THE UNIVERSITY OF MICHIGAN

INDUSTRY PROGRAM OF THE COLLEGE OF ENGINEERING

RELATIONSHIP BETWEEN UNSTEADY STATE WELL PERFORMANCE AND INSITU RESERVOIR CHARACTERISTICS

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A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy in the
University of Michigan
1963

July, 1963

IP-627

ACKNOWLEDGEMENTS

The author wishes to express his sincere appreciation to the following individuals and organizations for their contributions to this research:

Professor Donald L. Katz, co-chairman of the doctoral committee, for his advice and interest in the formulation of the problems investigated in the course of this research and for his aid in securing field well test data.

Associate Professor M. Rasin Tek, co-chairman of the doctoral committee, for his generous donation of time in discussing the mathematical problems which arose in the course of this research.

Professors G. Brymer Williams, Robert L. Kadlec and Bernard A. Galler, committee members, for their interest and valuable discussions.

Doctors J. E. Briggs, S. C. Jones, R. L. Nielsen, and J. R. Street and Messrs. H. B. Kristinsson, D. A. Saville, and J. L. Skinner for their interest and helpful suggestions.
tions.

The Michigan Gas Association and American Gas Association fellowships for their financial support.

The Northern Illinois Gas Company for their donation of aquifer pump test data.

Messrs. R. E. Carroll, D. L. Danford and the personnel of the Industry Program of the College of Engineering for their aid and production of the final form of this dissertation.

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NOMENCLATURE

Engineering units are underlined.

A	Cross sectional area normal to flow, cm^2, $\underline{(feet)^2}$
a	Scaling factor defined by Equation (III-55)
B	Term defined by Equation (V-4a)
C_i	Term defined by Equation (III-66)
c	Compressibility, vol/(vol)(atm), <u>vol/(vol)(psi)</u>
c (obs)	Observed compressibility, vol/(vol)(atm), <u>vol/(vol)(psi)</u>
D_i	Term defined by Equation (III-66)
E_i	Exponential integral, $Ei(-x) = -\int_{+x}^{\infty} \frac{1}{u} e^{-u} du$
h	Reservoir height, <u>feet</u>
h'	Caprock thickness, <u>feet</u>
h''	Bottom rock thickness, <u>feet</u>
I_o	Modified Bessel function, first kind, zeroth order
J_o	Bessel function, first kind, zeroth order
J_1	Bessel function, first kind, first order
K	Permeability, darcys, <u>millidarcys</u>
K (obs)	Observed permeability, <u>millidarcys</u>
K'	Permeability of caprock, <u>millidarcys</u>
K''	Permeability of bottom rock, <u>millidarcys</u>
K_o	Modified Bessel function, second kind, zeroth order
$K_{i,j}$	Term defined by Equation (III-71)
L	Distance between pumping well and linear fault, <u>feet</u>

l_1, L_1	Distance between pumping well and observation well, centimeters, <u>feet</u>
l_2, L_2	Distance between observation well and image well (See Figure IV-2), centimeters, <u>feet</u>
ln	Logarithm base e
log	Logarithm base 10
M	Dimensionless coefficient, defined by Equation (III-34) for radial system and Equation (III-93) for linear system
m	Slope of drawdown or build-up curve, <u>psi/cycle</u>
n	Number of homogeneous layers in porous media
P_b	Pressure base for gas measurement
P_D	Dimensionless pressure
$(P_D)_{gas}$	Dimensionless pressure for gas flow
P_t	Dimensionless pressure drop for well in infinite radial system (Table 10-5, Katz et al (50))
p	Pressure, atmospheres, <u>psia</u>
p_f	Flowing bottom hole pressure at shut-in, <u>psia</u>
p_o	Initial pressure, atm, <u>psia</u>
q, Q	Flow rate, reservoir conditions, cubic centimeters/second, <u>barrels/day</u>
Q_G	Gas flow rate, <u>SCF/day</u>
R	Dimensionless radius, $R = \frac{r}{rw}$
	Dimensionless length ratio, $R_D = \frac{l_2}{l_1}$
r	Distance from center of pumping well to point of pressure measurement, centimeters, <u>feet</u>
r_e	Radius of circular reservoir, <u>feet</u>
r_w	Radius of pumping well, <u>feet</u>
S	Storage coefficient, $S = \phi ch$, feet/psi

S_e	Skin effect
s	Direction of flow
T	Gas temperature, <u>°R</u>
T	Transmissibility, $T = Kh/\mu$, (millidarcy)(feet)/centipoise
T_b	Temperature base for gas measurement, <u>°R</u>
T_D	Dimensionless time defined by Equation (IV-13)
T_o	Dimensionless time of drawdown test, $T_0 = \frac{0.00633 K t_0}{\mu \phi c r^2}$
t	Time, seconds, <u>days</u>
t_D	Dimensionless time defined by Equation (III-33)
t_o	Duration of drawdown test, <u>days</u>
Δt	Time since cessation of drawdown test, <u>days</u>
u	Well function, $u = \frac{1}{4t_D}$
v_x, v_y, v_z	Fluid velocity in x, y, z direction, respectively
w	Variable defined by Equation (III-36)
x	Space coordinate
X_i	Term defined by Equation (III-70)
x_c	Linear reservoir characteristic length, <u>feet</u>
Y_i	Term defined by Equation (III-68)
y	Space coordinate
Z_i	Term defined by Equation (III-68)
z	Space coordinate
z	Gas compressibility factor
$\bar{z}$	Average gas compressibility factor

μ	Viscosity, centipoise
$\bar{\mu}$	Average viscosity, centipoise
ϕ	Porosity of porous media, fraction
λ_n	Roots of $J_1(\lambda) = 0$ given by Equation II-7a)
ρ	Density
∇	Del operator, $\nabla = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$
∇^2	Laplacian operator, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$
Φ	Force gradient, $\Phi = gz + \frac{p}{\rho}$

ABSTRACT

The purpose of this study was to investigate the relationship between unsteady state well performance and insitu characteristics of reservoirs. Certain insitu conditions often encountered in reservoirs were postulated and their effect on the unsteady state performance were obtained by calculations. The error in neglecting the non-linear term in the differential equation describing the flow of a slightly compressible liquid was evaluated. This information is important in interpreting reservoir drawdown and build-up pump test data.

A mathematical expression describing the pressure behavior in a reservoir was derived for the case where a gas field or a line of constant pressure is located near the vicinity of the pump test in an aquifer or oil field. The magnitude of the error in the evaluation of the insitu permeability and insitu compressibility was determined. A graphical method for locating the gas-water interface from drawdown tests on water wells located near a gas field is described.

The pressure behavior of a reservoir when leakage is occurring through the caprock, described by M. S. Hantush, was expressed in engineering units and extended to the case of leakage through both the cap and bottom rock and for pressure build-up tests in addition to drawdown tests. The magnitude of the error in the

measurement of insitu permeability and insitu compressibility was evaluated. The pressure behavior of an aquifer when leakage is occurring was observed to be very similar to the pressure behavior of an aquifer located near a gas field.

The magnitude of the error in neglecting the non-linear term in the differential equation describing the flow of a slightly compressible fluid in a porous media was evaluated by numerical means. The effect of neglecting this term was found to be negligible in all field data investigated. An equation giving the conditions when neglecting this term is not justified are presented.

Analyses of well test data from three fields for the insitu characteristics are presented.

I. INTRODUCTION

Before the optimum depletion schedule for an oil or gas reservoir can be determined, it is necessary to obtain numerical values for the reservoir characteristics. These characteristics include the reservoir permeability, effective compressibility, porosity, and fluid viscosity. In addition the nature of the reservoir limits, the amount of gas or oil contained in the field must be known before the optimum number of wells, well spacing, and production schedule can be determined.

Storage of natural gas in water sands has made it necessary to evaluate the characteristics of an aquifer prior to gas injection-production history. In order to evaluate the suitability of an aquifer for gas storage, it is necessary to determine if a suitable cap-rock exist and if there is sufficient permeability for the required gas deliverability.

It is reasonable therefore, that considerable research and money have been spent developing methods for evaluating reservoir and fluid characteristics and obtaining field data. Fluid characteristics can be determined by obtaining a sample of the fluid from the reservoir and evaluating its properties in a laboratory.

Determination of the reservoir rock properties can also be examined in laboratories, but it is not known how to average rock permeability in order to obtain the reservoir effective permeability. Thus it is necessary to perform well tests in the field in order to evaluate the effective permeability and compressibility.

Unsteady-state flow behavior must be analyzed since steady state conditions exist only prior to fluid flow tests in the reservoir. A pseudo-steady state can exist in a limited reservoir and is defined as the state when the pressure change with respect to time is constant and equal at all points in the reservoir. Available unsteady state flow equations are used to calculate the following information:

1. Insitu permeability and insitu compressibility in infinite aquifers of constant thickness
2. Insitu permeability and insitu compressibility in finite circular aquifers of constant thickness
3. Location of linear faults
4. Skin effects (reduced permeability within few feet of pumping well)

The equations and methods used to determine these properties are presented in the next section.

Unfortunately, the performance of many reservoirs cannot be described by the equations presented in the next section. This failure may be due to neglecting important parameters or failure of the mathematical equations to describe the flow. Effects of heterogenities in reservoirs may contribute to the "abnormal" behavior.

The validity of neglecting the non-linear term in the partial differential equation describing the flow of a slightly compressible fluid was evaluated. It was found that neglecting this term is usually justified.

The errors in the insitu permeability and insitu compressibility calculated from well test data if an infinite aquifer of uniform thickness is assumed are determined for the following cases:

1. A gas field or an external line of constant pressure is located near the test wells
2. Water is leaking through the cap and/or bottom rock into the aquifer.

Field well test data are analyzed in Section VI to demonstrate the application of available methods for determining insitu reservoir permeability and insitu reservoir compressibility.

II. LITERATURE SURVEY - DESCRIPTION OF EXISTING METHODS FOR THE DETERMINATION OF INSITU PERMEABILITY, INSITU COMPRESSIBILITY, AND RESERVOIR LIMITS

Drawdown and build-up well test results are used to estimate the insitu properties and geometry of underground reservoirs. These properties include insitu permeability and insitu compressibility as well as location of reservoir boundaries and reservoir limits (hydrocarbon volume). In addition, the effect of interference from neighboring hydrocarbon fields on a common aquifer can be predicted. Descriptions of several methods used in interpreting well tests are presented in this section.

A. Well Tests in Infinite Reservoirs

The pressure behavior of a well in an infinite reservoir during a constant rate drawdown pump test presented by Horner (35) and Theis (84) is given in engineering terms by

$$p = p_o + \frac{70.6 q \mu}{K h} Ei\left(\frac{-\phi c \mu r^2}{4(0.00633)Kt}\right) \qquad (II-1)$$

where:

c = compressibility, vol/(vol)(psi)

Ei = exponential integral, $Ei(-x) = -\int_x^{\infty} \frac{1}{u} e^{-u} du$

h = reservoir thickness, feet

K = permeability, millidarcys

p = pressure, psia

q = pumping rate, reservoir conditions, bbl/day

r = distance from center of pumping well to point of pressure measurement, feet

t = time, days

μ = viscosity, centipoise

ϕ = porosity, fraction

Equation (II-1), which assumes the pumping well radius is infinitesimal, was originally developed by Lord Kelvin and is known as the continuous point source solution. Values of the exponential integral are available in tables and graphs (19) (28) (43). If the value of the argument $\frac{\phi c \mu r^2}{4(0.00633)Kt}$ is less than .01, Equation (II-1) (19) (35) (51) (92) can be approximated by

$$p = p_o - \frac{162.6 q\mu}{Kh} \left\{ \log_{10}\left(\frac{0.00633Kt}{\phi\mu c r^2}\right) + 0.3513 \right\} \quad \text{(II-2)}$$

Thus if the well pressure is plotted versus log time, the insitu permeability is calculated from the slope by

$$m = - \frac{162.6 q\mu}{Kh} \quad \text{(II-3)}$$

and the insitu permeability from Equation (II-2) where m = pressure drop, psi/cycle. An example calculation of the insitu permeability and insitu compressibility is given by Katz et al (51). Equations (II-1) and (II-2) are approximate solutions which are valid for $\frac{0.00633Kt}{\phi c \mu r^2}$ greater than

1000. The pressure behavior for lower values of the argument is given by Van Everdigen and Hurst (86) and Chatas (12).

The pressure behavior of an infinite reservoir during a build-up test, obtained by superimposing a negative and equal production rate on Equation (II-1), (35) (51) (92) is

$$p = p_o + \frac{70.6 q \mu}{Kh} \left\{ Ei\left(- \frac{\phi \mu c r^2}{4(0.00633)K(t+\Delta t)}\right) - Ei\left(- \frac{\phi \mu c r^2}{4(0.00633)K\Delta t}\right)\right\} \qquad \text{(II-4)}$$

where:

Δt = time since cessation of pump test, days

t_o = duration of pump test, days

If $\frac{\phi \mu c r^2}{4(0.00633)Kt}$ is less than 0.01, Equation (II-3) can be approximated by

$$p = p_o - \frac{162.6 q \mu}{Kh} \log_{10}\left(\frac{t_o + \Delta t}{\Delta t}\right) \qquad \text{(II-5)}$$

Several examples of the determination of the insitu permeability from a build-up test are available in the literature (4) (35) (51) (71). If the pressure is plotted versus $\log_{10} \frac{t_o + \Delta t}{\Delta t}$ the insitu permeability is calculated from the slope by

$$K = - \frac{162.6 q \mu}{mh} \qquad \text{(II-6)}$$

The cumulative water influx for a constant terminal pressure drawdown is given by Van Everdigen and Hurst (86), Chatas (12), Katz et al (50) (51), and Jacob and Lohman (42).

Other methods of analysis, additional information and case studies for build-up and drawdown tests are given by Ammann (2), Arps (4), Bruce (9), Collins and Kolodzie (16), Dolan et al (20), Driscoll (21), Hazebroek et al (62), Maier (57), Matthews et al (58), Matthews and Stegemeier (59), Miller et al (62), Pirson and Pirson (72), Pitzer et al (73), Schrenkel (79), and Thomas (85).

Analyses of well tests and pressure behavior in gas fields are presented by Accord (1), Aronofsky and Jenkins (3), Bruce et al (8), Carter (10), Carter et al (11), Cornell and Katz (17), Cornell (18), Dykstra (22), Jones et al (45), Jones, L. G. (46, 47), Jones, P. (48, 49), Katz et al (50), Kidder (53), Layton (54), McMahon (61), Pottman et al (74), Smith (80), and Swift and Kiel (82).

B. Well Tests in Finite Reservoirs

The point source solution to the flow of a fluid into a single well of finite radius, developed by Muskat (66) is

$$p_w = p_o + \frac{141.6\, q\mu}{Kh} \left\{ \frac{3}{4} + \ln \frac{r_w}{r_e} - \frac{r_w^2}{2\, r_e^2} - \frac{2(0.00633 Kt}{\phi \mu c\, r_e^2} + 2\sum_{n=1}^{\infty} \frac{J_o\left(\frac{\lambda_n r_w}{r_e}\right) e^{-\frac{0.00633 Kt \lambda_n^2}{}}}{\lambda_n^2 \quad J_o^2(\lambda_n)} \right\} \qquad \text{(II-7)}$$

where:

$$\lambda_n = \text{roots of } J_1(\lambda) = 0 \qquad \text{(II-7a)}$$

J_0 = Bessel function, first kind, zeroth order

J_1 = Bessel function, first kind, first order

r_e = radius of circular reservoir, feet

r_w = radius of pumping well, feet

The corresponding approximate solution given by Horner (35) is

$$p = p_o + \frac{70.6 q\mu}{Kh}\left\{Ei\left(-\frac{\phi\mu c r^2}{4(0.00633)Kt}\right) - Ei\left(-\frac{\phi\mu c r_e^2}{4(0.00633Kt}\right) - \frac{4(0.00633)Kt}{\phi\mu c r_e^2}\exp\left(\frac{-\phi\mu c r_e^2}{4(0.00633Kt}\right)\right\} \qquad \text{(II-8)}$$

The last two terms of Equation (II-8) account for the effect of the external boundary. Thus Equations (II-6) or (II-7) are used to predict the pressure behavior of a reservoir during a <u>drawdown pump test</u>.

The pressure behavior during a <u>build-up test</u> is again found by superposition. If t_o is the total pumping time and Δt the time after cessation of pumping, the approximate point source solution (35) is

$$p = p_o - \frac{162.6 q\mu}{Kh}\left[\log_{10}\left(\frac{t_o+\Delta t}{\Delta t}\right) + 0.434\left\{Ei\left(\frac{\phi\mu c r_e^2}{4(0.00633Kt_o}\right) - \frac{4(0.00633)Kt_o}{\phi\mu c r_e^2}\exp\left(-\frac{\phi\mu c r_e^2}{4(0.00633Kt_o}\right)\right\}\right] \qquad \text{(II-9)}$$

Graphs of the last two terms in Equations (II-8) and (II-9) are given by Horner (35).

C. Effect of Linear Faults on Pressure Behavior

The method of images (35) is used to describe pressure behavior of a well producing near a linear fault. Thus if the distance from the production well to the fault is L, the pressure behavior for a drawdown test is given by

$$p_w = p_o + \frac{70.6 q\mu}{Kh}\left\{Ei\left(-\frac{\phi\mu c r_w^2}{4(0.00633)Kt}\right) + Ei\left(-\frac{\phi\mu c L^2}{0.00633Kt}\right)\right\} \quad \text{(II-10)}$$

and for a build-up test by

$$p_w = p_o + \frac{70.6 q\mu}{Kh}\left\{Ei\left(-\frac{\phi\mu c r_w^2}{4(0.00633)K(t_o+\Delta t)}\right) + Ei\left(-\frac{\phi\mu c L^2}{0.00633K(t_o+\Delta t)}\right) - Ei\left(-\frac{\phi\mu c r_w^2}{4(0.00633)K\Delta t}\right) - Ei\left(-\frac{\phi\mu c L^2}{0.00633K\Delta t}\right)\right\} \quad \text{(II-11)}$$

Pumping time and the distance L are usually large enough to allow Equation (II-11) over the first part of the build-up curve to be approximated by

$$p_w = p_o - \frac{162.6 q\mu}{Kh}\left\{\log_{10}\left(\frac{t_o+\Delta t}{\Delta t}\right) - 0.434\, Ei\left(-\frac{\phi\mu c L^2}{0.00633Kt_o}\right)\right\} \quad \text{(II-12)}$$

For large shut-in times, the pressure behavior can be calculated from

$$p_w = p_o - \frac{81.3 q\mu}{Kh} \log_{10}\left(\frac{t_o + \Delta t}{\Delta t}\right) \qquad \text{(II-13)}$$

Thus when the pressure is plotted versus $\log_{10}\left(\frac{t_o+\Delta t}{\Delta t}\right)$, the slope over the latter portion is twice the initial (35).

D. Interference of Several Wells Pumping in a Common Aquifer

The pressure behavior of wells producing on a common aquifer can be obtained by superposition of the separate effects. The pressure in an aquifer where other wells are producing is given by

$$p = p_o + \frac{70.6\mu}{Kh}\left\{ q\, Ei\left(-\frac{\phi\mu c r_w^2}{4(0.00633)Kt}\right) + \sum_{i=1}^{n} q_i\, Ei\left(-\frac{\phi\mu c L_i^2}{0.00633(4)Kt}\right)\right\} \qquad \text{(II-14)}$$

where:

q = production rate of given well, bbl/day

q_i = production rate interfering well, bbl/day

L_i = distance from given well to "i" interfering well, feet

n = number of interfering wells

r_w = radius of given well, feet

Additional procedures for predicting the effect of interference are given by Coats et al (14), Hurst (39), Mortada (63) (64), Parson (70), Stevens and Thodos (81), and Warren (90).

E. Skin Effects

The resistance to flow caused by a reduced permeability within a few feet of the pumping well is known as the "skin effect" and may be calculated by the following equation (88) (92)

$$S_e = 1.1515 \left\{ \frac{p_{1hr} - p_f}{m} - \log_{10}\left(\frac{K}{\phi \mu c r_w^2}\right) + 3.22 \right\} \quad \text{(II-15)}$$

where:

S_e = skin effect

p_{1hr} = pressure at shut-in time of one hour, psia

p_f = flowing bottom hole pressure at shut-in, psia

m = slope per cycle of straight-line portion of build-up curve, psia/cycle

Application of the skin effects are discussed by Arps (4), Brons and Marting (6), Hurst (37), Johnson et al (44), and Van Everdingen (88).

F. Other Topics

The effect of leakage on the performance of aquifers is discussed by Hantush (25) (28) (29) (31), Hantush and Jacob (30), Jacob (41), Walton (89) and Witherspoon et al (93).

Reservoirs with different permeabilities are analyzed by Henson et al (33), Katz (52), Lefkovits et al (55), Loucks and Guerrero (56), Mueller (65), and Warren (91).

Pressure behavior of partially penetrating wells was investigated by Hantush (27) and Nisle (69).

Water movement and pressure behavior in oil and gas storage reservoirs are described by Coats (13), Coats et al (14), Hutchinson and Sikora (40), Katz et al (50) (51), Katz (52), and Rzepczynski (77).

Compressibilities of reservoir rock have been measured by Fatt (23) and its effect on permeability by McLatchie et al (60).

Numerical methods were used to solve two phase flow by Nielsen (68).

Counter current gravity segregation was investigated by Briggs (5).

In summary, available solutions to flow of fluids in homogeneous reservoirs of constant thickness used in calculating insitu characteristics include:

1. Drawdown tests in infinite or finite reservoirs - insitu permeability and compressibility
2. Build-up tests in infinite or finite reservoirs - insitu permeability
3. Drawdown tests in infinite reservoirs with leakage - insitu permeability
4. Drawdown tests - reservoir hydrocarbon volume (reservoir limit test described by Jones (48) (49)).

Although one well is sufficient for calculating the insitu properties, two or more wells increase the reliability.

The dissertation extends these tests to include:

1. Build-up tests in infinite aquifer with leakage - insitu permeability
2. Drawdown tests in aquifers for locating gas fields or lines of constant pressure in vicinity of test wells.

III. DETERMINATION OF ERROR IN NEGLECTING THE NON-LINEAR TERM IN THE PARTIAL DIFFERENTIAL EQUATION DESCRIBING THE FLOW OF SLIGHTLY COMPRESSIBLE FLUIDS IN POROUS MEDIA

The purpose of this section is to define the conditions under which one is justified in neglecting the non-linear term in the differential equation describing the flow of slightly compressible liquids in porous media. A graph is presented showing the reservoir pressure behavior as a function of dimensionless time when the term is not neglected. A second graph gives the maximum values of dimensionless time for various values of the dimensionless coefficient of the non-linear term beyond which neglecting this term is not justified. Example problems illustrate how one may determine if neglecting the non-linear term is justified.

Katz et al (50) and Rowan and Clegg (76) noted that when a single equation of state is used in the derivation of the differential equation for flow of a slightly compressible liquid in a porous media, a non-linear term appears. Many authors have used two equations of state in deriving the differential equation for the flow of a slightly compressible liquid and thus avoided the term in their erroneous derivation.

A. Derivation of Non-Linear Differential Equation

The differential equation describing the flow of a slightly compressible liquid is obtained by combining the following three equations:

1. The equation of continuity
2. Darcys law
3. Equation of state for slightly compressible liquids

The continuity equation is obtained by writing a mass balance on a differential element showing the net change in the mass flow rate in and out of the element is equal to the rate of change of the mass in the element. In Cartesian coordinates, the continuity equation is

$$\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} + \frac{\partial(\rho v_z)}{\partial z} = -\phi\frac{\partial\rho}{\partial t} \qquad \text{(III-1)}$$

where:

t = time

x, y, z = space coordinates

v_x = fluid velocity in x direction

v_y = fluid velocity in y direction

v_z = fluid velocity in z direction

ρ = fluid density

t = time

ϕ = porosity of the porous media

Darcys law is an experimental observation that the velocity of a homogeneous fluid flowing through a porous media is proportional to the force potential Φ and inversely proportional to the viscosity.

$$\vec{V} = -\frac{K\rho}{\mu}\vec{\nabla}\Phi \qquad \text{(III-2)}$$

where:

$$\Phi = gz + \frac{p}{\rho}$$

K = permeability

p = pressure

μ = viscosity

ρ = density

If the effect of gravity is neglected, Equation (III-2) can be written

$$V_s = -\frac{K}{\mu}\frac{dp}{ds} \qquad \text{(III-3)}$$

where s is the direction of flow. Writing Equation (III-3) for each component in Cartesian coordinates yields

$$V_x = -\frac{K}{\mu}\frac{dp}{dx} \qquad \text{(III-4)}$$

$$V_y = -\frac{K}{\mu}\frac{dp}{dy} \qquad \text{(III-5)}$$

$$V_z = -\frac{K}{\mu}\frac{dp}{dz} \qquad \text{(III-6)}$$

The equation of state can be derived from the definition of compressibility

$$c = -\frac{1}{V}\frac{dV}{dp} = \frac{1}{\rho}\frac{d\rho}{dp} \qquad \text{(III-7)}$$

If the compressibility is independent of pressure, then integrating Equation (III-7) gives

$$c(p - p_0) = \ln\left(\frac{\rho}{\rho_0}\right) \qquad \text{(III-8)}$$

The equation of state is obtained by exponentiating Equation (III-8) and rearranging

$$\rho = \rho_o \, e^{c(p-p_o)} \tag{III-9}$$

Expanding Equation (III-1) yields

$$\frac{\partial \rho}{\partial x} v_x + \rho \frac{\partial v_x}{\partial x} + \frac{\partial \rho}{\partial y} v_y + \rho \frac{\partial v_y}{\partial y}$$

$$+ \frac{\partial \rho}{\partial z} v_z + \rho \frac{\partial v_z}{\partial z} = -\phi \frac{\partial \rho}{\partial t} \tag{III-10}$$

Applying the chain rule to Equation (III-10) gives

$$\frac{\partial \rho}{\partial p} \frac{\partial p}{\partial x} v_x + \rho \frac{\partial v_x}{\partial x}$$

$$+ \frac{\partial \rho}{\partial p} \frac{\partial p}{\partial y} v_y + \rho \frac{\partial v_y}{\partial y} \tag{III-11}$$

$$+ \frac{\partial \rho}{\partial p} \frac{\partial p}{\partial z} v_z + \rho \frac{\partial v_z}{\partial z} = -\phi \frac{\partial \rho}{\partial p} \frac{\partial p}{\partial t}$$

Differentiating Equation (III-9) gives

$$\frac{\partial \rho}{\partial p} = c \rho_o \, e^{c(p-p_o)} \tag{III-12}$$

Substituting Equations (III-4), (III-5), (III-6), and (III-12) in Equation (III-11) and cancelling $\rho_o \, e^{c(p-p_o)}$ in each term gives

$$- \frac{cK}{\mu} \left(\frac{\partial p}{\partial x}\right)^2 - \frac{K}{\mu} \frac{\partial^2 p}{\partial x^2} - \frac{cK}{\mu} \left(\frac{\partial p}{\partial y}\right)^2$$

$$- \frac{K}{\mu} \frac{\partial^2 p}{\partial y^2} - \frac{cK}{\mu} \left(\frac{\partial p}{\partial z}\right)^2 - \frac{K}{\mu} \frac{\partial^2 p}{\partial z^2} = -\phi c \frac{\partial p}{\partial t} \tag{III-13}$$

Thus rearranging Equation (III-13) gives the differential equation describing the flow of a slightly compressible fluid in a porous media

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} + c\left[\left(\frac{\partial p}{\partial x}\right)^2 + \left(\frac{\partial p}{\partial y}\right)^2 + \left(\frac{\partial p}{\partial z}\right)^2\right] = \frac{\mu \phi c}{K}\frac{\partial p}{\partial t} \qquad \text{(III-14)}$$

For linear flow, Equation (III-14) reduces to

$$\frac{\partial^2 p}{\partial x^2} + c\left(\frac{\partial p}{\partial x}\right)^2 = \frac{\mu \phi c}{K}\frac{\partial p}{\partial t} \qquad \text{(III-15)}$$

For radial flow, Equation (III-14) is given by

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} + c\left(\frac{\partial p}{\partial r}\right)^2 = \frac{\mu \phi c}{K}\frac{\partial p}{\partial t} \qquad \text{(III-16)}$$

Neglecting the non-linear terms in Equations (III-13), (III-14), and (III-15) yields diffusivity equations that most authors use in solving for the flow of a slightly compressible liquid in a porous media:

1. General

$$\nabla^2 p = \frac{\mu \phi c}{K}\frac{\partial p}{\partial t} \qquad \text{(III-17)}$$

2. Linear flow

$$\frac{\partial^2 p}{\partial x^2} = \frac{\mu \phi c}{K}\frac{\partial p}{\partial t} \qquad \text{(III-18)}$$

3. Radial flow

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \frac{\mu \phi c}{K}\frac{\partial p}{\partial t} \qquad \text{(III-19)}$$

B. Solution of Non-Linear Differential Equation for Radial Flow, Constant Terminal Rate Case

In addition to the differential equation derived for radial flow in part A

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} + c\left(\frac{\partial p}{\partial r}\right)^2 = \frac{\mu \phi c}{K}\frac{\partial p}{\partial t} \qquad \text{(III-16)}$$

one initial condition

$$p\,(r, 0) = p_0 \qquad \text{(III-20)}$$

and two boundary conditions

$$\left.\frac{\partial p}{\partial r}\right|_{r=r_w} = \frac{Q\mu}{AK} = \frac{Q\mu}{2\pi r h K}, \qquad t \geq 0 \qquad \text{(III-21)}$$

$$p(\infty, t) = p_0 \qquad \text{(III-22)}$$

are necessary to define the problem. The terms in these equations are defined below:

A = area perpendicular to flow, square centimeters

K = permeability, darcys

p = pressure, atmospheres

p_0 = initial pressure, atmospheres

Q = rate of production, cc/sec

r = radius, centimeters

r_w = radius of well, centimeters

t = time, seconds

μ = viscosity, centipoise

ϕ = porosity, fraction

Substitution of the dimensionless quantities

$$R = \frac{r}{r_w} \tag{III-23}$$

$$P_D = \frac{2\pi (p_o - p) h K}{\mu Q} \tag{III-24}$$

$$t_D = \frac{K t}{\mu \phi c r_w^2} \tag{III-25}$$

into Equations (III-16), (III-20), (III-21), and (III-22) yield the following dimensionless equations

$$\frac{\partial^2 P_D}{\partial R^2} + \frac{1}{R}\frac{\partial P_D}{\partial R} - \frac{c \mu Q}{2\pi h K}\left(\frac{\partial P_D}{\partial R}\right)^2 = \frac{\partial P_D}{\partial t_D} \tag{III-26}$$

$$P_D\ (R, 0) = 0 \tag{III-27}$$

$$\left.\frac{\partial P_D}{\partial R}\right|_{R=1} = -1 \tag{III-28}$$

$$P_D\ (\infty, t_D) = 0 \tag{III-29}$$

respectively.

Define the dimensionless coefficient

$$M = - \frac{c \mu Q}{2 \pi h K} \tag{III-30}$$

and substituting in Equation (III-26) yields

$$\frac{\partial^2 P_D}{\partial R^2} + \frac{1}{R}\left(\frac{\partial P_D}{\partial R}\right) + M\left(\frac{\partial P_D}{\partial R}\right)^2 = \frac{\partial P_D}{\partial t_D} \tag{III-31}$$

Thus the pressure can be calculated at any time by

$$p = p_o - \frac{\mu Q}{2 \pi h K} P_D \tag{III-32}$$

In Engineering field units, the dimensionless quantities defined by Equations (III-25) and (III-30) can be calculated from

$$t_D = \frac{0.00633 \; K t}{\mu \phi c r_w^2} \tag{III-33}$$

$$M = \frac{-141.2 \; c \mu q}{h K} \tag{III-34}$$

where:

c = compressibility, vol/(vol)(psi)

h = thickness of porous media, feet

K = permeability, millidarcys

q = flow rate, bbl/day

r = radius of well, feet

t = time, days

ϕ = porosity, fraction

μ = viscosity, centipoise

Thus the reservoir pressure in psia is calculated by

$$p = p_o - \frac{141.2\,\mu q}{hK} P_D \qquad \text{(III-35)}$$

where:

p_o = initial pressure, psia

p = pressure, psia

P_D = dimensionless pressure

The values for dimensionless pressure, P_D, as a function of dimensionless time, t_D, are solved in this section.

No analytic solution to the differential Equation (III-31) is known. Thus the Equation (III-31) along with the initial and boundary conditions, Equations (III-27), (III-28), and (III-29) were solved by replacing them by finite difference equations and solving the difference equations numerically on the IBM 7090.

Partial differential equations may be solved by an explicit or an implicit method. These methods are currently being used to solve reservoir problems for which analytical solutions are not available (8) (15) (33) (34) (65) (68) (75) (93).

1. Explicit Numerical Method

Equation (III-31) was solved numerically by the following method. Transform R using

$$w = 1 - e^{-(R-1)} \qquad \text{(III-36)}$$

so that the limits of R, $1 \leq R \leq \infty$ is transformed to the limits of w, $0 \leq w \leq 1$.

The values of each term in Equation (III-31) in terms of w are derived below. Differentiating Equation (III-36) gives

$$\frac{dw}{dR} = e^{-(R-1)} = 1 - w \qquad \text{(III-37)}$$

Thus*

$$\frac{\partial P}{\partial R} = \frac{\partial P}{\partial w}\frac{dw}{dR} = \frac{\partial P}{\partial w}(1-w) \qquad \text{(III-38)}$$

and

$$\frac{\partial^2 P}{\partial R^2} = \frac{\partial}{\partial w}\left[\frac{\partial P}{\partial w}(1-w)\right]\frac{dw}{dR}$$

$$= \left(\frac{\partial^2 P}{\partial w^2} - w\frac{\partial^2 P}{\partial w^2} - \frac{\partial P}{\partial w}\right)(1-w) \qquad \text{(III-39)}$$

Solving Equation (III-36) for R gives

$$R = 1 - \ln(1-w) \qquad \text{(III-40)}$$

Substitution of Equations (III-38), (III-39), and (III-40) in (III-31) gives

*Note that the subscript D is dropped in this section to avoid confusion with other subscripts to be introduced later.

$$(1-w)^2 \frac{\partial^2 P}{\partial w^2} - (1-w)\frac{\partial P}{\partial w} - \frac{(1-w)}{\ln(1-w)}\frac{\partial P}{\partial w}$$

$$+ M(1-w)^2\left(\frac{\partial P}{\partial w}\right)^2 = \frac{\partial P}{\partial t_D} \qquad \text{(III-41)}$$

which reduces to

$$(1-w)^2 \frac{\partial^2 P}{\partial w^2} - \left[(1-w) - \frac{(1-w)}{1-\ln(1-w)}\right]\frac{\partial P}{\partial w}$$

$$+ M(1-w)^2\left(\frac{\partial P}{\partial w}\right)^2 = \frac{\partial P}{\partial t_D} \qquad \text{(III-42)}$$

Equation (III-42) is solved in the region

$$0 \leq w \leq 1 \qquad \text{(III-43)}$$

and

$$0 \leq t_D \leq \infty \qquad \text{(III-44)}$$

by representing the region by a grid

$$w = i\Delta w \qquad \text{(III-45)}$$

and

$$t_D = j\Delta t_D \qquad \text{(III-46)}$$

where Δw and Δt_D are the increments of distance and time respectively. Thus $P_{i,j}$ defines the value of the pressure at the point $P_{i\Delta w,\, j\Delta t_D}$.

A finite difference representation of Equation (III-42) is

$$(1-i\Delta w)^2\left(\frac{P_{i+1,j}-2P_{i,j}+P_{i-1,j}}{\Delta w}\right)$$
$$-\left[(1-i\Delta w)-\frac{1-i\Delta w}{1-\ln(1-i\Delta w)}\right]\left(\frac{P_{i+1,j}-P_{i-1,j}}{2\Delta w}\right) \qquad \text{(III-47)}$$
$$+M(1-i\Delta w)^2\left(\frac{P_{i+1,j}-P_{i-1,j}}{2\Delta w}\right)^2=\frac{P_{i,j+1}-P_{i,j}}{\Delta t_D}$$

Solving Equation (III-47) for $P_{i,j+1}$ gives an explicit expression for $P_{i,j+1}$ that is, $P_{i,j+1}$ is equal to known quantities.

$$P_{i,j+1}=P_{i,j}+\frac{\Delta t_D}{(\Delta w)^2}\Big\{(1-i\Delta w)^2\left(P_{i+1,j}-2P_{i,j}+P_{i-1,j}\right)$$
$$-\frac{\Delta w}{2}\left[(1-i\Delta w)-\frac{(1-i\Delta w)}{1-\ln(1-i\Delta w)}\right]\left(P_{i+1,j}-P_{i-1,j}\right) \qquad \text{(III-48)}$$
$$+\frac{M}{4}(1-i\Delta w)^2\left(P_{i+1,j}-P_{i-1,j}\right)^2\Big\}$$

An analysis of the stability of the numerical Equation (III-48) (15) (34) requires that

$$\frac{\Delta t_D}{(\Delta w)^2} < \frac{1}{2} \qquad \text{(III-49)}$$

or on rearranging

$$\Delta t_D < \frac{1}{2}(\Delta w)^2 \qquad \text{(III-50)}$$

is a necessary condition for a stable solution. Thus the maximum size of the time step is limited by the choice of Δw.

The boundary conditions defined by Equation (III-28) in terms of w is

$$\left.\frac{\partial P}{\partial w}\right|_{w=o} = -1 \tag{III-51}$$

Equation (III-51) can be expressed in numerical form by using a three point Lagrangian fit for P and evaluating the derivative at w = o. Thus

$$\frac{-3\,P_{o,j+1} + 4\,P_{1,j+1} - P_{2,j+1}}{2\Delta w} = -1 \tag{III-52}$$

is the numerical expression for Equation (III-51).

The remaining boundary condition, Equation (III-29), in terms of w is

$$P\Big|_{w=1} = 0 \tag{III-53}$$

The initial condition, Equation (III-27) is

$$P_{i,o} = 0 \tag{III-54}$$

The MAD (Michigan Algorithm Decoder) program used to solve Equations (III-48), (III-52), (III-53), and (III-54) is given in Appendix A. This program was used to evaluate the values of dimensionless pressure for values of dimensionless time from 0 to 0.01. The results are presented in Figure (III-1). Values of dimensionless pressure for values of dimensionless time greater than 0.01 were determined by an implicit method described in the next part.

2. Implicit Numerical Method

The solution of Equation (III-31) by an implicit numerical method is developed below.

Equation (III-31) is transformed to the w plane by

$$w = 1 - e^{-a(R-1)} \tag{III-55}$$

Note that a scaling factor "a" is introduced in this transformation.

