THE UNIVERSITY OF MICHIGAN

COLLEGE OF ENGINEERING
Department of Meteorology and Oceanography

Technical Report

THE LAKE BREEZE CIRCULATION ALONG THE SHORELINE OF A LARGE LAKE

William J. Moroz

E. Wendell Hewson, Project Director

ORA Projects 05685 and 07143

supported by:

NATIONAL SCIENCE FOUNDATION
GRANT NO. GP-944
WASHINGTON, D. C.

and

DEPARTMENT OF HEALTH, EDUCATION, AND WELFARE
PUBLIC HEALTH SERVICE
DIVISION OF AIR POLLUTION
GRANT NO. AP-00380-01
WASHINGTON, D. C.

administered through:

OFFICE OF RESEARCH ADMINISTRATION ANN ARBOR

November 1965

This report was also a dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in The University of Michigan, 1965.

TABLE OF CONTENTS

	Page
LIST OF TABLES	v
LIST OF FIGURES	vi
ABSTRACT	ix
1. INTRODUCTION	
1.1 Statement of the problem	1
1.2 The approach to the problem	2
2. AN OBSERVATIONAL STUDY OF THE LAKE BREEZE ON THE EASTERN SHORE OF LAKE MICHIGAN	
2.1 Significance of the study	5
2.2 Coastline wind characteristics	6
2.3 The observations presented	8
2.4 Location of the study	10
2.5 The observational program	
2.5.1 Wind observations	11
2.5.2 Vertical temperature soundings	14
2.5.3 Auxiliary observations	15
2.6 The observed lake breeze	
2.6.1 Prevailing meteorological conditions in the Lake Michigan area	17
2.6.2 Temperature and moisture structure	18
2.6.3 Lake breeze flow over the land	23
2.6.4 Lake breeze flow over the water	28
2.6.5 Homogeneity of the lake breeze along the shoreline	31
2.6.6 Wind on opposite sides of Lake Michigan	32
2.7 Summary and conclusions	34
3. A SEMI BOUNDED NUMERICAL MODEL OF THE LAKE BREEZE	
3.1 Differences between the lake breeze and the sea breeze	61
3.2 Physical formulation of the model	65
3.3 Mathematical formulation of the model	66
3.4 The eddy transport terms	72
3.5 The boundary and initial conditions	74
3.6 Procedure for predicting lake breeze parameters	80
3.7 Results of a lake breeze forecast	83
3.8 Comparison with the observed lake breeze	88
3.9 Summary and conclusions	92

TABLE OF CONTENTS (Concluded)

Page

4. SUGGESTIONS FOR FUTURE WORK
   4.1 Extension of observations 101
   4.2 Extension of the numerical model 103
   4.3 Interaction of local winds and circulations within the lake 104

APPENDIX
   A. DRY BULB AND DEW POINT STRUCTURE IN THE VERTICAL FOR July 22, 1964 106

   B. DERIVATION OF PERTINENT EQUATIONS 108

BIBLIOGRAPHY 115

LIST OF TABLES

Table		Page
2.1	Relative Humidity in Percent, at various Distances Inland from the Lake Michigan shoreline July 23, 1964	21
2.2	Wind Speed and Direction at U.S. Weather Bureau Stations on Opposite Shores of Lake Michigan, July 23, 1964	33
3.1	Air Temperature Over the Land at a Location Well Inland from the Lakeshore	78

LIST OF FIGURES

Figures		Page
2.1	Location of observation stations near the Lake Michigan shoreline.	38
2.2	U.S. Weather Bureau Daily Weather Map for July 23, 1964.	39
2.3	U.S. Weather Bureau Daily Weather Map for July 10, 1963.	40
2.4	Surface dry bulb temperature at various distances inland from the lakeshore July 23, 1964.	41
2.5	Dry bulb and dew point temperature structure in the vertical for July 24, 1964.	42
2.6	Horizontal wind components at 1600EST July 24, 1964.	43
2.7	Horizontal wind components at 0900EST July 23, 1964.	44
2.8	Horizontal wind components at 1000EST July 23, 1964.	45
2.9	Horizontal wind components at 1100EST July 23, 1964.	46
2.10	Horizontal wind components at 1200EST July 23, 1964.	47
2.11	Horizontal wind components at 1300EST July 23, 1964.	48
2.12	Horizontal wind components at 1400EST July 23, 1964.	49
2.13	Horizontal wind components at 1500EST July 23, 1964.	50
2.14	Horizontal wind components at 1600EST July 23, 1964.	51

LIST OF FIGURES (continued)

Figure		Page
2.15	Horzional wind components at 1700EST July 23, 1964.	52
2.16	Horizontal wind components at 1800EST July 23, 1964.	53
2.17	Horizontal wind components at 1900EST July 23, 1964.	54
2.18	Horizontal wind components at 2000EST July 23, 1964.	55
2.19	Variation of wind speed and direction during the day at 110 m and at 1000 m above the surface July 23, 1964.	56
2.20	Horizontal wind components over the water at 0745EST and 0830EST about 5 km offshore July 10, 1963.	57
2.21	Horizontal wind components over the water at 1620EST and 1710EST about 16 km offshore July 10, 1963.	58
2.22	Horizontal wind components over the water at 1800EST and 1825EST about 5 km offshore July 10, 1963.	59
2.23	Wind direction change with onset of the lake breeze at various distances inland July 23, 1964.	60
3.1	Schematic diagram of the model plane for the semi bounded lake breeze.	94
3.2	The across shore wind component in the model plane 4 hrs after time zero.	95
3.3	The across shore wind component in the model plane 6 hrs after time zero.	96

LIST OF FIGURES (concluded)

Figure		Page
3.4	The vertical wind component and dry bulb temperature in the model plane 6 hrs after time zero.	97
3.5	The across shore wind component in the model plane 9 hrs after time zero.	98
3.6	The vertical wind component and dry bulb temperature in the model plane 9 hrs after time zero.	99
3.7	The across shore wind component in the model plane 12 hrs after time zero.	100
B.1	Dry bulb and dew point structure in the vertical for July 22, 1964.	107

ABSTRACT

This study was conducted to determine the physical features of a lake breeze wind system occurring at the shoreline of a large deep lake. In the first part of the study an observed lake breeze in the vertical plane normal to the shoreline is described at hourly intervals. The observed thermal structure of the atmosphere on a day when a clearly defined lake breeze occurred is also presented. In the second part a numerical model for a symmetric lake breeze is constructed and found to produce results in good agreement with the observations presented in part one.

The observations of the lake breeze circulation system were made at a location on the eastern side of Lake Michigan under circumstances when prevailing external meteorological conditions would exert minimum influence on the local thermal circulation. Over the land the depth of the layer of onshore flow is approximately 750 m and a maximum velocity of 5 to 7 m sec^{-1} is observed within 250 m of the surface directly over the lakeshore. Above the lake breeze current a well defined return flow is apparent by midafternoon. The layer of return flow is about twice as deep as the lake breeze and velocities in the return flow are proportionately lower. The local wind system affects large scale atmospheric flow patterns through a depth exceeding 2500 m above the surface.

The simultaneous existence of a lake breeze on opposite sides of the lake is demonstrated, at least near the surface, using climatological records. Homogeneity of the lake breeze along the shoreline and the orderly progress of the lake breeze front to a distance exceeding 16 km inland are clearly apparent in the data presented. Turning of the local wind in response to the Coriolis force is also demonstrated by the observations.

In the second part of the study a mathematical model of the lake breeze is presented

starting with the differential equations for motion and heating in the atmosphere, the hydrostatic equation and the equation of continuity. The finite dimension of the lake is incorporated into the model which is termed semi-bounded in contrast to the sea breeze model which has no horizontal restriction. The flow deviations attributable to differential heating over land and water are obtained using the techniques developed by Estoque for the unbounded sea breeze. The differential equations are represented by finite difference approximations and a solution for the lake breeze flow patterns is obtained using an IBM 7090 digital computer. Features of the development of the lake breeze in both model and observations are in good agreement.

1. INTRODUCTION

1.1 Statement of the problem

The existence of the Great Lakes system as a source of fresh water on the North American continent is becoming increasingly important. As a result of the availability of large quantities of water intense industrial and associated commercial and residential development has occurred in the Great Lakes Basin and the rate of development is accelerating. Despite the high population concentration along the shores of the lakes few intensive meteorological investigations have been conducted to determine the effects which the Great Lakes have in modifying local weather phenomena except those which can be done using climatological records.

The present investigation was undertaken to determine the extent and physical characteristics of lake breeze circulations and how they vary in space and time. A second objective was to construct a mathematical model of the lake breeze which can be used to guide further investigations. In this discussion the lake breeze is regarded as the onshore wind which occurs through the layer next to the surface of the earth near the lakeshore and which is caused by differential heating over land and water.

The modifying influence exerted by the presence of a large lake on local climatic patterns in its vicinity is an important factor in determining man's activities near the lake shore. This influence has aided in the establishment of a successful fruit belt along the shores of the southern Great Lakes. The existence of the snow belt (Thomas, 1964) along the shores of the lakes during winter months is to be considered in locating major highways and airports; and weather conditions in the vicinity of the lake influence flying conditions for both large and small aircraft. More recently the local circulation along a sea coast (Frenzel, 1962) or a lake shore (Hewson et al., 1960; Hewson, Gill and Walke, 1963) has been considered as a major factor influencing local atmospheric diffusion characteristics and pollutant transport.

1.2 The approach to the problem

Munn and Richards (1964) used climatological records to show that a lake breeze occurs at a location near Lake Huron. Their analysis of low level observations shows the existence of an internal micrometeorological boundary layer near the lake shore but the data were inadequate for an analysis of the lake breeze as a mesoscale flow system. The observations presented in this dissertation permit analysis of the lake breeze as a flow system changing in space and time, both over land and over water. To our knowledge no other paper dealing specifically with the lake breeze flow system has been published.

While no information dealing with the lake breeze circulation has been published a considerable literature on sea breezes is available which is valuable in constructing the initial lake breeze model. Although most sea breeze investigations describe the flow over a point as a function of time, Fisher (1960) and Frizzola and Fisher (1963) have observed the sea breeze as a dynamic system varying in space as well as in time.

The forces which act to govern lake breeze motions are essentially the same as those governing the sea breeze yet the lake breeze must be different from the sea breeze as a result of the finite dimension of the lake surface. The differences will be apparent in the flow patterns of the two systems and in the extent of modification of other meteorological variables.

The thermal circulation which occurs in the atmosphere along the shore or coast line will be influenced by mixing of air into the local circulation from the surrounding atmosphere and the properties of the surrounding atmosphere are in turn influenced by the nature of the underlying surface. Over the lake, where the horizontal extent of the water surface is relatively much smaller than that of the ocean, there is less opportunity for the air to take on characteristics normally associated with an underlying water surface and in the region of the Great Lakes the surrounding atmosphere retains properties characteristic of a continental air mass. Flow off the water is rapidly modified in a short trajectory over the land by intense turbulent mixing as well as by heating over the land during the day.

In a mathematical model of the lake breeze the bounded nature of the lake surface must be introduced and the model should be such that the effects of the general circulation winds on the local flow system can be included. In Chapter 3 a numerical model for a lake breeze which is symmetric on each side of the lake center line is presented. Development of the lake breeze is satisfactorily predicted by the model using an IBM 7090 digital computer for the case where external influences affecting the local flow system are minimal. Work is continuing to improve the model and to introduce the effects of a general circulation flow.

2. AN OBSERVATIONAL STUDY OF THE LAKE BREEZE ON THE EASTERN SHORE OF LAKE MICHIGAN

2.1 Significance of the study

The Great Lakes of North America exert a strong influence on local and large scale atmospheric behavior in the eastern central portion of the continent. Examples are given by Petterssen and Calabrese (1949), Schenfeld and Thomson (1965), Thomas (1964) and Moroz and Hewson (1965).

Lake breezes have long been known to exist along the shores of the Great Lakes yet no studies have been conducted to determine the characteristics or physical features of the wind system. The lake breeze is important in determining the local climatology of those regions bounding the Great Lakes. It affects both cloud amount and moisture evaporation and transport from the lake surface directly and may be important in affecting the circulations within the lake itself. Industrial development and population concentration is intense along the shores of the lakes and air pollution has become a significant factor in many of the larger population centers. Pollutant transport is frequently controlled or affected by the lake breeze circulation or a modified general circulation near the shoreline. Meteorologists are also interested in the sea or lake breeze as a mesoscale example of the conversion of spatial variations of thermal energy input to kinetic energy of atmospheric motions.

This study has been initiated to observe the physical characteristics of the lake breeze along the shore of a large lake in the Great Lakes Basin. A time series of observations in space is required to permit analysis of temporal variations of the thermal circulation and which will be suitable for comparison with a numerical model of the circulations.

2.2 Coastline wind characteristics

Sea and lake breezes at a given latitude would be expected to differ principally in dimension and intensity. In the absence of literature dealing with the lake breeze a brief review of sea breeze characteristics is presented here. While several authors have contributed to the knowledge of sea breezes, only the investigations of Fisher (1960) and Frizzola and Fisher (1963) were undertaken specifically to observe the development of the sea breeze in space and time.

In midlatitudes horizontal velocities in the sea breeze have maximum values of about 10 m sec^{-1} which are observed below 300 m (Fisher, 1960; Frizzola and Fisher, 1963). The depth of the sea breeze onshore flow is usually less than 1000 m (Frizzola and Fisher, 1963) and inland penetration is 30 to 50 km (Fisher, 1960; Wallington, 1963, 1965). A characteristic wind direction change occurs in a clockwise sense during the day in the northern hemisphere in response to the Coriolis force.

A return circulation should exist above the sea breeze but has never been clearly observed in midlatitudes. Sutcliffe (1937) notes the complete absence of the return circulation in a study of average winds using data from routine pilot balloon observations. The characteristics of the sea breeze will, of course, be strongly modified by prevailing general circulation patterns and it is probable that the return circulation is frequently masked by prevailing winds aloft.

In the tropics the sea breeze circulation has greater dimensions and the change of wind direction in time over the period of the day is not observed (Wexler, 1946). W. van Bemmelen (1922) at Batavia (Djakarta, 7^{o}S latitude) and Dixit and Nicholson (1964) at Bombay (19^{o}N latitude) did observe a reversed flow above the sea breeze. In the latter case the effects of the sea breeze return circulation were still clearly evident at heights exceeding 3000 m. The interpretation of van Bemmelen's results is made more complicated at a location where local mountain wind systems could occur.

2.3 The observations presented

The lake breeze arises in response to pressure or density differences in the horizontal occurring as a result of differential heating in the atmosphere over land and water. The extent and features of the flow system are thus strongly dependent upon heat distribution and the basic variables obtained to describe the system are wind and temperature through the region where the local circulation dominates atmospheric flow patterns. In this study the moisture distribution in the region of interest was also determined because it was felt initially that moisture content of the atmosphere could be used as a natural tracer to aid in determining lake breeze boundaries.

During the 1964 season observations were made over the land on the east side of Lake Michigan on fourteen days when clear skies and weak prevailing synoptic pressure gradients were forecast. On seven of these days a clearly defined lake breeze occurred and during the period from July 22 to July 24, 1964 a three day series of observations of a lake breeze flow system was obtained. An analysis of lake breeze flow patterns at one hour intervals over the land is presented in this chapter using data obtained July 23, 1964. This day is selected for presentation because external factors which might distort the local circulation were minimal. Unfortunately the vertical temperature structure of the atmosphere could not be obtained on July 23 and it is necessary to infer the temperature structure using observations made July 22 and July 24 in the same general air mass.

In order to describe the complete lake breeze flow system observations over the water are required. During the 1964 season measurements were made only over the land. To provide information about wind structure in the vertical over the water a few observations made from a research vessel July 10, 1963 in a well defined lake breeze are presented separately. Climatological records from surface stations are used to provide auxiliary information on the surface structure of the lake breeze flow system.

2.4 Location of the study

Observations of the lake breeze were made at a site on the eastern shore of Lake Michigan about midway between Muskegon and Holland, Michigan. The site selected offered several distinct advantages. Inland from the lake shore the land is flat and uniformly developed for agriculture. Lake Michigan is deep and hence relatively cold and temperature contrasts between land and water remain strong throughout the summer. The lake is long and narrow and observations at a point at the center of the long axis of the lake permit initial investigation of the lake breeze under conditions where curvature of the shoreline does not strongly modify flow patterns.

There are additional reasons for selecting a site on Lake Michigan. The Great Lakes Research Division of the University of Michigan operates three research vessels on this lake from which supporting data may be obtained. At Muskegon near the lake shore and at Grand Rapids 53 km inland surface meteorological observations are recorded at hourly intervals at U.S. Weather Bureau stations. These stations are located near an excellent road running perpendicular to the shoreline on which other observing stations could be conveniently located with a high degree of mobility between stations. Finally, through the courtesy of the City of Grand Rapids, an excellent site for a lakeshore station was provided on the lawn of their water filtration plant which is located at the lakeshore near the water intake.

The location of the study area relative to Lake Michigan is indicated for orientation purposes in the insert of Fig. 2.1.

2.5 The observational program

2.5.1 Wind observations

During the 1964 season pilot balloon observations were made over the land at stations along the road perpendicular to the lake shore ("the line of observation"). The observation stations were located at the lake shore and at intervals of 8 km inland along a line through the lake station. Fig. 2.1 is a sketch showing the location of the stations from which data were obtained for the study and the Lake Michigan shoreline near the site selected. The vertical plane intersecting the earth's surface along the line of observation is termed the "plane of observation" in this report.

Hourly wind fields of the lake breeze in the plane of observation were computed from data obtained by single theodolite tracking of pilot balloons released simultaneously at hourly intervals at each of the observing stations. Pilot balloons were inflated to have a nominal ascent rate of 90 m min^{-1}. Usually three stations were in operation. Initially balloons were tracked to a maximum height of 2200 m where it was considered gradient flow would dominate. However, during the course of the investigation it was found that it was necessary to track above 3000 m before wind direction and speed approached a steady value. Theodolite observations were made at 30 sec intervals to permit analysis for winds through thin layers in order that details of flow changes with height could be discerned. Just prior to balloon release surface wet and dry bulb temperatures were obtained using sling psychrometers.

Pilot balloon observations were analysed using a digital computer to obtain the alongshore and across shore components of flow. No weighting functions were used to smooth data although some smoothing is introduced by using the conventional technique of averaging over two layers to evaluate the wind at the mid-point of the layers. The inherent error of wind analysis using single theodolite observations in regions where convergence or divergence may be large is well recognized. However, comparison of results obtained for a few runs early in the program where double theodolite observations were made yielded velocities for a given layer, with very few exceptions, within 1 m sec^{-1} of those computed by single theodolite analysis techniques. The estimate of accuracy to $\pm$ 2 m sec^{-1} made by Frizzola and Fisher (1963) for single theodolite observations in the sea breeze is applicable to the results presented here.

During the summer of 1963 a few observations of winds aloft were made over the water from the deck of a 110 ft vessel operated by the Great Lakes Research Division of the University of Michigan. The ship was stopped during the observation period. The instrument used for tracking the balloon is a split image sextant to which a compass had been attached. The elevation angle is measured from the horizon and the azimuth angle is observed from the compass. The instrument is crude but effective for low level observations but the balloon cannot be tracked to heights exceeding 2000 m because the sextant has low magnification. Reliability of computed winds for a given layer is not as good as that attained using single theodolite data and some smoothing is required. However, data obtained by this method yield results in excellent agreement with those obtained by theodolite and the accuracy is believed to be satisfactory provided conclusions are not extended to small scale phenomena.

2.5.2 Vertical temperature soundings

Vertical wet and dry bulb temperature soundings to 2200 m were made using a Cessna 172 aircraft equipped with shielded, naturally ventilated thermistor probes with an associated indicator. The instrument has a time constant of a few seconds at aircraft speeds and hence provides reasonable detail of the temperature field but with very sharp contrasts smoothed.

As noted, on July 23 the aircraft was not available; however, observations of the dry bulb and wet bulb temperature structure in the vertical were made July 22 and July 24 at the same location. During the intervening period the characteristics of the external air mass remained relatively constant and on all three days, (July 22, 23 and 24, 1964) a lake breeze was observed.

The dry bulb and dew point temperature in the plane of observations for July 22 are presented in Appendix A from a previous paper (Moroz and Hewson, 1965); those for July 24 are given in Fig. 2.5 along with the wind components observed at the corresponding time (Fig. 2.6). The wind fields for July 23 are presented as typical of the lake breeze rather than those of the 22nd or 24th because on July 22 normal lake breeze development in the region of observation was interrupted by a thunderstorm in mid-afternoon, and on July 24 the lake breeze return current, which is a well defined feature on other days, is partially masked in the flow aloft.

2.5.3 Auxiliary observations

Auxiliary meteorological observations were obtained from U.S. Weather Bureau offices at Muskegon and Grand Rapids where routine observations are made hourly. The Muskegon station is located 6.5 km inland and 21 km north of the observation line and the Grand Rapids station is 10 km south of the observation line and 53 km inland; both stations are located at airports. In addition to these stations an existing 85 m tower belonging to radio station WJBL, Holland, and located 10.5 km inland and 18.5 km south of the observation line was instrumented with wind and temperature sensors at several levels.

Water temperatures reported here are taken from records of the City of Muskegon water pumping station whose intake is located 1.5 km offshore in 10 m of water. The temperatures recorded at the Muskegon pumping station were in good agreement with surface water temperatures obtained from ships operating within 50 km of the observation site and are considered to be representative on the broad scale for the region of interest.

2.6 The observed lake breeze

In the following discussion and in the figures, the across shore component of the wind is designated "u", positive inland to the east and the alongshore component is designated "v", positive to the north. The total wind is referred to as the "vector wind" and its magnitude as "velocity."

Observations over the land are presented for July 23, 1964. While the lake breeze circulation is not quite so intense as on some other days and vertical temperature soundings are not available for this day, large scale atmospheric features which might act to modify the local circulation are minimal. The 1964 program did not extend to observations over the lake but a few pilot balloon ascents made July 10, 1963 over the water in a clearly developed lake breeze circulation provide some detail for this region.

2.6.1 Prevailing meteorological conditions in the Lake Michigan area

U.S. Weather Bureau daily weather maps at 0100 EST for July 23, 1964 and for July 10, 1963 are shown in Figs. 2.2 and 2.3 for the region of interest. The inserts in each figure are the 500 mb height contours at 1900 and surface pressure contours at 1300 of the previous day to provide an indication of flow aloft and rates of change of surface weather features respectively.

On July 23, 1964 a broad ridge of high pressure at sea level extended southwest from an anticyclone centered over Labrador. Pressure gradients over the entire Great Lakes Basin were extremely weak. Surface winds in the Lake Michigan area were light and variable. At 500 mb winds over Lake Michigan were 5 m sec^{-1} and decreasing. The stationary front shown over lower Lake Michigan was extremely weak and lake breeze observations had been made July 22nd during the period of frontal passage without detecting any frontal characteristics. However, a thunderstorm did occur on July 22nd and interaction of the lake breeze and low level thunderstorm outflow has been described separately (Moroz and Hewson, 1965).

On July 10, 1963 (Fig. 2.3) a cold, surface anticyclone dominated local weather in the Lake Michigan area. Surface winds were again light in response to a very weak pressure gradient near the center of the anticyclone. At 500 mb over Lake Michigan winds were less than 20 m sec^{-1} at 1900 on July 9th decreasing to less than 12 m sec^{-1} at 1900 on July 10th.

2.6.2 Temperature and moisture structure

Surface dry bulb temperatures of July 23, 1964 are plotted in Fig. 2.4 for several stations located along the line of observation at various distances inland. The mean lake temperature change during the period of the observations was less than 0.5°C. Air temperature at Grand Rapids, which is beyond the inland penetration of the lake breeze under normal circumstances, became equal to the water temperature at about 0700 in response to heating which began at 0600. The temperature at Grand Rapids rose fairly smoothly to a maximum of 11^{o}C above lake temperature by 1600 and after 1700 decreased as radiational surface cooling became more pronounced.

At the lakeshore station air temperature increased rapidly after dawn until 1000. Between 1000 and 1100 a sharp drop of temperature occurs as a result of advection of cooler air off the water in the lake breeze. After 1100 the dry bulb temperature at the lake station held steady or fell irregularly until 1600 when afternoon radiational cooling results in a more rapid decrease.

At 8 km and at 16 km inland from the lake shore dry bulb temperature continued to increase until onset of the lake breeze between 1400 and 1500 at 8 km and between 1700 and 1800 at 16 km. Thereafter surface dry bulb temperature changed little until the effects of radiational cooling become significant. It is noteworthy that after onset of the lake breeze dry bulb temperatures at 8 km and 16 km inland continue to be within a degree or so of the temperature at 53 km inland whereas that at the lake shore is about 4^{o}C lower. This feature was also observed on other days and indicates intense modification of air off the water in a short trajectory over the land.

The relative humidity recorded at hourly intervals at the Lake shore station, at Muskegon 6.5 km inland, and at Grand Rapids 53 km inland is presented in Table 2.1. At the lake shore station a distinct increase of relative humidity occurs with onset of the lake breeze and these high values are maintained throughout the day. At Muskegon the sharp increase of relative humidity did not occur but a slight increase is noted between 1400 and 1500 during the period when the lake breeze front would be expected to pass this station. Grand Rapids records are presented as being representative of values at an inland location.

The significantly smaller increase of relative humidity a short distance inland from the lake shore after passage of the lake breeze front indicates that vertical mixing of drier air from aloft has occurred. It also suggests that moisture transport upward from the lake surface has been limited to a thin layer in a short trajectory of the air over the water. Bellaire (in press) has found that this is the case for heat transport over Lake Michigan when warm air passes over cooler water.

Dry bulb and dew point isotherms in the plane

TABLE 2.1

RELATIVE HUMDITY IN PERCENT, AT VARIOUS DISTANCES

INLAND FROM THE LAKE MICHIGAN SHORELINE

July 23, 1964

Time	Lake Shore	Muskegon 6.5 km inland	Grand Rapids 53 km inland
0800	69	69	79
0900	63	59	58
1000	54	51	48
1100	58	48	46
1200	67	45	45
1300	73	44	45
1400	72	44	44
1500	72	47	44
1600	75	47	42
1700	72	47	44
1800	74	47	47
1900	75	50	53
2000	75	55	63

of observation are presented in Fig. 2.5 for July 24, 1964. Figure 2.6 shows the wind field of July 24 at 1600 during the period of temperature soundings. Temperature observations made July 22 are given in Appendix A.

In Fig. 2.5 a surface inversion with top at about 200 m is observed over the water a short distance offshore where vertically downward motion may be most intense at this time as a result of strong horizontal divergence over the water. Inland, beyond 20 km, a dry adiabatic lapse rate is observed through the lowest 1000 m.

