A MATHEMATICAL MODEL FOR OPTIMAL STRENGTH AND ALIGNMENT OF A MARINE SHAFTING SYSTEM

by

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ABSTRACT

The design of a marine shafting system is modelled mathematically in order to perform optimization studies with respect to shaft strength as well as longitudinal and vertical positioning of the bearings. The objective criteria used are minimization of the bearing reaction influence numbers and even distribution of the bearing loading. Design trade-offs can be thus established. The problem is posed in a nonlinear programming formulation and is solved using a standard generalized reduced gradient method (GRG2) but in a specialized solution strategy. Two examples from actual ship designs are presented.

INTRODUCTION

A proper design of the propulsion system plays an important role in the good operation of marine vehicles. This article examines a detailed design procedure of the shafting system, specifically suited for subsequent optimization studies.

Optimal strength and alignment is defined to mean the calculation of shaft diameters, number of bearings and longitudinal and vertical positioning of these bearings, in such a way as to ensure a proper distribution of bearing loads and acceptable shaft stresses and deflections [1,2].

Traditionally an alignment plan, i.e. the identification of bearing longitudinal and vertical positioning, is achieved by indirect (or iterative) methods [3]. This assumes a given bearing longitudinal positioning. A common rule of thumb states the following minimum bearing distances [2]:

$$\ell/d \geq 12\text{-}14 \text{ for } d > 400 \text{ mm}$$

where ℓ/d is the relative bearing distance over shaft diameter. These iterative methods consist of perturbing a base-line design by arbitrary amounts dictated by the user's experience, until an acceptable plan for the vertical bearing positioning, is achieved by trial and error [3]. The base-line is usually taken to be that of straight alignment in which all bearing centers are concentric [4].

Almost every method, which gives an alignment plan, is based on the Reaction Influence Numbers, or RIN's [5]. They are essentially solutions of a number of equalibrium problems equal to the number of bearings. In each problem, the shaft is perturbed vertically by a unit amount at the location of a bearing and the changes in reactions at all bearings are the RIN's so computed [5,3].

Today the alignment study is usually conducted according to the fair curve alignment theory, the calculations being performed by a suitable computer program.[2]. In most cases, the analysis is still limited to the evaluation of the static condition (i.e. non-rotating shafting) at any average draught. There is however, a number of different external parameters such as reduction gear (if there is one) forces, propeller thrust etc., that may alter this static alignment condition considerably [2]. It is essential to be aware of these parameters and to take them into consideration in the design analysis as carefully as possible.

The purpose of this paper is to describe a model for finding an optimal longitudinal and vertical positioning of the bearings. The whole analysis will be performed for rotating shaft (propeller forces and moments, reduction gear forces and eccentric propeller thrust can be taken into account), but with one major simplification: the bearings will be considered as point bearings, in order to avoid at this stage all the difficulties arising from the highly nonlinear hydrodynamics of the bearing oil film [6]. Furthermore, the analysis will be restricted to the vertical plane which is more heavily loaded compared with the horizontal one.

MODEL OUTLINE

The final goal of the modeling effort is the development of a problem statement in the mathematical programming formulation

$$\begin{aligned} &\text{minimize} \quad f(\underset{\sim}{x}) \\ &\text{subject to} \\ &\qquad \underset{\sim}{h}(\underset{\sim}{x}) = \underset{\sim}{0} \\ &\qquad \underset{\sim}{g}(\underset{\sim}{x}) \leq \underset{\sim}{0} \end{aligned} \quad (1)$$

where the objective function $f(\underset{\sim}{x})$ is a scalar depending on the vector of design variables $\underset{\sim}{x} = (x_1, x_2, \ldots x_n)^T$, and $\underset{\sim}{h} = (h_1, h_2, \ldots h_m)^T$, $\underset{\sim}{g} = (g_1, g_2, \ldots g_k)^T$ are vector functions of $\underset{\sim}{x}$ used to represent the equality and inequality constraints of the problem.

In the above formulation (1) we assume that if several criteria are desirable as objectives (i.e. f really being a vector), then the objective function f represents an appropriate scalar combination of these criteria. Furthermore, the expressions of the constraint functions may not be in an explicit algebraic form, but may be defined through a set of numerical procedures.

The classification of design criteria as objectives or constraints is a rather subjective one and depends on the particular design application of the model. Furthermore, in a general problem statement several quantities may be considered as possibly varying, but in a particular application some of these may be taken as fixed, based on the design specifications. These quantities, considered as constant in a given model application, we shall call <u>parameters</u> and we will distinguish them from the problem variables x, with respect to which the optimization must be performed. The parameter vs. variable classification is also a rather subjective one, but it is generally suggested naturally by the statement of the design problem.

System Configuration

A general arrangement of a marine shafting system configuration is shown in Fig. 1, the main parts identified in the figure. This arangement is not the only one in use, but it does represent a great range of installed marine shafting systems [1]. In the present paper we are concerned primarily with the design of the intermediate shaft, in terms of both size and positioning. This assumes tacitly that we are at a stage in the design process, where the propeller design and the gear reduction design (if present) have been determined to a large extent.

It should be noted that although it may be desirable to include the configuration in the decision-making of an optimization model, this is generally impossible because the required mathematical expression of changes in topology is extremely difficult (except in special cases, such as truss structures). Therefore, the entire model is based only on the above configuration. However, a similar modeling procedure can be used for another configuration, if desired. The separately optimized design configurations can be then directly compared.

System Variables and Parameters

The main variables considered in the model are the following:

1. The diameters of propeller and intermediate shaft.

2. The longitudinal positions of the intermediate shaft bearings.

3. The vertically displaced positions of the bearings.

4. The longitudinal position of the reduction gear with respect to the main engine.

The design parameters, considered known and constant during the optimization process, are as follows:

1. The main engine type is known, e.g. turbine or diesel - with known number of strokes and cylinders - and the engine longitudinal position is known.

2. The propeller is known, e.g. of known weight, diameter size, mass centroid location. The forces and moments transmitted are assumed steady at the shaft RPM under consideration.

3. The power and thrust transmitted between the propeller and the remaining shafting system is given and the shaft RPM is considered fixed.

4. The material of the shaft has been selected.

5. The number of bearings to be used has been selected.

6. The use or not of a reduction gear component has been decided. If one is used, then its geometric and operational characteristics are known, e.g. weight, imposed forces, low speed shaft diameter, bearing position, allowable loading for bearings and shaft.

7. The position of the aft peak bulkhead is fixed.

Note that once the optimization study is completed for one set of parameter values, it may be desirable to reoptimize the system for another set of values, or for small changes in the parameters, to perform a sensitivity analysis. Results from sensitivity and parametric studies may be used by the designer to determine whether further study or remodeling may be necessary.

Design Requirements

The design requirements imposed on the shafting system are those selected by the designer as necessary for satisfactory and/or desirable performance. Some of them may be incorporated in the objective function in (1) and the remaining will be included as constraints.

In broad terms, the main design requirement is to achieve minimum cost, which can be broken down to parts cost, installation cost and operational/maintenance cost [1,5]. In a more technical sense this translates to the following more specific requirements:

1. Minimum diameter size for each shaft.
2. In-line positioning of stern tube bearings, i.e. coincidence of their centerlines.
3. The net downward loading for all bearings should not be excessive.
4. The loading for all bearings should be even, i.e. equal to the same percentage of their maximum allowable load.
5. The combined fatigue stress at critical shaft cross-sections should not exceed the fatigue strength of the material.
6. The shaft must be sufficiently flexible so that errors

due to uncertainty of loading and arbitrary changes in alignment (within limits) can be accommodated. This is important in order to acount for hull deflections at heavy seas, bearing weardown, or thermal variations.