Hence

$$\frac{\partial P}{\partial R} = a\,(1-w)\frac{\partial P}{\partial w} \tag{III-56}$$

and

$$\frac{\partial^2 P}{\partial R^2} = a^2\,(1-w)^2\frac{\partial^2 P}{\partial w^2} - a^2(1-w)\frac{\partial P}{\partial w} \tag{III-57}$$

Solving Equation (III-55) for R gives

$$R = \frac{1}{a}\left(a - \ln(1-w)\right) \tag{III-58}$$

The differential equation in the w plane is obtained by substituting Equations (III-56), (III-57), and (III-58) into Equation (III-31).

$$a^2(1-w)^2\frac{\partial^2 P}{\partial w^2} - a^2(1-w)\frac{\partial P}{\partial w} + \frac{a^2\,(1-w)}{a - \ln(1-w)}\frac{\partial P}{\partial w} + M\,a^2(1-w)^2\left(\frac{\partial P}{\partial w}\right)^2 = \frac{\partial P}{\partial t_D} \tag{III-59}$$

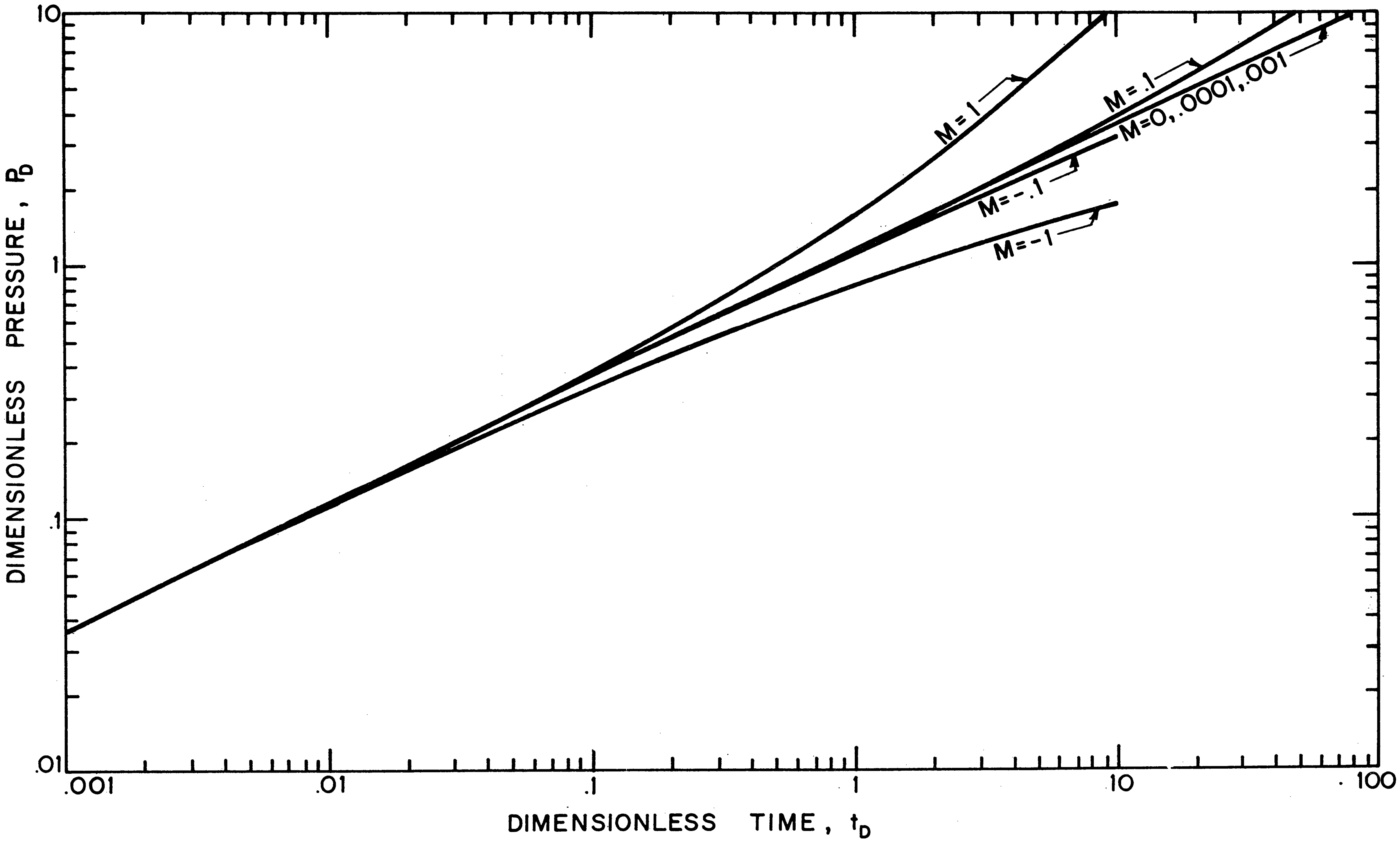

Figure III-1. Effect of Non-Linear Term on Dimensionless Pressure Drop versus Dimensionless Time for Radial System, Constant Terminal Rate Case.

where

$$0 \leq w \leq 1 \tag{III-60}$$

Expressing Equation (III-60) in difference form gives

$$\begin{aligned}
&a^2(1-i\Delta w)^2\left\{\frac{P_{i+1,j+1} - 2P_{i,j+1} + P_{i-1,j+1}}{\Delta w}\right\} \\
&- a^2(1-i\Delta w)\left\{\frac{P_{i+1,j+1} - P_{i-1,j+1}}{2\Delta w}\right\} \\
&+ \frac{a^2(1-i\Delta w)}{a - \ln(1-i\Delta w)}\left\{\frac{P_{i+1,j+1} - P_{i-1,j+1}}{2\Delta w}\right\} \\
&+ a^2 M(1-i\Delta w)^2\left\{\frac{P_{i+1,j+1} - P_{i-1,j+1}}{2\Delta w}\right\}^2 = \frac{P_{i,j+1} - P_{i,j}}{\Delta t_D}
\end{aligned} \tag{III-61}$$

Replace the non-linear term in Equation (III-61) by

$$(P_{i+1,j+1} - P_{i-1,j+1})^2 = (P_{i+1,j+1} - P_{i-1,j+1})(P^*_{i+1,j+1} - P^*_{i-1,j+1}) \tag{III-62}$$

where $P^*_{i+1,j+1} - P^*_{i-1,j+1}$ is estimated from previous time steps or previous iterations and corrected until

$$P^*_{i+1,j+1} - P^*_{i-1,j+1} \cong P_{i+1,j+1} - P_{i-1,j+1} \tag{III-63}$$

Substituting Equation (III-63) in Equation (III-61) and rearranging gives

$$P_{i+1,j+1}\left\{\frac{(1-i\Delta w)^2}{(\Delta w)^2}-\frac{(1-i\Delta w)}{2\Delta w}+\frac{(1-i\Delta w)}{(a-\ln(1-i\Delta w)(2\Delta w)}\right.$$

$$\left.+M\frac{(1-i\Delta w)^2}{4(\Delta w)^2}\left(P^*_{i+1,j+1}-P^*_{i-1,j+1}\right)\right\}$$

$$+P_{i,j+1}\left\{\frac{-2(1-i\Delta w)^2}{\Delta w}-\frac{1}{a^2\Delta t_D}\right\}$$

$$+P_{i-1,j+1}\left\{\frac{(1-i\Delta w)^2}{(\Delta w)^2}+\frac{(1-i\Delta w)}{2\Delta w}-\frac{(1-i\Delta w)}{(a-\ln(1-i\Delta w))2\Delta w}\right. \qquad \text{(III-64)}$$

$$\left.-M\frac{(1-i\Delta w)^2}{4(\Delta w)^2}\left(P^*_{i+1,j+1}-P^*_{i-1,j+1}\right)\right\}=-\frac{P_{i,j}}{a^2\Delta t_D}$$

Multiply Equation (III-64) by $(\Delta w)^2$ to obtain the implicit numerical equation for the differential equation

$$P_{i+1,j+1}\left\{(1-i\Delta w)^2-\frac{\Delta w(1-i\Delta w)}{2}+\frac{\Delta w(1-i\Delta w)}{2(a-\ln(1-i\Delta w))}\right.$$

$$\left.+\frac{M}{4}(1-i\Delta w)^2\left(P^*_{i+1,j+1}-P_{i-1,j+1}\right)\right\}$$

$$+P_{i,j+1}\left\{-2(1-i\Delta w)^2-\frac{(\Delta w)^2}{a^2\Delta t_D}\right\}$$

$$+P_{i-1,j+1}\left\{(1-i\Delta w)^2+\frac{\Delta w}{2}(1-i\Delta w)-\frac{\Delta w}{2}\frac{(1-i\Delta w)}{a-\ln(1-i\Delta w)}\right. \qquad \text{(III-65)}$$

$$\left.-\frac{M}{4}(1-i\Delta w)^2\left(P^*_{i+1,j+1}-P_{i-1,j+1}\right)\right\}$$

$$=-P_{i,j}\frac{(\Delta w)^2}{a^2\Delta t_D}$$

Equation (III-65) is solved by the following procedure. Assume

$$P_{i-1,j+1} = C_i \; P_{i,j+1} + D_i \qquad \text{(III-66)}$$

Substitution of Equation (III-66) into Equation (III-65) yields

$$P_{i+1,j+1}(Z_i) + P_{i,j+1}(Y_i) + C_i \; P_{i,j+1} + D_i(X_i) = K_{i,j} \qquad \text{(III-67)}$$

where

$$Z_i = (1-i\Delta w)^2 - \frac{\Delta w\,(1-i\Delta w)}{2} + \frac{\Delta w\,(1-i\Delta w)}{2\,(a-\ln(1-i\Delta w))}$$
$$+ \frac{M}{4}(1-i\Delta w)^2 \left(P^*_{i+1,j+1} - P^*_{i-1,j+1}\right) \qquad \text{(III-68)}$$

$$Y_i = -2\,(1-i\Delta w)^2 - \frac{(\Delta w)^2}{a^2\,\Delta t_D} \qquad \text{(III-69)}$$

$$X_i = (1-i\Delta w)^2 + \frac{\Delta w}{2}(1-i\Delta w) - \frac{\Delta w\,(1-i\Delta w)}{2\,(a-\ln(1-i\Delta w))}$$
$$- \frac{M}{4}(1-i\Delta w)^2 \left(P^*_{i+1,j+1} - P^*_{i-1,j+1}\right) \qquad \text{(III-70)}$$

$$K_{i,j} = -P_{i,j}\,\frac{(\Delta w)^2}{a^2\,\Delta t_D} \qquad \text{(III-71)}$$

Solving Equation (III-67) for $P_{i,j+1}$ gives

$$P_{i,j+1} = P_{i+1,j+1}\left\{\frac{-Z_i}{C_i X_i + Y_i}\right\} + \frac{K_{i,j} - D_i X_i}{C_i X_i + Y_i} \qquad \text{(III-72)}$$

Comparison of Equation (III-72) and Equation (III-68) yields

$$C_{i+1} = \frac{-Z_i}{C_i X_i + Y_i} \qquad \text{(III-73)}$$

and

$$D_{i+1} = \frac{K_{i,j} - D_i X_i}{C_i X_i + Y_i} \qquad \text{(III-74)}$$

Hence the recursion relationships are obtained for C_i and D_i which will allow the calculation of C_i and D_i ; i = 2, ..., imax provided C_1 and D_1 can be found.

The boundary condition, Equation (III-28) in the w plane is given by

$$\left.\frac{\partial P}{\partial w}\right|_{w=0} = -\frac{1}{a} \qquad \text{(III-75)}$$

Using the same procedure as in the derivation of Equation (III-52), Equation (III-75) can be approximated by

$$\frac{-3P_{0,j+1} + 4P_{1,j+1} - P_{2,j+1}}{2\Delta w} = -\frac{1}{a} \qquad \text{(III-76)}$$

and hence

$$P_{2,j+1} = \frac{2\Delta w}{a} - 3P_{0,j+1} + 4P_{1,j+1} \qquad \text{(III-77)}$$

Evaluating Equation (III-67) for i = 1 gives

$$P_{2,j+1}(Z_1) + P_{1,j+1}(Y_1) + P_{0,j+1}(X_1) = K_{1,j} \qquad \text{(III-78)}$$

since

$$P_{0,j+1} = C_1 P_{1,j+1} + D_1 \qquad \text{(III-79)}$$

Substituting Equation (III-77) into Equation (III-78) yields

$$\left(\frac{2\Delta w}{a} - 3P_{0,j+1} + 4P_{1,j+1}\right)Z_1 + P_{1,j+1}(Y_1) + P_{0,j+1}(X_1) = K_{1,j} \qquad \text{(III-80)}$$

or

$$P_{0,j+1}(X_1 - 3Z_1) + P_{1,j+1}(4Z_1 + Y_1) = K_{1,j} - \frac{2\Delta w Z_1}{a} \qquad \text{(III-81)}$$

Solving Equation (III-81) for $P_{0,j+1}$ yields

$$P_{0,j+1} = P_{1,j+1}\left\{\frac{-(4Z_1 + Y_1)}{X_1 - 3Z_1}\right\} + \frac{K_{1,j} - (2\Delta w Z_1/a)}{X_1 - 3Z_1} \qquad \text{(III-82)}$$

Comparison of Equation (III-82) with Equation (III-65) shows that

$$C_1 = \frac{-4Z_1 - Y_1}{X_1 - 3Z_1} \qquad \text{(III-83)}$$

and

$$D_1 = \frac{K_{1,j} - (2\Delta w Z_1/a)}{X_1 - 3Z_1} \qquad \text{(III-84)}$$

All values of C_i can now be calculated from Equations (III-73), (III-74), (III-83), and (III-84).

The last equation in the matrix has the form

$$P_{imax-1,j+1} = P_{imax,j+1}\, C_{imax} + D_{imax} \qquad \text{(III-85)}$$

The boundary condition for i = imax is

$$P_{imax,j+1} = 0 \qquad \text{(III-86)}$$

The resulting tri-diagonal matrix can be solved by the following procedure:

1. Set $P_{i,0} = 0$
2. Set $P^*_{i+1,j+1} - P^*_{i-1,j+1} = P_{i+1,j} - P_{i-1,j}$
3. Calculate C_1 and D_1
4. Calculate C_i, D_i for i = 2, ..., imax
5. Solve for $P_{i,j+1}$ from Equation (III-64)
6. Test $P_{i+1,j+1} - P_{i-1,j+1} \cong P_{i+1,j+1} - P_{i-1,j+1}$
7. If this condition is not satisfied, set

   $P^*_{i,j+1} = P_{i,j+1}$ for $i = 0, \cdots, imax$
8. Repeat steps 2 through 7
9. Proceed to next time step

The MAD program used to solve the equations in this part is given in Appendix B. The values of dimensionless pressure, P_D , evaluated for values of dimensionless time, t_D , from 0.01 to 100, are presented in Figure (III-1).

C. Solution of Non-Linear Differential Equation for Linear Flow, Constant Terminal Rate

The differential Equation (III-15) describing the linear flow of a slightly compressible liquid

$$\frac{\partial^2 p}{\partial x^2} + c\left(\frac{\partial p}{\partial x}\right)^2 = \frac{\mu \phi c}{K} \frac{\partial p}{\partial t} \tag{III-15}$$

along with the initial condition

$$p(x, 0) = p_o \tag{III-87}$$

and two boundary conditions

$$\left.\frac{\partial p}{\partial x}\right|_{x=0} = \frac{Q \mu}{A K} \quad , \qquad t \geq 0 \tag{III-88}$$

$$p(\infty, t) = p_o \tag{III-89}$$

define the constant terminal rate case in infinite linear porous media.

Substituting the dimensionless quantities

$$X = \frac{x}{x_c} \tag{III-90}$$

$$P_D = \frac{(p - p_o) A K}{\mu Q x_c} \tag{III-91}$$

$$t_D = \frac{Kt}{\mu \phi c x_c^2} \tag{III-92}$$

$$M = -\frac{c\mu Q x_c}{AK} \tag{III-93}$$

into Equations (III-26), (III-27), (III-28), and (III-29) gives the dimensionless equations to be evaluated, respectively.

$$\frac{\partial^2 P_D}{\partial X} + M\left(\frac{\partial P_D}{\partial X}\right)^2 = \frac{\partial P_D}{\partial t_D} \tag{III-94}$$

$$P_D(X,0) = 0 \quad , \qquad 0 \leqq X \leqq \infty \tag{III-95}$$

$$\left.\frac{\partial P_D}{\partial X}\right|_{X=0} = -1 \tag{III-96}$$

$$P_D(\infty, t_D) = 0 \tag{III-97}$$

Thus the pressure at any time is given by

$$p = p_0 - \frac{\mu Q x_c}{AK} P_D \tag{III-98}$$

The dimensionless terms defined by Equations (III-92) and (III-93) in engineering field units are:

$$t_D = \frac{0.00633\ Kt}{\mu \phi c x_c^2} \tag{III-99}$$

and

$$M = -\frac{887.6\ c \mu q x_c}{AK} \tag{III-100}$$

where:

A = cross sectional area normal to flow, (feet)

c = compressibility, vol/(vol)(psi)

K = permeability, millidarcys

q = flow rate, bbl/day

t = time, days

x_c = reservoir characteristic length (arbitrary), feet

ϕ = porosity, fraction

μ = viscosity, centipoise

Thus the reservoir pressure in psi can be calculated from

$$p = p_o - \frac{887.6\ \mu q x_c}{AK} P_D \tag{III-101}$$

As in previous case for radial flow, the linear flow case will be solved by numerical means using both implicit and explicit methods.

1. Explicit Numerical Method

Equation (III-94) is transformed by

$$w = 1 - e^{-X} \tag{III-102}$$

so that limits $0 \leq X \leq \infty$ are transformed to $0 \leq w \leq 1$. The values of each term in Equation (III-94) are given by

$$\frac{\partial P}{\partial X} = \frac{\partial P}{\partial w}\frac{dw}{dX} = \frac{\partial P}{\partial w}(1-w) \qquad \text{(III-103)}$$

and

$$\frac{\partial^2 P}{\partial X^2} = \frac{\partial}{\partial X}\left\{\frac{\partial P}{\partial w}(1-w)\right\} = (1-w)^2\frac{\partial^2 P}{\partial w^2} - (1-w)\frac{\partial P}{\partial w} \qquad \text{(III-104)}$$

Substituting Equations (III-103) and (III-104) in Equation (III-94) yields

$$(1-w)^2\frac{\partial^2 P}{\partial w^2} - (1-w)\frac{\partial P}{\partial w} + M(1-w)^2\left(\frac{\partial P}{\partial w}\right)^2 = \frac{\partial P}{\partial t_D} \qquad \text{(III-105)}$$

Equation (III-105) can be represented in difference form by

$$(1-i\Delta w)^2\left\{\frac{P_{i+1,j} - 2P_{i,j} + P_{i-1,j}}{(\Delta w)^2}\right\} - (1-i\Delta w)\left\{\frac{P_{i+1,j} - P_{i-1,j}}{2\Delta w}\right\}$$
$$+ M(1-i\Delta w)^2\left\{\frac{P_{i+1,j} - P_{i-1,j}}{2\Delta w}\right\} = \frac{P_{i,j+1} - P_{i,j}}{\Delta t_D} \qquad \text{(III-106)}$$

where i and j refer to the mess points of the distance and time increments respectively.

Rearranging Equation (III-106) and solving for $P_{i,j+1}$ gives

$$P_{i,j+1} = P_{i,j} + \frac{\Delta t_D}{(\Delta w)^2}\Big\{(1-i\Delta w)^2(P_{i+1,j} - P_{i,j} + P_{i-1,j})$$
$$- (1-i\Delta w)\frac{\Delta w}{2}(P_{i+1,j} - P_{i-1,j}) + \frac{M(1-i\Delta w)^2}{4}(P_{i+1,j} - P_{i-1,j})^2\Big\} \qquad \text{(111-107)}$$

the difference equation in implicit form.

Again an analysis of the stability (15) (34) requires that

$$\Delta t_D < \frac{1}{2} (\Delta w)^2 \tag{III-108}$$

The boundary condition given by Equation (III-96) is

$$\left.\frac{\partial P}{\partial w}\right|_{w=0} = -1 \tag{III-109}$$

which may be expressed in numerical form by

$$\frac{-3P_{0,j+1} + 4P_{1,j+1} - P_{2,j+1}}{2\Delta w} = -1 \tag{III-110}$$

The remaining boundary condition is

$$P_{I,j+1} = 0 \tag{III-111}$$

The initial condition is given by

$$P_{i,0} = 0 \tag{III-112}$$

Equations (III-107), (III-110), (III-111, and (III-112) were solved on the IBM 7090 computer for imax = 80. The MAD program is presented in Appendix C. Dimensionless pressure was evaluated for values of dimensionless time from 0 to 0.01. Values of dimensionless pressure for values of dimensionless time greater than 0.01 were found by an implicit method as discussed in the next part. These results are shown in Figure (III-2).

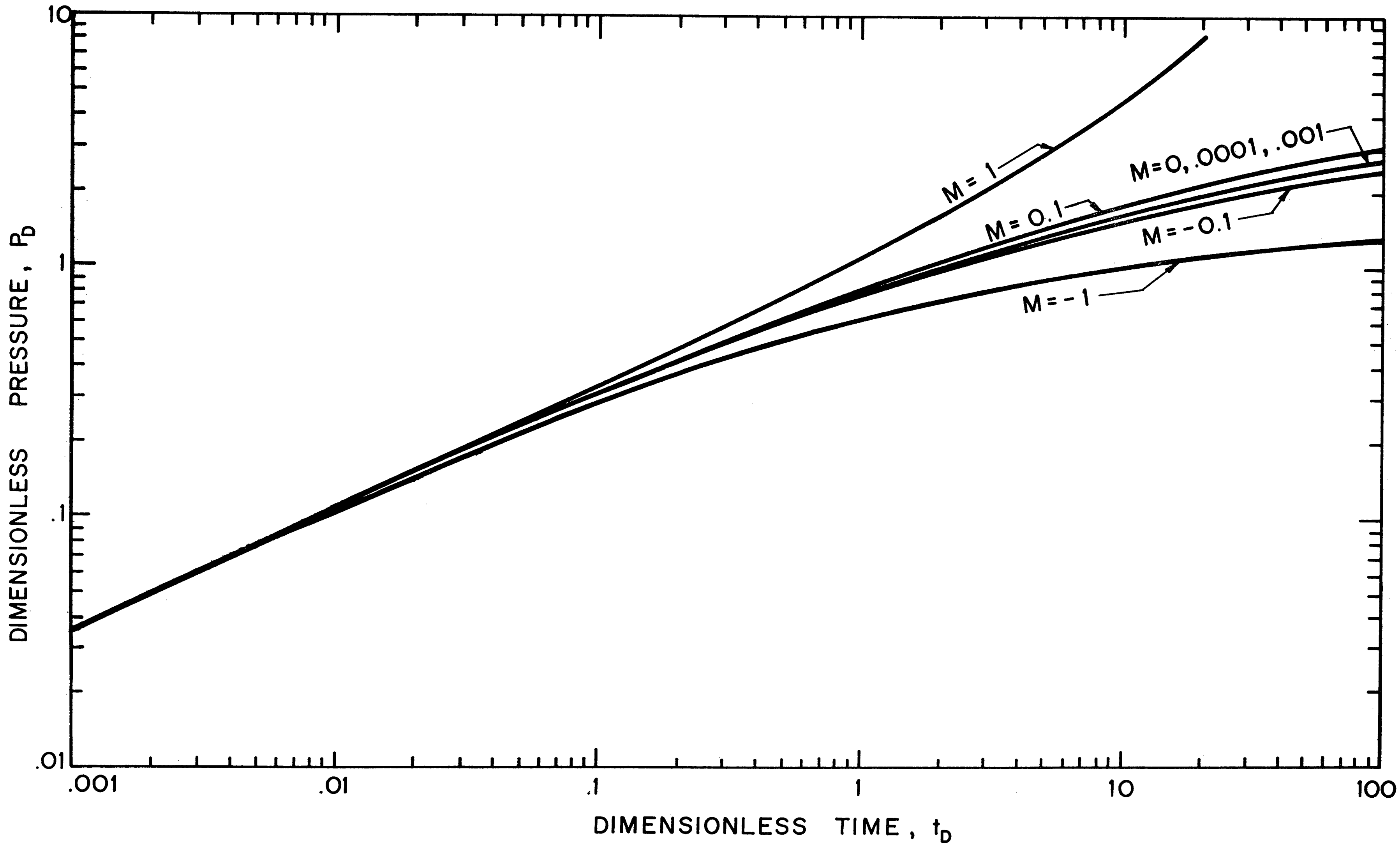

Figure III-2. Dimensionless Pressure Drop versus Dimensionless Time for Linear System, Constant Terminal Rate Case

2. Implicit Numerical Method

The non-linear differential Equation (III-92) is transformed by

$$w = 1 - e^{-aX} \qquad \text{(III-113)}$$

as follows.*

$$\frac{\partial P}{\partial X} = \frac{\partial P}{\partial w}\frac{dw}{dX} = a(1-w)\frac{\partial P}{\partial w} \qquad \text{(III-114)}$$

$$\frac{\partial^2 P}{\partial X^2} = \frac{\partial}{\partial X}\left(\frac{\partial P}{\partial X}\right)$$

$$= a^2(1-w)^2\frac{\partial^2 P}{\partial w^2} - a^2(1-w)\frac{\partial P}{\partial w} \qquad \text{(III-115)}$$

Substituting Equations (III-114) and (III-115) into Equation (III-94) yields

$$a^2(1-w)^2\frac{\partial^2 P}{\partial w^2} - a^2(1-w)\frac{\partial P}{\partial w}$$

$$+ Ma^2(1-w)^2\left(\frac{\partial P}{\partial w}\right)^2 = \frac{\partial P}{\partial t_D}, \qquad 0 \leqq w \leqq 1 \qquad \text{(III-116)}$$

A difference equation approximation of Equation (III-14) is given by

$$a^2(1-i\Delta w)^2\left\{\frac{P_{i+1,j+1} - 2P_{i,j+1} + P_{i-1,j+1}}{(\Delta w)^2}\right\} - a^2(1-i\Delta w)\cdot$$

$$\cdot\left\{\frac{P_{i+1,j+1} - P_{i-1,j+1}}{2\Delta w}\right\} + a^2M(1-i\Delta w)^2\left\{\frac{P_{i+1,j+1} - P_{i-1,j+1}}{2\Delta w}\right\}^2 = \frac{P_{i,j+1} - P_{i,j}}{\Delta t_D} \qquad \text{(111-117)}$$

*The subscript D is dropped as in the previous derivations.

Again the non-linear term in Equation (III-15) is replaced as in Equation (III-61) by

$$(P_{i+1,j+1} - P_{i-1,j+1})^2 = (P_{i+1,j+1} - P_{i-1,j+1})(P^*_{i+1,j+1} - P^*_{i-1,j+1}) \qquad \text{(III-62)}$$

where $(P^*_{i+1,j+1} - P^*_{i-1,j+1})$ is estimated from previous time steps or previous iterations.

Substituting of Equation (III-62) into Equation (III-117), rearranging and multiplying by $(\Delta w)^2$ gives

$$\begin{aligned} &P_{i+1,j+1}\left\{(1-i\Delta w)^2 - \frac{\Delta w(1-i\Delta w)}{2} + \frac{M}{4}(1-i\Delta w)^2 \cdot \right. \\ &\left. \cdot (P^*_{i+1,j+1} - P^*_{i-1,j+1})\right\} + P_{i,j+1}\left\{-2(1-i\Delta w)^2 - \frac{(\Delta w)^2}{a^2\Delta t_D}\right\} \\ &+ P_{i-1,j+1}\left\{(1-i\Delta w)^2 + \frac{\Delta w}{2}(1-i\Delta w) + \frac{M}{4}(1-i\Delta w)^2 \cdot \right. \\ &\left. \cdot (P^*_{i+1,j+1} - P^*_{i-1,j+1})\right\} = -P_{i,j}\left(\frac{(\Delta w)^2}{a^2\Delta t_D}\right) \end{aligned} \qquad \text{(III-118)}$$

The same procedure is used to solve Equation (III-118) as was used to solve Equation (III-65). Assume

$$P_{i-1,j+1} = C_i P_{i,j+1} + D_i \qquad \text{(III-119)}$$

Substituting Equation (III-119) in Equation (III-118) gives

$$\begin{aligned} &P_{i+1,j+1}(Z_i) + P_{i,j+1}(Y_i) \\ &+ (C_i P_{i,j+1} + D_i) X_i = K_{i,j} \end{aligned} \qquad \text{(III-120)}$$

where

$$\begin{aligned} Z_i = &(1-i\Delta w)^2 - \frac{\Delta w}{2}(1-i\Delta w) \\ &+ \frac{M}{4}(1-i\Delta w)^2 (P^*_{i+1,j+1} - P^*_{i-1,j+1}) \end{aligned} \qquad \text{(III-121)}$$

$$Y_i = -2(1 - i\Delta w)^2 - \frac{(\Delta w)^2}{a^2 \Delta t_D} \qquad \text{(III-122)}$$

$$X_i = (1 - i\Delta w)^2 + \frac{\Delta w}{2}(1 - i\Delta w) - \frac{c}{4}(1 - i\Delta w)^2 \left(P^*_{i+1,j+1} - P^*_{i-1,j+1}\right) \qquad \text{(III-123)}$$

$$K_{i,j} = -P_{i,j}\left(\frac{(\Delta w)^2}{a^2 \Delta t_D}\right) \qquad \text{(III-124)}$$

Solving Equation (III-120) for $P_{i,j+1}$ yields

$$P_{i,j+1} = P_{i+1,j+1}\left\{\frac{-Z_i}{C_i X_i + Y_i}\right\} + \frac{K_{i,j} - D_i X_i}{C_i X_i + Y_i} \qquad \text{(III-125)}$$

If Equation (III-125) is compared to Equation (III-119), it is obvious that

$$C_{i+1} = -\frac{Z_i}{C_i X_i + Y_i} \qquad \text{(III-126)}$$

and

$$D_{i+1} = \frac{K_{i,j} - D_i X_i}{C_i X_i + Y_i} \qquad \text{(III-127)}$$

Equations (III-126) and (III-127) are the recursion relationships used to calculate the values of C_i and D_i.

The boundary condition, Equation (III-88) in the w plane is

$$\left.\frac{\partial P}{\partial X}\right|_{X=0} = \left.\frac{\partial P}{\partial w}\frac{dw}{dX}\right|_{X=0} = -1 \qquad \text{(III-128)}$$

and hence

$$\left.\frac{\partial P}{\partial w}\right|_{w=0} = -\frac{1}{a} \qquad \text{(III-129)}$$

The remaining boundary condition, Equation (III-97), in the w plane is

$$P(1, t_D) = 0 \qquad \text{(III-130)}$$

The initial condition corresponding to Equation (III-95) is

$$P(w, 0) = 0 \quad , \qquad 0 \leq w \leq 1 \qquad \text{(III-131)}$$

The differential form of the Lagrangian three point formula evaluated at the initial point is used to approximate Equation (III-129).

$$\frac{-3P_{0,j+1} + 4P_{1,j+1} - P_{2,j+1}}{2\Delta w} = -\frac{1}{a} \qquad \text{(III-132)}$$

Rearranging Equation (III-132) gives

$$P_{2,j+1} = \frac{2\Delta w}{a} - 3P_{0,j+1} + 4P_{1,j+1} \qquad \text{(III-133)}$$

Evaluating Equation (III-120) at i = 1 gives

$$P_{2,j+1}(Z_1) + P_{1,j+1}(Y_1) + P_{0,j+1}(X_1) = K_{1,j} \qquad \text{(III-134)}$$

Substituting Equation (III-133) in Equation (III-134) gives

$$\left(\frac{2\Delta w}{a} - 3P_{0,j+1} + 4P_{1,j+1}\right)Z_1 + P_{1,j+1}(Y_1) + P_{0,j+1}(X_1) = K_{1,j} \qquad \text{(III-135)}$$

or

$$P_{o,j+1}(X_1 - 3Z_1) + P_{1,j+1}(4Z_1 + Y_1) = K_{1,j} - \frac{2\Delta w Z_1}{a} \qquad \text{(III-136)}$$

Solving Equation (III-136) for $P_{o,j+1}$ yields

$$P_{o,j+1} = P_{1,j+1}\left\{-\frac{4Z_1 + Y_1}{X_1 - 3Z_1}\right\} + \frac{K_{1,j} - 2\Delta w Z_1/a}{X_1 - 3Z_1} \qquad \text{(III-137)}$$

Comparison of Equation (III-137) with Equation (III-119) shows that

$$C_1 = \frac{-4Z_1 - Y_1}{X_1 - 3Z_1} \qquad \text{(III-138)}$$

and

$$D_1 = \frac{K_{1,j} - 2\Delta w Z_1}{X_1 - 3Z_1} \qquad \text{(III-139)}$$

All values of C_i can now be calculated from Equations (III-126), (III-127), (III-36), and (III-139).

The last equation in the matrix has the form

$$P_{imax-1,j+1} = P_{imax,j+1}(C_{imax}) + D_{imax} \qquad \text{(III-140)}$$

Since

$$P_{imax,j+1} = 0 \qquad \text{(III-141)}$$

The tri-diagonal matrix can now be solved, and the values of P found for i = 0, 1, 2, ..., imax.

The same procedure as outlined on page 33 for the radial model is used in this case.

The MAD program was used to evaluate values of dimensionless pressure, P_D, for several values of the coefficient M, for values of dimensionless time, t_D, from 0.01 to 100. These results are presented in Figure (III-2).

D. Analysis of Results and Application to Reservoir Problems

The percent error in the measurement of the pressure drop for a constant rate pump test in a radial flow system is shown in Figure (III-3). The value of dimensionless time for which the error is less than one percent may be approximated by

$$t_D \ (1\% \ \text{error}) \cong \frac{0.001}{M^2} \qquad \text{(III-142)}$$

The following two example problems show how to determine if neglecting the non-linear term in the differential equation describing the flow of a slightly compressible fluid in a porous media is justified.

Example Problem III-1

Consider an aquifer with the following physical properties and dimensions:

Compressibility, $c = 7 \times 10^{-6}$ vol/(vol)(psi)

Thickness, $h = 160$ feet

Permeability, $K = 200$ millidarcys

Porosity, $\phi = .21$

Viscosity, $\mu = 1$ centipoise

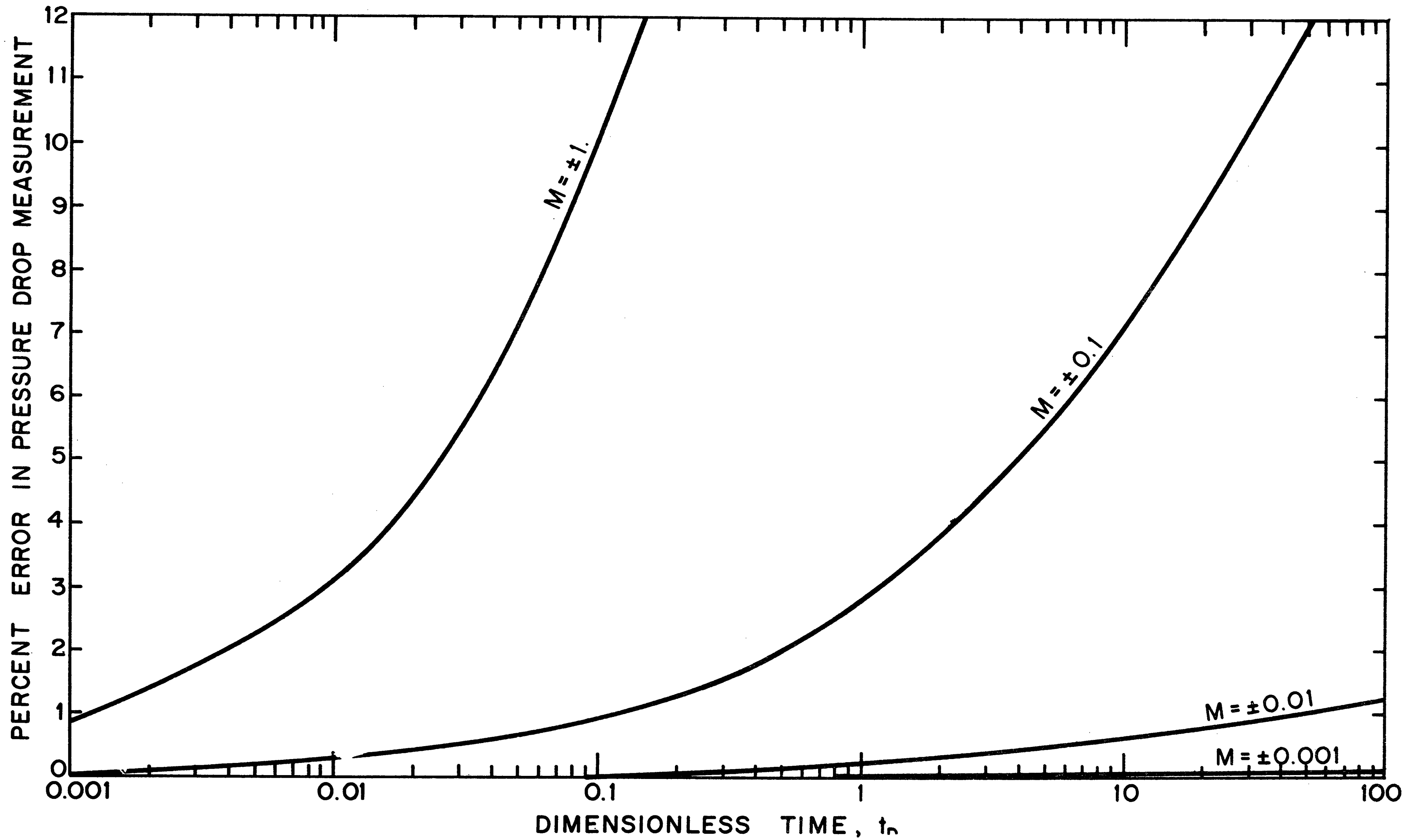

Figure III-3. Percent Error in Dimensionless Pressure Drop for Radial System, Constant Terminal Rate.

If a well with a one foot diameter, completed in the aquifer, is pumped at a constant rate of 300 bbl per day for three days, determine if one is justified in neglecting the non-linear term.

Solution

The coefficient M is calculated from Equation (III-34)

$$M = \frac{141.2\, c \mu q}{h K} = \frac{(141.2)(7\times10^{-6})(1)(300)}{(160)(200)} = 9.27 \times 10^{-6}$$

and the value of dimensionless time at the end of three days by Equation (III-30)

$$t_D = \frac{0.00633\, K t}{\mu \phi c r^2} = \frac{(0.00633)(160)(3)}{(1)(0.21)(7\times10^{-6})(0.5)^2} = 8.3 \times 10^6$$

The value of dimensionless time for which a one percent error in the calculation of pressure drop is made in neglecting the non-linear term is calculated from Equation (III-142)

$$t_D \cong \frac{0.001}{M^2} = \frac{0.001}{(9.27\times10^{-6})^2} = 1.16 \times 10^7$$

Neglecting the non-linear term results in less than a one percent error in the calculation of the pressure drop. Thus one is justified in neglecting the non-linear term and using solutions based on the diffusivity Equation (III-19).