Above 1500 m, directly over the shoreline, the dew point isotherms are displaced downward and a strong gradient of dew point temperature is observed. This feature is also apparent over the lake shore on July 22nd. Inland the vertical gradient of dew point temperature at this level weakens. Over the land the dew point isotherms dip downwards on the landward side of the lake shore. This could result from downward motion of drier air into the region of horizontal divergence landward of the center of maximum velocity in the return flow. Strong gradients of temperature or moisture were not observed in this study at the interface between the air flowing off the water and the return flow directly above it suggesting that strong vertical mixing occurs at this interface.

2.6.3 Lake breeze flow over the land

Figures 2.7 through 2.18 give the hourly fields of the u and v components of the wind at three stations along the line of observation for July 23, 1964. At 0900 (Fig. 2.7) near the lake shore, flow is toward the water in the land breeze below 1600 m. The velocity maximum in the land breeze occurs at 300 m with a strong component from the south. Above the surface layer of offshore wind the flow is onshore in the gradient wind and/or land breeze return current. Velocity increases upwards and the northerly component becomes more and more dominant at higher levels. At 1000 (Fig. 2.8) flow patterns are similar but weaker at all levels.

In Fig. 2.9 at 1100 a lake breeze has developed over the lakeshore between the surface and 400 m. A maximum velocity of 3 m sec^{-1} occurs at 100 m with a very weak southerly component. At the 8 km and 16 km stations the flow is still offshore (toward the shore-line) and strong horizontal convergence occurs at lower levels near the lake breeze frontal surface which is considered to coincide with the zero isotach bounding the region of onshore wind at the surface near the lake shore. Directly above the region of onshore flow a zone of intensified offshore flow is apparent indicating a merging of lake breeze return current into an inertially persisting land breeze. The land breeze seems to be displaced upwards and to occupy a deeper layer than previously, but above this, the onshore component of flow increases upwards as before. The maximum depth of the layer of offshore flow occurs directly over the lake shore where the lake breeze return current is strongest; this depth decreases rapidly inland.

At 1200 (Fig. 2.10) the lake breeze circulation has not changed significantly but aloft the land breeze is weakening. Flow in the lake breeze is still almost normal to the lake shore and convergence is very strong at the lake breeze front. The wind field for 1300 (Fig. 2.11) shows that the lake breeze has intensified but that it has not progressed significantly further inland.

By 1400, Fig. 2.12 indicates the lake breeze has deepened slightly to a height of just under 500 m over the lake shore but that progress of the lake breeze front inland is still less than 8 km. The v component of wind has decreased from greater than 0 at 1300 to -2 m sec^{-1} at 1400 showing the turning of the wind in a clockwise direction with a developing component of northerly flow in response to the Coriolis force. The lake breeze return current has still not emerged as the dominant feature above the onshore flow.

By 1500 (Fig. 2.13) a pronounced change in the dimension and intensity of the lake breeze onshore flow has occurred. Over the lakeshore onshore flow occurs through a depth exceeding 750 m. The onshore component of flow has increased to 4 m sec^{-1} between 75 m and 300 m above the surface. The vertical gradient of u is strong near the surface. The lake breeze front has advanced to 10 km inland and horizontal convergence at the frontal surface is pronounced. Directly over the central core of positive u at the lakeshore the offshore flow has intensified to greater than 2 m sec^{-1} through a deep layer. The direction of flow is almost opposite in the lake breeze from WNW, to that in the return current aloft which is from the SE.

The return current is clearly shown in the observations of Fig. 2.14 for 1600 as an east wind bounded above and below by a zero isotach. Flow above the return circulation is onshore as before in the gradient wind. Below the return current the lake breeze has begun to decrease both in intensity and depth but the alongshore velocity component has remained constant. Thus there is a further rotation of the lake breeze wind vector, from WNW to NW, as response to the Coriolis force becomes more pronounced. Figure 2.15 for 1700 is similar to that for 1600 except that a slight increase in lake breeze depth is indicated.

By 1800, Fig. 2.16, the lake breeze has decreased to about 350 m in depth with a maximum positive u of 2 m sec^{-1} centered at about 150 m. The wind direction has swung further alongshore, approaching NNW as indicated by the larger negative v component. The lake breeze front has penetrated well beyond 16 km inland. Strong offshore flow from the ESE persists between the top of the lake breeze and 2000 m but above this the general circulation flow remains onshore.

At 1900, Fig. 2.17, the center of the lake breeze appears to have shifted inland. Offshore flow occurs near the surface inland and Grand Rapids records indicate that the wind at this station is toward the lake shore at 1900. Gradient flow is becoming dominant above 2000 m but the lake breeze and its return circulation are still prominent below this level to a distance exceeding 16 km inland.

By 2000 a general weakening of the u-components in the lake breeze is apparent in Fig. 2.18 and flow has a strong alongshore component, being NNW below about 350 m. The return circulation persists aloft but above this again positive values of u occur but the balloon observations did not extend sufficiently far above the upper zero isotach to warrant plotting them.

The observations of the lake breeze presented here were made under conditions when external factors which might influence the local circulation were minimal, i.e., the external pressure gradient in the Lake Michigan region was very weak in the horizontal and winds aloft were exceptionally light. Figure 2.19 presents a summary of the temporal variations of wind speed and direction near the level of maximum velocities in the lake breeze and in the return circulation. The advance of the lake breeze inland with its onset between 1000 and 1100 at the lake shore station, between 1300 and 1400 at the 8 km station and between 1700 and 1800 at the 16 km station is clearly indicated by the directional change of the wind vector at 110 m (double shafted arrow). The change of wind direction with time as the effect of the Coriolis force becomes more pronounced and the low level convergence in the region of the lake breeze front are also clearly apparent. In the return flow aloft (single shafted arrow) the direction of flow is approximately reversed from that in the lake breeze. Wind speeds in the return flow are less than those in the lake breeze and the effect of the Coriolis force in causing a wind direction change is not as clearly evident.

2.6.4 Lake breeze flow over the water

The pilot balloon observations made from a single vessel on Lake Michigan on July 10, 1963 are in excellent agreement with more refined observations over the land. The synoptic situation characterized by a weak pressure gradient over Lake Michigan has been presented in Fig. 2.3. Figures 2.20 to 2.22 show the results of six pilot balloon ascents made over the water at various distances offshore. While the three pairs of soundings are not simultaneous they are felt to be sufficiently close in time during a period approaching steady flow that they have been treated as pairs of observations.

Observations over the water were made on the eastern side of Lake Michigan offshore from Manistee, Michigan, 140 km north of the line of observation. At this location the shoreline is oriented NNE-SSW and the lake width is 92 km. By analysing the data to obtain the components of the wind perpendicular to and along the lakeshore and by allowing for the difference in orientation of the shoreline these observations over the water become directly comparable with those made over the land during the 1964 season.

At 0745 a single ascent 3.2 km offshore given in Fig. 2.20 shows a weak land breeze from the surface to 800 m with flow toward the land above this. A second sounding 8 km offshore at 0830 indicated velocities below 1 m sec^{-1} from the surface to 1400 m where the balloon was lost.

Late in the afternoon about 1630 two balloons were tracked from stations about 16 km offshore, Fig. 2.21; these show positive u components through a depth of less than 500 m over the water with the depth of the onshore flow decreasing sharply toward the center of the lake. Above this surface layer the return circulation extends through a deeper layer. The reoriented lake breeze is from the NW, the same direction as over the land, Fig. 2.14, for about the same time of day.

Fig. 2.22, drawn for approximately 1815, shows a layer of onshore flow, also from the NW, to a height of about 700 m at a location 5 km offshore. A reversed flow from the SE occurs through a much deeper layer above the lake breeze. Despite the fact that the lake breeze should be weakening at this time of day as a result of the weakening gradient of temperature across the shoreline, the depth of the lake breeze currents are greater than those observed an hour earlier at 16 km offshore (Fig. 2.21). These observations indicate that over the water the depth of the lake breeze flow decreases with increasing distance from the shoreline.

2.6.5 Homogeneity of the lake breeze along the shoreline

The homogeneity of the lake breeze along the shoreline and the orderly progress of the lake breeze front inland are demonstrated in Fig. 2.23 for July 23, 1964. Wind directions for six stations in the region of observation are presented. Wind speeds for the stations are not shown; they are not directly comparable because of the different methods by which the data were obtained.

At the Muskegon and Grand Rapids U.S. Weather Bureau stations observations are made using wind vanes installed at a height of about 10 m. At the lake shore and at the 8 and 16 km stations, mean winds in the lowest 110 m above the surface as indicated by pilot balloon ascents are plotted. At Holland data are obtained from recording wind instruments installed at 19.5 m above the surface. Station locations relative to the shoreline and to the observation line are given in Fig. 2.23 with the station closest to the shoreline plotted at the top and each successive station downward being progressively further inland.

Wind direction changes at these stations in response to the lake breeze are remarkable for their consistency and agreement considering the various methods by which the data were obtained. The lake breeze occurs first at the lakeshore as expected. Thereafter the wind shift is observed to occur at the stations 6.5, 8, 10 and 16 km inland in ordered time sequence. Data for four lake breeze days show similar results. The wind direction change at Grand Rapids is not attributed to flow off the water as it did not occur on other days and because it occurred prior to the onset of the lake breeze at the 16 km station.

The clockwise turning of the lake breeze wind at the surface is also clearly apparent in these records. Initially the flow is almost directly off the water from 270° or perhaps with a small component from the south. The wind direction gradually changes through the day toward 360° in response to the Coriolis force.

2.6.6 Wind on opposite sides of Lake Michigan

Simple analysis suggests that the lake breeze will be oppositely directed on the eastern and western shores of Lake Michigan. Table 2.2 gives the wind speeds and directions on opposite sides of the lake

TABLE 2.2

WIND SPEED AND DIRECTION AT U.S. WEATHER BUREAU STATIONS
ON OPPOSITE SHORES OF
LAKE MICHIGAN JULY 23, 1964.

	Muskegon		Milwaukee	
Time	Speed knots	Direction degrees	Speed knots	Direction degrees
0900	6	080	4	030
1000	4	180	6	050
1100	6	180	5	030
1200	5	100	6	030
1300	6	360	6	120
1400	7	250	7	100
1500	7	290	7	100
1600	7	320	6	100
1700	6	290	5	100
1800	5	320	5	100
1900	5	320	4	090
2000	5	340	3	110
2100	3	060	3	090

between 0900 and 2100 July 23, 1964 taken directly from the Muskegon and Milwaukee U.S. Weather Bureau station records and shows that such is the case. These two stations are selected for comparison because winds are observed by similar techniques and both stations are located a few kilometers inland from the lake shore. At Muskegon onshore flow has direction 270° while at Milwaukee onshore flow is from 090°.

Wind speeds at both stations are approximately the same for the period considered but it is immediately obvious that flow has a strong onshore component from approximately opposite directions at the two stations, i.e., a lake breeze occurs simultaneously on both sides of the lake.

2.7 Summary and conclusions

The spatial and temporal variations of a lake breeze circulation system observed on the eastern side of Lake Michigan during a period of very weak gradient flow have been described. Horizontal temperature and moisture contrasts between air over the land and air off the water are pronounced near the lake shore but a few kilometers inland these contrasts are much weaker and in the vertical direction strong contrasts were not detected. The rapid modification of air properties in the horizontal and vertical described in Sec. 2.5.2 indicates that intense mixing occurs in the lake breeze and that the lake breeze circulation cannot be regarded as a closed cell.

The lake breeze observed was homogeneous along the lake shore, at least at the lower levels. Winds at the surface on both sides of the lake were onshore during the day so that surface divergence occurs over the water. A few observations of the wind field distribution in the vertical over the water indicate that the circulation system here has dimensions and characteristics similar to those over the land during the period when the lake breeze is fully developed.

Over land the lake breeze develops suddenly three to four hours after the air temperature over the land at an inland location becomes equal to lake water temperature. Qualitative observations indicate that the onset of the lake breeze at a given location occurs in a series of pulses and wind reversals after which a continuous onshore flow is established. The lake breeze front progresses less than 8 km inland in its initial stage of development. Thereafter the front progresses inland by surges after a period of intensification and development of the circulation system which may last several hours. This characteristic behavior was observed in all lake breezes analysed. Horizontal convergence occurs from both sides of the lake breeze front inland. Horizontal divergence over the water is indicated both by winds observed and by the presence of a warm layer of air near surface over the water which could be caused by adiabatic descent and warming as suggested in Fig. 2.5.

The depth of onshore flow in a fully developed lake breeze is about 750 m and horizontal onshore velocities exceeding 7 m sec^{-1} have been observed. Maximum flow intensity occurs late in the afternoon within 250 m of the surface directly over the lake shore. Maximum inland penetration of the lake breeze front may occur a few hours later in a weakening system. The region of onshore flow extends 25 to 30 km inland but does not reach 53 km inland for any of the cases observed.

Above the lake breeze during periods of light gradient flow aloft or when the thermal circulation at the surface near the lake shore is largely isolated from effects of the general circulation above, as in the case where an inversion occurs aloft at the top of a cold surface anticyclone, a very clear lake breeze return circulation develops. The return flow has about twice the depth of the layer of onshore flow and velocities are approximately half as large. Maximum velocities in the return flow occur directly above the corresponding maxima in the onshore flow near surface. On the eastern shore of Lake Michigan where normal gradient flow aloft is onshore, the offshore return circulation is sandwiched between two regions of flow in the opposite direction. General circulation flow patterns are influenced by the local lake breeze phenomenon to depths exceeding 2500 m.

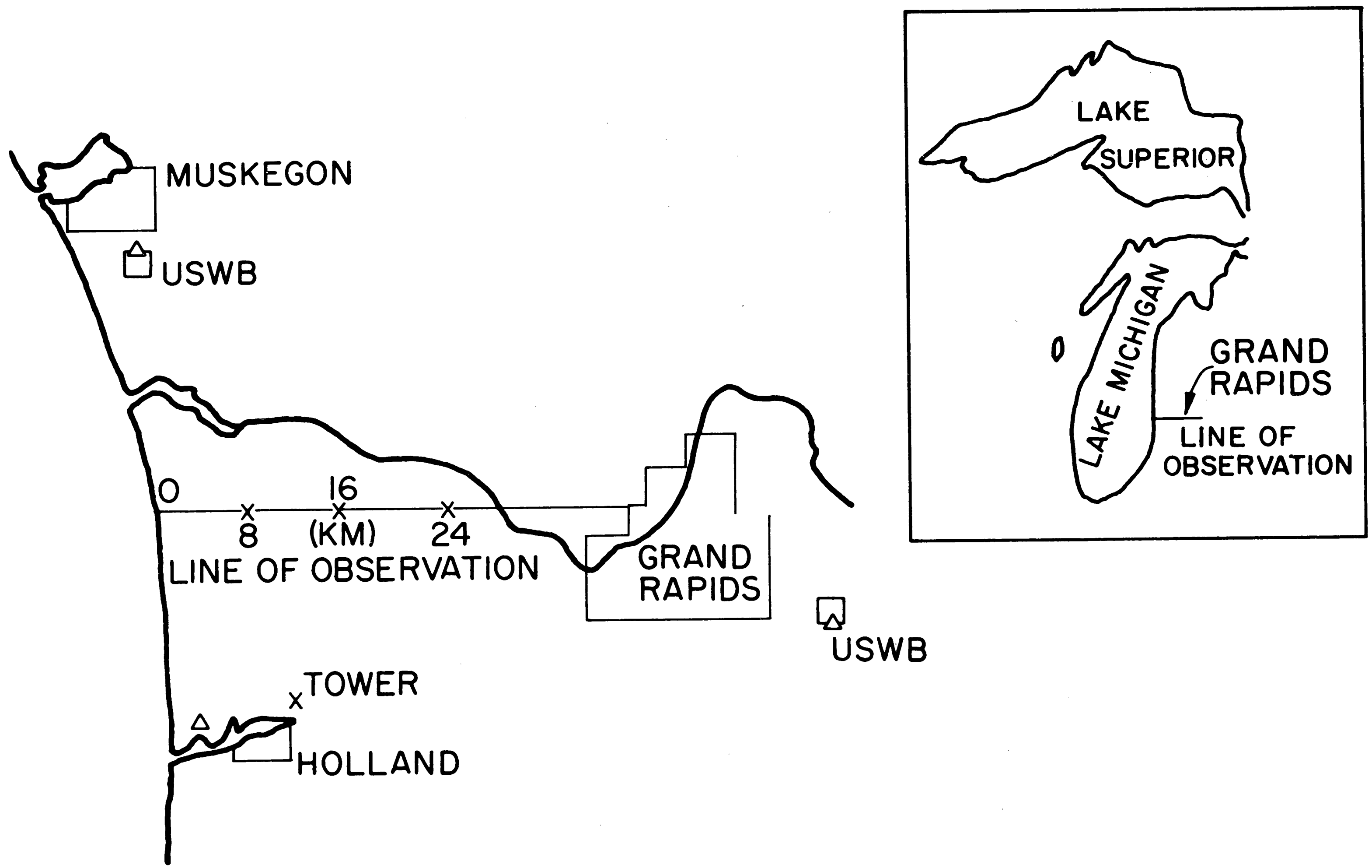

Fig. 2.1. Location of observation stations near the Lake Michigan shoreline. Insert shows the location of the line of observation relative to Lake Michigan.

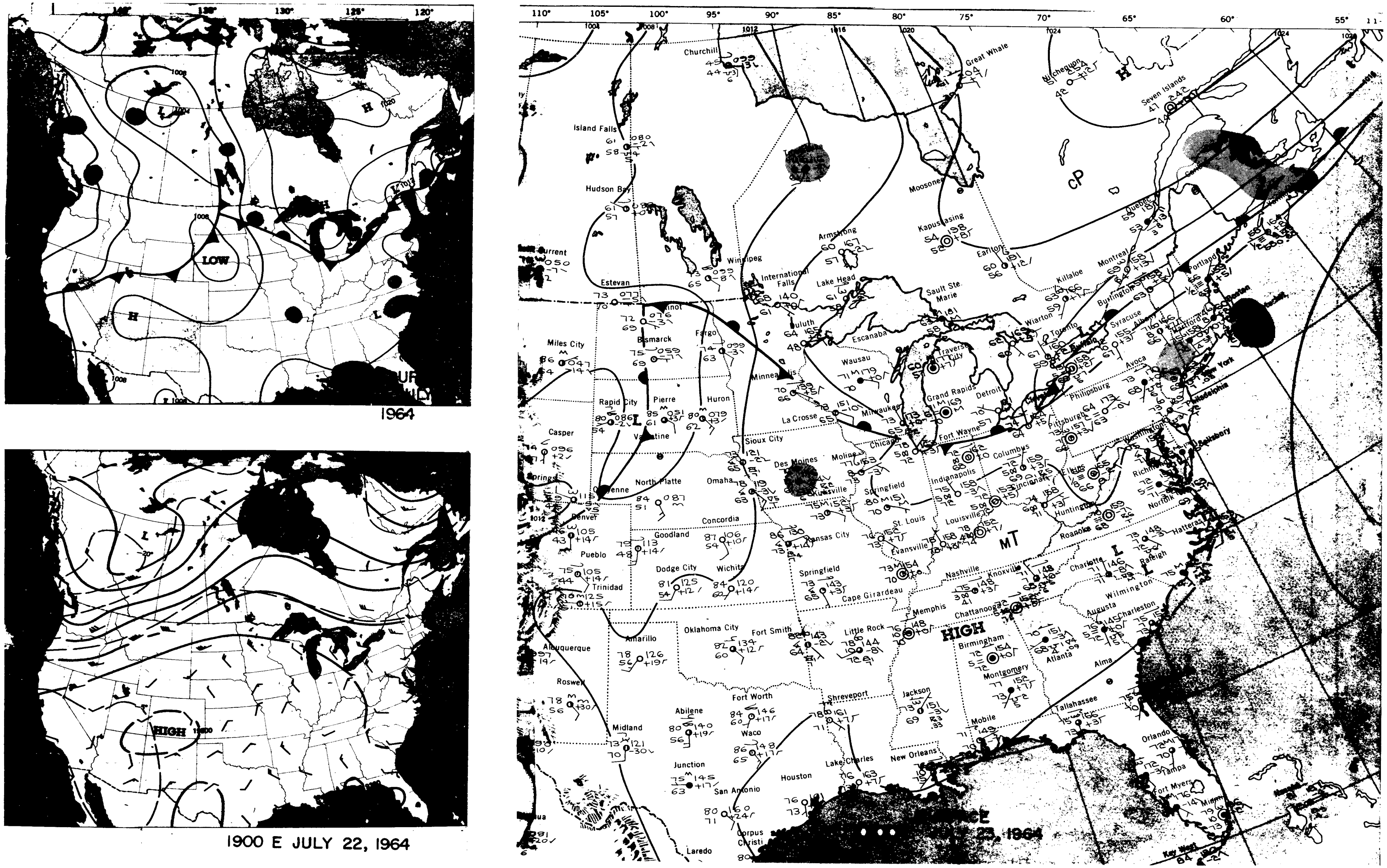

Fig. 2.2. U.S. Weather Bureau Daily Weather Map for July 23, 1964. Upper insert shows the sea level pressure contours at 1300EST July 22 and lower insert shows height contours for the 500 mb surface at 1900EST July 22, 1964.

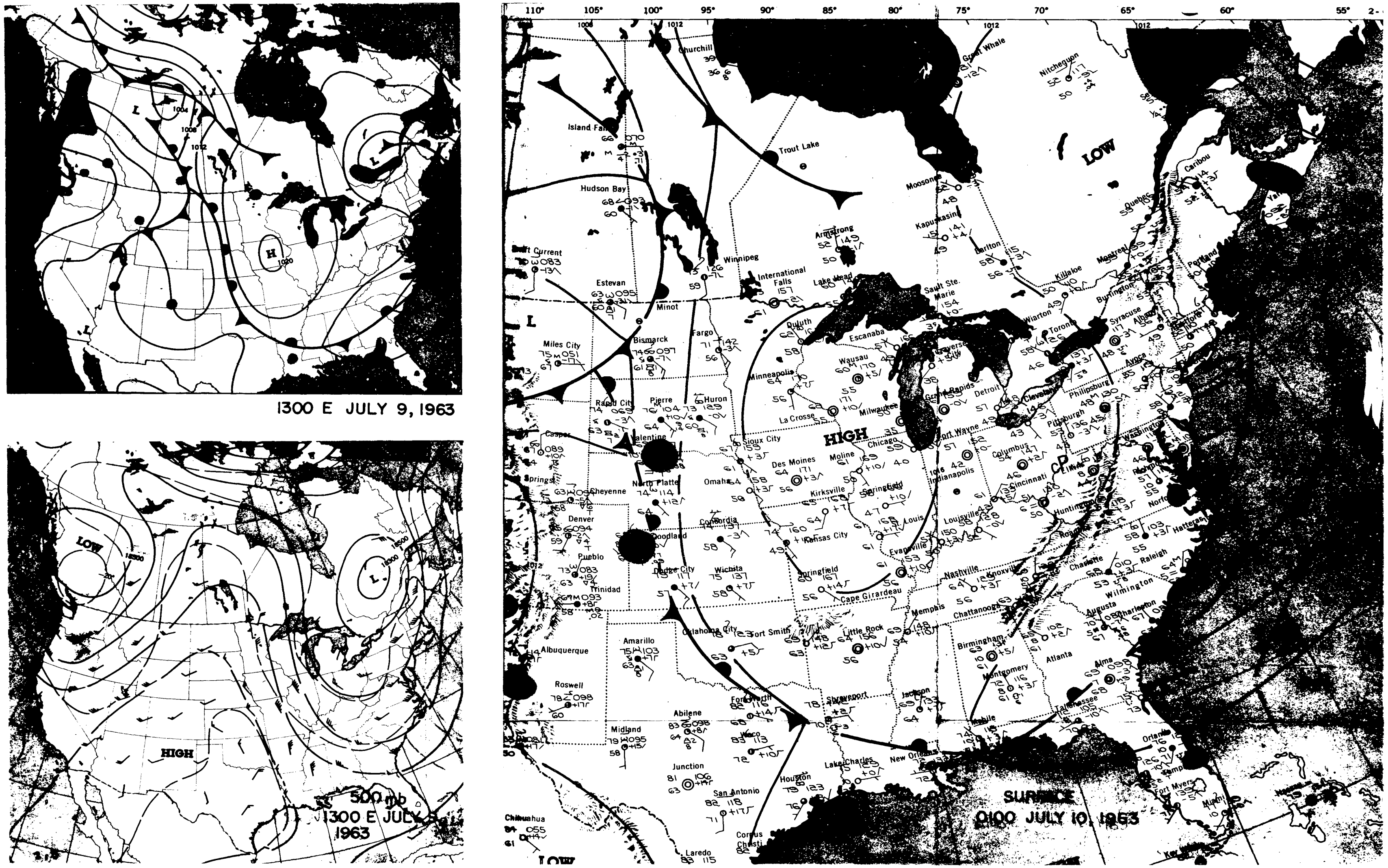

Fig. 2.3. U.S. Weather Bureau Daily Weather Map for July 10, 1963. Inserts are as in Fig. 2.2 but for July 9, 1963.

