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COLLAPSE OF A SPHERICAL BUBBLE IN A PRESSURE GRADIENT

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The question of how bubbles collapse under the influence of asymmetric conditions such as pressure gradients, initial shape perturbations, velocity relative to the bubble, and/or the presence of solid surfaces is crucial to the understanding of cavitation damage. Recent works by Gibson,(1) Yeh,(2) and Shima(3) as well as earlier analyses by Plesset(4) and Rattray(5) have studied the early stages of collapse under various combinations of the above conditions by the use of perturbation techniques based on the classical symmetric solution. Recently Plesset(6) has studied in a more complete manner the effect of nearby solid surfaces.

The present investigation is aimed at the development of a generalized computer analysis which allows the introduction of virtually any combination of the above conditions and solution in terms of bubble shape, wall velocities, and local liquid velocities and pressures through later stages in the collapse. As reported in a 1968 Cavitation Forum paper from this laboratory(7) the Marker-and-Cell (MAC) method was adopted to achieve this goal. The aforementioned paper includes brief discussions on the reasons for selection of this particular technique, how the MAC method works and its application to the classical Rayleigh spherical collapse problem. More complete details on the Marker-and-Cell method are found elsewhere(8).

At the time of writing we have investigated three problems: a) the spherical collapse, b) the growth of small perturbations during collapse, and c) collapse of an initially spherical bubble in a linear pressure gradient. This paper focuses on the last. The complete treatments are found in the first author's dissertation[9].

Gibson[1] defined a dimensionless physical parameter

$$\delta = R_o \frac{\partial p}{\partial z} \Big/ (p_\infty - p_c)$$

where R_o = the initial cavity radius

$\partial p/\partial z$ = the pressure gradient

p_∞ = the ambient pressure in the liquid at the axial position of the initial bubble center

p_c = the cavity internal pressure.

He concluded that for $|\delta| > 10^{-2}$, one would expect gross visible distortions in bubble shape during collapse. We selected two values for δ: 0.19 and 0.57. The former is identical to one of Gibson's perturbation cases, and both are typical of pressure gradients encountered in flowing systems.

In briefest summary, the MAC technique, as adopted for this study, is as follows:

Assuming an incompressible liquid, one writes the continuity equation

$$\vec{\nabla} \cdot \vec{u} = 0 \qquad \text{---(1)}$$

the momentum equation

$$\frac{\partial u}{\partial t} = -(\vec{u} \cdot \vec{\nabla})\vec{u} - \vec{\nabla} P + \nu \nabla^2 \vec{u} \qquad \text{---(2)}$$

and the divergence of the momentum equation

$$\frac{\partial}{\partial t} (\vec{\nabla} \cdot \vec{u}) = -\nabla^2 P + \nu \nabla^2 (\vec{\nabla} \cdot \vec{u}) - \vec{\nabla} \cdot (\vec{\nabla} \cdot (\vec{u}\vec{u})) \qquad \text{---(3)}$$

where u is the fluid velocity,

$P = p/\rho$ (p = the local pressure, ρ = the density) and ν is the kinematic viscosity. Eq. (1) has been used to simplify eq. (3). Eq. (2) and (3) are written in finite difference form in spherical coordinates, and axial symmetry is assumed. For each time step, eq. (3) is solved to find the pressure distribution in the liquid by an iteration procedure, then eq. (2) is evaluated to find the velocities at the next time step. Marker particles, used to represent the bubble surface, are than moved to their new positions, thus giving the new location of the bubble wall. The sequence is then repeated.

The numerical characteristics of a typical solution follow. The grid is composed of about 1400 cells with a radial extent of 5 R_o. 128 marker particles are used to represent the surface. Approximately 400 time-steps are necessary to bring the minimum bubble wall radius to 0.15 R. This requires 30 to 45 minutes of IBM-360 processing time. Accuracy and stability are best found by repeated testing of different grid finenesses and extents, and of different time step sizes.

Fig. 1 and 2 show bubble wall profiles collapsing in the two pressure gradients. Fig. 3 shows the velocity of the jet as a function of its radial position for both cases, as well as the Rayleigh bubble wall velocity for a corresponding radius reduction. Both profiles show the expected flattening on the high pressure side, and then continued acceleration of the bubble wall such that a high velocity jet begins to form. The calculation was terminated at this point because the computation time becomes very great for the large number of cells then included in the iteration scheme, and because the jet approaches the singularity at the center of the spherical coordinate system. Comparison of the two cases shows, as expected, a much more rapid development of the asymmetries in the more severe pressure gradient case, but surprisingly little difference in the two jet velocities at corresponding radial positions.