The above criteria and some additional secondary ones will be discussed in detail in the next sections describing the analytical formulation of the model.

<u>Model Subdivision</u>

The general design problem can be decomposed to two subproblems to be studied separately, for reasons of convenience that will become apparent later.

The two subproblems are:

1. Specification of propeller and intermediate shaft diameters; the diameter of a reduction gear -if present- is a parameter. Specification of the longitudinal position of each bearing of the intermediate shaft, and longitudinal position of the reduction gear; longitudinal positions of aft and forward stern tube bearings and location of reduction gear are parameters.
2. Specification of the vertical displacements of all bearings.

Clearly the above subproblems are not independent but they are linked through common variables. This interaction will be studied as part of the optimization procedure. In the next two sections we will develop detailed models for these two subproblems.

STRENGTH AND LONGITUDINAL BEARING POSITIONING MODEL

The model described in this section has two parts. The first deals with the development of a strength inequality constraint, which imposes an upper bound on some equivalent maximum stress. The second deals with appropriate expressions for the criteria of the Longitudinal Bearing Positioning (LBP) problem.

Combined Stresses

The maximum normal stress on a circular cross-section due to bending moment and axial force is [7]

$$\sigma = M_b D/2I_y + F/A \qquad (2)$$

where M_b is the bending moment, D is shaft diameter, I_y is moment of intertia, F is the axial force normal to the section and A is the cross-sectional area.

The maximum shear stress due to torsion is similarly [7] given by

$$\tau = M_t D/2I_p \qquad (3)$$

where M_t is the torsional moment and I_p is the polar moment of inertia.

The equivalent combined stress according to the maximum distortion energy criterion (von Mises) is

$$\sigma_{eq} = (\sigma^2 + 3\tau^2)^{1/2} \qquad (4)$$

Note that the velocity field imposed on the rotating propeller is changing during one full rotation and it is repeated periodically at a steady rate for steady RPM. It is then suggested [8] that a cosine-like variation in the propeller thrust, F, and torsional moment, M_t, may be employed, i.e.

$$M_t = M_{tm}(1 + \lambda \cos \nu\theta) \qquad (5)$$

$$F = F_m(1 + \mu \cos \nu\theta) \qquad (6)$$

where M_{tm}, F_m are mean values, ν is the number of propeller blades and λ , μ are the alternating components (% of mean values) of torsional moment and thrust respectively. Lacking a better estimate for λ and μ , the values suggested [8] in Table 1 may be used. Furthermore, the bending stress at any point on the shaft surface is given by

$$\sigma_\theta = (M_b D/2I_y)\cos\theta \qquad (7)$$

Now we can combine eq. (2) through (7) to obtain the equivalent stress [1]:

$$\sigma_{eq} = \left\{ \left[\frac{M_b D \cos\theta}{2I_y} + \frac{F(1 + \mu \cos \nu\theta)}{A} \right]^2 + 3\left[\frac{DM_t(1+\lambda \cos\nu\theta)}{2I_p} \right]^2 \right\}^{1/2} \qquad (8)$$

The maximum and minimum values are given for $\theta = 0$ and $\theta = \pi$ respectively, i.e.

$$\sigma_{max} = \left\{ \left[DM_b/2I_y + F(1 + \mu)/A\right]^2 + 3\left[DM_t(1 + \lambda)/2I_p\right]^2 \right\}^{1/2} \qquad (9)$$

$$\sigma_{min} = \left\{ \left[DM_b/2I_y - F(1 + (-1)^\nu \mu/A\right]^2 + 3\left[DM_t(1 + (-1)^\nu \lambda/2I_p\right]^2 \right\}^{1/2} \qquad (10)$$

This completes the development of the stress model.

Dynamic Strength

The equivalent stress calculated in (8) is alternating about a mean value, as in Fig. 2. Therefore fatigue failure must be considered and a criterion for dynamic strength must be used.

The Gerber criterion may be used as more appropriate [7], giving the inequality

$$\sigma_{ALT}/S_e + (\sigma_{ST}/S_{ut})^2 \leq 1 \qquad (11)$$

where the mean and alternating stress components are respectively

$$\sigma_{ST} = (\sigma_{max} + \sigma_{min})/2 \qquad (12)$$

$$\sigma_{ALT} = (\sigma_{max} - \sigma_{min})/2 \tag{13}$$

and S_e, S_{yt}, S_{ut} are the endurance limit, yield strength and ultimate strength, respectively, of the shaft material.

In addition to condition (11), the conservative requirement that the yield point is never exceeded should be imposed, i.e.

$$\sigma_{ALT} + \sigma_{ST} \leq S_{yt} \tag{14}$$

Shaft Strength Constraints

The two inequalites (11) and (14) represent the strength constraints on the shaft. Using (12) and (13) they are rewritten as

$$(SF)\ [(\sigma_{max} - \sigma_{min})/2S_e] + [(\sigma_{max} + \sigma_{min})/2S_{ut}]^2 \leq 1 \tag{15}$$

$$(SF)\ (\sigma_{max}/S_{yt}) \leq 1 \tag{16}$$

The safety factor SF was included to account for expected uncertainty in the precise evaluation of the loads magnitude. The values of M_b, M_t change substantially with the loading and operating conditions of the ship. Experimental data supplied by the U.S. Navy [9] can be used for an estimate of the safety factor, as in Table 2.

The equations (9) and (10) defining σ_{max} and σ_{min} will be included as equality constraints in model (1). Note that (15) and (16) will in general impose lower bounds on the shaft diameters. Additional lower bounds are given by Classification Rules, such as by Lloyd [10] and the American Bureau of Shipping [11]. These are generally expressed in the form

$$d_i \geq d_{imin};\ i = 1,2 \tag{16'}$$

where d_1 and d_2 are the intermediate and propeller shaft diameters.

Parameters for Longitudinal Bearing Positioning (LBP) Problem

The longitudinal positioning of the bearings is an important aspect of the shafting system design because it influences the bending moment distribution and therefore the minimum diameter requirements.

Considering the LBP as a subproblem examined separately, the following quantities must be taken as given parameters:

1. The external loading of the shafting system. This includes the propeller weight, forces and moments at the operating speed, the location of the center of gravity, reduction gear weight and transmitted force components.
2. The longitudinal positions of the two stern tube bearings are fixed, since the aft peak bulkhead position is determined from other vessel design considerations. However, the distance between these two bearings must be the greatest possible in order to minimize the maximum bending moment occuring between them [1,7].
3. The reduction gear is a compact component with a design essentially fixed by the manufacturer. Therefore, the relative position of the reduction gear bearings and the shaft diameter are fixed.

Objective and Constraints for the LBP Problem

In the model outline we described several criteria which must be satisfied by a good design with proper bearing positioning. These criteria are now refined and expressed analytically.

