Phase Report No. 1

THERMODYNAMIC EVALUATION OF THE INSTALLATION OF ELECTRICALLY CONDUCTIVE COATED PHOTOGRAPHIC WINDOWS ON THE McDonnell RF-101

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Project 2197

WRIGHT AIR DEVELOPMENT CENTER
WRIGHT-PATTERSON AIR FORCE BASE
CONTRACT NO. AF 33(616)-2268

October, 1954
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NOMENCLATURE

A  Surface area, square feet.

$C_p$  Specific heat of glass, Btu per pound, °F.

$C_{pa}$  Specific heat of air, Btu per pound, °F.

$e_s$  Vapor pressure of water at the surface temperature $T_s$, millimeters of mercury.

$e_\infty$  Vapor pressure of water at the free-stream temperature $T_\infty$, millimeters of mercury.

f  Factor defined in transient solution.

$h_i$  Convective heat-transfer coefficient on the cabin side of the window, Btu per hour, square foot, °F.

$h_o$  Convective heat-transfer coefficient through the windshield boundary layer, Btu per hour, square feet, °F.

$h_r$  Radiation heat-transfer coefficient, Btu per hour, square foot, °F.

k  Thermal conductivity, Btu per hour, square foot, °F foot.

$L_s$  Latent heat of evaporation at the surface temperature $T_s$, Btu per pound.

M  Dimensionless modulus defined in the text.

$M_\infty$  Mach number of free stream.

N  Dimensionless modulus defined in the text.

$N'$  Dimensionless modulus defined in the text.

$P_\infty$  Ambient static pressure, millimeters of mercury.

$Pr$  Prandtl number $(C_p\mu/k)$, nondimensional.

$q'$  Heat supplied $Q_e$ times $\frac{X}{k}$.

$Q_c$  Heat conducted through the window Btu per hour.

$Q_e$  Total heat supplied by the surface coating, Btu per hour.
NOMENCLATURE (cont.)

$Q_f$ Frictional heating.

$Q_i$ Heat transferred from the window to the cabin interior by free convection, Btu per hour.

$Q_o$ Heat transferred by convection from the window to the free stream, Btu per hour.

$Q_r$ Heat transferred by radiation from the window surface, Btu per hour.

$R$ Dimensionless modulus used in transient solution ($\frac{h_r\Delta X}{k}$).

$Re_x$ Reynolds number ($\frac{VpX}{\mu}$), nondimensional.

$r$ Recovery factor equal to $(Pr)^{1/3}$ for turbulent flow.

$T_c$ Temperature of cabin, °F.

$T_i$ Temperature of inner surface of window, °F.

$T_s$ Temperature of outer surface of window, °F.

$T_r$ Temperature of sky = -100°F.

$T_{ad}$ Adiabatic surface temperature, °F.

$T_\infty$ Free-stream temperature, °F.

$T_{n,\Delta t}$ Temperature of cell $n$ at time $\Delta t$.

$T^*$ Effective temperature of the boundary layer used for evaluating property values.

$W$ Weight flow of impinging water, pounds per hour, square foot.

$X$ Distance used Reynolds number, feet.

$\Delta X$ Cell thickness, feet.

$\Delta y$ Area for heat flow, square feet.

$Z$ Unheated starting length used in Rubisen's equation, feet.

$\Delta$ Increment.

$\epsilon$ Emmissivity of a surface, dimensionless.

$\Theta$ Time, hours.
NOMENCLATURE (cont.)

σ  Stefan-Boltzmann constant Btu per hour, square feet (degree Rankine)$^4$
    $= 0.173 \times 10^{-8}$.

μ  Viscosity of air, pounds per foot second.

ρ  Density of air, pounds per cubic foot.
SUMMARY

The steady-state heat requirements for maintaining the inner surface of the photographic windows of the RF-101 at 85°F at 50,000 feet altitude are computed as a function of the Mach number of flight when the electrically conductive coating is on either the inner or outer surface of the window. Transient temperature distributions through the windows are calculated for a prescribed dive.

