AN INVESTIGATION OF AN EXPERIMENTAL TECHNIQUE FOR DETERMINING THE TRAJECTORY OF A WATER DROPLET IN AN AIRSTREAM

A Master's Thesis

By

ALBERT OAKES MORTON
Lieutenant Commander, United States Navy

Project No. M992-D

WRIGHT AIR DEVELOPMENT CENTER, U. S. AIR FORCE
CONTRACT NO. AF 18(600)-51, E. O. NO. 462 BR-1

July, 1952
PREFACE

The Master's Thesis which follows was supported in part by U.S.A.F. Contract No. 18 (600)-51.

This report represents an initial investigation and is in the nature of a Progress Report. The work is being continued as a regular part of the Icing Research at the University of Michigan.

Myron Tribus
Director of Icing Research
ACKNOWLEDGMENT

The possibility of experimentally determining the trajectory of a water droplet in the airflow about a body was suggested by Dr. Myron Tribus, Director of Icing Research in the Engineering Research Institute, University of Michigan, and his continued interest in the investigation was a great help.

Especial thanks are due Dr. R. B. Morrison, Chief, Combustion Laboratory, Engineering Research Institute, and to his associates for their great help in the construction and gathering together of the experimental equipment used in the investigation.

Finally, the continued interest and helpful suggestions of the author’s faculty advisor, Professor A. M. Kuethe, were greatly appreciated.

The aircraft Icing Research Project, of which this investigation was a part, was supported by United States Air Force contracts through the Air Research and Development Command.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>SECTION</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preface</td>
<td>ii</td>
</tr>
<tr>
<td>Acknowledgment</td>
<td>iii</td>
</tr>
<tr>
<td>List of Symbols and Definitions</td>
<td>v</td>
</tr>
<tr>
<td>List of Figures and Tables</td>
<td>vi</td>
</tr>
<tr>
<td>Summary</td>
<td></td>
</tr>
<tr>
<td>Introduction</td>
<td>2</td>
</tr>
<tr>
<td>I General Considerations in Selecting the Experimental Technique</td>
<td>7</td>
</tr>
<tr>
<td>II Construction of a Droplet Generator</td>
<td>7</td>
</tr>
<tr>
<td>III Use of the Droplet Generator</td>
<td>11</td>
</tr>
<tr>
<td>IV Construction of a Device for Locating the Droplet Stream</td>
<td>14</td>
</tr>
<tr>
<td>V Use of the Hot Wire Anemometer Equipment</td>
<td>20</td>
</tr>
<tr>
<td>VI Wind tunnel</td>
<td>22</td>
</tr>
<tr>
<td>VII Experimental Results</td>
<td>23</td>
</tr>
<tr>
<td>VIII Discussion</td>
<td>29</td>
</tr>
<tr>
<td>IX Conclusions</td>
<td>30</td>
</tr>
<tr>
<td>Bibliography</td>
<td>32</td>
</tr>
</tbody>
</table>
LIST OF SYMBOLS AND DEFINITIONS

\( U \)  Free Stream Velocity, fps.

\( u, v \)  Components of Velocity in x and y directions respectively.

\( F \)  Force.

\( a \)  Droplet Radius, ft.

\( C_D \)  Drag Coefficient.

\( \rho \)  Density, Slugs/ft.\(^3\)

\( t \)  Time, Sec.

\( \psi \)  Dimensionless Parameter, Defined on p. 6

\( R_P \)  Reynolds Number based on relative droplet velocity.

\( R_U \)  Reynolds Number based on free stream velocity.

\( \mu \)  Micron = \( 10^{-6}\) Meter.

\( \mu_a \)  Air Viscosity.

\( \theta \)  Angle of Impingement of Droplet Stream, deg.

\( C \)  Characteristic length in flow field, ft.

\( K \)  Dimensionless Parameter = \( \frac{R_U}{\psi} \)

\( \phi \)  Dimensionless Parameter = \( R_U \times \psi \)

Subscripts:

\( a \)  Air.

\( d \)  Droplet.

\( o \)  Free Stream.

Superscripts: Numbers following words refer to the References in the Bibliography on p. 43.
LIST OF FIGURES AND TABLES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Representative Trajectories About a Cylinder</td>
<td>3</td>
</tr>
<tr>
<td>2. Photograph of Droplet Generator</td>
<td>9</td>
</tr>
<tr>
<td>3. Droplet Generator Schematic</td>
<td>10</td>
</tr>
<tr>
<td>4. Photomicrograph of Droplet Sample</td>
<td>13</td>
</tr>
<tr>
<td>5. Hot Wire Anemometer Circuit Schematic</td>
<td>16</td>
</tr>
<tr>
<td>6. Photograph of Wheatstone Bridge and Amplifier Equipment</td>
<td>17</td>
</tr>
<tr>
<td>7. Photograph of Hot Wire Assembly</td>
<td>18</td>
</tr>
<tr>
<td>8. Photograph of Test Section</td>
<td>24</td>
</tr>
<tr>
<td>9. Photograph of Droplet Generator Mounting</td>
<td>25</td>
</tr>
<tr>
<td>10. Plot of Experimental Results</td>
<td>28</td>
</tr>
</tbody>
</table>

Table I Experimental Results                                                                 | 27   |
AN INVESTIGATION OF AN EXPERIMENTAL TECHNIQUE
FOR
DETERMINING THE TRAJECTORY OF A WATER DROPLET IN AN AIRSTREAM

SUMMARY

The considerations in determining the trajectories of a water droplet in the airflow about a body are discussed. The problem is divided into two parts, one dealing with a method of producing a stream of water droplets of uniform size in a wind tunnel, and a second part dealing with the development of a device for determining the location of the stream of droplets. Both parts of the problem are discussed in detail. Sketches of equipment, circuit schematics, and photographs of equipment are included.