The deviation from symmetry also causes the centroid of the bubble to "migrate" towards the low pressure region, an often observed experimental phenomenon. Fig. 3 shows the jet velocity (actually the wall velocity on the axis of symmetry -- high pressure side) to differ from the predicted Rayleigh velocity at that radius only slightly until the later stages of collapse, when the jet does not accelerate as fast as the corresponding spherical bubble wall.

In summary, then, these calculations have shown the first clear analytic indications of high velocity jet formation in the collapse of cavities in pressure gradients. For $p_\infty - p_c = 1$ atm, the jet velocity at the conclusion of the calculation is 220 ft/sec. While the jet is still accelerating at the conclusion of the calculation, it does not seem likely that the velocity will continue to increase strongly as the jet becomes very narrow. Impingement of jets of liquid at the velocities obtained here are nearly of sufficient magnitude to cause material erosion.

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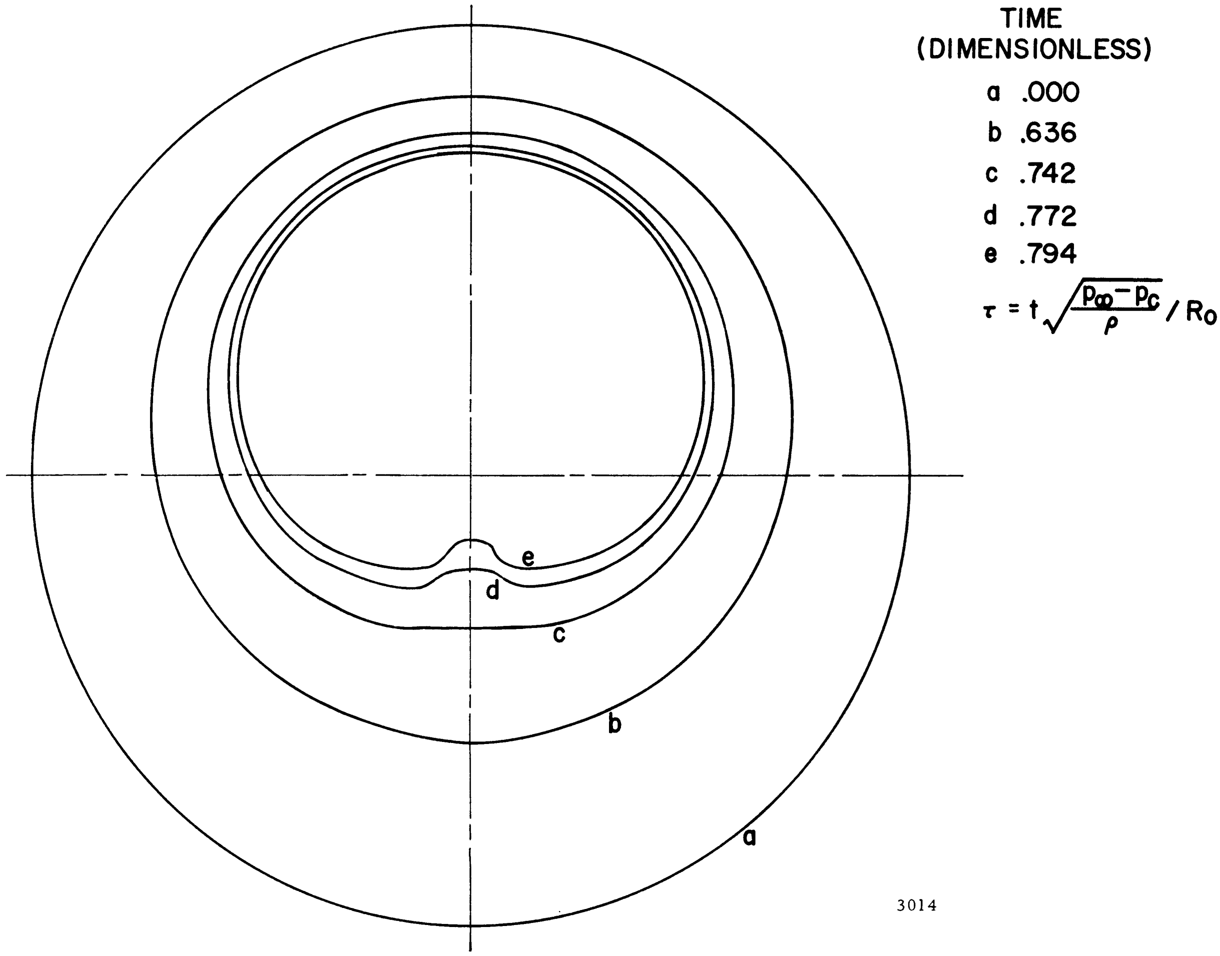