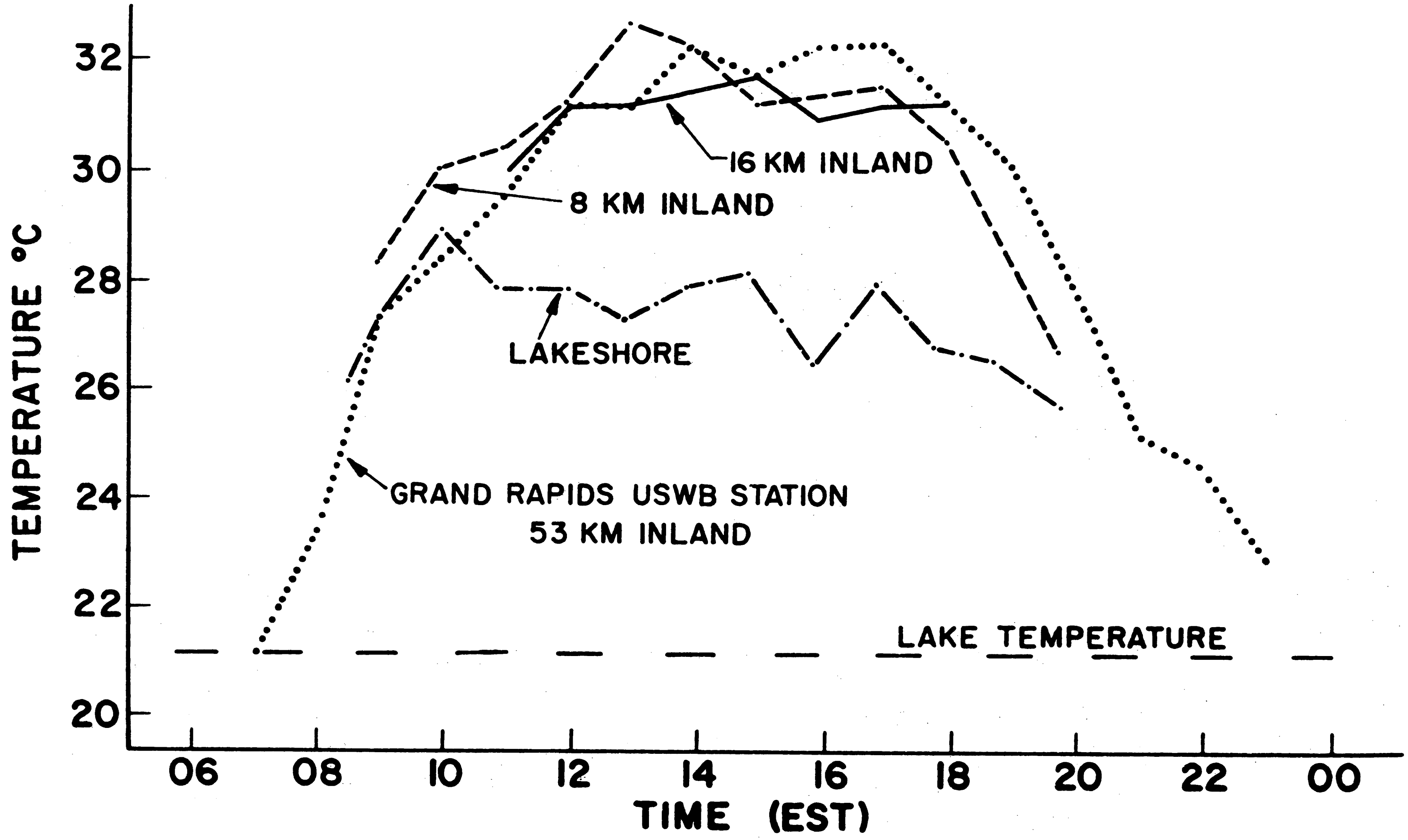

Fig. 2.4. Surface dry bulb temperature (^{o}C) at various distances inland from the lakeshore July 23, 1964.

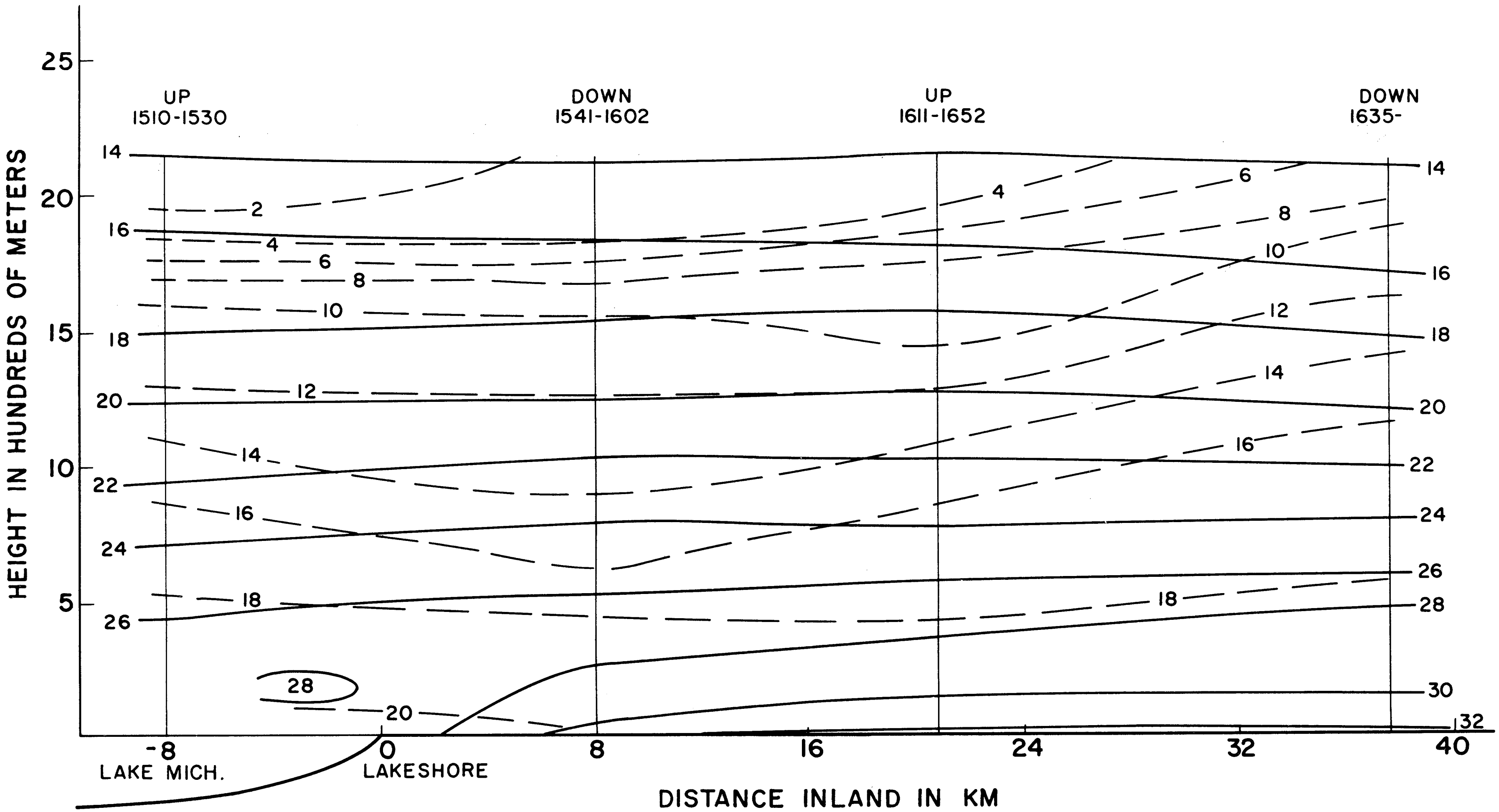

Fig. 2.5. Dry bulb (solid) and dew point (dashed) temperature (^{o}C) structure in the vertical for July 24, 1964.

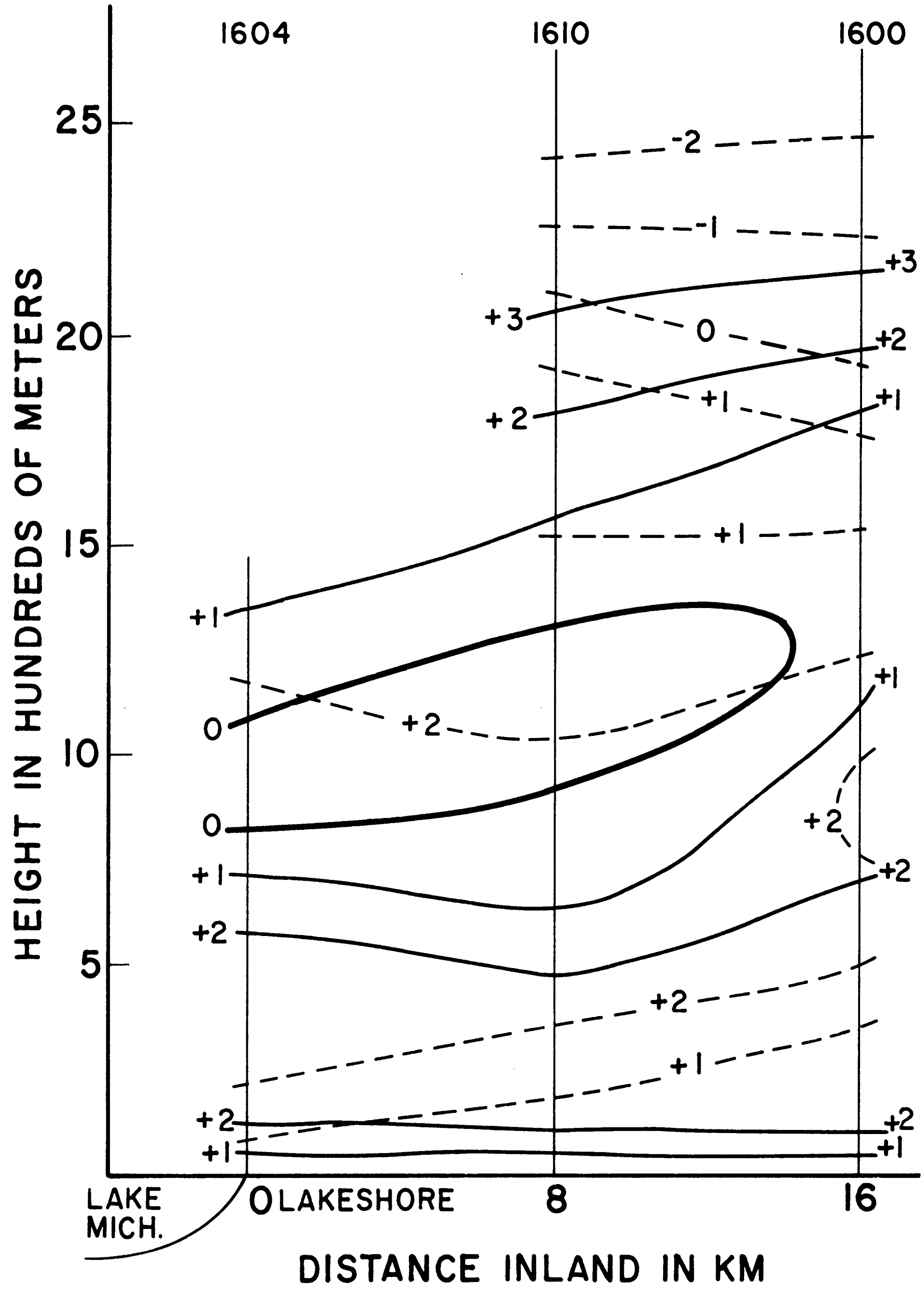

Fig. 2.6. Horizontal wind components at 1600EST July 24, 1964. Isotachs for the across shore component (u) are solid lines with values at the ends of the line in m sec^{-1}; those for the alongshore component (v) are dashed lines, with values within the line where possible, in m sec^{-1}. The time at which the balloon was released is indicated at the top of the fine vertical line for each station where observations were made. The length of the vertical line indicates the height to which the balloon was tracked.

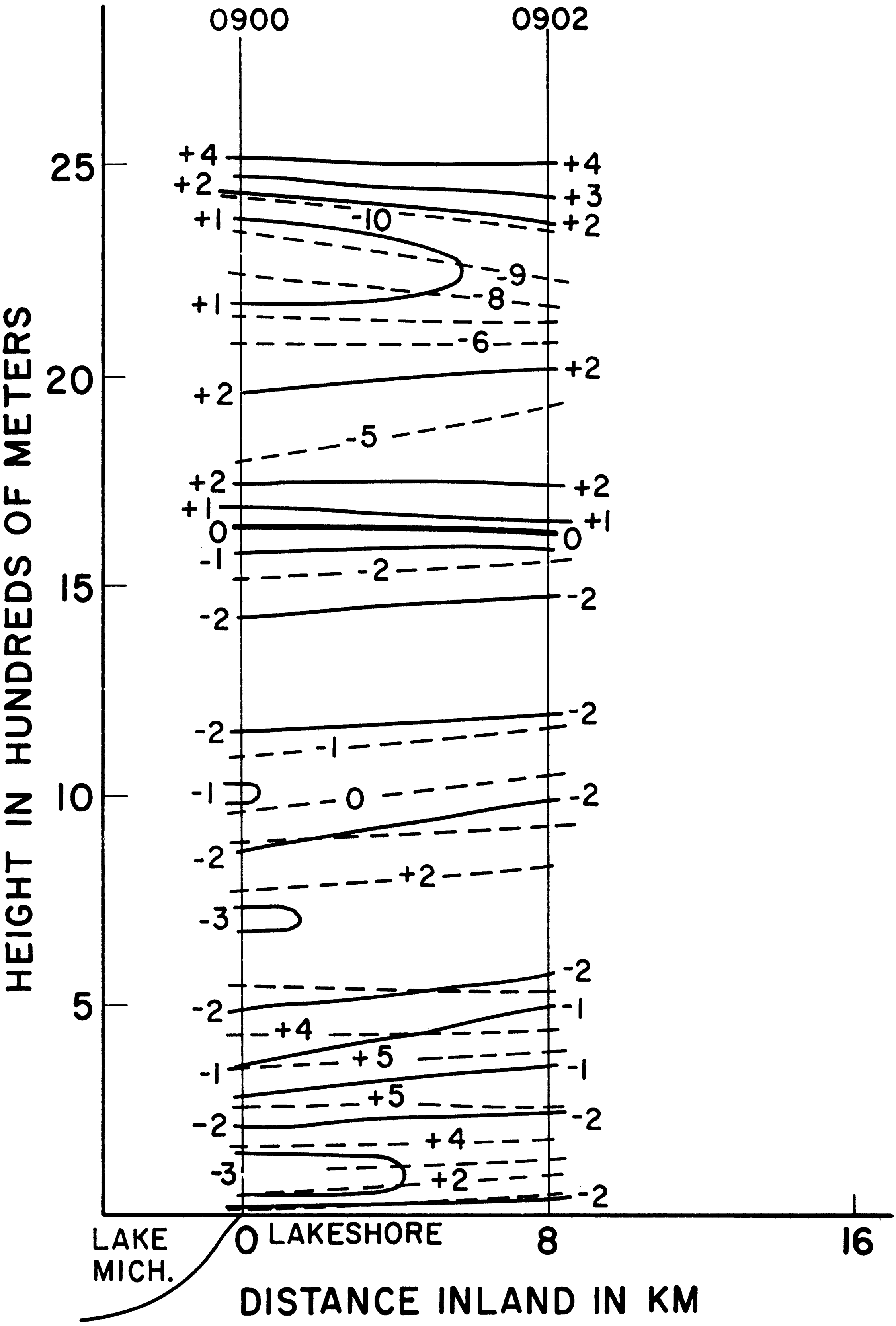

Fig. 2.7. Horizontal wind components at 0900EST July 23, 1964. Conventions of the figure are the same as those for Fig. 2.6.

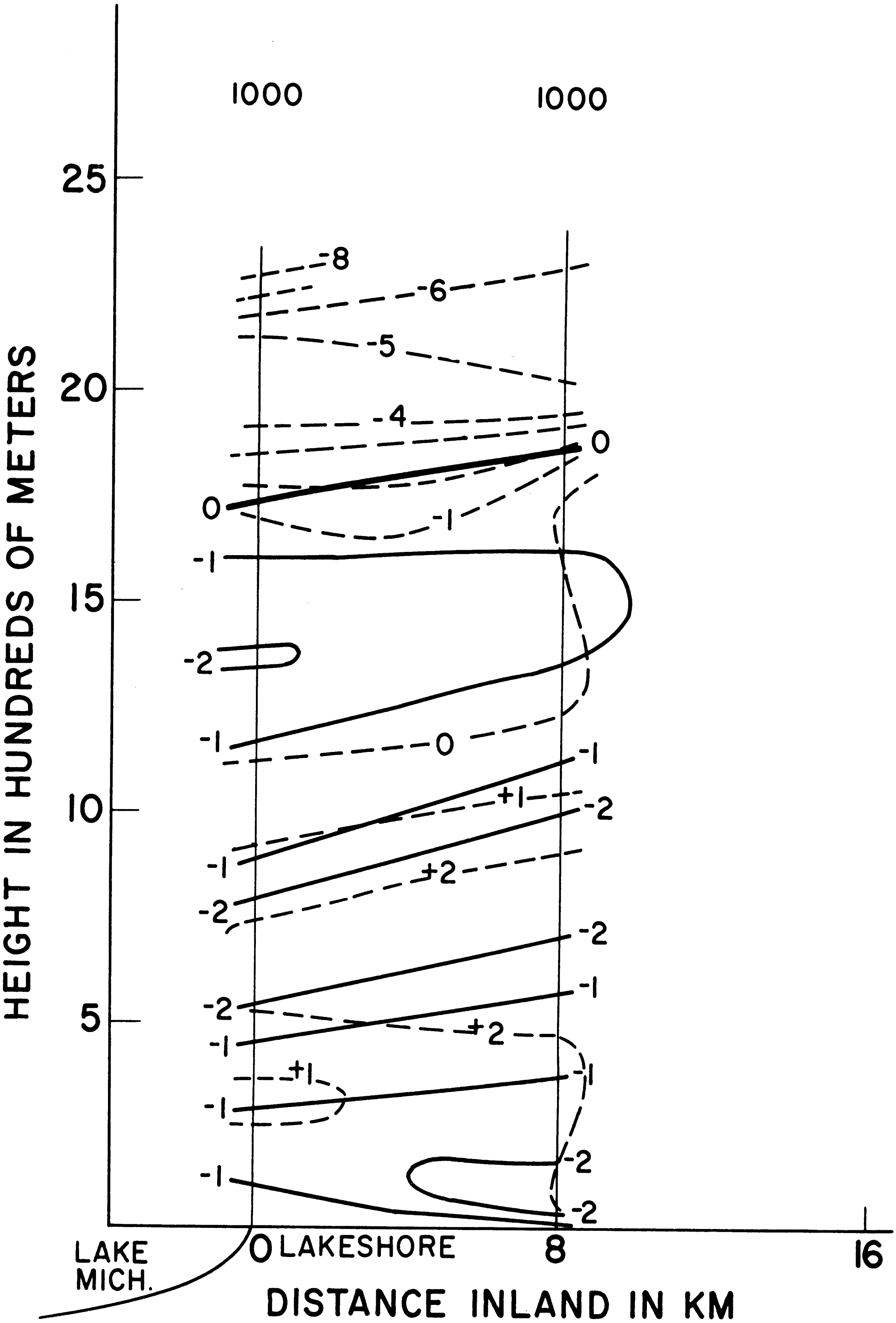

Fig. 2.8. Horizontal wind components at 1000EST July 23, 1964. Conventions of the figure are the same as those for Fig. 2.6.

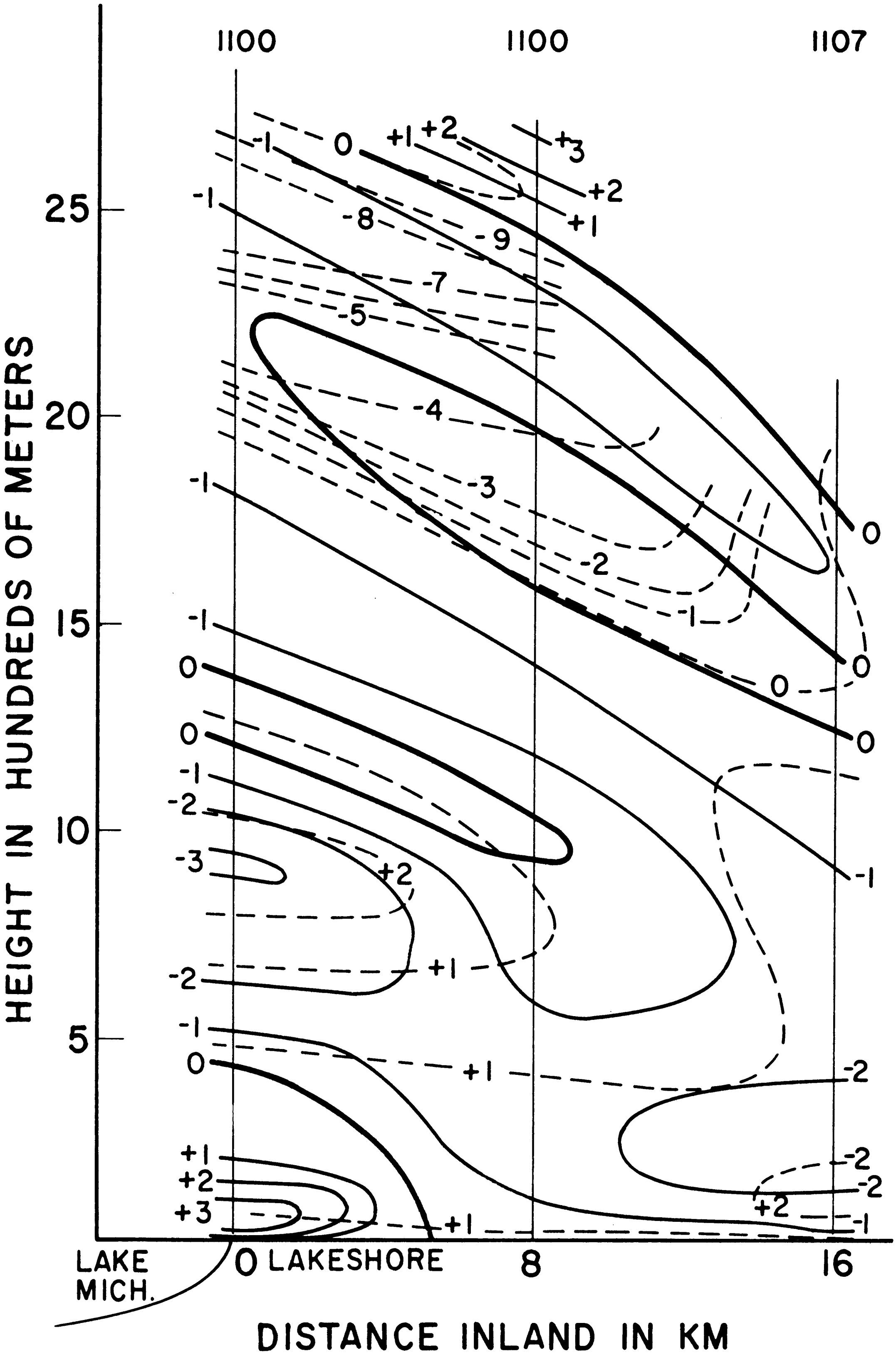

Fig. 2.9. Horizontal wind components at 1100EST July 23, 1964. Conventions of the figure are the same as those for Fig. 2.6.

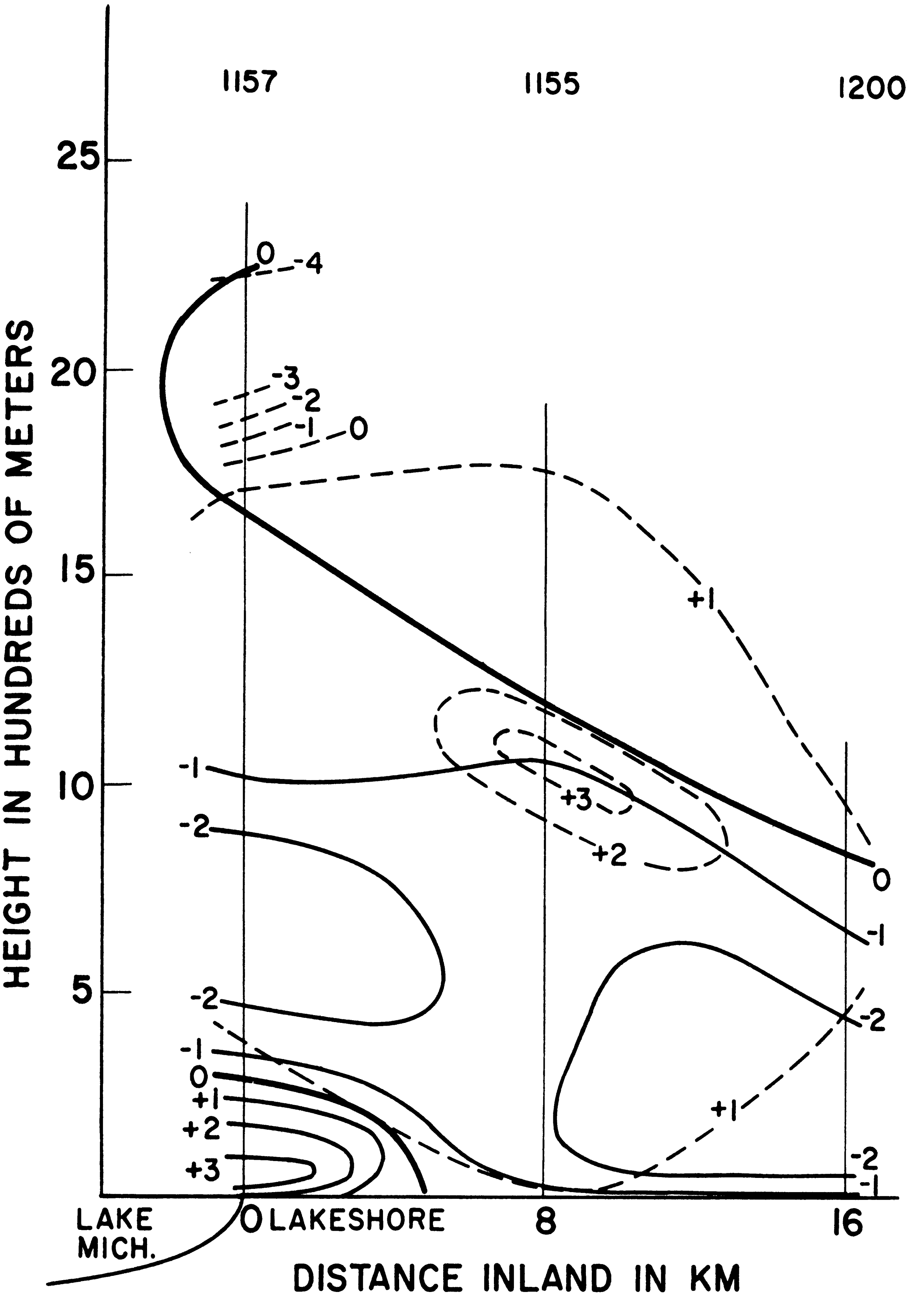

Fig. 2.10. Horizontal wind components at 1200EST July 23, 1964. Conventions of the figure are the same as those for Fig. 2.6.

