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Final Report

AN EXPERIMENTAL INVESTIGATION OF THE POSSIBILITY OF ACHIEVING A STANDING DETONATION WAVE

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ABSTRACT

The properties of gaseous detonation waves and the conditions that must be met to generate stable waves are described and discussed, as is the experimental facility which evolved as a result of these considerations. Preliminary experiments leading to the first known case of a stabilized detonation wave (probably very close to the Chapman-Jouguet case) are also described. In these experiments an ignition time delay has been detected between the shock wave and onset of combustion. It appears that this experimental technique has great potential in the study of chemical kinetics as well as in the study of stabilized detonation waves.

Preliminary analytical work on the stability of a plane detonation wave is discussed.

OBJECTIVE

The objective of this research program has been to stabilize experimentally and study a gaseous detonation wave.

INTRODUCTION

On July 1, 1954, The University of Michigan initiated work on a research contract with the Air Force Office of Scientific Research entitled "An Experimental Investigation of the Possibility of Achieving a Standing Detonation Wave." This contract was terminated on January 31, 1959, and this report represents a summary of the research findings. The major results obtained have been previously published, and consequently no attempt will be made herein to treat all aspects in detail. Only the general results of the study are presented here and the detailed results are left to the publications. The work on the stability of detonation waves, conducted by R. Ong, has never been published, however, and is discussed here in some detail. Further research on this problem is being conducted under a new contract, AF 49(638)-562.

The phenomenon of detonative combustion has been recognized since the latter part of the nineteenth century. Since then many investigators have pursued the problem, theoretically as well as experimentally. On theoretical grounds the phenomenon is fairly well understood, at least in the macroscopic sense, and simplified hydrodynamics along with thermodynamics predict the existence of strong as well as Chapman-Jouguet type of detonation waves. In the case of the former, the Mach number of the burned gases is subsonic relative to the front, whereas in the Chapman-Jouguet (C-J) case, this Mach number is unity. Weak detonations (burned gases moving supersonically relative to the front) in chemically reacting waves are ruled out on the basis of entropy considerations. Recently there has been active interest in the theoretical description of the structure of the detonation wave. Such studies serve to bring out the coupling between the combustion process and shock wave.

On the experimental side, the only stable detonations observed have been those of the C-J type. Strong waves have been detected but only at the onset of detonation and then they rapidly decay to the C-J type. Consequently all experimental studies to date have been restricted to the C-J detonation. Morrison[1] maintained that sufficiently high reservoir pressures in a shock tube should produce stable strong detonation waves and attempted to verify this. He was unsuccessful, possibly because of limited reservoir pressures available, or perhaps because of other less obvious stability considerations.

Thus it seems that ability to generate steady-state detonation waves would be highly advantageous. In such a system, it was reasoned, appropriate control of the "boundary conditions" on the wave could lead to stable strong detonations as well as to those of the C-J type. Other advantages would also accrue from a stabilized wave. For one thing, the instrumentation problem should be much simpler so that many more varied measurements could be obtained, resulting in a better understanding of the phenomenon. These measurements could conceivably include pressure, temperature, Mach number, ionization, composition and spectroscopy. Another possible use of stabilized detonation waves is in the application to hypersonic ramjets. This would allow burning at supersonic velocity and hence circumvent the great stagnation pressure loss which ordinarily occurs in the diffuser. Analysis and calculations were made for a hypothetical ramjet and the results reported in Ref. 2. Further, the standing detonation wave represents a powerful experimental tool for the study of chemical kinetics, transport phenomena, flame propagation, and fluid mechanics.

It is along these lines that this research program has been conceived and conducted. The approach used to achieve the desired goals and the results obtained in the course of this work are brought out in the body of this report.

CONDITIONS FOR STABILIZING A GASEOUS DETONATION WAVE

A rather thorough discussion of the dynamic conditions that must be met to stabilize a detonation wave is presented in Ref. 3. Only a cursory review of these considerations will be presented here.