Results of the calculations indicate that adequate power is provided for prevention of fog formation on the window surfaces when the inner surface is maintained at 85°F. Large temperature gradients will develop in the windows during the dive, and during the initial portion of the photographic run the outer surface temperature will rise rapidly.
THERMODYNAMIC EVALUATION OF THE INSTALLATION OF ELECTRICALLY CONDUCTIVE COATED PHOTOGRAPHIC WINDOWS ON THE McDonnell RF-101

INTRODUCTION

Present day aerial photographic missions have become a complex, costly, hazardous operation. In view of this fact, it is imperative that each mission be free from operational defects. In order to protect camera, lens, film, and other photographic equipment from the extreme conditions of temperature and pressure encountered during high altitude flight, it has become necessary to provide for photographic compartments equipped with optical quality windows. One of the many operational problems which results in mission failure is the fogging or frosting of the photographic windows. This report deals with a study of the heat required for fog prevention from the photographic windows of the McDonnell RF-101. The McDonnell RF-101 is unique in that it is the first military aircraft to be completely equipped with photographic windows which utilize a conductive, semitransparent coating applied directly on the surface of the glass. Heat is generated on the window surface by application of a suitable voltage source to bus bars which are located at two opposite edges of the window. The windows are reported by the manufacturer to be capable of delivering up to 2000 watts/ft² of power provided the heat is dissipated at reasonable surface temperatures.*

*Since this report was written, a specimen tested by this laboratory has failed at 1080 watts per foot square when it was immersed in a bath of city tap water maintained at 85°F. Libbey-Owens Ford personnel indicate that galvanic action caused by impurities in the water caused breakdown of the coating. Personnel at WADC also have experienced difficulty with specimen failure. The authors examined several of these specimens during the conference on the windows held at WADC September 20, 1954. Libbey-Owens Ford personnel concluded that surface scratches caused initiation of breakdown of the conductive coating. In order to improve the serviceability of the coated windows, Libbey-Owens Ford suggested that they apply a thicker outer protective film. This would decrease the light transmission of the windows but would improve the resistance of the surface to abrasion.
Preliminary conferences with Air Force Personnel indicated that (1) an outer surface temperature of 85°F was desired under all conditions of flight, (2) 8 kilowatts were reserved for the windows, (3) there were 8 windows to be heated, (4) the conductive coating was to be on the cabin side, and (5) no data were immediately available with respect to the placement of the windows in the aircraft. Preliminary computations were performed and a report was issued. The results reported indicated that the maximum heat requirements which occurred at 25,000 feet approached the limit of capacity of the conductive coating. Furthermore, since the heat was generated on the inner surface, an exceptionally high inner-surface temperature (272°F) was required to maintain the outer surface at 85°F. This combination of heat flux and inner-surface temperature was considered undesirable for the following reasons:

1. The windows would be required to operate at near full capacity; thus, no margin of safety for extreme or unusual conditions would be provided.

2. The high inner-surface temperature of the glass would result in radiation and conduction to the camera lens with the subsequent optical problem.

3. The large temperature gradient would probably cause failure of the window from thermal stress.

Recommendations were made to (1) to put the conductive coating on the outer surface (this would eliminate the problem of a high inner-surface temperature) and (2) to investigate the problem further. As a result of this preliminary analysis, further conferences were held with Air Force and McDonnell personnel. It was recommended that the criteria for the maintenance of fog-free windows be obtained from consideration of Mil-T-5842A Amendment 1. Examination of a graph of this specification indicates that the specification of an inner-surface temperature of 85°F is sufficient to establish the fact that the inner surface will remain fog free during the steady state at all altitudes, since the inner-surface temperature would always be above the dewpoint of the cabin air and thus there would be no driving force available for mass transfer to the window. Questions which arose at this point were:

1. If the inner-surface temperature were maintained at 85°F by thermostatic control, how would it respond to changes at the outer surface?

2. What would be the minimum temperature that the outer surface temperature would attain during a dive, if a constant Qe were applied to the inner surface?