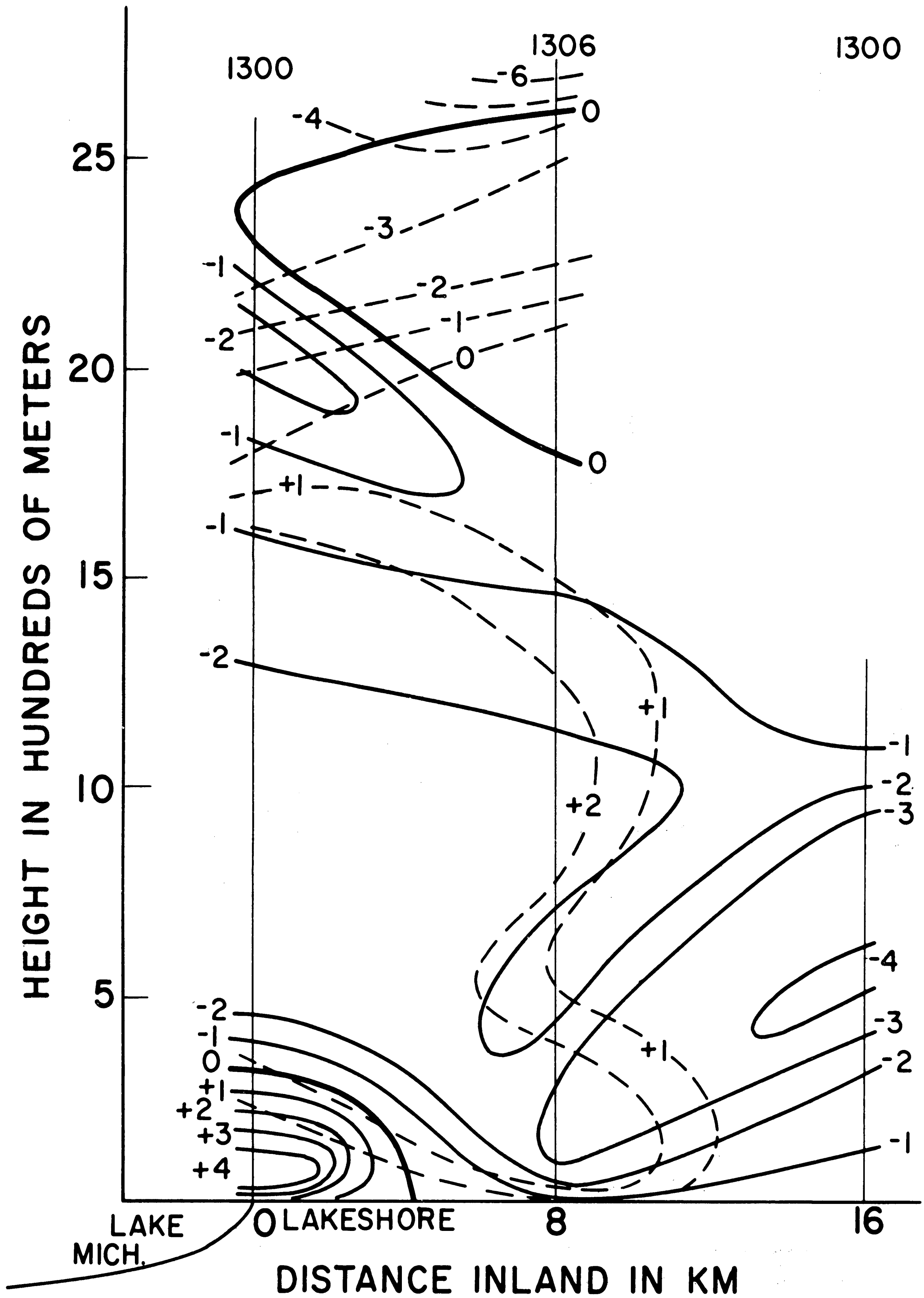

Fig. 2.11. Horizontal wind components at 1300EST July 23, 1964. Conventions of the figure are the same as those for Fig. 2.6.

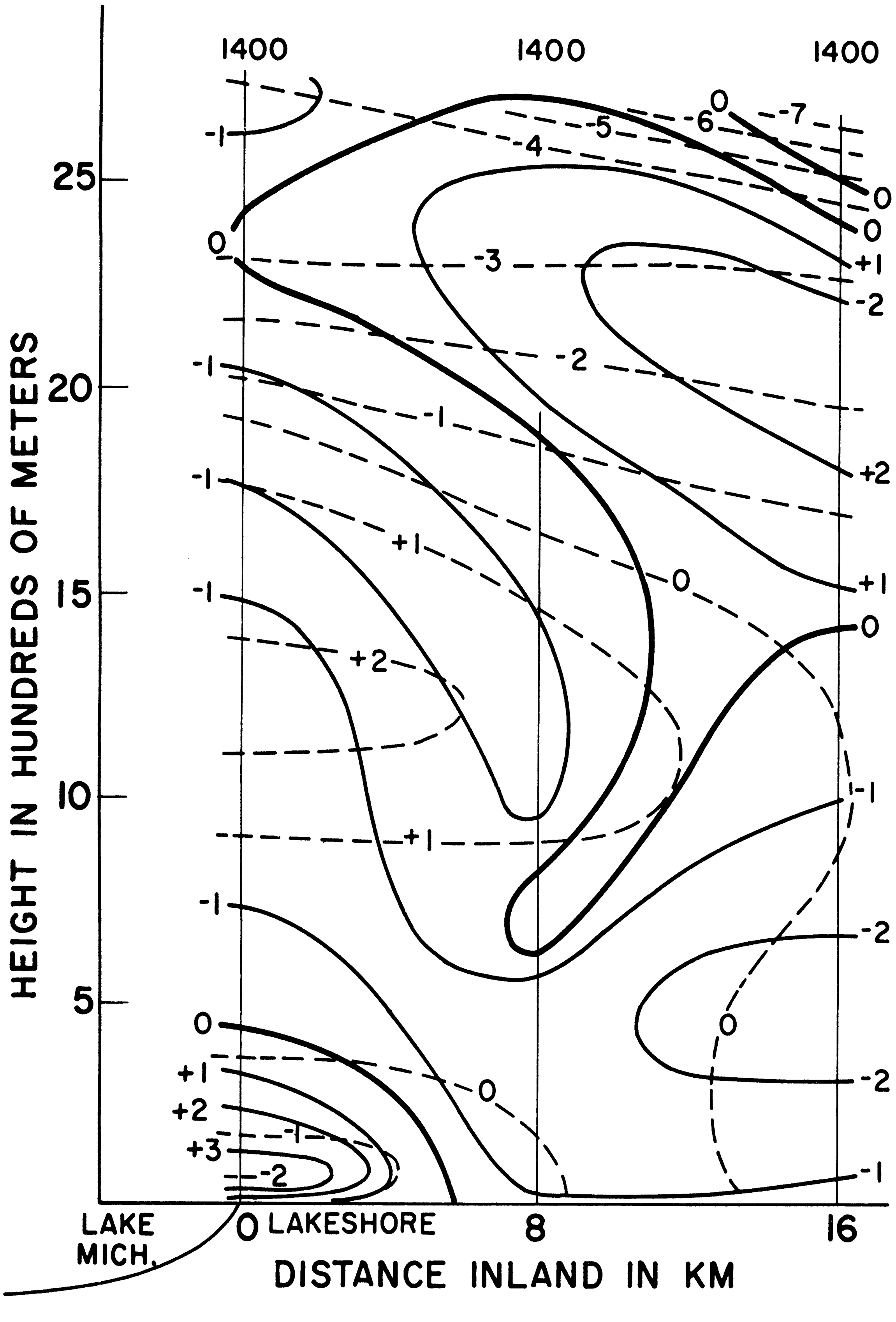

Fig. 2.12. Horizontal wind components at 1400EST July 23, 1964. Conventions of the figure are the same as those for Fig. 2.6.

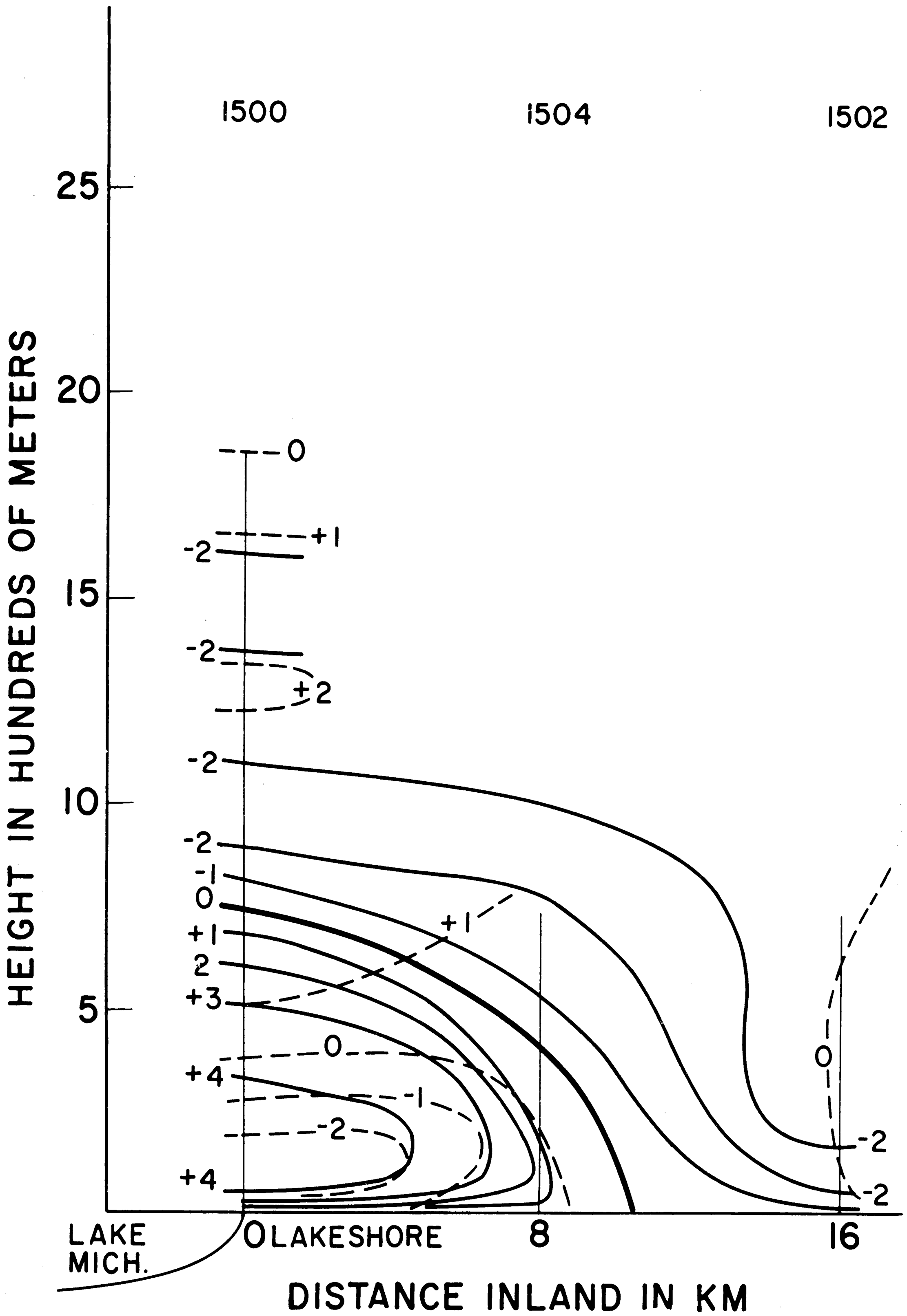

Fig. 2.13. Horizontal wind components at 1500EST July 23, 1964. Conventions of the figure are the same as those for Fig. 2.6.

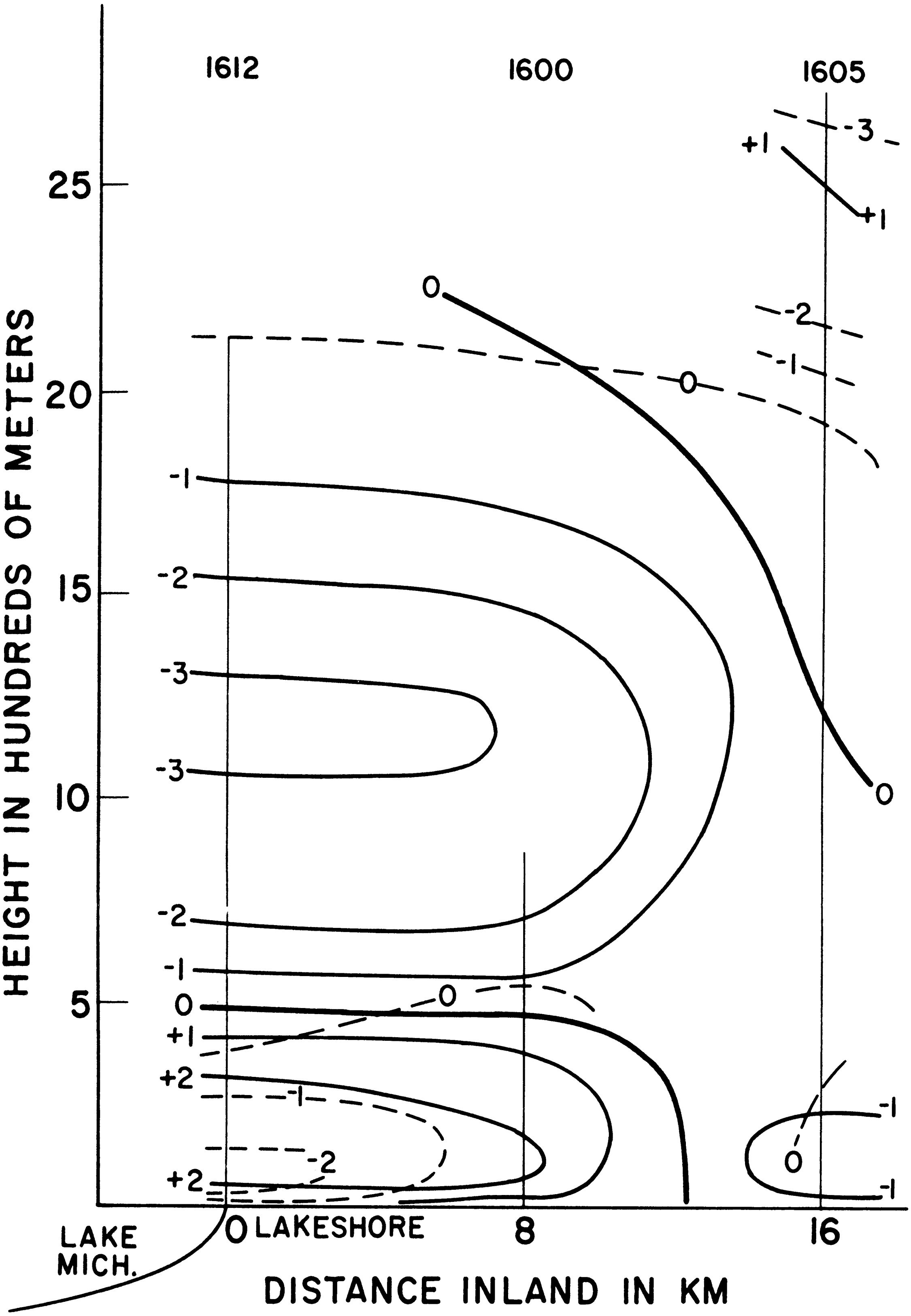

Fig. 2.14. Horizontal wind components at 1600EST July 23, 1964. Conventions of the figure are the same as those for Fig. 2.6.

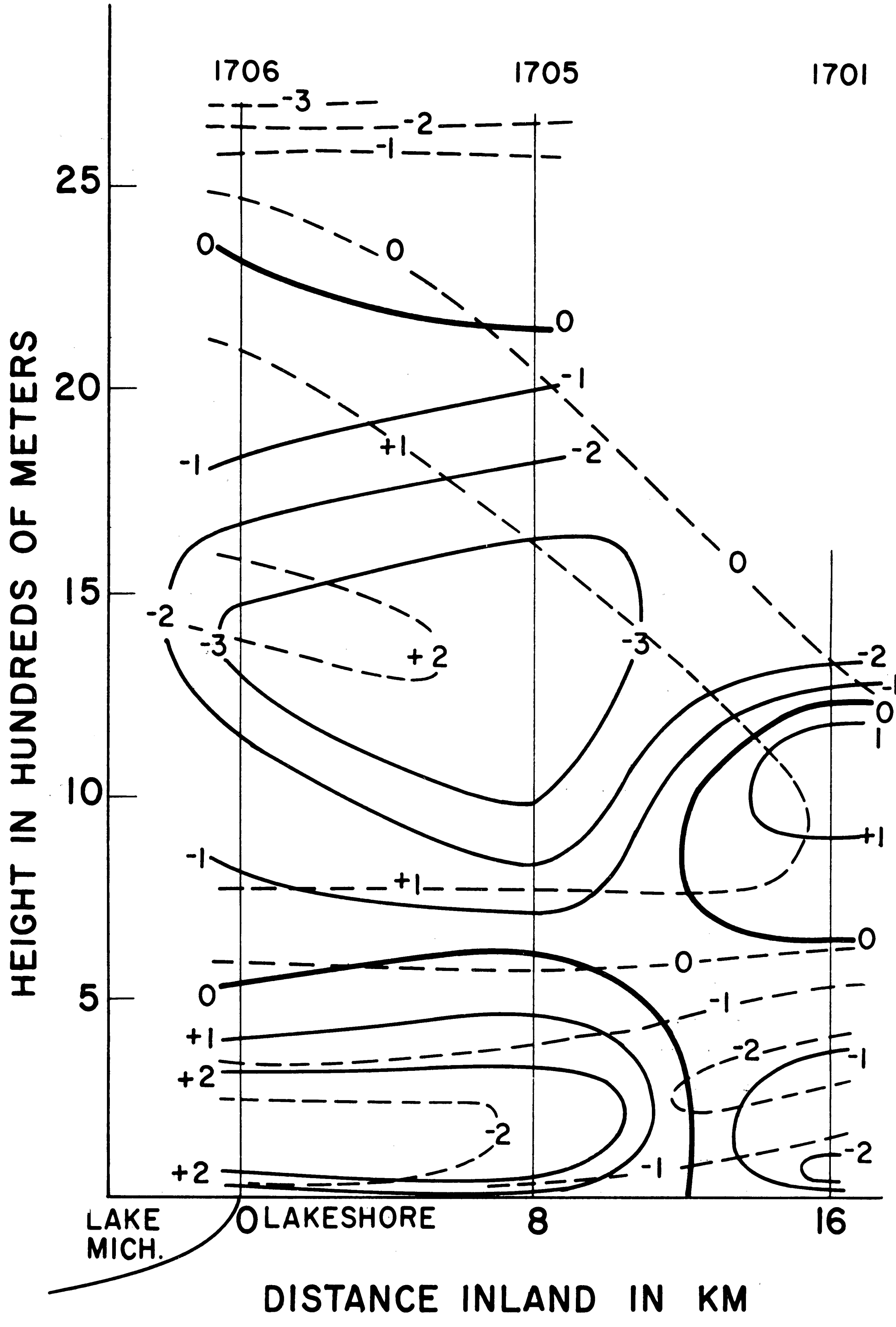

Fig. 2.15. Horizontal wind components at 1700EST July 23, 1964. Conventions of the figure are the same as those for Fig. 2.6.

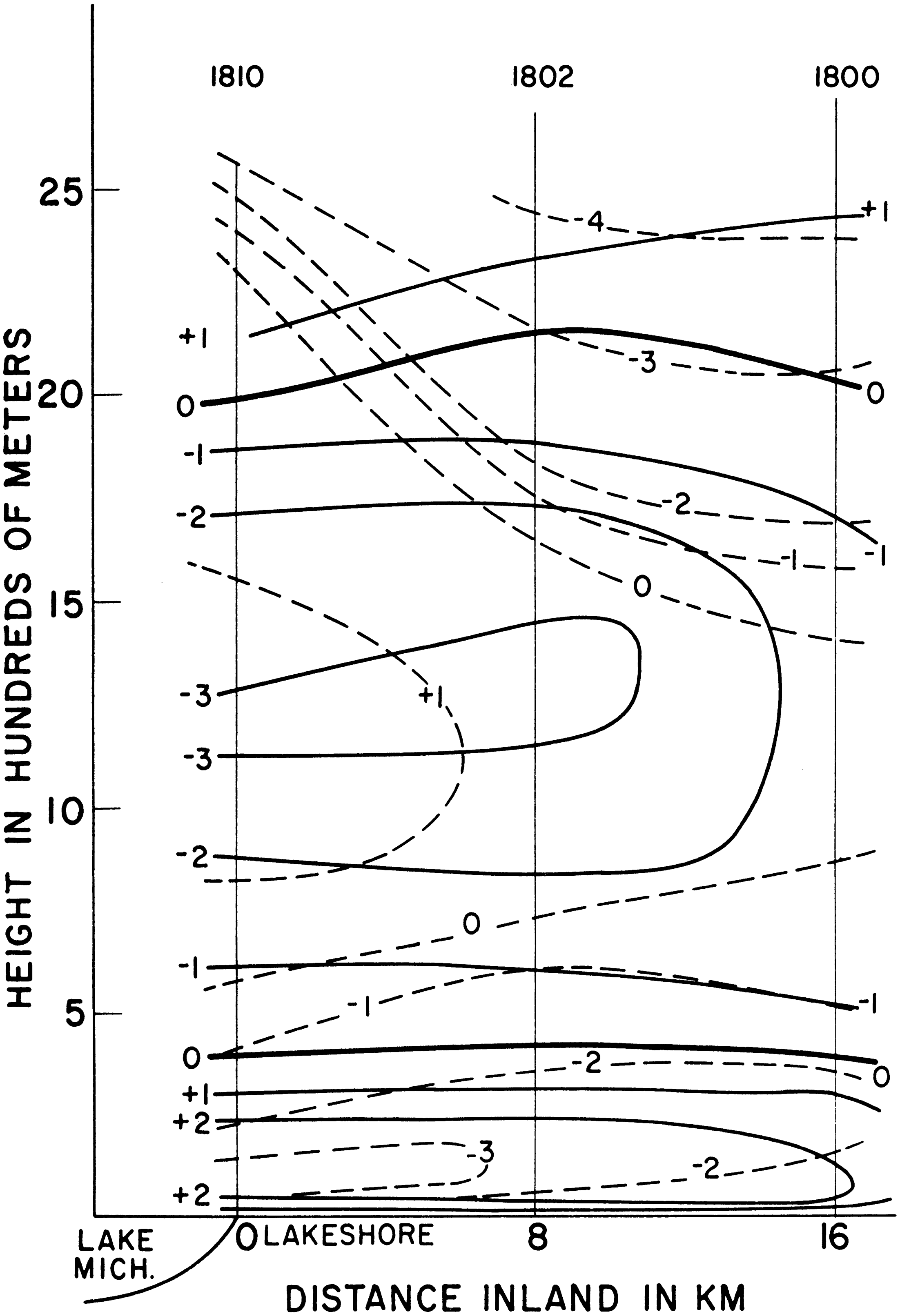

Fig. 2.16. Horizontal wind components at 1800EST July 23, 1964. Conventions of the figure are the same as those for Fig. 2.6.

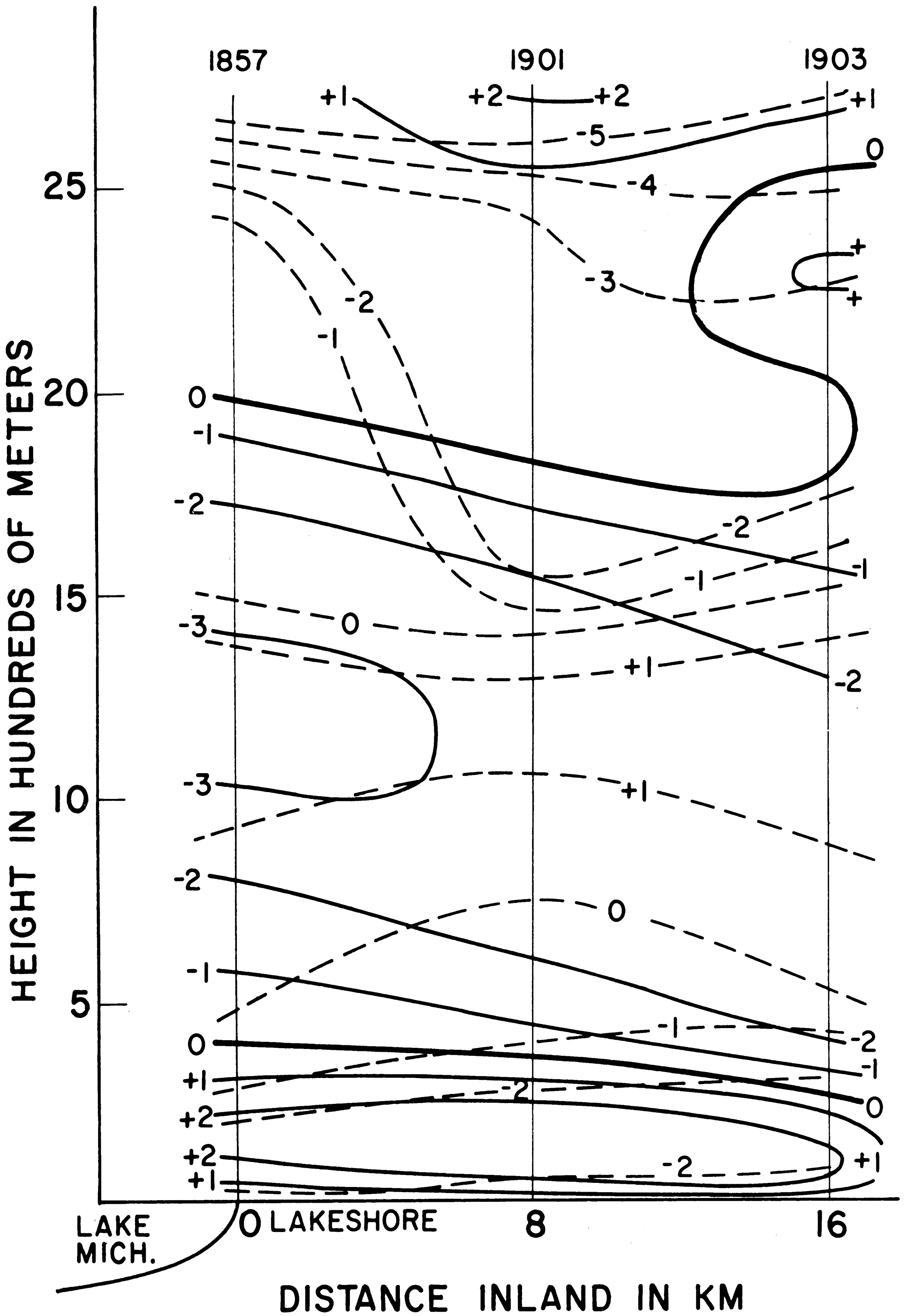

Fig. 2.17. Horizontal wind components at 1900EST July 23, 1964. Conventions of the figure are the same as those for Fig. 2.6.