It is concluded that the hot wire anemometer and the water droplet generator developed in this investigation provide a suitable means for experimentally determining the trajectory of a stream of water droplets in the flow about a body, at the air velocities used in the study. Air velocities up to 100 fps were used, and no difficulty was anticipated at higher velocities.

Possible alternate methods for determining the droplet trajectories are discussed, and their obvious disadvantages pointed out.

The experimental results obtained in checking analytically determined trajectories of droplets in the flow about a right circular cylinder are given and the significance of the results discussed.
INTRODUCTION

In designing an anti-icing or de-icing system for an aircraft, the designer must provide sufficient capacity in the system to give the necessary protection to the aircraft. One parameter of importance in the design is, quite naturally, the size of the area to be protected, and a second parameter is, the amount of protection to be given to those protected areas.

The early types of wing de-icers were the rubber "inflatable-shoe" type, which is alternately inflated and deflated to crack the ice so that the airstream could blow it off the wing. This type of icing protection system is practically obsolete and will not be considered in this report.

The later types of ice protection systems are the thermal protection systems, in which heat is supplied to the areas to be protected by one of several methods. In this type of icing protection system, the capacity of the system is more intimately associated with the size of the protected area and the amount of protection to be provided than in the inflatable-shoe type. In the thermal system, the amount of heat required for protection of a given area is directly related to the amount of ice being accumulated on the airplane.

It is the determination of the amount of ice to be accumulated, or the icing rate, that leads us to the problem of determining the trajectory of a water droplet in the airflow about a body.

Let us first consider a right circular cylinder placed transverse to an airstream containing water droplets. The water droplets have a much higher mass than the surrounding air, and therefore, will not follow the streamlines around the body, but will follow a somewhat less curved path. This is shown in Figure 1. It is seen that for some initial displacement, \( y_1 \), from the x axis, the droplet trajectory will be tangent to the cylinder. Those droplets having initial displacements less than \( y_1 \) will impinge on the cylinder and those having initial displacements greater than \( y_1 \) will miss the cylinder completely. Clearly then, all of the droplets included in the projected frontal area of the cylinder do not
REPRESENTATIVE TRAJECTORIES
ABOUT A CYLINDER

FIG. 1
impinge on the cylinder, and only those within the limiting trajectories contribute to the icing rate.

The problem then is to determine these limiting trajectories from which the icing rate may be predicted. For simple shapes where the equations of the streamlines are well known, the trajectories may be computed by stepwise integration, graphically, or by a differential analyzer. The trajectories about a cylinder were first calculated by Albrecht\(^1\), for the case of Stokes' Law regime \((C_D R_F/24 = 1)\). Later Langmuir and Blodgett\(^2\) calculated trajectories about a cylinder for the case where \(C_D\) varies with \(R_F\), a differential analyzer being used to generate the solutions.

The Langmuir-Blodgett trajectories were computed as follows:\(^3\)

Let \(u_a, v_a\) = components of air velocity in the \(x\) and \(y\) directions respectively.

\(u_d, v_d\) = components of droplet velocities in the \(x\) and \(y\) directions respectively.

Then \((u_a - u_d), (v_a - v_d)\) = components of the relative droplet velocity, \(P\).

The relative velocity of the droplet with respect to the airstream gives rise to a drag force:

\[
F_x = \frac{1}{2} \rho_a \pi a^2 C_D P (u_a - u_d)
\]

\[
F_y = \frac{1}{2} \rho_a \pi a^2 C_D P (v_a - v_d)
\]

where

\(a\) = droplet radius
\(C_D\) = drag coefficient
\(\rho_a\) = air density

These drag forces give rise to accelerations according to Newton's second law

\[
F_x = \max = \frac{4}{3} \pi a^3 \rho_a \frac{du_d}{dt}
\]
\[ F_y = m a_y = \frac{4}{3} \pi a^3 \rho_d \frac{dv}{dt} \]

where \( \rho_d \) = droplet density

Equating these forces gives

\[ \frac{du}{dt} = \frac{3}{8} \frac{\rho_a}{\rho_d} \frac{P(u_a - u_d)}{a} C_D \]

\[ \frac{dv}{dt} = \frac{3}{8} \frac{\rho_a}{\rho_d} \frac{P(v_a - v_d)}{a} C_D \]

The droplet trajectory is obtained by integrating the following equations simultaneously:

\[ (x - x_0) = \int \left[ \left( \frac{3}{8} \frac{\rho_a}{\rho_d} (u_a - u_d) \frac{PC_d}{a} \right) dt \right] dt \]

\[ (y - y_0) = \int \left[ \left( \frac{3}{8} \frac{\rho_a}{\rho_d} (v_a - v_d) \frac{PC_d}{a} \right) dt \right] dt \]

The first integration gives the droplet velocity and the second integration gives the droplet displacement. To simplify the computations, the above equations are made dimensionless as follows:

\[ \frac{x}{C} = \frac{x_0}{C} + \int \left\{ \left( \frac{9 \rho_a C}{\rho_d a} \right) \left( \frac{C_D P}{24 U} \right) \left( \frac{u_a - x}{U} \right) \right\} d\left( \frac{U_t}{C} \right) \]

\[ \frac{y}{C} = \frac{y_0}{C} + \int \left\{ \left( \frac{9 \rho_a C}{\rho_d a} \right) \left( \frac{C_D P}{24 U} \right) \left( \frac{v_a - y}{U} \right) \right\} d\left( \frac{U_t}{C} \right) \]