For the purpose of defining a scalar optimization problem, we must select one criterion as the most important one, to serve as the objective function of the problem. In the present model we select as main design objective a desirable lack of sensitivity of the shafting system with respect to changes in alignment (we mentioned earlier that, for example, hull deflections and bearing weardown will contribute to such alignment changes). This criterion then implies small bearing reaction influence numbers [2,5] and the objective function may be expressed as follows:

$$\text{minimize } f_1 = \sum_{i=1}^{N_L} \sum_{j=1}^{N_L} |SR_{ij}| \tag{17}$$

This expression is explained by reference to the simplified configuration of the shafting system shown in Fig. 3. In this figure the total number of bearings is designated by N_L. Bearings No. 1 and No. 2 are the reduction gear ones, while No. N_L-1 and No. N_L are the stern tube bearings. The diameters for the intermediate and propeller shaft respectively are designated by d_1 and d_2. The reduction gear shaft diameter d_G is a design parameter. The longitudinal positions of the reduction gear and intermediate shaft bearings are given by the variables x_j where $j=2,\ldots,N_L-2$ for a system employing a reduction gear and $j=1,\ldots, N_L-2$ without reduction gear. Recall that the relative position of reduction gear bearings is a parameter.

With the above notation, the reaction influence numbers SR_{ij} of the shafting system in (17) are expressed as

$$SR_{pq} = f(d_k, x_j) \; ; k=1, 2 \tag{18}$$

where the function f represents symbolically the results of a very accurate finite element analysis computation. The finite element program will be discussed separately in a sebsequent section.

We will now continue with description of the constraints.

For a given bearing number, positioning must be selected so that the maximum bending moments for each shaft portion remain small. This is achieved in part by requiring small shaft diameters, and the previously discussed strength constraints aimed at that. However, an additional requirement may be imposed by asking to have the ratio of the aft to forward stern tube bearing reaction between the values two and three(or some other bounds). Besides all bearings' reactions must be positive (upward), which is in fact a general requirement for the bearing loading. Thus, the inequality constraints

$$2R_{N_L-1} < R_{N_L} < 3R_{N_L-1} \tag{19}$$

$$R_j > 0 \quad ; j = 1, \ldots, N_L \tag{20}$$

must be imposed.

Note that when all bearings are in line, the aft stern tube bearing reaction is very large and positive, while the forward stern tube bearing reaction is usually negative. This implies a large bending moment for the propeller shaft, a situation to be avoided.

The above mentioned large bending moment is proportional to the aft stern tube bearing reaction, so a constraint that ensures a small such reaction should be included. A simple way to implement this, is to impose an upper bound on the reaction value. A suggest value for this upper bound is twice the propeller weight w_p, i.e.

$$R_{N_L} < 2W_p \tag{21}$$

Finally, from geometric considerations the longitudinal positioning variables must satisfy the sequencing

$$L_G \leq x_j \leq x_{j+1} \leq X_{N_L-1} \; ; \; j=2, \ldots N_L -2 \tag{22}$$

in the pressence of reduction gear, and

$$0 \leq x_j \leq x_{j+1} \leq X_{N_L-1} \; ; \; j=1, \ldots, N_L -2 \tag{23}$$

in the absence of reduction gear, L_G being the distance between the reduction gear bearings.

VERTICAL BEARING POSITIONING MODEL

In the Vertical Bearing Positioning (VBP) Model the variables are the vertical positions of the bearings δ_j, j=1,..., N_L-2.

The main concern in the VBP problem is proper allocation of bearing loadings q_j, where

$$q_j = R_j/L_jD_j \; ; \; j=1,\ldots, N_L \qquad (24)$$

Here L_j is the bearing length and D_j is the diameter taken as the sum of the shaft diameter and the diametral clearance of the oil film.

We define p_j to be the loading q_j of the jth bearing as a percentage of its maximum allowable loading q_{jmax}, i.e.

$$p_j \stackrel{\Delta}{=} q_j/q_{jmax} \qquad (25)$$

and $\bar{p}$ to be the arithmetic average of all the p_j's, i.e.

$$\bar{p} \stackrel{\Delta}{=} N_L^{-1} \sum_{j=1}^{N_L} p_j \qquad (26)$$

Then the requirement for the bearing loading to be evenly distributed suggests the following objective function

$$\text{minimize} \quad f_2 = \sum_{j=1}^{N_L} |p_j - \bar{p}| \qquad (27)$$

The attending requirement of having each loading within acceptable margins, is expressed by imposing upper and lower bounds on the loadings, i.e.

$$q_{jmin} < q_j < q_{jmax} \; ; \; j=1,\ldots,N_L \qquad (28)$$

where the bounds generally depend on the oil film characteristics and the L/D ratio of each bearing [6]. It is expected that actual values for q_{jmin} and q_{jmax} will be supplied by the bearing manufacturer. In the absence of better data, the values R_{jmin} =

500-1,000Kp are suggested [12] depending on bearing diameter size. For q_{jmax}, the values in Table 3 are suggested [8].

There are several explicit constraints arising from a desired shape of the shaft's elastic curve. First, note that the slopes $_{j}$ of the elastic curve at bearing locations must be restricted not to exceed a certain limit ϕ_{jmax}, which depends on the bearing type, i.e.

$$\phi_{jmax} \leq \phi_j \leq \phi_{jmax} \; ; \; j=1,\ldots,N_L \tag{29}$$

The value ϕ_{jmax} may be supplied by the manufacturer. Otherwise, the following expression can be used (see Fig. 4):

$$\phi_{max} = \arccos \frac{DD_i \pm (D^2D_i^2 + L^2D_i^2 + L^4)^{1/2}}{D_i^2 + L^2} \tag{30}$$

the symbols used being explained in Figure 4.

Next, observe that a good elastic curve must be smooth [4,5,1]. Expressing this analytically is a somewhat subjective operation. Examining two situations as in Fig. 5, it is evident that case (b) exhibits a certain desirable smoothness. We may then impose the following specific constraints: The vertical displacement of every bearing, but the two stern tube ones, must be positive(downward); the slope of the intermediate shaft elastic curve must decrease gradually up to the flange connecting with the main engine, where theoretically it must be zero, in order not to transmit bending moments from the shafting system to the engine. These constraints are expressed by the inequalities

$$0 \leq \delta_j \leq \delta_{j-1} \; ; \; j=2,\ldots,N_L - 1 \tag{31}$$

$$M_A \leq m \, M_{max} \tag{32}$$

Inequality (32) requires that the moment at point A (Fig. 3), for a specified small slope at that point, remains below a small percentage of the maximum bending moment on the stafting system, i.e. m is taken as a small number (a good value is 0.05) instead of zero.

Two additional geometric requirements relate to the elastic curve: The vertical displacements of the reduction gear bearings may be equal since, the reduction gear is considered compact, i.e.,

$$\delta_1 = \delta_2 \tag{33}$$

The equality (33) has been considered in our analysis; its absense though does not alter the results considerably. Furthermore, the line connecting the stern tube bearings serve as the datum for measuring vertical displasements. Therefore,

$$\delta_{N_L} = \delta_{N_L-1} = 0 \tag{34}$$

A final constraint must be included, if a reduction gear is employed in the system, namely that the two bearing reactions must be approximately equal. This will ensure that under operating conditions good contact is maintained between mating gear teeth [2,12]. We express this constraint by

$$|R_{V1} - R_{V2}| \leq \Delta R_V \tag{35}$$

where R_{V1}, R_{V2} are the vertical reactions and ΔR_V the allowable difference. A value for ΔR_V may be supplied by the gear manufacturer. In the absence of such data, it is suggested [12] that ΔR_V = 2300 - 6800kp depending upon the reduction gear size.