Chapman-Jougeut gaseous detonation waves propagate at a relatively high Mach number (3 to 10); this Mach number is defined as the ratio of the velocity of the wave to the speed of sound of the unburned combustible gases. Strong detonation waves propagate at even higher rates. The Chapman-Jouguet Mach number is dependent, of course, upon the particular fuel-oxidant combination employed. This determines the amount of energy addition via combustion. Also, there is some effect of static pressure and of static temperature. In particular, the C-J Mach number of detonation varies approximately as the inverse square root of the static temperature of the unburned gases. The pressure effect, although slight, is such that lower pressures lead to lower Mach numbers. This is mainly the result of the increased dissociation occurring at lower pressures behind the wave, which leads to lower effective heat release.

The thickness of the wave must also be considered. Operation at lower pressures or leaner mixture ratios tends to broaden the reaction zone. This thickening of the wave may become undesirable in the actual stabilization as it could lead to two-dimensional effects behind the wave, losses to the wall or surrounding stream, or modification of the combustion process as a result of reflected shocks. Such events would seriously complicate the assessment of the experimental results. On the other hand, if these conditions can be tolerated or avoided, the broadening of the wave would be advantageous to the study of the wave structure.

In view of the above, it was anticipated that if a detonatable gaseous mixture could be accelerated to a high Mach number without premature burning and at the right conditions of pressure and temperature, a detonation wave could be stabilized in a channel or on some suitable body in the stream (questions of stability will be discussed later). The achievement of high Mach number flows requires high pressure and temperature ratios. If the tests are solely for aerodynamic purposes, extreme stagnation conditions may ordinarily be avoided by testing at reduced pressure and temperature. However, with the added restrictions of chemical reaction, the freedom of choice for the testing conditions is smaller and leads to more demanding experimental apparatus. We are thus faced with the necessity of providing a high stagnation temperature and stagnation pressure combustible mixture, and accelerating this mixture to a high Mach number as indicated above. Obviously, the problem becomes much simpler for the lower Mach numbers of detonation, but even the lowest lead to severe conditions. The lower limit of the Mach number of detonation is usually in the neighborhood of 3 for most lean C-J gaseous mixtures. However, in these cases the reaction zone is extended so that more reasonable values for stabilization would appear to be 4.5 - 5.

It is instructive to consider the expansion of a detonatable gaseous mixture to a supersonic Mach number in a convergent-divergent nozzle, assuming that a normal Chapman-Jouguet detonation is initiated at some station. A requirement for this initiation is that the temperature behind the shock is sufficiently high to cause ignition of the gases. For a given stagnation temperature, the variation of static temperature with Mach number may be plotted as shown in Fig. 1. Curves for three different stagnation temperatures are shown. Superimposed on the same plot is the Mach number of Chapman-Jouguet detonation which is a function of the local static temperature. The particular values used correspond to a stoichiometric mixture of hydrogen and air. This assumes the inverse square root variation of detonation Mach number with static temperature and neglects pressure dependence. It is apparent that, at stations where the Mach number is lower than that corresponding to the intersection of the two curves, the detonation Mach number (corresponding to the local temperature) exceeds the local Mach number, and conceivably the wave would travel upstream and never stabilize. Hence, in this portion of the nozzle, steady detonation would not be allowed. On the other hand, a wave initiated at a section where the Mach number is greater than the intersection Mach number cannot be stable since the detonation Mach number is lower than the local Mach number. Thus the intersection Mach number represents the possible position for a stable configuration. However, this appears unlikely, as any perturbation of the wave would move it to an unstable position. Of course, the above reasoning is limited to plane C-J waves and does not account for boundary-layer effects. It is possible that the generation of strong waves and/or the presence of dissipative effects could alter these conclusions. At any rate, stabilizing an oblique detonation wave on a wedge was considered first. For this case the normal component of the attached wave would be the Mach number of detonation and the free-stream Mach number would have to be greater. Referring back to Fig. 1, this represents operating points to the right of the point of intersection. Stabilization of these detonation waves on a wedge leads to the interesting concept of a detonation polar. This was first pointed out by Siestrunck _et al._,[4] and was extended in Refs. 3 and 5. It is worth noting that different modes of detonation may theoretically be ob-