Now define

\[ \psi = \frac{9 \rho_a C}{\rho_d a} , \quad R_p = \frac{2 \rho_a P}{\mu a} , \quad R_u = \frac{2 \rho_a U}{\mu a} \]

where \( \mu a \) = air viscosity
\[ C = \text{characteristic length in flow field (cylinder radius)} \]

\[ U = \text{free stream velocity} \]

Then

\[
\frac{x}{C} = \frac{x_0}{C} + \int \left\{ \left( \frac{u}{R_u} \right) \left( \frac{C_p R_p}{24} \right) \left( \frac{u_a - x}{U} \right) \right\} \frac{d(U)}{C} \frac{d(U)}{C}
\]

\[
\frac{y}{C} = \frac{y_0}{C} + \int \left\{ \left( \frac{u}{R_u} \right) \left( \frac{C_p R_p}{24} \right) \left( \frac{v_a - y}{U} \right) \right\} \frac{d(U)}{C} \frac{d(U)}{C}
\]

As stated previously, these integrations must be accomplished by stepwise methods, graphically, or by a differential analyzer. This is necessary since \( C_p \) is a function of \( R_p \) and usually \( u_a/U \) and \( v_a/U \) are rather complicated functions of position in the flow field.

From the above discussion it is apparent that determining the trajectories for even the simplest of shapes is quite involved and tedious if approached from the analytical standpoint. Langmuir and Blodgett\(^2\) also computed the trajectories about a sphere and a ribbon placed normal to the airstream by much the same methods used for the cylinder.

The trajectories for airfoils have been investigated by a number of workers in the field\(^4,5,6\), most of whom use numerical and graphical methods that are long and tedious. Since the streamline pattern for most commonly used airfoils is well known, the problem can be approached analytically, as the velocities at each point in the flow can be determined.

In the case of an arbitrary shape for which the streamline pattern is not known or cannot be computed, the icing rate cannot be predicted from an analytical treatment. This is, in general, true for three dimensional flows not possessing axial symmetry. One might also want to know, for example, the icing rate on a radar dome or droppable fuel tank which is to be fitted to an airplane. In these cases one must resort to extrapolating data from other similar shapes, or go to some experimental method of determining the limiting trajectories from which an icing rate might be predicted.

It is the purpose of this investigation to evaluate an experimental technique for determining the trajectories.
SECTION I

GENERAL CONSIDERATIONS IN SELECTING THE EXPERIMENTAL TECHNIQUE

Preliminary discussions of various experimental techniques for determining water droplet trajectories in the flow about a body led to the selection of a technique involving the use of actual water droplets, and the development of a device for detecting their location in a simple two dimensional flow in a wind tunnel. Other techniques considered will be mentioned at the end of this report.

The trajectory problem, as approached herein, divides itself quite nicely into two parts. The first part involves finding a method of producing the droplets, and the second part involves finding a method of detecting their location in the flow. Each part of the problem will be considered by itself in a separate section.

In a wind tunnel study of this sort, it is desirable to have a stream as free of turbulence as possible, so that a stable flow may be used. The tunnel used in this study was acquired form a previous project and was reported to have a turbulence level of the order of 1/2%. This was considered satisfactory for the work to be undertaken and no special attempts were made to improve the stability of the flow. The availability of this equipment also influenced the selection of the technique to be used.

SECTION II

CONSTRUCTION OF A DROPLET GENERATOR

There is a considerable amount of information available on methods for producing droplets of small diameter. However, most of these methods are used for the production of very large numbers of droplets, such as the saturation of a large volume of air, fog generators, etc. As far as can be determined from the literature, Dimmock has done the only work with the production of a single stream of droplets which might be applicable to the present problem.

Ideally, what was desired in this study was a stream of droplets of uniform size, proceeding single file down the wind tunnel, and it was
felt that the method of producing such a droplet stream that held the most promise was that of a vibrating capillary.

In this system a glass tube is heated until plastic and then pulled apart to form a fine capillary tube. The capillary can be made as small as desired and usually the tube is pulled apart until the filament breaks. The capillary can then be broken off at the desired size, or broken off bit by bit until the desired droplet size is obtained. Generally, the hole in the tubing will carry on down to the smallest capillary that can be pulled, without sealing itself off.

It was found in the study that pyrex glass tubing with 1 mm wall thickness tended to fatigue quite rapidly with the vibrating system used, and breakage was quite high. Much more satisfactory results were obtained with thick walled glass tubing, but considerably more care was required in the drawing process to avoid overheating and the resultant sealing off of the hole. Tubing of approximately 1/4 inch O.D. and 1/10 inch wall thickness was used.

The actual production of the droplets is accomplished by supplying fluid to the tube and vibrating the capillary tip. This method of making droplets is quite well known and the various reports of its use seem to differ only in the method of vibrating the capillary tip.

Dimmock$^8$ produced vibrations by waxing a piece of steel hypodermic needle on the capillary and placing the tip in an alternating electrical field. This was considered impractical for wind tunnel use, where the vibrations were desired in the plane of the flow. Vonnegut and Neubauer$^9$ produced oscillations by applying an air jet to the capillary tip. In this case the tip describes an involved path, giving off a stream of droplets at each point in the path where the acceleration reaches sufficient magnitude. The droplets within each stream are quite uniform in size, but the size varies from stream to stream. This method was also considered impractical for wind tunnel use.

The method finally selected involved the application of an oscillatory force to the glass tube at a frequency which would induce a vibration in the tip. This was done by means of a modified, heavy duty, radio speaker. The speaker was the permanent magnet type, and the speaker cone was removed. An aluminum fixture was attached to the sleeve which ordinarily supports the cone, and this fixture extended approximately six inches from the original speaker cone base. The outer end of the fixture was built so that it could be securely clamped to a glass tube. To provide additional support for the fixture, a three wire suspension system attached to the speaker frame supports the fixture at its midpoint. The general construction of this apparatus is shown in Figure 2.
DROPLET GENERATOR SCHEMATIC

FIG. 3
To produce the oscillatory force, a signal of the desired frequency is taken from an audio oscillator, amplified by a 25 watt audio amplifier, and fed to the speaker.