3. Would the outer surface ever be below the ambient dewpoint during a rapid descent?
The first question is brought about by the fact that the forward oblique window will be subject to direct impingement of water droplets whenever the aircraft enters a cloud cover or encounters a rain storm. Since this window is 1/2-inch thick, there will be a considerable lag before the system calls for heat if the controls are at the inner surface. Furthermore, in order to provide sufficient heat at the outer surface to prevent icing, an extremely high inner-surface temperature might be required. Figure 6 shows that when 850 watts/ft² are applied to the inner surface, the inner surface rises to 240°F. This indicates that the conductive coating should be on the outer surface. The second question arises from an examination of the steady-state heat requirements obtained in the preliminary analysis which showed that approximately twice as much heat is required for the maintenance of a constant outer-surface temperature at 25,000 feet, as is required at 50,000 feet altitude. Thus, it was deemed advisable to calculate the outer-surface temperature during a dive with a constant heat flux applied to the window. The third question arises from the fact that the outer-surface temperature of the windows (when the coating is on the inside which is maintained at 85°F) will be fairly low initially at 50,000 feet at the beginning of the dive. Since the rate of descent was so great and the initial temperature so low it seemed possible that the window surface temperature might remain relatively constant while the aircraft dove into a zone with a higher dewpoint. This thought was corroborated by Air Force personnel who had seen aircraft land with the windows frosted over on the outer surface. Reports of British experience seemed to indicate that a similar phenomenon took place occasionally.

The problem was divided into two separate parts: (1) Analysis of the forward oblique window No. 7 and (2) analysis of the remaining windows 9, 5, 3, 11, and 13 considering the view-finder window as representative of the average heat loss from this group. Each analysis was performed for the steady state at 50,000 feet and for the transient state during a dive to 3,000 feet according to a prescribed flight plan.

METHOD OF ANALYSIS

Steady State (Conductive Coating on the Cabin Side)

In the steady state analysis, it was assumed that:

1. the average of the USAF hot and cold days applied;
2. there was turbulent flow over the windows;
3. constant fluid properties;
4. the cabin temperature was maintained at 55°F;
5. the inside-convection coefficient was 3 Btu/hr ft\(^2\) °F;

6. radiation was to the cabin interior, nocturnal and solar radiation were neglected; and

7. the center of the window could be used as the characteristic length in Rubesin's Equation (10) to obtain the average for the window.

The heat balance may be written as follows:

\[
Q_e = Q_c + Q_i + Q_r \tag{1}
\]

\[
Q_c = Q_o \quad \text{(solar and nocturnal radiation neglected).} \tag{2}
\]

Where \(Q_r\) is the radiation loss to the cabin

\[
Q_r = \sigma e A \left( T_i^4 - T_c^4 \right), \tag{3}
\]

and \(Q_i\) is the convection loss into the cabin,

\[
Q_i = h_i A \left( T_i - T_c \right), \tag{4}
\]

and \(Q_c\) is the heat conducted through the window.

\[
Q_c = \frac{kA}{L} \left( T_i - T_S \right) \tag{5}
\]

and \(Q_o\) is the convection loss to the outer atmosphere.

\[
Q_o = h_o A \left[ T_S - T_{ad} \right]. \tag{6}
\]

In the above equations, the value of \(\epsilon\) was taken from Eckert\(^2\) to be 0.935 and the value of \(k\) was obtained from the Boston Optical Research Laboratory as 0.556 Btu/hr ft\(^2\) °F/ft. The value of \(h_o\) was calculated from the equation of Rubesin\(^10\),

\[
h_o = \frac{0.0288}{X} k Pr^{1/3} Re_x \left\{ 1 - \left( \frac{Z}{X} \right)^{39/40} \right\}^{7/39} \tag{7}
\]

Properties of air were obtained from Eckert\(^2\) and evaluated at the effective temperature \(T^*\) (degree Rankine) where

\[
T^* = T_{\infty} + 0.58 \left( T_S - T_{\infty} \right) + 0.032 \left( T_{\infty} \right)^2. \tag{8}
\]
The adiabatic surface temperature was calculated by the equation,

\[ T_{ad} = T_\infty (1 + 0.2rM^2) \]  \( \text{(9)} \)

and the final equation is then

\[ Q_e = h_c A (T_s - T_{ad}) + 0.161A \left[ \left( \frac{T_a}{100} \right)^4 - \left( \frac{T_c}{100} \right)^4 \right] + h_A (T_1 - T_c) \]  \( \text{(10)} \)

The steady-state heat requirement for maintaining the forward oblique window at various outer-surface temperatures is plotted in Fig. 5. As a function of the Mach number of flight at 50,000 feet. From curves 4 through 8 the heat required for maintaining any desired outer-surface temperature between -30 and +85°F can be determined as a function of Mach number. The corresponding inner-surface temperature required can also be determined.

