ENGINEERING RESEARCH INSTITUTE
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PROGRESS REPORT NO. 2

(Period covering August 1, to October 31, 1951)

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During the period August through October, the main effort of this project was directed toward completing the application of the Ferrari linearized theory to the shock-wave-cylinder interaction study.

Briefly, the Ferrari approach is this: after separation of variables the potential function can be written in the form

$$\phi\ (x,r,\theta) = \sum_{n=0}^{\infty} \phi_n\ (r,x)\ r^n \cos n\theta,$$

where x, r, and θ are the usual cylindrical coordinates. It can be shown that to satisfy the potential equation each of the ϕ_n must have an integral representation of the form

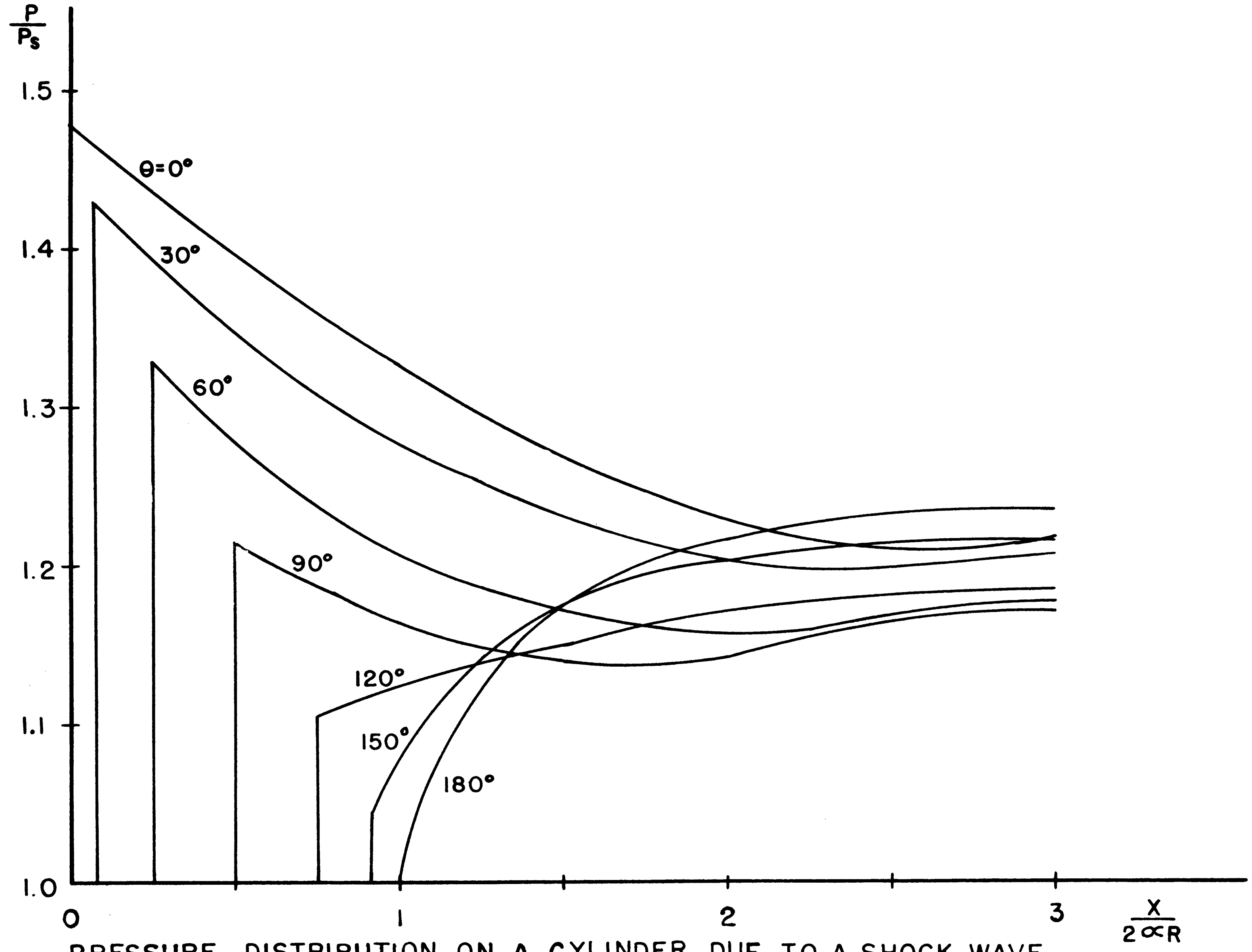
$$\phi_n = \int_{\cosh^{-1} \frac{x}{\alpha r}}^{0} f_n(x - \alpha r \cosh u)\ g_n(u)\ du \quad ,$$

where f_n is an unknown distribution function to be determined by the boundary conditions and g_n is a known function of u. The boundary condition (the known velocity normal to the surface caused by the shock wave) is expanded in a Fourier series with coefficients depending on x, and the velocity normal to the surface is found from the expression for ϕ. By equating these two series, an integral equation for each f_n is obtained. Since these integral equations cannot be solved exactly, a step-by-step numerical method is resorted to which gives f_n as a broken-line function. Using these approximate solutions for f_n, one can calculate numerical values for ϕ_n and $\partial\phi_n/\partial x$, which are necessary to compute the velocity components v_θ and v_x and hence the pressure. The numerical equations necessary to compute ϕ_n and $\partial\phi_n/\partial x$ have not been published, so that it was necessary to develop these expressions.

The coefficients in the Fourier series for the boundary conditions were computed and plotted to get some idea of the number of terms that would be necessary in order to get a reasonably close approximation. It seemed probable that terms up to and including the third harmonic could not be neglected and that terms after this could be dropped without seriously decreasing the accuracy. After that the distribution functions, f_n, were computed assuming that their slopes are constant in an interval of 1/4 the "intersection distance", the "intersection distance" being defined as the axial distance between the first intersection of the shock plane and the cylinder and the last point of intersection. Intervals of 1/8, 1/4, 1/2, and 1 times the intersection distance were also tried on some of the ϕ_n, and it was found that 1/4 is optimum with respect to time consumed and accuracy.

The values of ϕ_n and $\partial\phi_n/\partial x$ for n=0 to n=3 were computed numerically as a function of x, the distance along the axis of the cylinder. With these functions it was possible to find directly the values of the axial and tangential velocity on the surface of the cylinder. Knowing the velocity components on the surface, the streamline direction and the pressure can be computed. It should be noted that the expression for pressure cannot be linearized in the ordinary way in which the pressure coefficient depends only on v_x; for while v_x goes to zero asymptotically, there is an asymptotic pressure distribution due to the uniform crossflow which induces a tangential velocity v_θ but no axial velocity v_x. A faired curve of the pressure at several stations around the surface of the cylinder is given in Fig. 1. The curves are faired because, with only a finite number of Fourier terms taken into account, the calculations show some irregularity. The points just behind the "shock wave" are known from two-dimensional theory and the asymptotic values are also available as a check. The coordinate $x/2\alpha R$ is a multiple of the intersection distance and P_s is the ambient pressure in front of the "shock wave".

During this period the major part of the drawing on the wing-body interference model was completed. The stress calculations and the dimensions still remain to be checked.



PRESSURE DISTRIBUTION ON A CYLINDER DUE TO A SHOCK WAVE
(FERRARI THEORY)
FIG. 1