Example Problem III-2

A natural gas storage field is situated on a large aquifer. The gas field and aquifer have the following dimensions and physical properties:

Compressibility, c = 7×10^{-6} vol/(vol)(psi)

Thickness, h = 20 feet

Permeability, k = 200 millidarcys

Porosity, ϕ = 0.18

Water Viscosity, μ = 0.8 centipoise (reservoir conditions)

Radius of gas field, r_b = 3000 feet

If the gas field is produced for ninety days at a rate which causes a water influx rate of 20,000 bbl/day, determine if neglecting the non-linear term in the solution of the flow equation is justified.

Solution

Again the coefficient M is calculated from Equation (III-34)

$$M = \frac{141.2\, c \mu q}{K h} = \frac{(141.2)(7\times10^{-6})(0.8)(20,000)}{(200)(20)} = 0.000396$$

The value of dimensionless time at the end of 90 days is

$$t_D = \frac{0.00633\, K t}{\mu \phi c r_b^2} = \frac{(0.00633)(200)(90)}{(0.8)(0.18)(7\times10^{-6})(3,000)^2} = 17.4$$

Equation (III-142) is used to determine the value of dimensionless time above which an error greater than one percent would be made

$$t_D = \frac{0.001}{M^2} = \frac{0.001}{(3.96 \times 10^{-4})^2} = 6370$$

Thus the solutions based on the solution of the diffusivity equation can be used to calculate pressure drop.

The error in pressure measurement resulting from neglecting the non-linear term for a constant rate pump test in a linear flow system is shown in Figure (III-4). Again the value of dimensionless time for which the error is less than one percent may be approximated by

$$t_D \; (1\% \;\; error) \cong \frac{0.001}{M^2} \qquad \text{(III-142)}$$

Note that this is the same expression found in the radial case. An example problem demonstrates how to determine if the non-linear term may be neglected in the linear flow case.

Example Problem III-3

A gas field is situated on a large aquifer. The gas field and aquifer are located between two parallel faults so that the linear equations can be used to predict the pressure. The aquifer has the following dimensions and physical properties:

Cross sectional area to flow, A = 500,000 square feet

Compressibility, $c = 7 \times 10^{-6}$ vol/(vol)(psi)

Permeability, K = 50 millidarcys

Porosity, ϕ = 0.10

Water Viscosity, μ = 0.7 centipoise

Gas is injected into the field for thirty days causing a water efflux rate of 10,000 bbl/day. Justify neglecting the non-linear term in the differential equation used in determining the pressure behavior in the aquifer. Use 1,000 feet as the characteristic length.

Solution

The value of M, calculated from Equation (III-98), is

$$M = \frac{887.6\, c \mu q x_c}{KA} = \frac{(887.6)(7\times10^{-6})(0.7)(10,000)(1,000)}{(50)(500,000)}$$

$$= 1.73 \times 10^{-3}$$

Dimensionless time at the end of thirty days is given by

$$t_D = \frac{0.00633\, Kt}{\mu \phi c x_c^2} = \frac{(0.00633)(50)(30)}{(0.7)(0.1)(7\times10^{-6})(10^6)}$$

$$= 19.4$$

Equation (III-142) is used to determine the value of dimensionless time when the error reaches one percent

$$t_D \ (1\% \ \text{error}) \cong \frac{0.001}{M^2} = \frac{0.001}{(1.74\times10^{-3})^2} = 330$$

Since the value of dimensionless time at the end of thirty days is less than the value which results in a one

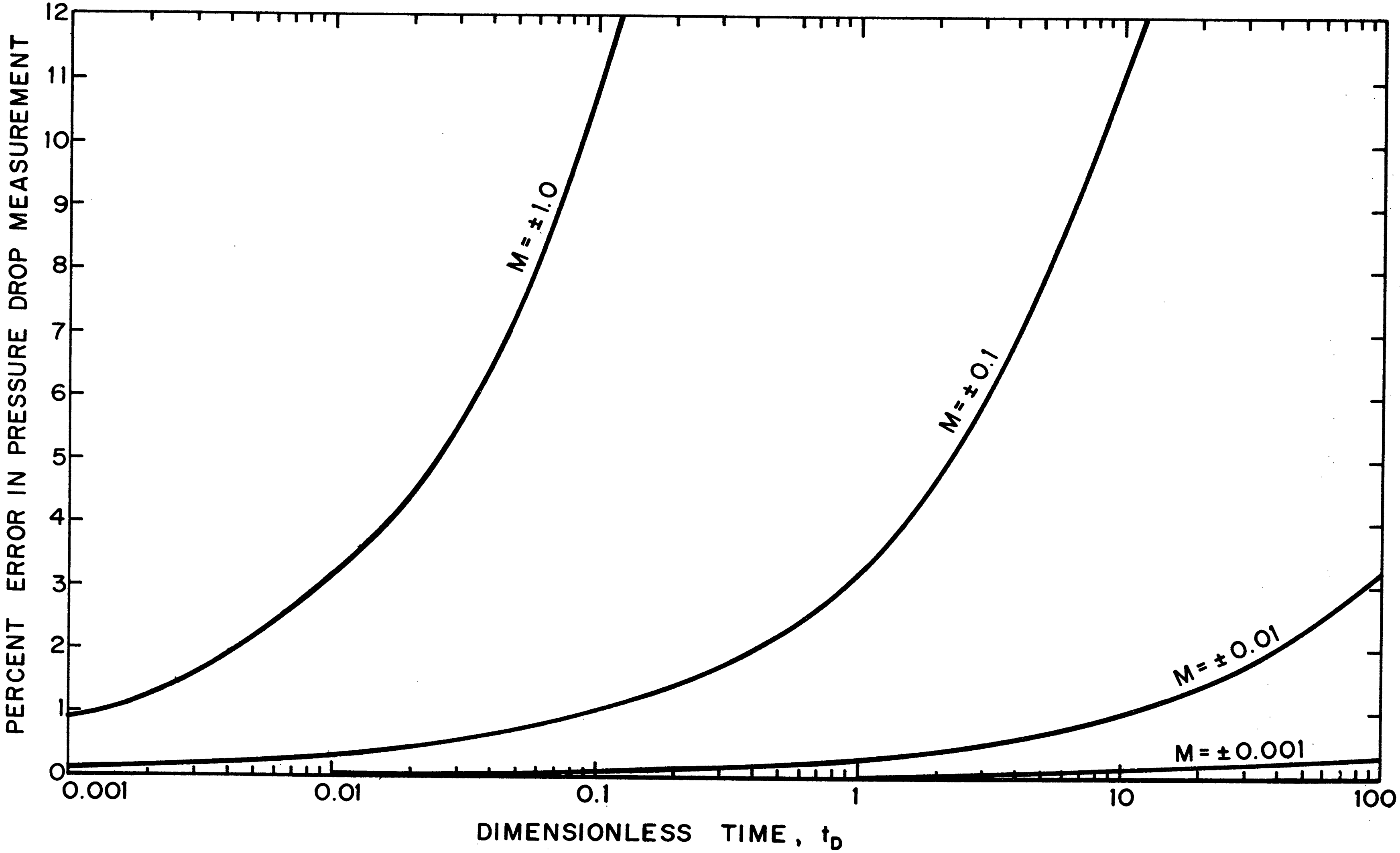

Figure III-4. Percent Error in Dimensionless Pressure Drop for Linear System, Constant Terminal Rate Case.

percent error in the calculation of pressure drop, solutions based on the diffusivity Equation (III-18) are valid. The same conclusion results for any selected value for the characteristic length.

IV. EFFECT OF NEIGHBORING GAS FIELD (OR LINE OF CONSTANT PRESSURE) ON INSITU PERMEABILITY AND COMPRESSIBILITY MEASUREMENTS IN AQUIFERS FOR CONSTANT RATE PUMP TESTS

Field data on pressure behavior of wells have been observed to deviate considerably from the behavior predicted by the methods described in Section II. Failure of the mathematical model to describe the reservoir is partially responsible for these deviations.

The presence of a gas field, an outcropping of the porous media on the bottom of a lake or stream, or a sudden increase in the permeability are a few conditions which may cause the deviation. Effect of these conditions on the measurement of the insitu permeability and compressibility will be evaluated in this section. Furthermore, the unsteady state pressure behavior of wells during constant rate pump tests will be described for these situations.

A. Prediction of Pressure Behavior During Drawdown or Build-up Pump Tests

Field pump test data from aquifers can be analyzed to determine if a gas field (or line of constant pressure) is located near a pumping or observation well if the theoretical pressure performance is known as a function of the distance between the pumping and observation wells in addition to the distance between the wells and the gas field. These relationships are developed below.

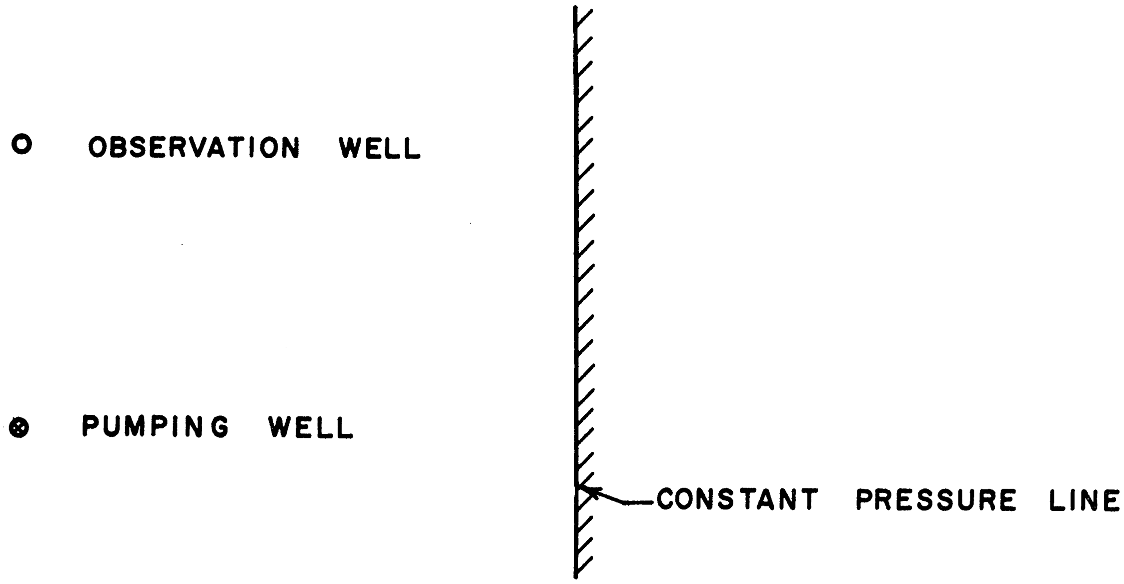

Figure IV-1. Pumping Well and Observation Well near a Line of Constant Pressure.

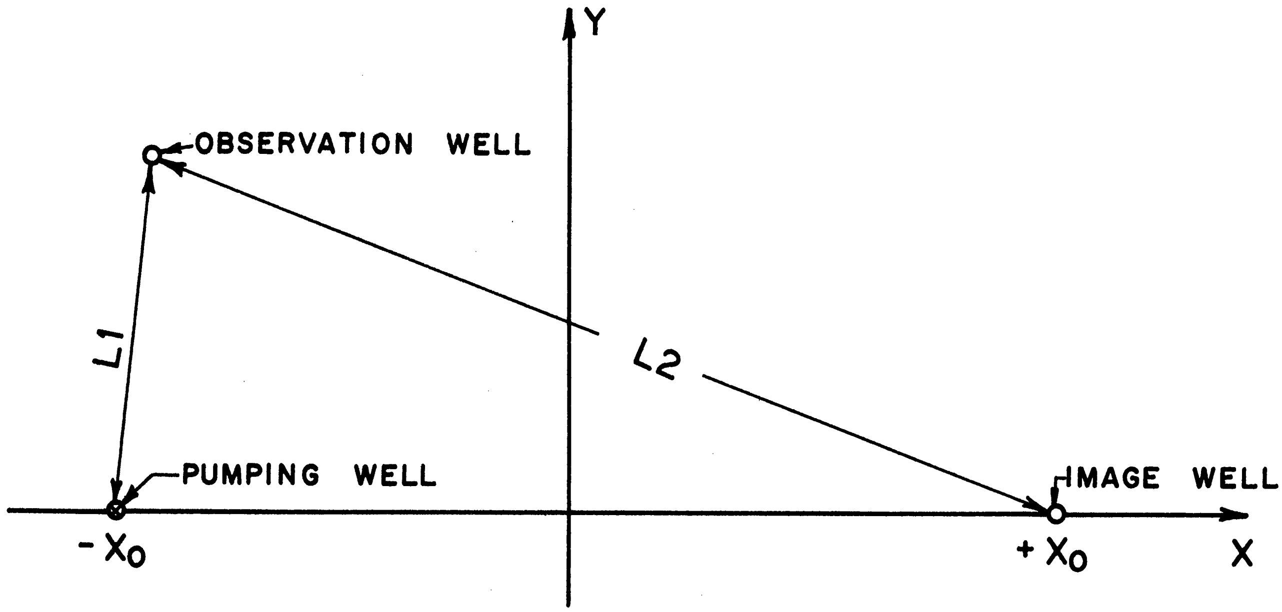

Figure IV-2. Mathematical Model and Location of Image Well.

1. Pressure Behavior of Observation Well During Drawdown and Build-up Tests

Figure (IV-1) shows a sketch of a pumping well and an observation well near a line of constant pressure.

The mathematical equations which describe the pressure behavior of an observation well during a drawdown pump test is given by the diffusivity equation

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = \frac{\mu \phi c}{K} \frac{\partial p}{\partial t} \qquad \text{(IV-1)}$$

the initial condition

$$p(x, y, 0) = p_o \qquad \text{(IV-2)}$$

and the boundary conditions

$$\lim_{x \to \infty} p(x, y, t) = p_o \qquad \text{(IV-3)}$$

$$\lim_{y \to \infty} p(x, y, t) = p_o \qquad \text{(IV-4)}$$

$$\frac{\partial p}{\partial r}(-x_o, 0, t) = \frac{q\mu}{2\pi K h \sqrt{(x+x_o)^2 + y^2}} \qquad \text{(IV-5)}$$

$$\frac{\partial p}{\partial r}(x_o, 0, t) = \frac{-q\mu}{2\pi K h \sqrt{(x-x_o)^2 + y^2}} \qquad \text{(IV-6)}$$

If the distance between the pumping well and the observation well is greater than 30 times the well radius, Mortada (63) showed that the point source solution presented by Horner (35) is valid. Thus the solution to Equations (IV-1) through (IV-6) is given by the addition of a point source at $x = -x_0$, $y = 0$ and a point sink at $x = x_0$, $y = 0$ (See Figure IV-2).

$$p - p_0 = \frac{Q\mu}{4\pi K h} Ei\left\{-\frac{\mu\phi c \ell_1^2}{4Kt}\right\} - \frac{Q\mu}{4\pi K h} Ei\left\{-\frac{\mu\phi c \ell_2^2}{4Kt}\right\} \qquad \text{(IV-7)}$$

where:

- c = compressibility, vol/(vol)(atm)
- Q = production rate, cc/sec
- μ = viscosity, centipoise
- K = permeability, darcys
- h = thickness, centimeters
- Ei = exponential integral, $Ei(-x) = -\int_{+x} \frac{1}{u} e \; du$
- p = pressure, atmospheres
- p_0 = initial pressure, atmospheres
- ℓ_1 = distance between pumping well and observation well, centimeters
- ℓ_2 = distance between observation well and image well (See Figure IV-2), centimeters
- ϕ = porosity, fraction

Replacing the terms in Equation (IV-7) by the following dimensionless quantities

$$P_D = \frac{2\pi K h (p - p_0)}{Q\mu} \qquad \text{(IV-8)}$$

$$T_D = \frac{Kt}{\mu \phi c \ell_1^2} \qquad \text{(IV-9)}$$

$$R_D = \frac{\ell_2}{\ell_1} \qquad \text{(IV-10)}$$

yields

$$P_D = -\frac{1}{2}\left\{ Ei\left(-\frac{1}{4T_D}\right) - Ei\left(-\frac{R_D^2}{4T_D}\right)\right\} \qquad \text{(IV-11)}$$

If field units are used, the expression for the dimensionless pressure (Equation IV-8) is given by

$$P_D = \frac{141.2\, Kh\,(p_0 - p)}{q\mu} \qquad \text{(IV-12)}$$

where:

h = reservoir thickness, feet

K = permeability, millidarcys

p = pressure, psia

p_o = initial pressure, psia

q = production rate, bbl/day

μ = viscosity, centipoise

and the expression for dimensionless time becomes

$$T_D = \frac{0.00633\ Kt}{\mu \phi c \ell_1^2} \qquad \text{(IV-13)}$$

where:

ϕ = porosity, fraction

c = compressibility, vol/(vol)(psi)

t = time, days

ℓ_1 = distance between pumping and observation well, feet

The distances in Equation (IV-10) must be expressed in the same units.

Numerical values of dimensionless pressure drop, P_D, as a function of dimensionless time, T_D, are presented in Figure (IV-3) and Table (IV-1) for values of R_D between 1 and 16 and T_D less than 1,000. Steady state values for the dimensionless pressure drop, P_D, as a function of dimensionless distance ratio, R_D, are presented in Figure (IV-4).

The pressure behavior for a <u>build-up</u> <u>pump</u> test is obtained by superimposing a negative flow rate on the time after the pumping well is shut-in. Thus

$$p - p_o = \frac{q\mu}{4\pi Kh}\left\{Ei\left(-\frac{\mu\phi c \ell_1^2}{4K(t_o+\Delta t)}\right) - Ei\left(-\frac{\mu\phi c \ell_2^2}{4K(t_o+\Delta t)}\right) - Ei\left(-\frac{\mu\phi c \ell_1^2}{4K\Delta t}\right) + Ei\left(-\frac{\mu\phi c \ell_2^2}{4K\Delta t}\right)\right\} \quad \text{(IV-14)}$$

or if dimensionless pressure, P_D, and field units are used

$$P_D = -\frac{1}{2}\left\{Ei\left(-\frac{\mu\phi c \ell_1^2}{4(0.00633)K(t_o+\Delta t)}\right) - Ei\left(-\frac{\mu\phi c \ell_2^2}{4(0.00633)K(t_o+\Delta t)}\right) - Ei\left(-\frac{\mu\phi c \ell_1^2}{4(0.00633)K\Delta t}\right) + Ei\left(-\frac{\mu\phi c \ell_2^2}{4(0.00633)K\Delta t}\right)\right. \quad \text{(IV-15)}$$

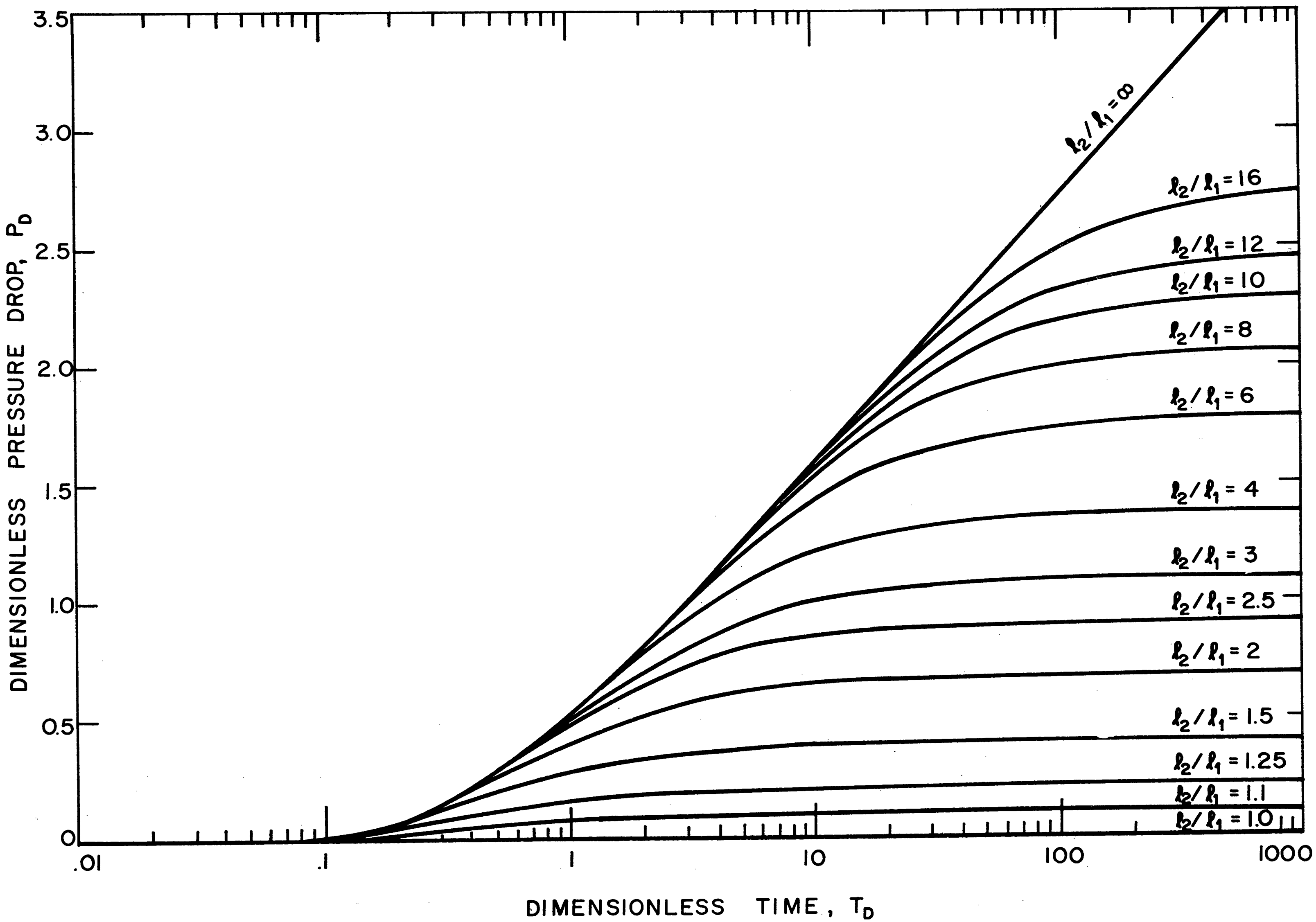

Figure IV-3. Dimensionless Pressure Drop for Observation Well in Radial System with Constant External Pressure Line.

TABLE IV -1 DIMENSIONLESS PRESSURE DROP, $P_D(R_D, t_D)$ FOR RADIAL SYSTEM WITH EXTERNAL LINE OF CONSTANT PRESSURE

DIM. TIME	DIMENSIONLESS PRESSURE DROP, P_D											
t_D	R_D = 1.1	R_D = 1.25	R_D = 1.5	R_D = 2	R_D = 2.5	R_D = 3	R_D = 4	R_D = 6	R_D = 8	R_D = 10	R_D = 12	R_D = 16
.01	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.02	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.03	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.04	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
.05	.0004	.0005	.0006	.0006	.0006	.0006	.0006	.0006	.0006	.0006	.0006	.0006
.06	.0010	.0014	.0015	.0015	.0015	.0015	.0015	.0015	.0015	.0015	.0015	.0015
.07	.0019	.0029	.0032	.0032	.0032	.0032	.0032	.0032	.0032	.0032	.0032	.0032
.08	.0031	.0049	.0055	.0056	.0056	.0056	.0056	.0056	.0056	.0056	.0056	.0056
.09	.0045	.0074	.0085	.0087	.0087	.0087	.0087	.0087	.0087	.0087	.0087	.0087
.10	.0061	.0103	.0121	.0125	.0125	.0125	.0125	.0125	.0125	.0125	.0125	.0125
.15	.0153	.0284	.0367	.0392	.0393	.0393	.0393	.0393	.0393	.0393	.0393	.0393
.20	.0243	.0473	.0652	.0729	.0734	.0735	.0735	.0735	.0735	.0735	.0735	.0735
.25	.0316	.0640	.0923	.1078	.1096	.1097	.1097	.1097	.1097	.1097	.1097	.1097
.30	.0380	.0787	.1171	.1420	.1458	.1462	.1463	.1463	.1463	.1463	.1463	.1463
.40	.0479	.1020	.1586	.2037	.2140	.2158	.2161	.2161	.2161	.2161	.2161	.2161
.50	.0549	.1192	.1903	.2554	.2743	.2789	.2799	.2799	.2799	.2799	.2799	.2799
.60	.0602	.1323	.2157	.2984	.3268	.3351	.3376	.3376	.3376	.3376	.3376	.3376
.70	.0644	.1425	.2358	.3343	.3722	.3851	.3898	.3901	.3901	.3901	.3901	.3901
.80	.0676	.1507	.2520	.3644	.4117	.4295	.4373	.4378	.4378	.4378	.4378	.4378
.90	.0702	.1574	.2656	.3902	.4462	.4693	.4806	.4817	.4817	.4817	.4817	.4817
1.00	.0723	.1630	.2769	.4124	.4764	.5047	.5203	.5221	.5221	.5221	.5221	.5221
2.00	.0833	.1909	.3351	.5322	.6514	.7225	.7876	.8111	.8121	.8121	.8121	.8121
2.50	.0852	.1965	.3476	.5602	.6953	.7813	.8683	.9084	.9113	.9114	.9114	.9114
3.00	.0868	.2009	.3566	.58[illegible]2	.7270	.8245	.9301	.9881	.9942	.9946	.9946	.9946
3.50	.0881	.2037	.3632	.5949	.7508	.8573	.9789	1.0546	1.0650	1.0659	1.0660	1.0660
4.00	.0890	.2063	.3683	.6063	.7693	.8832	1.0187	1.1110	1.1266	1.1283	1.1284	1.1284
5.00	.0902	.2095	.3751	.6226	.7961	.9213	1.0786	1.2016	1.2289	1.2334	1.2339	1.2339
6.00	.0910	.2117	.3803	.6336	.8147	.9479	1.1218	1.2710	1.3110	1.3195	1.3209	1.3210
7.00	.0916	.2133	.3838	.6417	.8282	.9676	1.1544	1.3258	1.3786	1.3920	1.3947	1.3952
8.00	.0921	.2145	.3864	.6477	.8385	.9828	1.1799	1.3702	1.4353	1.4542	1.4587	1.4597
9.00	.0924	.2155	.3885	.6527	.8467	.9948	1.2003	1.4072	1.4835	1.5083	1.5151	1.5169
10.00	.0927	.2162	.3902	.6568	.8533	1.0044	1.2171	1.4382	1.5251	1.5558	1.5652	1.5681
12.00	.0931	.2174	.3927	.6627	.8636	1.0193	1.2429	1.4872	1.5928	1.6355	1.6508	1.6569
14.00	.0935	.2182	.3945	.6670	.8706	1.0302	1.2618	1.5243	1.6459	1.6999	1.7216	1.7320
16.00	.0937	.2188	.3958	.6702	.8764	1.0384	1.2765	1.5534	1.6889	1.7529	1.7812	1.7967
18.00	.0939	.2193	.3969	.6727	.8808	1.0445	1.2879	1.5767	1.7240	1.7975	1.8322	1.8534
20.00	.0940	.2197	.3977	.6747	.8842	1.0498	1.2973	1.5960	1.7533	1.8352	1.8763	1.9036
40.00	.0947	.2214	.4016	.6838	.9001	1.0740	1.3406	1.6883	1.9009	2.0359	2.1220	2.2089
60.00	.0949	.2220	.4029	.6869	.9054	1.0821	1.3556	1.7215	1.9565	2.1161	2.2266	2.3552
80.00	.0950	.2223	.4035	.6885	.9081	1.0862	1.3632	1.7383	1.9858	2.1593	2.2845	2.4418
100.00	.0950	.2224	.4039	.6894	.9098	1.0887	1.3677	1.7490	2.0038	2.1862	2.3211	2.4985
200.00	.0952	.2228	.4047	.6913	.9130	1.0936	1.3770	1.7701	2.0409	2.2422	2.3994	2.6252
400.00	.0952	.2230	.4051	.6922	.9147	1.0961	1.3816	1.7809	2.0600	2.2721	2.4412	2.6960
600.00	.0953	.2230	.4052	.6925	.9152	1.0969	1.3832	1.7845	2.0664	2.2822	2.4556	2.7204
800.00	.0953	.2231	.4053	.6927	.9155	1.0974	1.3840	1.7863	2.0696	2.2872	2.4628	2.7335
1000.00	.0953	.2231	.4053	.6928	.9156	1.0976	1.3844	1.7874	2.0716	2.2903	2.4672	2.7412

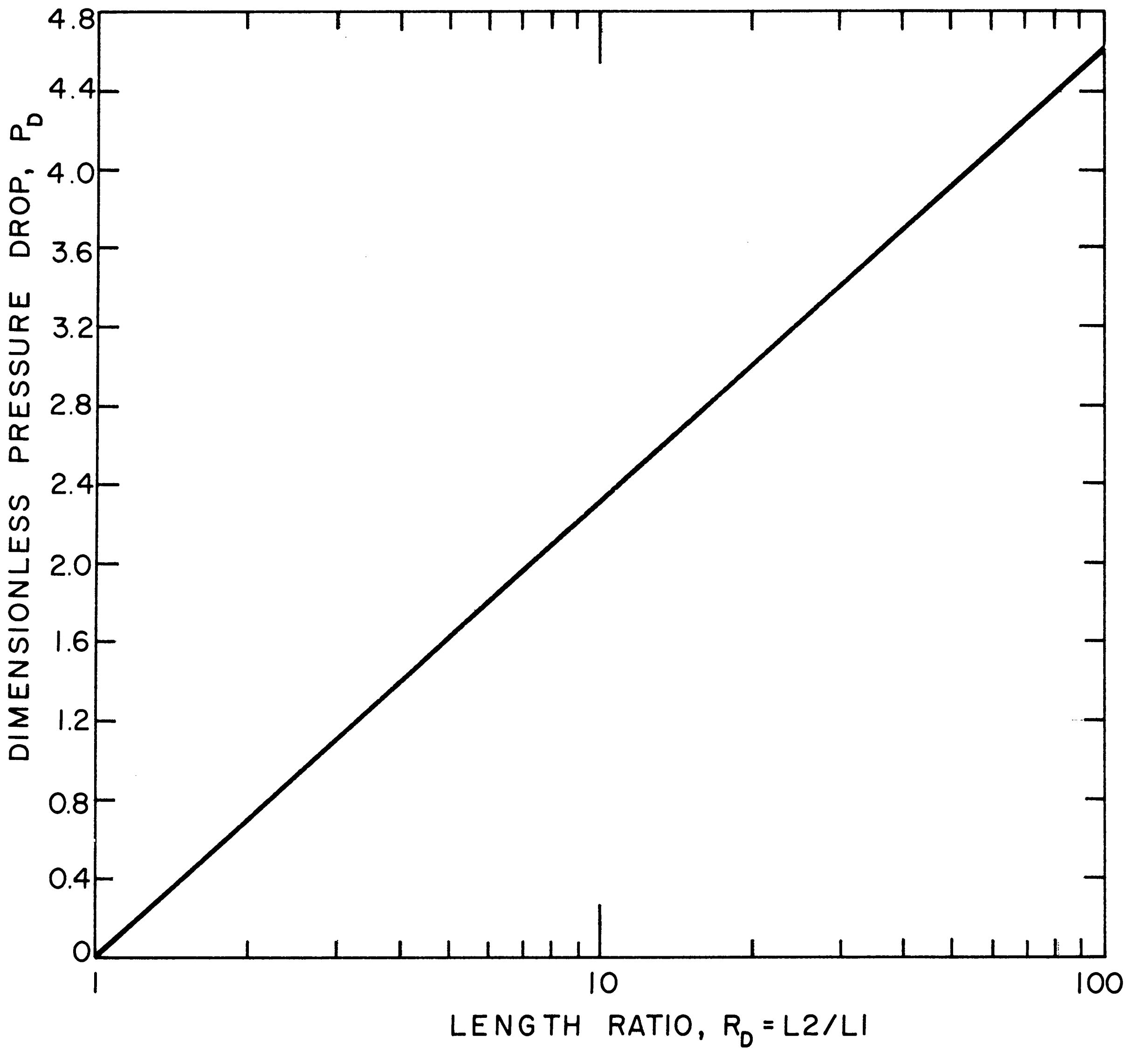

Figure IV-4. Steady State Dimensionless Pressure Drop for Radial Systems with Constant External Pressure Line.

The values for the exponential integral can be replaced by an equivalent form (19).

$$E_i(-x) = \ln x + 0.5772 + \sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n \cdot n!} \qquad \text{(IV-16)}$$

If the shut-in time is large so that $\left(\frac{\mu \phi c \ell_1^2}{4(0.00633) K \Delta t}\right)$ is less than 0.01, and the value of ℓ_2 is very large compared to ℓ_1, then

$$P_D \cong \frac{1}{2} \ln \left(\frac{t_o + \Delta t}{\Delta t}\right) = \frac{2.303}{2} \log_{10}\left(\frac{t_o + \Delta t}{\Delta t}\right) \qquad \text{(IV-17)}$$

Thus when a well in an infinite radial system is pumped, a straight line is obtained as illustrated in Figure (IV-5).

The effect of the presence of a line of constant pressure in the vicinity of the pumping and observation wells is shown in Figure (IV-6) for $R_D = 6$. Here a slope of 1.1515 is not obtained as in Figure (IV-5). Thus if ℓ_2 is not very large compared to ℓ_1, substituting Equation (IV-16) in Equation (IV-15) gives for large shut-in time

$$P_D = 0 \qquad \text{(IV-18)}$$

Hence an asymtope value of 0 is obtained for long pump tests as shown in Figure (IV-6).

2. Pressure Behavior of Pumping Well During Draw-down and Build-up Tests

The point source solution can not be used to describe the pressure behavior of a pumping well during a

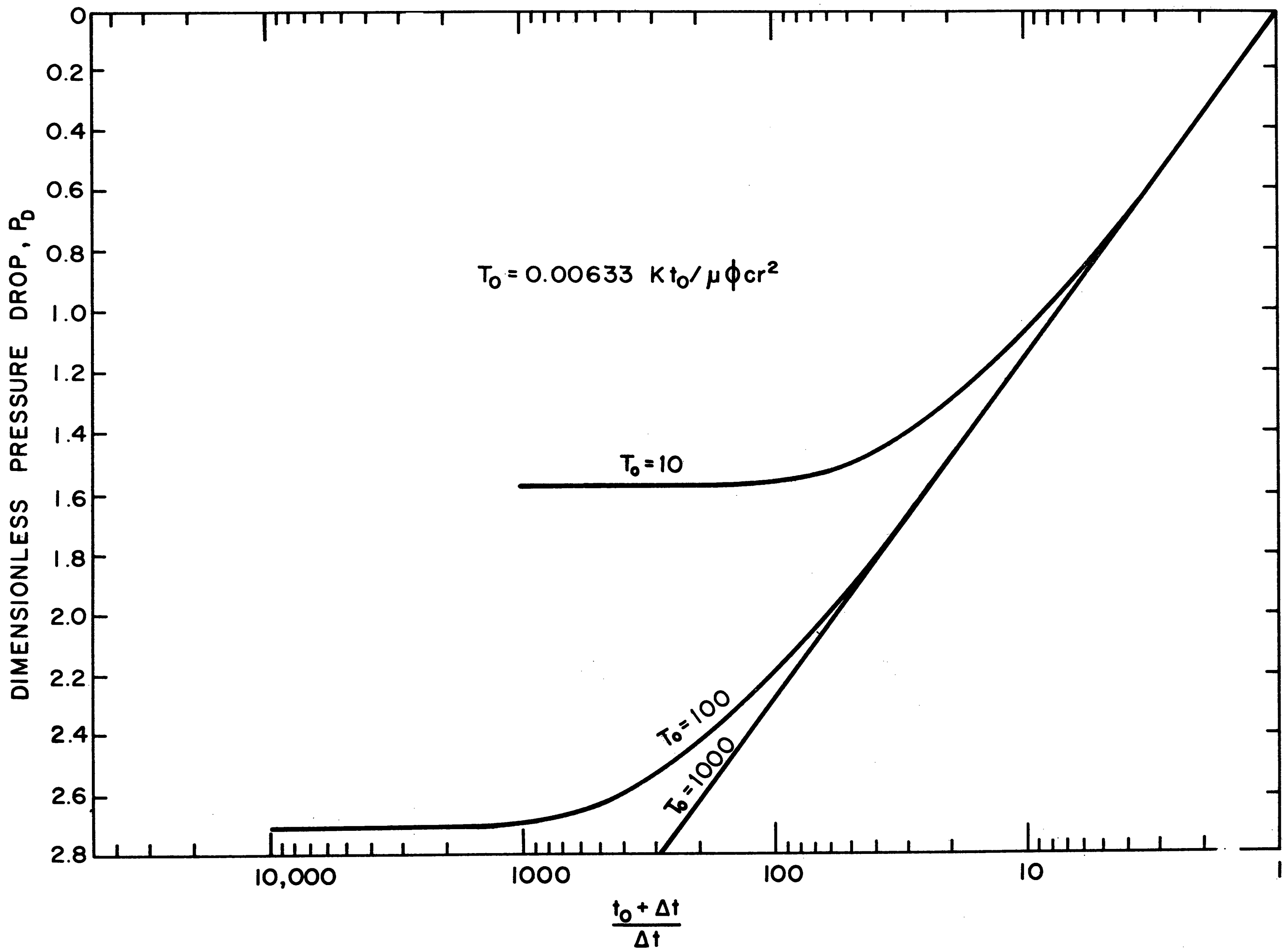

Figure IV-5. Pressure Build-Up for Observation Well in Infinite Radial System.