Above a free-stream Mach number of 1.0, one of two shock conditions may exist dependent on the geometry of the Pitot mast and the nose. An analysis was made with the assumption that a detached shock would occur and that normal shock conditions would exist in the vicinity of the forward oblique window. In addition, the window temperatures were determined for an attached shock wave with a 5° half-angle cone for the Pitot mast. The results of these calculations are shown in Figs. 7 and 8. In the transonic range between 1.0 and 1.3 these curves appear erratic. Above Mach 1.3, the curves smooth out and indicate that slightly less heat is required for a shock condition.

Calculations were performed for the remaining windows using a fixed value of 85°F for the inner-surface temperature. Figures 10 and 11 show the heat required and the corresponding outer-surface temperature, respectively, as a function of the free-stream Mach number.

The minimum steady-state heat requirement for maintaining the inner surface of the windows at 85°F during steady flight as 50,000 feet altitude in saturated air are shown in Table I below.

**TABLE I**

<table>
<thead>
<tr>
<th>Window</th>
<th>Area - ft²</th>
<th>Q Btu/hr ft²</th>
<th>KW</th>
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<tr>
<td>7</td>
<td>1.62</td>
<td>4000</td>
<td>1.17</td>
</tr>
<tr>
<td>3</td>
<td>0.945</td>
<td>1418</td>
<td>.42</td>
</tr>
<tr>
<td>11 - (2)</td>
<td>0.972</td>
<td>2920</td>
<td>.85</td>
</tr>
<tr>
<td>5</td>
<td>0.736</td>
<td>1105</td>
<td>.32</td>
</tr>
<tr>
<td>13 - (2)</td>
<td>0.805</td>
<td>2325</td>
<td>.68</td>
</tr>
<tr>
<td>9</td>
<td>0.313</td>
<td>470</td>
<td>.14</td>
</tr>
</tbody>
</table>

Total Power Required 3.56

Reserve Power for Anti-icing or Deicing 4.42

8.00
The Steady-State Case (Conductive Coating on the Outer Surface)

The assumptions for this case are the same as for the preceding case, except that radiation to the sky at -100°F is included. Figure 9 shows the heat requirements for the forward oblique window as a function of the Mach number for an inner-surface temperature of 85°F.

The Transient Case (Conductive Coating on the Inside)

The method of analysis used in the transient temperature calculations is an extension of the method of Gisintherer. The window was divided into five equal cells through the thickness of the window and the temperature at the center of a cell was assumed to be representative of the entire cell. The thermal conductivity and the specific heat of the glass were regarded as being constant and the flow of heat was regarded as one dimensional. The temperature of the cells were taken as \( T_1, T_2, T_3 \ldots T_n \) at time zero (corresponding to the steady-state temperature distribution at 50,000 feet) and as \( T_1^{\Delta\Theta}, T_2^{\Delta\Theta}, T_3^{\Delta\Theta} \ldots T_n^{\Delta\Theta} \) after a small time increment \( \Delta\Theta \) and in this analysis, the time increment is assumed to be so small that:

1. The initial temperature gradients \( \frac{T_n - 1 - T_n}{\Delta X} \) and \( \frac{T_n - T_n + 1}{\Delta X} \) can be used in this interval.

2. The temperature at point 2 is not affected by any change at the left of cell 1 and at the right of cell 2.

3. The heat capacity of cell 2 can be computed using the temperature at point 2.

The subdivision of the window is shown in Fig. 2.

