $W_f = 2080$ $W_r = 1930$

   Volare & Aspen: $W_f = 2345$ $W_r = 1835$

APPENDIX E

ANALOG COMPUTER SIMULATION OF VEHICLE RESPONSE TO STEP-STEERING INPUTS

E.1 Mathematical Model Used

A four-degree-of-freedom model was used in this simulation study. The four degrees of freedom were:

1) yaw rate, r
2) roll angle, ϕ
3) lateral velocity, v
4) wheel angle (front wheel), δ_f

Note: The assumption of a compliant steering system introduced the fourth degree of freedom (wheel angle, δ_f).

The differential equations of motion are:

1) Yaw Equation:

$$I_{zz_T} \dot{r} = aF_f - bF_r - m_s a_1 h \ddot{\phi} + AT_f + AT_r \tag{E.1}$$

2) Roll Equation:

$$I_{xs}\dot{p} + c_\phi p + K_\phi \phi = -(F_f - A_{y_f} mu_f)h_f - (F_r - A_{y_r} mu_r)h_r \tag{E.2}$$

3) Lateral Velocity-Sideslip Relationship:

$$\dot{v} + ur + \frac{m_s}{m} h \dot{p} = \frac{(F_{T_f} + F_{T_r})}{m} \tag{E.3}$$

4) Steering System:

$$\ddot{\delta}_f = \frac{K_{ss}}{I_s} \left(\frac{\delta_{sw}}{N_g} - \delta_f\right) - \frac{C_{Fs}}{I_s} \mathrm{sign}(\dot{\delta}_f) - \frac{F_s}{I_s}(x_{pf} + x_{mf}) - \frac{C_s}{I_s}(\dot{\delta}_f) \tag{E.4}$$

The constraint forces acting on the system are the lateral forces generated at the tire-road interface and are given by the following equations.

$$F_f = (\delta_f - \phi K_{frs} - \frac{v}{u} - \frac{ar}{u})C_{FD} + K_{\gamma\phi}\phi C_{\gamma} \quad (E.5)$$

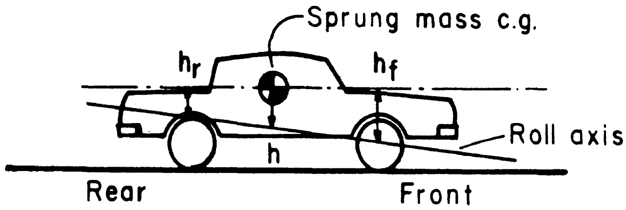
$$F_r = -(\frac{v}{u} - \frac{br}{u} + \phi K_{rrs})C_{RD} \quad (E.6)$$

E.2 Definition of Symbols Used in the Above Equations

Vehicle Parameters:

K_{ss}	Steering system stiffness
I_s	Steering system mass moment of inertia
N_g	Steering gear ratio
C_{Fs}	Coloumb friction in steering system
C_s	Viscous damping in steering system
x_{pf}	Pneumatic trail of front tires
x_{mf}	Mechanical trail of front tires
I_{zz_T}	Vehicle yaw moment of inertia
I_{xs}	Vehicle roll moment of inertia
C_{ϕ}	Roll viscous damping of sprung mass
K_{ϕ}	Roll stiffness
mu_f	Front unsprung mass
mu_r	Rear unsprung mass
m_s	Sprung mass
m	Total vehicle mass

h
h_f
h_r



a	Distance of total vehicle c.g. from front axle
b	Distance of total vehicle c.g. from rear axle
a_1	Distance of sprung mass c.g. from total vehicle c.g.
C_{FD}	Front tire cornering stiffness (of 2 front tires)
C_{RD}	Rear tire cornering stiffness (of 2 rear tires)
C_γ	Front camber stiffness
AT_f	Aligning torque of front tires
AT_r	Aligning torque of rear tires
K_{frs}	Front roll steer
K_{rrs}	Rear roll steer
$K_{\gamma\phi}$	Camber/unit roll

Variables:

δ_{sw}	Steering wheel angle
δ_f	Front wheel angle
F_f	Front lateral tire force
F_r	Rear lateral tire force
r	Yaw rate

p Roll rate ($p = \dot{\phi}$)

ϕ Roll angle of sprung mass

A_{y_f} Front unsprung mass lateral acceleration ($A_{y_f} = \dot{v} + ur + a\dot{r}$)

A_{y_r} Rear unsprung mass lateral acceleration ($A_{y_r} = \dot{v} + ur - b\dot{r}$)

u Forward velocity (this is a constant velocity model. "u" is a parameter of the simulation)

v Lateral velocity

Figure E.1 shows the analog circuit diagram and the differential equations of motion.

E.3 Vehicles Chosen for Simulation

Simulations were carried out for two vehicles:

1) Ford Pinto [Subcompact]

2) Plymouth Fury [Intermediate]

Five simulations were carried out on each of the vehicles—the five cases being:

