UM-HSRI-PF-73-3

WET TRACTION TEST PROGRAM

Final Report

DOT-HS-031-2-283

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February 1973

Prepared for:

Traction Studies Group
Safety Systems Laboratory
National Highway Traffic Safety Administration
U.S. Department of Transportation
Washington, D.C. 20590

TECHNICAL REPORT STANDARD TITLE PAGE

1. Report No.
UM-HSRI-PF-73-3

2. Government Accession No.

3. Recipient's Catalog No.

4. Title and Subtitle
WET TRACTION TEST PROGRAM

5. Report Date
February 1973

6. Performing Organization Code

7. Author(s)
R. E. Wild

8. Performing Organization Report No.
UM-HSRI-PF-73-3

9. Performing Organization Name and Address
The University of Michigan
Highway Safety Research Institute
Huron Parkway & Baxter Road
Ann Arbor, Michigan 48105

10. Work Unit No.

11. Contract or Grant No.
DOT-HS-031-2-283

12. Sponsoring Agency Name and Address
Safety Systems Laboratory
National Highway Traffic Safety Administration
U.S. Department of Transportation
Washington, D.C. 20590

13. Type of Report and Period Covered
Final Report

14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

This paper describes a test in which 86 tires (2 each of 43 different types representing 90% of the O.E. population) were measured for longitudinal and lateral traction capability on 3 different wetted surfaces. The resulting data was subjected to a simple but thorough statistical analysis. Findings of general interest emerged showing the fallibility of the "skid number" for characterizing a tire-pavement combination, the independent nature of lateral traction with respect to longitudinal traction, and the effects on traction of tire diameter and load rating. Traction uniformity between identical tires was found to be excellent, while the traction differences between tires of the same size but of different manufacture were found to be statistically significant. These latter two findings lead to the conclusion that traction grading on one of the three surfaces, concrete, is possible.

17. Key Words
Limits of resolution, standard deviation of the mean, traction uniformity, traction grading

18. Distribution Statement

19. Security Classif. (of this report)
None

20. Security Classif. (of this page)
None

21. No. of Pages

22. Price

Form DOT F 1700.7 (8-69)

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LIST OF SYMBOLS

$\mu_x P$	peak longitudinal traction
$\mu_x S$	locked-wheel longitudinal traction
$\mu_y P$	peak lateral traction
σ	the universe or "true value" standard direction of a particular measurement, based on a number of trials approaching infinity
$\hat{\sigma}$	the best estimate of σ, based on 10 trials
σ_s	the standard deviation of σ. $\hat{\sigma}$ is 95% certain of being within $\pm 2\sigma_s$ of σ.
σ_m	the universe or "true value" standard deviation of the mean. σ_m is obtained by dividing an ∞ number of measurements into groups of n, computing the mean of each group, and finding the standard deviation of these means. $\sigma_m = \sigma/\sqrt{n}$
$\hat{\sigma}_m$	the best estimate of σ_m, based on 10 trials. $\hat{\sigma}_m = \hat{\sigma}/\sqrt{10}$

WET TRACTION TEST PROGRAM

1. INTRODUCTION

This report presents the results obtained from a research study entitled "Wet Traction Test Program." The study was completed by the Highway Safety Research Institute (HSRI) of The University of Michigan for the Safety Systems Laboratory (SSL) of the National Highway Traffic Safety Administration (NHTSA).

The main feature of the study was a test program designed to provide SSL with longitudinal and lateral traction data for the most common tire types found on automobiles in the United States today. These data were required by SSL for purposes of making recommendations regarding both a minimum numeric for longitudinal traction and a procedure for grading overall tire traction quality.

2. TEST PARAMETERS AND METHODS

The tire tests were conducted at the Texas Transportation Institute (TTI) with the HSRI Mobile Tire Tester. Three surfaces were used: (1) existing portland cement concrete; (2) jennite flush seal; and (3) a crushed gravel hot mix (labeled "asphalt" in the data section). The skid numbers for these surfaces, as obtained by the Texas Highway Department using an ASTM tire with external watering at 40 mph, are: (1) 59; (2) 20; (3) 52.

Two each of the tires listed in Table 1 were tested in this program. The test loads were as close to 85 percent of the Tire and Rim Association rated maximum at 24 psi inflation pressure as could be achieved using a discrete-weight loading system. These loads are tabulated in Table 1 along with wheel-rim diameter and width. A tire identification code is also listed in Table 1. Note that all tires were tested at 24 psi.

TABLE 1

TIRES TESTED

[illegible]	Size	Rim	Test Load	SSL Code #
[illegible]	ER78-14	14x5"	990	AM 1 & 2
[illegible]tom [illegible]	G-78-15	15x6"	1155	AM 3 & 4
[illegible]stom [illegible]	H-78-15	15x6"	1305	AM 5 & 6
[illegible]er	600/13	13x5½"	735	AM 7 & 8
[illegible]stom	G-70-15	15x6"	1155	AM 9 & 10
[illegible]	G-60-15	15x7"	1155	AM 11 & 12
[illegible]	G-78-15	15x6"	1155	AM 13 & 14
[illegible]tudded)	G-78-15	15x6"	1155	AM 15 & 16 *
[illegible]elux [illegible]	E-78-14	14x5"	990	BM 1 & 2
[illegible]elux [illegible]	F-78-15	15x6"	1100	BM 3 & 4
[illegible] "500"	G-78-15	15x6"	1155	BM 5 & 6
[illegible] "500"	F-78-14	14x6"	1095	BM 7 & 8

Manufacturers Description	Size	Rim	Test Load	SSL Code #
Firestone Super Sport Wide Oval	F-70-14	14x6"	1095	BM 9 & 10
Firestone Town & Country All Position Belt	G-78-15	15x6"	1155	BM 11 & 12
Sears Allstate Radial	215R-15	15x6"	1305	CM 1 & 2
Sears Allstate Radial	195R-15	15x6"	1095	CM 3 & 4
Sears Allstate Dynaglass Silent Guard	F-78-14	14x6"	1095	CM 5 & 6
Sears Guardsman 78	E-78-14	14x5"	990	CM 7 & 8
Sears Allstate Guardsman 78	D-78-14	14x5"	940	CM 9 & 10
Sears Superwide 70	F-70-14	14x6"	1095	CM 11 & 12
Sears Silent Snowguard	G-78-15	15x6"	1155	CM 13 & 14
Sears Allstate Guardsman 78	C-78-15	13x5½"	890	CM 15 & 16
Bridgestone Radial	175-13	13x5½"	785	DM 1 & 2
Uniroyal Fastrak	H-78-15	15x6"	1305	DM 3 & 4
Uniroyal Fastrak	G-78-14	14x6"	1155	DM 5 & 6

[illegible] on concrete surface only.

TABLE 1 (Continued)

	Size	Rim	Test Load	SSL Code #
[illegible]	F-78-14	14x6"	1090	DM 7 & 8
[illegible]	L-78-15	15x6"	1450	DM 9 & 10
[illegible]	F-70-14	14x6"	1090	DM 11 & 12
[illegible]	GR-70-15	15x6"	1155	EM 1 & 2
[illegible]	GR-70-14	14x6"	1155	EM 3 & 4
[illegible]	C-78-13	13x5½"	890	EM 5 & 6
[illegible]	B-78-14	14x5"	835	EM 7 & 8
[illegible]	H-78-15	15x6"	1300	EM 9 & 10
[illegible]	F-78-15	15x6"	1090	EM 11 & 12
[illegible]	GR-60-15	15x7"	1155	EM 13
[illegible]	850/14	14x6"	1155	FM 1 & 2
[illegible]	215R-15	15x6"	1305	GM 1 & 2

Manufacturers Description	Size	Rim	Test Load	SSL Code #
Bridgestone Radial	175-14	14x5"	890	HM 1 & 2
Bridgestone Steel Radial	215R-15	15x6"	1305	HM 3 & 4
ASTM Control Tire	750-14	14x5"	1090	ZM 1 & 2
ASTM Control Tire	750-14	14x5"	1090	ZM 3 & 4
Goodyear Custom Power Cushion Polyglas - No Sipes	G-78-14	14x6"	1155	JM 1 & 2
Goodyear Custom Power Cushion Polyglas - with Sipes	G-78-14	14x6"	1155	JM 3 & 4

All tests were conducted at 40 mph. Water was applied by the Mobile Tire Tester's watering system at a rate such as to yield a water depth of 0.02 inches.

The longitudinal (braking) force and lateral force exerted by the test tire on the balance system were continuously measured as the longitudinal slip of the tire was varied from 0 to 100%. These data were obtained at slip angles of 0° and 8°, with ten test replications being made at each test condition.

3. TEST DATA

The traction forces generated by the test tire are measured in a plane both parallel and perpendicular to the plane of wheel rotation. These traction forces, i.e., the longitudinal and lateral components of the total shear force generated at the tire-road interface, can be reduced to an effective friction coefficient, "μ," by dividing the traction force component by the test value of the vertical load. In this report, the longitudinal (braking) traction coefficient is designated as μ_x and the lateral traction coefficient is designated as μ_y.

To satisfy the objectives of this study, two values of μ_x and μ_y were determined—peak values and the values established when the wheel is braked to a fully locked condition, i.e., longitudinal slip is 100 percent. The peak values of μ_x and μ_y were determined at whatever percent longitudinal slip they happened to occur. Peak values of μ_x typically occurred at about 10 percent longitudinal slip with peak values of μ_y generally occurring at zero longitudinal slip.

Appendix I presents the peak and locked-wheel values of μ_x and μ_y for each of the tires tested in this program. Peak values of longitudinal (braking) and lateral traction coefficients are designated as μ_xP and μ_yP, respectively, with locked wheel or "sliding" traction coefficients being designated as $\mu_x S$ and $\mu_y S$. The $\mu_x P$, $\mu_x S$, and $\mu_y P$ data from Appendix I is presented in the form of histograms in Figures 1, 2, and 3, displaying the range of O.E. tire traction.

