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THIRD QUARTERLY REPORT

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The past three months have been spent primarily in the investigation of simplicial homeomorphisms of the 3-sphere. The principal results obtained are as follows:

(I) Let f be a simplicial homeomorphism of the 3-sphere onto itself, preserving orientation. Let the period of f be n. Then f has period n at each nonfixed point of f.

(II) Let f be as in (I), and suppose that the fixed-point set F of f is a single simple closed polygon P. Then there is a tabular neighborhood T of P, such that (1) f, restricted to T, is homeomorphic to a latitudinal rotation of a solid torus onto itself, and (2) if N is the closure of the complement of T, and B is the boundary of N, then N has a decomposition into polyhedral 3-manifolds with boundary N_1, N_2, ..., N_n, such that the sets N_i are permuted by f, and such that each set N_i intersects B in a set homeomorphic to a plane annulus.

(III) Let f, n and P be as in (I) and (II), and suppose that P is unknotted. Then f is homeomorphic to a rotation.

These results (and the methods used in obtaining them) represent first steps in an attempt to verify a conjecture of P. A. Smith, to the effect that if P is as in (II), then P must be unknotted. There appears to be some ground for hoping that Smith's conjecture is related (methodologically, at least) to the Dehn Lemma and to the Poincare Conjecture. (See the proposal for the present project.)