The glass tube is rigidly supported by a modified laboratory test tube clamp at a point above the speaker attachment. This arrangement, shown in Figure 3, gives the glass tube the characteristics of a cantilever beam with an oscillatory force applied close to the built in end.

As for the vibrational characteristics of the tube, it was found that with a tube approximately 12 inches long the fundamental frequency was about 60 cps and the entire tube vibrated in the first mode. Amplitudes of vibration of the tip of as much as 1 inch were observed. The higher harmonics gave proportionately lower amplitudes at the tip, until at about 7000 cps the vibrations were not visually discernable. With quite long capillary tips, the tip itself acts as a beam with an end fixity less than unity, and as many as four nodes were observed in the tip. This situation was generally undesirable as it led very quickly to the breaking off of about 1/2 inch of the tip.

SECTION III

USE OF THE DROPLET GENERATOR

It has been previously stated that the goal in the construction of the droplet generator was a device capable of producing a stream of droplets proceeding single file down the wind tunnel. This specific goal was not achieved, but it is felt that a satisfactory compromise was reached so that the device could be used as intended.

Initially, it was expected that the capillary tip would oscillate in the plane of the applied oscillatory force, coincident with the plane of the flow; projecting one droplet upstream and one droplet downstream. The droplet projected upstream could be drawn off and not allowed to enter the test section, leaving only the downstream series of droplets going into the test section.

Experiments with the droplet generator showed that at the lower frequencies the tip would oscillate in a plane, but because of the amplitude of the oscillation, the droplets were thrown up at an angle to the horizontal. This resulted in a vertical spread of the droplet stream, and;
led to the investigation of higher frequencies.

At frequencies above 2000 cps it was found that the droplets left the tip in a horizontal plane, but that the oscillation was no longer planar. This is believed to be caused by the capillary tip itself acting as a beam with some end fixity less than unity, and having no preferred direction of oscillation. Generally the tip could not be made axially symmetric, so that after the applied impulses were transmitted down the glass tube they had lost their planar character, and the resultant oscillation gave off droplets in all directions. This resulted in a thin layer of droplets in the test section, the vertical spread being caused by these droplets initially projected upstream being turned back downstream, and being thrown out of the horizontal plane in the process. The droplet layer could be made less than 1/2 cm. in thickness, and this was felt to be acceptable.

The droplet size desired was arbitrarily chosen as 50\(\mu\), the feeling being that the troubles involved in the steady production of very small droplets (10\(\mu\)) might prove unduly bothersome, and that the Langmuir-Blodgett\(^2\) date was applicable to droplets of 50\(\mu\) in diameter.

The method used to measure the droplet size in this investigation was devised by Mr. Bruce Toms of the Icing Research Group, and is based on similar methods used by other investigators. The method consists of pouring a thin layer of castor oil into a Petri dish, and covering the castor oil with a layer of commercial Xylene. The droplets are allowed to fall into the dish, and the dish is then placed in a photomicrograph for examination. The Xylene has a much lower viscosity and a somewhat lower specific gravity than water, and the droplets will fall through the Xylene, coming to rest on the castor oil and remaining spherical in shape. The droplets, being covered with Xylene, have little tendency to coalesce, even though they become tangent to one another. Figure 4 is a photomicrograph of a representative sample of the droplets. It was gratifying to find that the droplets were quite uniform in size, and especially that there were no very large droplets present, as is usually the case with spray nozzles.

The principal disadvantage of the castor oil-Xylene method of catching the droplets is that it is not adapted for use in an airstream because the Xylene blows out of the dish. However, there are methods of catching droplets in an airstream, using an oiled slide. The Shell Oil Company markets an oil under the trade name of Spirax No. 250, which is well suited for this purpose. Its principal disadvantage is that the droplets evaporate rapidly, making photography quite difficult. Another method is to coat a glass slide with heated petroleum jelly and place it in the stream. The droplets are caught in the heated jelly and captured there as the jelly hardens, thus permitting examination at a later time.
The actual measurement of the droplet size is carried out by use of a haemacytometer, which has a grid etched on its surface, the smallest division being 50μ. The grid was photographed at the same magnification used to photograph the droplets, and then the grid negative and the droplet negative were superimposed on one print, as shown in Figure 4.

There appear to be three parameters in the droplet generator that affect the droplet size. These are (1) the frequency of oscillation, (2) size of the capillary, and (3) the hydrostatic head. Of these, the first two are the more important, and with a given capillary the droplet size is very sensitive to small frequency changes in the vicinity of a harmonic frequency of the tip. Time did not permit an investigation of the effect of each of these parameters.

An important problem encountered in the use of the droplet generator was that of the water crawling up the outside of the capillary before being projected into the airstream as a droplet. The effect of this occurrence was to increase the vertical thickness of the droplet layer, which was most undesirable. Ranz and Marshall10 experienced a similar problem in their work, and solved it by coating the outside of the capillary with an antiewetting agent. This was tried with the oscillating capillary, but with little success. The water would not crawl up the outside of the tip when it was not being vibrated, but would as soon as the vibration started. It was noted, however, that when the applied frequency was exactly equal to the harmonic frequency of the tip that this crawling up the outside of the tip was not present, the water apparently being given sufficient acceleration at the tip to be projected into the stream.

SECTION IV

CONSTRUCTION OF A DEVICE FOR LOCATING THE DROPLET STREAM

In discussing the possible methods for detecting the location of the droplet stream, it was felt that visual means were impractical, photographic means held some promise, and that a hot wire anemometer was the most promising.