DESCRIPTION OF A FINITE ELEMENT PROGRAM FOR THE SHAFT ALIGNMENT CALCULATIONS

It was mentioned in the previous sections that a numerical computation is used for the calculation of the shaft elastic curve, bearing reactions and reaction influence numbers under a current set of constraints imposed by the optimization procedure. The program used for that purpose must be very accurate in order for the reduced gradient optimization method subsequently used [14] to work properly. For example, the partial derivatives of the constraints and objective function with respect to the variables, must be as accurate as possible.

For the above reason, a finite element program has been developed for the shafting system analysis. The problem has been formulated as a two-dimensional vertical plane one. The analysis is structured to accept typical shaft alignment program input of shaft lengths and diameters, material densities, and external loads. All the bearings are taken as point supports with or without lateral and rotational flexibility. Buoyancy effects of water or bearing lubrication oil can be included. External loads can be introduced from components such as gear weight, gear teeth loading, couplings, oil distribution boxes, thrust collars, couplings etc. In addition to propeller weight, the steady eccentric thrust, lateral forces and moments can be included.

The shafting system has been modeled by beam elements, including bending, shear, axial loading and all their coupling effects. For the beam differential element shown in Fig. 6, the following governing differential equation is used:

$$EI\frac{d^4v}{dx^4} + F\frac{d^2v}{dx^2} = P_y \qquad (36)$$

where E: Young's modulus

G: shear modulus

I: area moment of inertia of beam element

A: uniform shaft cross-sectional area

F: axial force on element; positive in compression

p_y: uniform, distributed load

v: displacement in y direction

The finite element analysis for a shafting system composed of elements of this type has been derived using Galerkin's method, the most widely accepted of the weighted residuals methods. This variational approach is essentially a special case of the Ritz method.

NUMERICAL RESULTS

The two subproblems, presented in the previous sections were not constructed only on the basis of engineering analysis, but also for computational reasons. From the design viewpoint, the two objective functions are competing so that a true vector optimization problem is involved. The multiobjective methods are not particularly useful in the present context, because the intuitive understanding of the competing design objectives already exists. Thus a simpler way can be used, namely a sequential procedure where each subproblem is solved separately while feasibility of the other is maintained. From the computational viewpoint, this approach is dictated by the fact that the solution is so sensitive to vertical positioning perturbations that no good initial guess provides appropriate feasible convergence for the combined problem (with composite/weighted objective function). This was verified by computational experiments using the GRG2 code version [14] of the generalized reduced gradient method.

The optimization procedure then starts by generating a feasible initial point. This is necessary, not only because of the computational complications mentioned above, but also because there may be <u>no</u> feasible design for the given set of parameters. The initial point is generated by using a very simplified version of the second subproblem (vertical bearing positioning, abbr. VBP) in order to circumvent the difficulty of large sensitivities.

After a feasible initial point is found, the main optimization procedure starts by solving the first subproblem (strength and longitudinal bearing positioning, abbr. SLBP). The inputs assume equal longitudinal intermediate shaft bearing positioning, reduction gear position at the far right end of the system (near the main engine) and diameters at the bounds given by the classification rules. The initial vertical positioning is held constant during the SLBP optimization. The results obtained are used as the initial point for the VBP optimization. Thus one outer iteration is completed. The procedure is repeated until convergence is achieved within a practical tolerance.

The results for two examples of actual ship designs are described now. First we present the problem specifications and then the results of the optimization study. Since both examples are from the literature, direct comparison with the previously proposed designs is possible.

Example 1: Bulk cargo ship with the main bearing on the forward end of the bullgear ([12], p. 472), Fig. 7.

The variables are as follows:

d_1, d_2: intermediate and propeller shaft diameters respectively.

x_2, $x_3, \ldots, x_6$: longitudinal position of the reduction gear and the intermediate shaft bearings respectively.

δ_1, $\delta_2, \ldots,$ δ_6: vertical positions of the reduction gear and intermediate shaft bearings respectively (positive downwards).

The parameters and the values used for the bearings are given below:

	BRLN	OILG	QMAX	QMIN
#1	103.0	0.3	0.0896	0.006
#2	62.7	0.3	0.039825	0.001
#3	36.0	0.25	0.0427	0.0018
#4	36.0	0.25	0.0427	0.0018
#5	36.0	0.25	0.0427	0.0018
#6	36.0	0.25	0.0427	0.0018
#7	18.75	0.28	0.097	0.0034
#8	18.75	0.28	0.097	0.0034

where

BRLN: bearing length in inches

OILG: bearing diametral oil clearance in inches

QMAX: bearing maximum allowable loading in $klbf/in^2$

QMIN: bearing minimum allowable loading in $klbf/in^2$

The remaining parameters are:

W_p = 49.381 klbf

F = 244.0 klbf

d_G = reduction gear diameter = 24 in.

ΔR_V = allowable reduction gear bearing reactions difference = 5.0 klbf

λ = alternating component of torsional moment = 3.5% of mean torsional moment.

μ = alternating component of thrust = 6% of mean thrust

S_e = endurance limit = 27.0 $klbf/in^2$

S_{yt} = yield strength = 30.0 $klbf/in^2$

S_{ut} = ultimate strength = 60.0 $klbf/in^2$

E = Young Modulus = 30000 $klbf/in^2$

G = Shear Modulus = 12000 $klbf/in^2$

SF_{prop} = Safety factor for the prop. shaft = 2.0

SF_{int} = Safety factor for the interim. shaft = 1.75

DENS = Shaft material density = 0.000283 $klbf/in^3$

The results of the optimization study are shown in Tables 4 and 5.

Example 2 SD14 bulk cargo ship, constructed at the Hellenic Shipyards, Greece [1], Fig. 8.

The variables and bearing description are the same as in Example 1. The parameter values are given below:

W_p = 19.8414 klbf

F = 170.0 klbf

d_G = 15.9212 in

ΔR_V = 5.0 klbf

λ = 12%

μ = 6%

S_e = 27.0 $klbf/in^2$

S_{yt} = 30.0 $klbf/in^2$

S_{ut} = 60.0 $klbf/in^2$

E = 30000 $klbf/in^2$

G = 12000 $klbf/in^2$

SF_{prop} = 2.0

SF_{int} = 1.75

DENS = 0.000283 $klbf/in^3$

The bearing parameters are

	BRLN	OILG	QMAX	QMIN
#1	39.37	0.1	0.085	0.00285
#2	19.685	0.1	0.036	0.0059
#3	14.1732	0.1	0.046	0.01
#4	14.1732	0.1	0.046	0.01
#5	14.1732	0.1	0.046	0.01
#6	14.1732	0.1	0.046	0.01

#7	15.748	0.1	0.044	0.007
#8	15.748	0.1	0.044	0.007

The results of the optimization study are shown in Tables 6 and 7.

DISCUSSION

A simple but extremely useful remark is that the optimum SLBP must give almost equal bearing spacing. Furthermore, the bearings must be gathered somewhat aft in order to provide good support to the propeller cantilever. It can be seen from the presented examples that these rules of thumb are satisfied.

The objective function (17) of the SLBP problem decreases always from the one iteration to the next, since it is independent of the variables of the VBP problem. Hence theoretically it must converge to a lower value than the one initially given. The same does not happen with the objective function (27) of the VBP problem. Since it does depend on the SLBP variables, the final design might have a greater objective function from the initial one. This is actually expected, because we try to minimize simultaneously two competing objectives resulting to the above observed conflict. Of course it is entirely possible to introduce subjective preference, e.g. weighting factors, in order to bias the solution towards either criterion.