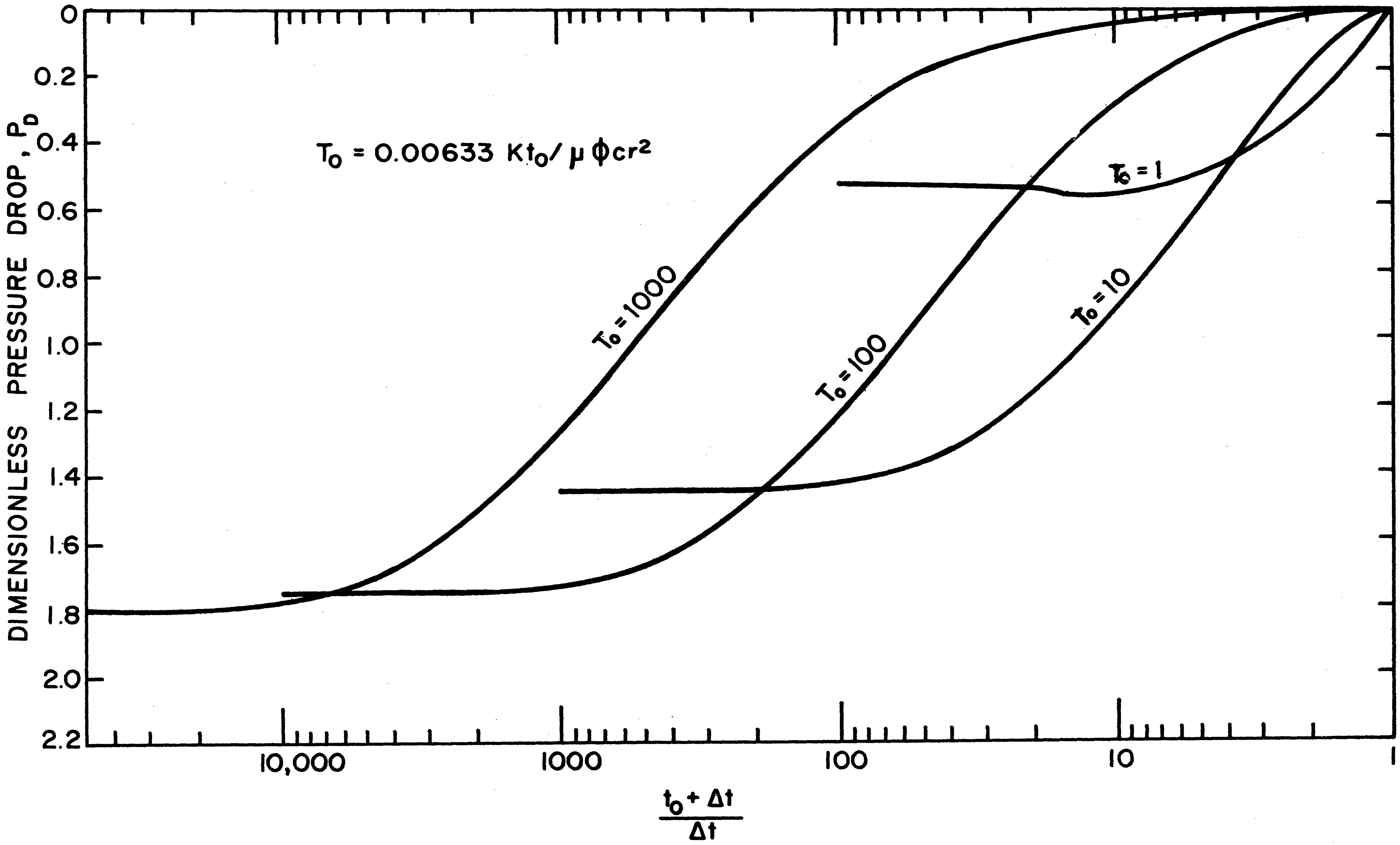

Figure IV-6. Pressure Build-Up for Observation Well in Radial System with External Line of Constant Pressure, $R_D = 6$.

drawdown test for values of dimensionless time below 1000. The dimensionless pressure drop, P_D for a pumping well is given by

$$P_D = P_t(t_D) + \frac{1}{2} Ei\left(-\frac{R_D^2}{4t_D}\right) \qquad \text{(IV-19)}$$

where:

$$t_D = \frac{0.00633\,K\,t}{\mu\,\phi\,c\,r_w^2} \qquad \text{(IV-20)}$$

$$R_D = \frac{l_2}{r_w}$$

P_t = dimensionless pressure drop for well in infinite radial system (Table 10-5, Katz et al (50), Chatas (12), Van Everdigen and Hurst (86))

The engineering units are listed below Equations (IV-12) and IV-13). For values of dimensionless time above 1000, Equation (IV-11) can be used to predict the ideal pressure behavior at the well.

Values of dimensionless pressure, P_D, versus dimensionless time, t_D, are given in Figure (IV-7).

The pressure performance of the pumping well during a build-up test near a line of constant pressure will be similar to that of a well in an infinite radial system if the pumping time is short. The build-up curve for a pumping well where R_D = 100 is shown in Figure (IV-8).

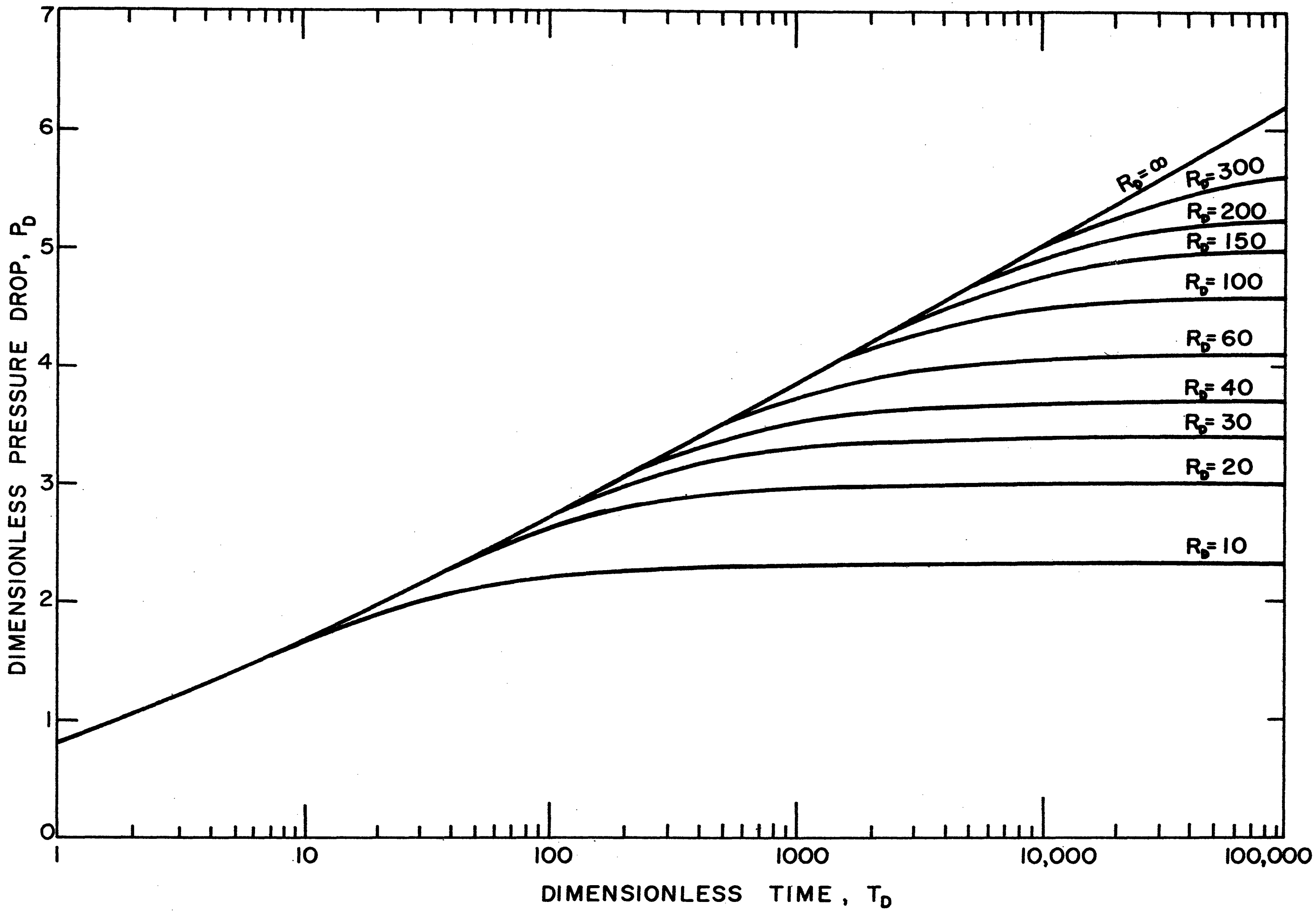

Figure IV-7. Dimensionless Pressure Drop for Pumping Well in Radial System with Constant External Pressure Line.

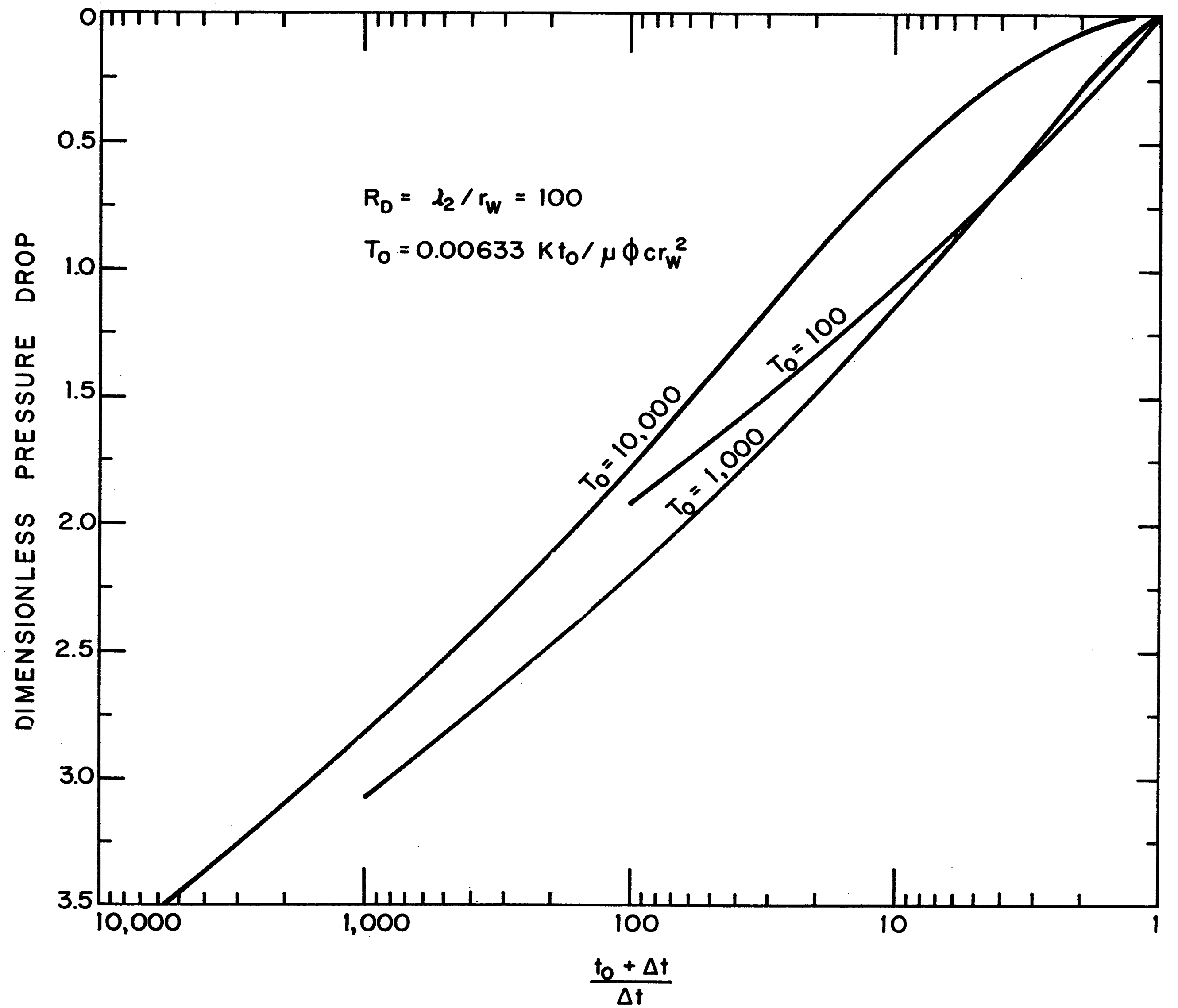

Figure IV-8. Pressure Build-Up for Pumping Well in Radial System with Constant External Pressure Line, R_D = 100.

B. <u>Evaluation of Error in the Measurement of Insitu Permeability and Insitu Compressibility</u>

The insitu permeability and insitu compressibility are obtained for infinite radial reservoirs by plotting pressure versus $\log_{10}$ pumping time. The insitu permeability can be obtained from the slope of this plot. (51) (92)

$$m = -162.6 \frac{q\mu}{Kh} \qquad \text{(IV-21)}$$

where:

m = slope of pressure vs $\log_{10}$ time, psia/cycle

q = pumping rate, bbl/day

K = permeability, millidarcys

h = reservoir thickness, feet

μ = viscosity, centipoise

Solving Equation (IV-21) for K yields

$$K = -\frac{162.6\, q\mu}{mh} \qquad \text{(IV-22)}$$

The insitu compressibility is obtained by evaluating the expression for the pressure drop

$$p_o - p = -\frac{70.6\, q\mu}{Kh} Ei\left\{-\frac{\mu \phi c \ell_1^2}{4(0.00633)Kt}\right\} \qquad \text{(IV-23)}$$

for the compressibility. The terms in Equation (IV-23) are defined below Equations (IV-12) and (IV-13). Thus solving Equation (IV-23) for c gives

$$c = -\frac{4(0.00633)Kt}{\mu \phi \ell_1^2} Ei\left\{-\frac{(p_o - p)Kh}{70.6\, q\mu}\right\} \qquad \text{(IV-24)}$$

Unfortunately, there may be considerable error in the determination of the insitu permeability and insitu compressibility if there is an external line of constant terminal pressure in the vicinity of the pumping and observation wells. The magnitude of this error will be shown below.

The numerical values for the slope of a plot of dimensionless pressure drop, P_D, versus dimensionless time, T_D, when an external line of constant pressure is present can be obtained by writing Equation (IV-11) in integral form

$$P_D = \frac{1}{2}\int_{\frac{1}{4T_D}}^{\infty} \frac{1}{u} e^{-u} du - \frac{1}{2}\int_{\frac{R_D^2}{4T_D}}^{\infty} \frac{1}{u} e^{-u} du \qquad \text{(IV-25)}$$

and differentiating with respect to $\log_{10} T_D$

$$m = \frac{d P_D}{d(\log_{10} T_D)} = \frac{2.303}{2}\left(e^{-\frac{1}{4T_D}} - e^{-\frac{R_D^2}{4T_D}}\right) \qquad \text{(IV-26)}$$

The error due to the external line of constant pressure is given by the last term in Equation (IV-26).

The error in the measurement of the insitu permeability is a minimum if the maximum value of the slope from Equation (IV-26) is used. The value of dimensionless time, T_D, when the slope, m, is a maximum for a given value of the dimensionless length ratio, R_D, is obtained by differentiating Equation (IV-26) with respect to $\log_{10} T_D$, setting the resulting expression equal to zero, and solving for T_D. Thus

$$\frac{dm}{d(\log_{10} T_D)} = \frac{(2.303)^2}{8\,T_D}\left\{e^{-\frac{1}{4T_D}} - R_D^2\, e^{-\frac{R_D^2}{4T_D}}\right\} = 0 \qquad \text{(IV-27)}$$

and

$$T_D = \frac{R_D^2 - 1}{4 \ln R_D} \qquad \text{(IV-28)}$$

where

$\ln$ = logarithm base e

The minimum error in the measurement of insitu permeability for a given value of the dimensionless length ratio, R_D, is obtained by the following steps:

1. Use Equation (IV-28) to find T_D.
2. Find the slope for this value of T_D from equation (IV-26).
3. Calculate the minimum error in K from equation (IV-29).

$$\frac{K(obs)}{K} = \frac{1}{e^{-\frac{1}{4T_D}} + e^{-\frac{R^2}{4T_D}}} \qquad \text{(IV-29)}$$

where $K(obs)$ = observed permeability, millidarcys. A plot of the ratio of observed permeability to the true permeability, $\frac{K(obs)}{K}$ versus the dimensionless length ratio, R_D, is shown in Figure (IV-9). Note that the observed permeability is always greater than the true permeability. Thus if $\frac{K(obs)}{K}$ equals 1.3, the observed permeability is 30 percent higher than the true permeability.

The error in the measurement of insitu compressibility which corresponds to the error in the measurement of insitu permeability is obtained by the following analysis.

If an external boundary of constant pressure is present, the dimensionless pressure drop is given by Equation (IV-11)

$$P_D = -\frac{1}{2}\left\{ Ei\left(-\frac{1}{4T_D}\right) - Ei\left(-\frac{R_D^2}{4T_D}\right)\right\} \qquad \text{(IV-11)}$$

In the absence of an external boundary ($R_D = \infty$), the dimensionless pressure drop is given by

$$P_D = -\frac{1}{2}\ Ei\left(-\frac{1}{4T_D}\right) \qquad \text{(IV-30)}$$

Thus if no external line of constant pressure is assumed when there is such a line, the error in the measurement of insitu compressibility is as follows. Equation (IV-30) is assumed to give the correct pressure drop when the actual pressure drop is given by Equation (IV-11), thus

$$Ei\left(-\frac{1}{4\,T_{D(obs)}}\right) = Ei\left(-\frac{1}{4T_D}\right) - Ei\left(-\frac{R_D^2}{4T_D}\right) \qquad \text{(IV-31)}$$

Solving Equation (IV-31) for $-\frac{1}{4T_{D(obs)}}$ gives

$$-\frac{1}{4T_{D(obs)}} = Ei^{-1}\left\{Ei\left(-\frac{1}{4T_D}\right) - Ei\left(-\frac{R_D^2}{4T_D}\right)\right\} \qquad \text{(IV-32)}$$

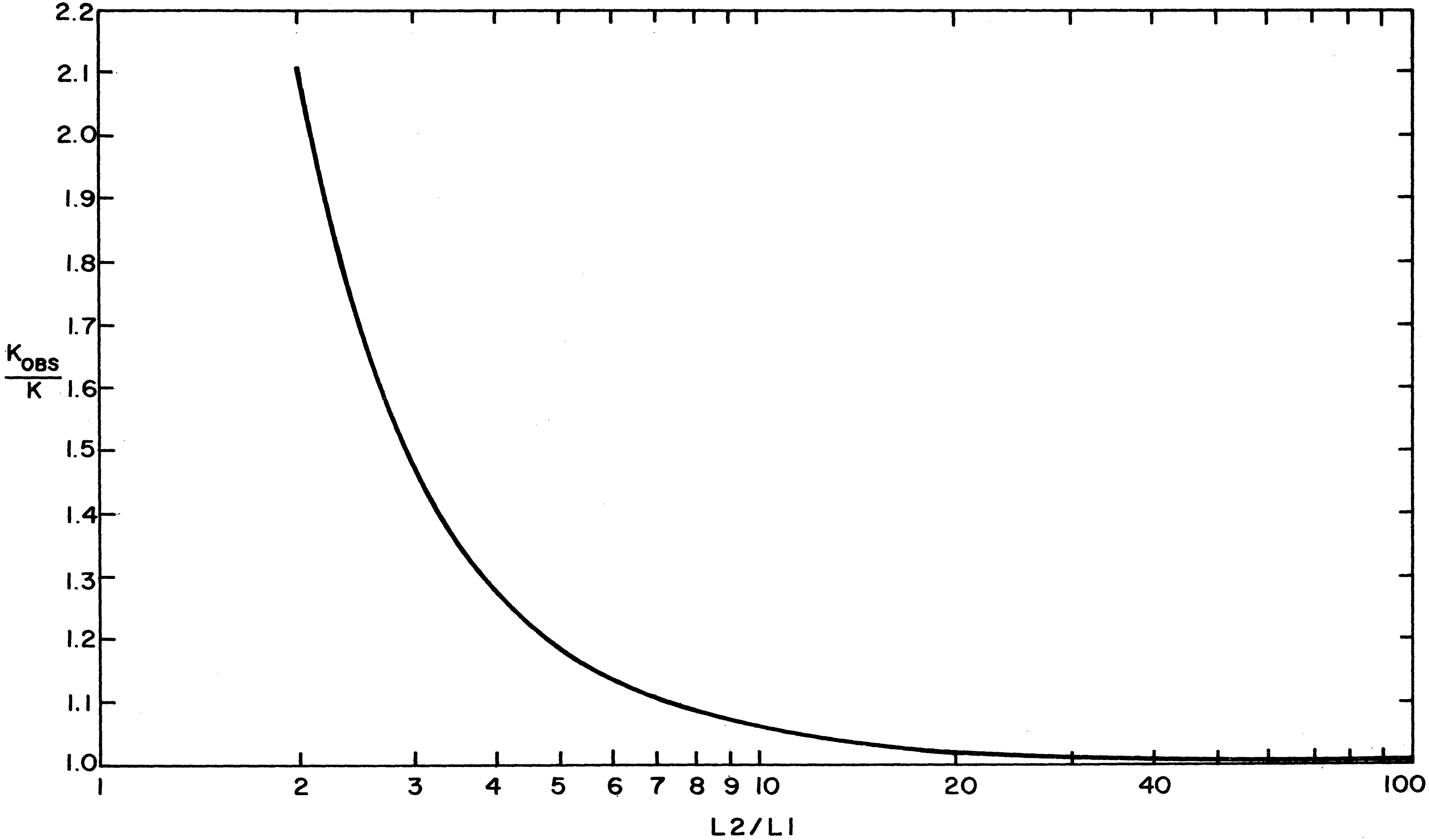

Figure IV-9. Determination of the Minimum Error in the Measurement of Insitu Permeability Resulting from an External Line of Constant Pressure.

Expand T_D(obs) yielding

$$-\frac{1}{4}\left(\frac{K}{K(obs)}\right)\left(\frac{c(obs)}{c}\right)\left(\frac{1}{T_D}\right) = Ei^{-1}\left\{Ei\left(-\frac{1}{4T_D}\right) - Ei\left(-\frac{R_D^2}{4T_D}\right)\right\} \quad \text{(IV-33)}$$

Solving Equation (IV-33) for $\frac{c(obs)}{c}$ gives

$$\frac{c(obs)}{c} = -4\left(\frac{K(obs)}{K}\right)(T_D)\, Ei^{-1}\left\{Ei\left(-\frac{1}{4T_D}\right) - Ei\left(-\frac{R_D^2}{4T_D}\right)\right\} \quad \text{(IV-34)}$$

where c(obs) is the measured value of the insitu compressibility.

The ratio of observed compressibility to the true compressibility, $\frac{c(obs)}{c}$, versus the dimensionless length ratio, R_D, is shown in Figure (IV-10). As in the case for the observed permeability, the observed compressibility is always greater than the actual compressibility.

C. Description of Graphical Method for Locating External Line of Constant Pressure (or Gas-Water Interface)

The location of the external line of constant pressure (or the gas-water interface if a large gas field is located in the vicinity of the well test) can be determined graphically if the following information is available:

1. The true effective permeability (Obtain from pressure data from the pumping well or core data)
2. The observed permeability at an observation well at a known distance from the pumping well (Obtain from pump test)

3. The observed permeability at a second observation well at a known distance from the pumping well (Obtain from pump test)

The procedure for locating the external line of constant pressure or the gas-water interface is as follows. For the purpose of discussion, assume the wells are located as shown in Figure (IV-11).

1. Calculate the value of $\frac{K(obs)}{K}$ for observation well No. 1.
2. Using the value of $\frac{K(obs)}{K}$ from step 1, read the value of R_D from Figure (IV-9).
3. If the distance from the pumping well to observation well No. 1 is "a" feet, then the distance from observation well No. 1 to the image well with respect to observation well No. 1 is (R_Da) feet. Draw a circle of radius (R_Da) around observation well No. 1.
4. Determine the locus of all points located midway between pumping well and the circle of radius (R_Da) (Shown as locus No. 1 in Figure (IV-11)).
5. Calculate the value of $\frac{K(obs)}{K}$ for observation well No. 2.
6. Using the value of $\frac{K(obs)}{K}$ from step 5, read the value of R_D for observation well No. 2.

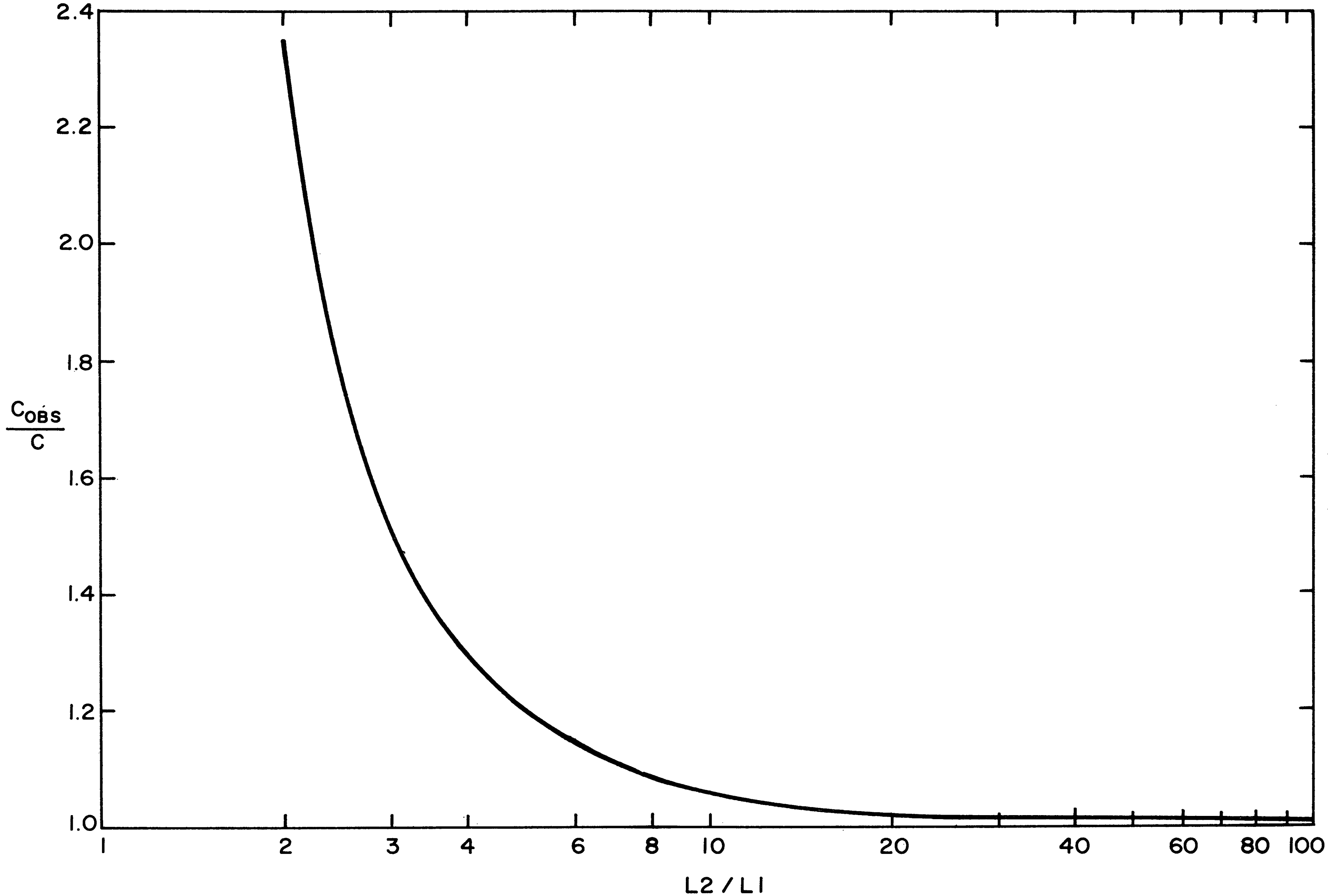

Figure IV-10. Determination of Error in Measurement of Insitu Compressibility Resulting from External Line of Constant Pressure.

7. If the distance from the pumping well to observation well No. 2 is "b" feet, the distance from observation well No. 2 to the new image well is ($R_D b$). Draw a circle of ($R_D b$) around observation well No. 2.
8. Determine the locus of all points located midway between the pumping well and the circle of radius $R_D b$. (Shown as locus No. 2 in Figure (IV-11).
9. The location of the external line of constant pressure (or the gas-water interface) is given by the intersection of locus No. 1 and locus No. 2. (This is shown as points A and B in Figure (IV-11).)
10. If the exact location is required, i.e., whether the external line of constant pressure is located at point A or point B, then, pump either observation well No. 1 or observation well No. 2, determine the $\frac{K(obs)}{K}$ at the other observation well and repeat steps No. 1 through No. 4. Locus No. 3, obtained by this method, will intercept either point A or point B.

D. <u>Example Problems</u>

Several example problems are presented to illustrate the application of the material in this section.

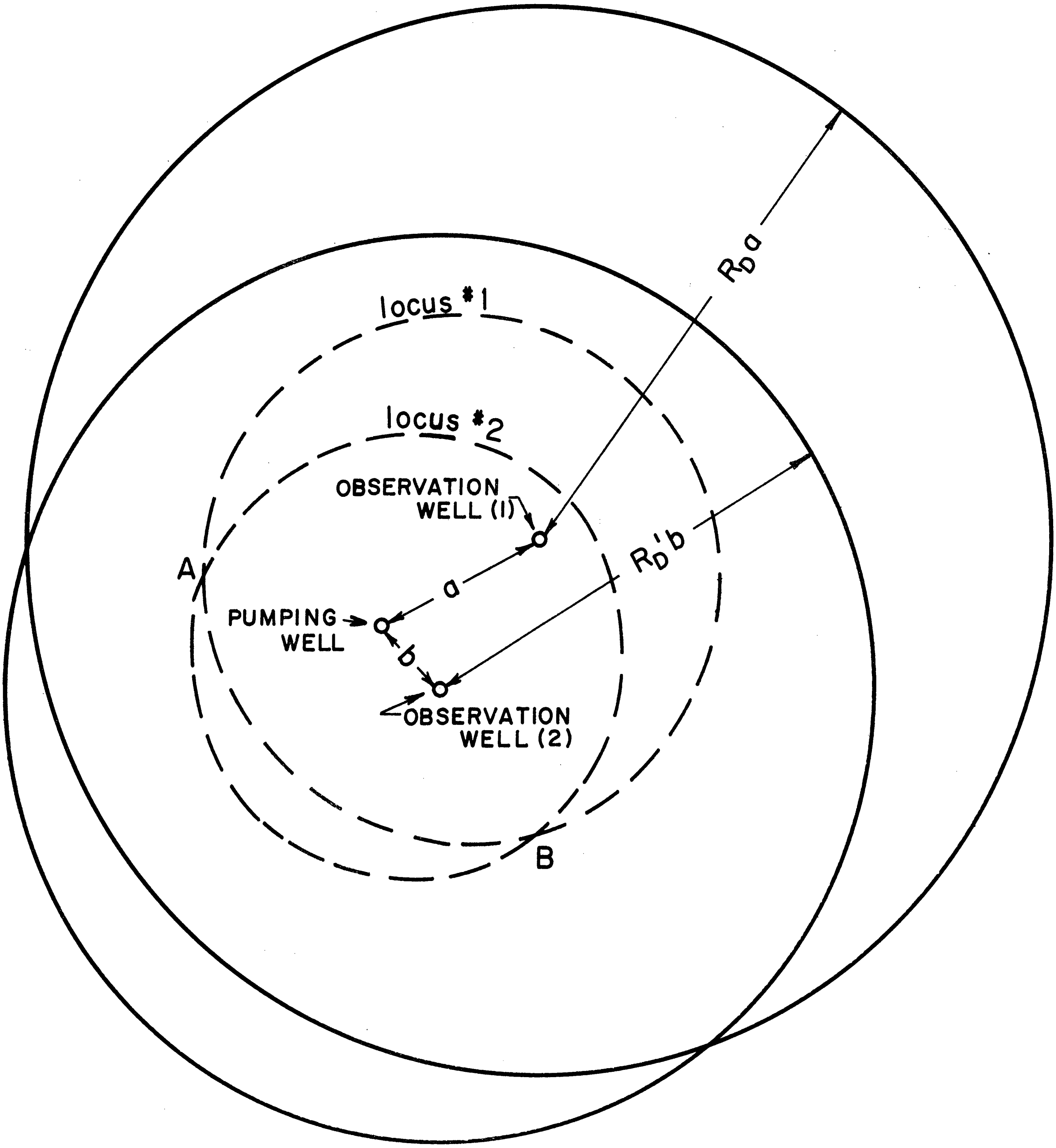

Figure IV-11. Illustration of Graphical Method for Determining Location of External Line of Constant Pressure or Gas-Water Interface.

Problem IV-1

The distance between an observation well and pumping well in an aquifer is 200 feet. The pumping well is located 300 feet from the edge of a large gas field. The locations of the wells in reference to the gas field are shown in Figure (IV-12). If the true value for the insitu permeability is 100 millidarcys, determine the insitu permeability that would be measured at the observation well.

Solution

The value of L1 and L2 are shown in Figure (IV-12).

$$L1 = 200 \text{ feet}$$

$$L2 = 824 \text{ feet}$$

Calculate the ratio

$$\frac{L2}{L1} = \frac{824}{200} = 4.12$$

and read the value of $\frac{K(obs)}{K}$ from Figure (IV-9)

$$\frac{K(obs)}{K} = 1.24$$

Thus the value of the observed permeability is

$$K(obs) = 1.24\ K$$

$$= 1.24\ (100)$$

$$= 124 \text{ millidarcys}$$

Problem IV-2

If the true value of the insitu compressibility for the aquifer described in Problem (IV-1) is 7×10^{-6} vol/(vol)(psi), determine the value of insitu compressibility that would be measured at the observation well.

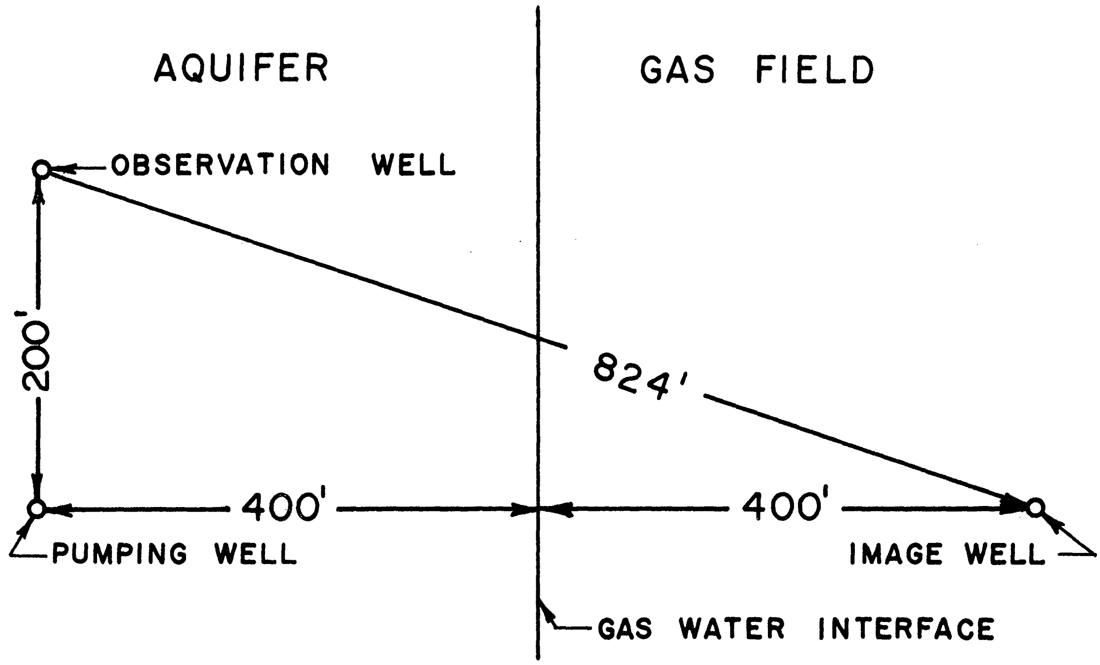

Figure IV-12. Location of Wells for Problem IV-1.

Solution

The value of $\frac{L2}{L1}$ = 4.12 is obtained from Problem (IV-1).

Read the value of $\frac{c(obs)}{c}$ from Figure (IV-10).

$$\frac{c(obs)}{c} = 1.28$$

The observed value of the insitu compressibility is

$$c(obs) = 128c$$

$$= 1.28\ (7 \times 10^{-6})$$

$$c(obs) = 8.96 \times 10^{-6}\ \text{vol/(vol)(psi)}$$

V. EFFECT OF LEAKAGE INTO THE PERMEABLE STRATA THROUGH THE CONFINING CAP AND BOTTOM ROCK ON THE INSITU PERMEABILITY AND INSITU COMPRESSIBILITY MEASUREMENTS IN AQUIFERS OR OIL FIELDS

Leakage through the confining cap and bottom rock and its effect on the pressure behavior of well tests and the resulting insitu permeability and insitu compressibility is evaluated in this section. The equations describing leakage are presented in engineering units so that comparison can be made with previous work in analysis of reservoir performance. Furthermore, the effect of leakage is shown to be similar to the effect of an external line of constant pressure discussed in the previous chapter. Thus it may be difficult or even impossible to differentiate between the two effects by a single well test. Pressure behavior for both drawdown pump tests and build-up pump tests are presented. Example problems at the end of the section illustrate the application of this material to field data.

Hantush (28) (31) and Hantush and Jacob (25) examined the problem of leakage during pump tests in underground aquifers. Their results are presented in terms of transmissibility, coefficients of storage, and well functions; terms used by hydrologists. These results are transformed into engineering terms such as dimensionless pressure drop, dimensionless time, permeability, compressibility, viscosity, and porosity so that their work can be compared with previous accomplishments in the field. The superposition principle is used to extend these results to build-up pump tests.