The design and use of the hot wire anemometer is quite well known in the field of velocity and turbulence measurements, 11,12,13 and the same general ideas apply to the problem of detecting the location of a droplet stream. The operation of a hot wire anemometer is based on the change in electrical resistance with change in temperature of a thin wire. The measure of this change in resistance with temperature is called the
thermal coefficient of resistivity and is expressed in ohms per ohm per degree C. Of the materials suited for use in a hot wire device, tungsten was chosen for this investigation, and has a temperature coefficient of resistivity of .005 ohms\(^2\) per ohm per degree C. Electrolytic iron has a higher thermal coefficient than tungsten, but would present corrosion problems when used with moisture present and its availability as a small diameter wire is uncertain.

In this study it was proposed to place the hot wire in the wind tunnel test section in such a manner that it could be moved about. Then when the hot wire was being struck by the water droplets, the wire would be cooled and its resistance lowered. The lowered resistance of the hot wire would cause a change in the current in the circuit, and this change in current would be an indication that the wire was in the droplet stream.

The success of the method outlined above depends to a very great extent on the relative magnitudes of the effect of the tunnel turbulence and the effect of the water droplets on the wire. It must be borne in mind that although the velocity shifts due to turbulence are a relatively small percentage of the total velocity, they affect a much larger portion of the wire than a \(50\sqrt{V}\) water droplet would. This results, it was found, in the effect of the turbulence being considerably greater than the effect of the droplet stream. This fact was not appreciated in the early stages of the investigation and resulted in some faulty assumptions concerning the hot wire anemometer. For example, it was assumed that since the water droplet would be much colder than the hot wire, it would have a sizable effect on the wire in changing its resistance. This assumption neglected the fact that, for example, with a hot wire 1/2 inch long, a droplet of 50\(\sqrt{V}\) diameter is only 0.39% of the length of the wire.

Based on this assumption, it was felt that the hot wire anemometer would not need to be as sensitive or elaborate as those used in turbulence work, and a relatively simple circuit was devised. In this circuit the change in current due to the change in hot wire resistance was displayed on a cathode-ray oscilloscope. This system worked well in still air, but when placed in the wind tunnel the turbulence effects completely masked the effect of the droplets.

Further consideration of the problem led to the conclusion that it might be possible to integrate out the effect of the turbulence, since over a period of time the turbulence would be as much positive as it would negative and the time integral would be zero. In contrast to this, the droplets produce an undirectional effect which would be cumulative in the integrating process.

The circuit selected was essentially a Wheatstone Bridge circuit,
HOT WIRE ANEMOMETER CIRCUIT

AMPLIFIER CIRCUIT

FIG. 5
WHEATSTONE BRIDGE AND AMPLIFIER EQUIPMENT

FIG. 6
where the hot wire is one leg of the bridge and the change in current in
the hot wire unbalances the bridge. This unbalance is amplified and dis-
played on a microammeter. Figure 5 is a schematic of the bridge and am-
plifier circuits, and Figure 6 is a photograph of the bridge and amplifier
equipment. The amplifier circuit, suggested by Dr. R. B. Morrison and
Mr. Henry Hicks of the Combustion Laboratory, Engineering Research
Institute, uses a 6SN7 electron tube as an amplifier. The 6SN7 has a gain
of 20, and appeared to give better results than either the 6AS7 or 6SL7
tubes, both of which were tried. The two VR150 tubes shown are used to
stabilize the B+ voltage, and are connected in series to give a B+ voltage
of 300 volts. All resistors shown have power ratings of 10 watts or more.

The hot wire assembly is shown in Figure 7. The assembly is
mounted on a heavy iron base for stability, and a micrometer is used to
position the hot wire vertically. A travel of 5 cm is available with the
micrometer, and scale readings of 10^{-3} cm are possible. The horizontal
positioning is accomplished manually by moving the entire hot wire assem-
bly on its base to the desired position.

The arms of the hot wire probe are made of 1/16 inch welding
rod, and the .001 inch tungsten wire is fastened across the tips of the
arms. The .001 inch wire was chosen because it has sufficient strength
to withstand the impact of the droplets, is not too difficult to work with,
and was available. A smaller size tungsten wire (.0003 inch diameter)
was tried but proved to have little tensile strength and was difficult to
work with. Tungsten wire cannot be soft soldered directly and another
means must be used to fasten it to the probe. Some workers have copper
plated the tips of the tungsten wire and then soft soldered it in place.
A somewhat simpler method was used in this investigation, wherein a thin
coating of silver solder is tinned on the tips of the arms of the probe.
Then, while the silver solder is still molten, the tungsten wire is
placed across the tips so that it imbeds itself in the silver solder, and
is held securely in place when the silver solder hardens.

It is not clear whether the silver solder and tungsten actually
form a bond or whether the tungsten is only cast into the silver solder.
In any event, the electrical bond was, without exception, very good and the
simplicity of the procedure made it very attractive. The use of induction
heating and the precision equipment usually found in electron tube labor-
atories would provide a somewhat neater joint, but was not readily avail-
able.
SECTION V

USE OF THE HOT WIRE ANEMOMETER EQUIPMENT

The objective of the hot wire anemometer equipment is to accurately establish the position of a stream of droplets relative to some fixed datum. In this investigation a horizontal flat plate was positioned beneath the wind tunnel test section for use as a reference datum. The hot wire assembly, on its base, rests on this plate. The plate was checked for flatness and found to be satisfactory, and was mounted in a level position.

For this investigation it was decided to mount the model horizontally, and the measurement desired was the vertical displacement of the droplet stream from a horizontal, longitudinal axis through the center of the model. By referring the model position to the flat plate, measurement could be made of the height of the droplet stream above the flat plate with the hot wire equipment. This measurement could then be translated into a vertical displacement from the reference axis.