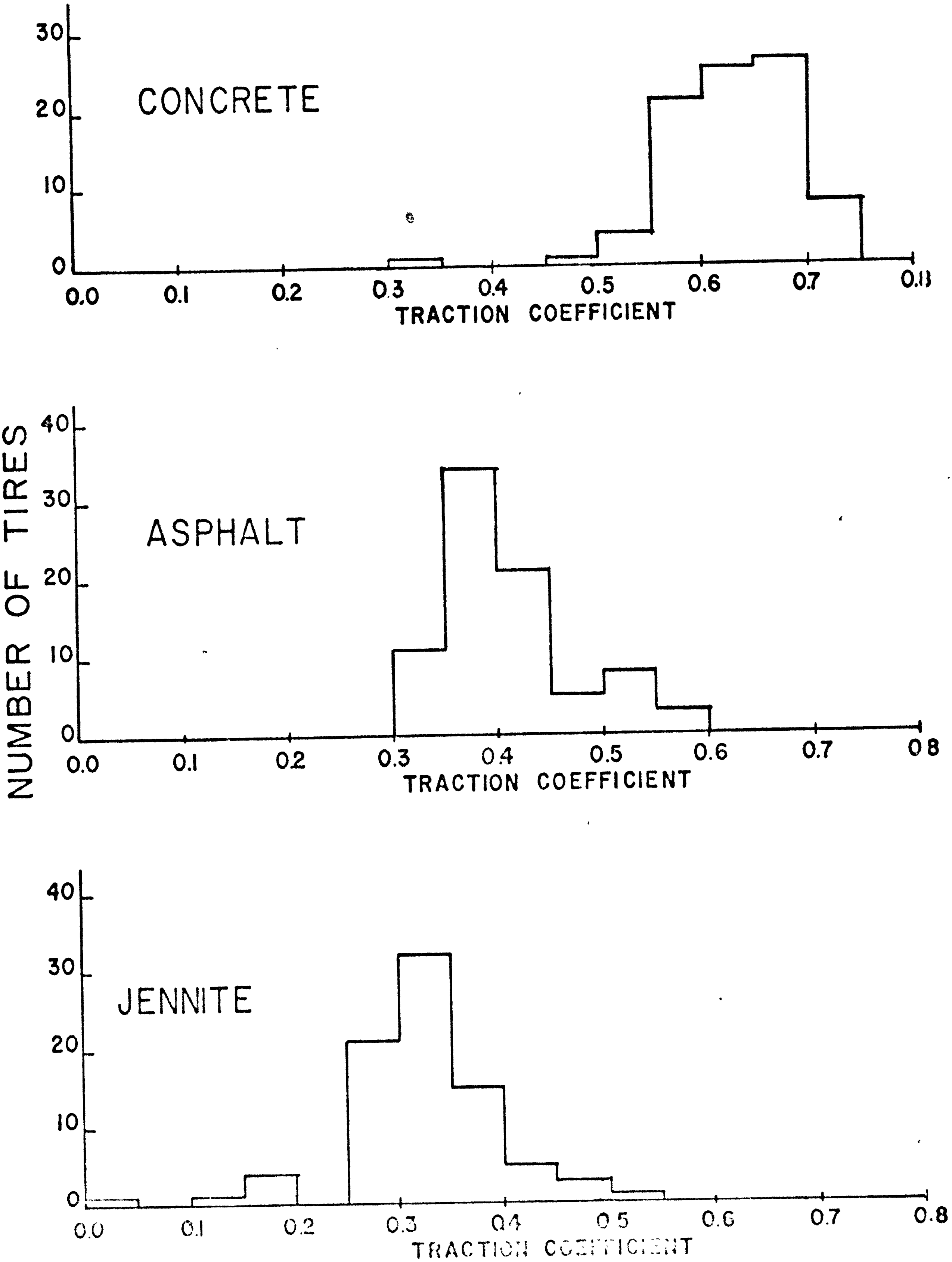
Each traction coefficient listed in Appendix I is the mean of ten replications. Below each value of the mean coefficient tabulated in Appendix I, the estimate of the standard deviation, $\hat{\sigma}$, computed from these ten replications is given, where $\hat{\sigma}$ is defined as

$$\hat{\sigma} = \sqrt{\frac{\sum_{i=1}^{n} (\bar{\mu} - \mu_i)^2}{n-1}}$$

where

$\bar{\mu}$ is the mean value of the traction coefficient

n is the number of replications


CONCRETE
30
20
10
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0.2
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0.5
0.6
0.7
0.8
TRACTION COEFFICIENT
NUMBER OF TIRES
ASPHALT
40
30
20
10
0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
TRACTION COEFFICIENT
JENNITE
40
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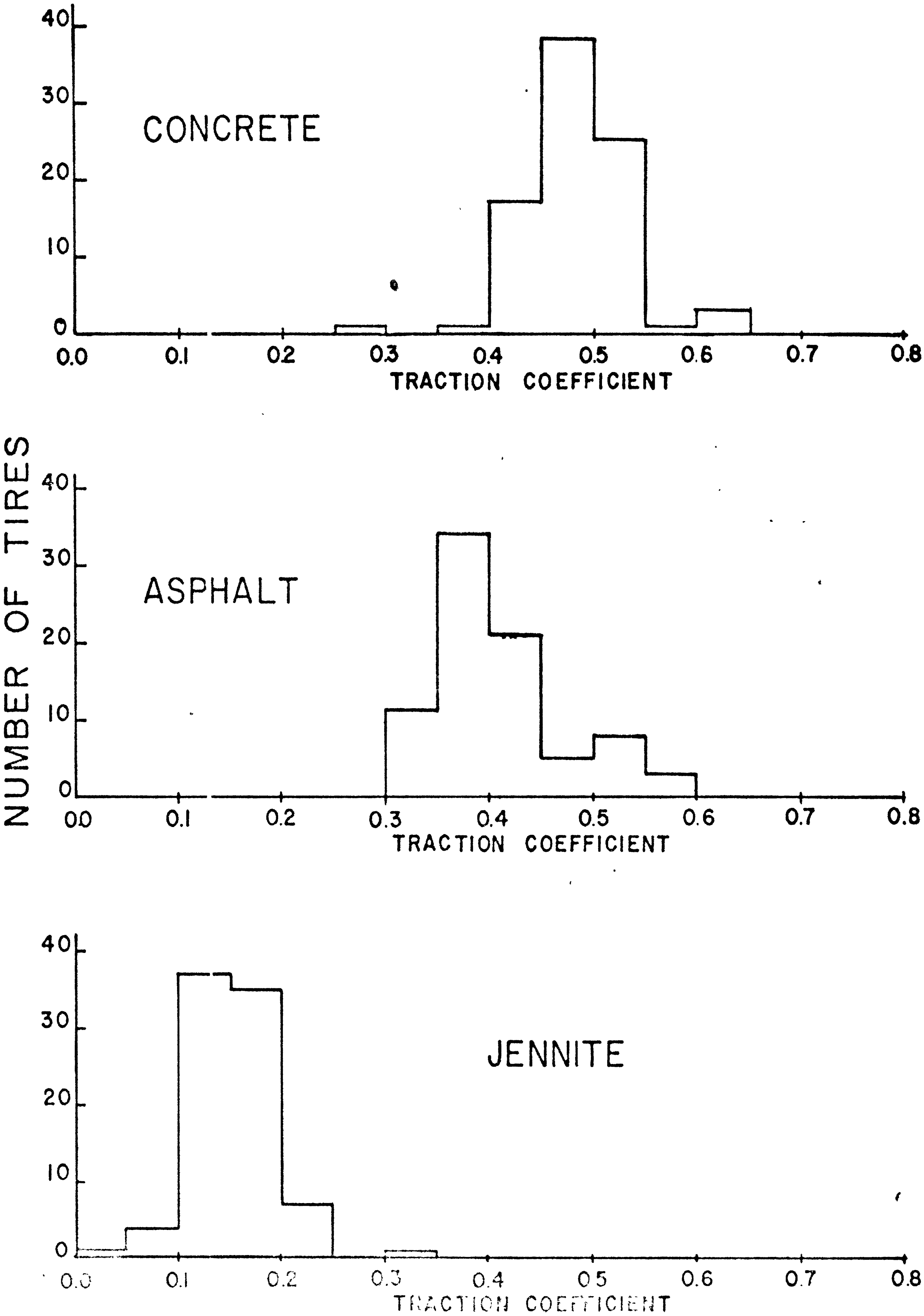
TRACTION COEFFICIENT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NUMBER OF TIRES
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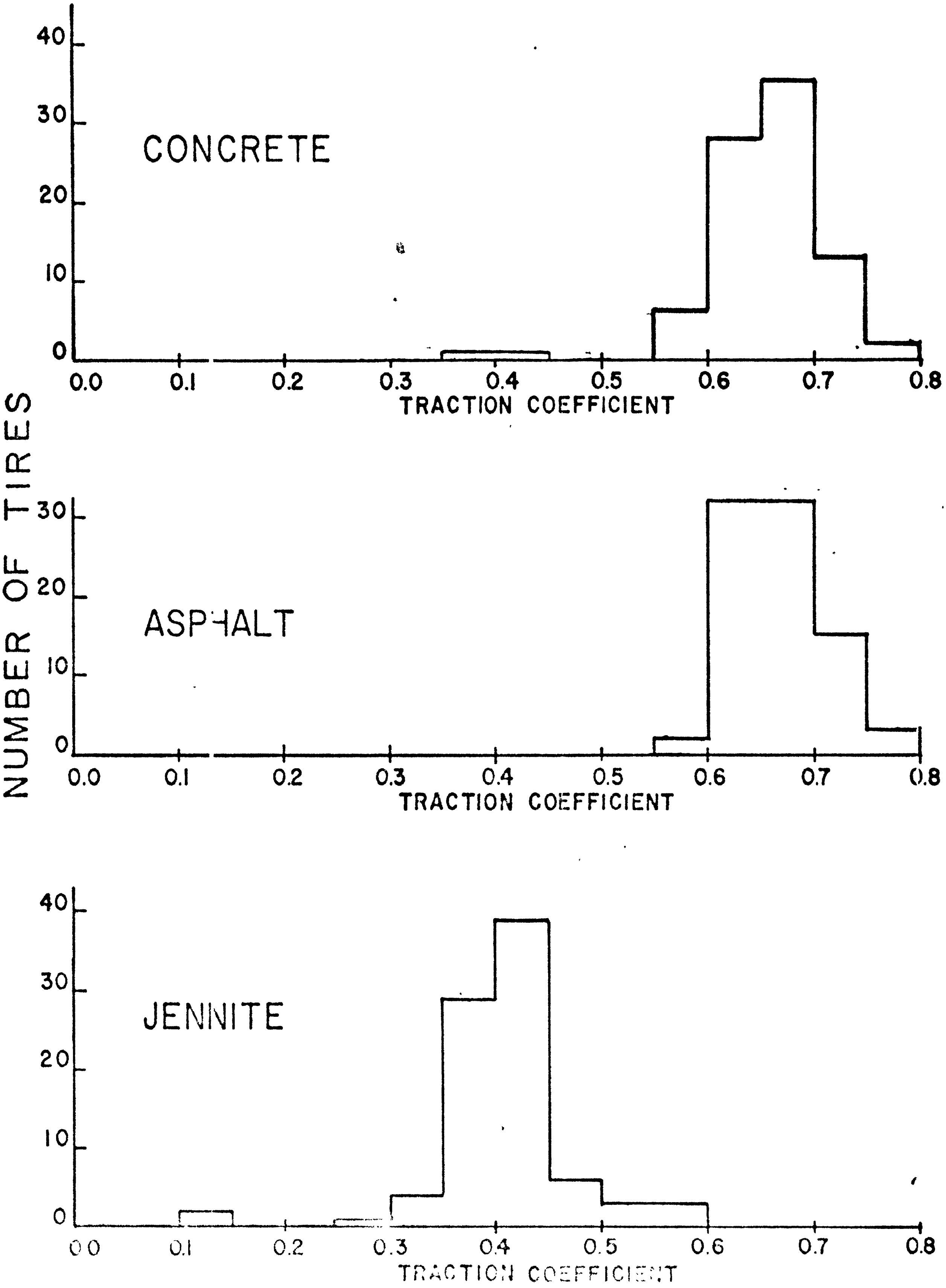
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JENNITE
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0.5
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0.7
0.8
TRACTION COEFFICIENT

