ENGINEERING RESEARCH INSTITUTE
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QUARTERLY REPORT NO. 1

AFFINE STRUCTURES IN EUCLIDEAN MANIFOLDS

Covering Period June 2 to September 1, 1952

By

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Project 2051

ORDNANCE CORPS, U.S. ARMY
CONTRACT NO. DA-20-018 ORD-12277

September, 1952

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I. It has been shown that the knot-type of a polygon in Euclidean 3-space E^3 is invariant under orientation-preserving homeomorphisms of E^3 onto itself. That is, if P is a polygon in E^3, and f is an orientation-preserving homeomorphism of E^3 onto E^3, then the knot-types of P and f(P) are the same. In "Affine Structures in 3-manifolds: V. The Triangulation Theorem and Hauptvermutung" (Annals of Mathematics, 56, 96-113 (1952), Theorem 6) it was shown that any semi-locally tamely imbedded simple closed curve J in E^3 can be thrown onto a polygon by an orientation-preserving homeomorphism f of E^3 onto itself. The knot-type of such a curve J can then be defined as that of the polygon f(J); and by the result stated above, the definition is unique—that is, it does not depend on the choice of f. It is clear, also, that the knot-types thus defined for semi-locally tamely imbedded curves are invariant under orientation-preserving homeomorphisms of E^3 onto itself. We have, therefore, a true generalization of the classical theory, the classical combinatorial definitions of knot and knot-type being replaced by topologically invariant definitions.

It appears probable that the methods used in obtaining the above result are capable of further applications, for example, to show that every locally tamely imbedded set in a triangulated 3-manifold is tamely imbedded. Publication will therefore be postponed until this possibility has been investigated further.

II. In collaboration with O. G. Harrold, the following results have been obtained: We say that a set M in E^3 is locally polyhedral at a point p if p has a closed neighborhood N which intersects M in a polyhedron. K will denote a 2-sphere; I denotes the interior of K, and E denotes the exterior of K, compactified at infinity.

Theorem 1. If K is locally polyhedral except at one point, then both E and I are simply connected, and either the closure of E or the closure of I is a topological 3-cell.

Theorem 2. If K is locally polyhedral except at three points, then either E or I is simply connected.

This will probably be forthcoming in the *Annals of Mathematics*.

III. It has been shown that if M is a compact 3-manifold which is the sum of two sets, each of which is homeomorphic to Euclidean 3-space E^3, then M is a 3-sphere. This is the solution of a problem originally proposed by J. W. Alexander. This will probably be submitted to the *Annals of Mathematics*.

IV. The chief investigator has spent a great deal of time working on Dehn's Lemma and the Poincare Conjecture, but no tangible results have been obtained so far.