The use of the Wheatstone Bridge and amplifier circuits was relatively simple, once the circuit was understood. The first step, with the meter resistance in and the meter disconnected from the bridge circuit, was to activate the amplifier circuit by applying 6 volts (AC or DC) to the heater of the 6SN7, and then applying the B+ voltage of 300 volts to the plates of the tube. With two VR150 tubes in series, it is necessary to apply approximately 330 volts to the circuit to fire the VR tubes and obtain a regulated 300 volts on the plates of the tube. The applied voltage could then be reduced to about 310 volts, at which voltage the tube drew a current of about 30 milliamperes. During the warmup period of the tube constant attention to the meter reading was required in order to keep the meter reading on scale. With the meter disconnected from the bridge circuit, the meter reading is controlled by the two potentiometers marked MB1 and MB2 on the amplifier circuit of Figure 4. MB1 and MB2 are connected in parallel, MB1 giving coarse control and MB2 giving fine control. These two potentiometers control the current in the plate circuit so that when the meter reading is zero, the currents in each leg of the plate circuit are equal. After the tube operation has stabilized and the meter reading is reduced to zero by means of the potentiometers MB1 and MB2, the 40,000 ohm meter resistance can be cut out of the circuit, and the meter reading again reduced to zero. This having been done, the amplifier circuit is ready for use.

To make ready the bridge circuit, the current through the bridge
is adjusted to the desired value by means of the voltage divider arrangement on the 12 volt battery supply. This step is taken with the hot wire in position in the wind tunnel, and the tunnel running. The amount of current through the hot wire depends on the length of the wire, and the proper amount is best determined by trial and error. The "proper" amount of current is considered to be that current which will heat the tungsten wire to just below a red heat. It was found that if the tungsten wire was allowed to reach a red heat, the surface of the wire oxidized very rapidly. This oxidation effectively reduced the cross sectional area of the wire, which raises the current density, which in turn again overheats the same spot in the wire, causing further oxidation, etc. This process occurs so rapidly that once it had started the current through the wire could not be reduced rapidly enough to prevent burning off the wire. As a remedy for this situation, gold plated wire can be used, where the gold prevents the oxidation of the tungsten. It was found desirable to cut off the current to the hot wire whenever the hot wire was removed from the tunnel or the tunnel shut down.

Having established the current in the hot wire, the amplifier circuit is connected to the bridge circuit and the meter reading reduced to zero by adjusting potentiometers BB1 and BB2, Figure 5. These potentiometers are also connected in parallel to give both coarse and fine control. Caution is necessary when first connecting the bridge and amplifier circuits together, because if the bridge is badly unbalanced, the current through the meter may be excessive. To avoid this, it is desirable to switch in the meter resistance before connecting the two circuits. As an additional precaution it might be well to parallel the SPST switch connecting the bridge and amplifier circuits with a momentary contact switch so the bridge balance could be checked without endangering the meter quite so much. When the bridge has been balanced to give a zero meter reading, the meter resistance may be cut out and the hot wire anemometer equipment is ready for use.

In using the hot wire anemometer to determine the vertical position of the droplet stream in the test section, the horizontal position of the stream is first established by cross lighting the test section with a photoflood light and looking upstream into the tunnel from the end of the diffuser. Using this technique, the shape of the stream is also determined and fine adjustments can be made in the audio oscillator frequency to obtain the best droplet stream. The hot wire assembly is then positioned manually so that it is in a vertical plane with the droplet stream, and the hot wire is raised or lowered, by means of the micrometer, until the meter reading indicates the wire is in the stream. Normally the meter reading has been adjusted to zero with the hot wire out of the stream. Then as the wire enters the stream, the change in current causes the meter reading to increase. It was found that with a thin layer of droplets and the maximum permissible current in the hot wire, the
meter reading would go from zero to more than half scale in traversing the droplet layer. Also, by traversing the layer slowly the center of the layer could be determined quite accurately. This assumes that the distribution of the droplets in the layer is such that the droplets are concentrated in the center of the layer.

It was also found that if the droplet layer had any appreciable vertical dimension (more than 1 cm) the meter readings were quite unreliable and no measurement could be made of the edge or center of the layer.

The hot wire anemometer circuit as constructed for this investigation appeared to damp out the effects of tunnel turbulence quite satisfactorily, so that the turbulence presented no difficulties.

Using the hot wire close to a body in the flow required a somewhat different technique than that outlined above. As might be expected, the streamlines close to a body are quite close together, indicating high velocity gradients. As the hot wire crosses the streamlines, the rate of cooling of the wire changes rapidly, which unbalances the bridge circuit in the same manner as the droplet stream, resulting in a steady change in meter reading. To detect the droplet stream under these conditions, the wire must be moved slowly so that the discontinuity in meter movement due to the effect of the droplet stream may be seen. This is most readily done by moving the wire away from the body so that the velocity is decreasing, giving a decrease in meter reading. Then when the hot wire enters the stream, the meter reading will stabilize or may even increase slightly. During the above process, as the meter reading reaches the end of the scale, it is necessary to stop the hot wire movement, rebalance the bridge circuit to move the meter needle to the other end of the scale, and then continue with the hot wire traverse. It was found that when quite close to a body, this process had to be repeated very often, making the whole procedure quite tedious.

SECTION VI

WIND TUNNEL

The wind tunnel used in this investigation was constructed for use in the spray work of Flederman and Hansen, and is completely described therein.

Air is drawn through the system by a 36 inch Propellair Fan, which
draws 6000 cfm at 1800 RPM, with 1 inch of water pressure drop across the
fan. The fan is powered by a Reeves Variable Speed Control whose speed
may be continuously varied from 580 to 2900 RPM, with a maximum of 5HP
available at 2900. The coupling between the fan and the Reeves unit is
effected by a twin Vee belt pulley system, with a 3:5 pulley ratio to
decrease the RPM at the fan to its design maximum speed of 1800 RPM.