[illegible]

4. FINDINGS

The results of this test program can be divided into two parts: (1) a discussion of findings of general interest and (2) a discussion of the feasibility of a traction grading procedure. Basic to both of these discussions, however, is a statistical analysis of the data in Appendix I.

Each of the means in Appendix I is considered to arise from a randomly distributed variable, namely, the ten individual traction measurements, and is thus subject to the simple statistics of normal or Gaussian distributions. Care must be taken in applying these statistics to any collection of these means, however, since the means are independent, each being produced by a different tire (although we would like to use Gaussian statistics to discuss the collection of means because of its relatively wide understanding and ease of interpretation). A Chi-Square, χ^2, distribution is generally used to describe a collection of independent random variables, $Z_1(k)$, $Z_2(k)$, $Z_3(k) \ldots Z_n(k)$, but for $n > 40$ this distribution is nearly Gaussian. For the data in this paper $n = 85$ or 87, and thus we can safely assume that the individual means will be normally distributed. As a check, the distributions in Figures 1, 2, and 3 appear to be Gaussian, and we shall proceed to analyze them as such.

4.1 GENERAL FINDINGS

Table 2 contains the means about which the O.E. tire sample (Figures 1-3) is centered. One notable feature of these means is that on all pavements, the free-rolling value of μ_y at $\alpha=8°$ is higher than the tire's maximum longitudinal capability, $\mu_x P$; and it is possible that $\mu_y P$ could be even greater at $\alpha=10°$ or $12°$. This property of the means is repeated in a tire-by-tire comparison using Appendix I. In virtually every one of the 259 individual cases (84 tires earch on asphalt and jennite and 86 tires on concrete) the value of $\mu_y P$ at $\alpha=8°$ exceeds that of $\mu_x P$ at $\alpha=0°$.

TABLE 2

THE MEAN VALUE ABOUT WHICH THE O.E. TIRE SAMPLE, FIGURES 1-3, IS CENTERED

	$\mu_x P$	$\mu_y P$	$\mu_x S$
Concrete	.62	.65	.48
Asphalt	.57	.66	.40
Jennite	.32	.40	.15

This result verifies many similar observations made over a long period of time at HSRI when examining the traction properties of many individual tires under a wide variety of loads, speeds, and surfaces.

Another feature of the O.E. population means in Table 2 is that $\mu_y P$ on asphalt exceeds that on concrete while both longitudinal measurements, $\mu_x P$, and the venerable skid number, $\mu_x S$, show the asphalt to be a lower coefficient pavement. This "paradox" suggests that tire traction cannot be characterized by a longitudinal measurement alone. In fact, if we again look at the individual cases in Appendix I, we find that the rank correlations between $\mu_y P$ and $\mu_x S$ and $\mu_y P$ and $\mu_x P$ are very poor on all pavements.

Several interesting comparisons between tires can be made using the data tabulated in Appendix I with the aid of the tire descriptions given in Table 1. For instance, Table 3 indicates that 15" diameter tires display a clear-cut trend of traction improvement over their 14", but otherwise identical, counterparts. In Table 4, a comparison is made between pairs of tires that are identical save for load rating (i.e., letter size) and which are loaded to 85 percent of the rated maximum at 24 psi. It is seen that the tires with higher load ratings generally exhibit higher

TABLE 3

PERCENT INCREASE IN TRACTION BETWEEN TIRES IDENTICAL BUT FOR DIAMETER

TIRE AND SIZE		Concrete $\mu_x P$	Concrete $\mu_x S$	Concrete $\mu_y P$	Asphalt $\mu_x P$	Asphalt $\mu_x S$	Asphalt $\mu_y P$	Jennite $\mu_x P$	Jennite $\mu_x S$	Jennite $\mu_y P$
EM 3, 4	14"									2%
EM 1, 2	15"	12%	6%	5%	18%	16%	13%	8%	27%	
JM 1,2	14"									
AM 3,4	15"	1%	5%		18%	6%	9%	1%	6%	2%

TABLE 4

PERCENT INCREASE IN TRACTION BETWEEN TIRES IDENTICAL BUT FOR LOAD RATING (LETTER SIZE)

TIRE AND SIZE		Concrete $\mu_x P$	Concrete $\mu_x S$	Concrete $\mu_y P$	Asphalt $\mu_x P$	Asphalt $\mu_x S$	Asphalt $\mu_y P$	Jennite $\mu_x P$	Jennite $\mu_x S$	Jennite $\mu_y P$
AM 3,4	G	1%		6%	12%	2%	5%	4%		
AM 5,6	H									
CM 3,4	F						1%		7%	
CM 1,2	H	9%	3%	2%	9%	1%		1%		6%
CM 9,10	D									8%
CM 7,8	E	7%	5%	3%	7%	3%	5%	7%	8%	
EM 11,12	F				4%		2%			
EM 9,10	H	10%	4%	2%		1%		29%	28%	16%

traction levels than their mates with a lower load rating. The results in Tables 3 and 4 were obtained by averaging the two means of a traction coefficient from a pair of identical tires (e.g., EM-3 and EM-4) and comparing this average to that yielded by a second pair of tires that are alike with respect to the manufacturer and model, but of a different diameter (e.g., EM-1 and EM-2) or load rating.

Table 5 was generated in a comparable manner. Note that a studded snow tire is compared to an identical, but non-studded, snow tire. The limits of confidence, ± two standard deviations of the mean, are included. The presence of the studs results in slightly lower values of $\mu_x P$ and $\mu_y P$ and does not affect $\mu_x S$.

TABLE 5

COMPARISON OF A STUDDED AND NON-STUDDED SNOW TIRE ON CONCRETE

	$\mu_x P$	$\mu_x S$	$\mu_y P$
Studded	.60 ± .01	.50 ± .01	.62 ± .01
Non-Studded	.61 ± .02	.50 ± .01	.65 ± .02

No one type of tire construction stands out as producing traction levels consistently above or below the overall O.E. means. In fact, as Tables 6(a), (b), and (c) show, the mean traction levels of the bias-ply, bias-belted, and radial-ply subgroups were all very close to one another and thus close to the mean levels of the whole O.E. sample.

TABLE 6

MEAN TRACTION LEVELS OF TIRE CONSTRUCTION SUBGROUPS OF THE O.E. SAMPLE

Construction	Number of Samples	$\mu_x P$	$\mu_x S$	$\mu_y P$
Bias	28	.60	.47	.63
Bias-Belted	38	.63	.50	.65
Radial	20	.62	.47	.67

a. Concrete

Construction	Number of Samples	$\mu_x P$	$\mu_x S$	$\mu_y P$
Bias	28	.57	.42	.67
Bias-Belted	36	.58	.41	.66
Radial	20	.56	.38	.66

b. Asphalt

Construction	Number of Samples	$\mu_x P$	$\mu_x S$	$\mu_y P$
Bias	28	.29	.15	.38
Bias Belted	36	.33	.15	.43
Radial	20	.33	.15	.41

c. Jennite

The upper and lower bounds on the range of tire traction coefficients exhibited by this O.E. sample are shown in Table 7. The traction levels achieved by the FM-1 and FM-2 tires on concrete and jennite are shown in parentheses since these tires are not typical of O.E. equipment, having a completely smooth tread. (On asphalt, in which the aggregate particles were large and protruded above the water layer, FM-1 and FM-2 acquitted themselves extremely well, ranking very near the top.) Table 7 shows that the highest traction levels in the O.E. tire field are separated from the lowest levels by amounts corresponding to 20 or 30 "skid numbers."