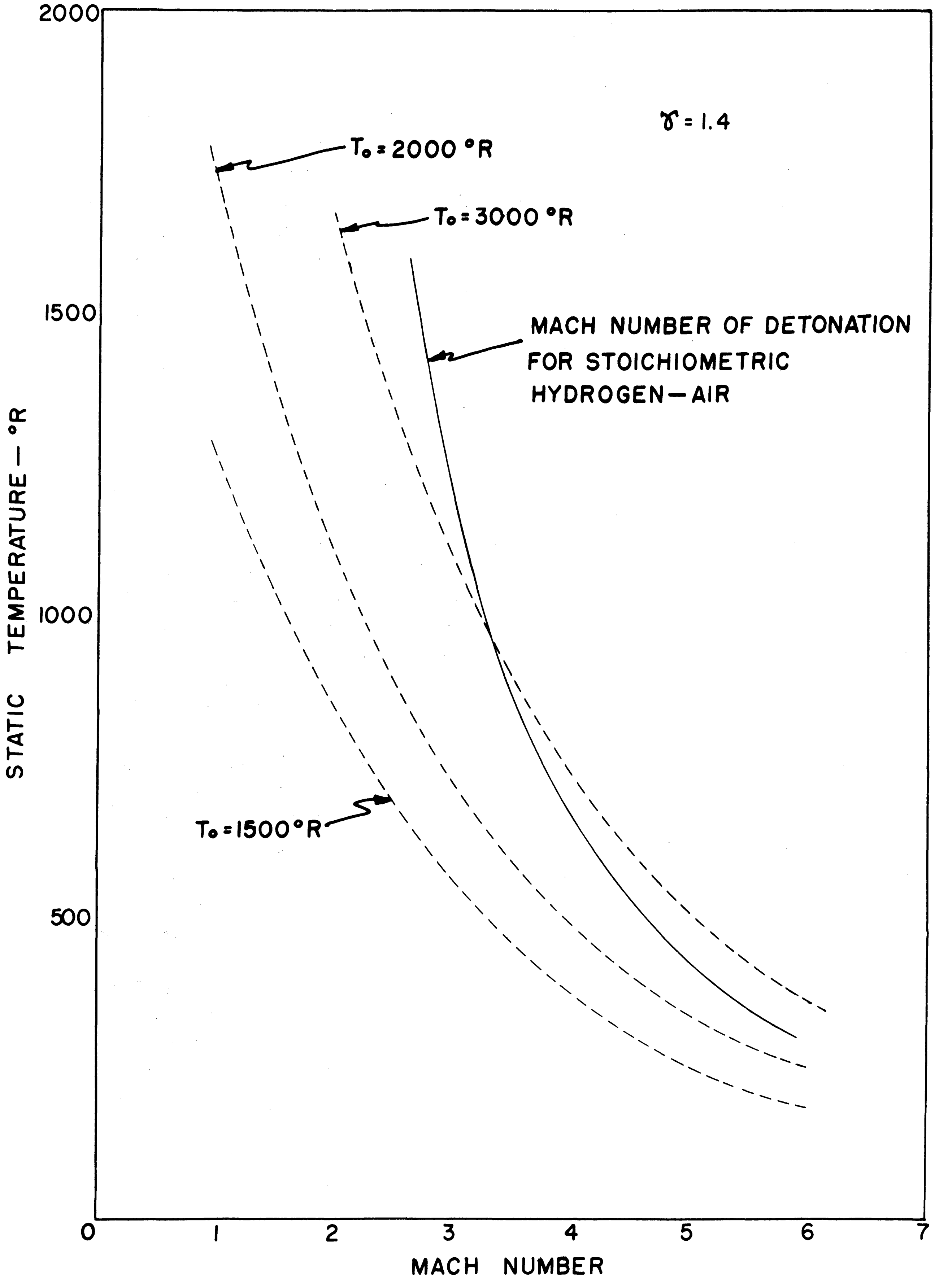

Fig. 1. Chapman-Jouguet detonation waves in a convergent-divergent channel.