Using these assumptions, the heat balance for cell 1 can be written as

\[
\begin{align*}
    h_o \Delta y^2 (T_o - T_s) \Delta \Theta + h_o (T_{ad} - T_x) \Delta y^2 \Delta \Theta + \frac{k \Delta y^2}{\Delta X} (T_2 - T_s) \Delta \Theta + \\
    h_r \Delta y^2 (T_r - T_s) \Delta \Theta &= \frac{\rho c_p \Delta y^2 \Delta X}{2} (T_s^{\Delta\Theta} - T_s) . \quad (11)
\end{align*}
\]

By dividing through by \( \frac{k \Delta y^2 \Delta \Theta}{\Delta X} \) and defining

\[
N = \frac{h_o \Delta X}{k} ; \quad R = \frac{h_r \Delta X}{k} ; \quad M = \frac{\rho c_p \Delta X^2}{k \Delta \Theta} . \quad (12)
\]
the following is obtained for $T_1 = T_3$:

$$T_{1,Δθ} = \frac{2N}{M} T_{ad} + \left(\frac{M - 2N - 2R - 2}{M}\right) T_1 + \frac{2}{M} T_2 + \frac{2R}{M} T_2.$$  \hspace{1cm} (13)

The balance on cell 2 is simply

$$T_{2,Δθ} = \frac{1}{M} T_1 + \frac{M - 2}{M} T_2 + \frac{1}{M} T_3.$$  \hspace{1cm} (14)

The balance on cells 3, 4, and 5 are of the same type,

$$T_{n,Δθ} = \frac{1}{M} T_{n - 1} + \frac{M - 2}{M} T_n + \frac{1}{M} T_{n + 1},$$  \hspace{1cm} (15)

or defining the $f$ factors,

$$T_{n,Δθ} = f_{n - 1,n} T_{n - 1} + f_{n,n} T_n + f_{n + 1,n} T_{n + 1},$$  \hspace{1cm} (16)

where $f_{n - 1,n} = \frac{1}{M}$; $f_{n,n} = \frac{M - 2}{M}$; $f_{n + 1,n} = \frac{1}{M}$.

At cell 6, the balance is written:

$$\frac{kA\Delta Y^2}{AX} (T_5 - T_3) Δθ + h_1A\Delta Y^2 (T_C - T_3) Δθ + h_1A\Delta Y^2 (T_C - T_3) +$$

$$Q_e A\Delta Y^2 Δθ = \frac{ρC_p A\Delta Y^2 Δθ}{2} (T_e, Δθ - T_3).$$  \hspace{1cm} (17)

Dividing through by $\frac{kA\Delta Y^2 Δθ}{AX}$ and defining $N' = \frac{h_1A\Delta X}{k}$,

M as before; $R = \frac{h_pA\Delta X}{k}$; and $q' = \frac{Q_eA\Delta X}{k}$;

we obtain:

$$T_{e,Δθ} = \frac{2}{M} T_5 + \left(\frac{M - 2N - 2R - 2}{M}\right) T_3 + \frac{2}{M} (N' + R) T_C + q'.$$  \hspace{1cm} (18)

We define

$$\frac{2}{M} = f_{5,e}; \quad \frac{M - 2N - 2R - 2}{M} = f_{3,e}; \quad \frac{2}{M} (N' + R) = f_{c,e};$$

then

$$T_{e,Δθ} = f_{5,e} T_5 + f_{3,e} T_3 + f_{c,e} T_C + q'.$$  \hspace{1cm} (19)
The Transient Case (with Conductive Coating on the Outside)

This analysis is the same as the preceding one except that the term $Q_e$ is added to cell 1 rather than cell 6. The equation for cell 1 becomes

$$T_1, \Delta \theta = f_{x,1} T_{ad} + f_{1,1} T_1 + f_{2,1} T_2 + f_r T_r + q', \quad (20)$$

where

$$f_{x,1} = \frac{2N}{M}; \quad f_{2,1} = \frac{2}{M};$$

$$f_{1,1} = \frac{M - 2N - 2R - 2}{M}; \quad f_r = \frac{2R}{M}.$$ 

The equations for the inner cells remains unchanged while the equation for cell 6 is changed to

$$T_6, \Delta \theta = f_{5,6} T_5 + f_{6,6} T_6 + f_{c,6} T_c, \quad (21)$$

where

$$f_{x,1} = \frac{2N}{M}; \quad f_{1,1} = \frac{M - 2N - 2}{M}; \quad f_{c,6} = \frac{2(N' + R)}{M}.$$ 