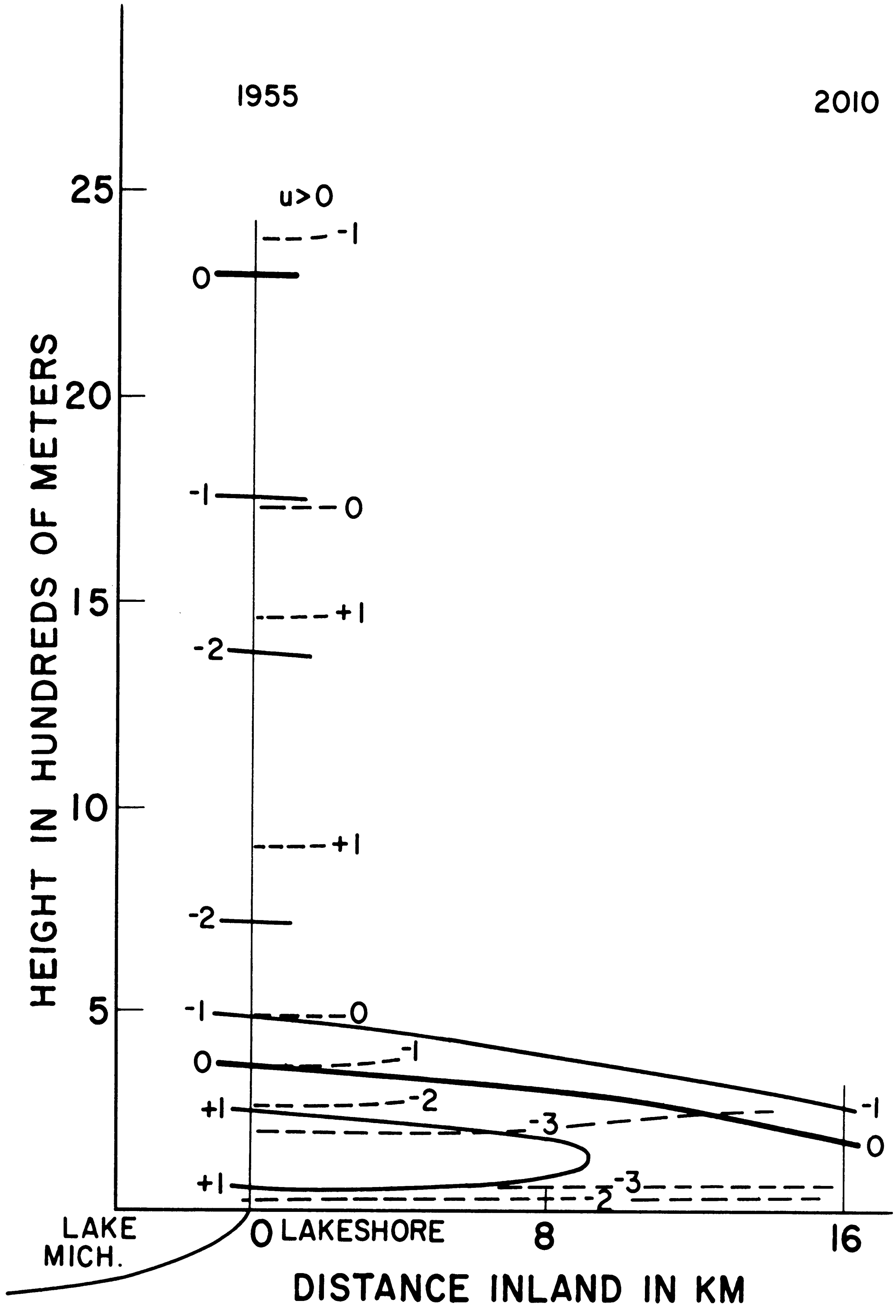

Fig. 2.18. Horizontal wind components at 2000EST July 23, 1964. Conventions of the figure are the same as those for Fig. 2.6.

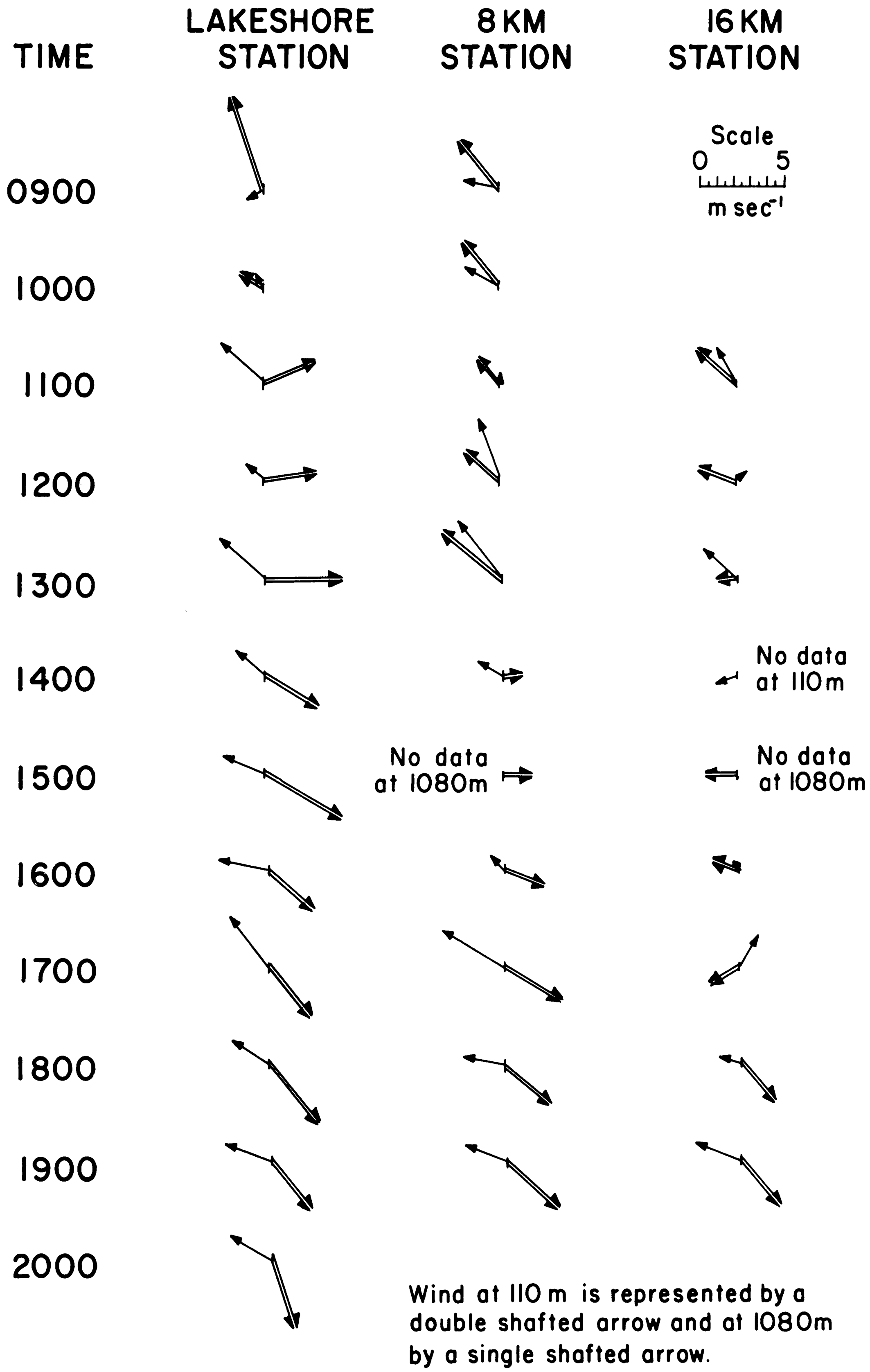

Fig. 2.19. Variation of wind speed and direction during the day at 110 m (double shafted arrow) and at 1000 m (single shafted arrow) above the surface July 23, 1964.

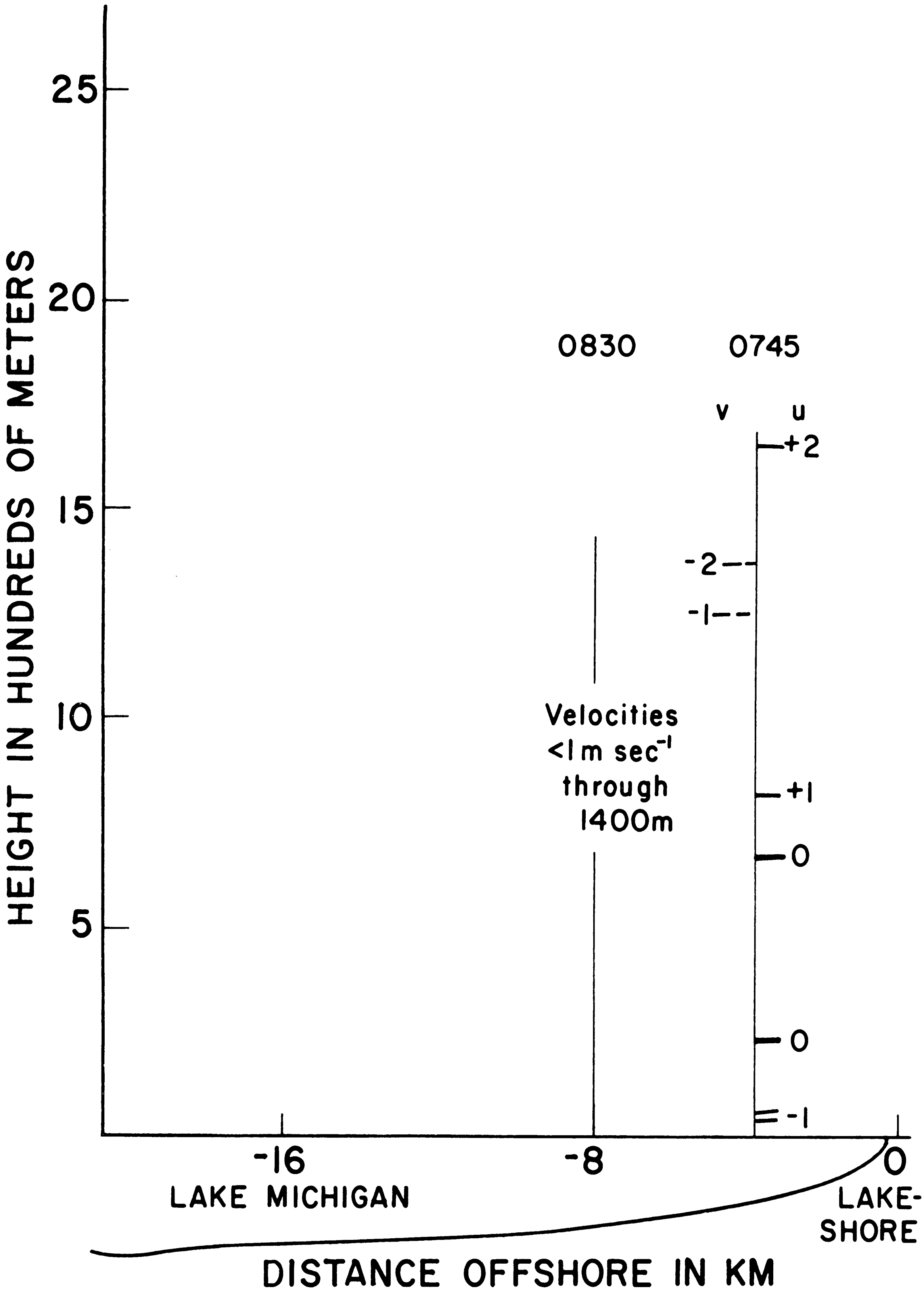

Fig. 2.20. Horizontal wind components over the water at 0745EST and 0830EST about 5 km offshore July 10, 1963. Conventions of the figure are the same as those for Fig. 2.6.

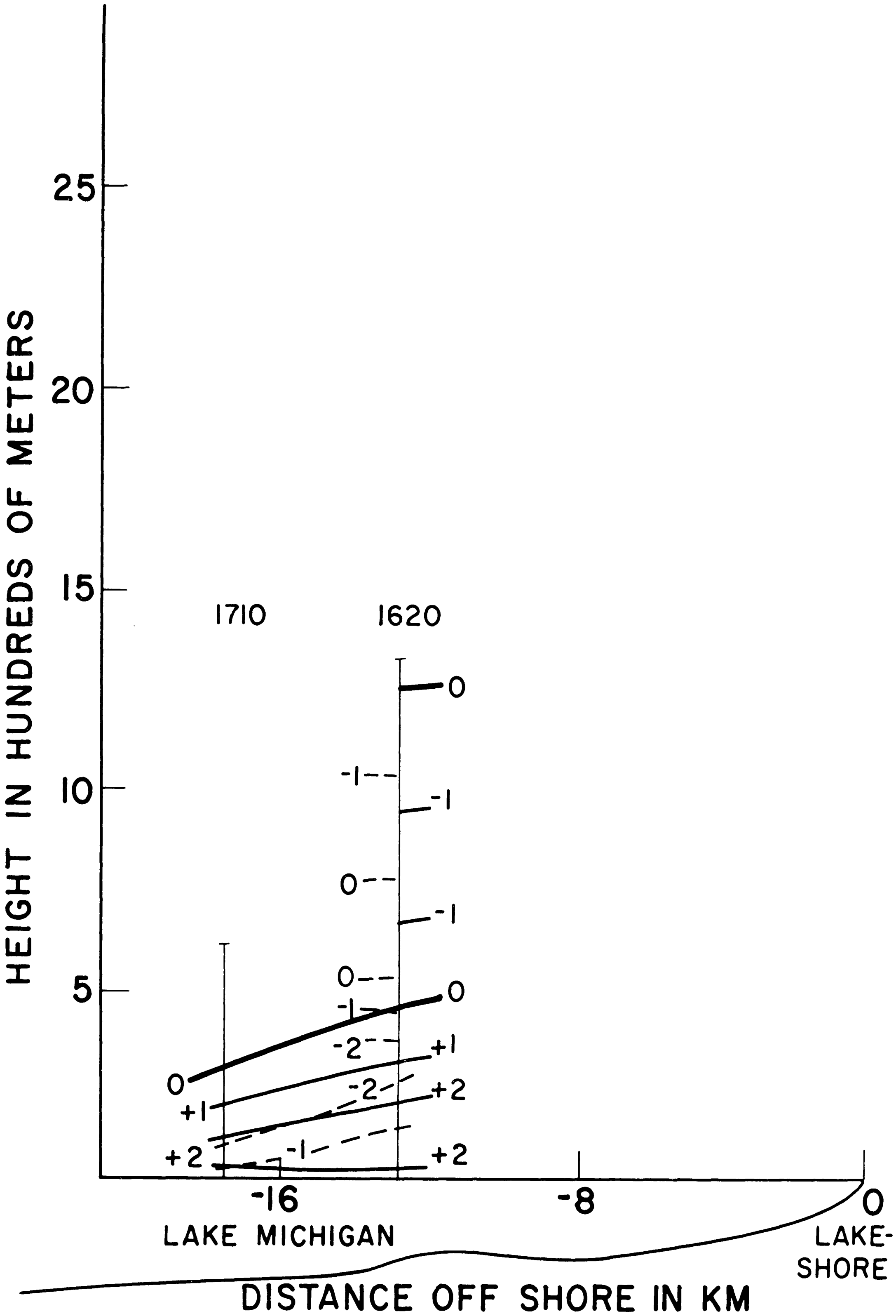

Fig. 2.21. Horizontal wind components over the water at 1620EST and 1710EST about 16 km offshore July 10, 1963. Conventions of the figure are the same as those for Fig. 2.6.

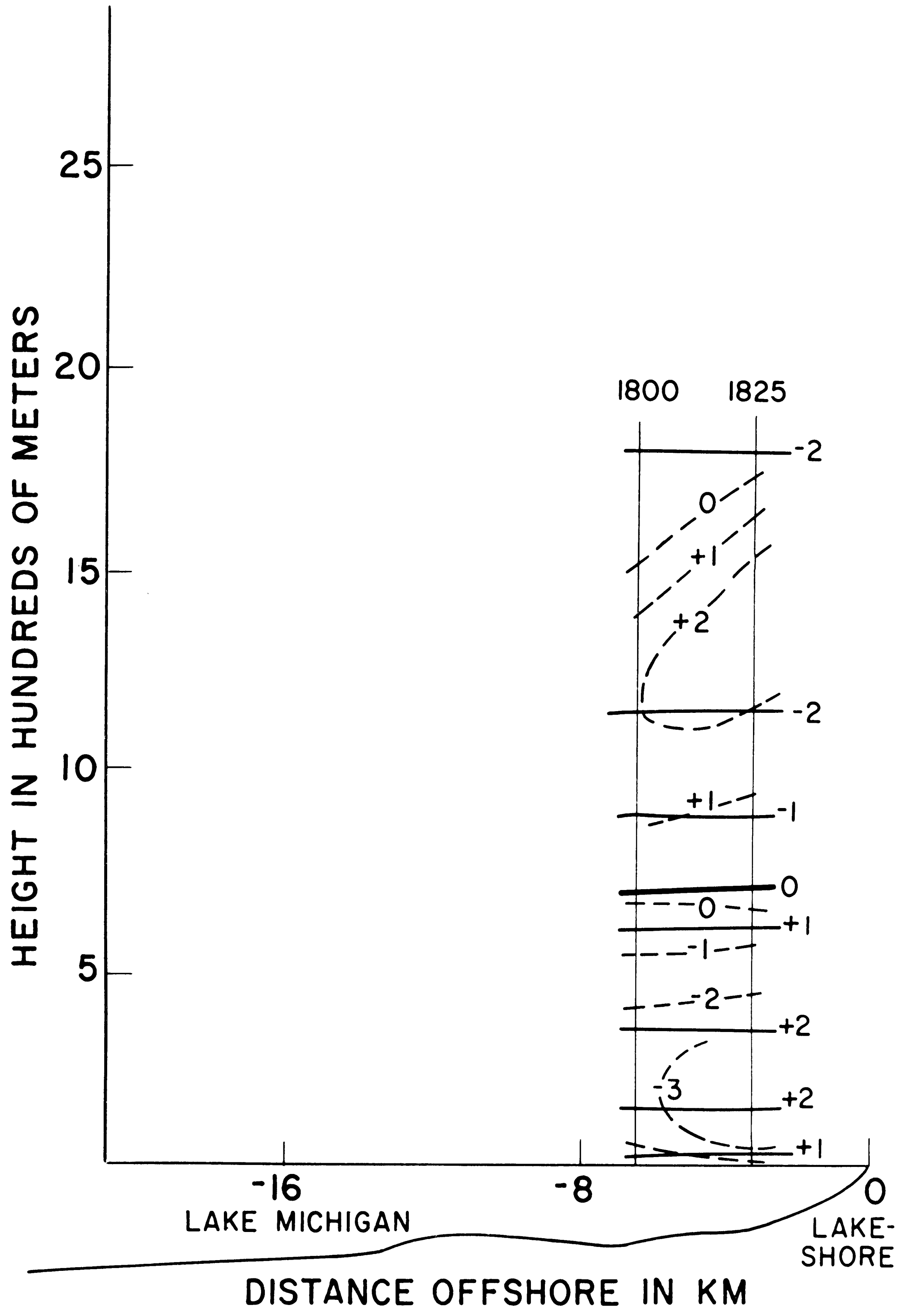

Fig. 2.22. Horizontal wind components over the water at 1800EST and 1825EST about 5 km offshore July 10, 1963. Conventions of the figure are the same as those for Fig. 2.6.

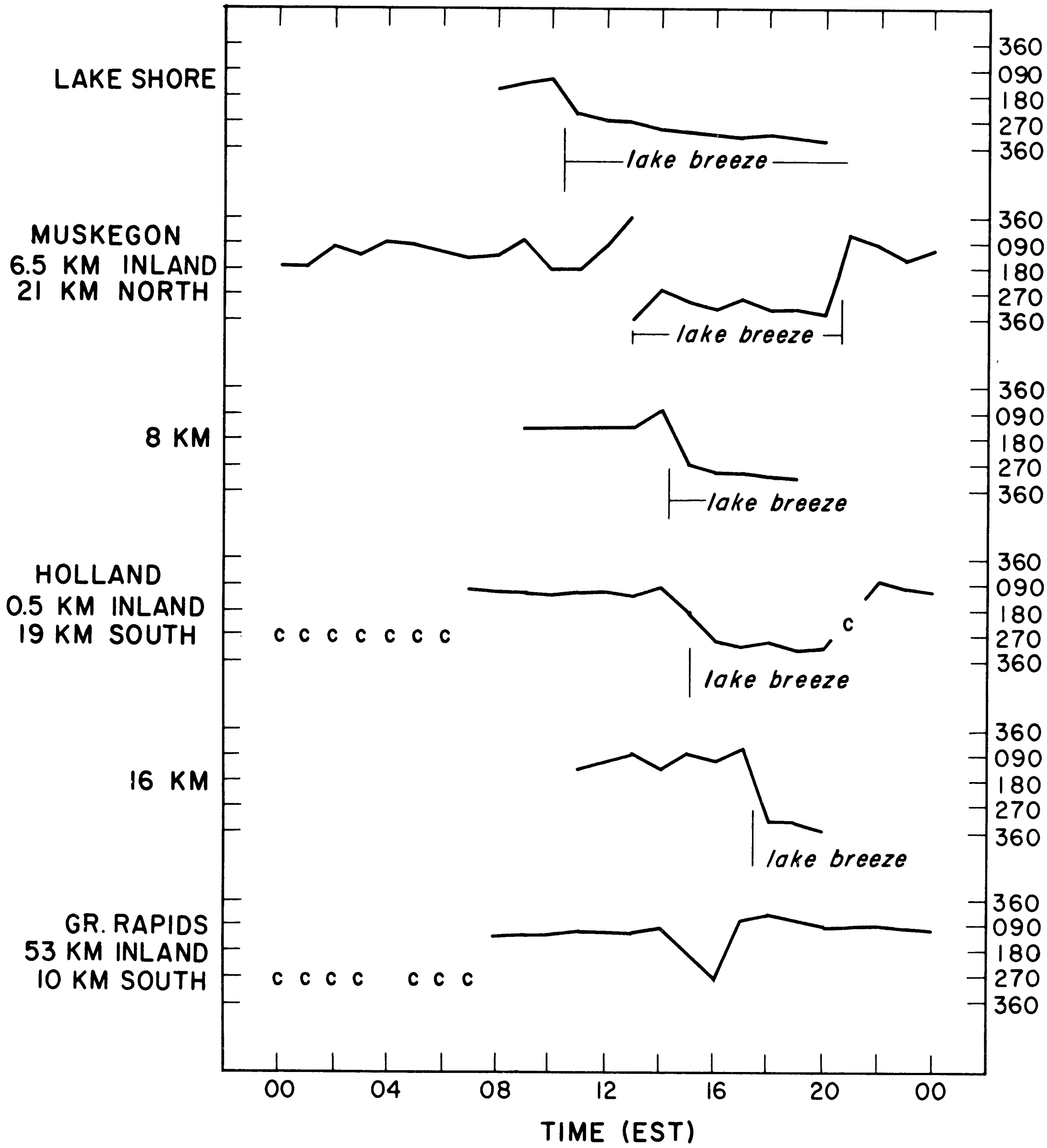

Fig. 2.23. Wind direction change with onset of the lake breeze at various distances inland July 23, 1964.

3. A SEMI-BOUNDED NUMERICAL MODEL OF THE LAKE BREEZE

3.1 Differences between the lake breeze and the sea breeze

In Chapter 2 an observed lake breeze at a location along the long axis on the eastern side of Lake Michigan was described. Under conditions of minimum gradient flow and external influence the lake breeze starts 3 to 4 hours after the temperature over the land at an inland station becomes equal to the lake water temperature (time zero). Maximum development occurs 8 to 9 hours after time zero. Progress of the lake breeze front inland is irregular.

At the time of maximum development, for the conditions noted, the depth of the layer of onshore flow is about 750 m with maximum velocity of 5 to 7 sec^{-1} occurring within 250 m of the surface. The horizontal extent of a fully developed lake breeze is somewhat less than 70 km, 35 km on each side of the shore. In contrast to sea breeze observations, in midlatitudes, a lake breeze return circulation is clearly apparent. Depth of the return flow is approximately twice that of the lake breeze and return velocities are proportionately lower. The effect of the local circulation over the shoreline is found to extend to heights exceeding 2500 m above the surface.

Over the water horizontal divergence occurs at the surface and horizontal convergence is indicated aloft through the layer of return flow. Moderately intense downward motion must occur over the lake. Inflow from the surrounding atmosphere into the lake breeze circulation as a whole must also occur.

The forces which act to govern lake breeze motions are essentially similar to those governing the sea breeze, viz., pressure gradient forces arising in response to temperature contrasts resulting from differential heating over land and water, the apparent forces due to the earth's rotation and frictional forces arising principally as a result of turbulent transport of momentum in the atmosphere. The major differences between the lake breeze and the sea breeze will occur as a result of the finite dimension of the water surface. A mathematical model of the lake breeze will thus be similar to that for the sea breeze with suitable modification to incorporate the effects of the bounded water surface.

Early analytical treatments of Haurwitz (1947), Schmidt (1947) and Defant (1951) dealt with the sea breeze occurring in response to an assumed pressure or heating function. The equations were necessarily linearized and the effects of the non-linear advection and turbulence terms could not be suitably included. The development of the high speed electronic computer made the retention of the non-linear terms possible with correspondingly more realistic modeling.

Pearce (1955), in an initial numerical sea breeze model, showed that the rotational non-divergent velocity components arising in response to a specified heating function represented the observed sea breeze. Fisher (1961) using similar prognostic equations for velocity introduced surface friction and the turbulent transport of heat and momentum to predict the sea breeze occurring when surface temperatures varied according to observed values.

Estoque (1960) treated the sea breeze circulation as a local disturbance on the general circulation flow. The eddy diffusivity for vertical transport of heat and momentum is evaluated in terms of the existing local fields of velocity and temperature in the lower layers of the model. Heating is achieved using a surface potential temperature over the land which is a specified sinusoidal function of time. Estoque's treatment enabled him subsequently (1962) to superimpose a large scale flow on the model and to observe the effect of surface temperature discontinuities near a coast in modifying this large scale flow.

In this chapter we will model the lake breeze by appropriate adaptations of the techniques developed by Estoque and by introducing suitable boundary conditions. A symmetric lake breeze on each side of the lake is considered limiting the model to the case of no external influence.

For the symmetric lake breeze, circulation patterns on each side of the lake are identical and horizontal flow across the lake centerline must be zero. Thus the model is physically bounded in the horizontal over the water and for this reason is termed "semi bounded." The differential equations for motion and heating are represented by finite difference approximations and a numerical solution for lake breeze characteristics is obtained using a digital computer. The equations required for the solution are the equations for horizontal motion, the hydrostatic equation, the continuity equation and Taylor's (1915) heat diffusion equation.

3.2 Physical formulation of the model

In the semi infinite sea breeze models the circulation is considered as occurring over a straight infinite coastline with land and sea extending an infinite distance on each side of the coast. For the lake breeze, to eliminate lake end effects, we will here consider a lake having straight parallel shores of sufficient length that the lake breeze is homogenous in the alongshore direction. This condition is satisfied for a finite water surface having the configuration of Lake Michigan (Chapt. 2) and should be valid for almost all of the Great Lakes having major axes considerably larger than their minor axes. The assumption of homogeneity along the lake shore implies no change in the alongshore direction, considerably simplifying the equations and permitting modeling in two dimensional

Cartesian coordinates. For our model x is positive inland to the east, z is positive upward and y is positive alongshore to the north.