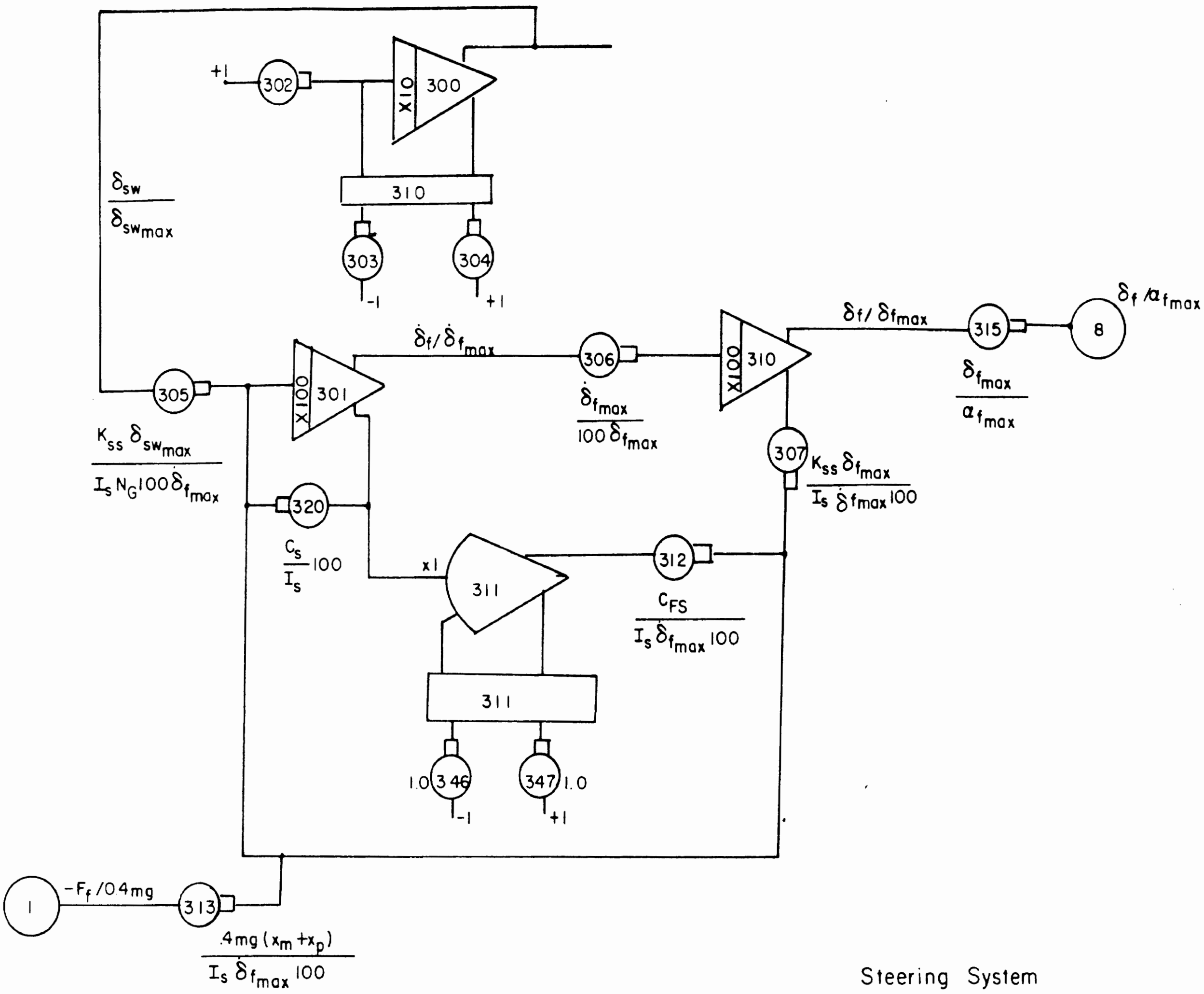
1) Baseline model - car equipped with manual steering, bias tires, and without stabilizing bar

2) With stabilizer - with addition of front stabilizer for Pinto and addition of rear stabilizer for Fury