The Transient Case (with Conductive Coating on the Outside During a Dive Through a Cloud)

The equation at cell 1 now includes all the terms above, plus the evaporation, and the kinetic energy of the impinging droplets.

$$T_1, \Delta \theta = f_{x,1} T_{ad} + f_{1,1} T_1 + f_{e,1} (e_\infty - e_1) + f_w (T_\infty + T_{aw}) + f_{2,1} T_2 + f_r T_r + q', \quad (22)$$

where

$$f_{x,1} = \frac{2N}{M} \quad \text{N} = \frac{h_0 \Delta X}{k}$$

$$f_{1,1} = \frac{M - 2N - 2R - 2W - 2}{N} \quad \text{M} = \frac{\delta C_p \Delta X^2}{N}$$

$$f_{e,1} = \frac{2N H}{M P_\infty} \quad \text{B} = \frac{W C_p \Delta X}{k}$$

$$f_{2,1} = \frac{2}{M} \quad \text{H} = \frac{0.662 L_1}{C_p a}.$$
\[ f_w = \frac{2B}{M} \quad R = \frac{h_0AX}{k} \]

The results of these calculations are shown in Figs. 12 through 15. Figure 12 represents windows 5, 9, 11, 13, and 3 and shows the inner- and outer-surface temperatures as a function of altitude starting with an inner-surface temperature of 85°F for an 0.8 free-stream Mach number dive.

The temperature distribution through these windows at various altitudes is shown in Fig. 16. The remaining curves (Figs. 13, 14, and 15) are for the forward oblique window. The cases examined are for maintaining the inner-surface temperature at 85°F when the coating is on either the inner or the outer surface. The temperature distribution through the forward oblique window at various altitudes is shown in Figs. 17 and 18.

During the dive to sea level, the aircraft may encounter a cloud cover and be subjected to icing conditions. Since most icing clouds are reported to exist below 12,000 feet, a cloud cover was assumed to exist from 15,000 feet down to 5,000 feet for purposes of transient calculations. The meteorological nature of the cloud was chosen as follows:

1. liquid water content, 0.2 grams per cubic meter, and
2. mean drop size, 20 microns.

Such an icing condition may be considered to be very mild. The water catch rate of the forward oblique window was estimated from Langmuir's data for ribbons. The window was considered to intercept the same amount as a ribbon of the projected area of the window. The results of this calculation is shown in Fig. 13.

**CONCLUSIONS**

An examination of Figs. 12 through 15, which show the temperature histories of the windows during rapid descent, indicates that an inner surface temperature of 85°F before entering the dive is adequate for fog prevention. This observation is based on the fact that the curve drawn according to the Mil-T-5842A Addendum 1 (USAF specifications for the temperature of the atmosphere to be used in the unsteady state) never intersects the transient curves of either the inner- or the outer-surface temperatures of the windows. When the inner-surface temperature before the dive is much below 85°F, fogging of the inner surface is possible as can be seen from Fig. 14.

The minimum power requirement for maintaining the inner surface of all the photographic windows at 85°F during the steady state at 50,000 feet is 3.58 kilowatts. This is the heat loss to the outer air stream (considered saturated with moisture) and to the cabin itself by convection and radiation.
Since 8 kilowats of power have been reserved for application to the photographic windows, a reserve of 4,42 kilowatts is available for icing protection.

All the windows in the lower side of the fuselage will probably intercept some water during a dive through a cloud cover since the windows will have a vertical component during the pull-out. However, the water catch rate of the forward oblique window will be much more severe. The three modes of icing protection generally applied to aircraft are:

1. **anti-icing** by complete evaporation of all the impinging water,

2. **anti-icing** by partial evaporation of the impinging water but maintenance of the surface temperature well above freezing, and

3. **de-icing** by allowing ice to form, and then shedding periodically as solid ice sheets.