Air enters the tunnel from a plenum chamber through a bell
mouth, into a 3 foot square, 1 foot long settling chamber containing five
40 mesh screens. These screens serve to reduce the initial turbulence.
A convergent section connects the 3 foot square settling chamber with a
1 foot square test section, giving an area reduction of 9 to 1.

The test section is 3 feet long and 1 foot square in cross
section, and is constructed of 1/4 inch thick Plexiglass. A rectangular
access door is provided in the side of the test section, and the bottom
of the test section is slotted to permit insertion of the hot wire probe.
Figure 8 is a photograph of the test section.

A 9 foot long diverging diffuser made of sheet metal with a
wooden liner connects the test section with the fan housing. The dif-
funser is a transition section which varies from a 1 foot square at the
test section to a 36 inch diameter circle at the fan housing.

The droplet generator is mounted on the framework of the up-
stream end of the test section, Figure 9. The glass tube enters the top
of the tunnel 1 foot upstream of the test section, at a point where the
cross sectional area is just over 1 square foot.

The model, a right circular cylinder in this case, is mounted
at the downstream end of the test section. The cylinder was mounted
eccentrically so that it could be rotated about a longitudinal axis, thus
changing its vertical position.

SECTION VII

EXPERIMENTAL RESULTS

To evaluate the effectiveness of the hot wire anemometer equip-
ment developed in this investigation, the equipment was used to determine
the trajectory of a water droplet in the flow about a right circular cy-
linder. The experimental results were then to be compared with the analy-
tically determined trajectories of Langmuir-Blodgett\(^2\).
The equipment was used in the same manner as described in previous sections. The vertical displacement of the droplet stream from the reference axis was determined at three and at seven diameters upstream of the cylinder. As predicted by the Langmuir-Blodgett\textsuperscript{2} data there was no discernable change in the initial displacement between these two stations.

The second item to be determined was the angle $\theta$ from the center of the cylinder at which the droplet stream impinges on the cylinder. This measurement was made visually by observing the impingement point and then measuring the angle from the center of the cylinder with a protractor. Since the droplet generator did not produce a single stream of droplets the resulting stream covered a small area on the cylinder rather than impinging at a point. This introduces a possibility of error in approximating the center of this impingement area.

Table I gives the experimental results obtained, and the corresponding trajectory data from Langmuir-Blodgett\textsuperscript{2}. Figure 10 is a plot of the data of Table I. The experimental data obtained was for a value of $K$ of 16, whereas the closest Langmuir-Blodgett\textsuperscript{2} data is for a value of $K$ of 10. A $K$ of 10 corresponds to a smaller droplet diameter, which would move the Langmuir-Blodgett curve to the right, giving a greater spread in values. It is seen that for the two cases considered, the initial displacements are in quite poor agreement, but that impingement angle of the tangent trajectory agrees quite well in the two cases.

It is felt that the reasons for this disagreement in initial displacements are two fold. First, in the computation of the Langmuir-Blodgett trajectories the drag coefficients used were those determined experimentally for large spheres. These drag coefficients may be described as "steady state" drag coefficients, because the spheres were not under any great acceleration. Computations by Dr. J. E. Broadwell of the Icing Research Group indicate that the droplet would experience accelerations of the order of $10^4$ or $10^5$ g, even on the limiting trajectory where the curvature of the trajectory is small. The validity of a "steady state" drag coefficient under such accelerations is questionable. Indeed, one might even question whether the droplet would remain intact under these accelerations or whether it would break into smaller particles.

The second source of error is in that the drag coefficient of a droplet is affected by evaporation of the droplet. The effect of evaporation is to lubricate the surface of the droplet, reducing the drag on the droplet and giving less curvature to the droplet path. Since the air in the wind tunnel was not saturated it is reasonable to assume that the droplet was evaporating and that the trajectories determined are valid. Hansen\textsuperscript{14} determined the drag coefficients of evaporating fuel droplets, and found that with a highly volatile droplet such as hexane the drag coefficient was reduced by a factor of as much as 10.
#### TABLE I

**EXPERIMENTAL RESULTS**

Computation of Constants:

\[ \rho_a = \text{Air Density} = 2.378 \times 10^{-3} \text{ slugs/ft.}^3 \]

\[ \rho_d = \text{Droplet Density} = 1.935 \text{ slugs/ft.}^3 \]

\[ a = \text{Droplet Radius} = 35 \mu = 1.15 \times 10^{-4} \text{ ft.} \]

\[ C = \text{Cylinder Radius} = 8.72 \times 10^{-2} \text{ ft.} \]

\[ U = \text{Free Stream Velocity} = 92 \text{ fps.} \]

\[ Ma = \text{Air Viscosity} = 3.719 \times 10^{-7} \text{ slugs/ft. sec.} \]

\[ R_U = \frac{2a\rho_a U}{Ma} = 135 \quad \psi = \frac{9\rho_a C}{\rho_d a} = 8.38 \]

\[ K = \frac{R_U}{\psi} = 16 \quad \phi = R_U \times \psi = 1130 \]