When the classification rules' constraints (16') are not present, and the number of bearings is considerably greater than two (number of stern tube bearings), the diameter of the intermediate shaft d_1 reduces considerably while the diameter of the propeller shaft d_2 increases. When d_1 decreases, the intermediate shaft becomes more flexible and so the influence numbers corresponding to its bearings reduce. The reactions of the stern tube bearings, because of the reaction influence numbers SR_{ij}, are (Fig.3):

$$R_{N_L} = \hat{R}_{N_L} + \sum_{j=1}^{N_L-2} SR_{N_L,j}\,\delta_j$$

$$R_{N_L-1} = \hat{R}_{N_L-1} + \sum_{j=1}^{N_L-2} SR_{N_L-1,j}\,\delta_j$$

where $\hat{R}_i$ is the reaction of ith bearing when all bearings are vertically in straight line. The stern tube reactions above satisfy almost always as active one of the constraints

$$2R_{N_L-1} \leq R_{N_L} \leq 3\, R_{N_L-1} \qquad (19)$$

Since the δ_i's remain constant during the optimization procedure of subproblem 1 and the SR_{ij}'s are reduced, then R_N and R_{N_L-1} no longer satisfy (19). This is the reason why d_2 has a tendency to increase; doing so will increase R_N and R_{N_L-1} and the propeller load will not produce a great difference between them, since the propeller shaft has become much stiffer. This necessitates the presence of an upper limit for d_2 when (16') is absent, since a great difference between d_1 and d_2 is not only impractical but also undesirable because of stress concentration considerations. Constraint (21) provides such an upper limit; of course another one representing better a particular common practice might be used.

In real operating conditions, a considerable aft stern tube bearing wear-out is about 0.75 - 1 mm [12]. This hypothetical lowering of the stern tube bearing will not be incorporated in the optimization procedure since it can be subtracted afterwards from the assumed zero stern tube vertical displacement, thus defining a new datum line for the vertical displacements of all bearings.

About 2.5% of the shaft power is converted to heat in the reduction gear as well [12]; the result of this heating, at the normal operating temperature, is the rise of the reduction gear bearings. In common practice cases, it is about 0.6mm [12]. This need not be taken into account originally but treated in a similar way as the stern tube bearing wear-out of the previous paragraph.

In all the optimization analysis presented here, nothing has been mentioned about the vibrational (torsional, longitudinal and lateral) characteristics of the shafting system. They were not incorporated because they are difficult, tedious and rather expesive to compute in iterative procedures required by optimization techniques. This happens because they can be calculated accurately only through other separate numerical computations. For this reason, every optimal design found by the method of this paper should be checked afterwards for vibration performance in order to be accepted by the designer. If this current "optimum" design is not acceptable according to vibration criteria, a redesign of the shafting system has to be performed taking into account possible limitations imposed by vibrational characteristics.

ACKNOWLEDGEMENT

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NOMENCLATURE

σ	:	normal stress on a circular cross section
M_b	:	bending moment
D	:	shaft diameter
I_y	:	moment of inertia
F	:	axial force normal to the cross section; positive in compression
τ	:	maximum shear stress due to torsion
M_t	:	torsional moment
I_p	:	polar moment of inertia
σ_{eq}	:	equivalent combined stress
$M_{t\mu}, F_{\mu}$	:	mean values of the torsional mement and axial force respectively
λ, μ	:	alternating components (% of mean values) of torsional moment and thrust respectively.
σ_{max}, σ_{min}	:	maximum and minimum normal stress on a circular cross section
σ_{ALT}	:	alternating stress component
σ_{st}	:	mean stress component
S_e	:	endurance limit of the material
S_{yt}	:	yield strength of the material
S_{ut}	:	ultimate strength of the material
E	:	Young's modulus of the material
G	:	Shear modulus of the material
SF	:	safety factor
d_1	:	intermediate shaft diameter
d_2	:	propeller shaft diameter

d_G	:	reduction gear shaft diameter
N_L	:	number of bearings
SR_{ij}	:	reaction influence number of the ith bearing with respect to the jth bearing.
x_j	:	longitudinal position of the jth bearing
R_j	:	reaction of the jth bearing
W_p	:	propeller weight
L	:	total length of the shafting system
q_j	:	loading of jth bearing
q_{jmax}, q_{jmin}	:	maximum and minimum loading of jth bearing
ϕ_j	:	slope of the elastic curve at the jth bearing location
δ_j	:	vertical displacement of the jth bearing; positive downwards
R_{V1}, R_{V2}	:	vertical reactions of the reduction gear bearings
R_V	:	allowable vertical reaction difference for the reduction gear bearings
p_y	:	uniform, distributed load in the y direction
v	:	displacement in the y direction
θ	:	total rotation angle of the elastic curve
V_y	:	shear force in the y direction
M_z	:	bending moment around the z axis

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Table 1: Values for Alternating Components of Torsional Moment and Thrust [4]

	No. of prop. blades	Supports	λ%	μ%
Multi-screw ships	3	struts	2-5	7- 3
	4	struts	2-5	3-9
	5	struts	2-4	3-9
	3	bossings	4-8	6-12
	4	bossings	4-6	5-10
	5	bossings	4-5	4-8
Single-Screw Ships	3	-	7-12	8-12
	4	-	10-15	3-8
	5	-	6-10	3-8

Table 2 : Safety Factor Values for Strength Calculations [9]

Shaft	Surface Ships	Ice breaker Ships	Submarines	
			Single-screw	multi-screw
Propeller Shaft	2.0	3.5	2.25	2.0
Interme-diate Shaft	1.75	2.25	2.0	1.75

Table 3 : Upper Bound Values for Bearing Loading [8]

	Aft Stern Tube bearing	Forward stern tube bearing	Intermediate shaft bearing with L/D=1
q_{max}	6.3 kp/cm^2	2.8 kp/cm^2	3 kp/cm^2 for lubric ring 5 kp/cm^2 for lubric disc

Table 4 : Optimal Design for Example 1

		LBP subproblem							
	Variables	x_2*	x_3	x_4	x_5	x_6	d_1*	d_2	OBJ
with classification rules limits for d_1 and d_2	Initial	102.125	241.325	380.725	520.125	659.525	21.0	23.0	61.78
	1st Iter.	65.081	223.619	380.291	520.823	659.595	20.9907	22.9907	54.58
	2nd. Iter.	65.081	223.619	380.291	520.829	659.601	20.99	22.99	54.57
without classification rules limits for d_1 and d_2.	Initial	102.125	241.325	380.725	520.125	659.525	21.0	23.0	61.78
	1st Iter.	65.085	202.001	358.513	482.229	607.625	17.4963	30.0892	39.62
	2nd Iter.	65.085	213.515	384.359	501.913	615.691	16.5842	30.3529	34.65

		VBP subproblem						
	Variables	δ_1*	δ_2	δ_3	δ_4	δ_5	δ_6	OBJ
with classification rules limits for d_1 and d_2.	Initial	0.0	0.0	0.0	0.0	0.0	0.0	335.18
	1st Iter.	0.752689	0.752782	0.723077	0.613102	0.424235	0.180889	181.1
	2nd. Iter.	0.752689	0.752769	0.723067	0.613099	0.424235	0.18089	181.08
without classification rules limits for d_1 and d_2.	Initial	0.0	0.0	0.0	0.0	0.0	0.0	335.18
	1st Iter.	0.939355	0.939482	0.898725	0.72625	0.473997	0.191765	179.11
	2nd. Iter.	1.02117	1.02132	0.971752	0.760516	0.486957	0.19458	184.3

* x_j : longitudinal position of jth bearing from main engine connecting flange in inches

d_1 : intermediate shaft diameter in inches

d_2 : propeller shaft diameter in inches

δ_j : vertical displacement of jth bearing from the line defined by the two stern tube bearings, in inches, (positive downwards)

Table 5(a) : Bearing Reactions for Example 1

Bearing	Reactions in klbf*		
	Initial	Final 1 **	Final 2
#1	35.326	35.393	37.488
#2	40.202	40.593	32.390
#3	12.419	13.817	11.314
#4	14.814	10.825	0.793
#5	10.39	1.283	1.008
#6	28.816	1.621	1.425
#7	-57.812	28.326	33.500
#8	130.178	82.133	98.522

* Shear deformation and thrust effects are included.