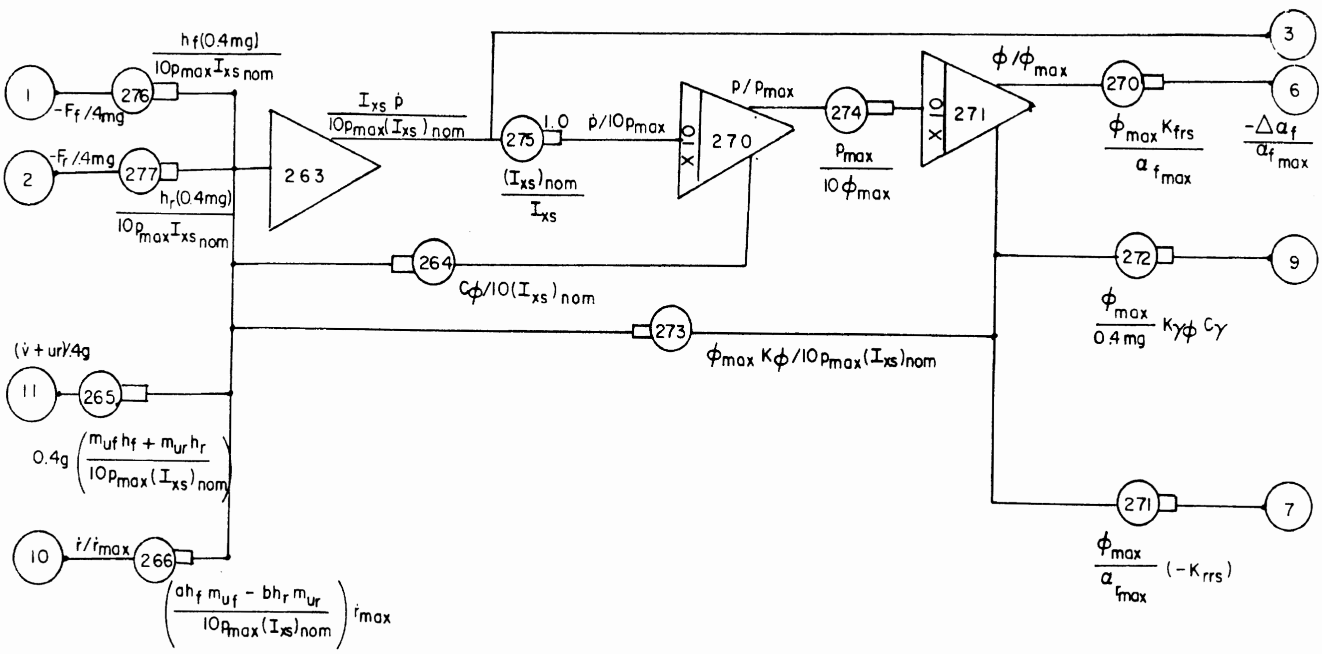
3) With power steering

4) With radial tires

5) With stabilizer, power steering, and radial tires

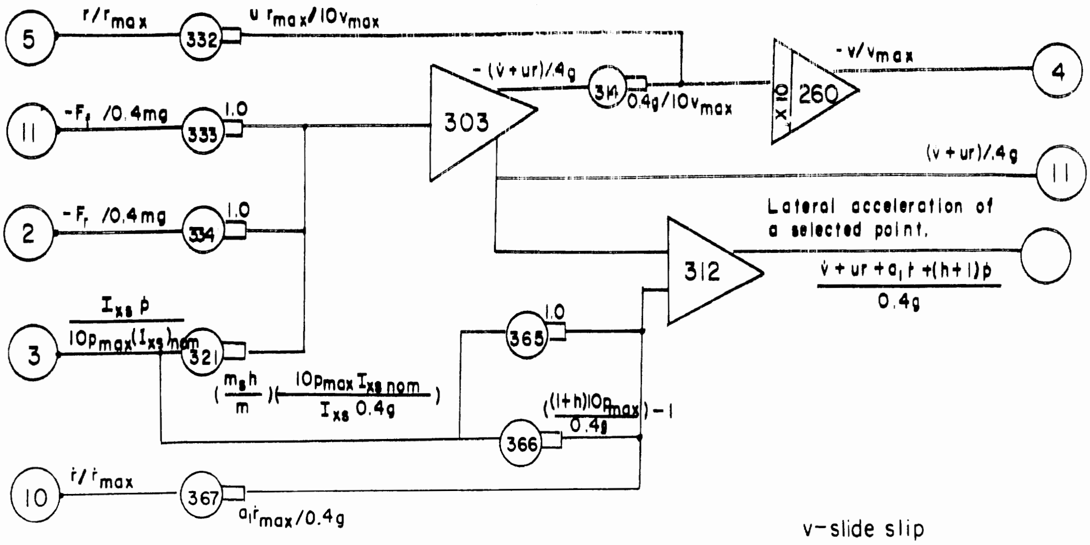


Steering System

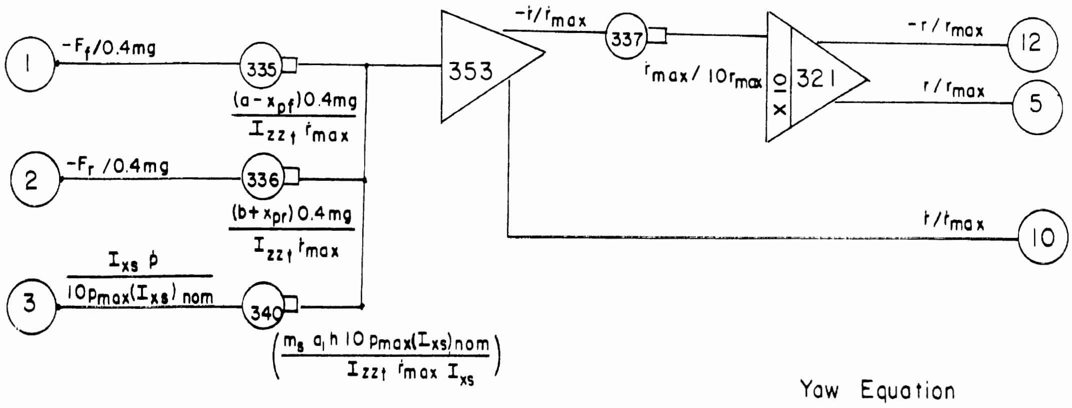


Roll Equation

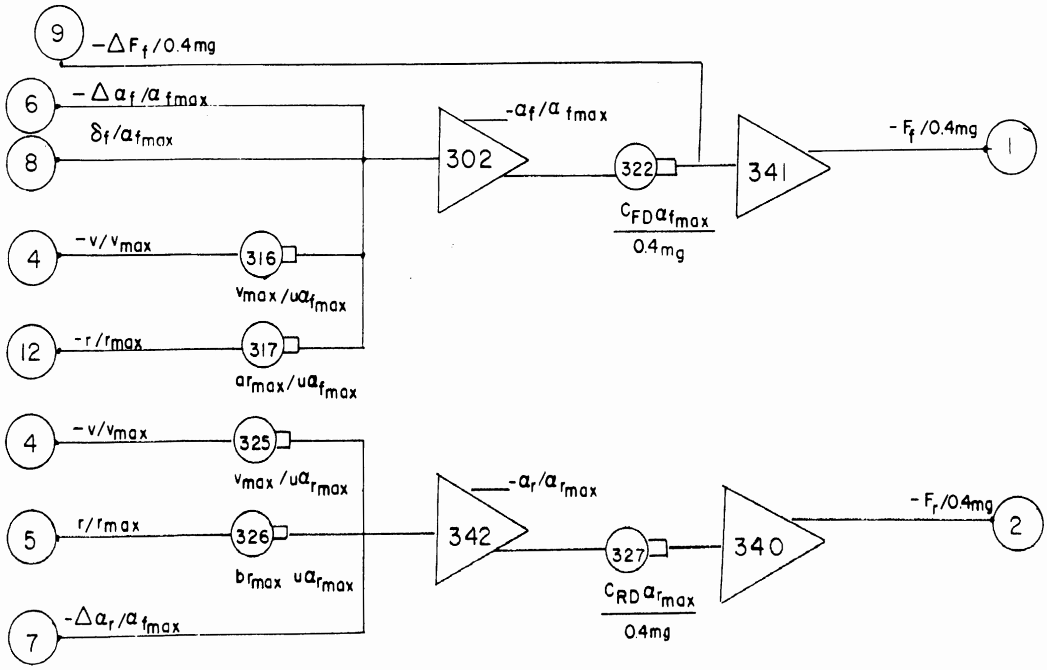
Figure E.1. Analog circuit diagram.



v-slide slip



Yaw Equation



Slip Angle & Lateral Force

Figure E.1 (Cont.)

Table 1 and Table 3 contain the parameters used in the simulation of the Fury and Pinto, respectively, while Table 2 and Table 4 contain the values of coefficients for the analog computer setup.

E.4 Results

The steady-state yaw rate gains at 50 mph forward velocity were found to be:

$$r/\delta_{sw}\Big|_{50}$$

Car	Baseline	With Stabilizer	With Power Steering	With Radial tires	With radial tires, power steering and stabilizer
Pinto	0.1208	0.1499	0.1541	0.1625	0.2422
Fury	0.1166	0.1208	0.1683	0.1333	0.1916

Table E.1

Plymouth Fury

	Symbol	Units	1	2	3	4	5
Steering	I_s	in·lb·sec^2	35*				
	C_s	in-lb/rad/sec	2000*				
	C_{fs}	in-lb	300*				
	K_{ss}	in-lb/deg	2000*				
	N_G	—	28.8		18.8		18.8
	X_p	inches	1.62			1.84	1.84
	X_m	inches	-0.112		0.169		0.169
Roll Equation	I_{xs}	lb-ft-sec^2	350				
	$C\phi$	ft-lb/rad/sec	800				
	$K\phi$	ft-lb/deg	1004	1161			1161
	h_f	ft	1.84*				
	h_r	ft	1.06*				
	gm_{uf}	lb	235				
	gm_{ur}	lb	380				
	K_{frs}		0.02				
	K_{rrs}		-0.05				
	$K_{\gamma\phi}$		0.88				
	$C\gamma$	lb/deg	73			14	14
v-side slip	u	ft/sec	73.3				
	gm_s	lb	3650				
	mg	lb	4265				
	$I_{xs\,nom}$	ft-lb-sec^2	350				
	h	ft	1.48*				
Lateral Force	a	ft	4.52				
	b	ft	5.29				
	I_{zzt}	ft-lb-sec^2	3050				
	a_1	ft	0.26				
	X_{pf}	inches	1.62			1.84	1.84
	X_{pr}	inches	1.34			1.56	1.56
	AT_f	ft-lb/deg	46			57	57
	AT_r	ft-lb/deg	37			45	45
	C_f	lb/deg	341			370	370
	C_r	lb/deg	332			346	346

NOTES:

1 = Baseline

2 = With rear stabilizer

3 = Power steering

4 = Radial tires GR78x15

5 = Power steering, rear stabilizer, & radial tires

* Values estimated based on Pinto data.