A. Prediction of Pressure Behavior During Drawdown Tests

The differential equation describing the flow of slightly compressible fluids in a porous media with leakage (25) is given by

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} - \frac{p-p_o}{B^2} = \frac{\mu \phi c}{0.00633K}\frac{\partial p}{\partial t} \qquad \text{(V-1)}$$

The initial condition is

$$p(r, 0) = p_o \qquad \text{(V-2)}$$

and the boundary conditions are

$$p(\infty, t) = p_o \qquad t \geq 0 \qquad \text{(V-3)}$$

and

$$\lim_{r \to 0} r\frac{\partial p}{\partial r} = -\frac{q\mu}{141.2\,Kh} \qquad \text{(V-4)}$$

where:

$B = \sqrt{Kh/\left(\frac{K'}{h'} + \frac{K''}{h''}\right)}$, feet (V-4a)

c = compressibility, vol/(vol)(psia)

h = thickness of permeable zone, feet

h' = thickness of caprock, feet

h'' = thickness of bottom rock, feet

K = permeability, millidarcys

K' = permeability of caprock, millidarcys

K'' = permeability of bottom rock, millidarcys

p = pressure, psia

p_o = initial pressure, psia

q = flow rate, bbl/day

r = distance from center of pumping well to point of pressure measurement, feet

t = time, days

ϕ = porosity, fraction

μ = viscosity, centipoise

The solution to Equations (V-1) through (V-4) is given by Hantush and Jacob (25) as

$$\frac{p - p_o}{\left(\frac{q\mu}{282.4Kh}\right)} = 2K_o\left(\frac{r}{B}\right) - I_o\left(\frac{r}{B}\right)\left[Ei\left(-\frac{r^2}{4B^2u}\right)\right] + \exp\left(-\frac{r^2}{4B^2u}\right)\Bigg\{0.5772 + \ln u + \left[Ei(-u)\right] - u + u\left[I_o\left(\frac{r}{B}\right) - 1\right] \Big/ \left(\frac{r^2}{4B^2}\right) - u^2 \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{(-1)^{n+m}(n-m+1)!}{(n+2)!^2}\left(\frac{r^2}{4B^2}\right)^m u^{n-m}\Bigg\} \quad \text{(V-5)}$$

where:

u = well function, $u = \frac{1}{4t_D}$

I_o = Modified Bessel function, first kind, zeroth order

K_o = Modified Bessel function, second kind, zeroth order

$t_D = 0.00633Kt/\mu\phi c r^2$

Equation (V-5) written in terms of engineering units is

$$P_D = 4\,K_0\left(\frac{r}{B}\right) - 2\,I_0\left(\frac{r}{B}\right)\left[-Ei\left(-\frac{r^2 t_D}{B^2}\right)\right]$$

$$+2\left[\exp\left(-\frac{r^2 t_D}{B^2}\right)\right]\Bigg\{0.5772 - \ln(4t_D)$$

$$+\left[-Ei\left(-\frac{1}{4t_D}\right)\right] - \frac{1}{4t_D} + \frac{1}{4t_D}\,\frac{I_0\left(\frac{r}{B}\right)-1}{\left(\frac{r^2}{4B^2}\right)} \qquad \text{(V-6)}$$

$$-\frac{1}{16\,t_D}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{(-1)^{n+m}(n-m+1)!}{(n+2)!^2}\left(\frac{r^2}{4B^2}\right)^m\left(\frac{1}{4t_D}\right)^{n-m}\Bigg\}$$

where

$$P_D = \frac{141.2\,(p - p_0)\,Kh}{q\,\mu} \qquad \text{(V-6a)}$$

The dimensionless pressure drop, P_D, is shown as a function of dimensionless time, t_D, with parameters of $\left(\frac{r}{B}\right)$ in Figure (V-1). Note that the general pressure behavior is similar to that observed with the external line of constant terminal pressure, Figure (IV-3). Extensive tables of u versus $\left(\frac{r}{B}\right)$ are given by Hantush (28).

For large values of dimensionless time, t_D, a steady state pressure drop is obtained. This pressure drop in dimensionless terms is shown in Figure (V-2) as a function of $\left(\frac{r}{B}\right)$.

B. Prediction of Pressure Behavior During Build-up Tests

The pressure response for build-up well tests is obtained by superimposing a negative flow rate of the same

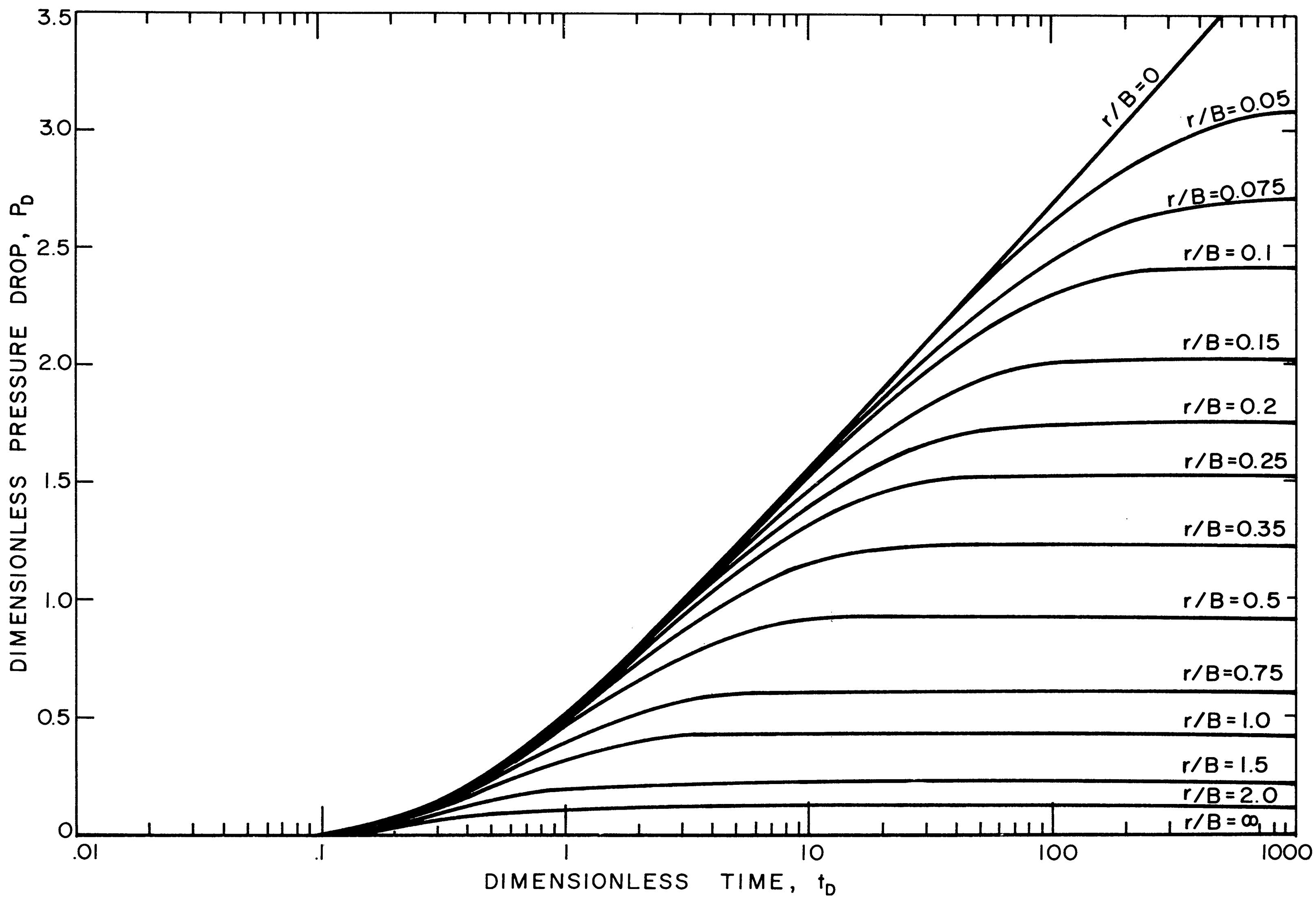

Figure V-1. Dimensionless Pressure Drop for Radial System with Leakage through Caprock and/or Bottom Rock.

magnitude on the drawdown test after the pumping well is shut-in. Thus the pressure behavior during build-up can be predicted by

$$
\begin{aligned}
P_D = {} & 4 K_o\left(\frac{r}{B}\right) - 2 I_o\left(\frac{r}{B}\right)\left[Ei\left(\frac{-r^2 t_D}{B^2}\right)\right] \\
& + 2\left[\exp\left(\frac{-r^2 t_D}{B^2}\right)\right]\Bigg\{0.5772 - \ln(4t_D) \\
& + \left[-Ei\left(-\frac{1}{4t_D}\right)\right] - \frac{1}{4t_D} + \frac{1}{4t_D}\,\frac{I_o\left(\frac{r}{B}\right)-1}{\left(\frac{r^2}{4B^2}\right)} \\
& - \frac{1}{16 t_D^2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{(-1)^{n+m}(n-m+1)!}{(n+2)!2}\left(\frac{r^2}{4B^2}\right)^m\left(\frac{1}{4t_D}\right)^{n-m}\Bigg\} \\
& - 4 K_o\left(\frac{r}{B}\right) + 2 I_o\left(\frac{r}{B}\right)\left[Ei\left(\frac{-r^2 t_D'}{B^2}\right)\right] \qquad \text{(V-7)} \\
& - 2\left[\exp\left(\frac{-r^2 t_D'}{B^2}\right)\right]\Bigg\{0.5772 - \ln(4t_D') \\
& + \left[-Ei\left(-\frac{1}{4t_D'}\right)\right] - \frac{1}{4t_D'} + \frac{1}{4t_D'}\,\frac{I_o\left(\frac{r}{B}\right)-1}{\left(\frac{r^2}{4B^2}\right)} \\
& - \frac{1}{16(t_D')^2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{(-1)^{n+m}(n-m+1)!}{(n+2)!2}\left(\frac{r^2}{4B^2}\right)^m\left(\frac{1}{4t_D'}\right)^{n-m}\Bigg\}
\end{aligned}
$$

where:

$$t_D' = \frac{0.00633\ K\Delta t}{\mu \phi c r^2} \qquad \text{(V-8)}$$

$$t_D = \frac{0.00633\ K t}{\mu \phi c r^2} \qquad \text{(V-9)}$$

Δt = time since well shut-in, days

t = time since pump test started, days

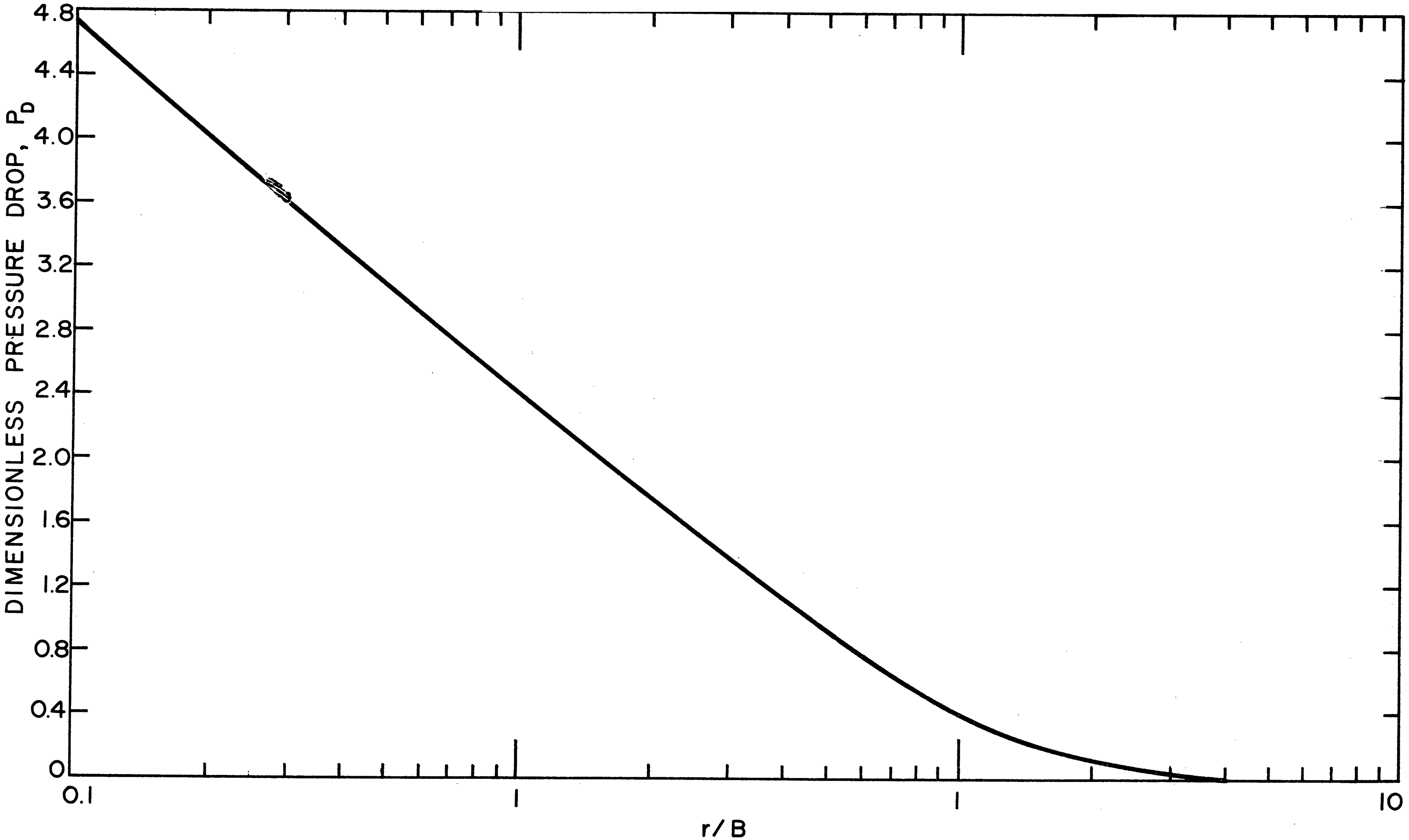

Figure V-2. Steady State Dimensionless Pressure Drop for Radial Systems with Leakage through Confining Caprock and/or Bottom Rock.

The effect of leakage in build-up tests is shown in Figure (V-3) for $\frac{r}{B} = 0.2$. Comparison of Figure (V-3) with Figure (IV-6) shows that the effect of leakage on the pressure drop is very similar to the effect of an external line of constant pressure. Thus additional observation wells are necessary to differentiate between the two effects.

C. Evaluation of Error in the Measurement of Insitu Permeability and Insitu Compressibility

Standard methods used to obtain insitu permeability and insitu compressibility are described in Section II.

The numerical value of the slope if leakage is occurring from a plot of dimensionless pressure drop versus dimensionless time is obtained by expressing Equation (V-6) in integral form (28)

$$P_D = \frac{1}{2}\int_{\frac{1}{4t_D}}^{\infty} \frac{1}{u} \exp\left(-u - \frac{r^2}{4B^2u}\right) du \qquad \text{(V-10)}$$

and differentiating with respect to $\log_{10} t_D$

$$m = \frac{dP_D}{d(\log_{10} t_D)} = \frac{2.303}{2} e^{-\left(\frac{1}{4t_D} + \frac{r^2 t_D}{B^2}\right)} \qquad \text{(V-11)}$$

If $B = \infty$ (no leakage), then the slope is

$$m = \frac{2.303}{2} e^{-\frac{1}{4t_D}} \qquad \text{(V-12)}$$

which is the expression for the slope of radial flow in an infinite, radial, porous media.

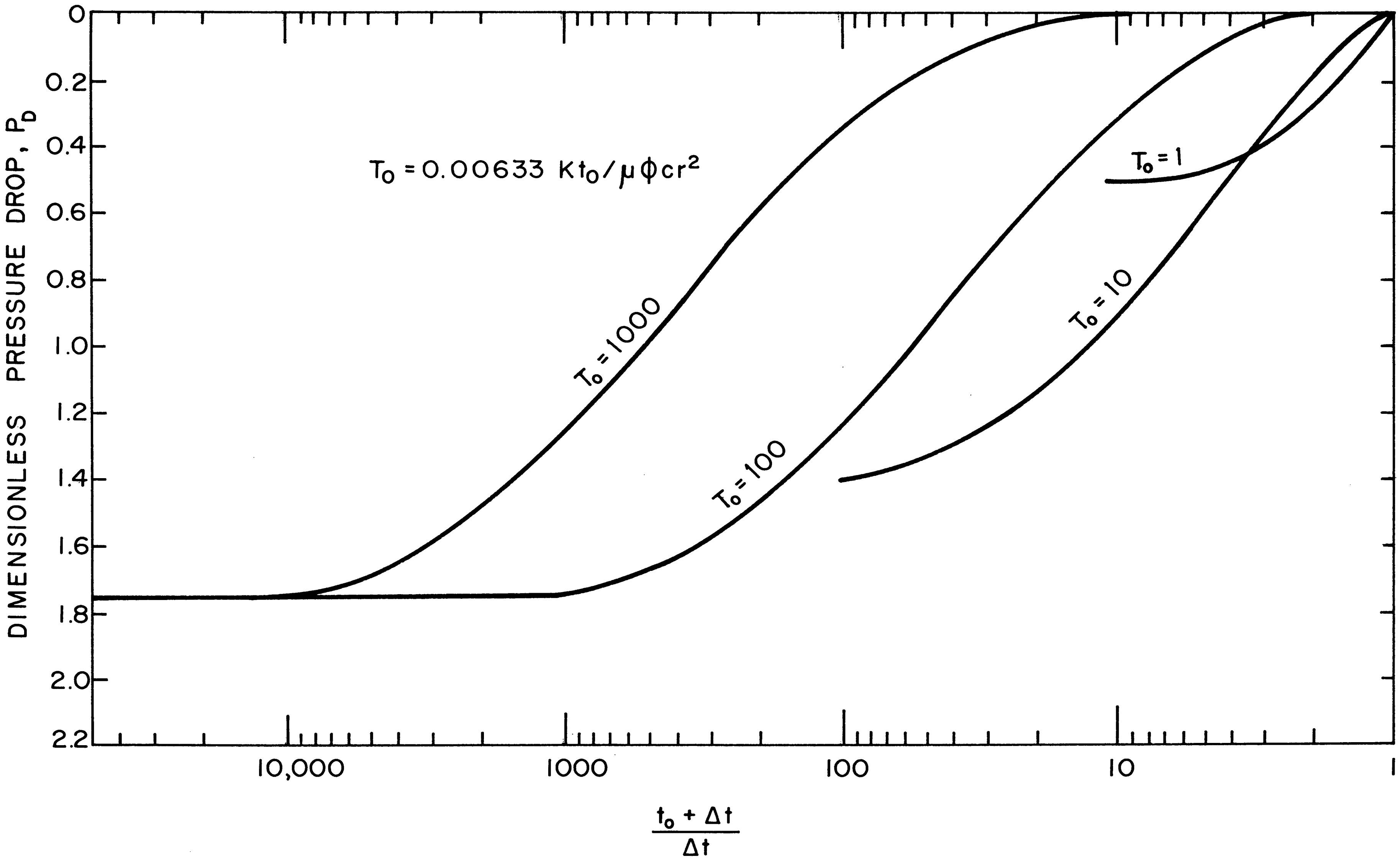

Figure V-3. Pressure Build-Up for Observation Well in Radial System with Leakage, $\frac{r}{B} = 0.2$.

The minimum error in the measurement of insitu permeability is obtained by using the slope at the point of inflection as shown by Figure (V-1). The value of dimensionless pressure at this point is

$$t_D = \frac{1}{2(\frac{r}{B})} \qquad \text{(V-13)}$$

Substitution of Equation (V-13) in Equation (V-11) gives the slope at the point of inflection

$$m_{max} = \frac{2.303}{2} e^{-\frac{r}{B}} \qquad \text{(V-14)}$$

Thus the ratio of the observed insitu permeability to the true insitu permeability at the point of inflection is given by

$$\frac{K(obs)}{K} = \frac{1}{e^{-\frac{r}{B}}} = e^{\frac{r}{B}} \qquad \text{(V-15)}$$

The values of $\frac{K(obs)}{K}$ versus $\frac{r}{B}$ obtained at the point of inflection are shown in Figure (V-4).

The pressure drawdown at the inflection point defined by Equation (V-13) is

$$P_D = \frac{1}{2} K_o \left(\frac{r}{B}\right) \qquad \text{(V-16)}$$

If no leakage is occurring, the pressure at the inflection point is

$$P_D = -\frac{1}{2} Ei\left\{-\frac{1}{4t_D}\right\} \qquad \text{(V-17)}$$

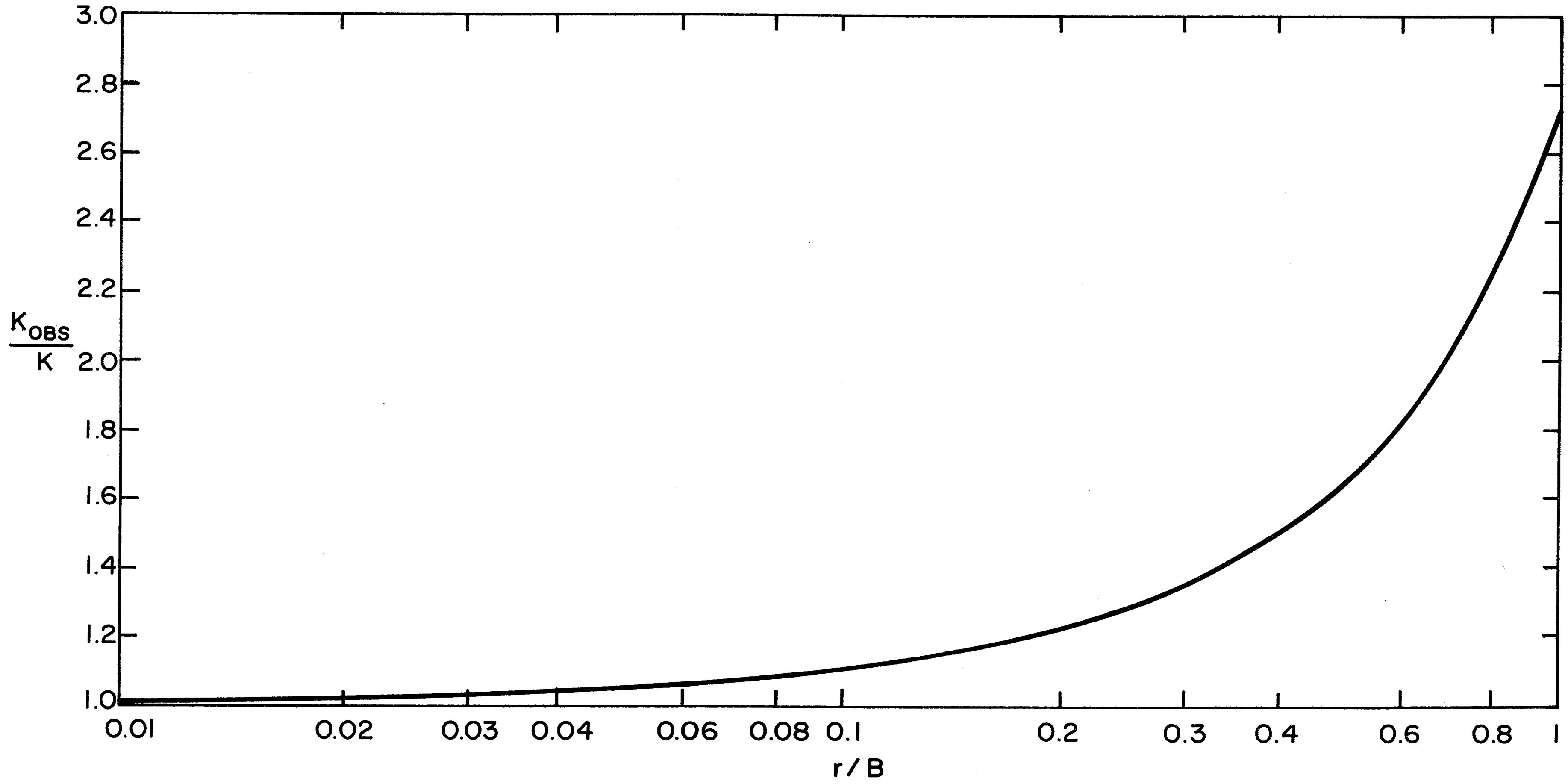

Figure V-4. Ratio of Observed Permeability to Actual Permeability versus $\frac{r}{B}$ for Radial System with Leakage.

Setting Equation (V-16) equal to Equation (V-17) gives

$$\frac{1}{2} K_o\left(\frac{r}{B}\right) = -\frac{1}{2} Ei\left(-\frac{1}{4t_{D(obs)}}\right) \qquad \text{(V-18)}$$

Solving for $\frac{1}{t_{D(obs)}}$ gives

$$\frac{1}{4t_{D(obs)}} = -Ei^{-1}\left\{-K_o\left(\frac{r}{B}\right)\right\} \qquad \text{(V-19)}$$

and hence

$$\frac{c\,(obs)}{c}\;\frac{K}{K\,(obs)}\;\frac{1}{t_D} = -4\,Ei^{-1}\left\{-K_o\left(\frac{r}{B}\right)\right\} \qquad \text{(V-20)}$$

Solving Equation (V-20) for $\frac{c\,(obs)}{c}$ gives

$$\frac{c\,(obs)}{c} = \left(\frac{K\,(obs)}{K}\right)\left(4t_D\right)\left(-Ei\left\{-K_o\left(\frac{r}{B}\right)\right\}\right) \qquad \text{(V-21)}$$

Thus at the point of inflection, the ratio of the observed compressibility to the actual compressibility is given by

$$\frac{c\,(obs)}{c} = \frac{2e^{\frac{r}{B}}}{\left(\frac{r}{B}\right)}\left[-Ei\left\{-K_o\left(\frac{r}{B}\right)\right\}\right] \qquad \text{(V-22)}$$

Values of the ratio of observed compressibility to the actual compressibility $\frac{r}{B}$ using the slope obtained at the point of inflection in a plot of dimensionless pressure drop versus $\log_{10}$ time is shown in Figure (V-5).

D. Example Problems

Two examples are given to illustrate the application of methods developed in this section.

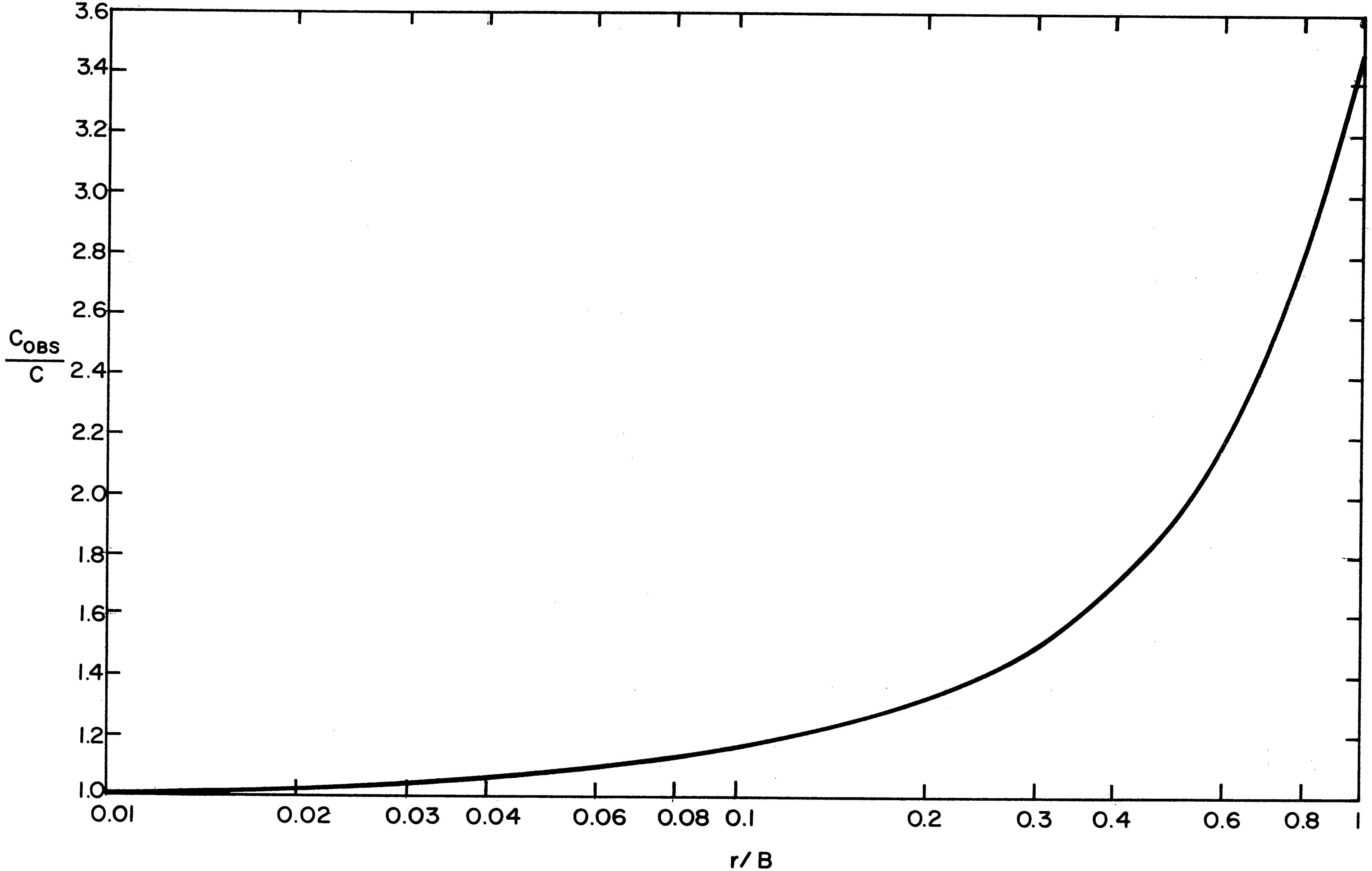

Figure V-5. Ratio of Observed Compressibility to Actual Permeability versus $\frac{r}{B}$ Measured at Inflection Point for Radial System with Leakage.

Problem V-1

A well completed in an aquifer is pumped at a rate of 40 gal/min. The aquifer is 20 feet thick and has a permeability of 200 millidarcys. The caprock is 5 feet thick and has a permeability of 0.001 millidarcys. The bottom rock is 10 feet thick and has a permeability of 0.5 millidarcys. If the pressure is measured at an observation well 141 feet from the pumping well, determine the value of insitu permeability and insitu compressibility if the maximum slope of the drawdown curve is used and the pressure drop at the point of inflection is used to calculate the insitu compressibility. Assume the true compressibility of the formation is 8×10^{-6} vol/(vol)(psi).

Solution

Calculate the value of B using Equation (V-4a)

$$B = \sqrt{Kh/\left(\frac{K'}{h'} + \frac{K''}{h''}\right)} = \sqrt{\frac{(200)(20)}{\left(\frac{0.001}{5} + \frac{0.5}{10}\right)}} = 282 \text{ feet}$$

Thus

$$\frac{r}{B} = \frac{141}{282} = 0.5$$

Use Figure (V-4) or Equation (V-15) to find $\frac{K(obs)}{K}$ for $\frac{r}{B} = 0.5$

$$\frac{K(obs)}{K} = 1.65$$

$$K(obs) = 1.65\ (200)$$

$K(obs) = 330$ millidarcys, the observed permeability

Use Figure (V-5) to find $\frac{c(obs)}{c}$ for $\frac{r}{B} = 0.5$

$$\frac{c(obs)}{c} = 1.92$$

$$c(obs) = 1.92\ (8 \times 10^{-6})$$

$$c(obs) = 1.536 \times 10^{-5}\ \text{vol/(vol)(psi)}$$

This example shows that if the bottom rock is permeable, the error in the insitu permeability and insitu compressibility can be large.

Problem V-2

An 92 foot thick aquifer is being considered for storage of natural gas. Core data shows that a 20 foot caprock with a permeability of 0.02 millidarcys lies above the aquifer. The core data also showed a 10 foot, tight sandstone below the aquifer has a permeability of 1 millidarcy. A well was pumped at 10 gal/min. Pressure data obtained from an observation well 100 feet from the pumping well was used to measure the insitu permeability. If the measured value of the insitu permeability is 165 millidarcys, what is the true permeability of the aquifer?

Solution

The true value of the permeability must be found by trial and error.

Trial No. 1

Assume $\frac{r}{B} = 0.5$

Use Figure (V-4) or Equation (V-15) to find $\frac{K(obs)}{K}$

$$\frac{K(obs)}{K} = 1.65$$

Thus

$$B = \frac{r}{0.5} = \frac{100}{0.5} = 200$$

$$B^2 = 40{,}000$$

and

$$K = \frac{K\,(obs)}{1.65} = \frac{165}{1.65} = 100 \text{ millidarcys}$$

Check value of K using Equation (V-4a)

$$\frac{Kh}{\left(\frac{K'}{h'} + \frac{K''}{h''}\right)} = 40{,}000$$

$$K = \frac{40{,}000}{h}\left(\frac{K'}{h'} + \frac{K''}{h''}\right)$$

$$= \frac{40{,}000}{92}\left(\frac{0.02}{20} + \frac{1}{10}\right)$$

$$= 44 \text{ millidarcys}$$

Since the two values of K do not agree, it is necessary to assume a new value of $\frac{r}{B}$ and repeat the calculations.

<u>Trial</u> <u>No.</u> <u>2</u>

Assume $\frac{r}{B} = 0.3$

Use Figure (V-4) or Equation (V-15) to find $\frac{K\,(obs)}{K}$

$$\frac{K\,(obs)}{K} = 1.35$$

Thus

$$B = \frac{r}{0.3} = \frac{100}{0.3} = 333$$

$$B^2 = 111{,}000$$

and

$$K = \frac{K(obs)}{1.35} = \frac{165}{1.35} = 122 \text{ millidarcys}$$

This value is checked using Equation (V-4a)

$$Kh = 111,000 \left(\frac{K'}{h'} + \frac{K''}{h''} \right)$$

$$K = \frac{111,000}{92} \left(\frac{0.02}{20} + \frac{1}{10} \right)$$

$$= 122 \text{ millidarcys}$$

Since the two values of K agree, K = 122 millidarcys is the correct value for the permeability.

VI. INTERPRETATION OF FIELD WELL TEST DATA

The pressure behavior of one gas field and two aquifers are analyzed in this section. Replacement of p by p^2 in the equations for flow of slightly compressible fluids to describe the flow of gas when the pressure drawdown is less than ten percent is justified. Analysis of well drawdown and build-up tests in the two aquifers will illustrate many problems involved in the interpretation of reservoir pressure data.

A. Field A

Pressure data from a drawdown test on a gas well completed in the deep Frio trend of Southwest Texas was presented by Accord (1). These data are analyzed using a procedure similar to the procedure for analyzing the flow of slightly compressible liquids. Since the pressure drop was less than 10 percent of the initial pressure, the equations for a slightly compressible fluid can be used if p is replaced by p^2 (17) (50) (92). Justification of this procedure is given below:

1. Approximate Equations Describing Gas Flow in Porous Media for Constant Rate Tests

Darcys law for the radial flow of gas in porous media is given by

$$q = \frac{KA}{\mu} \frac{dp}{dr} \qquad \text{(VI-1)}$$

where:

A = cross sectional area normal to flow, square centimeters

q = gas flow rate, flow conditions, cc/sec

K = permeability, darcys

p = pressure, atmospheres

r = radial distance, centimeters

μ = viscosity, centipoise

The gas law can be used to convert the gas flow rate to standard conditions. Thus

$$q = Q_G \, z \left(\frac{p_b}{p}\right)\left(\frac{T}{T_b}\right) \tag{VI-2}$$

where:

P_b = pressure base for gas measurement, atmospheres

Q_G = flow rate at standard condition, cc/sec

T = temperature at flow conditions, °K

T_b = temperature base for gas measurement, °K

z = gas compressibility factor, flow condition

The area normal to flow at a given radius is

$$A = 2\pi r h \tag{VI-3}$$

where:

h = reservoir thickness, centimeters

Substituting Equations (VI-2) and (VI-3) in Equation (VI-1) and rearranging gives

$$Q_G = \frac{\pi K T_b r h}{\mu T p_b} \frac{2 \bar{z} p dp}{dr} \qquad \text{(VI-4)}$$

which simplifies to

$$\frac{dp^2}{dr} = \frac{p_b}{\pi T_b} \frac{\bar{\mu} \bar{z} T Q_G}{h K r} \qquad \text{(VI-5)}$$

The boundary condition at the well bore is given by

$$\left. \frac{dp^2}{dR_D} \right|_{R_D=1} = \frac{p_b}{\pi T_b} \frac{\bar{\mu} \bar{z} T Q_G}{h K} \qquad \text{(VI-6)}$$

where

$$R_D = \frac{r}{r_w} \qquad \text{(VI-7)}$$

The diffusivity equation for the flow of gas in a porous media, derived by Katz et al (50), is

$$\frac{\partial^2 p^2}{\partial R_D^2} + \frac{1}{R_D} \frac{\partial p^2}{\partial R_D} = \frac{\mu \phi r^2}{K p} \frac{\partial p^2}{\partial t} \qquad \text{(VI-8)}$$

The initial condition is

$$p\,(R_D, 0) = p_o \,, \qquad R_D \geqq 0 \qquad \text{(VI-9)}$$

or

$$p^2\,(R_D, 0) = p_o^2 \,, \qquad R_D \geqq 0 \qquad \text{(VI-10)}$$

The remaining boundary condition is

$$\lim_{r \to \infty} p = p_o \qquad t \geqq 0 \qquad \text{(VI-11)}$$

or

$$\lim_{r \to \infty} p^2 = p_o^2 \qquad t \geq 0 \qquad \text{(VI-12)}$$

Define dimensionless pressure and dimensionless time for the flow of gas as follows

$$(P_D)_{gas} = \frac{\pi (p_o^2 - p^2) T_b h K}{p_b \bar{\mu} \bar{z} T Q_G} \qquad \text{(VI-13)}$$

$$(t_D)_{gas} = \frac{2 K p t}{\mu \phi r^2} \qquad \text{(VI-14)}$$

Substituting Equations (VI-13) and (VI-14) in Equations (VI-8), (VI-6), (VI-10), and (VI-12) yield respectively

$$\frac{\partial^2 (P_D)_{gas}}{\partial P_D} + \frac{1}{R_D} \frac{\partial (P_D)_{gas}}{\partial R_D} = \frac{\partial (P_D)_{gas}}{\partial (t_D)_{gas}} \qquad \text{(VI-15)}$$

$$\left. \frac{\partial (P_D)_{gas}}{\partial R_D} \right|_{R_D=1} = -1 \qquad \text{(VI-16)}$$

$$P_D (R_D, 0) = 0 \qquad \text{(VI-17)}$$

$$\lim_{R_D \to \infty} (P_D)_{gas} = 0 \qquad (t_D)_{gas} \geqq 0 \qquad \text{(VI-18)}$$

Equations (VI-15), (VI-16), (VI-17), and (VI-18) are identical to the equations of flow of a slightly compressible fluid except that p is replaced by p^2.

If p^2 is plotted versus $\log_{10} t$ and engineering units are used, the slope of the drawdown curve is given by (50)

$$m = -\frac{1,424 \bar{\mu} \bar{z} T Q_G}{h K} \qquad \text{(VI-19)}$$

where:

h = reservoir thickness, feet

K = permeability, millidarcys

m = slope, $(psi)^2$ /cycle

p_o = initial pressure, psia

Q_G = gas flow rate, SCF/day, for T_b = 60°F, p_b = 14.7 psia

T = reservoir temperature, °R

$\bar{z}$ = average gas compressibility

$\bar{\mu}$ = average gas viscosity, centipoise

The error in the application of this method is due to the change in p given in Equation (VI-14).

2. Analysis of Pressure Drawdown Data

The pressure drawdown data and values of p^2 are given in Table (VI-1). Note that the pressure drop in this test is less than 10 percent of the initial pressure.