TABLE 7

RANGE OF O.E. TIRE TRACTION
(TIRES FM-1 OR FM-2 IN PARENTHESES)

	μ_x^P	μ_x^S	μ_y^P	
Concrete	.73	.61	.77	Maximum
	.52 (.34)	.41 (.29)	.57 (.38)	Minimum
Asphalt	.74	.57	.79	
	.44	.31	.58	
Jennite	.51	.32	.56	
	.17 (.08)	.08 (.05)	.27 (.12)	

4.2 FEASIBILITY OF TRACTION GRADING

It appears that a viable tire traction grading procedure can be established if the limits of resolution possessed by the measurement procedure do not encompass so large a part of the total range of tire traction as to render the bulk of the tire population indistinguishable from one another.

The limits of resolution are the bounds within which one tire cannot be distinguished from another on the basis of traction. These limits, once found, will preclude any changes in ranking should the traction measurements be repeated. The limits of resolution are a combination of the precision of the measurement and the repeatability of the measurement. For the purpose of this paper, the precision of a measurement of μ on the Mobile Tire Tester is between ±.003 and ±.005,* depending on vertical load. This uncertainty is at least a factor of 10 smaller than the uncertainty due to statistical fluctuations such as random environmental disturbances, and will therefore not be considered further.

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*These values result directly from the resolution limits of trace amplitude on the data recorder, a Honeywell Visicorder. They do not include the uncertainty in the calibration factors, C_x and C_y, by which these amplitudes were multiplied to obtain μ. In this study, however, C_x and C_y were held constant (370 and 390 pounds per inch, respectively) and therefore all determinations of μ_x or μ_y were equally influenced, and tire-to-tire comparisons restricted to this measuring system (the Mobile Tire Tester) are unaffected by any error in C_x or C_y. Indeed, the goals of this research program could have been achieved without introducing any calibration factors, but the reader will certainly feel more at ease comparing tires on the basis of traction coefficient rather than on the basis of inches of light beam deflection. If the data in Appendix I, however, is to be compared to data obtained on another measuring system, uncertainties in the calibration factors become quite important. For this reason, a complete analysis of measurement precision is carried out in Appendix II.

Since we are dealing with a Gaussian distribution of *means* (c.f. Appendix I and Figures 1-3), we cannot directly use the values of standard deviation, $\hat{\sigma}$, given in Appendix I to arrive at the limits of resolution. The $\hat{\sigma}$ values tabulated in Appendix I apply to the 10 individual measurements taken in each case and are indicative of the range within which a subsequent *single* measurement will fall, namely, 95 out of every 100 successive single measurements of a particular traction coefficient will be within $\pm\, 2\hat{\sigma}$ of each other. Any future measurement, however, will most certainly be quoted as the *mean* of a *set* of measurements and will be related to the values presented here by $\hat{\sigma}_m$, the standard deviation of the mean, where

$$\hat{\sigma}_m = \hat{\sigma}/\sqrt{n} \qquad (2)$$

For all of the means quoted in this study, n=10. Therefore, if the values of $\hat{\sigma}$ in the Appendix are divided by $\sqrt{10}$, $\hat{\sigma}_m$ results, and we can expect that 95 out of every 100 future determinations of the means in Appendix I will be within $\pm\, 2\hat{\sigma}_m$ of each other. Thus the limits of resolution are $\pm\, 2\hat{\sigma}_m$.

Since we are discussing the feasibility of establishing a traction standard, it seems appropriate to go one step beyond merely defining $\pm\, 2\hat{\sigma}_m$ to be the limits of resolution. Any value of $\hat{\sigma}_m$ computed from Appendix I must be regarded as an estimate of the universe σ_m, being based on a value of $\hat{\sigma}$ taken from only one set of measurements rather than, say, 10^4 sets. In order to assess the accuracy of our estimate of σ_m, we must find σ_s, the standard deviation of the standard deviation. This procedure is contained in Appendix III, and the results assure us that a value of $\hat{\sigma}_m$ computed from a value of $\hat{\sigma}$ in Appendix I, using Equation (2), will be accurate.

This being the case, 771 values of $\hat{\sigma}_m$ were computed from Appendix I, one from each value of $\hat{\sigma}$ for $\mu_x P$, $\mu_y P$, and $\mu_x S$ for all tires on all surfaces. These values of $\hat{\sigma}_m$ were collected into nine groups (three traction coefficients x three pavements) and each group was analyzed for mean value, $\bar{\hat{\sigma}}_m$, and standard deviation, $\hat{\sigma}_{ms}$. The results are presented in Table 8.

TABLE 8

MEAN VALUES OF COLLECTIONS OF $\hat{\sigma}_m$ PLUS STANDARD DEVIATIONS OF THE COLLECTIONS

	$\mu_x P$	$\mu_x S$	$\mu_y P$	
Concrete	.0103	.0064	.0086	$\bar{\hat{\sigma}}_m$
	.0041	.0026	.0029	$\hat{\sigma}_{ms}$
Asphalt	.0114	.0105	.0108	
	.0038	.0036	.0042	
Jennite	.0139	.0066	.0130	
	.0041	.0037	.0038	

For the $\mu_x P$-concrete combination in Table 8, 97.5% of the values of $\hat{\sigma}_m$ obtained were below .02, i.e., $\bar{\hat{\sigma}}_m + 2\hat{\sigma}_{ms} = .0185 = .02$. However, 84% of the values, $\bar{\hat{\sigma}}_m + \hat{\sigma}_{ms}$ were below .01. These values result in limits of resolution, $\pm 2\hat{\sigma}_m$, of ±.04 and ±.02, respectively. These calculations represent the "worst case" limits of resolution for $\mu_x P$ on concrete and indicate the maximum effect that the spread in the 10 individual measurements will have on the repeatability of the mean value obtained from them. Most tires will have limits of resolution below these values and will therefore better lend themselves to a traction grading procedure. The "worst case" limits of resolution for 97.5% and for 84% of the O.E. sample are given for all nine cases in Table 9.

TABLE 9

LIMITS OF RESOLUTION FOR TRACTION GRADING

	μ_xP	μ_xS	μ_yP	
Concrete	±.04	±.02	±.02	97.5%
	±.02	±.02	±.02	84%
Asphalt	±.04	±.04	±.04	
	±.04	±.02	±.04	
Jennite	±.04	±.02	±.04	
	±.04	±.02	±.04	

Having established the limits of resolution, we turn now to the question of how much of the O.E. population will be covered by these limits. Utilizing the information in Table 7 or the histograms in Figures 1-3, it is apparent that the limits of resolution are small enough to render the bulk of the total population distinguishable from one another. Thus the O.E. tire population can be graded according to traction. The exact percentages of the O.E. population covered by the 97.5% limits of resolution from Table 9 are given in Table 10. The percent covered by the 84% limits, if different from the 97.5% limits, is given in parentheses.

TABLE 10

PERCENT OF TOTAL O.E. TRACTION RANGE INCLUDED WITHIN THE LIMITS OF RESOLUTION SHOWN IN TABLE 9

	μ_xP	μ_xS	μ_yP
Concrete	38% (19%)	20%	20%
Asphalt	27%	31% (15%)	38%
Jennite	24%	17%	28%

Rather than grading the O.E. population as a whole, however, a procedure that would be more useful to the consumer would be to grade tires of the same size, since any one tire buyer would be unlikely to choose among tires of more than one size. Inspection of the data in Appendix I shows that statistically valid gradations are possible in nearly all tire sizes tested if the pavement is limited to concrete. [The larger $\pm 2\hat{\sigma}_m$ limits on asphalt and jennite nullify all but a few differences in traction on these pavements within any one tire size.] Table 11 displays the maximum and minimum tractions obtained within each tire size on concrete. In only three cells in the table did all of the tires of one size produce tractions that were within the $\pm 2\hat{\sigma}_m$ limits, thereby rendering the tires indistinguishable. Statistically valid gradations are possible in all other cells.

TABLE 11

MAXIMUM AND MINIMUM TRACTION LEVELS ON CONCRETE FOR VARIOUS TIRE SIZES

Tire Size Tested	μ_x^P	μ_x^S	μ_y^P	
C-13	.59	.48	.57	Maximum (Best Tire)
	.52	.42	.52	Minimum (Worst Tire)
E-14	No Differences	.48	No Differences	
		.44		
F-14	.67	.51	.67	
	.58	.45	.60	
G-14	.69	.51	.72	
	.62	.49	.64	
F-15	.64	.49	.69	
	.56	.45	.59	
G-15	.73	.61	.77	
	.65	.50	.68	
H-15	.70	.52	.69	
	.57	.43	.60	
G-15 Snow Tires	.68	.54	No Differences	
	.60	.48		

The fact that two tires of each type were tested enables us to make a beginning assessment of the influence that tire non-uniformity with respect to traction will have on the feasiblity of a traction grading procedure. On the 43 pairs tested on concrete, the two "identical" tires in 37 of these pairs, or 86%, were within the ±.02 limits of resolution for $\mu_x P$ and were thus truly identical in peak braking force production. The occurrence of identical pairs for $\mu_x S$ and $\mu_y P$ on concrete was 86% and 91%, respectively. In the light of this very high incidence of identicalness we can surmise that the testing of more than two specimens each would have further reduced the number of pairs whose members exhibited tractions differing by more than the limits of resolution. Thus, it appears that tire quality control with respect to traction is quite good, and tire-to-tire variations in traction between supposedly identical tires are so small that they would not seem to be a factor in the establishment of a tire traction grading procedure. More research in this area is warranted, however, to confirm this indication.

4.3 CONCLUSIONS

The data shows that tire traction grading is possible on concrete. The traction differences between tires are large enough and the variability of the measurement is small enough so that classifications can be established. These classifications must take into account that the longitudinal performance of a tire correlates very poorly with the lateral performance, and thus lateral as well as longitudinal measurements of tire shear force must be made.