Table E.2

Plymouth Fury

Page 1 of 2

	Pot#		1	2	3	4	5
Roll Equation	264	$C\phi/10(I_{xs})_{nom}$	.2286				
	265	$0.4g(m_{uf} \cdot h_f + m_{ur} \cdot h_r)/10p_{max}(I_{xs})_{nom}$	.1060				
	266	$(ah_f m_{uf} - bh_r m_{ur})\dot{r}_{max}/10p_{max}(I_{xs})_{nom}$	.0045				
	270	$\phi_{max} K_{frs}/\alpha_{fmax}$	.0175				
	271	$\phi_{max} K_{rrs}/\alpha_{rmax}$	.0436				
	272	$(\phi_{max} K_{y\phi} C_y)/0.4mg$	.1506			.0288	.0288
	273	$\phi_{max} K\phi/10p_{max}(I_{xs})_{nom}$	1.258	1.4547			1.4547
	274	$p_{max}/10\phi_{max}$	1.289				
	275	$(I_{xs})_{nom}/I_{xs}$	1.0				
	276	$(h_f 0.4mg)/(10p_{max}(I_{xs})_{nom})$	.9839				
	277	$(h_r 0.4mg)/(10p_{max}(I_{xs})_{nom})$	.5667				
Steering system	302		1.0				
	303		1.0				
	304		1.0				
	305	$\delta_{swmax} K_{ss}/(I_s N_g 100\dot{\delta}_{fmax})$	.5951		.9116		.9116
	306	$\delta_{fmax}/100\dot{\delta}_{fmax}$	0.25				
	307	$(K_{ss}\delta_{fmax})/I_s\dot{\delta}_{fmax}100$	1.3096				
	312	$C_{fs}/I_s\dot{\delta}_{fmax}100$	.0428				
	313	$0.4mg(x_m + x_p)/I_s\dot{\delta}_{fmax}100$	.3685		.4372	.4162	.4896
	315	$\delta_{fmax}/\alpha_{fmax}$	1.0				
	320	$C_s/I_s 100$	.5714				
	346		1.0				
	347		1.0				

NOTES:

1 = Baseline

2 = With rear stabilizer

3 = Power steering

4 = Radial tires GR78x15

5 = Power steering, rear stabilizer & radial tires.

u = 73.33 ft/sec

v_{max} = 4 ft/sec

p_{max} = 0.900 rad/sec

ϕ_{max} = 0.06981 rad

$\dot{r}_{max}$ = 2.6178 rad/sec^2

r_{max} = 0.1745 rad/sec

δ_{fmax} = 0.08 rad.

$\dot{\delta}_{fmax}$ = 2.0 rad./sec.

α_{fmax} = 0.07997 rad.

δ_{swmax} = 1.047 rad.

$\alpha_{r max}$ = 0.07997 rad.

Plymouth Fury

pg. 2 of 2

	Pot #		1	2	3	4	5
v-side slip	314	$0.4g/10v_{max}$	.322				
	321	$(m_s h/m)(10p_{max}(I_{xs})_{nom})/(I_{xs}\,0.4g)$	.8967				
	332	$u\,r_{max}/10\,v_{max}$	.3199				
	333		1.0				
	334		1.0				
	365		1.0				
	366	$((1+h)10p_{max}/0.4g)-1$	.6830				
	367	$a_1\dot{r}_{max}/0.4g$	.0528				
Yaw Equation	335	$(a-x_{pf})0.4mg/I_{zzt}\,\dot{r}_{max}$	.9369			.9330	.9330
	336	$(b+x_{pr})0.4mg/I_{zzt}\,\dot{r}_{max}$	1.1541			1.1580	1.1580
	337	$\dot{r}_{max}/10\,r_{max}$	1.500				
	340	$(m_s a_1 h\,10p_{max}(I_{xs})_{nom})/(I_{zzt}\dot{r}_{max}I_{xs})$	.0492				
Slip-Lateral Force	316	$v_{max}/u\alpha_{f\,max}$	.6821				
	317	$a\,r_{max}/u\,\alpha_{f\,max}$	.1345				
	322	$(C_{fD}\,\alpha_{f\,max})/0.4mg$	.9159			.9937	.9937
	325	$v_{max}/u\alpha_{r\,max}$	.6821				
	326	$b\,r_{max}/u\,\alpha_{r\,max}$	.1574				
	327	$C_{RD}\,\alpha_{r\,max}/0.4\,mg$	.8917			.9293	.9293

NOTES:

1 = Baseline

2 = With rear stabilizer

3 = Power steering

4 = Radial tires GR78x15

5 = Power steering, rear stabilizer & radial tires.

$u = 73.33$ ft/sec

$v_{max} = 4$ ft/sec

$p_{max} = 0.900$ rad/sec

$\phi_{max} = 0.06981$ rad

$\dot{r}_{max} = 2.6178$ rad/sec^2

$r_{max} = 0.1745$ rad/sec

$\delta_{f_{max}} = 0.08$ rad.

$\dot{\delta}_{f_{max}} = 2.0$ rad/sec.

$\alpha_{f_{max}} = 0.07997$ rad.

$\delta_{sw_{max}} = 1.047$ rad.

$\alpha_{r\,max} = 0.07997$ rad.