For the vertical structure a two layer model similar to that of Rossby and Montgomery (1935) is assumed. Next to the earth a shallow boundary layer in which shear stress remains relatively constant with height is postulated. Above this is the transitional boundary layer in which the effects of vertical turbulent transport gradually diminish with height to negligible magnitude relative to other terms in the equations of motion. At the top of the model, therefore, balanced flow is assumed to prevail. Following the notation of the computer program Z(1) is taken at the ground, Z(2) is taken at the top of the constant flux layer and Z(N) is at the top of the model.

Figure 3.1 is a schematic diagram of the plane of the semi bounded model normal to the lake shoreline.

3.3 Mathematical formulation of the model

In the Estoque model of the sea breeze the local mesoscale atmospheric quantitites **Q** are represented as deviations Q' on the specified general circulation values of the variables Q_g, which are considered to be independent of turbulent transport.

$$Q = Q_g + Q'$$

The relevent variables in this model are pressure p, temperature T, potential temperature θ, the across shore and alongshore velocity components u and v respectively and vertical velocity w.

In the general equations of motion applied near the earth's surface frictional forces are important. For our purposes it is assumed that internal friction in the atmosphere arises principally as a result of turbulent transport of momentum in the vertical, that turbulent momentum transport in the horizontal is negligible in comparison and that the coefficient of turbulent transport is the same for both horizontal components of velocity. Internal friction is represented by a term of the form

$$\mathbb{F} = \frac{\partial}{\partial Z} \left[K_M \frac{\partial \mathbf{V}}{\partial Z}\right] \qquad (1)$$

where $\mathbb{F}$ and $\mathbf{V}$ are horizontal force and velocity vectors respectively, and K_M is the eddy diffusivity for momentum.

Similarly, the heat flux as given in Taylor's (1915) heat diffusion equation is

$$\frac{\partial}{\partial Z}\left[K_H \frac{\partial \theta}{\partial Z}\right] \tag{2}$$

where K_H is the eddy diffusivity for heat.

Through the surface boundary layer the fluxes of momentum and heat remain relatively constant, a condition expressed by

$$\frac{\partial}{\partial Z}\left[K_M \frac{\partial \mathbb{V}}{\partial Z}\right] = \frac{\partial}{\partial Z}\left[K_H \frac{\partial \theta}{\partial Z}\right] = 0; \; Z(1) \leq Z \leq Z(2) \tag{3}$$

where Z(2) is taken at the top of the layer of constant flux. The equation of horizontal motion for the general circulation may be written in the form

$$\frac{d\mathbb{V}_g}{dt} = -\frac{1}{\rho_g} \nabla p_g - f \, \mathbb{k} \times \mathbb{V}_g$$

while the equation for the net flow in the transitional layer below is

$$\frac{d\mathbb{V}}{dt} = -\frac{1}{\rho} \nabla p - f \, \mathbb{k} \times \mathbb{V} + \frac{\partial}{\partial Z}\left[K_M \frac{\partial \mathbb{V}}{\partial Z}\right]$$

where $\mathbb{V}$ and $\mathbb{V}_g$ are horizontal velocity vectors;

f is the Coriolis parameter $2\,\Omega \sin \phi$, in which

Ω is the angular velocity of the earth and

ϕ is the latitude;

∇ is the horizontal operator $\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right]$; and

$\mathbb{k}$ is the unit vertical vector.

The rest of the notation is standard.

By removing the external general circulation contribution (subscript g) in the transitional layer from the net flow (no subscript) an expression is obtained for the sea breeze deviation flow (primed). The prediction equations for the horizontal lake breeze velocities are obtained by expanding the total derivative, expressing ρ in terms of p and T, taking $\frac{1}{\rho} \nabla p_g \approx \frac{1}{\rho_g} \nabla p_g$ and writing the equations in scalar form.

Thus

$$\frac{\partial u'}{\partial t} = - u' \frac{\partial u_g}{\partial x} - u \frac{\partial u'}{\partial x} - w' \frac{\partial u_g}{\partial Z} - w \frac{\partial u'}{\partial Z} - \frac{RT}{p} \frac{\partial p'}{\partial x} + fv' + \frac{\partial}{\partial Z} [K_M \frac{\partial u}{\partial Z}] \tag{4}$$

$$\frac{\partial v'}{\partial t} = - u' \frac{\partial v_g}{\partial x} - u \frac{\partial v'}{\partial x} - w' \frac{\partial v_g}{\partial Z} - w \frac{\partial v'}{\partial Z} - fu' + \frac{\partial}{\partial Z} [K_M \frac{\partial v}{\partial Z}] \tag{5}$$

where homogeneity in the y direction has been assumed.

Following a similar procedure, the prediction equation for potential temperature is

$$\frac{\partial \theta'}{\partial t} = -u' \frac{\partial \theta_g}{\partial x} - u \frac{\partial \theta'}{\partial x} - w' \frac{\partial \theta_g}{\partial Z} - w \frac{\partial \theta'}{\partial Z} + \frac{\partial}{\partial Z} [K_H \frac{\partial \theta}{\partial Z}] \tag{6}$$

Combining the hydrostatic approximation, the defining equation for potential temperature and the equation of state yields a relation for pressure in terms of predicted potential temperature

$$\frac{\partial}{\partial Z}\left[p^{R/C_p}\right] = -\frac{g}{C_p}\, p_o^{R/C_p}\, \frac{1}{\theta} \tag{7}$$

Details of the derivation of Eq. 7 are given in Appendix B. Using this expression and knowing p and T, p' and T' in the lake breeze may be evaluated.

Use of the hydrostatic equation for motions on the scale of the lake breeze can be justified by an order of magnitude comparison of terms as follows.

The equation of motion in the vertical for a flat surface and neglecting the Coriolis component (Petterssen, 1956) is written

$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial Z} - g + F_Z$$

Wallington (1963) has observed vertical velocities as high as 8 m sec^{-1} at heights exceeding 1000 m at the inland boundary of a sea breeze. Assuming uniform acceleration these values would suggest

$$\frac{dw}{dt} \approx 10 \text{ cm sec}^{-2}$$

and if one assumes that eddy transport of vertical motion may be represented by a relation similar to that for horizontal momentum, for uniform acceleration to a vertical velocity of 10 m sec^{-1} in a distance of 1000 m

$$\frac{\partial^2 w}{\partial z^2} \approx 10^{-5} \ cm^{-1} \ sec^{-1}$$

and

$$F_Z = K_M \frac{\partial^2 w}{\partial z^2} \approx K_M \cdot 10^{-5}$$

A large value of K_M near the ground for highly turbulent motion is 10^6 cm^2 sec^{-1} and the magnitude of the eddy flux should decrease upward above some level near the surface. Thus we may reasonably consider $\frac{dw}{dt}$ and F_Z at least one order of magnitude smaller than g.

With these assumptions the vertical equation of motion reduces to the hydrostatic equation

$$\frac{\partial p}{\partial z} = - \rho g \tag{8}$$

The final relation required for complete determination of the velocity field is the continuity equation. For a compressible atmosphere the equation of continuity may be written to a high degree of accuracy (Kibel, 1957)

$$\nabla \cdot \mathbb{V} = -\frac{1}{\rho}\frac{\partial}{\partial Z}[\rho w]$$

Thus, in a compressible atmosphere, the equation of continuity in terms of the deviation and general circulation quantities becomes

$$\frac{\partial w'}{\partial Z} + w'\frac{\partial}{\partial Z}[\ln\rho] + w_g\frac{\partial}{\partial Z}[\ln\rho - \ln\rho_g] = -\nabla \cdot \mathbb{V}' \quad (9)$$

On the scale of the model w_g may have similar magnitude to w' but

$$\ln\frac{\rho}{\rho_g} \ll |\ln\rho|$$

Thus the 3rd term may be neglected and Eq. 9 reduces to

$$\frac{\partial w'}{\partial Z} + w'\frac{\partial}{\partial Z}\ln[\rho_g + \rho'] \approx -\nabla \cdot \mathbb{V}' \quad (10)$$

in which $\frac{\partial}{\partial Z}[\ln\rho]$ is almost constant. Equation 9 is derived in Appendix B.

3.4 The eddy transport terms

While much recent evidence suggests that the eddy diffusivity for heat is not the same as that for momentum, we will here, a priori, specify $K_M/K_H = 1$ for simplicity of solution so that $K_M = K_H = K$. Further, near the surface, the effects of eddy transport of heat and momentum are dominant while aloft in the free atmosphere these effects become small relative to other forces. The relative magnitude of the eddy transport term should thus decrease upward above the surface boundary layer. This is achieved by allowing K to decrease linearly from its value at the top of the surface boundary layer to zero at the top of the model where the general circulation dominates atmospheric behavior.

Within the surface boundary layer turbulent transport varies with atmospheric stability and the nature of the underlying surface. Priestly (1959) has defined the free and forced convection regimes for heat transport and, from observations, has estimated that transition from one regime to the other occurs at a Richardson number of

$$Ri = \frac{g}{T} \frac{\partial\theta/\partial Z}{[\partial V/\partial Z]^2} \approx -0.03$$

For the free convection regime Priestly derives an expression for the eddy diffusivity for heat in the form

$$K_H = h\ z^2\ [\frac{g}{T} \frac{\partial\theta}{\partial Z}]^{\frac{1}{2}} \quad ; \quad Ri \leq -0.03 \qquad (11)$$

where h is an empirical constant ≈ 0.9. Estoque (1959) obtains an eddy diffusivity for momentum for the forced convection regime of the form

$$K_M = k^2 z^2 \; [1 + \alpha \, Ri]^2 \; \frac{\partial V}{\partial Z} \tag{12}$$

where k is Von Karman's constant ≈ 0.4. This expression is found to produce results in general agreement with observation using $\alpha = -3$.

3.5 The boundary and initial conditions

The matching equations for velocity and potential temperature at the top of a surface boundary layer of depth Z(2) for the forced convection regime are derived in Appendix B. The equivalent equations for the free convection regime have been given by Estoque (1960). Monin and Obukhov (1954) have suggested an average depth of 50 m for the layer of constant flux and Yamamoto and Shimanuki (1964) have found the layer extends to about 200 m over a large city. Above the surface boundary layer a uniform grid spacing of 100 m is used in the vertical. Observation indicates that the total depth of the lake breeze model should exceed 2500 m and 3000 m has been determined as the level at which wind speed and direction become relatively constant aloft. Thus in the computer program Z(1) is taken at the surface, Z(2) = 50 m and Z(32) = 3050 m.

In the horizontal, grid spacing was determined using the relation

$$X(M) = X(M-1) + A + B[M-2] ; \quad M \geq 2 \tag{13}$$

where A and B are constants,

round parentheses are used for subscripting,

square brackets indicate arithmetic operations,

and M is the number of the grid point starting from the shoreline where M=1 and x=0 in Eq. 13.

This expression yields an expanding grid with smallest spacing at the shoreline where gradients are strongest. For our model A=B=3 providing reasonable detail near the lake shore with a lake half width of 63 km, in good agreement with the actual lake half width of about 67 km at the point on Lake Michigan where the observations over the land presented in Chapter 2 were obtained.

In the actual model the horizontal grid numbers start at the center of the lake. The grid spacing decreases from 18 km at the lake centerline X(1), to 3 km at the shoreline X(7), then increases again inland to X(16). The model extends considerably further inland than it does over the water because there is no physical reason to restrict the horizontal extent over the land and because it is desirable to avoid influencing model characteristics by boundary condition specifications. The effects of relaxing the horizontal extent of the model and of using an expanding grid on the vertical are currently being investigated.

Q(t,m,n) is the value of the variable Q at time t, at the mth grid point on the x axis and the nth grid point on the z axis.

The symmetric lake breeze will occur only if external effects arising from the characteristics of the general circulation are negligible within the region where the lake breeze occurs. Accordingly

$$u_g = v_g = w_g = 0 \quad \text{for all } t,\ m \text{ and } n.$$

At the surface

$$u = v = w = 0 \qquad \text{for all } t \text{ and } m \text{ at } n=1.$$

At the top of the model the perturbation quantities, Q' become negligible and

$$Q' = 0 \quad \text{for all t and m at n = 32.}$$

At the inland boundary of the model it is assumed that the lateral rate of change of the perturbation of any dependent variable, Q', becomes very small, i.e.,

$$\frac{\partial Q'}{\partial x} = 0 \quad \text{for all t and n at m = 16.}$$

At the centerline of the lake

$$u' = v' = 0 \quad \text{for all t and n at m = 1}$$

and the lateral derivatives $\frac{\partial u'}{\partial x}$ and $\frac{\partial v'}{\partial x}$ have the same value on both sides of m = 1. Further $\frac{\partial p'}{\partial x} = \frac{\partial \theta'}{\partial x} = \frac{\partial T'}{\partial x} = 0$ for all t and n at m = 1.

At the surface, over the water the potential temperature is taken to be constant, with the value

$$\theta = 294^{\circ}K \text{ for all t, } 1 \leq M \leq 6,\ n = 1.$$

Over the land, surface potential temperature is specified according to Table 3.1 which gives the hourly observed temperature 53 km inland from the lake shore where surface temperatures are not affected by air flowing off the water. For simplicity intermediate values of surface temperature are obtained by linear interpolation.

At the lake shore, surface potential

TABLE 3.1

AIR TEMPERATURE OVER THE LAND AT
A LOCATION WELL INLAND FROM THE LAKESHORE

Time Hrs.	Temperature °K	Time Hrs.	Temperature °K
0*	294.0**	10	301.3
1	295.8	11	299.9
2	297.3	12	298.1
3	298.9	13	296.7
4	299.9	14	295.5
5	300.8	15	294.3
6	301.4	16	293.3
7	301.9	17	292.4
8	302.1	18	291.8
9	301.9	19	291.2

* Time zero corresponds to the time when air temperature inland equals lake water temperature.

** These values represent a mean of observations on four lake breeze days taken relative to a lake temperature of 294°K.

temperature is smoothed linearly as follows

$$\theta(t,7,1) = \frac{\theta(t,6,1) + \theta(t,8,1)}{2} .$$

An initial lapse rate, $\gamma = 7.5 \times 10^{-3}$ $^{\circ}C\ m^{-1}$ is specified in accordance with observed values prior to the onset of the lake breeze in the morning. The initial fields of temperature, pressure and potential temperature are uniform in the horizontal and are given by

$$\left.\begin{aligned} T_g &= T_o - \gamma Z \qquad &(14)\\ p_g &= p_o[T/T_o]^{g/R\gamma} \qquad &(15)\\ \theta_g &= T[p/p_o]^{-R/C_p} \qquad &(16)\end{aligned}\right\} \begin{array}{l}\text{at } t = 0\\ \text{for } 1 \le m \le 16,\\ 3 \le n \le 32\end{array}$$

where T_o and p_o are temperature and pressure respectively at the surface, Z(1). The values of T_g, p_g and θ_g are retained throughout the forecast period as representative of the unchanging general circulation.

The initial values of the perturbation quantities are uniformly zero throughout the plane of the model, i.e.,

$$Q' = 0 \qquad \text{at } t = 0 \text{ for all m and n.}$$

Computations were made for a latitude of 45°.

3.6 Procedure for predicting lake breeze parameters

The usual technique of representing a prognostic equation by its finite difference approximation using centered space and time differences with the exception of the first step is employed. With this technique Phillips (1956) has pointed out that the second order turbulent transport terms will lead to computational instability. For this reason it is necessary to stabilize the second order terms using the finite difference scheme of Dufort and Frankel (Richtmeyer, 1957) which is unconditionally stable.

A precise analysis of the model equations to determine the optimum time step for numerical advance in time is impractical. While some estimate of the time increment is possible final selection is made on the basis of trial. A time step of 5 min was selected for the bounded lake breeze model based upon experience with the sea breeze model and on the basis of computer time required for the computations. Despite the larger central grid distance used in the horizontal it was found necessary to differentiate the equation of continuity with respect to z as suggested by Estoque in order to achieve computational stability. In the finite difference expression for Eq. 10 the difference of ln ρ over a 200 m layer is found by hand computation to be -0.0195 with a variation of $\pm$ 0.0003 from bottom to top of the model for the prescribed initial conditions and this value will change only slightly throughout the life of the model. The value is thus taken constant

$$\frac{\partial}{\partial z} [\ln \rho] = C$$

and differentiating Eq. 10 with respect to height leads to

$$\frac{\partial^2 w'}{\partial z^2} + C \frac{\partial w'}{\partial z} = - \frac{\partial}{\partial z} [\frac{\partial u'}{\partial x}] \tag{17}$$

for a lake breeze which is homogeneous in the y direction. The second term in Eq. 17 is found to have negligible influence in the determination of w' and may be neglected if desired.

The procedure used in predicting the lake breeze features is as follows.

1. Compute the vertical fields of p, T and θ for $3 \leq n \leq 32$ at $t = 0$ using Eqs. 14 to 16 and specified surface conditions.

2. Compute θ and K at Z(2) using the relations appropriate for the prescribed convective regime. In this model, initially Ri is infinite with a zero velocity field everywhere and initial values of θ and K must be specified at Z(2).

3. Evaluate the field of K for $3 \leq n \leq 31$ where K decreases linearly with height.

4. Predict the new values of u', v' and θ' through the transitional layer $3 \leq n \leq 31$, by stepping forward in time using Eqs. 4, 5, and 6.

5. Using the new values of u' and v' at Z(3) determine $\mathbb{V}$ at Z(3) for each m. Assuming constant wind direction below Z(3) determine the new $\mathbb{V}$ at Z(2) using the relations applicable for the prevailing convective regime. Obtain the new u' and v' at Z(2) using the relation

$$u(t,m,2) = \frac{\mathbb{V}(t,m,2)}{\mathbb{V}(t,m,3)} [u(t,m,3)]$$

and a similar relation for v. Obtain the new $\theta'(t,m,2)$ directly using the equations of Sec. 3.5 and the known values of $\theta(t,m,3)$ and $\theta(t,m,1)$.

6. Using Eq. 17 and the new field of u' evaluate w' at each of the grid points.

7. Using the new field of θ' determine θ at each of the grid points and the corresponding field of p using Eq. 7.

8. The new values of T are determined using the known fields of p and θ and the defining equation for θ.

9. Repeat steps 2 through 9 until the desired forecast is obtained.

3.7 Results of a lake breeze forecast

Using the numerical model, a lake breeze has been predicted employing a 16 x 32 grid in the plane normal to the shoreline. The results of the forecast for a lake 126 km wide are presented in Figs. 3.2 through 3.7. For compactness, the horizontal extent of the figures has been restricted to the region where significant changes occur.

In Fig. 3.2, four hours after time zero when the forecast was started (time zero is the time at which the air temperature inland is equal to the water temperature), the lake breeze maximum onshore velocity is 1 m sec^{-1} directly over the shoreline at a height of 150 m. The lake breeze has penetrated 7 to 8 km inland from the shoreline and extends through a layer 300 m deep over the land. Convergence occurs from both directions at the lake breeze frontal surface near the ground. Above the layer of onshore flow a weaker but deeper return flow occurs. The circulation system at this time is restricted to the region near the lakeshore and predicted vertical velocities are small.

Figure 3.3 shows the forecast lake breeze 6 hr after time zero. The maximum velocity in the onshore flow has increased to 2 m sec^{-1} centered at a point 3 km inland from the shoreline at a height of 150 m. A secondary maximum is apparent over the water about 7 km offshore. A weak shoreward flow is shown inland from, but near, the frontal surface. The depth of the lake breeze has increased to 450 m but the flow at 9 km inland is still offshore and very weak. The region of positive u-component extends considerably further out over the water than it does over the land but velocities are smaller over the water. Above the layer of onshore flow the return circulation is well defined through a deep layer. Horizontal velocities in the return flow are less than half those in the lake breeze.

Figure 3.4 represents the vertical velocity and temperature structure in the lake breeze at the time corresponding to Fig. 3.3. Over the land upward motion occurs while over the water air is descending. A strong horizontal temperature gradient occurs at the shoreline near the surface. The lapse rate of temperature has become superadiabatic over the land through a depth of 1200 m or so above the earths' surface.

Over the land maximum surface temperature occurs 8 hr after time zero (Table 3.1). Figures 3.5 and 3.6 show the velocity and temperature structure in the lake breeze 9 hr after the start of the forecast. In Fig. 3.5 the onshore velocity has increased to a maximum of 3 m sec^{-1} at a point about 5 km inland. The two velocity maxima shown in Fig. 3.3 are still apparent near the shoreline. The region of onshore flow extends inland to 13 km and through a depth of 500 m. Over the lake the flow is toward the shoreline from distances greater than 30 km offshore but velocities remain weaker than those over the land. The return flow aloft has intensified to 1 m sec^{-1}

but the depth of the layer has not changed significantly. A reversed cell has developed in Fig. 3.5 inland from the lake breeze front and model instability is becoming noticeable in the waves on the isotachs over the water.

Figure 3.6 shows the vertical velocity and temperature structure at the time for which Fig. 3.5 is prepared. Waves have also developed in the isotherms near the shoreline. The horizontal gradient of temperature remains strongest near the shoreline at the surface. The temperature maxima at low levels about 9 km inland occur where upward motion is strongest and where a parcel lifted adiabatically in a superadiabatic atmosphere should be warmer than its surroundings. A corresponding dip in the isotherms is observed over the shoreline at low levels where downward motion is most intense.

The wind components normal to the shoreline 12 hr after heating over the land commenced are indicated in Fig. 3.7. In this figure the waves in the isotachs directly over the shoreline have become more pronounced than those for 3 hr earlier but the principal features of the lake breeze are still clear. The across shore component of velocity has remained constant over the land with the central core of maximum velocity shifted further inland but over the water the secondary region of maximum onshore flow has undergone intense development. The lake breeze front has advanced 16 km inland and the reversed cell has intensified. In the return flow the velocity has increased to 2 m sec^{-1} centered at a height of 1600 m above the surface directly over the shoreline. The velocity remains greater than 1 m sec^{-1} to a height exceeding 2000 m. The return current above the lake breeze appears to be merging with the offshore flow in the reversed cell near the surface suggesting mixing of air from inland sources into the lake breeze return current.

Prediction of the lake breeze using the numerical model was continued to 16 hr after time zero but beyond 12 hr the effects of computational approximations and inaccuracies become increasingly dominant. By 16 hr after time zero the across shore component of velocity has decreased near the shoreline but the cell over the water continues to intensify and velocities in this cell reach 10 m sec^{-1}. At the same time the reversed cell over the land continues to develop. The net effect of a velocity decay near the shoreline with intensification in the boundary regions of the lake breeze is to distort appreciably the isotachs so that those velocities arising from error and model instability mask the pattern of the lake breeze itself. The figure for 16 hr after the start of the prediction is not given for this reason.

3.8 Comparison with the observed lake breeze

The lake breeze predicted by the numerical model for a lake of the width of Lake Michigan develops much as the observed lake breeze of July 23, 1964. If we consider that time zero for the numerical model corresponds to a time between 0700 and 0800 for the observed lake breeze comparisons between observation and prediction can be made.

Over the land the predicted lake breeze is slightly weaker than that observed along the Lake Michigan shoreline in mid-afternoon and the inland penetration of the forecast lake breeze is somewhat less than that observed. The return flow aloft is clearly shown in the theoretical model and exhibits characteristics in good agreement with those observed. Convergence from both sides of the lake breeze front occurs in both model and observations. The existence of the reversed cell which develops inland in the model will be extremely difficult to verify in the real atmosphere under circumstances where prevailing external winds are never truly zero. There is, however, some suggestion that the cell does exist in the observations presented previously. The overall depth of the lake breeze circulation system predicted by the model corresponds well with that observed.

Over the water light onshore flow is predicted below 1000 m from distances exceeding 30 km offshore in the numerical model. Observations made July 10, 1963 over Lake Michigan suggest that the actual lake breeze circulation is more symmetrical on each side of the shoreline under light gradient wind conditions. Additional observations are required over the water to define more adequately lake breeze characteristics in this region.

In the predicted lake breeze the central core of maximum velocity in the layer of onshore flow shifts inland with time. This feature is not as pronounced in the actual flow system and has been seldom observed, even in a prevailing onshore flow, and then only late in the day when the lake breeze is dying. The cores of maximum velocity which develop over the water and further inland after 16 hr of model time are considered to be due to errors inherent in the numerical model representation and computational instability.

The least acceptable feature of the model from a theoretical standpoint is the treatment of the turbulent transport of heat and momentum which resolves itself in an empirical formulation. A sufficient number of observations have been made over land near the earth's surface to permit reasonable representation through this region but, above the surface boundary layer of constant flux, the representation is more questionable. While allowance for variation of eddy diffusivity with atmospheric stability has been made no allowance for variations from water to land nor for variations in depth of the constant flux layer with either stability or nature of the underlying surface have been incorporated. The model has been found to be relatively insensitive to changes of eddy diffusivity as large as an order of magnitude except in the very early stages. Over land, eddy diffusivities between 10^4 and 10^5 $cm^2\ sec^{-1}$ at the top of the constant flux layer were computed in the unstable atmosphere. A minimum eddy diffusivity of $5 \times 10^2\ cm^2\ sec^{-1}$ was arbitrarily specified in setting up the model to avoid unrealistic conditions. This value was used over the water at some distance from the shoreline where winds remained very light and Richardson numbers remained large.

3.9 Summary and conclusions

The above analysis indicates that satisfactory agreement between the development of the lake breeze observed and that predicted by the numerical model has been achieved. Development of the numerical model to improve representation and to lengthen the permissible forecast time to include a realistic lake breeze decay is continuing. The existing model has been tested on a smaller horizontal grid (for a lake of width of Lake Ontario) and produces results not significantly different from those presented here. Use of the smaller grid also indicates that the model results are not appreciably distorted near the lakeshore by reducing the horizontal extent of the model plane over land. With this option, and by reducing the number of grid points on the vertical through use of an expanded grid, the lake breeze can be predicted for both sides of the lake simultaneously on the IBM 7090 computer without exceeding computer storage facilities. Subsequently the effect of a superimposed general circulation wind on the lake breeze flow system on both sides of the lake may be investigated.

An additional feature of the local wind system along the shores of a large lake may also be examined using the numerical model. Of the Great Lakes System, Lakes Superior, Michigan and Ontario frequently remain open all winter at least near the center of the lake. This is also true of some fairly broad stretches of the St. Lawrence River. During late fall and particularly early winter there are periods under light general circulation winds when an intense land breeze should occur, with associated upward motion over the lake. This land breeze can be studied using the present and future models and the model concepts can be extended to include the effects of air mass modification as very cold air passes over the warmer water during winter when effects are more pronounced and most easily observed.