In addition, this study shows that, on wet surfaces, the maximum lateral force capability of most tires exceeds the maximum braking force capability. Also, the "skid number" is shown to classify the asphalt pavement as significantly "slipperier" than the concrete surface, while lateral force measurements show the asphalt to be equal or superior to the concrete on the basis of traction.

APPENDIX I

μ_x, μ_y AND STANDARD DEVIATIONS

CONCRETE

TIRE	0°		8°		8°		VALUE
	$\mu_x P$	$\mu_x S$	$\mu_y P$	$\mu_y S$	$\mu_x P$	$\mu_x S$	
AM-1	.59	.44	.62	.09	.51	.43	MEAN
	.024	.019	.045	.017	.029	.026	$\hat{\sigma}$
AM-2	.58	.44	.63	.10	.52	.44	
	.031	.019	.039	.003	.034	.035	
AM-3	.67	.52	.69	.12	.63	.51	
	.027	.018	.031	.021	.018	.017	
AM-4	.73	.50	.72	.12	.67	.54	
	.027	.014	.026	.010	.022	.021	
AM-5	.69	.52	.66	.13	.63	.51	
	.032	.022	.029	.007	.019	.015	
AM-6	.68	.50	.67	.12	.62	.51	
	.020	.018	.016	.010	.015	.019	
AM-7	.58	.45	.64	.11	.51	.42	
	.027	.010	.045	.017	.014	.015	
AM-8	.58	.44	.67	.11	.51	.44	
	.023	.026	.043	.017	.021	.016	
AM-9	.71	.54	.73	.11	.65	.56	
	.021	.029	.027	.010	.026	.025	
AM-10	.73	.55	.74	.11	.66	.57	
	.030	.020	.030	.013	.025	.025	
AM-11	.71	.61	.77	.11	.69	.63	
	.043	.022	.022	.012	.021	.044	
AM-12	.70	.61	.77	.10	.69	.60	
	.046	.038	.028	.012	.011	.028	

<u>CONCRETE</u>

	0°		8°		8°		
TIRE	$\mu_x P$	$\mu_x S$	$\mu_y P$	$\mu_y S$	$\mu_x P$	$\mu_x S$	VALUE
AM-13	.60	.49	.66	.10	.57	.48	MEAN
	.020	.019	.026	.005	.015	.019	σ
AM-14	.62	.50	.64	.10	.58	.50	
	.029	.015	.025	.004	.016	.011	
AM-15	.59	.50	.61	.10	.58	.51	
	.011	.010	.020	.010	.010	.011	
AM-16	.61	.51	.64	.10	.57	.50	
	.016	.012	.038	.014	.020	.008	
BM-1	.60	.47	.66	.10	.54	.45	
	.026	.018	.031	.015	.031	.036	
BM-2	.59	.44	.65	.11	.54	.45	
	.010	.012	.031	.009	.029	.032	
BM-3	.65	.49	.61	.12	.55	.45	
	.041	.015	.024	.006	.028	.022	
BM-4	.59	.46	.62	.11	.56	.48	
	.027	.011	.023	.011	.020	.017	
BM-5	.66	.51	.69	.11	.65	.54	
	.044	.021	.025	.012	.035	.026	
BM-6	.65	.54	.70	.11	.63	.52	
	.046	.015	.022	.013	.027	.022	
BM-7	.67	.49	.65	.12	.61	.52	
	.031	.017	.021	.011	.023	.021	
BM-8	.66	.51	.67	.12	.61	.51	
	.023	.021	.024	.010	.014	.015	

CONCRETE

	0°		8°		8°		
TIRE	$\mu_x P$	$\mu_x S$	$\mu_y P$	$\mu_y S$	$\mu_x P$	$\mu_x S$	VALUE
BM-9	.63	.46	.64	.12	.58	.47	MEAN
	.025	.011	.028	.015	.026	.015	$\hat{\sigma}$
BM-10	.64	.49	.65	.11	.59	.49	
	.033	.021	.022	.013	.028	.026	
BM-11	.68	.54	.66	.10	.61	.50	
	.027	.020	.033	.014	.014	.020	
BM-12	.65	.52	.67	.11	.63	.54	
	.032	.026	.025	.009	.024	.028	
CM-1	.70	.49	.69	.11	.62	.50	
	.049	.020	.015	.014	.019	.018	
CM-2	.68	.49	.69	.11	.62	.48	
	.047	.016	.022	.013	.033	.020	
CM-3	.63	.47	.69	.12	.56	.46	
	.028	.014	.022	.014	.020	.014	
CM-4	.64	.48	.66	.10	.58	.50	
	.031	.014	.046	.018	.024	.037	
CM-5	.60	.46	.66	.12	.60	.51	
	.040	.016	.023	.015	.018	.014	
CM-6	.63	.48	.65	.11	.58	.48	
	.026	.018	.012	.024	.029	.019	
CM-7	.57	.43	.65	.12	.56	.48	
	.015	.022	.030	.011	.020	.013	
CM-8	.60	.48	.64	.11	.55	.47	
	.025	.014	.023	.010	.014	.014	

CONCRETE

	0°		8°		8°		
TIRE	μ_x^P	μ_x^S	μ_y^P	μ_y^S	μ_x^P	μ_x^S	VALUE
CM-9	.53	.42	.65	.10	.52	.44	MEAN
	.029	.023	.060	.010	.025	.028	$\hat{\sigma}$
CM-10	.56	.45	.60	.10	.50	.42	
	.029	.015	.033	.014	.028	.029	
CM-11	.60	.46	.61	.10	.54	.46	
	.026	.025	.018	.006	.037	.037	
CM-12	.60	.47	.60	.09	.52	.44	
	.030	.014	.028	.012	.024	.045	
CM-13	.65	.48	.64	.09	.60	.50	
	.026	.012	.029	.026	.017	.015	
CM-14	.65	.50	.67	.11	.61	.50	
	.017	.017	.027	.014	.014	.018	
CM-15	.59	.48	.57	.10	.52	.45	
	.037	.018	.024	.009	.019	.013	
CM-16	.54	.42	.59	.11	.52	.45	
	.026	.018	.030	.016	.015	.013	
DM-1	.56	.41	.72	.10	.55	.45	
	.040	.037	.042	.007	.034	.026	
DM-2	.57	.44	.70	.09	.57	.48	
	.035	.030	.036	.012	.029	.033	
DM-3	.63	.51	.63	.12	.61	.51	
	.057	.023	.016	.019	.016	.025	
DM-4	.67	.52	.63	.13	.62	.52	
	.048	.017	.018	.018	.014	.020	

CONCRETE

	0°		8°		8°		
TIRE	$\mu_x P$	$\mu_x S$	$\mu_y P$	$\mu_y S$	$\mu_x P$	$\mu_x S$	VALUE
DM-5	.65	.51	.65	.11	.59	.51	MEAN
	.031	.014	.031	.010	.017	.013	$\hat{\sigma}$
DM-6	.66	.48	.70	.12	.60	.51	
	.041	.014	.033	.009	.024	.016	
DM-7	.60	.47	.62	.10	.55	.46	
	.039	.014	.031	.010	.018	.020	
DM-8	.58	.45	.66	.12	.56	.48	
	.026	.011	.022	.015	.017	.017	
DM-9	.68	.54	.70	.13	.63	.54	
	.030	.021	.016	.012	.014	.018	
DM-10	.63	.51	.67	.11	.58	.50	
	.027	.018	.024	.011	.032	.012	
DM-11	.65	.50	.64	.12	.60	.52	
	.039	.007	.027	.014	.020	.021	
DM-12	.64	.50	.65	.12	.59	.48	
	.025	.010	.022	.014	.022	.029	
EM-1	.66	.51	.68	.10	.64	.55	
	.029	.017	.027	.008	.030	.028	
EM-2	.72	.54	.68	.09	.62	.53	
	.050	.033	.036	.024	.022	.028	
EM-3	.62	.50	.64	.09	.58	.49	
	.032	.019	.037	.011	.021	.015	
EM-4	.63	.49	.65	.09	.59	.49	
	.025	.016	.031	.015	.019	.021	

CONCRETE

	0°		8°		8°		
TIRE	$\mu_x P$	$\mu_x S$	$\mu_y P$	$\mu_y S$	$\mu_x P$	$\mu_x S$	VALUE
EM-5	.52	.42	.62	.10	.49	.45	MEAN
	.051	.016	.039	.015	.061	.024	$\hat{\sigma}$
EM-6	.54	.47	.59	.10	.52	.40	
	.039	.026	.035	.005	.023	.018	
EM-7	.57	.43	.57	.10	.52	.45	
	.024	.012	.024	.006	.034	.030	
EM-8	.57	.47	.57	.10	.51	.44	
	.044	.029	.027	.010	.028	.029	
EM-9	.61	.48	.60	.11	.60	.51	
	.025	.020	.019	.011	.021	.020	
EM-10	.63	.49	.61	.12	.58	.49	
	.037	.023	.018	.013	.018	.019	
EM-11	.56	.45	.59	.12	.56	.46	
	.030	.016	.026	.019	.017	.022	
EM-12	.58	.48	.60	.12	.54	.45	
	.051	.032	.027	.011	.026	.026	
EM-13	.73	.60	.72	.09	.65	.58	
	.038	.032	.025	.019	.024	.026	
EM-14	.66	.48	.63	.08	.59	.47	
	.029	.025	.033	.013	.012	.019	
FM-1	.47	.38	.38	.06	.34	.30	
	.105	.085	.022	.008	.031	.036	
FM-2	.34	.29	.41	.07	.35	.31	
	.048	.038	.065	.016	.035	.028	
GM-1	.59	.44	.66	.11	.55	.45	
	.029	.022	.024	.010	.020	.030	