<table>
<thead>
<tr>
<th>Experimental Data</th>
<th>Langmuir-Blodgett Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>K = 16</td>
<td>K = 10</td>
</tr>
<tr>
<td>( \phi ) = 1130</td>
<td>( \phi ) = 1100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Initial Displacement</th>
<th>Angle of Impingement</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y ) - in.</td>
<td>( \theta ) - deg.</td>
</tr>
<tr>
<td>.354</td>
<td>20</td>
</tr>
<tr>
<td>.730</td>
<td>48</td>
</tr>
<tr>
<td>.787</td>
<td>64</td>
</tr>
<tr>
<td>.850</td>
<td>70</td>
</tr>
<tr>
<td>.925</td>
<td>76 Tangent</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Initial Displacement</th>
<th>Angle of Impingement</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y ) - in.</td>
<td>( \theta ) - deg.</td>
</tr>
<tr>
<td>.10</td>
<td>7.6</td>
</tr>
<tr>
<td>.30</td>
<td>33.0</td>
</tr>
<tr>
<td>.50</td>
<td>37.7</td>
</tr>
<tr>
<td>.70</td>
<td>63.6</td>
</tr>
<tr>
<td>.728</td>
<td>76.1 Tangent</td>
</tr>
</tbody>
</table>
COMPARISON OF TRAJECTORY DATA

LANGMUIR–BLODGETT
Φ = 1100 K = 10

LANGMUIR–BLODGETT
Φ = 1130 K = 16

EXPERIMENTAL

ANLGE OF IMPINGEMENT, θ - DEG.

INITIAL DROPLET STREAM DISPLACEMENT, Y-IN.

FIG. 10
To verify the effect of evaporation on the droplet trajectory it has been suggested that a salt be added to the water to reduce its vapor pressure. The reduction in vapor pressure is a function of the molecular weight of the salt, and lithium chloride is suggested as being one of the lighter salts. Time did not permit an investigation of the trajectories of an aqueous solution of lithium chloride, although a 90% saturated solution of lithium chloride and water was prepared and found to be too viscous to flow through the capillary tip. A somewhat less saturated solution might be made to flow through the capillary, but difficulty might be experienced in obtaining the desired droplet size.

The use of another fluid for the droplets has also been suggested and work is going ahead along this line. Using one of the Dowtherm fluids is currently being investigated. The important properties of the fluid selected are a low vapor pressure and a viscosity comparable with that of water. These two properties do not ordinarily go together.

SECTION VIII

DISCUSSION

The approach to the problem of experimentally determining the trajectory of a water droplet used in this investigation has definite drawbacks which have been brought out in previous sections. While the methods and the associated equipment will give acceptable trajectory data, there are other methods worthy of mention.

The first method to be considered is based on a photographic determination of the trajectory in a two dimensional flow. It was felt that a fluorescent dye could be added to the water, and then by lighting the test section with an ultra-violet lamp, the droplets would give off some light. It is true that a 50 μ droplet might give off so small an amount of light that it could not be traced visually, but a time exposure with a sensitive film should show the droplet path. Any unsteadiness in the droplet stream would tend to obliterate the true trajectory, and the method would not be applicable to three dimensional flows.

Another method is that of using solid particles to simulate the droplets and tracing their trajectories. Glass spheres are available commercially in very small sizes and a method similar to the droplet generator developed herein might be used to introduce them into the stream. The probable advantage of this method is that a true stream of individual
droplets could be obtained in this manner. Whether the effect of the glass spheres striking the hot wire would produce enough disturbance in the current to give an indication is problematical, but another method of locating the stream could likely be devised.

The last method to be mentioned involves the use of water as the flow medium and metal or glass spheres as the droplets. The technique would be similar to that described herein, though a different detection device would be required. Lower velocities would be used and the flow would be truly incompressible.

SECTION IX

CONCLUSIONS

The results of the investigation reported herein indicate that the methods and associated equipment used are satisfactory for the determination of the trajectory of a water droplet in either a two or three dimensional flow. Since the droplet stream has a definite thickness, the displacement measurements cannot be taken as precision measurements.

The extension of the method to a three dimensional flow would involve using a similar hot wire probe, mounted horizontally, to determine the horizontal displacement of the stream from some reference datum. The stream would have to be as fine as possible in both dimensions in this case.

The writer has two specific suggestions for further investigation, aimed at improving the techniques given in this report. The first is an investigation of the dynamics of droplet formation from a vibrating capillary in an airstream. In the spinning disc type of droplet generator the water leaves the disc as a filament and then breaks into droplets, and Dimmock\(^6\) shows a photograph of the water leaving the vibrating capillary as a filament and then breaking into droplets. It is believed that in an airstream with an appreciable velocity, this filament will not form and the water will leave the capillary tip as a droplet, and that further investigation might indicate a way to obtain a single stream of droplets.

The second suggestion is to investigate further the amplifier and Wheatstone bridge circuits used herein with the idea of improving the sensitivity of the hot wire to the droplet stream and decreasing the sensitivity to turbulence effects. Computations by Mr. Henry Hicks of the Combustion Laboratory, Engineering Research Institute, indicate that for
a Wheatstone bridge circuit of the type used herein there is an optimum relationship of the various resistances in the circuit which will produce the maximum unbalance in the bridge for a given change in resistance of one leg of the bridge. These computations indicate that for maximum sensitivity, the resistance of each leg of the bridge should be equal, and that the amplifier circuit resistance should be twice the resistance of one leg. Time did not permit an experimental verification of these computations.

The use of the hot wire anemometer equipment in compressible flows should present no additional difficulties other than the possible disturbance of the flow due to the presence of the hot wire probe. The present wind tunnel used in this investigation would likely be far too turbulent for velocities much greater than 100 fps.


3. Tribus, Myron, "Modern Icing Technology". Lecture Notes, Engineering Research Institute, University of Michigan, Jan. 1952.


5. Guibert, A., Janssen, E., and Robbins, Wm., "Determination of the Rate, the Area, and the Distribution of Impingement of Waterdrops on Various Airfoils from Trajectories Obtained on the Differnetial Analyzer". NACA RM 9A05, Feb. 1949. An addendum to this report has recently been issued.