Table E.3

Pinto

Group	Symbol	Units	1	2	3	4	5
Steering	I_s	in·lb·sec^2	30				
	C_s	in-lb/rad/sec	1608				
	C_{fs}	in-lb	240				
	K_{ss}	in-lb/deg	1500				
	N_G	—	24.1		19.04		19.04
	X_p	inches	1.41			1.31	1.31
	X_m	inches	0.26				
Roll Equation	I_{xs}	lb-ft-sec^2	235				
	$C\phi$	ft-lb/rad/sec	480				
	$K\phi$	ft-lb/deg	400	750			750
	h_f	ft	1.84				
	h_r	ft	1.06				
	gm_{uf}	lb	147				
	gm_{ur}	lb	219				
	K_{frs}		0.1				
	K_{rrs}		-0.1				
	$K\gamma\phi$		0.8				
	$C\gamma$	lb/deg	50			7.0	7.0
v-side slip	u	ft/sec	73.33				
	gm_s	lb	2665				
	gm	lb	3031				
	$I_{xs\ nom}$	ft·lb·sec^2	235				
	h	ft	1.48				
Lateral Force	a	ft	3.77				
	b	ft	4.11				
	I_{zzt}	ft-lb-sec^2	1500				
	a_1	ft	0.13				
	X_{pf}	inches	1.41			1.31	1.31
	X_{pr}	inches	1.1			1.05	1.05
	AT_f	ft-lb/deg	28			33	33
	AT_r	ft-lb/deg	21			23	23
	C_f	lb/deg	244			302	302
	C_r	lb/deg	232			264	264

NOTES:

1 = Baseline

2 = With front stabilizer

3 = Power steering

4 = Radial tires

5 = Power steering front stabilizer, radial tires

Table E.4

Pinto

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	Pot#		1	2	3	4	5
Roll Equation	264	$C\phi/10(I_{xs})_{nom}$	.204				
	265	$0.4g(m_{uf} \cdot h_f + m_{ur} \cdot h_r)/10 p_{max}(I_{xs})_{nom}$	.1225				
	266	$(a h_f m_{uf} - b h_r m_{ur})\dot{r}_{max}/10 p_{max}(I_{xs})_{nom}$	.0033				
	270	$\phi_{max} K_{frs}/\alpha_{fmax}$	.0873	.0625			.0625
	271	$\phi_{max} K_{rrs}/\alpha_{rmax}$	.0873	.0625			.0625
	272	$(\phi_{max} K_{\gamma\phi} C_{\gamma})/0.4\,mg$	.1320	.0945		.0185	.0133
	273	$\phi_{max} K\phi/10 p_{max}(I_{xs})_{nom}$	.975	1.3094			1.3094
	274	$p_{max}/10\phi_{max}$	1.0	1.3962			1.3962
	275	$(I_{xs})_{nom}/I_{xs}$	1.0				
	276	$(h_f\, 0.4mg)/(10 p_{max}(I_{xs})_{nom})$	1.3599				
	277	$(h_r\, 0.4mg)/(10 p_{max}(I_{xs})_{nom})$	.7834				
Steering system	302		1.0				
	303		1.0				
	304		1.0				
	305	$\delta_{sw_{max}} K_{ss}/(I_s N_g 100\, \dot{\delta}_{fmax})$	.7793		.9848		.7386
	306	$\delta_{f_{max}}/100\dot{\delta}_{fmax}$	0.2				
	307	$(K_{ss}\,\delta_{fmax})/I_s\,\dot{\delta}_{fmax}\,100$	1.432				
	312	$C_{fs}/I_s\,\dot{\delta}_{fmax}\,100$	0.05				
	313	$0.4mg(x_m + x_p)/I_s\,\dot{\delta}_{fmax}\,100$	.4218			.3965	.3965
	315	$\delta_{f_{max}}/\alpha_{fmax}$	1.0				
	320	$C_s/I_s\,100$	0.536				
	346		1.0				
	347		1.0				

NOTES:

1 = Baseline

2 = With front stabilizer, $\phi_{max} = 0.05$

3 = Power steering

4 = Radial tires

5 = Power steering, front stabilizer & radial tires. $\delta_{sw} = 45°$, $\phi_{max} = 0.05$

$u = 73.33$ ft/sec

$v_{max} = 4$ ft/sec

$p_{max} = 0.6981$ rad/sec

$\phi_{max} = 0.06981$ rad

$\dot{r}_{max} = 2.6178$ rad/sec^2

$r_{max} = 0.1745$ rad/sec

$\delta_{f_{max}} = 0.08$ rad.

$\dot{\delta}_{f_{max}} = 1.6$ rad/sec

$\alpha_{f_{max}} = 0.07997$ rad.

$\delta_{sw_{max}} = 1.047$ rad.

$\alpha_{r\,max} = 0.07997$ rad.

Pinto

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	Pot #		1	2	3	4	5
v-side slip	314	$0.4g/10v_{max}$	.322				
	321	$(m_s h/m)(10p_{max}(I_{xs})_{nom})/(I_{xs}0.4g)$	.7053				
	332	$u r_{max}/10v_{max}$	.3198				
	333		1.0				
	334		1.0				
	365		1.0				
	366	$((1+h)10p_{max}/0.4g)-1$	.3442				
	367	$a_1\dot{r}_{max}/0.4g$	.0528				
Yaw Equation	335	$(a-x_{pf})0.4mg/I_{zzt}\dot{r}_{max}$	1.1277			1.1303	1.1303
	336	$(b+x_{pr})0.4mg/I_{zzt}\dot{r}_{max}$	1.2972			1.297	1.297
	337	$\dot{r}_{max}/10r_{max}$	1.5				
	340	$(m_s a_1 h\, 10p_{max}(I_{xs})_{nom})/(I_{zzt}\dot{r}_{max}I_{xs})$	.0283				
Slip-Lateral Force	316	$v_{max}/u\alpha_{f\,max}$	.6821				
	317	$a\, r_{max}/u\alpha_{f\,max}$	.1122				
	322	$(C_{fD}\alpha_{f\,max})/0.4mg$	.922			1.1412	1.1412
	325	$v_{max}/u\alpha_{r\,max}$	.6821				
	326	$b r_{max}/u\alpha_{r\,max}$	.1223				
	327	$C_{RD}\alpha_{r\,max}/0.4mg$	.8768			0.9977	0.9977

NOTES:

1 = Baseline

2 = With front stabilizer, $\phi_{max} = 0.05$

3 = Power steering

4 = Radial tires

5 = Power steering, front stabilizer & radial tires. $\delta_{sw} = 45°$, $\phi_{max} = 0.05$

$u = 73.33$ ft/sec

$v_{max} = 4$ ft/sec

$p_{max} = 0.6981$ rad/sec

$\phi_{max} = 0.06981$ rad

$\dot{r}_{max} = 2.6178$ rad/sec^2

$r_{max} = 0.1745$ rad/sec

$\delta_{f\,max} = 0.08$ rad.

$\dot{\delta}_{f\,max} = 1.6$ rad/sec

$\alpha_{f\,max} = 0.07997$ rad.

$\delta_{sw\,max} = 1.047$ rad.

$\alpha_{r\,max} = 0.07997$ rad.