** Final results (1) with and (2) without the Classification Rules limits for the diameters.

Table 5(b) : Influence Numbers for Example 1 . Bearings list in Sequence #1 to #8.

	Reaction influence numbers in klbf per 0.001 in bearing movement down*							
Initial	-1.238528			SYMMETRIC				
	1.965498	-3.286883						
	-0.913568	1.828058	-1.694124					
	0.234611	-0.637042	1.147412	-1.481458				
	-0.060439	0.164112	-0.462967	1.095255	-1.478287			
	0.015798	-0.042898	0.121016	-0.456123	1.120017	-1.60506		
	-0.004268	0.011589	-0.032694	0.123227	-0.478114	1.282184	-1.593388	
	0.000896	-0.002434	0.006867	-0.025883	0.100423	-0.434935	0.691463	-0.336398
Final 1**	-1.125508							
	1.707791	-2.706118						
	-0.73782	1.382106	-1.207907	SYMMETRIC				
	0.202239	-0.499013	0.869317	-1.236458				
	-0.058998	0.145575	-0.386186	1.005283	-1.44397			
	0.015614	-0.038527	0.102205	-0.433467	1.114859	-1.609065		
	-0.004199	0.010361	-0.027485	0.116568	-0.47661	1.283277	-1.592697	
	0.000881	-0.002175	0.005769	-0.024469	0.100047	-0.434896	0.690785	-0.335943
Final 2	-0.537756							
	0.831695	-1.337701						
	-0.360322	0.667044	-0.517434	SYMMETRIC				
	0.094974	-0.230397	0.359766	-0.59593				
	-0.036201	0.08782	-0.188728	0.582326	-0.932215			
	0.009436	-0.022891	0.049194	-0.261307	0.715216	-1.015665		
	-0.003192	0.007743	-0.016641	0.088392	-0.398909	1.043119	-1.570779	
	0.001366	-0.003313	0.00712	-0.037822	0.17069	-0.517102	0.850266	-0.471205

* Shear deformation and thrust effects are included.

** Final results (1) with and (2) without the Classification Rules limits for the diameters.

Table 6: Optimal Design for Example 2

	LBP subproblem								
	Variables	x_2*	x_3	x_4	x_5	x_6	d_1*	d_2	OBJ
with classification rules limits for d_1 and d_2	Initial	52.322	214.37	376.418	538.466	700.512	13.846	17.11	12.48
	1st Iter.	44.34	210.768	377.704	538.136	700.228	13.838	17.1016	12.13
	2nd Iter.	44.34	210.766	377.71	538.16	700.244	13.836	17.1	12.12
without classification rules limits for d_1 and d_2.	Initial	52.322	214.37	376.418	538.466	700.512	13.846	17.11	12.48
	1st Iter.	44.384	211.492	377.568	532.224	688.808	13.44	20.681	11.47
	2nd Iter.	44.384	228.774	412.102	553.784	698.072	13.176	20.066	10.52

	VBP subproblem							
	Variables	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	OBJ
with classification rules limits for d_1 and d_2.	Initial	0.0	0.0	0.0	0.0	0.0	0.0	429.8
	1st Iter.	1.37564	1.37587	1.32583	1.165	0.83488	0.34081	201.7
	2nd Iter.	1.37557	1.37579	1.32577	1.165	0.83488	0.34081	201.6
without classification rules limits for d_1 and d_2	Initial	0.0	0.0	0.0	0.0	0.0	0.0	429.8
	1st Iter.	1.55971	1.55991	1.49258	1.26704	0.87512	0.3486	191.7
	2nd Iter	1.7384	1.7386	1.64837	1.3431	0.90329	0.35417	228.7

*x_j : longitudinal position of j-th bearing from main engine connecting flange in inches.

d_1 : intermediate shaft diameter in inches

d_2 : propeller shaft diameter in inches

δ_j : vertical displacement of j th bearing from the line defined by the two stern tube bearings, in inches, (positive downwards).

Table 7(a): Bearing Reactions for Example 2.

	Reactions in klbf*		
Bearing	Initial	Final 1**	Final 2
#1	7.222	10.798	11.146
#2	14.872	11.035	10.981
#3	6.790	8.274	7.908
#4	7.102	6.155	4.302
#5	6.227	2.468	1.740
#6	9.481	2.114	2.146
#7	-18.556	11.781	12.857
#8	52.453	32.902	36.340

* Shear deformation and thrust effects are included.

** Final results (1) with and (2) without the classification rules limits for the diameters.

Table 7(b) : Influence Numbers for Example 2. Bearings Listed in Sequence #1 to #8.

	Reaction influence numbers in klbf per 0.001 in bearing movement down*.							
Initial	-0.491471							
	0.661319	-0.910203						
	-0.214766	0.335043	-0.218487					
	0.056807	-0.108964	0.14456	-0.183382				
	-0.015074	0.028913	-0.058766	0.135951	-0.183653			
	0.004186	-0.008029	0.016318	-0.059107	0.145919	-0.224851		
	-0.001523	0.002922	-0.005938	0.021509	-0.08109	0.232061	-0.365701	
	0.000522	-0.001002	0.002036	-0.007374	0.027799	-0.106497	0.197759	-0.113244
Final 1 **	-0.47953							
	0.640913	-0.875332						
	-0.204114	0.315224	-0.201865					
	0.054658	-0.103361	0.135657	-0.177397				
	-0.015078	0.028513	-0.056762	0.135195	-0.184624			
	0.004139	-0.007826	0.01558	-0.058796	0.145908	-0.223808		
	-0.001505	0.002846	-0.005666	0.021383	-0.080936	0.23079	-0.363766	
	0.000517	-0.000977	0.001945	-0.00734	0.027783	-0.10586	0.196854	-0.112795
Final 2	-0.372032							
	0.485379	-0.64472						
	-0.144616	0.215079	-0.129915					
	0.042938	-0.076539	0.097855	-0.155682				
	-0.014689	0.026183	-0.04834	0.139533	-0.208264			
	0.003895	-0.006943	0.012818	-0.062051	0.162111	-0.237623		
	-0.001412	0.002517	-0.004647	0.022494	-0.091184	0.244163	-0.380496	
	0.000537	-0.000956	0.001766	-0.008548	0.03465	-0.11637	0.208565	-0.119643

* Shear deformation and thrust effects are included.