Figure 1. Bubble Wall Profile for Spherical Bubble Collapsing in Pressure Gradient, $\sigma = .19$

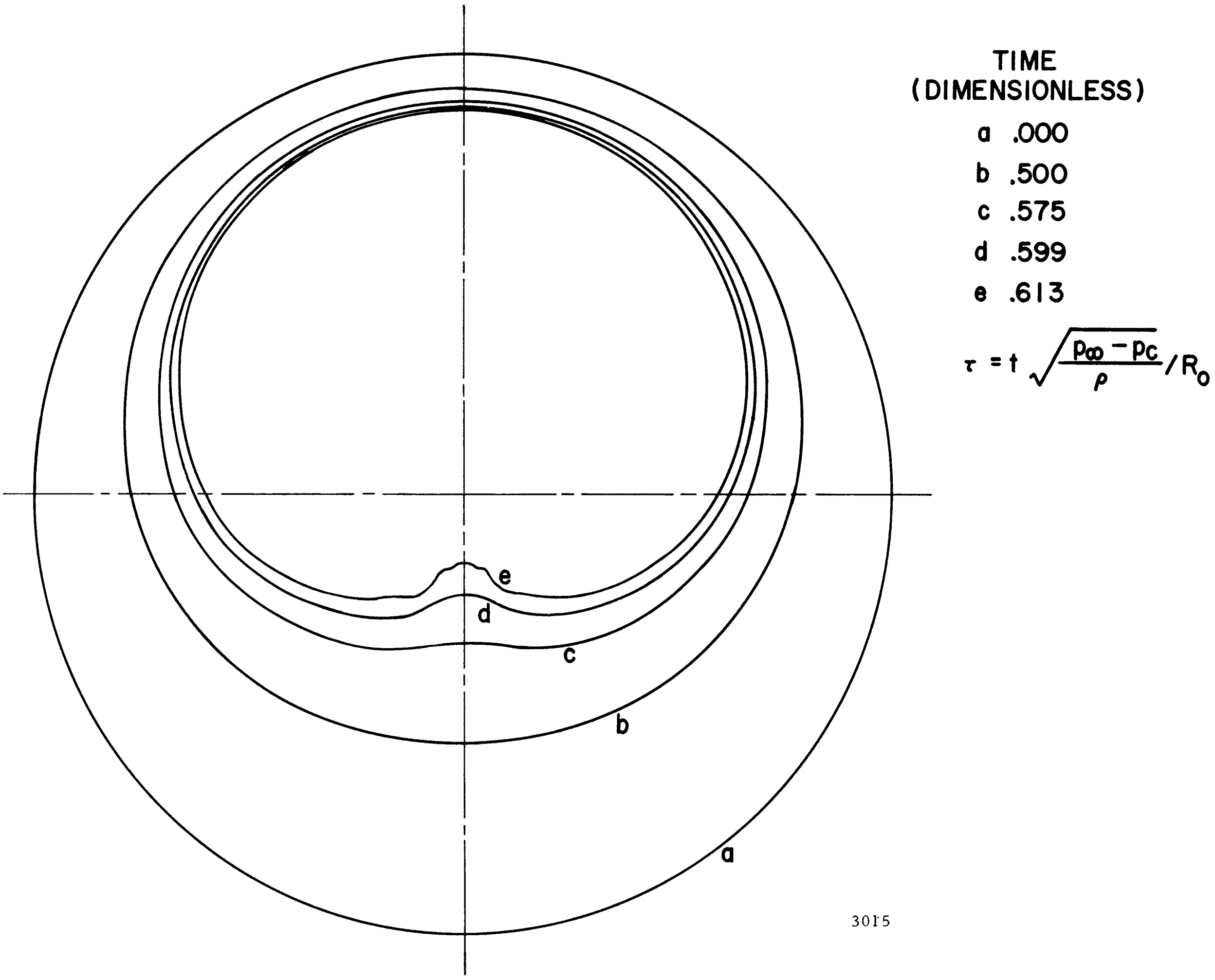

Figure 2. Bubble Wall Profile for Spherical Bubble Collapsing in Pressure Gradient, $\sigma = .57$