APPENDIX F

CONSULTATION VISITS AND DEMONSTRATION

This appendix documents (1) the consultation visits made to vehicle manufacturers and (2) the "hands-on" demonstration conducted at the end of the program.

The three consultation visits were conducted in the following manner. First, HSRI explained the elements of the project. Then, industry personnel presented their general views. And, finally, a question and answer discussion took place. The meetings were conducted informally and no formal statements were solicited from industry personnel.

The first consultation visit was made on June 7, 1977, to General Motors Corporation Proving Grounds in Milford, Michigan. The attendees from General Motors were:

R. Humphries
R. Rasmussen
T. Bundorf
R. Riefe
S. Anderson

The second consultation visit was made on June 10, 1977, to the Chrysler Engineering Offices in Highland Park, Michigan. The attendees from Chrysler were:

C. Kennedy
E. Kramer
M. Agar

The third consultation visit was made on August 19, 1977, to the Ford Motor Company Headquarters in Dearborn, Michigan. Those in attendance included:

L. Forbes
E. Farber
W. Bergman
T. Richardson
P. Schuba
R. Freeman
A. Anderson
S. Chiang

A demonstration, providing the attendees the opportunity to drive modified and unmodified versions of 1977 models of the Pinto, Skylark, and Fury, was held on August 31, 1977, at the Chrysler Proving Grounds in Chelsea, Michigan. Expert drivers from General Motors, Ford, and Chrysler were in attendance at the demonstration. In addition to HSRI personnel, the following people attended the demonstration:

Stan Anderson	Saginaw Steering Gear, GM
Dick Riefe	Saginaw Steering Gear, GM
Roy Nagel	GM Environmental Activities Staff
Brian Repa	GM Research Labs
Richard Rasmussen	GM Proving Ground
David A. Finch	Modern Engineering Service Company
George Yee	Ford Motor Company
Edward G. Kramer	Chrysler
Francis DiLorenzo	NHTSA/CARD
Shang-li Chiang	Ford Motor Company

The schedule for the demonstration was as follows:

STEERING CONTROLLABILITY CHARACTERISTICS
(Project No. DOT-HS-6-01409)

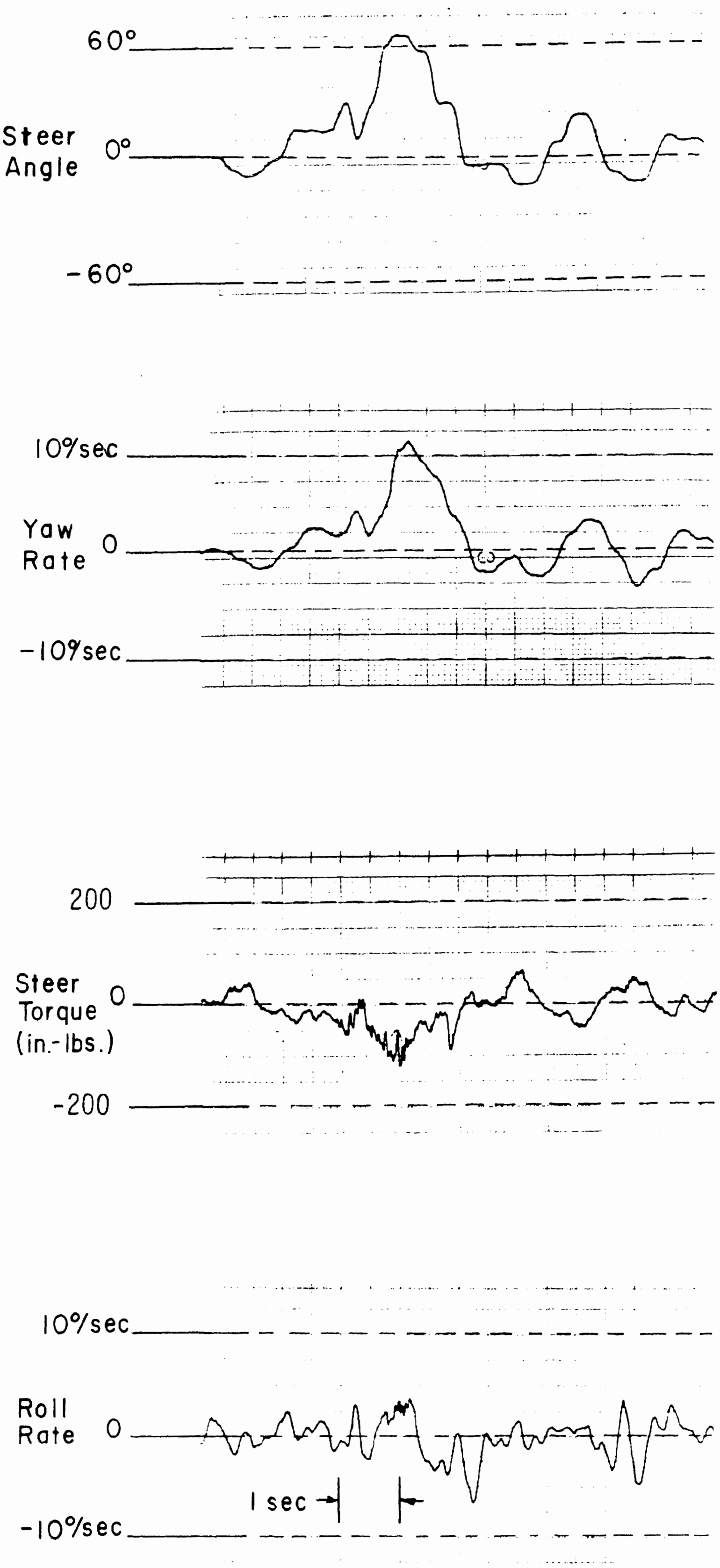
SCHEDULE FOR AUGUST 31, 1977

8:30 a.m.	(HSRI)	Welcome and Introduction of Contract Technical Manager
9:00		Description of the Project and the Format of the Demonstration
10:00		Leave for Chrysler Proving Grounds in Vans Provided by University of Michigan
11:00		Arrive at Test Area for Explanation and Demonstration of Equipment
12:00		Picnic Lunch
12:30 p.m.		Designated Drivers Try the Cars
2:30		Other Drivers Try the Cars
3:30		Vans Depart Chrysler Proving Grounds for HSRI
4:15		Arrive Back at HSRI - Participants Depart in Their Cars