CONCRETE

	0°		8°		8°		
TIRE	$\mu_x P$	$\mu_x S$	$\mu_y P$	$\mu_y S$	$\mu_x P$	$\mu_x S$	VALUE
GM-2	.56	.44	.68	.10	.50	.40	MEAN
	.024	.013	.021	.017	.018	.020	$\hat{\sigma}$
HM-1	.60	.46	.69	.10	.56	.45	
	.021	.012	.040	.012	.022	.021	
HM-2	.61	.44	.70	.09	.56	.46	
	.033	.021	.021	.020	.024	.026	
HM-3	.57	.43	.64	.10	.51	.42	
	.026	.018	.026	.007	.013	.018	
HM-4	.57	.44	.64	.11	.52	.44	
	.019	.021	.032	.008	.019	.022	
ZM-1	.62	.47	.61	.11	.58	.47	
	.051	.020	.018	.018	.020	.018	
ZM-2	.63	.45	.70	.13	.59	.48	
	.054	.044	.015	.011	.015	.017	
ZM-3	.65	.47	.65	.12	.58	.52	
	.033	.024	.026	.009	.019	.038	
ZM-4	.65	.48	.63	.12	.59	.47	
	.038	.039	.023	.009	.032	.017	
JM-1	.69	.50	.72	.10	.67	.52	
	.037	.019	.026	.010	.018	.020	
JM-2	.69	.49	.69	.11	.64	.54	
	.025	.022	.008	.011	.024	.041	
JM-3	.68	.49	.68	.08	.61	.49	
	.016	.010	.023	.024	.034	.043	
JM-4	.67	.49	.71	.12	.63	.52	
	.024	.028	.018	.015	.023	.027	

ASPHALT

	0°		8°		8°		
TIRE	$\mu_x P$	$\mu_x S$	$\mu_y P$	$\mu_y S$	$\mu_x P$	$\mu_x S$	VALUE
AM-1	.52	.35	.64	.08	.43	.33	MEAN
	.040	.040	.041	.012	.048	.043	∂
AM-2	.48	.32	.66	.10	.43	.32	
	.030	.025	.033	.003	.027	.023	
AM-3	.64	.43	.73	.10	.54	.42	
	.047	.022	.030	.014	.021	.026	
AM-4	.70	.43	.72	.10	.59	.44	
	.057	.021	.030	.010	.024	.022	
AM-5	.60	.41	.69	.12	.54	.42	
	.032	.026	.010	.008	.024	.019	
AM-6	.60	.43	.69	.10	.55	.43	
	.023	.038	.018	.010	.038	.028	
AM-7	.49	.38	.64	.11	.43	.35	
	.033	.023	.041	.014	.024	.010	
AM-8	.45	.34	.66	.10	.44	.35	
	.032	.025	.039	.014	.042	.018	
AM-9	.63	.44	.66	.09	.57	.46	
	.034	.020	.091	.013	.027	.028	
AM-10	.66	.44	.70	.09	.55	.45	
	.034	.020	.036	.011	.030	.037	
AM-11	.69	.57	.79	.10	.63	.59	
	.025	.026	.018	.008	.034	.033	
AM-12	.65	.52	.75	.10	.57	.50	
	.023	.050	.029	.005	.021	.036	

ASPHALT

	0°		8°		8°		
TIRE	$\mu_x P$	$\mu_x S$	$\mu_y P$	$\mu_y S$	$\mu_x P$	$\mu_x S$	VALUE
AM-13	.66	.53	.67	.10	.59	.50	MEAN
	.022	.038	.031	.011	.019	.045	$\hat{\sigma}$
AM-14	.63	.52	.64	.11	.62	.60	
	.020	.034	.017	.012	.014	.014	
AM-15	--	--	--	--	--	--	
	--	--	--	--	--	--	
AM-16	--	--	--	--	--	--	
	--	--	--	--	--	--	
BM-1	.50	.34	.61	.09	.42	.32	
	.031	.026	.039	.012	.032	.023	
BM-2	.47	.31	.61	.10	.43	.33	
	.030	.019	.034	.015	.041	.045	
BM-3	.58	.42	.63	.10	.50	.38	
	.043	.039	.039	.010	.014	.021	
BM-4	.54	.36	.63	.11	.51	.39	
	.054	.021	.029	.008	.033	.029	
BM-5	.67	.54	.75	.10	.66	.57	
	.037	.036	.029	.008	.029	.022	
BM-6	.68	.56	.73	.10	.61	.49	
	.024	.035	.029	.010	.031	.048	
BM-7	.62	.43	.67	.11	.57	.42	
	.030	.034	.020	.010	.016	.021	
BM-8	.56	.41	.71	.12	.54	.43	
	.036	.025	.025	.013	.035	.031	

ASPHALT

	0°		8°		8°		
TIRE	$\mu_x P$	$\mu_x S$	$\mu_y P$	$\mu_y S$	$\mu_x P$	$\mu_x S$	VALUE
BM-9	.55	.39	.67	.11	.52	.40	MEAN
	.052	.038	.033	.006	.040	.025	$\hat{\sigma}$
BM-10	.54	.39	.67	.10	.50	.38	
	.041	.026	.038	.008	.048	.029	
BM-11	.69	.54	.70	.11	.67	.53	
	.011	.031	.030	.010	.015	.036	
BM-12	.70	.54	.71	.11	.64	.53	
	.032	.034	.017	.012	.024	.033	
CM-1	.61	.39	.68	.10	.56	.42	
	.046	.033	.032	.015	.027	.032	
CM-2	.64	.40	.68	.09	.52	.39	
	.042	.033	.029	.015	.027	.024	
CM-3	.58	.38	.69	.10	.48	.36	
	.041	.043	.047	.016	.038	.036	
CM-4	.57	.40	.68	.08	.50	.38	
	.041	.072	.052	.013	.041	.045	
CM-5	.57	.39	.72	.10	.55	.42	
	.047	.061	.032	.015	.039	.029	
CM-6	.57	.39	.69	.10	.53	.39	
	.047	.045	.027	.025	.034	.031	
CM-7	.50	.33	.68	.10	.49	.38	
	.018	.026	.036	.013	.034	.028	
CM-8	.56	.40	.69	.10	.50	.39	
	.031	.048	.027	.008	.039	.028	

ASPHALT

	0°		8°		8°		
TIRE	μ_x^P	μ_x^S	μ_y^P	μ_y^S	μ_x^P	μ_x^S	VALUE
CM-9	.50	.35	.67	.08	.47	.34	MEAN
	.027	.049	.039	.015	.018	.025	$\hat{\sigma}$
CM-10	.49	.36	.64	.08	.45	.32	
	.048	.042	.021	.017	.022	.025	
CM-11	.53	.37	.62	.09	.47	.38	
	.024	.014	.025	.009	.042	.029	
CM-12	.53	.38	.59	.08	.48	.37	
	.041	.028	.027	.013	.022	.018	
CM-13	.67	.47	.67	.09	.62	.47	
	.028	.024	.026	.024	.014	.022	
CM-14	.69	.50	.70	.10	.61	.47	
	.025	.032	.023	.014	.021	.029	
CM-15	.55	.41	.64	.09	.49	.41	
	.024	.026	.061	.012	.023	.028	
CM-16	.51	.38	.62	.10	.49	.42	
	.025	.028	.026	.010	.025	.033	
DM-1	.51	.33	.68	.09	.48	.39	
	.059	.068	.054	.011	.050	.035	
DM-2	.45	.35	.62	.08	.48	.37	
	.035	.043	.060	.020	.044	.061	
DM-3	.58	.40	.65	.10	.51	.40	
	.040	.049	.034	.012	.012	.018	
DM-4	.57	.43	.64	.11	.52	.40	
	.054	.037	.037	.008	.023	.026	

ASPHALT

	0°		8°		8°		
TIRE	$\mu_x P$	$\mu_x S$	$\mu_y P$	$\mu_y S$	$\mu_x P$	$\mu_x S$	VALUE
DM-5	.55	.39	.64	.09	.47	.38	MEAN
	.033	.025	.040	.015	.025	.024	σ
DM-6	.56	.38	.68	.10	.50	.38	
	.051	.036	.020	.009	.028	.014	
DM-7	.51	.36	.62	.10	.47	.37	
	.032	.019	.043	.011	.035	.032	
DM-8	.50	.36	.64	.12	.48	.39	
	.034	.028	.035	.008	.023	.034	
DM-9	.63	.47	.72	.11	.54	.44	
	.036	.036	.051	.012	.033	.028	
DM-10	.57	.45	.68	.12	.53	.44	
	.022	.021	.026	.008	.032	.030	
DM-11	.54	.37	.63	.10	.51	.39	
	.042	.034	.031	.014	.058	.032	
DM-12	.55	.37	.66	.09	.52	.37	
	.054	.043	.061	.014	.075	.026	
EM-1	.63	.43	.70	.08	.58	.43	
	.035	.036	.034	.008	.031	.022	
EM-2	.66	.43	.72	.09	.57	.44	
	.049	.026	.022	.009	.031	.036	
EM-3	.52	.37	.61	.08	.44	.37	
	.029	.040	.026	.012	.027	.034	
EM-4	.57	.37	.65	.07	.49	.36	
	.030	.030	.013	.013	.033	.018	

ASPHALT

	0°		8°		8°		
TIRE	$\mu_x P$	$\mu_x S$	$\mu_y P$	$\mu_y S$	$\mu_x P$	$\mu_x S$	VALUE
EM-5	.44	.33	.63	.09	.46	.34	MEAN
	.029	.020	.032	.007	.037	.025	$\hat{\sigma}$
EM-6	.46	.35	.58	.09	.44	.31	
	.037	.036	.051	.014	.037	.019	
EM-7	.53	.35	.64	.09	.50	.34	
	.032	.029	.030	.008	.043	.026	
EM-8	.50	.34	.64	.10	.46	.34	
	.035	.024	.041	.012	.029	.040	
EM-9	.53	.39	.63	.10	.50	.39	
	.037	.044	.017	.008	.040	.031	
EM-10	.54	.38	.63	.10	.47	.38	
	.040	.037	.014	.012	.025	.025	
EM-11	.55	.37	.64	.11	.53	.41	
	.044	.027	.049	.021	.062	.048	
EM-12	.56	.39	.64	.11	.51	.39	
	.027	.026	.022	.009	.050	.025	
EM-13	.69	.55	.71	.09	.59	.51	
	.032	.033	.019	.015	.026	.044	
EM-14	.74	.50	.70	.08	.61	.47	
	.040	.025	.017	.018	.010	.030	
FM-1	.73	.49	.69	.08	.57	.45	
	.054	.034	.054	.012	.037	.043	
FM-2	.64	.41	.71	.10	.62	.46	
	.084	.052	.062	.017	.037	.046	
GM-1	.53	.36	.66	.12	.50	.40	
	.021	.028	.034	.024	.043	.038	