** Final results (1) with and (2) without the Classification Rules limits for the diameters.

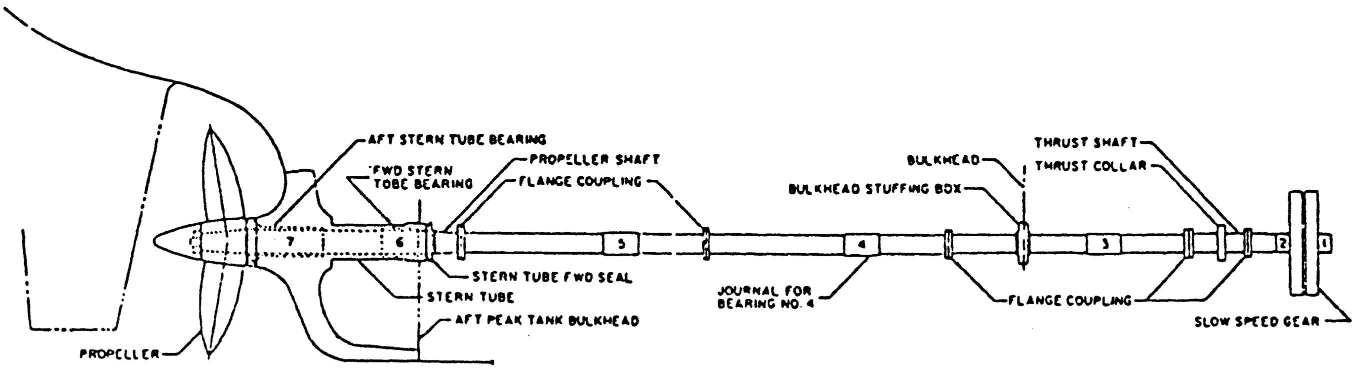


Figure 1

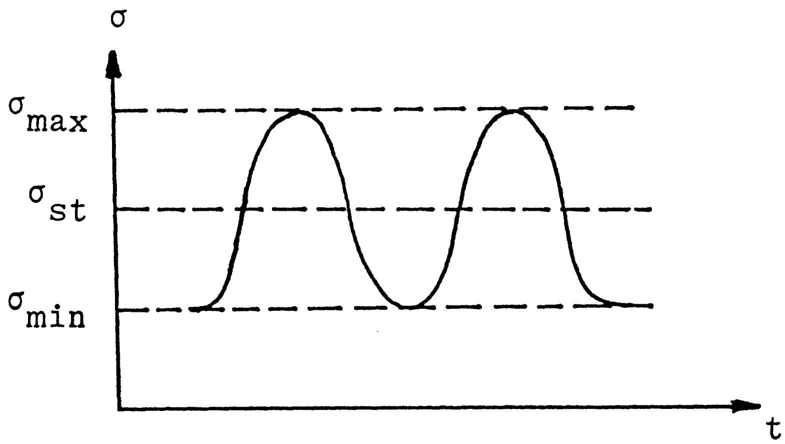


Figure 2

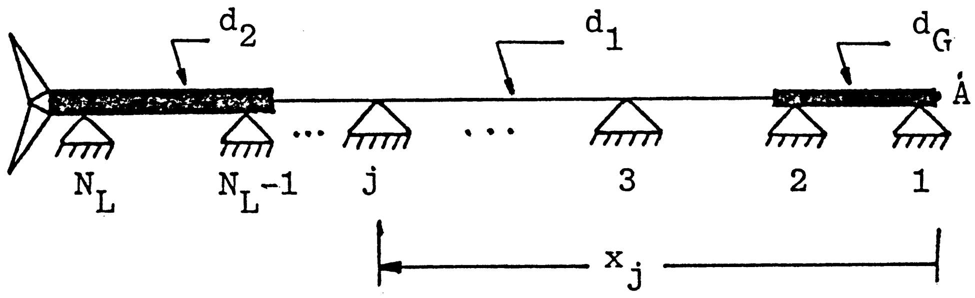


Figure 3

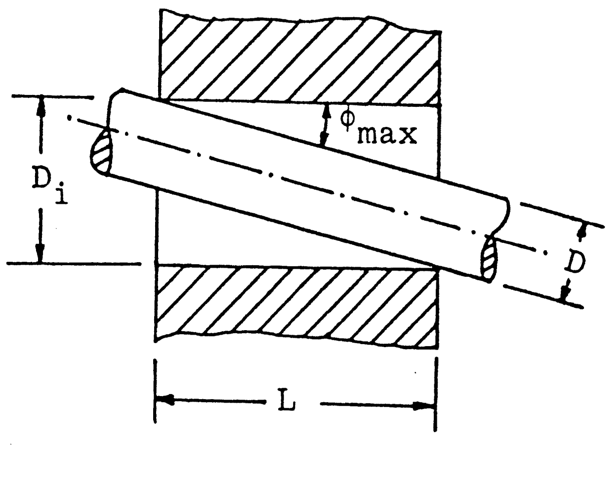
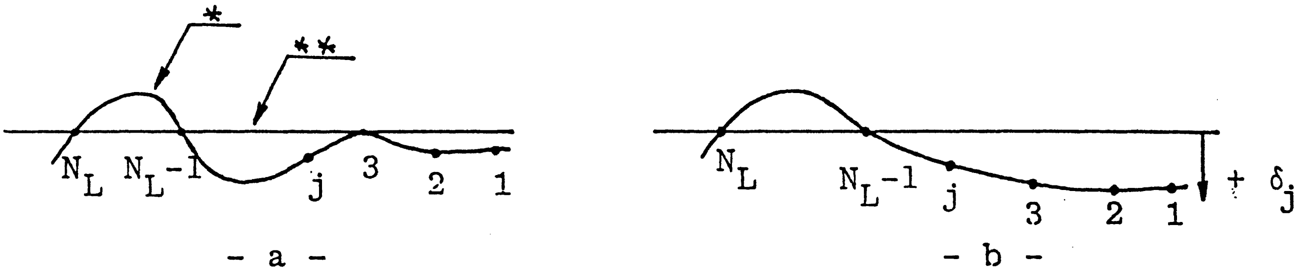