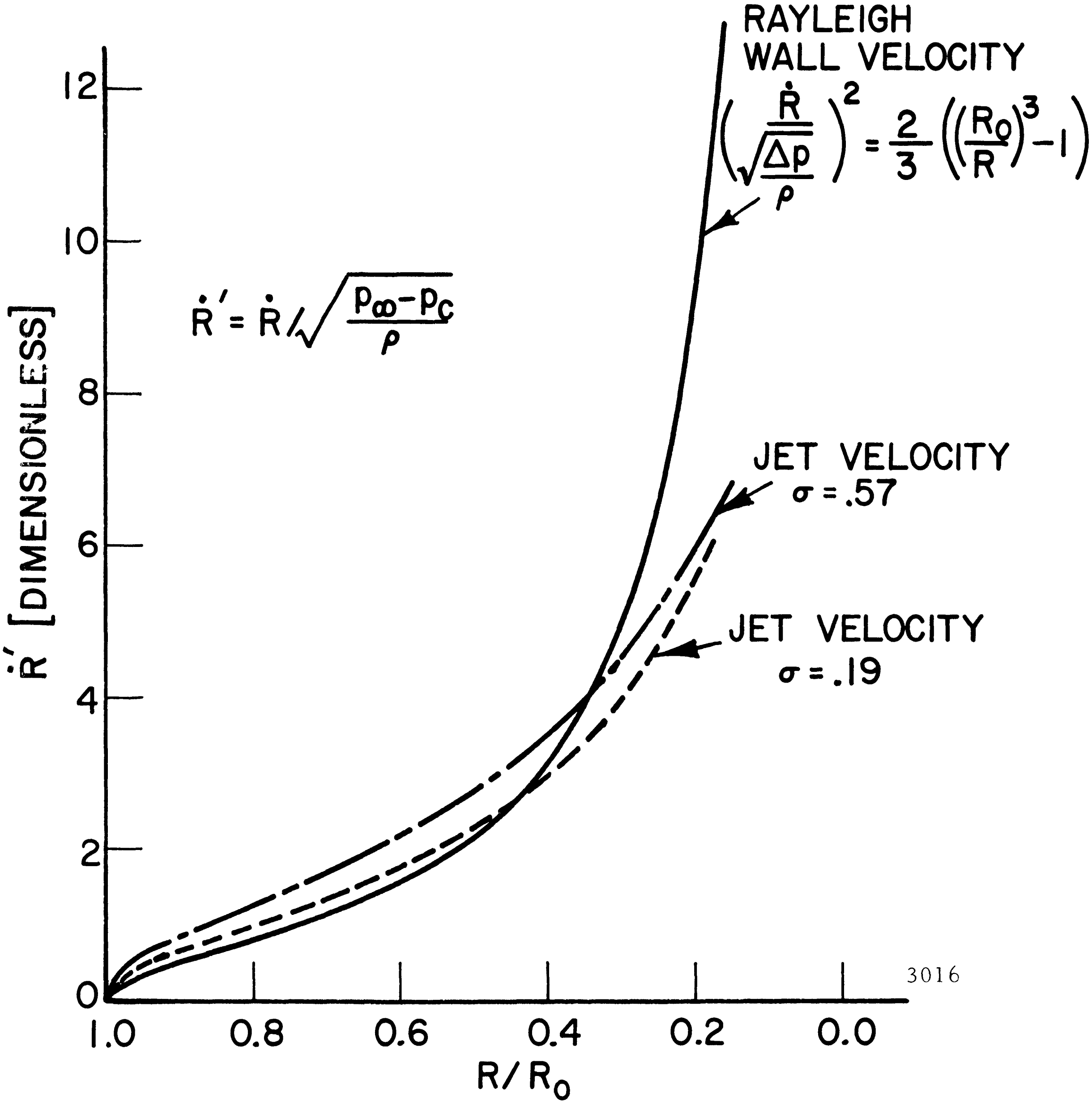

Figure 3. Jet Velocity as Function of Radial Position