Other reservoir data needed to analyze the pump test data are:

Average viscosity, $\bar{\mu}$ = 0.0362 centipoise

Average gas compressibility factor, $\bar{z}$ = 1.3

Reservoir temperature, T = 729 °R

Reservoir thickness, h = 8 feet

The gas flow rate for the pressure drawdown test is 600 SCF/day.

The slope over the initial drawdown of a plot of p^2 versus $\log_{10} t$ in Figure (VI-1) is used to calculate the insitu permeability. The sharp changes in the slope over the latter history reflects the effect of reservoir faults as discussed in Section II.

Thus using

$$m = -1{,}300{,}000 \text{ psi}^2/\text{cycle}$$

the insitu permeability to gas is calculated by rearranging Equation (VI-19)

$$K = \frac{-1424\,\bar{\mu}\,\bar{z}\,T\,Q_G}{h\,m} \qquad \text{(VI-20)}$$

$$K = \frac{-1424\,(0.0362)(1.3)(729)(600)}{8\,(-1{,}300{,}000)} = 2.9 \text{ md}$$

It is interesting to note that when the question for a slightly compressible fluid was applied to this data directly by Accord (1), a value of 3.0 millidarcys was obtained for the insitu permeability to gas.

TABLE VI-1

PRESSURE DATA FOR FLOW TEST ON GAS WELL IN FIELD A (ACCORD (1))

Flow Time Hours	Pressure, psia (well-bottom)	$(\text{Pressure})^2$, $(\text{psia})^2$ (well-bottom)
0.000	9340	87.24×10^6
0.117	9122	83.21
0.250	9085	82.54
0.500	9059	82.07
0.750	9049	81.88
1	9038	81.69
2	9018	81.32
3	8997	80.95
4	8976	80.57
5	8960	80.28
6	8950	80.10
7	8934	79.82
8	8924	79.64
9	8914	79.46
10	8908	79.35
11	8903	79.26
12	8893	79.09
14	8867	78.62
16	8846	78.25
18	8830	77.97
20	8815	77.70
22	8794	77.33
24	8778	77.05
28	8745	76.48
32	8690	75.52
36	8648	74.79
40	8625	74.39
44	8590	73.79
48	8560	73.27
56	8500	72.25
64	8460	71.57
72	8431	71.08

B. <u>Field B</u>

Drawdown and build-up test data from an aquifer located in St. Peter Sandstone in the Illinois Basin are presented below. Values for the physical properties of the sandstone obtained from core data are porosity 14.5 percent, water viscosity 1 centipoise, sand thickness 164 feet, and permeability 168 millidarcys.

Well (B-1) located in the aquifer was pumped at a rate of 1028 barrels per day. Pressures were observed at the Pumping Well (B-1), three Observation Wells (B-2) (B-3) B-6) completed in the aquifer, and two Observation Wells (B-4) (B-5) completed in the first permeable sand above the aquifer. Locations of the wells are shown in Figure (VI-2) and pressure measurements are listed in Table (VI-2) for the drawdown test and in Table (VI-3) for the build-up test. The distance between the Pumped Well (B-1) and the observation wells are:

Well (B-2) - 1021 feet

Well (B-3) - 1760 feet

Well (B-4) - 1232 feet

Well (B-5) - 1320 feet

Well (B-6) - 6600 feet

1. <u>Drawdown Test</u>

A plot of the pressure change versus $\log_{10}$time for the drawdown test is shown in Figure (VI-3). The

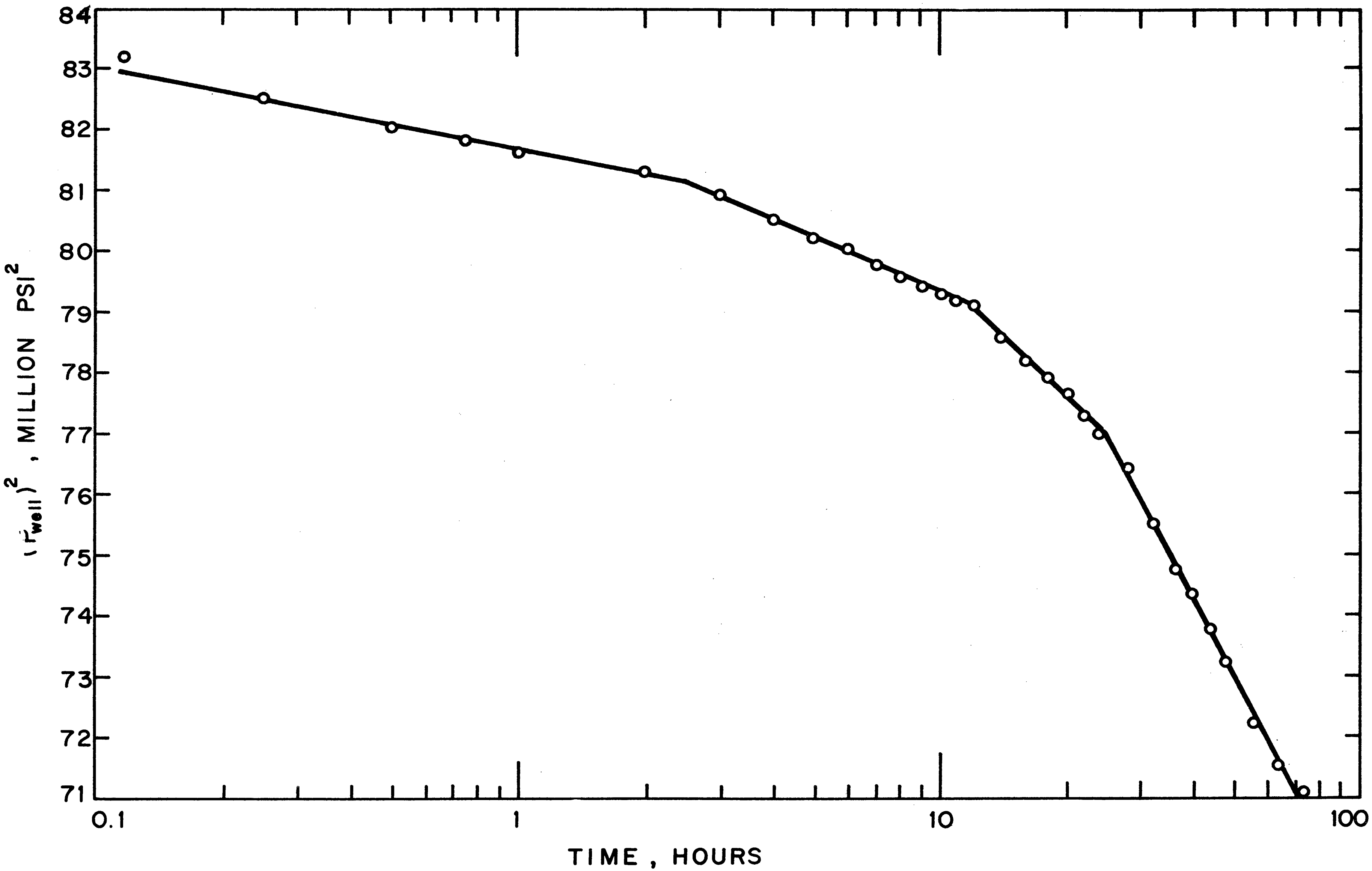

Figure VI-1. Well Pressure versus Time for Flow Test on Gas Well in Field A.

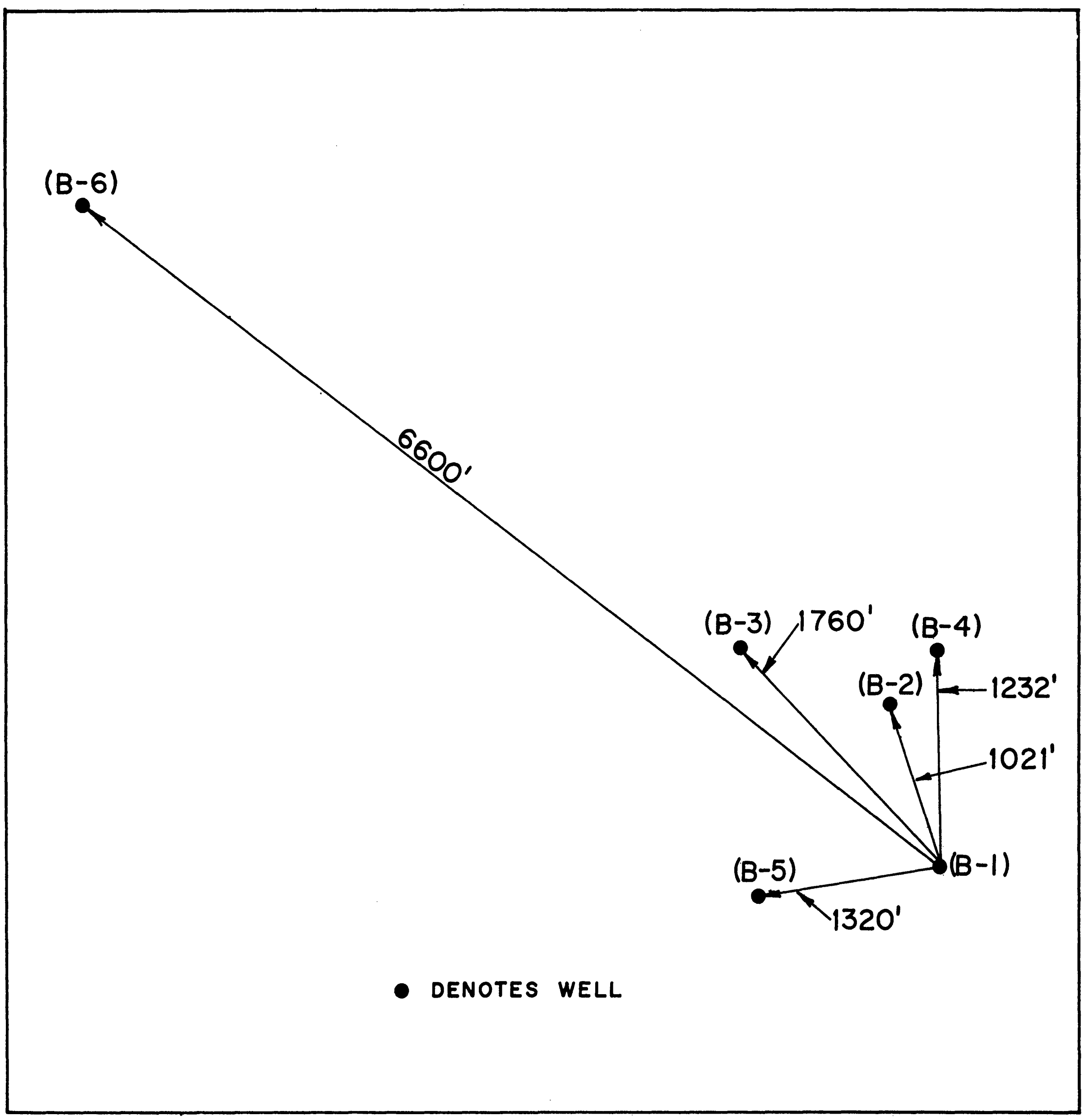

Figure VI-2. Location of Wells in Field B.

TABLE VI-2

FIELD B - PRESSURE DRAWDOWN DATA

Pumping Time, Hours	Pressure Drop, Feet of Water Well B-1	Well B-2	Well B-3	Well B-4	Well B-5	Well B-6
0	0	0	0	0	0	0
0.017	60					
0.033	80					
0.050	95					
0.067	120					
0.083	135					
0.100	146					
0.117	151					
0.113	159					
0.150	164					
0.162	169					
0.250	183					
0.333	198	0.05				
0.420	201	0.09	0.01			
0.50	202	0.20	0.03			
0.58		0.43	0.09			
0.67		0.60	0.13			
0.75	204	0.77	0.19	0.02		
0.82		1.05	0.25	0.02		
0.91		1.20	0.34	0.02		
1.00	205	1.50	0.42	0.04		
1.08		1.73	0.52			
1.16		1.93	0.61			
1.25	204	2.05	0.66	0.04		
1.33		2.30	0.79			
1.42		2.45	0.88			
1.50	206	2.61	0.98	0.07	0.01	
1.58		2.79				
1.67		2.95				
1.75	208	3.05	1.26	0.12	0.01	
1.82		3.20				
1.91		3.35				
2.00	210	3.50	1.48	0.14	0.01	
2.25		3.80				
2.50	210	4.16	1.94	0.19	0.01	
2.75		4.45				
3.0	210	4.70	2.31	0.46	0.01	
3.5		5.05	2.65	0.55	0.01	
4	210	5.52	2.93	0.72	0.02	
5	210	6.15	3.43	1.05	0.10	
6	212	6.55	3.78	1.37	0.12	
7	208	7.00	4.14	1.92	0.14	
8	205	7.30	4.41	2.22	0.24	
9	208	7.65	4.67	2.62	0.31	
10	208	7.85	4.87	2.84	0.36	
11		7.85	4.87		0.35	
12	209	8.30	5.31	3.44	0.62	0.10
14	210	8.70	5.64	3.96	0.75	0.10
16	210	9.00	5.94	4.29	0.89	0.12
18	212	9.30	6.22	4.55	1.05	0.18
20	211	9.60	6.48	4.81	1.35	0.25
23		10.13	6.92		1.35	0.31
24	210	10.05	6.92	5.28	1.64	0.35
30	205	10.50	7.41	5.87	2.39	0.45
35	205	10.89	7.73	6.26	2.85	0.67
40	209	11.05	7.93	6.54	3.40	0.66
45	203	11.40	8.28	6.78	3.80	0.88
47		11.40	8.28	6.78	3.80	0.76
50	210	12.00	8.53	7.13	4.33	1.00
55	202	12.10	8.68	7.27	4.80	1.05
59		12.25	8.93	7.47	5.14	0.96
60	200	12.25	8.93	7.47	5.14	1.24
65			9.18			1.25
70	200	12.30	9.18	7.73	5.99	1.26
71		12.30		7.73	5.99	1.16
72	205	12.18	9.32	7.87	6.14	1.30

TABLE VI-3

FIELD B- PRESSURE BUILD-UP DATA

Shut-In Time, Hours		Pressure Drop, Feet of Water Well B-1	Well B-2	Well B-3	Well B-4	Well B-5	Well B-6
0.050	1441	30	12.18	9.33			
0.100	721	31	12.18	9.33			
0.162	445	34	12.18	9.34		6.17	
0.250	291	34	12.18	9.34	7.87	6.17	
0.333	219	31	12.18	9.34		6.20	
0.42	172	31	12.18	9.33		6.19	
0.50	145	29	12.15	9.31		6.19	
0.58	125	28	12.05	9.24	7.87	6.19	
0.67	119	27	11.83	9.20	7.87	6.19	
0.75	97	26	11.60	9.13	7.87	6.21	
0.82	89	25	11.40	9.07	7.87	6.23	
0.91	80	24	11.10	8.99	7.87	6.23	
1.00	73	22	10.87	8.91	7.87	6.24	
1.25	59	21	10.55	8.57	7.87	6.25	1.39
1.5	49	20	10.01	8.43	7.85	6.26	1.42
1.75	42		9.57	8.21	7.84	6.28	1.42
2.0	37		9.27	7.98	7.77	6.31	1.42
2.5	29.8		8.69	7.58	7.70	6.34	1.42
3.0	25.0		8.15	7.23	7.60	6.36	
3.5	21.6		8.08	6.95	7.47	6.37	
4	19.0		7.48	6.63	7.37	6.38	
5	15.4		6.82	6.28	7.01	6.46	1.42
6	13.0		6.43	5.88	6.72	6.48	1.39
7	11.3		6.05	5.53	6.35	6.39	1.39
8	10.0		5.60	5.31	6.05	6.47	1.39
9	9.0		5.42	5.03	5.81	6.49	1.39
10	8.2		5.25	4.98	5.62	6.49	1.39
11	7.6		4.49				1.26
12	7.0		4.94	4.67	5.14	6.52	1.47
14	6.2		4.65	4.45	4.78	6.48	1.55
16	5.5		4.37	4.17	4.47	6.45	1.52
18	5.0	9.55	4.10	3.98	4.19	6.40	1.52
20	4.6		3.95	3.81	3.99	6.34	1.51
23	4.1	9.03	3.84				1.36
24	4.0		3.79	3.65	3.73	6.22	1.59
30	3.4		3.40	3.30	3.39	6.05	1.55
35	3.0		3.13	3.06	3.11	5.89	
40	2.8	8.99	2.95	2.85	2.89	5.64	1.50
45	2.6	8.80	2.82	2.77	2.83	5.47	1.49
47	2.54		2.84	2.58	2.77		1.41
50	2.44	8.66	2.84	2.82	2.77	5.14	1.60
55	2.31	8.50	2.37	2.56	2.64	5.14	1.52
59	2.22			2.43			
60	2.20	8.42	2.39	1.43	2.42	4.96	1.46
65	2.11	8.35	2.37	2.41	2.42	4.76	1.44
70	2.03	8.25	2.35	2.28	2.31	4.52	1.43
71	2.02			2.13			
83	1.87			2.03			
95	1.76					3.78	

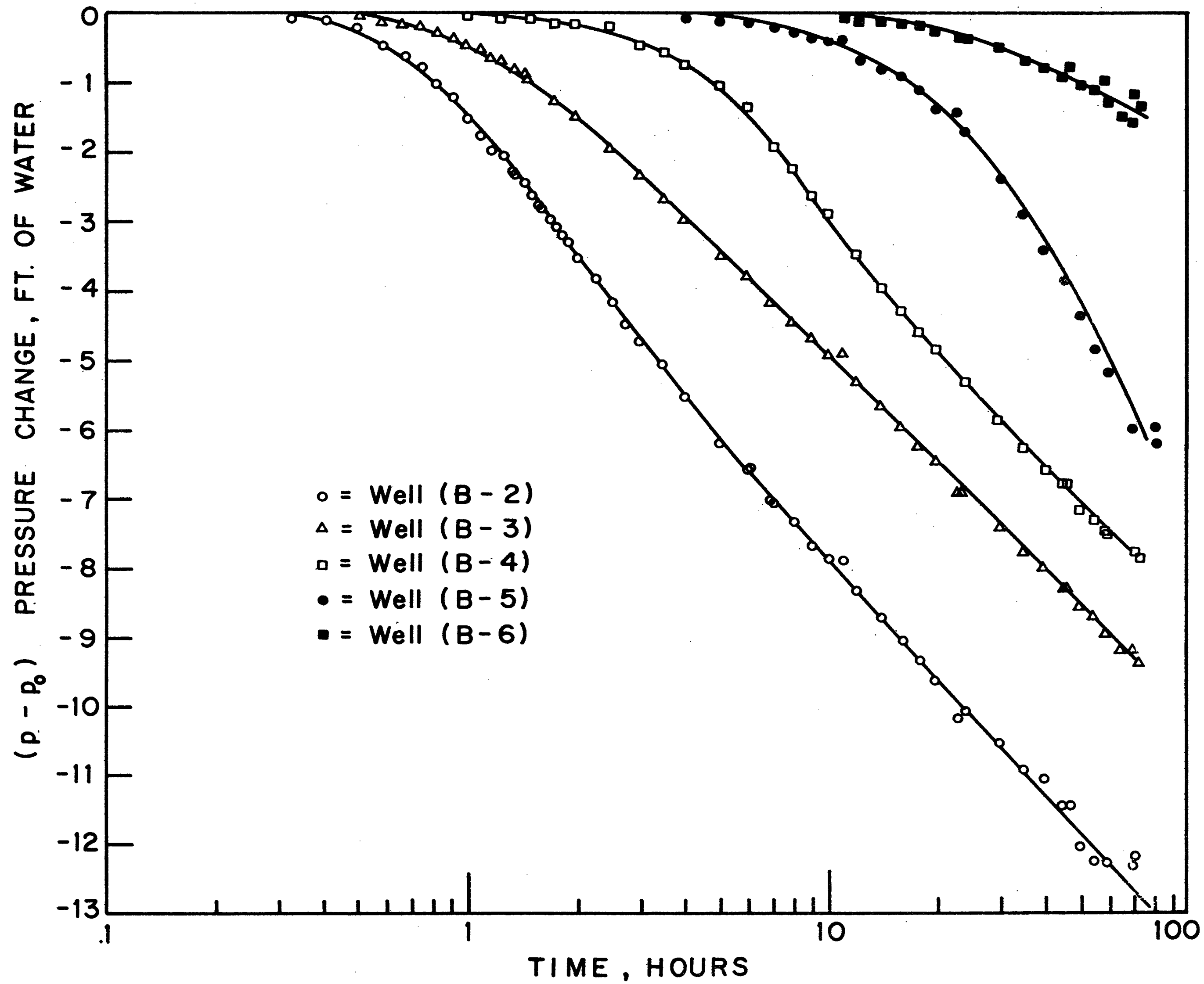

Figure VI-3. Pressure Curves for Drawdown Test on Field B.

pressure drops in Wells (B-4) and (B-5) show that there is communication between the St. Peter sandstone and the next sandstone above the St. Peter. The change in slope of the drawdown curve in Wells (B-2) and (B-4) indicates that there is pressure communication in the vicinity of these wells. See Figure (VI-2). Data from Wells (B-2) and (B-3) were used to determine the insitu permeability and insitu compressibility.

The slope of the drawdown curve for Wells (B-2) and (B-3) are -6.5 and -5.0 feet of water/cycle, respectively or -2.82 and -2.17 psi/cycle respectively.

The insitu permeability calculated from Equation (II-6), is

$$K = -\frac{162.6 q \mu}{m h} \qquad \text{(II-6)}$$

Thus for Well (B-2)

$$K = \frac{-162.6\,(1028)(1)}{-2.82\,(164)} = 361 \text{ millidarcys}$$

and for Well (B-3)

$$K = \frac{-162.6\,(1028)(1)}{-2.17\,(164)} = 470 \text{ millidarcys}$$

The insitu compressibility is obtained by solving Equation (II-1) for c

$$c = \frac{-4(0.00633)Kt}{\phi \mu r^2} \; Ei^{-1}\left\{\frac{p - p_o}{\left(\frac{70.6 q \mu}{Kh}\right)}\right\} \qquad \text{(VI-21)}$$

A drawdown of -6.5 feet of water (-2.82 psi) at 6 hours (0.25 days) for Well (B-2) was used to calculate the insitu compressibility for Well (C-2).

$$c = \frac{-4(0.00633)(361)(0.25)}{(0.145)(1)(1021)^2} \; Ei^{-1}\left\{\frac{-2.82}{\frac{(70.6)(1028)(1)}{(361)(164)}}\right\}$$

$$c = 9.06 \times 10^{-7} \; vol/(vol)(psi)$$

Using the drawdown of -9.3 feet of water (-4.03 psi) at 72 hours (3 days), for Well (B-3) the insitu compressibility is given by

$$c = \frac{-4(0.00633)(470)(3)}{(0.145)(1)(1760)^2} \; Ei^{-1}\left\{\frac{-4.03}{\frac{(70.6)(1028)(1)}{(470)(164)}}\right\}$$

$$c = 6.35 \times 10^{-7} \; vol/(vol)(psi)$$

This value is checked using the drawdown -4.14 feet of water (-1.79 psi) at 7 hours (0.292 days) for Well (B-3)

$$c = \frac{-4(0.00633)(470)(0.292)}{(0.145)(1)(1760)^2} \; Ei^{-1}\left\{\frac{-1.79}{\frac{(70.6)(1028)(1)}{(470)(164}}\right\}$$

$$c = 7.17 \times 10^{-7} \; vol/(vol)(psi)$$

These results for the insitu compressibility are not only inconsistent, they are less than the compressibility of the water itself. These data strongly indicate that in addition to leakage, the formation is heterogeneous and can not be represented by a single pseudo homogeneous porous media. The pressure drawdown for a layered reservoir can be calculated from

$$p = p_o + 70.6\mu \sum_{i=1}^{n} \frac{q_i}{K_i h_i} Ei\left\{\frac{-\phi c \mu r^2}{4(0.00633Kt)}\right\} \qquad \text{(VI-22)}$$

where:

h_i = height of the ith layer, feet

K_i = permeability of the ith layer, millidarcys

n = number of distinct homogeneous layers

q_i = flow rate from the ith layer, bbl/day

If the heterogeneities in a reservoir are known, then the pressure behavior can be determined. Unfortunately the converse is not true, since there may be an infinite number of combinations of stratifications, leakage, faults, etc. which will give essentially identical pressure behavior. Furthermore the flow may not be radial and forcing the data to fit this model may lead to large errors in the determination of the insitu properties. Applications of hemispherical model and thick sand model (bottom water drive) to reservoir well test data are given by Katz et al (51).

Barometric pressures were not recorded during this test. Failure to correct the pressure data for barometric changes can cause serious error in interpreting pump test data when the pressure drawdown is only a few feet of water.

2. Build-up Test

The pressure change versus $\log_{10}\left(\frac{\Delta t}{t_o + \Delta t}\right)$ is shown in Figure (VI-4) for the build-up test. The slope of the curves for Wells (B-2) and (B-3) is

$$m = -6 \text{ ft. of water/cycle}$$

$$m = -2.6 \text{ psi/cycle}$$

The insitu permeability calculated from Equation (II-6) is

$$K = -\frac{162.6\, q\, \mu}{m\, h}$$

$$K = -\frac{162.6\,(1028)(1)}{(-2.6)(164)}$$

$$K = 393 \text{ millidarcys}$$

C. Field C

Drawdown and build-up test data from a second aquifer in the Illinois Basin are analyzed. Location of the pumping Well (C-1) and three observation Wells (C-2) (C-3) (C-4) in the Mt. Simon formation are shown in Figure (VI-5).

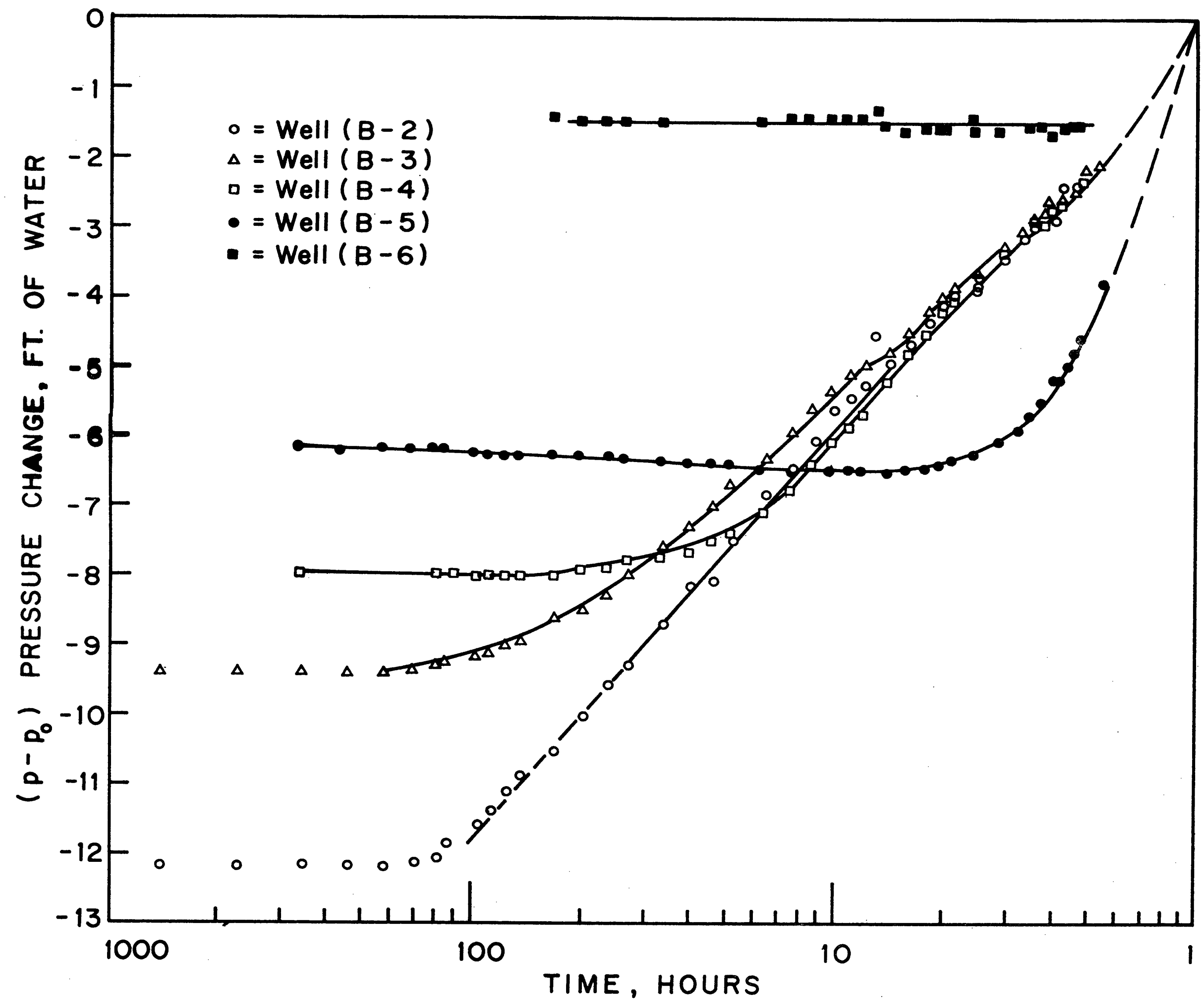

Figure VI-4. Pressure Curves for Build-Up Test on Field B.

Well (C-1 was pumped at a rate of 2740 barrels per day (80 gallons per minute). Pressure observations corrected for changes in barometric pressure, at the Pumping Well (C-1) and the three Observation Wells (C-2), (C-3), and (C-4) are recorded in Table (VI-4). The pressure behavior, corrected for changes in barometric pressure, is given in Table (VI-5) for the build-up test. The horizontal distances between the Pumping Well (C-1) and the observation wells are

Well (C-2) - 4611 feet
Well (C-3) - 2869 feet
Well (C-4) - 6934 feet

The distances between the pumping well and the observation wells when the vertical displacement of the formation is considered is

Well (C-2) - 4612 feet
Well (C-3) - 2880 feet
Well (C-4) - 6934 feet

The aquifer is several thousand feet thick and contains several non-continuous shale streaks. Core data show the permeability of the sandstone varies from a few millidarcys to several darcys. Thus the meaning of insitu permeability and insitu compressibility is questionable in such a heterogeneous formation. Pump test actually measure the transmissibility, T, and storage coefficient, S, where these terms are defined by

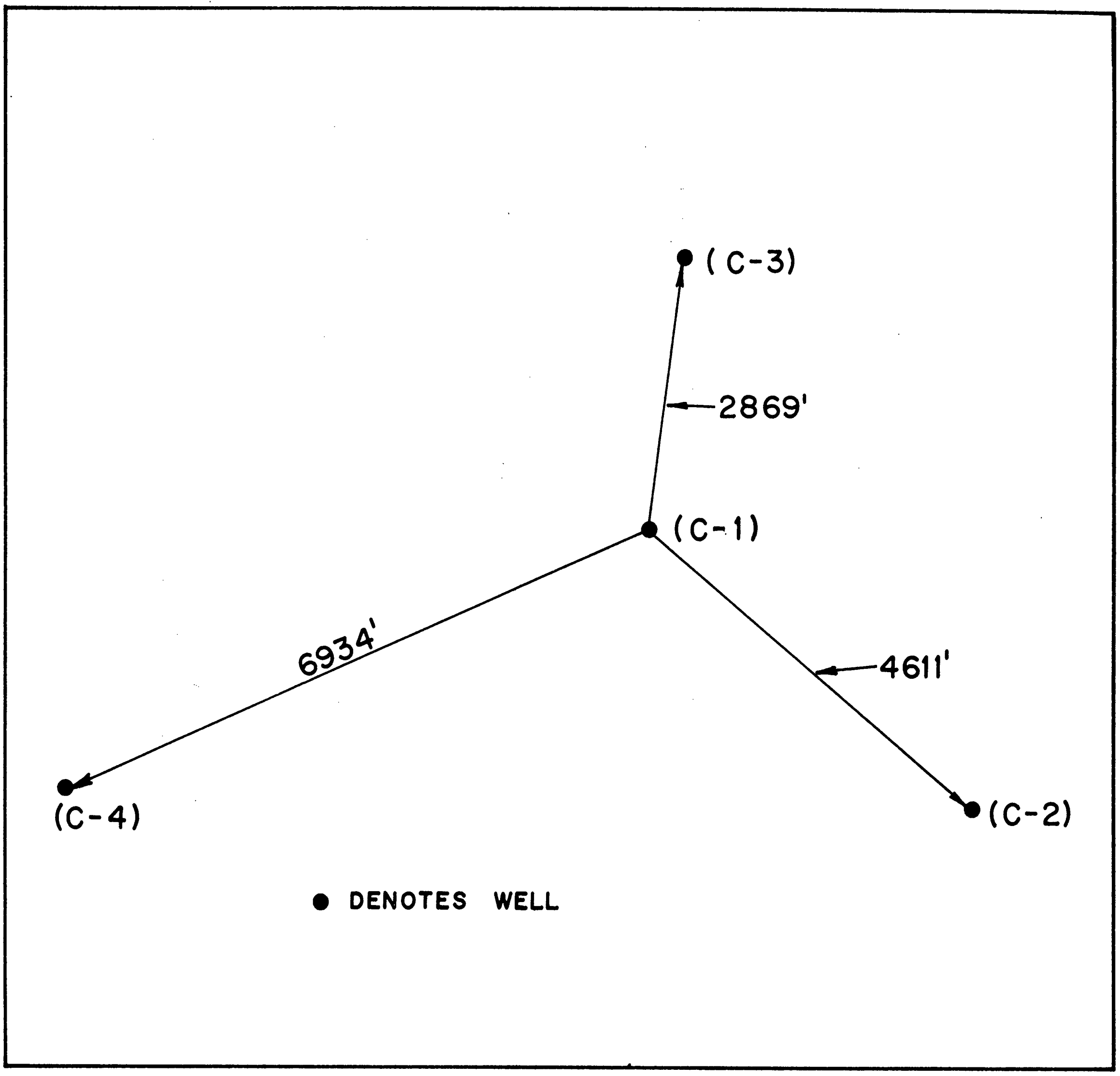

Figure VI-5. Location of Wells in Field C.

TABLE VI-4

FIELD C — PRESSURE DRAWDOWN DATA

Pumping Time, Hours	Pressure Drop, Feet of Water (Corrected for Barometric Pressures) Well C-1	Well C-2	Well C-3	Well C-4
0.0	0			
0.1	398			
0.2	561			
0.3	634			
0.4	706			
0.5	746			
0.75	794			
1	811			
2	847			
3	847			
4	847			
4.25	847	0.03	0.01	
5	847		0.03	
6	847	0.02	0.05	
7	847	0.04	0.10	
8	847	0.09	0.16	
9	847	0.14	0.22	
10	847	0.18	0.29	
11	847	0.27	0.35	
12	847	0.32	0.39	
13	847	0.39	0.43	
14	847	0.44	0.44	
15	847	0.48	0.45	
17	847	0.54	0.45	
18	847	0.59	0.46	
20	842	0.67	0.49	
23		0.87	0.60	
25	837	0.96	0.65	
27		1.08	0.70	
30	846	1.16	0.72	
32		1.25	0.78	0.11
35	852	1.49	0.95	0.16
40	883	1.68	1.03	0.19
45	844	1.79	1.01	0.28
50	837	1.98	1.13	0.37
52		2.00	1.16	0.41
55	837	2.11	1.19	0.44
57	832	2.25	1.28	0.53
65	832	2.41	1.46	0.72
70	848	2.62	1.47	0.74
75	837	2.66	1.55	0.82
80	840	2.75	1.56	.85
90	842	2.92	1.68	.94
114	837	3.24	1.88	1.17
126		3.39	1.99	1.28
138	835		2.03	1.41
150	837	3.45	2.08	1.43
162	819	3.53	2.12	1.46
174	826	3.65	2.20	1.53
186	826	3.76	2.28	1.59
198	821	3.91	2.42	1.59
210	821	3.92	2.41	1.72
222		4.04	2.55	1.84
234		3.91	2.43	1.83
246		4.02	2.56	1.92
258	778	4.05	2.55	1.83
264	773	4.16	2.64	1.93
270		4.17	2.63	1.93
282		4.21	2.64	1.92
288		4.30	2.73	2.02
294		4.37	2.71	1.99
306		4.33	2.75	2.02
312	792	4.42	2.92	2.17
318		4.42	2.83	2.12
330		4.50	2.93	2.23
336		4.53	2.99	2.23
342		4.50	2.93	2.19
354	773	4.62	3.03	2.28
360	778	4.65	3.06	2.31
366	776	4.71	3.03	2.27
378	778	4.66	3.09	2.33
384	789	4.65	3.07	2.31
390	789	4.72	2.98	2.32
402	778	4.69	3.12	2.33
408	778	4.66	3.09	2.30
414	778	4.66	3.09	2.30
426	778	4.75	3.19	2.37
438		4.83	3.25	2.41
450	789	4.96	3.38	2.52
480	789	5.10	3.47	2.65
510	789	5.03	3.30	2.61
528	794		3.49	2.64
750	771	5.30	3.73	2.79
870	779	5.65	4.04	3.18
990	834	5.66	4.03	3.02
1152	798	5.75	4.25	3.27

TABLE VI-5

FIELD C - PRESSURE BUILD-UP DATA

Shut-in Time, Hours	Pressure Drop Feet of Water (corrected)			
	Well C-1	Well C-2	Well C-3	Well C-4
6	193	5.67	4.15	3.25
18	65	5.08	3.73	3.17
24	49	4.76	3.54	
30	39.4	4.56	3.52	3.11
42	28.5	4.11	3.34	2.95

$T = Kh/\mu$, millidarcy feet/centipoise

$S = \phi ch$, feet/psia

Except for the inclusion of viscosity, these terms are not new and have been used by hydrologists for decades (28) (42) (84).

The pressure for a drawdown test in terms of transmissibility, T, and storage coefficient, S, is calculated by

$$p = p_0 + \frac{70.6q}{T} Ei\left\{-\frac{Sr^2}{4(0.00633Tt)}\right\} \qquad \text{(VI-23)}$$

The storage coefficient, S, can be obtained by solving Equation (VI-23) for S. Thus

$$S = \frac{4(0.00633)Tt}{r^2} Ei^{-1}\left\{\frac{(p-p_0)T}{70.6q}\right\} \qquad \text{(VI-24)}$$

Comparison of Equations (II-1) and (VI-23) shows that the transmissibility, T, can be calculated from the slope of the drawdown or build-up test by

$$T = -\frac{162.6q}{m} \qquad \text{(VI-25)}$$

The expression for dimensionless time in engineering units can be written as

$$t_D = \frac{0.00633Tt}{Sr^2} \qquad \text{(VI-26)}$$

1. Drawdown Test

Pressure change versus log time is plotted in Figure (VI-6) for the drawdown test. Data from each of the three observation wells are used to determine the insitu transmissibility, T, and storage coefficient, S. Equation (VI-25) is used to calculate the insitu transmissibility, and Equation (VI-24) is used to calculate the storage coefficient. The slopes for the drawdown test shown in Figure (VI-6) are

Well (C-2), m = -3.05 ft water/cycle = -1.34 psi/cycle

Well (C-3), m = -2.5 ft water/cycle = -1.08 psi/cycle

Well (C-4), m = -2.2 ft water/cycle = -0.95 psi/cycle

The corresponding values for the insitu transmissibility given by Equation (VI-25)

$$T = -\frac{162.6\, q}{m} \qquad \text{(VI-25)}$$

are calculated for each of the three observation wells.