ASPHALT

	0°		8°		8°		
TIRE	$\mu_x P$	$\mu_x S$	$\mu_y P$	$\mu_y S$	$\mu_x P$	$\mu_x S$	VALUE
GM-2	.55	.40	.66	.10	.45	.37	MEAN
	.047	.043	.049	.012	.031	.030	δ
HM-1	.52	.35	.64	.08	.47	.34	
	.030	.032	.050	.013	.043	.023	
HM-2	.50	.34	.62	.07	.46	.33	
	.026	.020	.051	.015	.022	.026	
HM-3	.49	.35	.60	.08	.40	.30	
	.013	.043	.045	.019	.023	.017	
HM-4	.46	.31	.61	.10	.41	.33	
	.020	.014	.040	.008	.012	.017	
ZM-1	.53	.39	.61	.09	.50	.38	
	.025	.028	.024	.017	.026	.031	
ZM-2	.56	.41	.67	.12	.50	.39	
	.064	.064	.035	.008	.024	.031	
ZM-3	.56	.39	.63	.11	.50	.43	
	.037	.027	.035	.027	.055	.039	
ZM-4	.58	.38	.63	.11	.52	.38	
	.052	.029	.024	.029	.031	.030	
JM-1	.57	.40	.68	.08	.52	.40	
	.024	.020	.027	.011	.010	.029	
JM-2	.57	.41	.65	.09	.51	.41	
	.039	.035	.029	.014	.046	.036	
JM-3	.62	.45	.69	.08	.58	.47	
	.043	.035	.032	.019	.039	.044	
JM-4	.62	.42	.69	.10	.57	.45	
	.029	.025	.030	.015	.028	.032	

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TIRE	0°		8°		8°		
	$\mu_x P$	$\mu_x S$	$\mu_y P$	$\mu_y S$	$\mu_x P$	$\mu_x S$	VALUE
AM-1	.31	.16	.35	.04	.19	.13	MEAN
	.055	.034	.046	.015	.018	.022	$\hat{\sigma}$
AM-2	.32	.17	.36	.06	.20	.14	
	.082	.053	.036	.014	.040	.020	
AM-3	.32	.16	.42	.06	.22	.14	
	.037	.020	.043	.011	.040	.011	
AM-4	.43	.18	.45	.06	.29	.18	
	.077	.061	.040	.014	.044	.028	
AM-5	.39	.18	.44	.07	.26	.18	
	.036	.013	.039	.010	.034	.014	
AM-6	.33	.16	.43	.06	.25	.18	
	.048	.021	.042	.011	.030	.011	
AM-7	.31	.18	.40	.07	.20	.15	
	.025	.016	.020	.013	.010	.016	
AM-8	.28	.13	.41	.07	.21	.15	
	.032	.022	.031	.007	.020	.013	
AM-9	.41	.18	.48	.07	.27	.20	
	.033	.006	.053	.014	.052	.024	
AM-10	.40	.18	.43	.06	.24	.19	
	.022	.013	.048	.014	.036	.018	
AM-11	.51	.23	.56	.07	.38	.23	
	.047	.011	.035	.012	.040	.027	
AM-12	.45	.23	.52	.06	.30	.20	
	.050	.035	.049	.012	.052	.011	

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	0°		8°		8°		
TIRE	$\mu_x P$	$\mu_x S$	$\mu_y P$	$\mu_y S$	$\mu_x P$	$\mu_x S$	VALUE
AM-13	.32	.18	.39	.05	.20	.13	MEAN
	.033	.017	.057	.012	.036	.024	$\hat{\sigma}$
AM-14	.33	.18	.44	.06	.29	.23	
	.048	.012	.053	.011	.031	.026	
AM-15	--	--	--	--	--	--	
	--	--	--	--	--	--	
AM-16	--	--	--	--	--	--	
	--	--	--	--	--	--	
BM-1	.29	.14	.38	.05	.19	.13	
	.040	.017	.024	.011	.022	.021	
BM-2	.26	.14	.42	.06	.21	.13	
	.032	.021	.052	.010	.051	.028	
BM-3	.33	.20	.39	.07	.22	.13	
	.055	.037	.026	.007	.036	.016	
BM-4	.28	.13	.40	.06	.23	.15	
	.033	.015	.031	.010	.034	.020	
BM-5	.41	.22	.43	.06	.30	.19	
	.027	.030	.024	.020	.039	.019	
BM-6	.43	.32	.42	.07	.26	.17	
	.045	.038	.046	.010	.045	.023	
BM-7	.36	.16	.42	.07	.28	.19	
	.064	.019	.031	.008	.047	.018	
BM-8	.32	.16	.42	.07	.25	.16	
	.042	.017	.056	.006	.046	.009	

JENNITE

	0°		8°		8°		
TIRE	$\mu_x P$	$\mu_x S$	$\mu_y P$	$\mu_y S$	$\mu_x P$	$\mu_x S$	VALUE
BM-9	.28	.13	.36	.07	.20	.13	MEAN
	.050	.013	.035	.008	.043	.015	$\hat{\sigma}$
BM-10	.31	.16	.37	.06	.20	.14	
	.057	.022	.047	.007	.034	.012	
BM-11	.37	.18	.47	.07	.29	.18	
	.028	.018	.022	.011	.025	.019	
BM-12	.36	.19	.43	.07	.26	.16	
	.035	.015	.030	.008	.043	.026	
CM-1	.34	.14	.45	.07	.27	.17	
	.035	.012	.046	.013	.039	.028	
CM-2	.35	.15	.42	.06	.23	.14	
	.033	.015	.041	.009	.039	.023	
CM-3	.35	.14	.42	.06	.19	.13	
	.051	.022	.049	.010	.048	.016	
CM-4	.33	.17	.40	.04	.21	.13	
	.056	.043	.057	.011	.036	.020	
CM-5	.30	.13	.43	.07	.27	.17	
	.058	.021	.055	.013	.060	.044	
CM-6	.32	.14	.40	.07	.23	.14	
	.057	.006	.037	.014	.030	.015	
CM-7	.28	.12	.40	.07	.22	.16	
	.039	.023	.044	.013	.036	.028	
CM-8	.30	.14	.36	.06	.19	.13	
	.037	.015	.037	.011	.034	.013	

JENNITE

	0°		8°		8°		
TIRE	$\mu_x P$	$\mu_x S$	$\mu_y P$	$\mu_y S$	$\mu_x P$	$\mu_x S$	VALUE
CM-9	.26	.11	.42	.04	.21	.13	MEAN
	.042	.016	.044	.010	.043	.021	∂
CM-10	.28	.13	.40	.05	.20	.12	
	.040	.023	.046	.006	.042	.028	
CM-11	.32	.14	.37	.04	.19	.14	
	.043	.019	.027	.013	.021	.020	
CM-12	.30	.14	.34	.05	.20	.13	
	.045	.012	.039	.006	.036	.026	
CM-13	.29	.14	.37	.06	.22	.15	
	.027	.018	.062	.025	.048	.018	
CM-14	.32	.17	.39	.06	.23	.14	
	.036	.011	.028	.013	.027	.014	
CM-15	.36	.19	.40	.05	.25	.17	
	.025	.023	.036	.009	.031	.016	
CM-16	.32	.14	.40	.06	.23	.17	
	.038	.027	.018	.017	.020	.016	
DM-1	.25	.11	.39	.05	.20	.14	
	.059	.023	.032	.016	.036	.021	
DM-2	.26	.09	.40	.04	.23	.14	
	.048	.032	.029	.010	.045	.029	
DM-3	.35	.16	.42	.07	.27	.18	
	.030	.012	.053	.005	.053	.014	
DM-4	.35	.19	.39	.07	.23	.16	
	.062	.059	.042	.000	.043	.020	

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	0°		8°		8°		
TIRE	$\mu_x P$	$\mu_x S$	$\mu_y P$	$\mu_y S$	$\mu_x P$	$\mu_x S$	VALUE
DM-5	.28	.14	.38	.05	.21	.15	MEAN
	.036	.017	.028	.011	.042	.031	$\hat{\sigma}$
DM-6	.35	.14	.43	.06	.23	.15	
	.043	.015	.049	.007	.047	.018	
DM-7	.31	.16	.36	.05	.19	.12	
	.059	.038	.026	.011	.024	.020	
DM-8	.26	.12	.41	.08	.24	.16	
	.040	.016	.051	.010	.060	.037	
DM-9	.34	.18	.48	.07	.29	.18	
	.045	.016	.042	.005	.030	.025	
DM-10	.29	.17	.42	.07	.23	.18	
	.024	.012	.040	.010	.030	.010	
DM-11	.30	.15	.41	.08	.23	.16	
	.047	.037	.060	.010	.060	.021	
DM-12	.31	.15	.39	.06	.21	.13	
	.047	.017	.048	.008	.033	.020	
EM-1	.38	.21	.43	.05	.26	.18	
	.076	.064	.064	.005	.051	.014	
EM-2	.32	.17	.39	.04	.24	.15	
	.054	.020	.041	.010	.043	.027	
EM-3	.28	.14	.44	.04	.25	.16	
	.032	.010	.045	.010	.049	.014	
EM-4	.37	.16	.40	.04	.20	.15	
	.037	.016	.060	.011	.042	.008	