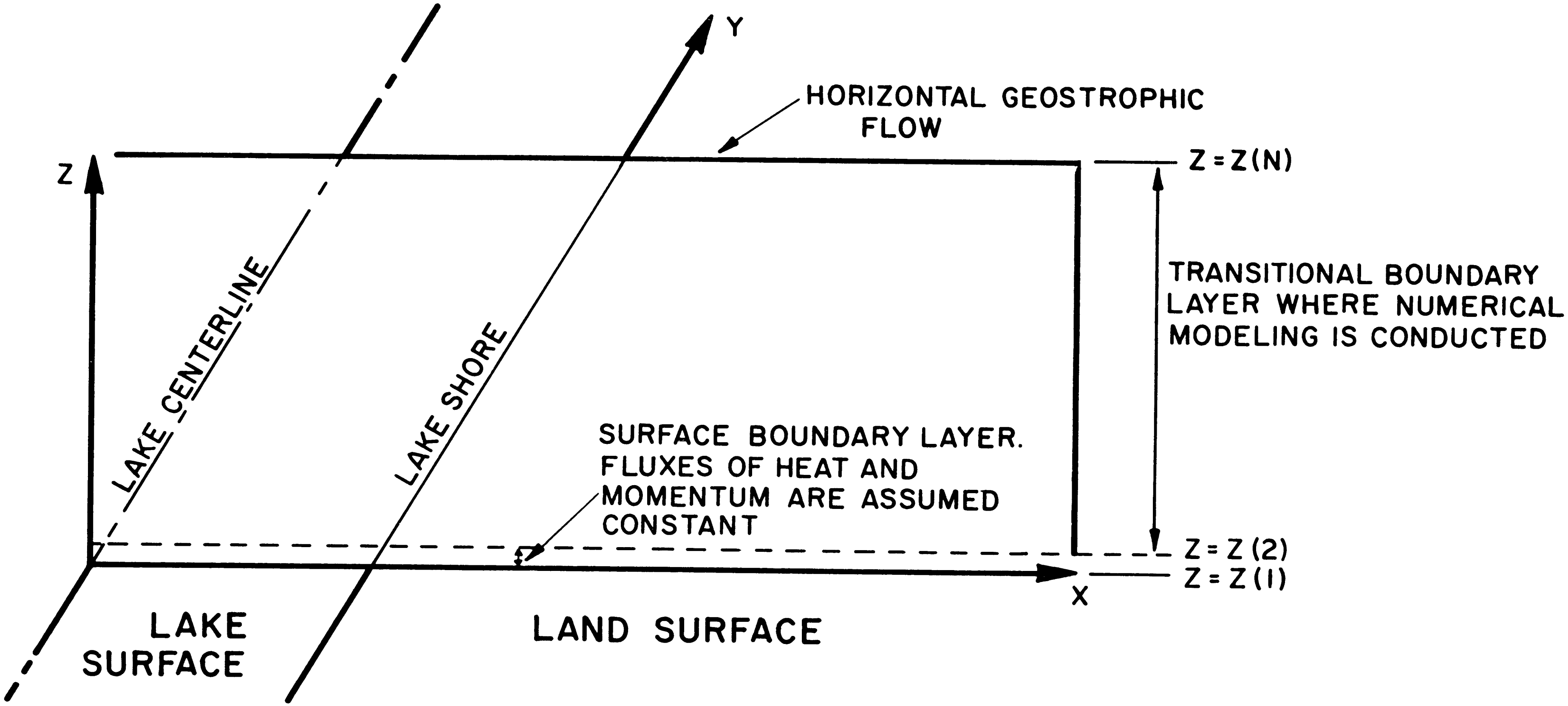

Fig. 3.1. Schematic diagram of the model plane for the semi bounded lake breeze.

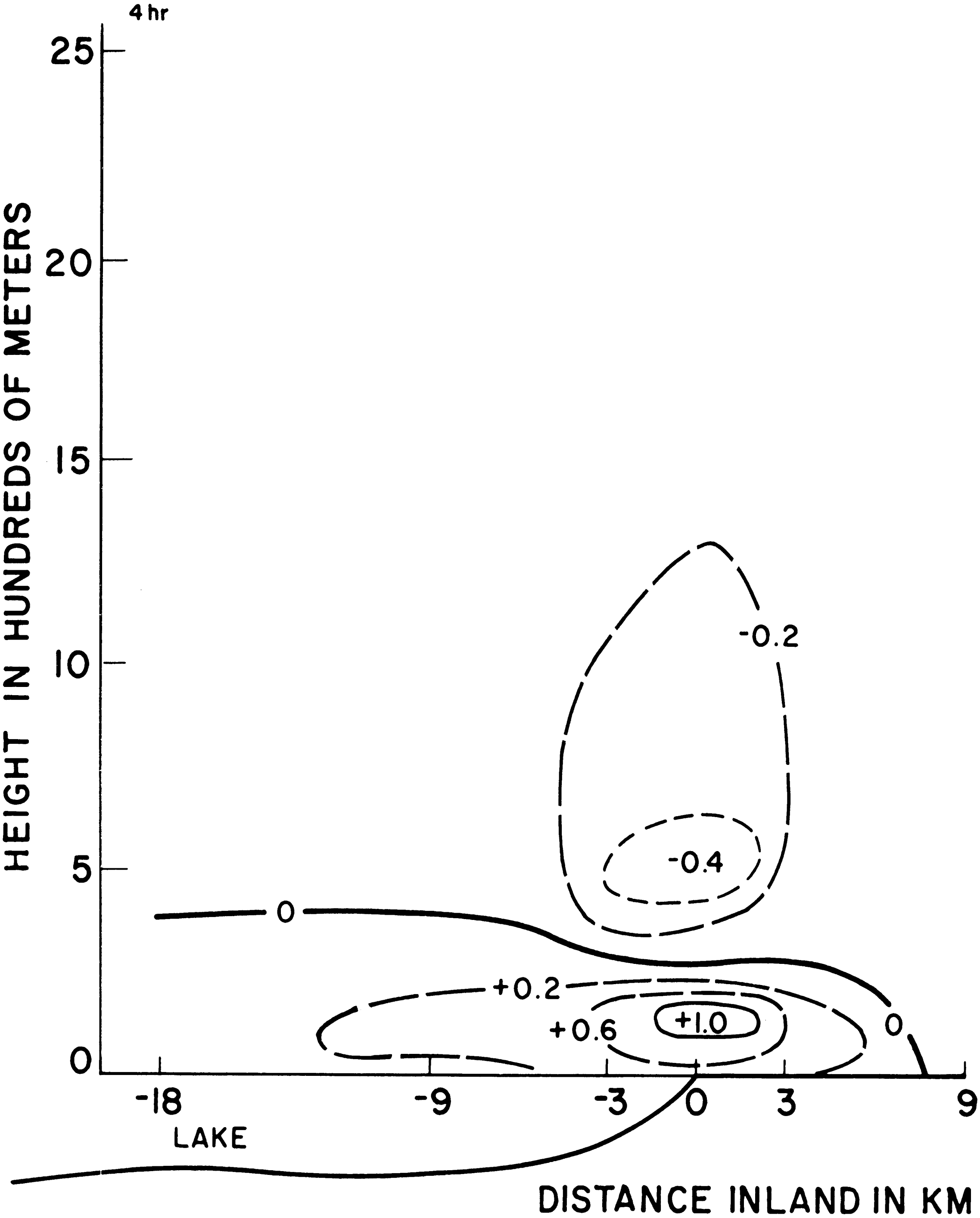

Fig. 3.2. The across shore wind component (u, in m sec^{-1}) in the model plane 4 hrs after time zero.

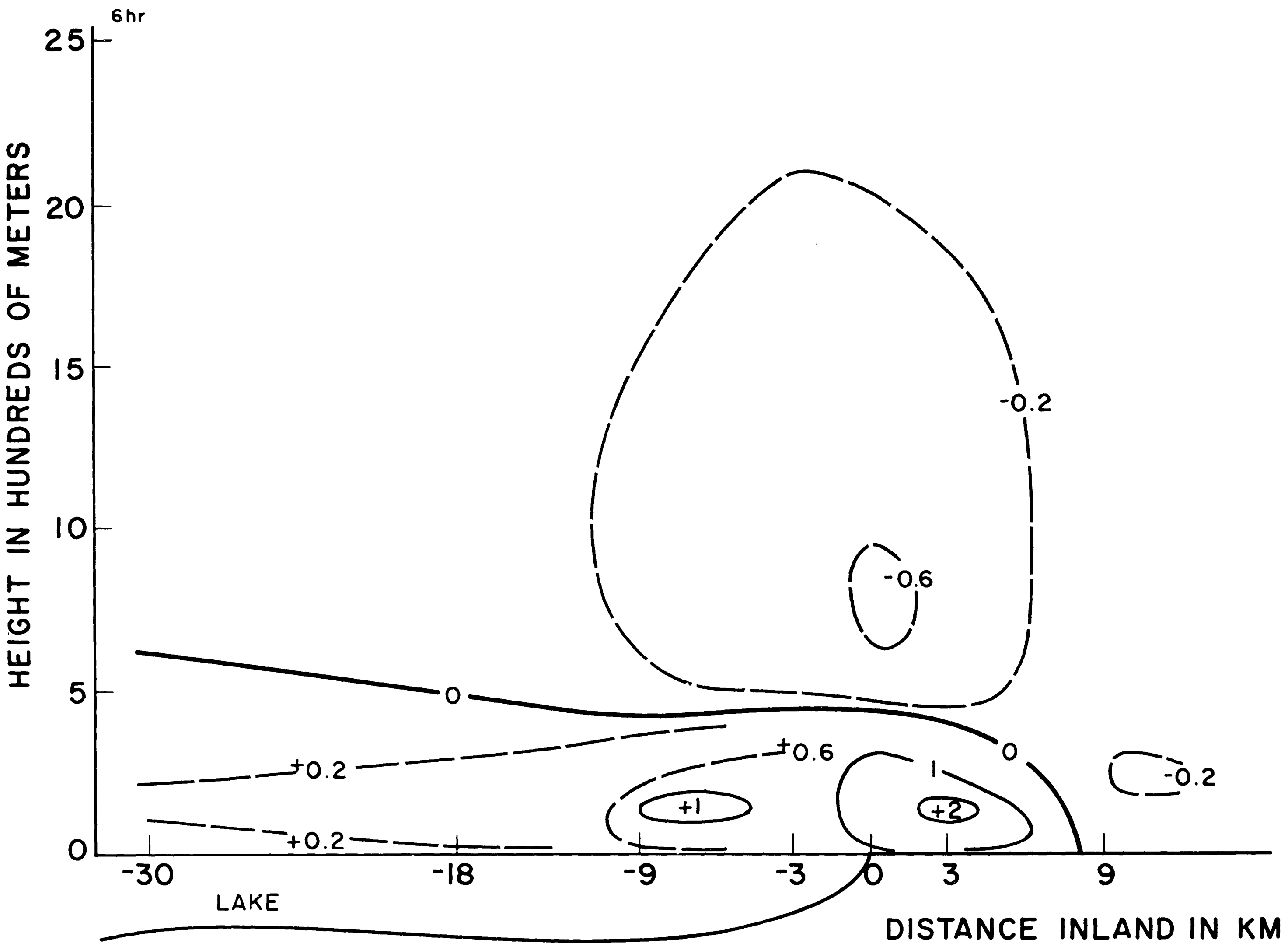

Fig. 3.3. The across shore wind component (u, in m sec^{-1}) in the model plane 6 hrs after time zero.

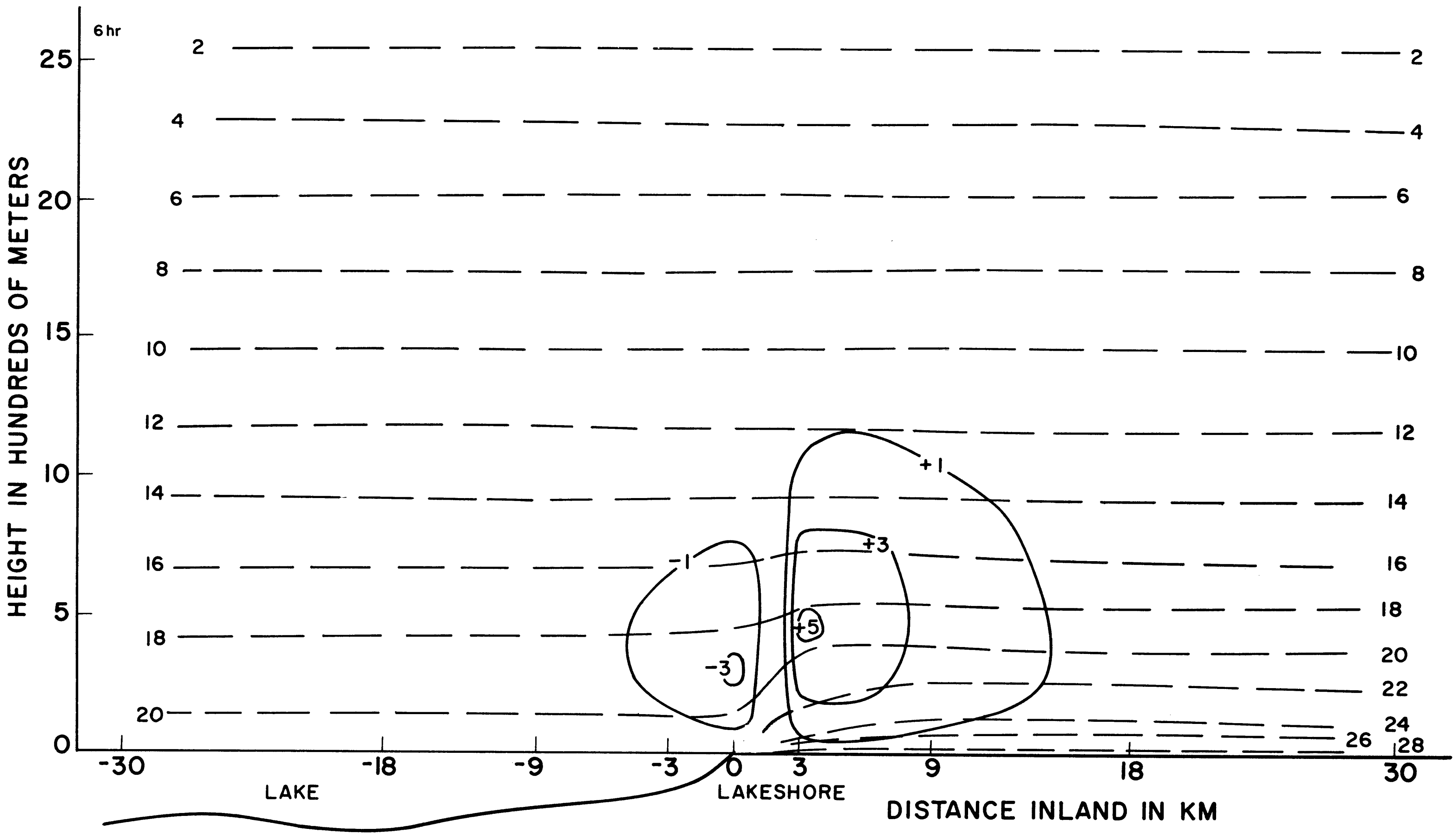

Fig. 3.4. The vertical wind component in cm sec^{-1} (solid line) and dry bulb temperature in ^{o}C (dashed) in the model plane 6 hrs after time zero.

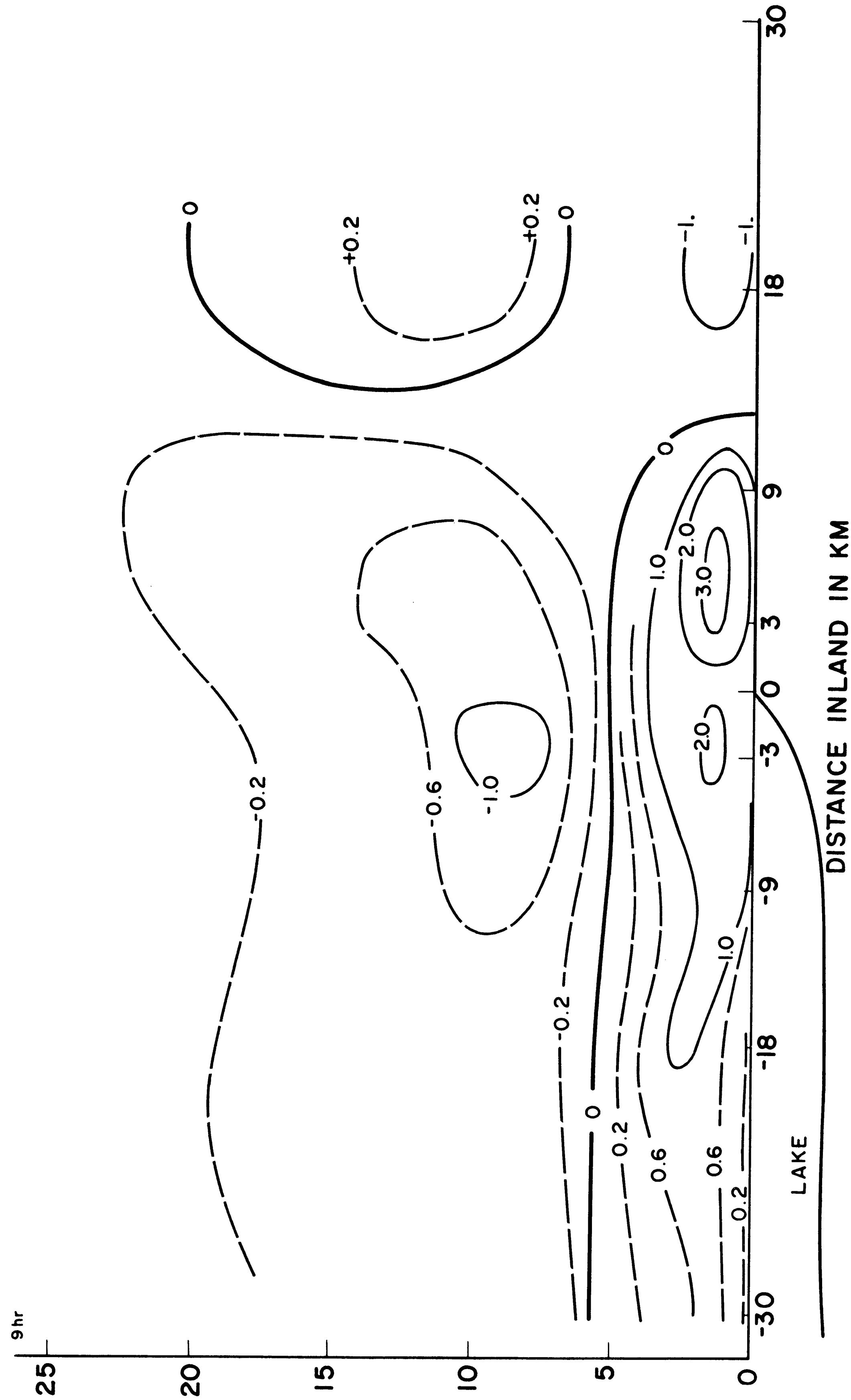

Fig. 3.5. The across shore wind component (u, in m sec^{-1}) in the model plane 9 hrs after time zero.

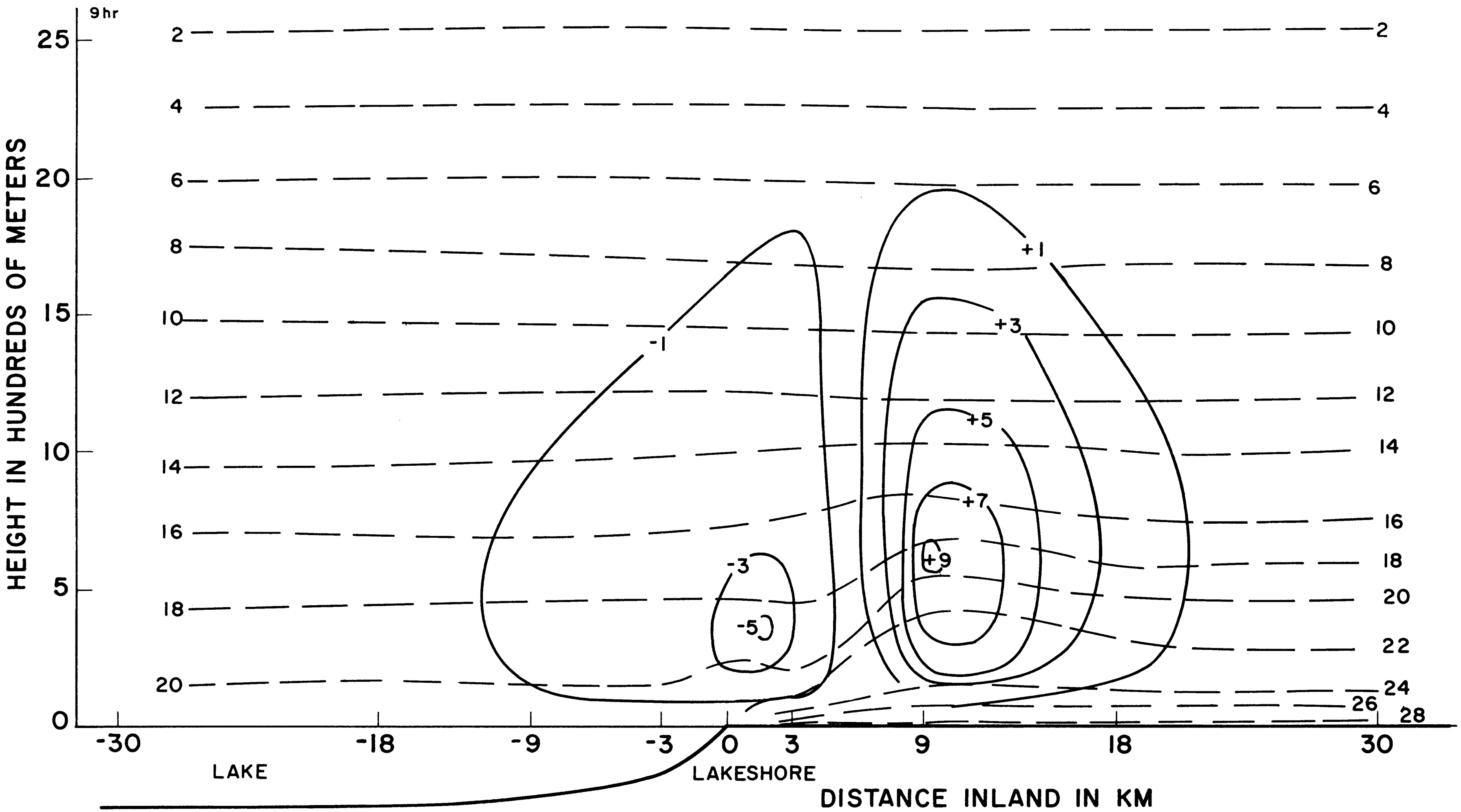

Fig. 3.6. The vertical wind component in cm sec^{-1} (solid line) and dry bulb temperature in ^{o}C (dashed line) in the model plane 9 hrs after time zero.

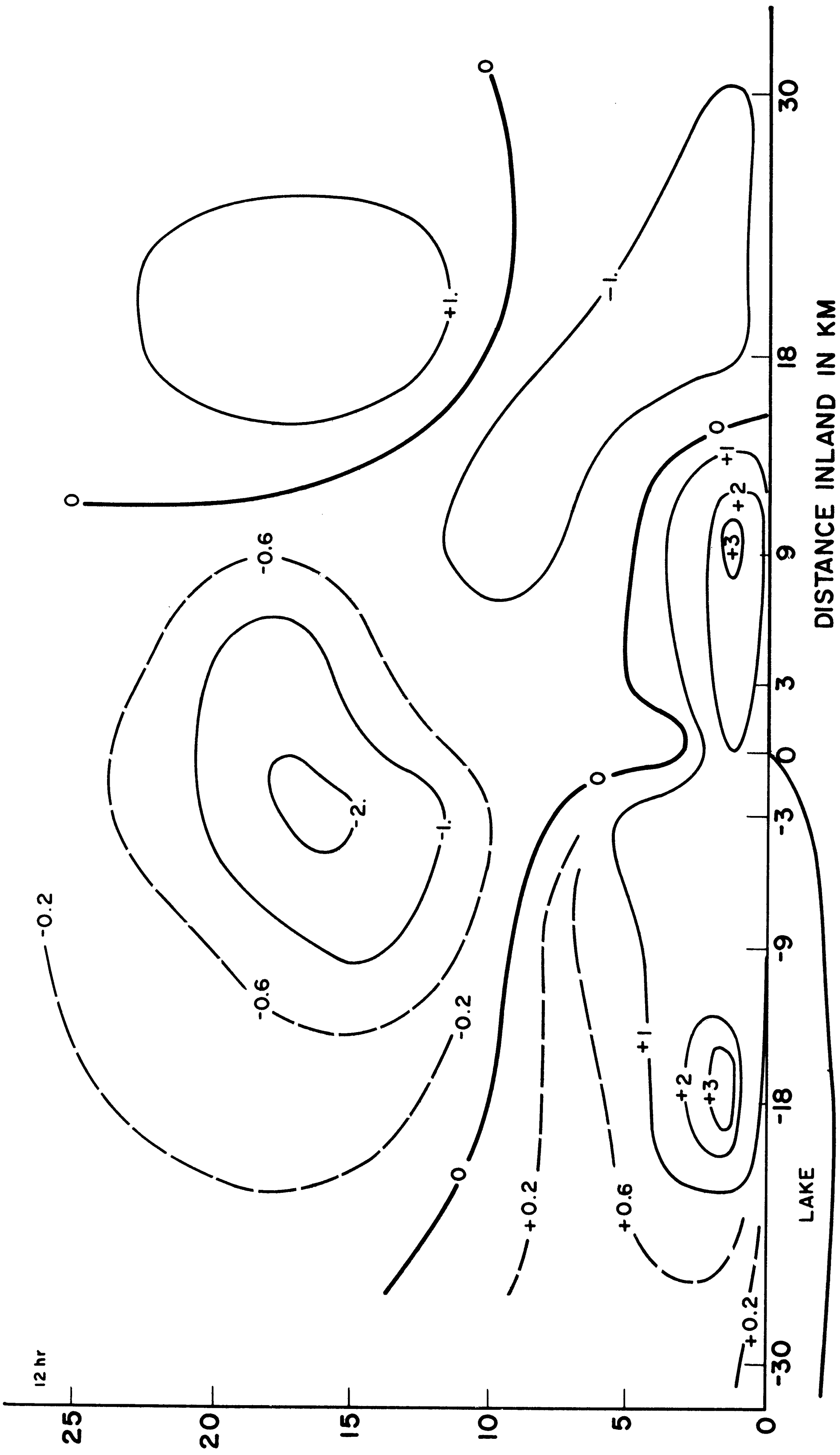

Fig. 3.7. The across shore wind component (u, in m sec^{-1}) in the model plane 12 hrs after time zero.