Well (C-2), $T = \frac{162.6(2740)}{-1.34}$ = 332,000 millidarcy feet/centipoise

Well (C-3), $T = \frac{-162.6(2740)}{-1.08}$ = 412,000 millidarcy feet/centipoise

Well (C-4), $T = \frac{-162.6(2740)}{-0.95}$ = 468,000 millidarcy feet/centipoise

The average insitu transmissibility for the three wells is

T = 404,000 millidarcy feet/centipoise

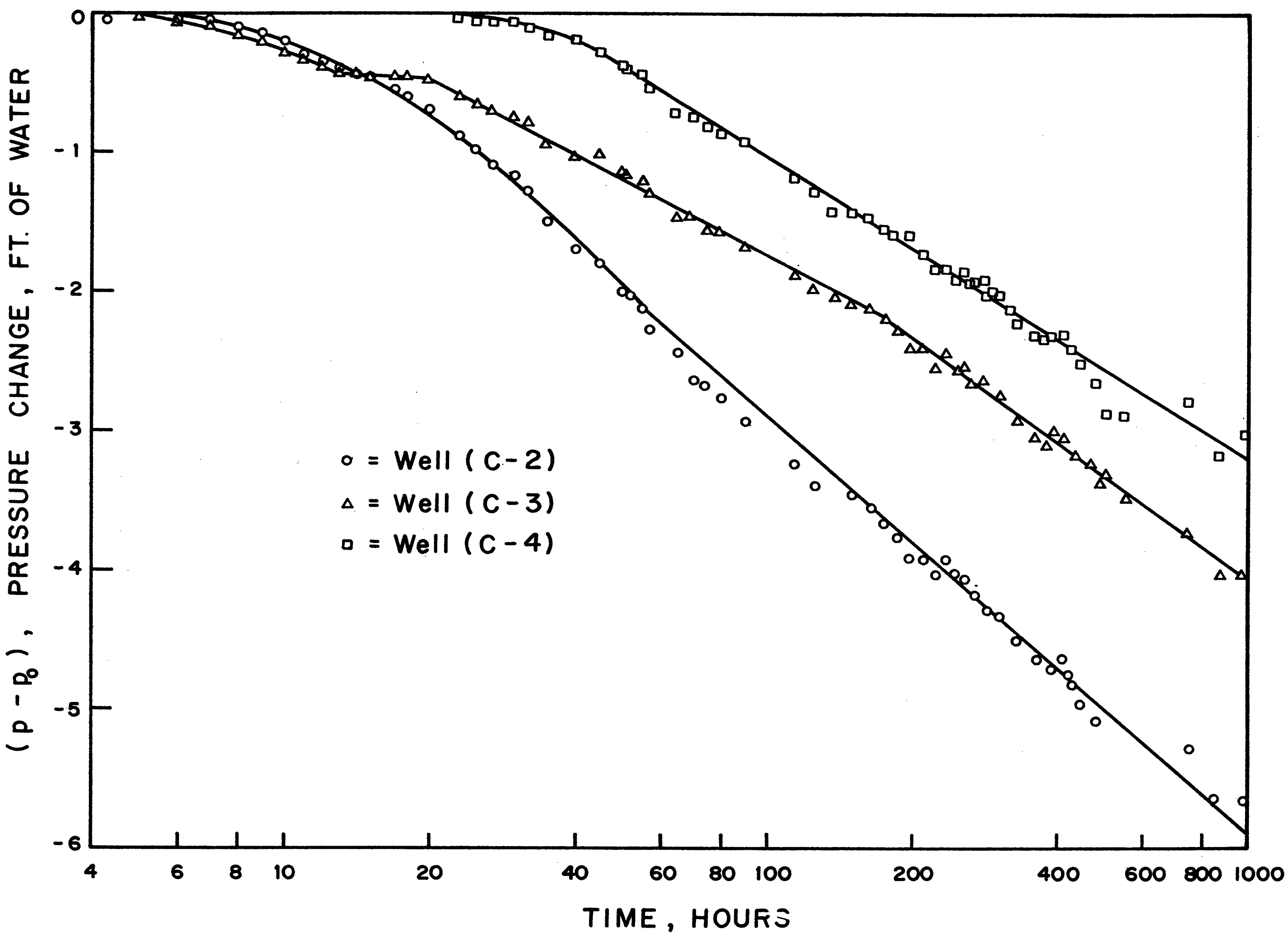

Figure VI-6. Pressure Curves for Drawdown Test on Field C.

Since the value for the aquifer thickness, h, is questionable, solving the expression for the transmissibility

$$T = \frac{Kh}{\mu}$$

for the insitu permeability is not recommended for this field.

The values for the storage coefficient, S, are calculated by Equation (VI-24)

$$S = \frac{4(0.00633)Tt}{r^2} Ei^{-1}\left\{\frac{(p-p_o)T}{70.6q}\right\}$$

for each of the three observation wells. Values for the drawdown at 720 hours (30 days) are read from Figure (VI-6).

Well (C-2)

$$S = \frac{4(0.00633)(332,000)(30)}{(4612)^2} Ei^{-1}\left\{\frac{(-5.5)(0.433)(332,000)}{70.6\ (2740}\right\}$$

$$S = 0.00012 \text{ feet centipoise / psia}$$

Well (C-3)

$$S = \frac{4(0.00633)(412,000)(30)}{(2880)^2} Ei^{-1}\left\{\frac{-3.7(0.433)(412,000)}{70.6\ (2740)}\right\}$$

$$S = 0.00072 \text{ feet centipoise / psia}$$

Well (C-4)

$$S = \frac{4(0.00633)(468,000)(30)}{(6934)^2} Ei^{-1}\left\{\frac{-2.9(0.433)(468,000)}{70.6\ (2740)}\right\}$$

$$S = 0.00021 \text{ feet centipoise / psia}$$

The average value for the storage coefficient, S, for Field C is

$$S_{ave} = 0.00035 \text{ (feet)(centipoise)/psia}$$

The expression for the storage coefficient $S = \mu \phi ch$ should not be solved for the insitu compressibility since the effective value for the aquifer thickness, h, is not known for Field C.

2. Build-up Test

The build-up curves for Wells (C-2), (C-3), and (C-4) are shown in Figure (VI-7). The curves are extrapolated to $\frac{t_o + \Delta t}{\Delta t} = 1$ in order to obtain the slope. The slope for these curves is -2.8 ft. of water/cycle or -1.21 psi/cycle.

The insitu transmissibility, given by Equation (VI-25) is

$$T = \frac{-162.6(2740)}{-1.21}$$

$$T = 368{,}000 \text{ millidarcy feet/centipoise}$$

This value is within the range of values obtained for the insitu transmissibility from the drawdown test.

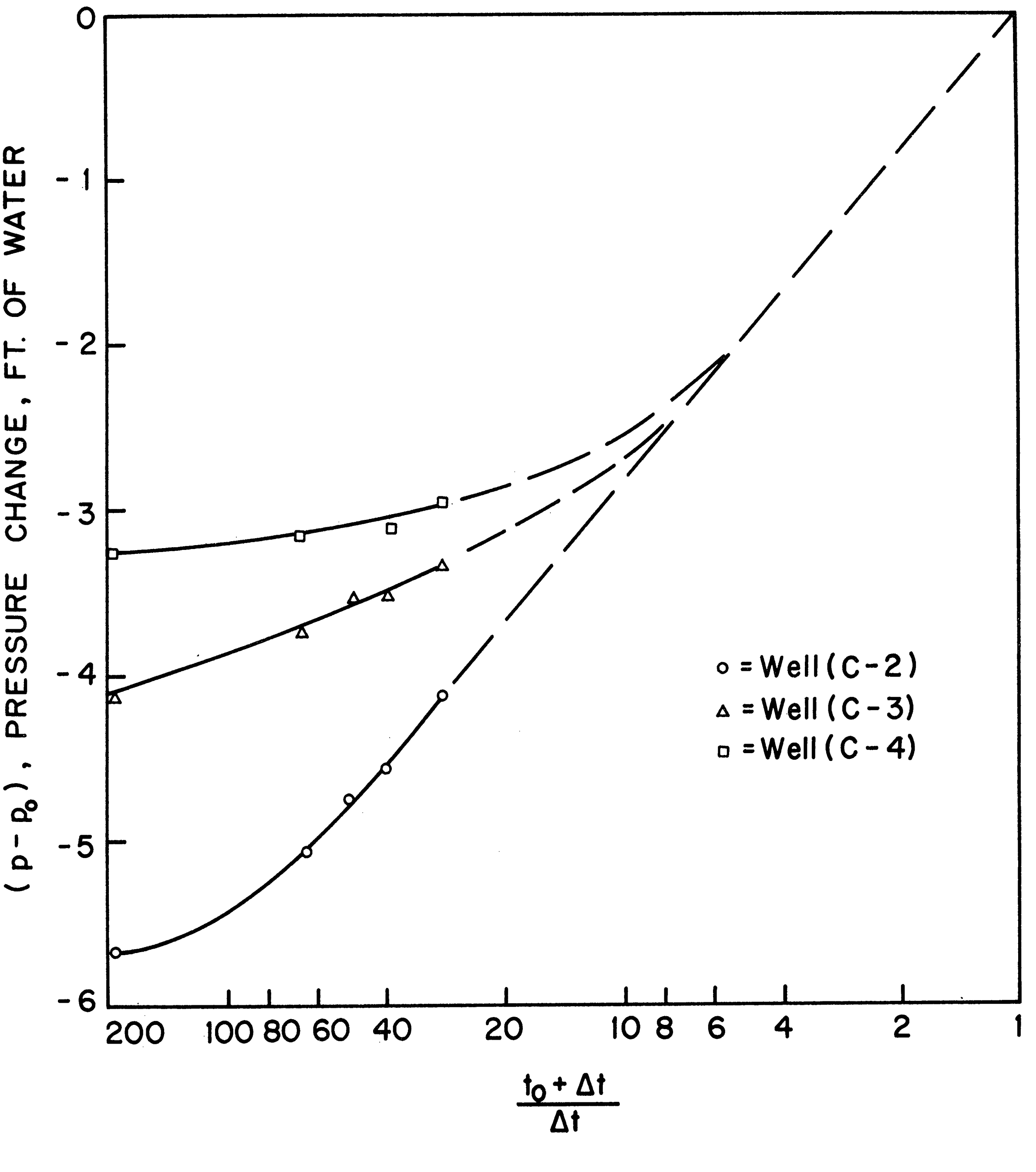

Figure VI-7. Pressure Curves for Build-Up Test on Field C.

VII. SUMMARY, CONCLUSIONS, AND RESULTS

Several problems involving the unsteady state behavior of fluids in underground strata were investigated in this dissertation. These problems include:

1. Evaluation of the error in neglecting the non-linear term in the partial differential equation describing the flow of a slightly compressible liquid in a porous media.
2. Determination of mathematical expressions describing the unsteady state pressure behavior for radial flow in a reservoir during drawdown and build-up pump tests when gas fields or lines of constant pressure are located in the vicinity of the well.
3. Determination of mathematical expressions in engineering units describing the unsteady state pressure behavior for radial flow in a reservoir during drawdown tests when leakage is occurring through the confining cap and bottom rock and extended these results to build-up tests.
4. Illustration of many difficulties encountered when actual field data are analyzed.

The following results and conclusions were obtained in this study.

1. The effect of neglecting the non-linear term in evaluating the unsteady state pressure behavior is negligible for both radial and linear systems when

$$t_D \leqq 0.001\ M^{-2}$$

where:

$$t_D = \frac{0.00633\ K\ t}{\mu \phi c r^2}$$

$$M = \frac{-141.2\ c \mu q}{Kh} \text{ for radial flow and}$$

$$\frac{-887.6 c \mu q x_c}{AK} \text{ for linear flow}$$

A = cross sectional area normal to flow, (feet)

c = compressibility, vol/(vol)(psi)

h = thickness of porous media, feet

K = permeability, millidarcys

q = flow rate, bbl/day

r = radius of well, feet

t = time, days

x_c = reservoir characteristic length, feet

ϕ = porosity, fraction

μ = viscosity, centipoise

2. The pressure behavior during a drawdown test when a gas field is located nearby is given by

$$p = p_o + \frac{70.6\, q\mu}{Kh}\left\{Ei\left(-\frac{\mu\phi c\, l_1^2}{4(0.00633Kt)}\right) - Ei\left(-\frac{\mu\phi c\, l_2^2}{4(0.00633Kt)}\right)\right\}$$

where:

Ei = exponential integral

l_1 = distance from pumping well to point of distance measurement, feet

l_2 = distance from point of pressure measurement to image of pumping well, feet

The observed insitu permeability and insitu compressibility are larger than their true values and the magnitude of these errors are given in Figures (IV-9) and (IV-10).

3. The pressure behavior during a build-up test when a gas field is located in the vicinity of the test is

$$p = p_o + \frac{70.6\, q\mu}{Kh}\left\{Ei\left(-\frac{\mu\phi c\, l_1^2}{4(0.00633)K(t_o+\Delta t)}\right) - Ei\left(\frac{\mu\phi c\, l_2^2}{4(0.00633)K(t_o+\Delta t)}\right) - Ei\left(-\frac{\mu\phi c\, l_1^2}{4(0.00633)K\Delta t}\right) + Ei\left(-\frac{\mu\phi c\, l_2^2}{4(0.00633)K\Delta t}\right)\right\}$$

where:

t_o = length of drawdown test, days

Δt = time since cessation of pumping, days

4. The pressure performance versus dimensionless time when leakage is occurring through the cap and bottom rock was evaluated for drawdown and build-up tests. These results are presented in Figures (V-1) and (V-3). The complex expressions for the pressure behavior are given by Equation (V-6) for the drawdown and Equation (V-7) for the build-up. The error in the measurement of insitu permeability and insitu compressibility is shown in Figures (V-4) and (V-5).
5. Two fields characteristics, transmissibility, T, and storage coefficient, S, should be evaluated for reservoirs in which it is difficult or impossible to obtain the reservoir thickness. These characteristics defined by

$$T = \frac{Kh}{\mu}$$

and

$$S = \phi c h$$

can be used to predict the reservoir performance.
6. A graphical method for locating an external line of constant pressure or a gas-water interface is described.

VIII. RECOMMENDATIONS FOR FUTURE WORK

Methods have been presented in this dissertation for predicting the pressure behavior of constant rate pump tests when (1) a gas field is near or a line of constant pressure is present and (2) leakage is occurring through the cap and bottom rock. The usefulness of this work needs to be substantiated by field data and extended to the ase of constant pressure well tests. Specific problems and tests which would enhance our understanding of unsteady state reservoir behavior are given below.

1. Drawdown and build-up well tests should be conducted near the edge of a gas field to evaluate the usefulness of the material presented in Section IV on pressure behavior near a gas field. These tests could be performed near the edge of a gas storage field during the few months preceding or after the gas withdrawal season. It is further recommended that both constant rate and constant pressure pump tests be performed.
2. The pressure behavior for constant pressure drawdown and build-up tests should be determined for the case when a gas field is located in the vicinity of the test site.

It probably will be necessary to solve the partial differential equation, boundary, and initial conditions by numerical methods.

3. The pressure behavior for constant pressure drawdown and build-up tests need to be evaluated when leakage is occurring through the cap and bottom rock. Again numerical methods will probably be necessary to obtain the desired mathematical solutions.

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APPENDIX A

COMPUTER PROGRAM FOR NUMERICAL EXPLICIT METHOD USED IN DETERMINING THE ERROR IN NEGLECTING THE NON-LINEAR TERM IN THE PARTIAL DIFFERENTIAL EQUATION DESCRIBING RADIAL FLOW OF A SLIGHTLY COMPRESSIBLE FLUID IN POROUS MEDIA

A description of the IBM 7090 computer program used to determine the error in neglecting the non-linear term in the partial differential equation describing the flow of a slightly compressible fluid for radial flow is presented in this appendix. This appendix includes the MAD (Michigan Algorithm Decoder) program, the program nomenclature, the flow diagram, a list of information required, and an example problem illustrating the use of the program. This program was used to calculate values of dimensionless pressures for values of dimensionless time below 0.01.

The following information is required in the computer program:

1. The number of length increments. (Eighty length increments were found to be satisfactory, $\Delta w = \frac{1}{80}$.
2. The value of the time increments. (Δt = 0.00005 is an acceptable value of Δt for $\Delta w = \frac{1}{80}$. Stability requirements demand that $(\Delta t)/(\Delta w)^2$ is less than 0.5.)
3. The number of time increments to be evaluated.
4. The number of M coefficients to be evaluated. (The value of M is defined by Equation (III-34)).

5. The numerical values of the M coefficient to be evaluated.

The computer program uses the information to calculate values of dimensionless pressure versus dimensionless time for each value of the dimensionless coefficient M.

Deck Assembly

1. IBM center control cards

2. Radial, Explicit Numerical Program (MAD or Binary Deck)

3. Input Data

The input data is read in "simplified input format".

For Example,

IMAX = 80, DELT = 0.00005, MMAX = 2, JMAX = 49, M(1) = 0. , 1.*

Definitions of the terms used in the simplified input are given below in the nomenclature.

Flow Sheet

The instruction J = a, b, J > c in the flow sheet means that the set of calculations is calculated for J = a and repeated in increments of b until the condition J > c is satisfied. For J = a, b, J > c refers to iterative calculations for a given box. Through d, J = a, b, J > c refers to iterative calculations through "circle d". The Flow Sheet is presented at end of Appendix.

Nomenclature Used in IBM Program and Flow Sheet

ADIM	Term used to dimension P(I,J) vector
A	$\Delta t/(\Delta w)^2$
DELT	Δt
DELW	Δw
DT	$\Delta t/(F)^2$
F	Scaling coefficient "a". In this dissertation, F = 1.0
I	Space index
IMAX	Number of space increments
J	Time index
JMAX	Number of time increments
K1	Term used in program, K1 = $1-i\Delta w$
M	Dimensionless coefficient defined by Equation (III-34)
MMAX	Number of dimensionless coefficients
P	Dimensionless pressure
R	Counter used in IBM program
X2	Term defined by X2 = $P_{i+1,j} - P_{i-1,j}$
X3	Term defined by X3 = $(X2)^2$
X4	Absolute value of X2
Z	Value of dimensionless time at a given time step, Z = $j\Delta t$

IBM Center Control Cards

MAD Program

```
      M.C. MILLER                         Q203N              002   045   000
      M.C. MILLER                         Q203N              002   045   000
    $COMPILE MAD, EXECUTE, DUMP, PUNCH OBJECT,PRINT OBJECT
              R  EVALUATION OF NON-LINEAR TERM IN NON-LINEAR,
              R SECOND ORDER, PARTIAL DIFFERENTIAL EQUATION
              R FOR FLOW OF SLIGHTLY COMPRESSIBLE LIQUIDS
              R IN INFINITE RADIAL MODEL
              R
              RSOLVED BY DIFFERENCE EQUATIONS,EXPLICIT FORM
              R
               DIMENSION P( 8100,ADIM),M(10)
               INTEGER I,J,K,L,R,N,IMAX,MMAX,JMAX
               VECTOR VALUES ADIM=2,101,100
               F=1.
      S4       READ DATA,IMAX,DELT,M,MMAX,JMAX
               DELW=1./IMAX
               A=DELT/DELW/DELW
                 DT =DELT/F/F
               PRINT RESULTS DELT,IMAX,MMAX,JMAX, A,M(1)...M(MMAX)
               P(IMAX,  1)=0.
               THROUGH S6,FOR R=1,1,R.G.MMAX
               THROUGH S1,FOR I=0,1,I.G.IMAX
      S1       P(I,0)=0.0
               Z=0.
               J=0
               PRINT COMMENT $1EVALUATION OF NON-LINEAR TERM IN NON-LINEAR,
              1SECOND ORDER, PARTIAL DIFFERENTIAL EQUATION FOR FLOW OF SLIGH
              2TLY$
               PRINT COMMENT $0COMPRESSIBLE LIQUIDS IN INFINITE RADIAL MODEL
              1$
               PRINT RESULTS M(R),DELT,DELW,A,F
               PRINT COMMENT $0  N      TIME     PRESSURE$
      S5       THROUGH S2,FOR I=1,1,I.G.(IMAX-1)
               X2=P(I+1,0)-P(I-1,0)
               X4=.ABS.(X2)
               WHENEVER X4 .L. 1.E-10, X2=0.
               X3=X2*X2
               K1=(1.-I*DELW)
               P(I,  1)=P(I,0)+A*(K1*K1*(P(I+1,0)-2.*P(I,0)+P(I-1,0))
              1-DELW/2.*(K1-K1/(F -ELOG.(K1)))*X2
              2+M(R)*K1*K1/4.*X3)
               WHENEVER P(I,  1).L.1.E-20,P(I,1)=0.
      S2       CONTINUE
               P(0, 1)=(2.*DELW/F+4.*P(1, 1)-P(2,  1))/3.
               J=J+1
               Z=Z+DT
               PRINT FORMAT RSLT1,J,Z,P(0,1)
               VECTOR VALUES RSLT1=$I4,F9.6,F12.7*$
               WHENEVER J.G.JMAX,TRANSFER TO S6
               THROUGH S3, FOR I=0,1,I.G.IMAX
      S3       P(I,0)=P(I,1)
               TRANSFER TO S5
      S6       CONTINUE
               TRANSFER TO S4
               END OF PROGRAM
```

Data Input

```
    $ DATA
    IMAX=80,   DELT=.00005,   MMAX=2,   JMAX= 49,   M(1)=0.,1.*
```

IBM Program Print Out

EVALUATION OF NON-LINEAR TERM IN NON-LINEAR, SECOND ORDER, PARTIAL DIFFERENTIAL EQUATION FOR FLOW OF SLIGHTLY

COMPRESSIBLE LIQUIDS IN INFINITE RADIAL MODEL

M(1) = .000000,   DELT = 5.000000E-05,   DELW = .012500,   A = .32000

F = 1.000000

N	TIME	PRESSURE
1	.000050	.0083333
2	.000100	.0118008
3	.000150	.0142833
4	.000200	.0163656
5	.000250	.0181936
6	.000300	.0198440
7	.000350	.0213605
8	.000400	.0227713
9	.000450	.0240960
10	.000500	.0253485
11	.000550	.0265395
12	.000600	.0276771
13	.000650	.0287680
14	.000700	.0298173
15	.000750	.0308294
16	.000800	.0318081
17	.000850	.0327563
18	.000900	.0336766
19	.000950	.0345715
20	.001000	.0354427
21	.001050	.0362922
22	.001100	.0371213
23	.001150	.0379315
24	.001200	.0387240

25	.001250	.0394998
26	.001300	.0402600
27	.001350	.0410055
28	.001400	.0417369
29	.001450	.0424552
30	.001500	.0431609
31	.001550	.0438547
32	.001600	.0445371
33	.001650	.0452086
34	.001700	.0458699
35	.001750	.0465212
36	.001800	.0471630
37	.001850	.0477958
38	.001900	.0484198
39	.001950	.0490354
40	.002000	.0496430
41	.002050	.0502428
42	.002100	.0508351
43	.002150	.0514202
44	.002200	.0519983
45	.002250	.0525697
46	.002300	.0531345
47	.002350	.0536931
48	.002400	.0542455
49	.002450	.0547920
50	.002500	.0553328

IBM Program Print Out

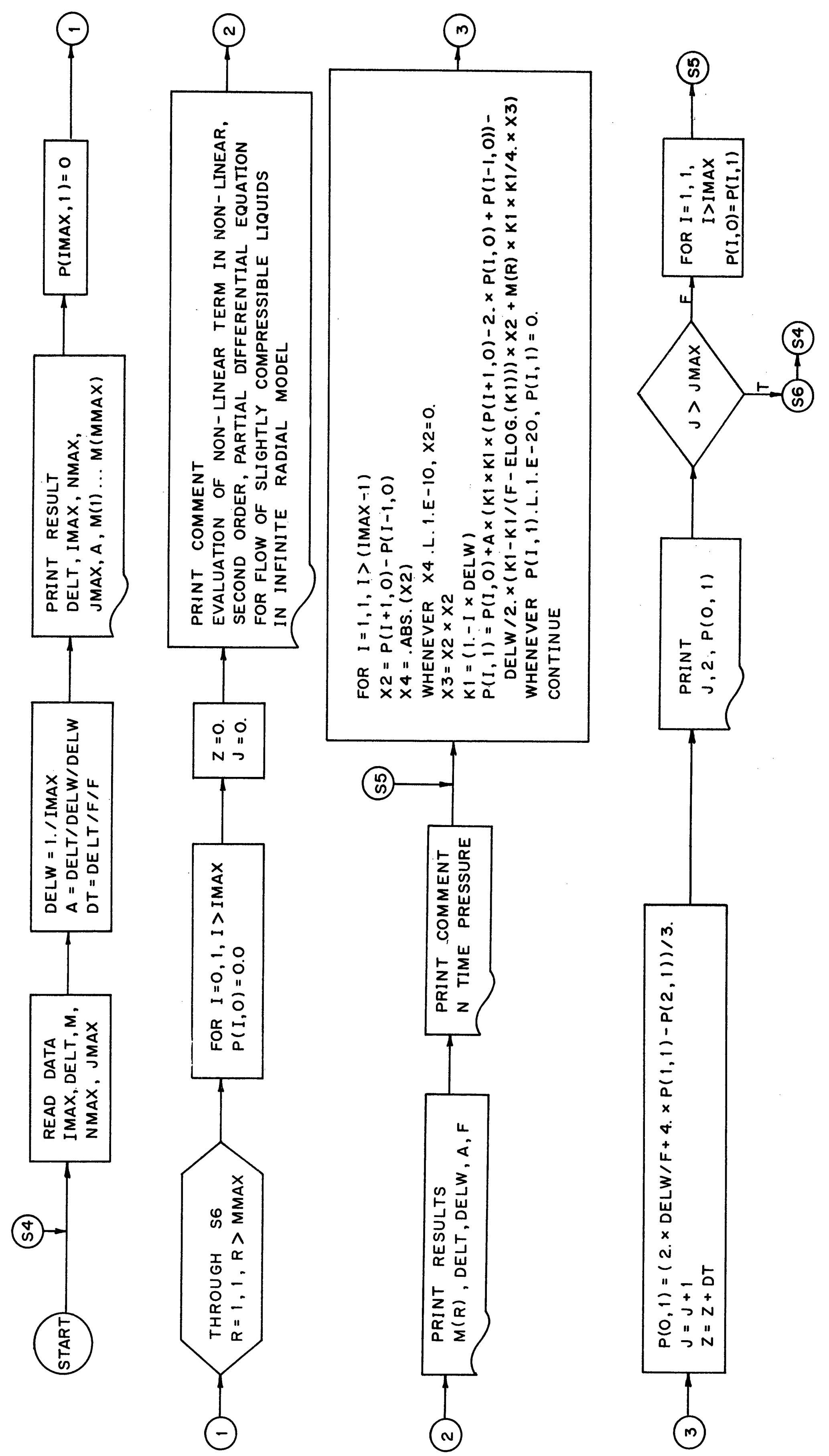
EVALUATION OF NON-LINEAR TERM IN NON-LINEAR, SECOND ORDER, PARTIAL DIFFERENTIAL EQUATION FOR FLOW OF SLIGHTLY COMPRESSIBLE LIQUIDS IN INFINITE RADIAL MODEL

M(2) = 1.000000, DELT = 5.000000E-05, DELW = .012500, A = .320000

F = 1.000000

N	TIME	PRESSURE
1	.000050	.0083333
2	.000100	.0118080
3	.000150	.0143028
4	.000200	.0163989
5	.000250	.0182415
6	.000300	.0199070
7	.000350	.0214391
8	.000400	.0228659
9	.000450	.0242067
10	.000500	.0254755
11	.000550	.0266830
12	.000600	.0278372
13	.000650	.0289448
14	.000700	.0300110
15	.000750	.0310400
16	.000800	.0320356
17	.000850	.0330008
18	.000900	.0339383
19	.000950	.0348503
20	.001000	.0357387
21	.001050	.0366054
22	.001100	.0374517
23	.001150	.0382792
24	.001200	.0390890
25	.001250	.0398822
26	.001300	.0406597
27	.001350	.0414225
28	.001400	.0421714
29	.001450	.0429071
30	.001500	.0436302
31	.001550	.0443414
32	.001600	.0450413
33	.001650	.0457303
34	.001700	.0464090
35	.001750	.0470779
36	.001800	.0477372
37	.001850	.0483875
38	.001900	.0490290
39	.001950	.0496622
40	.002000	.0502873
41	.002050	.0509046
42	.002100	.0515145
43	.002150	.0521171
44	.002200	.0527128
45	.002250	.0533017
46	.002300	.0538842
47	.002350	.0544603
48	.002400	.0550303
49	.002450	.0555944
50	.002500	.0561528

FLOW DIAGRAM



APPENDIX B

COMPUTER PROGRAM FOR NUMERICAL IMPLICIT METHOD USED IN DETERMINING THE ERROR IN NEGLECTING THE NON-LINEAR TERM IN THE PARTIAL DIFFERENTIAL EQUATION DESCRIBING RADIAL FLOW IN A SLIGHTLY COMPRESSIBLE FLUID IN POROUS MEDIA

An implicit numerical method was used to calculate values of dimensionless pressure for values of dimensionless time above 0.01. A description of the IBM 7090 computer program used in this calculation to determine the error in neglecting the non-linear term in the partial differential equation describing the flow of a slightly compressible fluid for radial flow is presented below. As in the previous appendix, the MAD (Michigan Algorithm Decoder) program, the program nomenclature, the flow sheet, a list of required information, and an example problem illustrating the use of the program are given.

The following information is required in the computer program:

1. The number of length increments. (Eighty length increments were found to be satisfactory, $\Delta w = \frac{1}{80}$.)
2. The number of time increments to be evaluated.
3. The numerical value for each time increment.
4. The number of M coefficients to be evaluated. (The value of M is defined by Equation (III-34)
5. The numerical values of the M coefficients to be evaluated.
6. The allowable error between the assumed and calculated pressures.

7. The maximum number of iterations allowed for a given time step.

The above information is used by the computer program to calculate values of dimensionless pressure versus dimensionless time for each value of the dimensionless coefficient M.

Deck Assembly

1. IBM center control cards

2. Radial, Implicit Numerical Program (MAD or Binary Deck)

3. Input Data

The input data is read in the "simplified input format". For example,

IMAX = 80, JMAX = 51, MMAX = 2, M(1) = 0., 1., B = 20,

MAXDIF = 1.E-6, TIME(1) = 0.0001, etc.*

Definition of these terms used in the simplified input are given in the nomenclature below.

Flow Sheet

The instruction J = a, b, J >c in the flow sheet means that the set of calculations is calculated for J = a and repeated in increments of b until the condition J >c is satisfied. For J = a, b, J >c refers to iterative calculations for a given box. Through d, J = a, b, J >c refers to iterative calculations through "circle d". The Flow Sheet is presented at end of Appendix.

Nomenclature used in IBM Program and Flow Sheet

A	TIME (I)/$(\Delta w)^2$
AA	C(I) * P(I)
B	Maximum number of iterations for a given time step
BB	C (I) * X (I)
C (I)	Terms defined by Equations (III-73) and (III-83)
D (I)	Terms defined by Equations (III-74) and (III-84)
DD	1-iΔw
DELW	Length increment, Δw
DP	Calculated Pressure Drop
DPSTAR	Assumed Pressure Drop
F	Scaling factor "a" defined by Equation (III-55)
I	Space index
I3	Counter used in IBM program
IMAX	Number of space increments
J	Time index
JMAX	Number of time increments
K (I)	Terms defined by Equation (III-71)
K1,K2, K3,K4	Terms used in IBM program
L	Counter used in IBM program
M	Dimensionless coefficient defined by Equation (III-34)
MAXDIF	Allowable difference between assumed and calculated pressure drop
MMAX	Number of dimensionless coefficients
N	I - 1
P	Dimensionless pressure

POLD	Dimensionless pressure from previous time increment
R	I + 1
TIME	Dimensionless time
X (I)	Terms defined by Equation (III-70)
XT	TIME (J) / F / F
Y (I)	Terms defined by Equation (III-69)
Z (I)	Terms defined by Equation (III-68)

IBM Center Control Cards

MAD Program

```
M.C. MILLER                    Q203N              002    045    000
M.C. MILLER                    Q203N              002    045    000
$COMPILE MAD, EXECUTE, DUMP, PUNCH OBJECT,PRINT OBJECT
              R EVALUATION OF NON-LINEAR TERM IN NON-LINEAR,
              RSECOND ORDER, PARTIAL DIFFERENTIAL EQUATION
              RFOR FLOW OF SLIGHTLY COMPRESSIBLE LIQUIDS
              RIN INFINITE RADIAL MODEL
              R
              RSOLVED BY DIFFERENCE EQUATIONS, IMPLICIT FORM
              R
               EXECUTE FTRAP.
               DIMENSION P(500),M(500),D(500),X(500),Y(500),Z(500),PSTAR(500
              1),K(500),DP(500),DPSTAR(500),C(500),POLD(500),TIME(200)
               INTEGER I,J,R,L,  N,IMAX,JMAX,MMAX,ALMAX,I1,I2,B,I3,M1
               F=1.
               READ DATA,IMAX,JMAX,M(1)...M(MAX),MMAX,B,TIME(1)...TIME(MAX)
               DELW=1./IMAX
               THROUGH S14, FOR L=1,1,L.G.MMAX
               THROUGH S1,FOR I=0,1,I.G.IMAX
 S1            P(I  )=0.0
               A=TIME(1)/DELW/DELW
               PRINT COMMENT$1EVALUATION OF NON-LINEAR TERM IN NON-LINEAR, S
              1ECOND ORDER,PARTIAL DIFFERENTIAL EQUATION FOR FLOW OF SLIGHTL
              2Y$
               PRINT COMMENT $ COMPRESSIBLE LIQUIDS IN INFINITE RADIAL MODEL
              1$
               PRINT COMMENT $0SOLVED BY DIFFERENCE EQUATIONS,IMPLICIT FORM$
               PRINT RESULTS M(L),       DELW, MAXDIF,B ,IMAX,F
               PRINT COMMENT $0  N        TIME     PRESSURE    BN          P(
              1IMAX-1)$
               J=0
               I3=1
 S2            THROUGH S3,FOR I=0,1,I.G.IMAX
               POLD(I) = P(I)
 S3            PSTAR(I  )=P(I  )
 S12           THROUGH S4,FOR I=1,1,I.G.IMAX-1
               DD=1.-I*DELW
               K1=DD*DD
               K2=DD*DELW/2.
               K3=(M(L)/4.)*K1         *(PSTAR(I+1   )-PSTAR(I-1))
               K4=K2/(F-ELOG.(DD))
               Z(I)=K1-K2+K3+K4
               Y(I)=-2.*K1-1./A
               X(I)=K1+K2-K3-K4
 S4            K(I)=-POLD(I)/A
               C(1)=-(4.*Z(1)+Y(1))/(X(1)-3.*Z(1))
               D(1)=(K(1)-2.*DELW/F
              1                *Z(1))/(X(1)-3.*Z(1))
               THROUGH S6,FOR I=1,1,I.G.IMAX-1
               R=I+1
               BB=C(I)*X(I)
               C(R)=-Z(I)/(BB           +Y(I))
 S6            D(R)=(K(I)-D(I)*X(I))/(BB        +Y(I))
               P(IMAX   )=0.
               THROUGH S7,FOR I= IMAX     ,-1,I.L.1
               AA=C(I)*P(I)
 S7            P(I-1   )=AA          +D(I)
               WHENEVER M(L) .E.0., TRANSFER TO S10
              RTEST
              R
               THROUGH S8,FOR I=1,1,I.G.(IMAX-1)
               R=I+1
               N=I-1
               DP(I)=P(R   )-P(N   )
               DPSTAR(I)=PSTAR(R   )-PSTAR(N   )
               WHENEVER.ABS.(DP(I)-DPSTAR(I)).G.MAXDIF,TRANSFER TO S9
 S8            CONTINUE
               TRANSFER TO S10
 S9            I3=I3+1
               WHENEVER I3.G.B,TRANSFER TO S15
               THROUGH S11,FOR I=0,1,I.G.IMAX
 S11           PSTAR(I   )=P(I   )
               TRANSFER TO S12
 S10           J=J+1
               XT=TIME(J)/F/F
               WHENEVER XT       .G. 1000., TRANSFER TO S5
               PRINT FORMAT RSLT1,J,XT,P(0),I3,P(IMAX-1)
               VECTOR VALUES RSLT1=$I4,F12.4,F12.7,I5,1PE20.7*$
               TRANSFER TO S13
 S5            PRINT FORMAT RSLT2,J,XT       ,P(0),I3 ,P(IMAX-1)
               VECTOR VALUES RSLT2=$I4,1PE12.3,F12.7,I5,1PE20.7*$
 S13           WHENEVER J.E.JMAX,TRANSFER TO S14
               A=(TIME(J+1)-TIME(J))/DELW/DELW
               I3=1
               TRANSFER TO S2
 S15           PRINT COMMENT $1  TOO MANY INTERATIONS IN PSTAR CALCULATIONS$
               PRINT RESULTS P(0)...P(IMAX),PSTAR(0)...PSTAR(IMAX)
 S14           CONTINUE
               END OF PROGRAM
```

Data Input

```
$DATA
IMAX=80, JMAX=51,MMAX=2, M(1)=0.,1.,
B=20, MAXDIF=1.E- 6,
TIME(1)=.0001,.00015,.0002,.0003,.0004,.0005,.0006,.0007,.0008,.0009,
.001,.0015,.002,.003,.004,.005,.006,.007,.008,.009,
.01,.015,.02,.03,.04,.05,.06,.07,.08,.09,
.1,.15,.2,.3,.4,.5,.6,.7,.8,.9,1.,1.5,2.,3.,4.,5.,6.,7.,8.,9.,10.,
15.,20.,30.,40.,50.,60.,70.,80.,90.,100.,150.,200.,300.,400.,500.,600.,
700.,800.,900.,1000.,1500.,2000.,3000.,4000.,5000.,6000.,7000.,8000.,
9000.,10000.*
```

IBM Program Print Out

EVALUATION OF NON-LINEAR TERM IN NON-LINEAR, SECOND ORDER,PARTIAL DIFFERENTIAL EQUATION FOR FLOW OF SLIGHTLY COMPRESSIBLE LIQUIDS IN INFINITE RADIAL MODEL