The first method of anti-icing protection is the most desirable from a photographic standpoint, since the windows would be dry and ready for instant use on exit from the cloud cover. Unfortunately, this method of anti-icing is impossible at the high-flight speeds at which this aircraft is scheduled to operate. Approximately 12-1/2 kilowatts would be required to evaporate the water impinging on the forward oblique window alone during a mild icing condition (0.2 grams/m³ liquid water content and $T_\infty = -5^\circ F$) such as has been examined here. The second mode of anti-icing by partial evaporation requires considerably less heat than above, since only a fraction of the impinging water is evaporated; the remainder runs off and the surface remains wet. For this reason, it is possible to maintain the windows ice free with the power supplied for the light icing condition examined; however, it may not be possible in clouds of greater concentration or if clouds should be encountered at higher altitudes. Clouds with liquid water content up to five times the one assumed are commonly encountered.

Transient calculations were carried out for the partial evaporation mode of protection. These calculations are intended as a guide only and may be somewhat in error due to the severity of the step function in the numerical method of computation. The steady-state temperature was determined by the method of Gray and found to be of the same order of magnitude (43°F as compared to the minimum transient value of 47.5°F obtained for the case with the conductive coating on the inner surface of the forward oblique window). The third mode of protection known as de-icing requires considerably less heat than either of the preceding methods since the water is allowed to freeze until an ice cap is formed (usually up to 1/4-inch thick). Then the power is turned on until a water film sufficient to allow release of the icecap is formed. Thus, in this method a reduction in the heat load required for icing protection of as much as 10 to 1 over that required for anti-icing might be realized. Present
day de-icing equipment is designed to operate at 15 to 20 watts/in\(^2\) or 2160 to 2880 watts/ft\(^2\). Recent investigations in the field of cyclic de-icing indicate it may be possible to accomplish efficient de-icing at somewhat lower power densities by the utilization of a heater which has a high thermal efficiency. Thus, although the present design power loads are slightly above the recommended operating limits of the conductive coated windows, the increase in thermal efficiency of the system might reduce the power load required to within the capacity of the conductive coatings.

The problem of bonding of the icecap at the unheated edges of the windows, bus bars, window frames, and even fuselage might cause bridging and failure to shed. However, experience has shown that dome shaped objects, wings, etc., which are not equipped with burn-through strips do not shed the icecap even though they may be completely free from any bonds to the surface. In many instances it has been found that an icecap bonded at the edges will not shed, but water melts from under the icecap and leaves the icecap bridging the heated area. If the aerodynamic forces are not great enough to cause fracture of the bridged icecap, failure of the heater is possible. The most practical icing protection is offered when the conductive coating is on the outer surface of the windows, since heat can be immediately supplied to the surface for either anti-icing or de-icing. Furthermore the inner surface temperature can be maintained at a reasonable value when the coating is on the outer surface. Although thermodynamic reasoning dictates that the conductive coating should be on the outer surface, the physical endurance of the coating is such that it would probably not stand up under the severe abrasion and galvanic action of the impinging water. Thus, it appears that physical limitations of the coating dictate that the coating should be on the inner surface. Thus, only marginal icing protection will be provided for those mild cases which can be accomplished with the heat required for defogging since high inner-surface temperatures result from greater power loads.