4. SUGGESTIONS FOR FUTURE WORK

4.1 Extension of observations

Observations of the lake breeze show that for a lake of width exceeding 100 km the physical dimensions of the circulation occurring along the lakeshore are approximately similar to the dimensions of the sea breeze. Under light gradient wind conditions the lake breeze occurs simultaneously on both sides of the lake, at least at low levels, and at midlatitude locations a lake breeze return current is clearly apparent. The lake breeze and its associated return current influence the general circulation under light gradient wind conditions to a height of 3000 m or more.

Observations of the lake breeze circulation system are currently being extended to reveal details of modification of air flowing off the water. The depth of the layer through which the air is modified in its trajectory over the water and the rate at which this air is modified as it passes over the land are of particular interest because analysis of these details will permit interpretation of mixing or diffusion characteristics near the lakeshore both over the water and over the land. Further study of the circulation system over the water and on the opposite shore of the lake are required to permit a more adequate description of circulation systems which occur simultaneously on each side of the lake.

Vertical motions in the lake breeze require further investigations. In the sea breeze, over the land, Wallington (1961, 1963, 1965) has provided excellent general detail of upward motion derived from glider flights by himself and others made near the sea breeze front. This technique of observation might be extended to model glider flights, radio controlled from the surface, to provide observations of the upward motion over the land. Over the water downward motion must prevail. Some measure of the intensity of this downward motion may be obtained by radar tracking of constant volume tetroons equipped with a transponder. This observational technique has been used successfully (Angell and Pack, 1961 and Pack 1962) for trajectory analysis and would provide information not otherwise available, particularly over the water where glider observations cannot be made.

4.2 Extension of the numerical model

The mathematical model of the lake breeze successfully predicts lake breeze development but some experimentation and modification are desirable to improve model stability using a reasonable time step and to extend the period of forecast to include lake breeze decay and development of the land breeze. This work is currently in progress. Further testing of the model is desirable in an atmosphere where the lapse rate changes with height. Data are already available to permit comparison of the flow patterns predicted by the model with the observed flow system for the case where an inversion exists aloft.

The numerical lake breeze model must be extended to incorporate the flow system on both sides of the lake and to study the lake breeze under conditions where a synoptic general circulation pressure gradient occurs. Analysis incorporating the effects of a prevailing wind in the surrounding atmosphere will, of course, include the case where the general circulation flow is only modified by the local surface and temperature discontinuity at the lakeshore and no true lake breeze occurs. The equations of the current model can be used without change for this extension but changes in the grid system must be made in order to accomodate the larger number of grid points on the horizontal using presently available facilities. The model should finally be extended to a three dimensional coordinate system so that the lake breeze along the shoreline of a lake which is bounded in two dimensions in the horizontal can be studied. The principal deterrent to modelling the lake breeze in three dimensions is currently computer storage capacity. However, experimentation with the model and with the model grid system may also lead to some acceptable solution to this problem using present equipment.

4.3 Interaction of local winds and circulations within the lake

Lastly, study of the local winds along a lakeshore may lead to studies of circulations within the lake itself. During the course of this investigation it was found that, as a result of overturning or upwelling, lake surface temperature changes of 5°C over a period of 24 hours are not uncommon. A temperature variation of the same order of magnitude is also frequently observed across the width of the lake. It is apparent that lake surface temperatures are considerably more dependent upon the immediately preceding synoptic weather than ocean water temperatures are and in this respect the lake breeze is more complex than the sea breeze. It may be that the lake breeze circulation itself strongly influences flow patterns in the lake and that the two phenomena should be studied simultaneously.

APPENDIX A

DRY BULB AND DEW POINT STRUCTURE IN THE VERTICAL FOR JULY 22, 1964

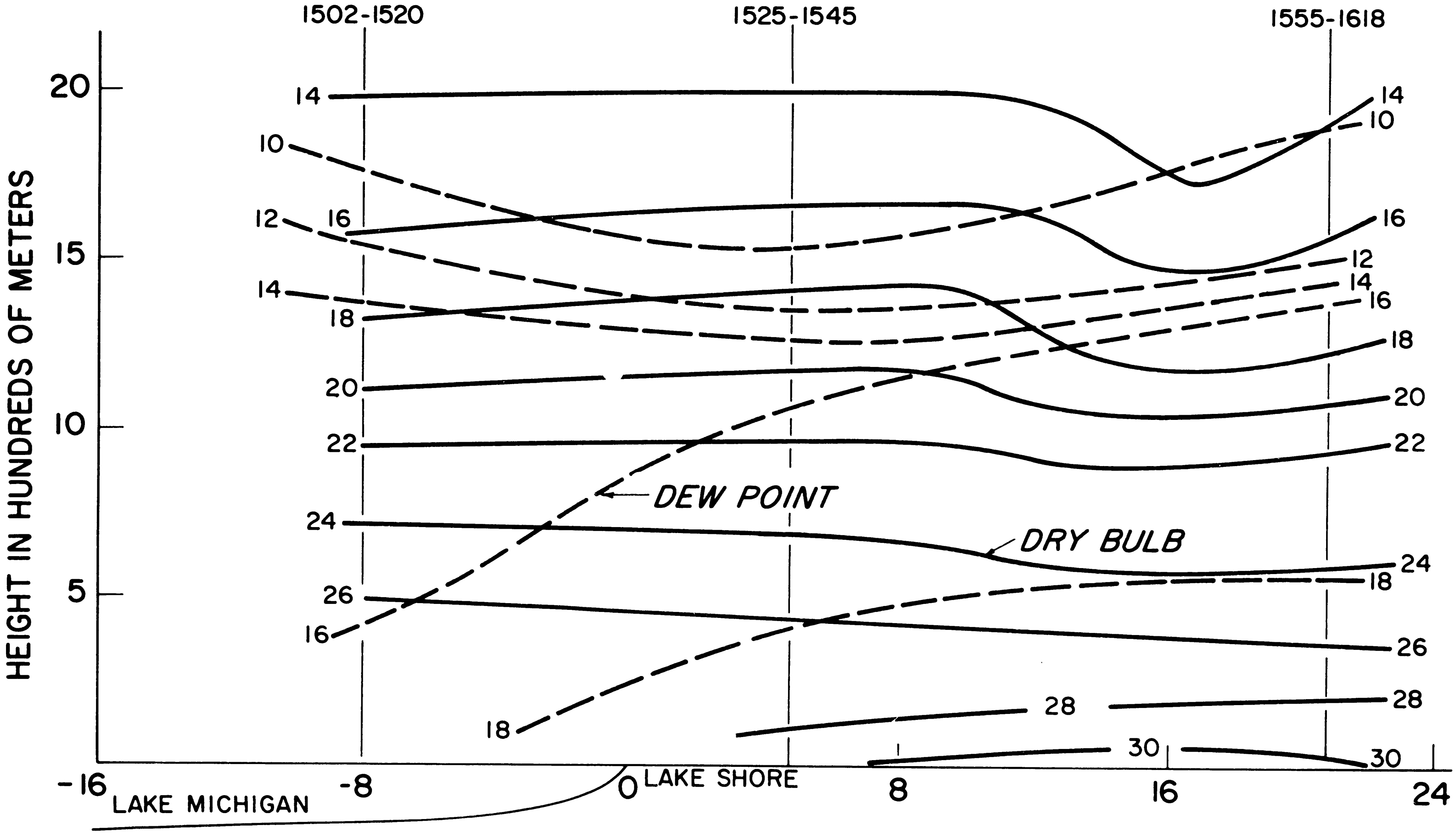

Fig. B.1. Dry bulb (solid) and dew point (dashed) temperature (^{o}C) structure in the vertical for July 22, 1964.

APPENDIX B

DERIVATION OF PERTINENT EQUATIONS

APPENDIX B

I. Derivation of Eq. 8, Chapter 3:

The hydrostatic approximation has been shown to be valid for lake breeze motions

$$\frac{\partial p}{\partial Z} = - \rho g \qquad \text{Eq. 8, Chapter 3}$$

Substituting for ρ from the equation of state

$$\rho = \frac{p}{RT}$$

and for T from the defining equation for potential temperature

$$T = \theta \left[\frac{p}{p_o}\right]^{R/C_p}$$

where p_o = 1000 mb = assumed surface pressure initially

$$\frac{\partial p}{\partial Z} = -\frac{g}{R}\frac{1}{\theta}\, p_o^{R/C_p}\, p^{1-R/C_p} = -\frac{g}{R}\, p_o^{R/C_p}\, \frac{p^{C_v/C_p}}{\theta}$$

where g, C_p, R and p_o are constants.

Let $\quad \frac{g}{R}\, p_o^{R/C_p} = A$

then $\quad p^{-C_v/C_p}\, \frac{\partial p}{\partial Z} = - \frac{A}{\theta}$

or $\quad \frac{\partial}{\partial Z}\left[p^{1-C_v/C_p}\right] = - \frac{A}{\theta} \cdot \frac{R}{C_p}$

then $\quad \frac{\partial}{\partial Z}\left[p^{R/C_p}\right] = - \frac{g}{C_p}\, p_o^{R/C_p}\, \frac{1}{\theta} \qquad (7)$

II. Derivation of Eq. 9, Chapter 3:

Kibel (1957), by an order of magnitude comparison of terms, has shown that for a compressible atmosphere the equation of continuity with good accuracy may be written

$$-\frac{1}{\rho}\frac{\partial}{\partial Z}[\rho w] = \nabla \cdot \mathbf{V} \tag{1}$$

Expanding the derivative this equation becomes

$$\frac{\partial w}{\partial Z} + \frac{w}{\rho}\frac{\partial \rho}{\partial Z} = -\nabla \cdot \mathbf{V} \tag{2}$$

Similarly, for the general circulation Eq. 1 becomes

$$\frac{\partial w_g}{\partial Z} + \frac{w_g}{\rho_g}\frac{\partial \rho_g}{\partial Z} = -\nabla \cdot \mathbf{V}_g \tag{3}$$

Subtracting Eq. 3 from Eq. 2 to get the deviation quantities

$$\frac{\partial}{\partial Z}[w-w_g] + \frac{w}{\rho}\frac{\partial \rho}{\partial Z} - \frac{w_g}{\rho_g}\frac{\partial \rho_g}{\partial Z} = -\nabla \cdot [\mathbf{V}-\mathbf{V}_g]$$

But the deviation quantities Q' are defined by $Q' = Q - Q_g$ where Q may be w, ρ or $\mathbf{V}$ here. Thus

$$\frac{\partial}{\partial Z}[w'] + w\frac{\partial}{\partial Z}[\ln\rho] - w_g\frac{\partial}{\partial Z}[\ln\rho_g] = -\nabla \cdot \mathbf{V}'$$

which may be written

$$\frac{\partial w'}{\partial Z} + w'\frac{\partial}{\partial Z}[\ln\rho] + w_g\frac{\partial}{\partial Z}[\ln\rho - \ln\rho_g] = -\nabla \cdot \mathbf{V}'$$

which is Eq. 9 of Chapter 3.

III i) Derivation of the equations for velocity and potential temperature at the lower boundary of the transitional layer Z(2), for the forced convection regime:

For the forced convection regime

$$K = k^2 [Z+Z_o]^2 \ [1+ \alpha Ri]^2 \frac{\partial V}{\partial Z} \tag{1}$$

where Z_o is taken as the bottom of the constant flux layer very near the ground. For $Z_o \ll Z$ Eq. 1 reduces to Eq. 12 of the text. Through the layer of constant fluxes of heat and momentum, $Z_o \leq Z \leq Z(2)$,

$$\frac{\partial}{\partial Z} \left[K \frac{\partial V}{\partial Z}\right] = 0 \tag{2}$$

$$\frac{\partial}{\partial Z} \left[K \frac{\partial \theta}{\partial Z}\right] = 0 \tag{3}$$

Integrating Eqs. 2 and 3

$$K \frac{\partial V}{\partial Z} = V_*^2 \tag{4}$$

and

$$K \frac{\partial \theta}{\partial Z} = V_* \theta_* \tag{5}$$

where V_* and θ_* are constants of integration.

Substituting in Eq. 4 for K from Eq. 1

$$k^2 \ [Z+Z_o]^2 \ [1+ \alpha \ Ri]^2 \left[\frac{\partial V}{\partial Z}\right]^2 = V_*^2 \tag{6}$$

Thus $$[1 + \alpha\ Ri]\ \frac{\partial V}{\partial Z} = \frac{V_*}{k[Z+Z_o]} \tag{7}$$

But $$Ri = \frac{g}{\bar{\theta}}\ \frac{\partial\theta/\partial Z}{[\partial V/\partial Z]^2}$$

where $\bar{\theta}$ is the mean potential temperature over the layer. Equation 7 may thus be written

$$\left[1 + \frac{\alpha g}{\bar{\theta}}\ \frac{[\partial\theta/\partial Z]}{[\partial V/\partial Z]^2}\right] \frac{\partial V}{\partial Z} = \frac{V_*}{k[Z+Z_o]}\ ,$$

Thus $$\frac{\partial V}{\partial Z} + \frac{\alpha g}{\bar{\theta}}\ \frac{\partial\theta/\partial Z}{\partial V/\partial Z} = \frac{V_*}{k[Z+Z_o]}$$

Substituting in the 2nd term for $\frac{\partial\theta}{\partial Z}$ and $\frac{\partial V}{\partial Z}$ from Eqs. 4 and 5 and rearranging gives

$$\frac{\partial V}{\partial Z} = \frac{V_*}{k[Z+Z_o]} - \frac{\alpha g}{\bar{\theta}}\ \frac{\theta_* V_*}{[V_*]^2} \tag{8}$$

Integrating Eq. 8 with respect to Z yields

$$V = \frac{V_*}{k}\ \ln[Z+Z_o] - \frac{\alpha g}{\bar{\theta}}\ \frac{\theta_* V_*}{[V_*]^2}\ Z + C_1 \tag{9}$$

At Z = 0, V = 0. Solving for C_1 and substituting in Eq. 9

$$V = \frac{V_*}{k}\ \ln\left[\frac{Z+Z_o}{Z_o}\right] - \frac{\alpha g}{\bar{\theta}}\ \frac{\theta_*}{V_*}\ Z \tag{10}$$

Following a similar procedure

$$\theta - \theta_{Z(1)} = \frac{\theta_*}{k}\ \ln\left[\frac{Z+Z_o}{Z_o}\right] - \frac{\alpha g}{\bar{\theta}}\ \frac{\theta_*^{2}}{V_*^{2}}\ Z \tag{11}$$

Following Rossby and Montogmery (1935), in order to fit the solutions for velocity and potential temperature at the level Z(2), continuity of wind speed and direction, the potential temperature and the vertical derivatives of velocity and potential temperature are assumed through the layer $Z(1) \leq Z \leq Z(3)$ where $Z(3) = Z(2) + \Delta Z$. Thus

$$\frac{V_{Z(3)} - V_{Z(2)}}{\Delta Z} = \left.\frac{\partial V}{\partial Z}\right|_{Z(2)+Z_o} \tag{12}$$

where the LHS represents the finite difference approximation to the vertical derivative of velocity through the layer $Z(2) \leq Z \leq Z(3)$ and the RHS represents the derivative at the level $Z = Z(2) + Z_o$. Substituting in Eq. 12 for $V_{Z(2)}$ using Eq. 10

$$\frac{V_{Z(3)}}{\Delta Z} = \frac{V_*}{k[Z(2)+Z_o]} - \frac{\alpha g}{\bar{\theta}} \frac{\theta_*}{V_*} + \left[\frac{V_*}{k[Z(2)+Z_o]}\right] \cdot \left[\frac{Z(2)+Z_o}{\Delta Z} \ln \frac{Z(2)+Z_o}{Z_o}\right] - \frac{\alpha g}{\bar{\theta}} \frac{\theta_*}{V_*} \frac{Z(2)+Z_o}{\Delta Z} \tag{13}$$

But

$$\frac{\theta_*}{V_*} = \frac{\theta_* V_*}{[V_*]^2} = \frac{K\partial\theta/\partial Z}{K\partial V/\partial Z}$$

and through the layer $Z(1) \leq Z \leq Z(3)$

$$\frac{\theta_*}{V_*} \text{ is represented by } \frac{\Delta\theta}{\Delta V} = \frac{\theta_{Z(3)} - \theta_{Z(1)}}{V_{Z(3)}}$$

Substituting in Eq. 13 and combining terms

APPENDIX B (concluded)

$$\frac{\mathbb{V}_{Z(3)}}{\Delta Z} = \frac{\mathbb{V}_*}{k[Z(2)+Z_o]}\left[1 + \frac{Z(2)+Z_o}{\Delta Z}\ln\frac{Z(2)+Z_o}{Z_o}\right] - \frac{\alpha g}{\bar{\theta}}\left[\frac{\theta_{Z(3)}-\theta_{Z(1)}}{\mathbb{V}_{Z(3)}}\right]\left[1 + \frac{Z(2)+Z_o}{\Delta Z}\right]$$

Rearranging and cancelling terms

$$\mathbb{V}_* = \frac{k[Z(2)+Z_o]\mathbb{V}_{Z(3)}}{\Delta Z+[Z(2)+Z_o]\ln\frac{Z(2)+Z_o}{Z_o}}\left\{1 + \left[\frac{\alpha g}{\bar{\theta}}\right]\cdot\left[\frac{[\theta_{Z(3)}-\theta_{Z(1)}]}{[\mathbb{V}_{Z(3)}]^2}[Z(3)+Z_o]\right]\right\} \tag{14}$$

and

$$\theta_* = \frac{k[Z(2)+Z_o][\theta_{Z(3)}-\theta_{Z(1)}]}{\Delta Z+[Z(2)+Z_o]\ln\frac{Z(2)+Z_o}{Z_o}}\left\{1 + \left[\frac{\alpha g}{\bar{\theta}}\right]\cdot\left[\frac{[\theta_{Z(3)}-\theta_{Z(1)}]}{[\mathbb{V}_{Z(3)}]^2}[Z(3)+Z_o]\right]\right\} \tag{15}$$

Knowing $\mathbb{V}_*$ and θ_*, $\mathbb{V}_{Z(2)}$ and $\theta_{Z(2)}$ may be evaluated.

III ii) Evaluation of K for the forced convection regime

Substitute for $\partial\mathbb{V}/\partial Z$ from Eq. 8 into Eq. 4 to get

$$K_{Z(2)} = \mathbb{V}_*\left[\frac{1}{k[Z(2)+Z_o]} - \frac{\alpha g}{\bar{\theta}}\frac{\theta_*}{[\mathbb{V}_*]^2}\right]^{-1}$$

BIBLIOGRAPHY

Angell, J.K. and D.H. Pack, 1962: Analysis of low-level constant volume balloon (tetroon) flights from Wallops Island. J. of Atm. Sc., 19, 87-98.

Atlas, D., 1960: Radar detection of the sea breeze. J. of Met., 17, 244-258.

Barnett, K.M. and O. Clarkson, 1965: The relation of the time interval to accuracy of double theodolite observations. Mon. Weath. Rev., 93, 377-379.

Bellaire, F.R., 1965 (in press): The modification of warm air moving over cold water. Proc. of 8th Conf. of Great Lakes Res. Inst., Univ. of Mich., Ann Arbor, Mich.

Bilham, E.G., 1934: The sea breeze as a climatic factor. J. of State Med., London, 42, 40-50.

Craig, R.A., I. Katz and R.J. Harney, 1945: Sea breeze cross-section from psychrometric measurements. Bull. Am. Met. Soc., 26, 405-410.

Defant, F., 1951: Local winds. Compendium of Meteorology. Am. Met. Soc., 655-672.

DeJong, H.M., 1964: Application of the theory of adjustment to double theodolite observations. J. of Appl. Met., 3, 624-632.

Dixit, C.M. and J.R. Nicholson, 1964: The sea breeze at and near Bombay. Indian J. of Met. and Geophy., 15, 603-608.

Estoque, M.A., 1959: A preliminary report on a boundary layer numerical experiment. G.R.D. Res. Notes No. 20, Bedford, Mass.

________, 1960: A theoretical investigation of the sea breeze. Quar. J. of Roy. Met. Soc., 86, 523-534.

BIBLIOGRAPHY (continued)

Estoque, M.A., 1962: The sea breeze as a function of the prevailing synoptic situation. J. of Atm. Sc., 19, 244-250.

________, 1963: A numerical model of the atmospheric boundary layer. J. of Geophy. Res., 68, 1103-1113.

Fisher, E.L., 1960: An observational study of the sea breeze. J. of Met., 17, 645-658.

________, 1961: A theoretical study of the sea breeze. J. of Met., 18, 216-233.

Frenzel, C.W., 1962: Diurnal wind variations in central California. J. of Appl. Met., 1, 405-412.

Frizzola, J.A. and E.L. Fisher, 1963: A series of sea breeze observations in the New York City area. J. of Appl. Met., 2, 722-729.

Haurwitz, B., 1947: Comments on the sea breeze circulation. J. of Met., 4, 1-8.

Hewson, E.W., G.C. Gill, E.W. Bierly and I. Spickler, 1960: Meteorological analysis. Rep. no. 2515-5P, Univ. of Mich. Res. Inst., Ann Arbor, Mich.

Hewson, E.W., G.C. Gill and G.J. Walke, 1963: Smoke plume photography study, Big Rock Point Nuclear Plant, Charlevoix, Mich. Rep. no. 04015-3-P, Univ. of Mich. Res. Inst., Ann Arbor, Mich.

Jeffreys, H., 1922: On the dynamics of the wind. Quart. J. Roy. Met. Soc., 48, 29-47.

Kibel, I.A., 1963: Hydrodynamical methods of short period weather forecasting. Gostekhizdat, Moscow, 1957, published in the English version by Pergamon Press, London.

Kobayasi, von T. and T. Sasaki, 1932: Über Land und Seewinde. Beitrage zur Physik der freien Atmosphare, 19, 17-21.

BIBLIOGRAPHY (continued)

Laughton, J.K., 1873: Land and sea breezes. Quart. J. Roy. Met. Soc., 1, 203-207.

Monin, A.S. and A.M. Obukhov, 1954: Basic laws of turbulent mixing in the ground layer of the atmosphere. Akademiia Nauk SSR, Leningrad, Geofizicheskii Institut, Trudy, 24, 163-187.

Moroz, Wm.J. and E.W. Hewson, to be published. The mesoscale interaction of a lake breeze and low level outflow from a thunderstorm. Accepted for publication by J. of Appl. Met.

Mukammel, E.I., 1963: Ozone as a cause of tobacco injury. Met. Branch, Dept. of Transport, Canada.

Munn, R.E. and T.L. Richards, 1964: The lake breeze, a survey of the literature and some applications to the Great Lakes. Proc. of 7th Conf. of Great Lakes Res. Inst., Univ. of Mich., Ann Arbor, Mich.

Pack, D.H., 1962: Air trajectories and turbulence statistics from weather radar using tetroons and radar transponders. Mon. Weath. Rev., 90, 491-506.

Pearce, R.P., 1955: The calculation of a sea-breeze circulation in terms of the differential heating across the coastline. Quart. J. Roy. Met. Soc., 81, 351-381.

_______, 1962: A simplified theory of the generation of sea breezes. Quart. J. Roy. Met. Soc., 88, 20-29.

Petterssen, S., 1956: Weather analysis and forecasting, Volume I, McGraw-Hill Book Co., New York.

_______, and P.A. Calabrese, 1959: On some weather influences due to warming of the air by the Great Lakes in winter. Sc. Rep. no. 7, Univ. of Chic., Chicago, Illinois.

BIBLIOGRAPHY (continued)

Priestly, C.H.B., 1959: Turbulent transfer in lower atmosphere. Univ. of Chic. Press, Chicago 37, 130 pp.

Richtmeyer, R.D., 1957: Difference methods for intial value problems. Interscience Pub. Inc., New York, 238 pp.

Rossby, C.G., 1932: A generalization of the theory of the mixing length with applications to atmospheric and oceanic turbulence. M.I.T. Met. Papers, 1, no. 4.

________, and R.B. Montgomery, 1934: The layer of frictional influence in wind and ocean currents. M.I.T. Pap. in Phys. Oceanog. and Met., 3, no. 3.

Scarborough, J.B., 1955: Numerical mathematical analysis, Johns Hopkins Press, Baltimore, Md., 554 pp.

Schmidt, F.H., 1947: An elementary theory of the land and sea breeze circulation. J. of Met., 4, 9-15.

Shenfeld, L. and F.D. Thompson, 1961: The thunderstorm of Aug. 9th, 1961, at Hamilton, Ontario. Tech. Rep. 417, Met. Branch, Dept. of Transport, Canada.

________, 1965: Local severe thunderstorms in the Toronto-Hamilton area. Paper E-3, First Can. Conf. on Micromet., Apr. 12-14, Toronto, Canada, in press.

Shuman, F.G., 1957: Numerical methods in weather prediction, I, the balance equation. Mon. Weath. Rev., 85 329-332.

Stevenson, R.E., 1964: The influence of a ship on the surrounding air and water temperature. J. of Appl. Met., 3, 115-118.

BIBLIOGRAPHY (continued)

Stow, Rev. F.W., 1873: Land and sea breezes. Quart. J. Roy. Met. Soc., 1, 208-211.

Suttcliffe, R.C., 1937: The sea breeze at Felixstowe - a statistical investigation of pilot balloon ascents up to 5500 feet. Quart. J. Roy. Met. Soc., 63, 137-148.

Taylor, G.I., 1915: Eddy motion in the atmosphere. Phil. Trans. Roy. Soc., London, A, 215, 1-26.

Thompson, P.D., 1956: A theory of large scale disturbances in nongeostrophic flow. J. of Met., 13, 251-261.

________, 1961: Numerical weather analysis and prediction. The Macmillan Co., New York, 170 pp.

Thomas, M.K, 1964: A survey of Great Lakes snowfall. Proc. of 7th Conf. of Great Lakes Res. Inst., Univ. of Mich., Ann Arbor, Mich.

Thyer, N., 1962: Double theodolite pibal evaluation by computer. J. of Appl. Met., 1, 66-68.

van Bemmelen, Von W., 1922: Land und Seebrise in Batavia. Beitrage zur Physik der freien Atmosphare, 10, 169-177.

Wallington, C.E., 1961: Meteorology for glider pilots. John Murrey Publ., London. 284 pp.

________, 1963: Convection at the sea breeze front: Cumulus Dynamics. Pergamon Press, 119-125.

________, 1965: Gliding through a sea breeze front, Weather, 20, 140-144.

Watts, A.J., 1955: Sea breezes at Thorney Island. Met. Mag., 84,42-48.

Wexler, R., 1946: Theory and observations of land and sea breezes. Bull. of Am. Met. Soc., 27, 272-287.

BIBLIOGRAPHY (concluded)

Willet, H.C., 1944: Descriptive meteorology. Academic Press, New York, 264-266.

Yamamoto, G. and A. Shimanuki, 1964: Profiles of wind and temperature in the lowest 250 meters in Tokyo. Sc. Rep. Tohoku Univ., Geoph. Inst., Sendai, Japan, 111-114.