tained by adjustment of the wedge angle. Thus for any given flow condition the generation of a C-J detonation corresponds to only one wedge angle. Other angles lead to strong detonations. Further, certain combinations of conditions are excluded on the detonation hodograph which are analogous to the prohibited zones on the Hugoniot curve.

Practical considerations, such as boundary-layer detonation-wave interactions, combustion in the boundary layer, extended reaction zone, etc., led us to abandon the wedge stabilization technique for the initial studies. Instead we adopted a means of stabilizing the wave in the open jet of an underexpanded nozzle, discussed in the next section.

Another possibility for stabilizing C-J as well as strong detonation waves is to position the wave in a double-throat tunnel by appropriate control of the reservoir and receiver pressures. We expect to investigate this aspect in the new research program.

EXPERIMENTAL ARRANGEMENT

As indicated above, the experimental difficulties associated with generating the correct conditions for stabilizing a detonation wave are chiefly attributable to the high Mach numbers of detonation. Accordingly, simplifications can be made if this Mach number can be minimized. Primarily for this reason hydrogen-air was chosen as the particular fuel combination to be studied. Under standard conditions this mixture has a relatively low Mach number of detonation of 4.8. However, this still indicates high stagnation temperatures and pressures, sufficiently high, in fact, to preclude the mixing of fuel and air under stagnation conditions. The actual experimental arrangement adopted is shown schematically in Fig. 2. High-pressure air is drawn from high-pressure (2500 psi) storage tanks, throttled to about 600 psi, and passed through a regenerative pebble-type heat exchanger previously heated to 2100°F. There is little pressure drop across the heat exchanger and the high-pressure, high-temperature air is introduced to the stagnation chamber of a convergent-divergent axisymmetric mixing nozzle. Unheated hydrogen is reduced in pressure from a manifold of bottles and introduced at sonic velocity at the throat of the nozzle through a centered coaxial needle. The nozzle is shown in Fig. 3. The air and hydrogen mix supersonically in the divergent part of the nozzle. The divergence of the nozzle causes a rapid pressure and temperature drop, thereby precluding premature combustion at the mixing zone. The entire nozzle is operated highly underexpanded, that is, with the exit pressure much greater than ambient pressure. Consequently, further isentropic expansion of the gases occurs in the core of the open jet. The established normal shock wave or Mach disc serves to ignite the mixture and leads to detonation under the proper conditions.

The air heat exchanger, mentioned above, consists of a steel cylinder 14 ft long, 2 ft in diameter, lined with ceramic material, and filled with alumina