Figure 4



* elastic curve
** reference line

Figure 5

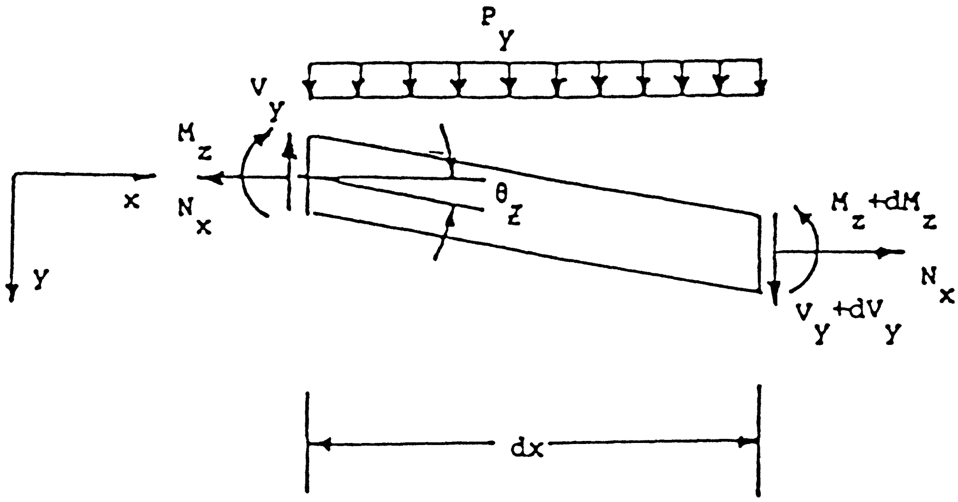
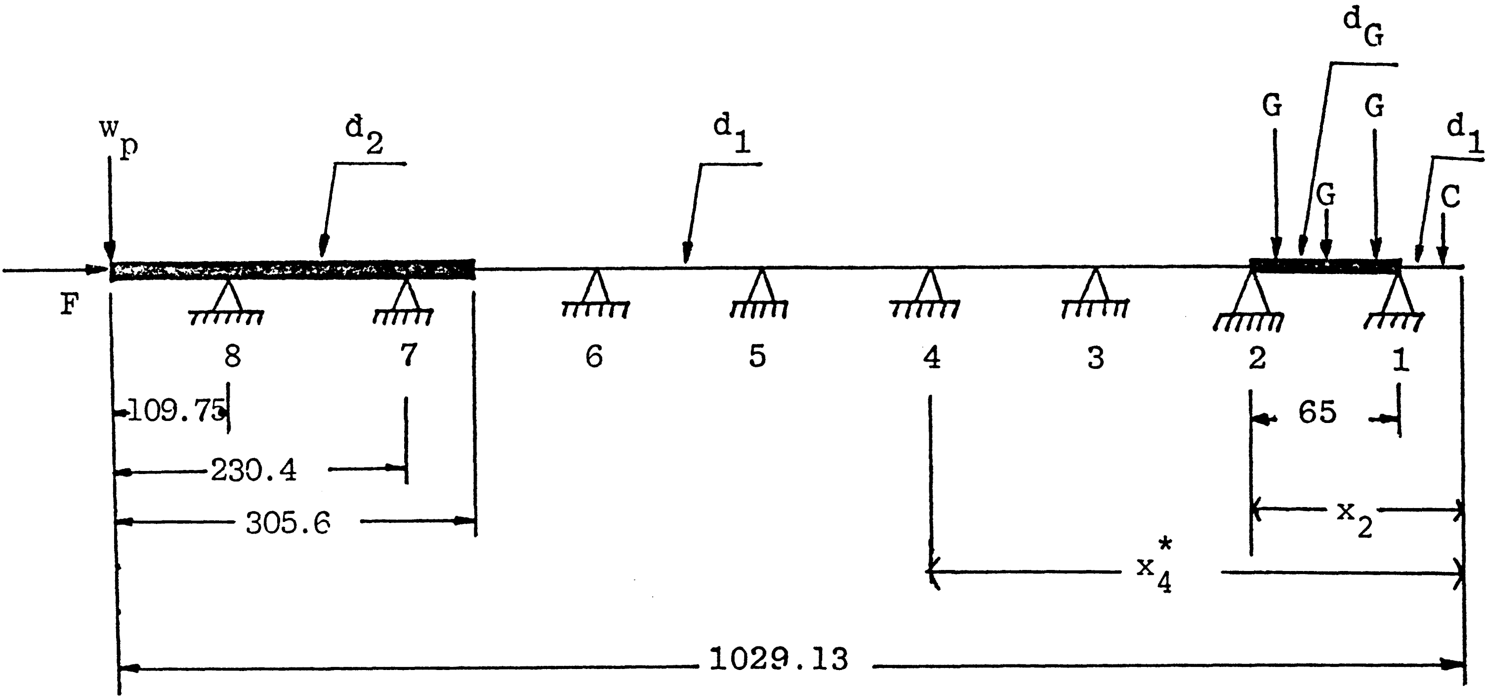


Figure 6



w_p = Propeller Load

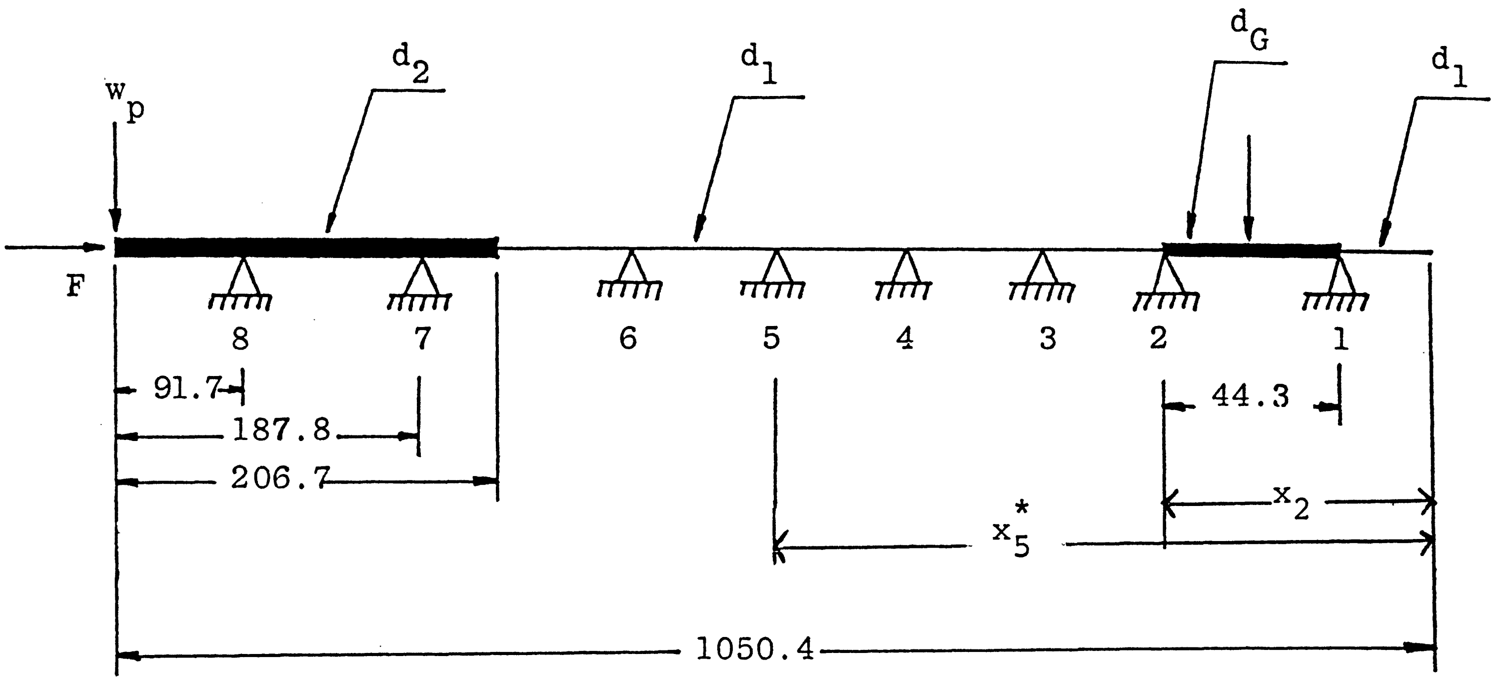
F = Propeller Thrust

G = Reduction Gear Loading = 17.9 klbf

C = Thrust Collar Weight = 2.2 klbf

* : All lengths in inches

Figure 7



w_p = Propeller Load

F = Propeller Thrust

G = Reduction Gear Loading = 15.74 klbf

* : All lengths in inches

Figure 8