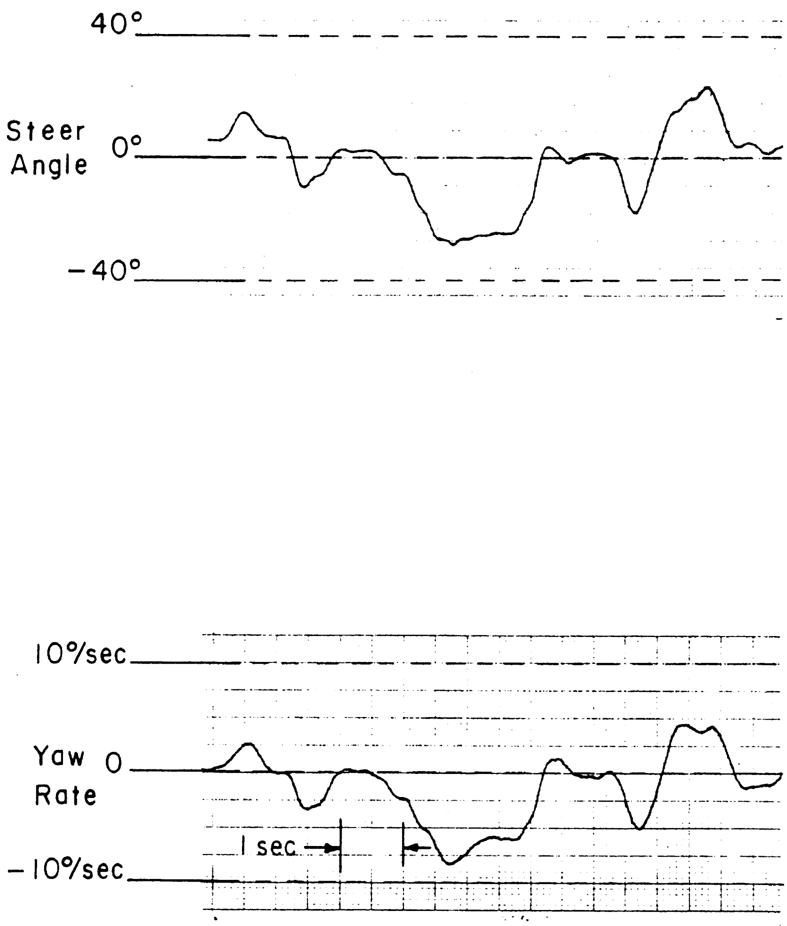
APPENDIX G

SAMPLE TEST DATA

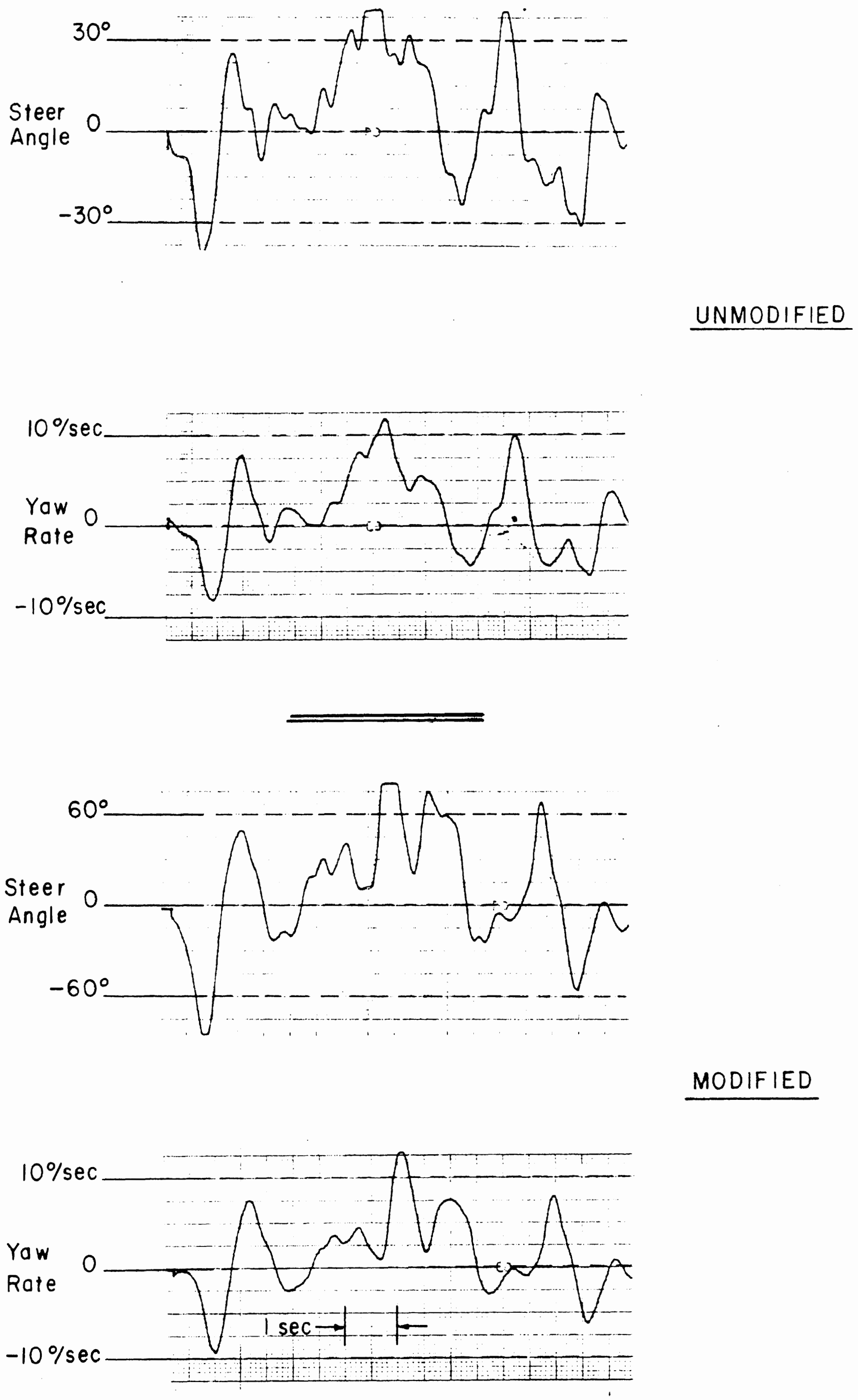
This appendix contains sample test data representative of the data recorded during each test series and subsequently stored on magnetic tape.