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	0°		8°		8°		
TIRE	$\mu_x P$	$\mu_x S$	$\mu_y P$	$\mu_y S$	$\mu_x P$	$\mu_x S$	VALUE
EM-5	.26	.11	.36	.06	.20	.14	MEAN
	.033	.007	.037	.011	.039	.014	$\hat{\sigma}$
EM-6	.29	.14	.37	.05	.22	.14	
	.042	.021	.033	.018	.028	.022	
EM-7	.31	.12	.38	.06	.20	.13	
	.050	.015	.029	.011	.036	.039	
EM-8	.27	.14	.39	.05	.19	.13	
	.043	.025	.038	.011	.033	.026	
EM-9	.35	.16	.38	.06	.25	.18	
	.049	.019	.037	.013	.032	.027	
EM-10	.32	.16	.43	.06	.27	.18	
	.037	.016	.035	.010	.033	.021	
EM-11	.27	.12	.35	.07	.18	.14	
	.050	.010	.037	.009	.030	.015	
EM-12	.25	.13	.35	.06	.18	.13	
	.054	.015	.039	.013	.031	.018	
EM-13	.49	.21	.51	.06	.34	.20	
	.053	.019	.035	.010	.049	.017	
EM-14	.47	.22	.45	.04	.31	.20	
	.039	.026	.023	.015	.030	.025	
FM-1	.10	.06	.12	.03	.09	.07	
	.035	.016	.017	.010	.018	.022	
FM-2	.08	.05	.14	.04	.09	.07	
	.028	.010	.027	.012	.014	.012	
GM-1	.30	.14	.43	.06	.22	.15	
	.032	.014	.044	.018	.040	.016	

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	0°		8°		8°		
TIRE	μ_x^P	μ_x^S	μ_y^P	μ_y^S	μ_x^P	μ_x^S	VALUE
GM-2	.29	.14	.38	.05	.17	.12	MEAN
	.036	.025	.048	.000	.047	.019	$\hat{\sigma}$
HM-1	.29	.13	.39	.04	.21	.13	
	.049	.016	.041	.011	.038	.014	
HM-2	.30	.12	.39	.05	.20	.13	
	.031	.009	.066	.009	.039	.022	
HM-3	.34	.17	.39	.05	.20	.14	
	.055	.017	.023	.011	.020	.014	
HM-4	.33	.14	.37	.06	.19	.14	
	.085	.020	.040	.011	.017	.015	
ZM-1	.17	.08	.27	.05	.15	.11	
	.045	.017	.036	.015	.038	.019	
ZM-2	.18	.10	.30	.07	.16	.13	
	.038	.015	.053	.011	.032	.022	
ZM-3	.18	.10	.33	.05	.18	.13	
	.047	.025	.061	.012	.038	.021	
ZM-4	.18	.10	.30	.06	.19	.12	
	.025	.017	.061	.009	.035	.017	
JM-1	.36	.15	.43	.06	.26	.17	
	.056	.024	.059	.006	.053	.021	
JM-2	.38	.17	.42	.05	.25	.16	
	.042	.030	.042	.012	.043	.014	
JM-3	.34	.15	.41	.04	.26	.16	
	.053	.012	.063	.016	.057	.038	
JM-4	.33	.13	.43	.07	.22	.15	
	.049	.008	.044	.004	.024	.014	

APPENDIX II

MEASUREMENT PRECISION ON THE MOBILE TIRE TESTER

A physical quantity f(x,y,z) calculated from the measurements of several variables will be in error due to uncertainties, dx, dy, and dz, in the measurements of x, y, and z. This error in f(x,y,z) is

$$df = f(x + dx, y + dy) - f(x,y) \qquad \text{(II-1)}$$

$$\begin{aligned} f(x + dx, y + dy) = f(x,y) &+ \frac{\partial f}{\partial x} dx + \frac{\partial^2 f}{\partial x^2} \frac{dx^2}{2!} + \ldots \\ &+ \frac{\partial f}{\partial y} dy + \frac{\partial^2 f}{\partial y^2} \frac{dy^2}{2!} + \ldots \\ \text{(Taylor's series)} \qquad &+ \frac{\partial f}{\partial z} dz + \frac{\partial^2 f}{\partial z^2} \frac{dz^2}{2!} + \ldots \qquad \text{(II-2)} \end{aligned}$$

Keeping only the first order terms in the expansion, substitution of Equation (II-2) into Equation (II-1) results in

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz \qquad \text{(II-3)}$$

The physical quantity under consideration is

$$u = \frac{a \cdot C}{F_z}$$

where

a = amplitude of trace on chart recorder

C = calibration factor (pounds/inch)

F_z = normal load (lead weights)

From Equation (II-3)

$$du = \frac{C}{F_z} da + \frac{a}{F_z} dC + \frac{a \cdot C}{F_z^2} dF_z \qquad \text{(II-4)}$$

(The sign of the dF_z term is not mathematically correct, but will result in a maximum error, du.)

Typical maximum values of the coefficients in Equation (II-4) are:

C_x = 370 lbs/in (calibration factor for longitudinal force)

C_y = 390 lbs/in (calibration factor for lateral force)

F_z = 1200 lbs

a = 3 inches

dF_z = 1 lb (lead weight weighed using Lebow load cell

da = .02 inches (Lufkin steel rule graduated in .01")

dC must be calculated separately, being the slope of a least squares straight line through a series of points. dC will be a function of calibration force uncertainty, 1 lb., chart recorder resolution, .02", and transducer nonlinearity. A procedure for calculating the standard deviation of the slope of a least squares straight line is given in Appendix 2 of Reference 1. The equation is

$$dC = \sigma = \sqrt{\frac{\sum_{i=1}^{n} (da_i)^2}{n-2}} \times \sqrt{\frac{n}{n \sum_{i=1}^{n} F_{c_i}^{\;2} - (\sum_{i=1}^{n} F_{c_i})^2}} \qquad \text{(II-5)}$$

where

n = number of calibration points

F_c = applied calibration force

The substitution of mobile tire tester calibration data into Equation (II-5) results in

$$dC_x = 2.46 \quad \text{and} \quad dC_y = 2.89$$

The uncertainty in μ_x and μ_y can now be calculated from Equation (II-4), and the results are:

$$d\mu_x = .00616 + .00615 + .00079 = .013$$

$$d\mu_y = .00650 + .00722 + .00083 = .014$$

APPENDIX III

DETERMINATION OF THE ACCURACY OF THE ESTIMATE OF σ_m

Any quantity computed from a randomly distributed variable will itself be randomly distributed, and thus each of the values of $\hat{\sigma}$ in Appendix I is a member of a Gaussian distribution which has a mean, σ, and a standard deviation, σ_s. [This mean, σ, is the "true value" which will yield the corresponding "true value" of σ_m which we desire.] Although σ is unattainable, requiring a number of trials approaching ∞, σ_s can be found, and with a value for σ_s, we can obtain a measure of how close our value of $\hat{\sigma}$ based on one set of 10 trials is to the universe standard deviation, σ. The value of σ_s is given by [1]

$$\sigma_s = \frac{\hat{\sigma}}{\sqrt{2(n-1)}} = \frac{\hat{\sigma}}{\sqrt{18}} \tag{3}$$

and the values of $\hat{\sigma}$ in Appendix I are 95 percent certain of being within ± $2\sigma_s$ of σ.

The standard deviations, $\hat{\sigma}$, of μ_xP, μ_xS, and μ_yP produced by measurements on all three pavements were analyzed to yield the mean value, $\bar{\sigma}$, and the standard deviation, $\hat{\sigma}_s$, for each of the nine groupings shown in Table A.

TABLE A

MEAN VALUES OF COLLECTIONS OF $\hat{\sigma}$ PLUS STANDARD DEVIATIONS OF THESE COLLECTIONS

	μ_xP	μ_xS	μ_yP	
Concrete	.0325	.0202	.0274	$\bar{\sigma}$
	.0129	.0083	.0092	$\hat{\sigma}_s$
Asphalt	.0361	.0331	.0341	
	.0122	.0114	.0134	
Jennite	.0441	.0210	.0413	
	.0129	.0116	.0119	

Consider, for example, the results in the upper left cell of Table A. We see that 97.5% of the values of $\hat{\sigma}$ for $\mu_x P$ were below .0583, which is $\hat{\sigma} + 2\hat{\sigma}_s$. This "worst case" $\hat{\sigma}$ of .0583 yields a worst case value of σ_s, using Equation (3), of .0183. Now the furthest that 95% of the values of $\hat{\sigma}_m$ could be from the desired but unattainable σ_m is $2\sigma_s/\sqrt{10}$.* For $\mu_x P$ concrete, the highest value of $2\sigma_s/\sqrt{10}$ is .0087. The lowest value is .00100. Thus the values of $\hat{\sigma}_m$ computed from the values of $\hat{\sigma}$ in Appendix I for $\mu_x P$ on concrete will be different from the "true values," σ_m, by between .00 and .01, rounding to two significant figures in accordance with the measurement precision. The same limits of confidence on $\hat{\sigma}_m$ result for the other eight combinations of pavement and traction variables and therefore a value of $\hat{\sigma}_m$ computed from a $\hat{\sigma}$ value in Appendix I, using Equation (2), will differ from the particular universe σ_m by a maximum of .01. We can thus have confidence that the values of $\hat{\sigma}_m$ so obtained are accurate.

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*This statement results from the following computation:

$$\hat{\sigma}_m = \frac{\hat{\sigma}}{\sqrt{n}} = \frac{\hat{\sigma}}{\sqrt{10}} \qquad \hat{\sigma} = \sigma - 2\sigma_s \text{ To } \sigma + 2\sigma_s$$

$$\hat{\sigma}_m = \frac{\sigma \pm 2\sigma_s}{\sqrt{10}} = \sigma_m \pm \frac{2\sigma_s}{\sqrt{10}}$$

REFERENCE

1. D.C. Baird, Experimentation: An Introduction to Measurement Theory and Experimental Design, Prentice-Hall, Englewood Cliffs, N. J., 1962, p. 34.