During the dive from 50,000 to 3,000 feet and until the end of the photographic run, rather rapid temperature changes take place in the heated windows. Figures 16, 17, and 18 show the temperature variation through the windows at various altitudes. A comparison of Figs. 17 and 18 shows that the temperature variations through the forward oblique window are greater at all altitudes when the coating is on the inner surface (Fig. 17) than when it is on the outer surface (Fig. 18). Although these curves are computed for power on to temperatures near 150\(^\circ\)F, the control cutout point would probably be between 85 and 110\(^\circ\)F. By the end of the photographic run, the outer-surface temperature of the windows will be near the equilibrium surface temperature of 162.1\(^\circ\)F for a Mach 0.8 run at 3,000 feet. When the coating is on the inner surface, a change occurs from a 51\(^\circ\)F outer-surface temperature at 15,000 feet to one of 126.5\(^\circ\)F after 10 seconds at 3,000 feet; a change of 75.5\(^\circ\)F, takes place in an interval of 40 seconds. Very little control over the outer-surface temperature can be achieved in this case, since the inner-surface temperature varies only about 1\(^\circ\)F, which is probably less than the limit of accuracy of the of the thermostat. When the coating is on the outer surface the changes which
take place are less severe initially. If the control point were maintained between 91 and 92°F, then the maximum temperature variation during the dive would be from 75°F at 25,000 feet to 91.75°F at 16,000 feet. The surface temperature would then remain at 91.75°F until about 12,000 feet, then rise rapidly (about 3°F/sec) until 3,000 feet is attained. At 3,000 feet the temperature will rise to near the equilibrium temperature at an ever decreasing rate. Essentially the same reasoning can be applied to the other windows. The point to be questioned here then, is whether the untempered windows will withstand the thermal shock. Should the windows shatter in flight, considerable damage would result.

Finally, it should be pointed out that the results described here are for a prescribed flight plan which fixed the rate of descent and Mach number of flight. The results can be expected to differ appreciably if there are significant deviations from said flight plan.
BIBLIOGRAPHY


Fig. 1. Edge view of window.

Fig. 2. Division of window into cells.
Fig. 3. Equilibrium surface temperature of the forward oblique window at 50,000 feet as a function of the free stream Mach number; Q = 0.
Forward Oblique Window
Cabin Temperature = 55°F
Window Area = 1.62 sq.ft.
Window Thickness = .5 in.
Pressure Altitude = 50,000 ft.

\[ Q_c \text{ for } T_s = -40^\circ F \]

\( -20^\circ \quad 0^\circ \quad 20^\circ \quad 40^\circ \quad 60^\circ \quad 80^\circ \quad 85^\circ \)

Cabin Convection Loss - \( Q_i \)
Cabin Radiation Loss - \( Q_r \)

**Fig. 4.** Heat losses from the forward oblique window for various inner and outer surface temperatures.
Fig. 5. Heat added to the inner surface of the forward oblique window versus free-stream Mach number with outer surface temperature as parameter.
Fig. 6. Heat added to the inner surface of the forward oblique window versus free-stream Mach number with inner surface temperature as parameter.
Fig. 7. Heat added to the inner surface of the forward oblique window versus free-stream Mach number with outer surface temperature as parameter-shock analysis.
Fig. 8. Heat added to the inner surface of the forward oblique window versus free-stream Mach number with inner surface temperature as parameter—shock analysis.
Fig. 9. Heat added to the outer surface of the forward oblique window versus free-stream Mach number when the inner surface is maintained at 85°F.
Fig. 10. Heat added to the inner surface of the viewfinder window versus free-stream Mach number when the inner surface is maintained at 85°F.
Fig. 11. Outer surface temperature of the view-finder window versus Mach number when the inner surface is maintained at 85°F.
Fig. 12. Temperature of the inner and outer surface of the viewfinder window during a dive at 0.8 Mach number when the coating is on the inner surface.
Fig. 13. Temperature of the inner and outer surface of the forward oblique window during a dive at 0.8 Mach number when the coating is on the outer surface; \( Q = 2443 \text{ Btu per hour square feet} \).
Fig. 14. Temperature of the inner and outer surface of the forward oblique window during a dive at 0.6 Mach number when the coating is on the outer surface; $Q = 2000$ Btu per hour square feet.
Fig. 15. Temperature of the inner and outer surface of the forward oblique window during a dive at 0.8 Mach number when the coating is on the inner surface.
Fig. 16. Temperature distribution through the view-finder window at various altitudes, $Q = 1375$ Btu per hour square feet.
Fig. 17. Temperature distribution through the forward oblique window at various altitudes when the coating is on the inner surface, $Q = 1615$ Btu per hour
Fig. 18. Temperature distribution through the forward oblique window at various altitudes when the coating is on the outside, $Q = 2443$. 

Forward Oblique Window
Coating Outside
Cabin Temperature = 55°F
Mach Number = 0.8

Temperature - °F

Distance Through Window - Inches