SOLVED BY DIFFERENCE EQUATIONS,IMPLICIT FORM

M(1) = .000000, DELW = .012500, MAXDIF = 10.000000E-07, B =

IMAX = 80, F = 1.000000

N	TIME	PRESSURE	BN	P(IMAX-1)
1	.0001	.0133011	1	.0000000E 00
2	.0001	.0154438	1	.0000000E 00
3	.0002	.0173404	1	.0000000E 00
4	.0003	.0205250	1	.0000000E 00
5	.0004	.0233026	1	.0000000E 00
6	.0005	.0257895	1	.0000000E 00
7	.0006	.0280575	1	.0000000E 00
8	.0007	.0301540	1	.0000000E 00
9	.0008	.0321117	1	.0000000E 00
10	.0009	.0339541	1	.0000000E 00
11	.0010	.0356991	1	.0000000E 00
12	.0015	.0429614	1	.0000000E 00
13	.0020	.0492553	1	.0000000E 00
14	.0030	.0596028	1	.0000000E 00
15	.0040	.0685006	1	.0000000E 00
16	.0050	.0763816	1	.0000000E 00
17	.0060	.0835065	1	.0000000E 00
18	.0070	.0900438	1	.0000000E 00
19	.0080	.0961090	1	3.4972836E-38
20	.0090	.1017849	1	2.5337114E-37
21	.0100	.1071333	1	1.4480600E-36
22	.0150	.1290299	1	7.2678235E-24
23	.0200	.1477552	1	1.4805471E-22
24	.0300	.1779687	1	6.1525390E-18
25	.0400	.2035498	1	9.2093271E-17
26	.0500	.2259100	1	7.3227713E-16
27	.0600	.2458918	1	4.0990127E-15
28	.0700	.2640369	1	1.8094378E-14
29	.0800	.2807147	1	6.6971315E-14
30	.0900	.2961888	1	2.1592918E-13
31	.1000	.3106538	1	6.2244555E-13
32	.1500	.3682615	1	4.5184912E-09
33	.2000	.4163586	1	3.5444978E-08
34	.3000	.4912657	1	1.5180347E-06
35	.4000	.5527991	1	7.7374445E-06
36	.5000	.6051975	1	2.4491473E-05
37	.6000	.6509620	1	6.0029113E-05
38	.7000	.6916831	1	1.2458737E-04
39	.8000	.7284333	1	2.2967137E-04
40	.9000	.7619696	1	3.8725779E-04
41	1.0000	.7928471	1	6.0905688E-04
42	1.5000	.9098961	1	4.0091186E-03
43	2.0000	1.0036693	1	9.7457930E-03
44	3.0000	1.1416780	1	2.7637878E-02
45	4.0000	1.2499011	1	4.8928528E-02
46	5.0000	1.3384766	1	7.1398470E-02
47	6.0000	1.4131096	1	9.3642391E-02
48	7.0000	1.4772550	1	1.1485905E-01
49	8.0000	1.5331367	1	1.3464326E-01
50	9.0000	1.5822681	1	1.5283222E-01
51	10.0000	1.6257329	1	1.6940413E-01

IBM Program Print Out

EVALUATION OF NON-LINEAR TERM IN NON-LINEAR, SECOND ORDER,PARTIAL DIFFERENTIAL EQUATION FOR FLOW OF SLIGHTLY COMPRESSIBLE LIQUIDS IN INFINITE RADIAL MODEL

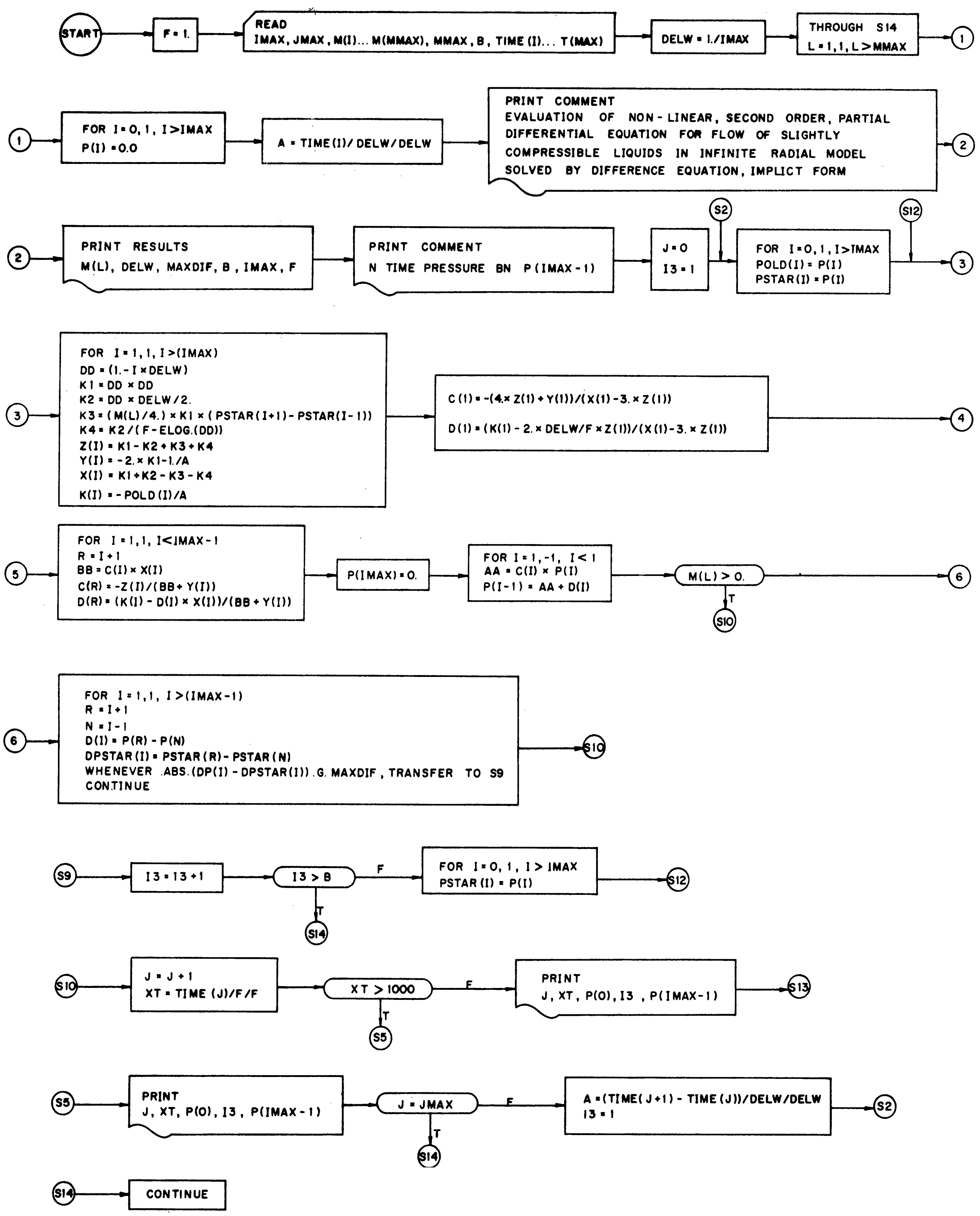
SOLVED BY DIFFERENCE EQUATIONS,IMPLICIT FORM

M(2) = 1.000000, DELW = .012500, MAXDIF = 10.000000E-07, B = 2

IMAX = 80, F = 1.000000

N	TIME	PRESSURE	BN	P(IMAX-1)
1	.0001	.0133231	3	.0000000E 00
2	.0001	.0154783	3	.0000000E 00
3	.0002	.0173884	2	.0000000E 00
4	.0003	.0206026	3	.0000000E 00
5	.0004	.0234113	3	.0000000E 00
6	.0005	.0259301	3	.0000000E 00
7	.0006	.0282309	2	.0000000E 00
8	.0007	.0303605	2	.0000000E 00
9	.0008	.0323517	2	.0000000E 00
10	.0009	.0342279	2	.0000000E 00
11	.0010	.0360069	2	.0000000E 00
12	.0015	.0434415	3	.0000000E 00
13	.0020	.0499087	3	.0000000E 00
14	.0030	.0606041	3	.0000000E 00
15	.0040	.0698504	3	.0000000E 00
16	.0050	.0780802	3	.0000000E 00
17	.0060	.0855540	3	.0000000E 00
18	.0070	.0924401	3	.0000000E 00
19	.0080	.0988541	3	3.5976035E-38
20	.0090	.1048786	3	2.6073725E-37
21	.0100	.1105753	3	1.4907704E-36
22	.0150	.1342009	3	7.7363950E-24
23	.0200	.1546447	3	1.5785186E-22
24	.0300	.1882595	3	6.7221543E-18
25	.0400	.2172161	3	1.0092352E-16
26	.0500	.2429319	3	8.0487968E-16
27	.0600	.2662525	3	4.5187569E-15
28	.0700	.2877217	3	2.0006314E-14
29	.0800	.3077101	3	7.4267524E-14
30	.0900	.3264824	3	2.4016695E-13
31	.1000	.3442339	3	6.9438660E-13
32	.1500	.4179798	4	5.5360370E-09
33	.2000	.4819923	4	4.4000771E-08
34	.3000	.5880698	4	2.0264578E-06
35	.4000	.6803159	4	1.0607364E-05
36	.5000	.7630929	4	3.4408661E-05
37	.6000	.8389783	4	8.6384336E-05
38	.7000	.9096145	4	1.8362337E-04
39	.8000	.9761125	4	3.4670887E-04
40	.9000	1.0392574	4	5.9883187E-04
41	1.0000	1.0996277	4	9.6483438E-04
42	1.5000	1.3621151	6	8.0618963E-03
43	2.0000	1.6004583	6	2.2088459E-02
44	3.0000	2.0277319	8	8.2024909E-02
45	4.0000	2.4276088	8	1.7708651E-01
46	5.0000	2.8113426	9	3.1038776E-01
47	6.0000	3.1851252	9	4.8506430E-01
48	7.0000	3.5526329	9	7.0424624E-01
49	8.0000	3.9161483	9	9.7126147E-01
50	9.0000	4.2771963	10	1.2903542E 00
51	10.0000	4.6369280	10	1.6683324E 00

FLOW DIAGRAM



APPENDIX C

COMPUTER PROGRAM FOR NUMERICAL EXPLICIT METHOD USED IN DETERMINING THE ERROR IN NEGLECTING THE NON-LINEAR TERM IN THE PARTIAL DIFFERENTIAL EQUATION DESCRIBING LINEAR FLOW OF A SLIGHTLY COMPRESSIBLE FLUID IN POROUS MEDIA

A description of the IBM 7090 computer program used to determine the error in neglecting the non-linear term in the partial differential equation describing the flow of a slightly compressible fluid for linear flow is presented below. Although this program is very similar to the method used for radial flow described in Appendix A, this appendix is enclosed for completeness. The MAD (Michigan Algorithm Decoder) program, the program nomenclature, the flow diagram, a list of required information, and an example problem illustrating the use of the program are given.

The following information is required in the computer program:

1. The number of length increments. (Eighty length increments were found to be satisfactory, $\Delta w = \frac{1}{80}$.)

2. The value of the time increment. ($\Delta t = 0.00005$ is an acceptable value of Δt for $\Delta w = \frac{1}{80}$. Stability requirements demand that $(\Delta t)/(\Delta w)^2$ is less than 0.5.)

3. The number of time increments to be evaluated.

4. The number of M coefficients to be evaluated. (The value of M is defined by Equation (III-100).)

5. The numerical values of the M coefficients to be evaluated.

This information is used by the computer to calculate values of dimensionless pressure versus dimensionless time for each value of the dimensionless coefficient M.

Deck Assembly

1. IBM center control cards
2. Linear, Explicit Numerical Program (MAD or Binary Deck)
3. Input Data

The input data is read in "simplified input format". For example,

IMAX = 80, DELT = 0.00005, MMAX = 2, JMAX = 49, M(1) = 0., 1. *

Definitions of the terms used in this input are given below in the nomenclature.

Flow Sheet

The instruction J = a, b, J > c in the flow sheet means that the set of calculations is calculated for J = a and repeated in increments of b until the condition J > c is satisfied. For J = a, b, J > c refers to iterative calculations for a given box. Through d, J = a, b, J > c refers to iterative calculations through "circle d". The Flow Sheet is presented at end of Appendix.

Nomenclature used in IBM Program and Flow Sheet

ADIM	Term used to dimension P(I,J) vector
A	$\Delta t/(\Delta w)^2$
DELT	Δt

DELW	Δw
DT	$\Delta t/(F)^2$
F	Scaling coefficient "a". In this dissertation, F = 1.0.
I	Space index
IMAX	Number of space increments, Δw
J	Time index
JMAX	Number of time increments
K1	Term used in program, K1 = $1 - i\Delta w$
M	Dimensionless coefficient defined by Equation (III-100)
MMAX	Number of dimensionless coefficients
P	Dimensionless Pressure
R	Counter used in IBM program
X2	Term defined by X2 = $P_{i+1,j} - P_{i-1,j}$
X3	Term defined by X3 = $(X2)^2$
X4	Absolute value of X2
Z	Value of dimensionless time at given time step, Z = $j\Delta t$

IBM Center Control Cards

```
M.C. MILLER                        Q203N               002   045   000
M.C. MILLER                        Q203N               002   045   000
$COMPILE MAD, EXECUTE, DUMP, PUNCH OBJECT,PRINT OBJECT
```

MAD Program

```
            R  EVALUATION OF NON-LINEAR TERM IN NON-LINEAR,
            R SECOND ORDER, PARTIAL DIFFERENTIAL EQUATION
            R FOR FLOW OF SLIGHTLY COMPRESSIBLE LIQUIDS
            R IN INFINITE LINEAR MODEL
            R
            RSOLVED BY DIFFERENCE EQUATIONS,EXPLICIT FORM
            R
             DIMENSION P( 8100,ADIM),M(10)
             INTEGER I,J,K,L,R,N,IMAX,MMAX,JMAX
             VECTOR VALUES ADIM=2,101,100
             F=1.
 S4          READ DATA,IMAX,DELT,M,MMAX,JMAX
             DELW=1./IMAX
             A=DELT/DELW/DELW
              DT =DELT/F/F
             PRINT RESULTS DELT,IMAX,MMAX,JMAX, A,M(1)...M(MMAX)
             P(IMAX,  1)=0.
             THROUGH S6,FOR R=1,1,R.G.MMAX
             THROUGH S1,FOR I=0,1,I.G.IMAX
 S1          P(I,0)=0.0
             Z=0.
             J=0
             PRINT COMMENT $1EVALUATION OF NON-LINEAR TERM IN NON-LINEAR
            1SECOND ORDER, PARTIAL DIFFERENTIAL EQUATION FOR FLOW OF SLIGH
            2TLY$
             PRINT COMMENT $0COMPRESSIBLE LIQUIDS IN INFINITE LINEAR MODEL
            1$
             PRINT RESULTS M(R),DELT,DELW,A,F
             PRINT COMMENT $0  N     TIME    PRESSURE$
 S5          THROUGH S2,FOR I=1,1,I.G.(IMAX-1)
             X2=P(I+1,0)-P(I-1,0)
             X4=.ABS.(X2)
             WHENEVER X4 .L. 1.E-10, X2=0.
             X3=X2*X2
             K1=(1.-I*DELW)
             P(I,  1)=P(I,0)+A*(K1*K1*(P(I+1,0)-2.*P(I,0)+P(I-1,0))
            1-DELW/2.*K1*X2
            2+M(R)*K1*K1/4.*X3)
             WHENEVER P(I,  1).L.1.E-20,P(I,1)=0.
 S2          CONTINUE
             P(0, 1)=(2.*DELW/F+4.*P(1, 1)-P(2,  1))/3.
             J=J+1
             Z=Z+DT
             PRINT FORMAT RSLT1,J,Z,P(0,1)
             VECTOR VALUES RSLT1=$I4,F9.6,F12.7*$
             WHENEVER J.G.JMAX,TRANSFER TO S6
             THROUGH S3, FOR I=0,1,I.G.IMAX
 S3          P(I,0)=P(I,1)
             TRANSFER TO S5
 S6          CONTINUE
             TRANSFER TO S4
             END OF PROGRAM
$ DATA
IMAX=80,   DELT=.00005,   MMAX=2,   JMAX= 49,  M(1)=0.,1.*
```

Data Input

IBM Program Print Out

EVALUATION OF NON-LINEAR TERM IN NON-LINEAR, SECOND ORDER, PARTIAL DIFFERENTIAL EQUATION FOR FLOW OF SLIGHTLY

COMPRESSIBLE LIQUIDS IN INFINITE LINEAR MODEL

M(1) = .000000, DELT = 5.000000E-05, DELW = .012500, A = .320000

F = 1.000000

N	TIME	PRESSURE
1	.000050	.0083333
2	.000100	.0118225
3	.000150	.0143279
4	.000200	.0164336
5	.000250	.0182852
6	.000300	.0199593
7	.000350	.0214997
8	.000400	.0229345
9	.000450	.0242831
10	.000500	.0255596
11	.000550	.0267747
12	.000600	.0279364
13	.000650	.0290513
14	.000700	.0301248
15	.000750	.0311610
16	.000800	.0321638
17	.000850	.0331361
18	.000900	.0340806
19	.000950	.0349995
20	.001000	.0358949
21	.001050	.0367684
22	.001100	.0376217
23	.001150	.0384560
24	.001200	.0392726

25	.001250	.0400725
26	.001300	.0408568
27	.001350	.0416263
28	.001400	.0423818
29	.001450	.0431241
30	.001500	.0438539
31	.001550	.0445717
32	.001600	.0452782
33	.001650	.0459738
34	.001700	.0466590
35	.001750	.0473344
36	.001800	.0480002
37	.001850	.0486570
38	.001900	.0493050
39	.001950	.0499446
40	.002000	.0505762
41	.002050	.0511999
42	.002100	.0518162
43	.002150	.0524253
44	.002200	.0530273
45	.002250	.0536226
46	.002300	.0542114
47	.002350	.0547939
48	.002400	.0553702
49	.002450	.0559407
50	.002500	.0565053

IBM Program Print Out

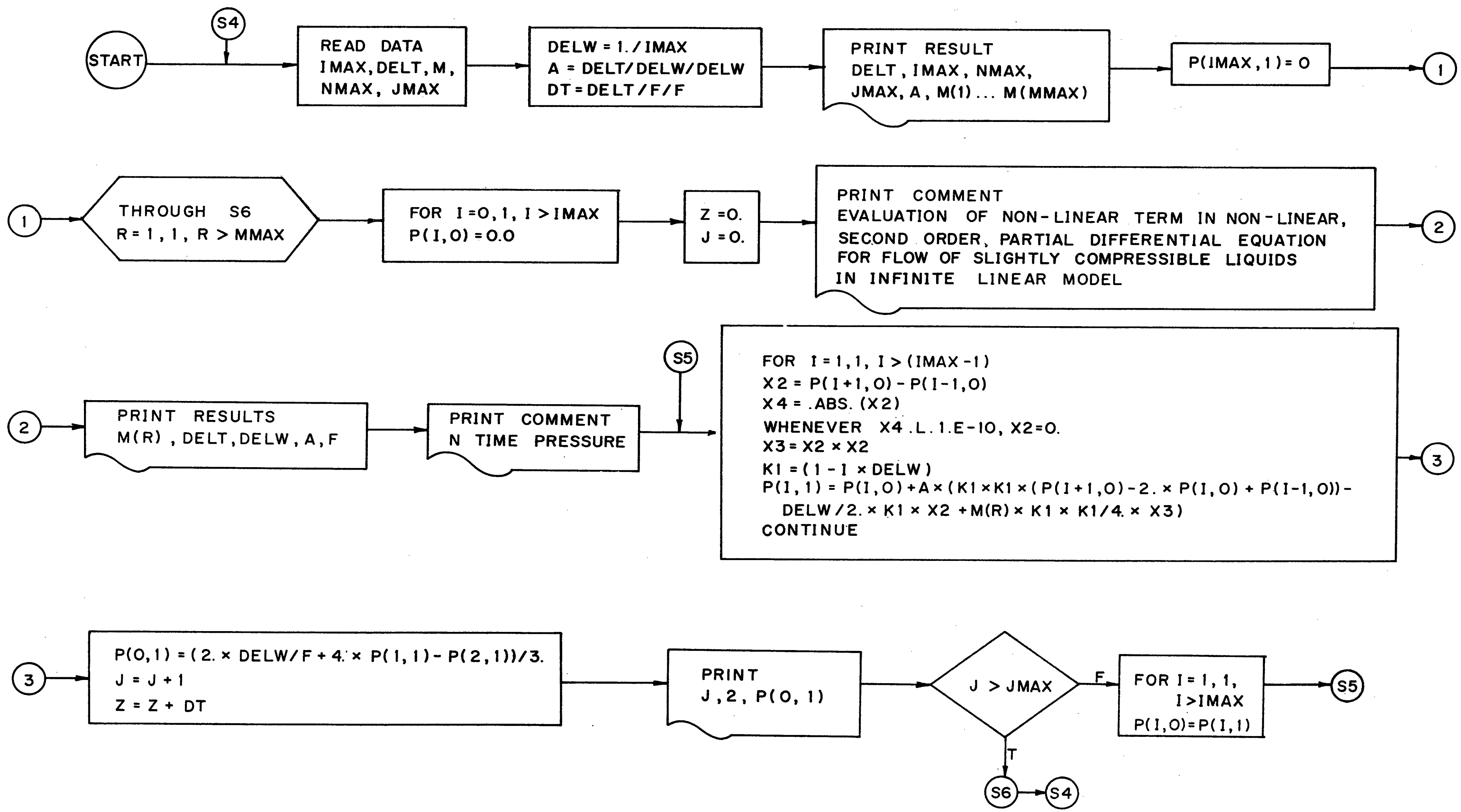
EVALUATION OF NON-LINEAR TERM IN NON-LINEAR, SECOND ORDER, PARTIAL DIFFERENTIAL EQUATION FOR FLOW OF SLIGHTLY COMPRESSIBLE LIQUIDS IN INFINITE LINEAR MODEL

M(2) = 1.000000, DELT = 5.000000E-05, DELW = .012500, A = .32000

F = 1.000000

N	TIME	PRESSURE
1	.000050	.0083333
2	.000100	.0118297
3	.000150	.0143475
4	.000200	.0164670
5	.000250	.0183333
6	.000300	.0200228
7	.000350	.0215790
8	.000400	.0230299
9	.000450	.0243950
10	.000500	.0256881
11	.000550	.0269199
12	.000600	.0280986
13	.000650	.0292306
14	.000700	.0303211
15	.000750	.0313747
16	.000800	.0323947
17	.000850	.0333844
18	.000900	.0343464
19	.000950	.0352829
20	.001000	.0361959
21	.001050	.0370871
22	.001100	.0379580
23	.001150	.0388100
24	.001200	.0396444
25	.001250	.0404621
26	.001300	.0412643
27	.001350	.0420516
28	.001400	.0428251
29	.001450	.0435853
30	.001500	.0443330
31	.001550	.0450688
32	.001600	.0457933
33	.001650	.0465069
34	.001700	.0472102
35	.001750	.0479036
36	.001800	.0485875
37	.001850	.0492624
38	.001900	.0499285
39	.001950	.0505863
40	.002000	.0512360
41	.002050	.0518779
42	.002100	.0525124
43	.002150	.0531396
44	.002200	.0537599
45	.002250	.0543734
46	.002300	.0549804
47	.002350	.0555811
48	.002400	.0561757
49	.002450	.0567644
50	.002500	.0573474

FLOW DIAGRAM



APPENDIX D

COMPUTER PROGRAM FOR NUMERICAL IMPLICIT METHOD USED IN DETERMINING THE ERROR IN NEGLECTING THE NON-LINEAR TERM IN THE PARTIAL DIFFERENTIAL EQUATION DESCRIBING LINEAR FLOW OF A SLIGHTLY COMPRESSIBLE FLUID IN POROUS MEDIA

An explicit numerical method was used to calculate values of dimensionless pressure for values of dimensionless time above 0.01. Although this computer program is very similar to Appendix B it is included for completeness. The MAD (Michigan Algorithm Decoder) program, the program nomenclature, the flow sheet, a list of required information, and an example problem illustrating the use of the IBM 7090 program are described below.

The following information is required in the computer program:

1. The number of length increments. (Eighty length increments were found to be satisfactory, $\Delta w = \frac{1}{80}$.)
2. The number of time increments to be evaluated.
3. The numerical value for each time increment.
4. The number of M coefficients to be evaluated. (The value of M is defined by Equation (III-100).)
5. The numerical values of the M coefficients to be evaluated.
6. The allowable error between the assumed and calculated pressures.
7. The maximum number of iterations allowed for a given time step.

The computer program uses this information to calculate the numerical values of dimensionless time versus dimensionless for each value of the dimensionless coefficient M.

Deck Assembly

1. IBM center control cards
2. Linear, Implicit Numerical Program (MAD or Binary Deck)
3. Input Data

The input data is read in the "simplified input format". For example,

IMAX = 80, JMAX = 51, MMAX = 2, M(1) = 0., 1., B = 20, MAXDIF = 1.E-6, TIME(1) = 0.0001, etc.*

Flow Sheet

The instruction J = a, b, J > c in the flow sheet means that the set of calculations is calculated for J = a and repeated in increments of b until the condition J > c is satisfied. For J = a, b, J > c refers to iterative calculations for a given box. Through d, J = a, b, J > c refers to iterative calculations through "circle d". The Flow Sheet is presented at end of Appendix.

Nomenclature used in IBM Program and Flow Sheet

A	$TIME(I)/(\Delta w)^2$
AA	C(I) * X (I)
B	Maximum number of iterations for a given time step
BB	C(I) * X(I)
C(I)	Terms defined by Equations (III-126) and (III-138)
D(I)	Terms defined by Equations (III-127) and (III-139)

DD	$1 - i\Delta w$
DELW	Length increment, Δw
DP	Calculated pressure drop
DPSTAR	Assumed pressure drop
F	Scaling factor "a" defined by Equation (III-113)
I	Space index
I3	Counter used in IBM program
IMAX	Number of space increments
J	Time index
JMAX	Number of time increments
K(I)	Terms defined by Equation (III-124)
K1,K2,K3	Terms used in IBM program
L	Counter used in IBM program
M	Dimensionless coefficient defined by Equation (III-100)
MAXDIF	Allowable difference between assumed and calculated pressure drop
MMAX	Number of dimensionless coefficients
N	I - 1
P	Dimensionless pressure
POLD	Dimensionless pressure from previous time increment
PSTAR	Assumed value for dimensionless pressure
R	I + 1
TIME	Dimensionless time
X(I)	Terms defined by Equation (III-123)
XT	TIME (J) / F / F
Y(I)	Terms defined by Equation (III-122)
Z(I)	Terms defined by Equation (III-121)

IBM Center Control Cards

MAD Program

```
 M.C. MILLER                      Q203N                002    045    000
 M.C. MILLER                      Q203N                002    045    000
$COMPILE MAD, EXECUTE, DUMP, PUNCH OBJECT,PRINT OBJECT
          R EVALUATION OF NON-LINEAR TERM IN NON-LINEAR,
          RSECOND ORDER, PARTIAL DIFFERENTIAL EQUATION
          RFOR FLOW OF SLIGHTLY COMPRESSIBLE LIQUIDS
          RIN INFINITE LINEAR MODEL
          R
          RSOLVED BY DIFFERENCE EQUATIONS, IMPLICIT FORM
          R
           EXECUTE FTRAP.
           DIMENSION P(500),M(500),D(500),X(500),Y(500),Z(500),PSTAR(500
          1),K(500),DP(500),DPSTAR(500),C(500),POLD(500),TIME(200)
           INTEGER I,J,R,L,  N,IMAX,JMAX,MMAX,ALMAX,I1,I2,B,I3,M1
           F=1.
           READ DATA,IMAX,JMAX,M(1)...M(MAX),MMAX,B,TIME(1)...TIME(MAX)
           DELW=1./IMAX
           THROUGH S14, FOR L=1,1,L.G.MMAX
           THROUGH S1,FOR I=0,1,I.G.IMAX
S1         P(I  )=0.0
           A=TIME(1)/DELW/DELW
           PRINT COMMENT$1EVALUATION OF NON-LINEAR TERM IN NON-LINEAR, S
          1ECOND ORDER,PARTIAL DIFFERENTIAL EQUATION FOR FLOW OF SLIGHTL
          2Y$
           PRINT COMMENT $ COMPRESSIBLE LIQUIDS IN INFINITE LINEAR MODEL
          1$
           PRINT COMMENT $0SOLVED BY DIFFERENCE EQUATIONS,IMPLICIT FORM$
           PRINT RESULTS M(L),     DELW,  MAXDIF,B ,IMAX,F
           PRINT COMMENT $0  N         TIME     PRESSURE   BN             P(
          1IMAX-1)$
           J=0
           I3=1
S2         THROUGH S3,FOR I=0,1,I.G.IMAX
           POLD(I) = P(I)
S3         PSTAR(I  )=P(I   )
S12        THROUGH S4,FOR I=1,1,I.G.IMAX-1
           DD=(1.-I*DELW)
           K1=DD*DD
           K2=DD*DELW/2.
           K3=(M(L)/4.)*K1          *(PSTAR(I+1  )-PSTAR(I-1))
           Z(I)=K1-K2+K3
           Y(I)=-2.*K1-1./A
           X(I)=K1+K2-K3
S4         K(I)=-POLD(I)/A
           C(1)=-(4.*Z(1)+Y(1))/(X(1)-3.*Z(1))
           D(1)=(K(1)-2.*DELW/F
          1                   *Z(1))/(X(1)-3.*Z(1))
           THROUGH S6,FOR I=1,1,I.G.IMAX-1
           R=I+1
           BB=C(I)*X(I)
           C(R)=-Z(I)/(BB        +Y(I))
S6         D(R)=(K(I)-D(I)*X(I))/(BB        +Y(I))
           P(IMAX  )=0.
           THROUGH S7,FOR I= IMAX    ,-1,I.L.1
           AA=C(I)*P(I)
S7         P(I-1  )=AA          +D(I)
           WHENEVER M(L) .E.0., TRANSFER TO S10
          RTEST
          R
           THROUGH S8,FOR I=1,1,I.G.(IMAX-1)
           R=I+1
           N=I-1
           DP(I)=P(R  )-P(N  )
           DPSTAR(I)=PSTAR(R  )-PSTAR(N  )
           WHENEVER.ABS.(DP(I)-DPSTAR(I)).G.MAXDIF,TRANSFER TO S9
S8         CONTINUE
           TRANSFER TO S10
S9         I3=I3+1
           WHENEVER I3.G.B,TRANSFER TO S14
           THROUGH S11,FOR I=0,1,I.G.IMAX
S11        PSTAR(I  )=P(I  )
           TRANSFER TO S12
S10        J=J+1
           XT=TIME(J)/F/F
           WHENEVER XT      .G. 1000., TRANSFER TO S5
           PRINT FORMAT RSLT1,J,XT,P(0),I3,P(IMAX-1)
           VECTOR VALUES RSLT1=$I4,F12.4,F12.7,I5,1PE20.7*$
           TRANSFER TO S13
S5         PRINT FORMAT RSLT2,J,XT     ,P(0),I3 ,P(IMAX-1)
           VECTOR VALUES RSLT2=$I4,1PE12.3,F12.7,I5,1PE20.7*$
S13        WHENEVER J.E.JMAX,TRANSFER TO S14
           A=(TIME(J+1)-TIME(J))/DELW/DELW
           I3=1
           TRANSFER TO S2
S14        CONTINUE
           END OF PROGRAM
```

Data Input

```
$DATA
IMAX=80,  JMAX=51,MMAX=2,  M(1)=0.,1.,
B=20, MAXDIF=1.E- 6,
TIME(1)=.0001,.00015,.0002,.0003,.0004,.0005,.0006,.0007,.0008,.0009,
.001,.0015,.002,.003,.004,.005,.006,.007,.008,.009,
.01,.015,.02,.03,.04,.05,.06,.07,.08,.09,
.1,.15,.2,.3,.4,.5,.6,.7,.8,.9,1.,1.5,2.,3.,4.,5.,6.,7.,8.,9.,10.,
15.,20.,30.,40.,50.,60.,70.,80.,90.,100.,150.,200.,300.,400.,500.,600.,
700.,800.,900.,1000.,1500.,2000.,3000.,4000.,5000.,6000.,7000.,8000.,
9000.,10000.*
```

IBM Program Print Out

EVALUATION OF NON-LINEAR TERM IN NON-LINEAR, SECOND ORDER,PARTIAL DIFFERENTIAL EQUATION FOR FLOW OF SLIGHTLY COMPRESSIBLE LIQUIDS IN INFINITE LINEAR MODEL

SOLVED BY DIFFERENCE EQUATIONS.IMPLICIT FORM

M(1) = .000000, DELW = .012500, MAXDIF = 10.000000E-07, B = 20

IMAX = 80, F = 1.000000

N	TIME	PRESSURE	BN	P(IMAX-1)
1	.0001	.0133468	1	.0000000E 00
2	.0001	.0155127	1	.0000000E 00
3	.0002	.0174327	1	.0000000E 00
4	.0003	.0206648	1	.0000000E 00
5	.0004	.0234902	1	.0000000E 00
6	.0005	.0260250	1	.0000000E 00
7	.0006	.0283411	1	.0000000E 00
8	.0007	.0304857	1	.0000000E 00
9	.0008	.0324915	1	.0000000E 00
10	.0009	.0343821	1	.0000000E 00
11	.0010	.0361753	1	.0000000E 00
12	.0015	.0436779	1	.0000000E 00
13	.0020	.0502114	1	.0000000E 00
14	.0030	.0610352	1	.0000000E 00
15	.0040	.0704066	1	.0000000E 00
16	.0050	.0787589	1	.0000000E 00
17	.0060	.0863525	1	.0000000E 00
18	.0070	.0933564	1	1.1088234E-38
19	.0080	.0998863	1	9.2418500E-38
20	.0090	.1060250	1	6.0052935E-37
21	.0100	.1118342	1	3.3806798E-36
22	.0150	.1359989	1	1.7167136E-23
23	.0200	.1569611	1	3.4995945E-22
24	.0300	.1915496	1	1.4700575E-17
25	.0400	.2214234	1	2.2032544E-16
26	.0500	.2480053	1	1.7540985E-15
27	.0600	.2721461	1	9.8308289E-15
28	.0700	.2943941	1	4.3449056E-14
29	.0800	.3151238	1	1.6100764E-13
30	.0900	.3346030	1	5.1974005E-13
31	.1000	.3530298	1	1.4999970E-12
32	.1500	.4296129	1	1.1288304E-08
33	.2000	.4960114	1	8.8980350E-08
34	.3000	.6055173	1	3.9127982E-06
35	.4000	.7000678	1	2.0126939E-05
36	.5000	.7841834	1	6.4224688E-05
37	.6000	.8605650	1	1.5861864E-04
38	.7000	.9309516	1	3.3162597E-04
39	.8000	.9965303	1	6.1570762E-04
40	.9000	1.0581490	1	1.0454163E-03
41	1.0000	1.1164367	1	1.6554139E-03
42	1.5000	1.3586604	1	1.1625442E-02
43	2.0000	1.5686593	1	2.9132885E-02
44	3.0000	1.9149747	1	8.7942625E-02
45	4.0000	2.2138803	1	1.6229092E-01
46	5.0000	2.4794863	1	2.4530167E-01
47	6.0000	2.7200514	1	3.3206388E-01
48	7.0000	2.9407316	1	4.1933646E-01
49	8.0000	3.1449153	1	5.0509205E-01
50	9.0000	3.3349338	1	5.8812507E-01
51	10.0000	3.5124638	1	6.6776519E-01

IBM Program Print Out

EVALUATION OF NON-LINEAR TERM IN NON-LINEAR, SECOND ORDER,PARTIAL DIFFERENTIAL EQUATION FOR FLOW OF SLIGHTLY COMPRESSIBLE LIQUIDS IN INFINITE LINEAR MODEL

SOLVED BY DIFFERENCE EQUATIONS,IMPLICIT FORM

M(2) = 1.000000, DELW = .012500, MAXDIF = 10.000000E-07, B = 2C

IMAX = 80, F = 1.000000

N	TIME	PRESSURE	BN	P(IMAX-1)
1	.0001	.0133689	3	.0000000E 00
2	.0001	.0155474	3	.0000000E 00
3	.0002	.0174812	2	.0000000E 00
4	.0003	.0207432	3	.0000000E 00
5	.0004	.0236002	3	.0000000E 00
6	.0005	.0261676	3	.0000000E 00
7	.0006	.0285169	2	.0000000E 00
8	.0007	.0306954	2	.0000000E 00
9	.0008	.0327355	2	.0000000E 00
10	.0009	.0346607	2	.0000000E 00
11	.0010	.0364887	2	.0000000E 00
12	.0015	.0441688	3	.0000000E 00
13	.0020	.0508817	3	.0000000E 00
14	.0030	.0620681	3	.0000000E 00
15	.0040	.0718052	3	.0000000E 00
16	.0050	.0805253	3	.0000000E 00
17	.0060	.0884888	3	.0000000E 00
18	.0070	.0958643	3	1.1404954E-38
19	.0080	.1027672	3	9.5095649E-38
20	.0090	.1092804	3	6.1818725E-37
21	.0100	.1154652	3	3.4815888E-36
22	.0150	.1415240	3	1.8302682E-23
23	.0200	.1643947	3	3.7373441E-22
24	.0300	.2028455	3	1.6111119E-17
25	.0400	.2366256	3	2.4224841E-16
26	.0500	.2671563	3	1.9348174E-15
27	.0600	.2952857	3	1.0878372E-14
28	.0700	.3215597	3	4.8233255E-14
29	.0800	.3463504	3	1.7931378E-13
30	.0900	.3699235	3	5.8071772E-13
31	.1000	.3924754	3	1.6814848E-12
32	.1500	.4902508	4	1.4069004E-08
33	.2000	.5783575	4	1.1261847E-07
34	.3000	.7330062	5	5.4064626E-06
35	.4000	.8742882	5	2.8750841E-05
36	.5000	1.0066097	5	9.4679370E-05
37	.6000	1.1325689	5	2.4133995E-04
38	.7000	1.2538115	5	5.2107086E-04
39	.8000	1.3714461	5	9.9980067E-04
40	.9000	1.4862493	5	1.7556938E-03
41	1.0000	1.5987870	5	2.8774391E-03
42	1.5000	2.1330223	8	2.8702732E-02
43	2.0000	2.6523208	8	8.6282647E-02
44	3.0000	3.6680869	11	3.9015816E-01
45	4.0000	4.6748586	13	9.5703807E-01
46	5.0000	5.6780592	13	1.8492870E 00
47	6.0000	6.6798893	14	3.1628510E 00

FLOW DIAGRAM