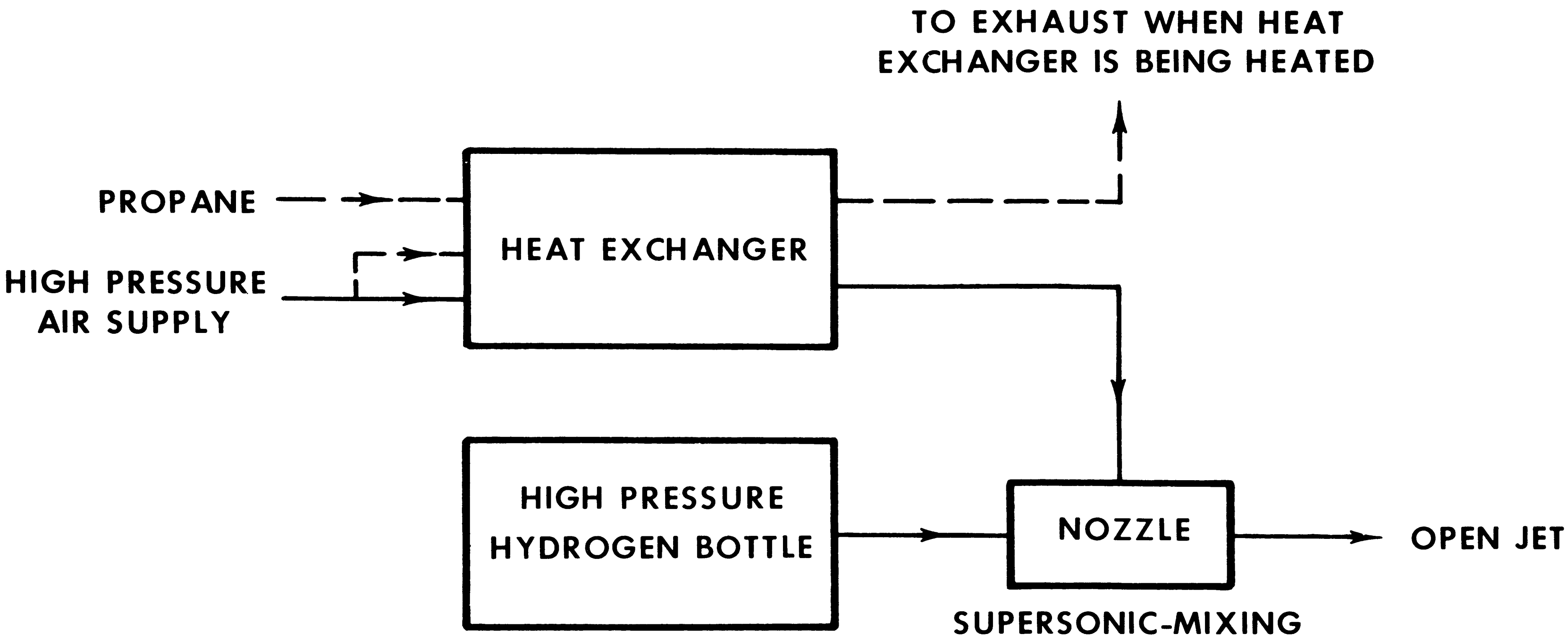

Fig. 2. Block diagram of experimental setup.