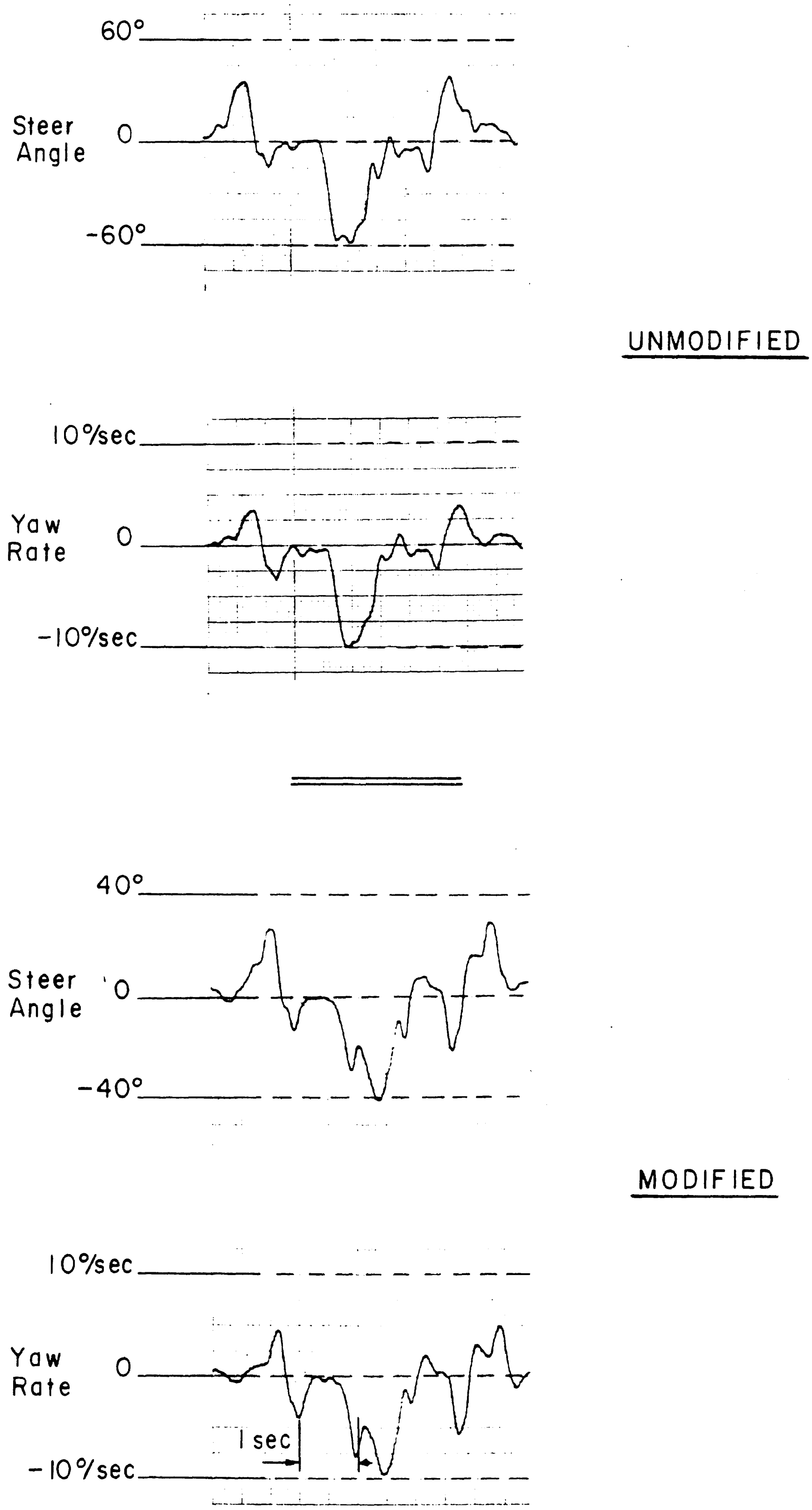
DRIVER COURSE - FORD PINTO - UNMODIFED



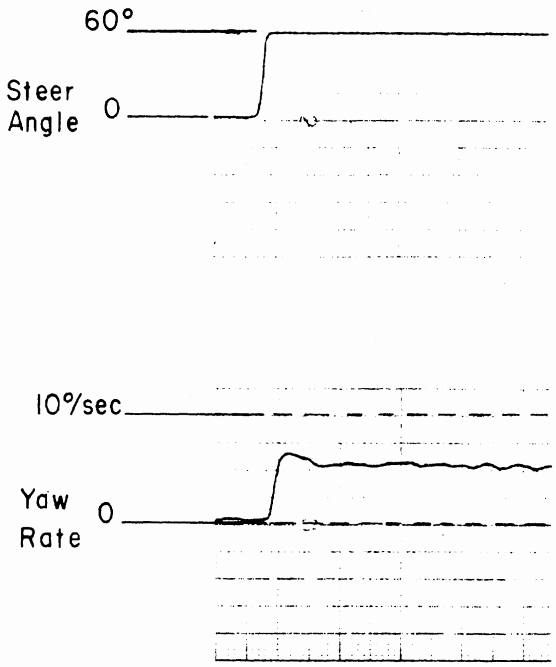
DRIVER COURSE - FORD PINTO - MODIFIED



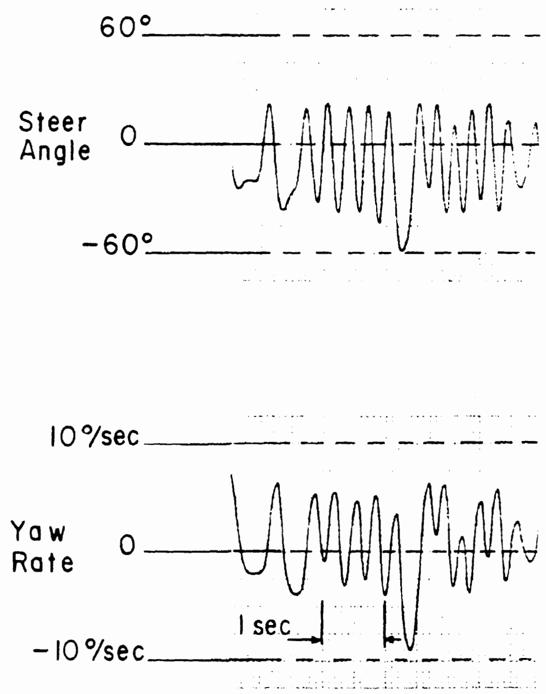
DRIVER COURSE - BUICK SKYLARK



DRIVER COURSE – PLYMOUTH FURY



STEP-STEER TEST - BUICK SKYLARK - MODIFIED



PULSE-STEER TEST - BUICK SKYLARK - UNMODIFIED