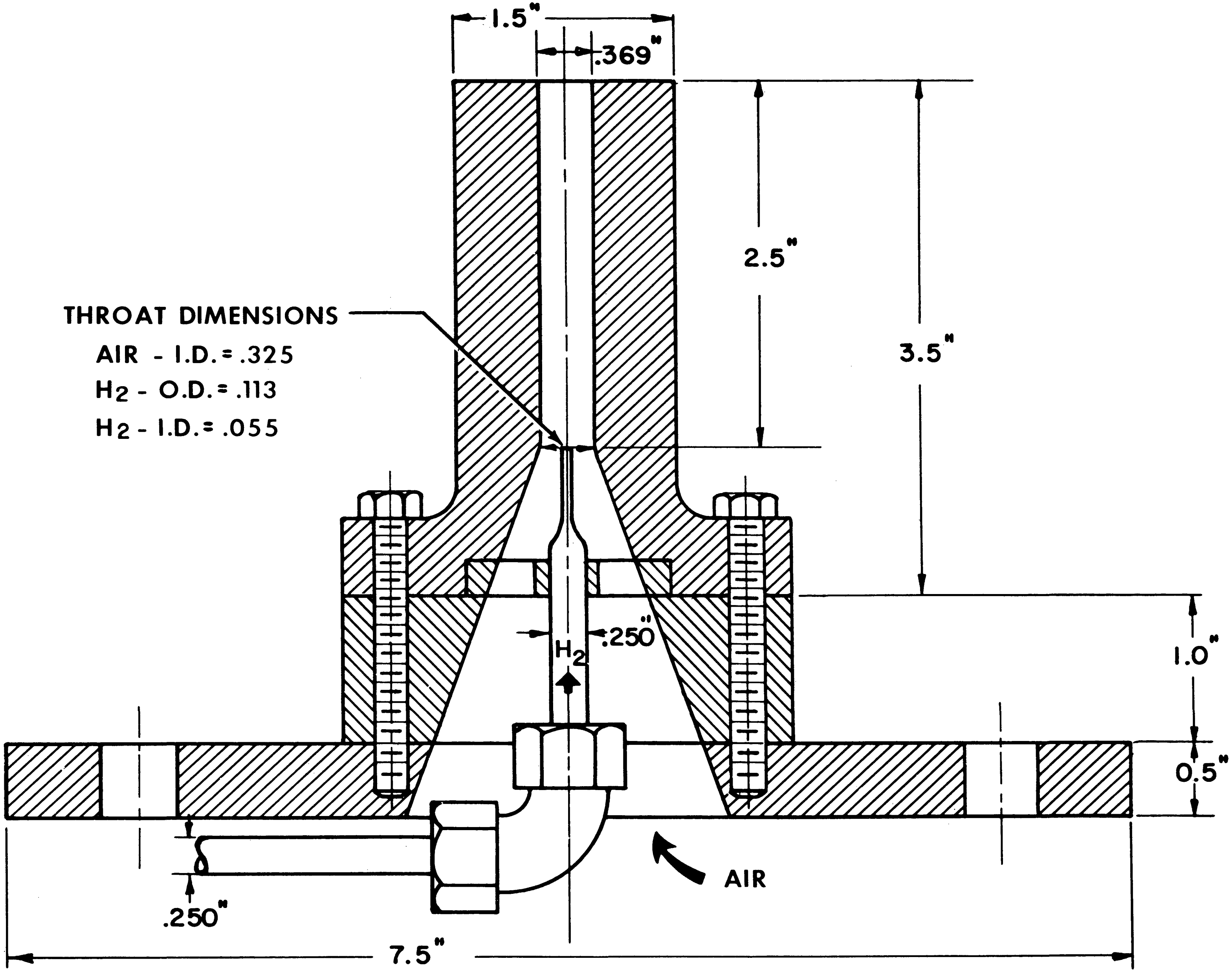

Fig. 3. Mixing nozzle.

pebbles. It is heated directly by the combustion products of a propane-air flame and requires 6 to 8 hours of heat-up time. It is designed for pressures up to 1000 psi and temperatures up to 3000°F. A detailed theoretical analysis is reported in Ref. 6 and the experimental evaluations of the heat exchanger in Ref. 7. It should be pointed out that, while the heat exchanger is of the blow-down type, for run times of interest here the blow-down temperature remains essentially constant.

The structure of the highly underexpanded free jet is shown in Fig. 4. The flow in the region bounded by the exit plane of the nozzle, the intercepting shock, and the normal shock (Mach disc) undergoes isentropic expansion so that the Mach number increases rapidly in the stream direction. All the flow variables in this region, as well as the hydrogen concentration, are changing in both the radial and axial directions. For small-scale jets, such as in this case, these changes occur quite rapidly in the flow direction. This region is terminated by the Mach disc which reduces the flow from supersonic to subsonic velocity. Further, it discontinuously increases the temperature and pressure. Under correct operating conditions, the flow conditions immediately downstream of the Mach disc are sufficient to allow ignition of the mixture. This configuration then becomes a standing detonation wave under certain restrictive conditions. These restrictive conditions will be elaborated upon in the discussion of the experimental results.

EXPERIMENTAL RESULTS AND DISCUSSION

It was stated in the introduction that one advantage accruing from steady-state detonation waves would be the ease of increasing the variety and accuracy of experimental measurements. One price paid for this, however, is the greater difficulty involved in ascertaining the initial conditions of the wave. Specifically, these include the initial fuel-air ratio, static temperature, static pressure, and Mach number. Accordingly, early experiments centered around determination of these quantities in the open jet. The measurements are discussed in detail in a paper presented at the Seventh Symposium on Combustion[8] and will be repeated only briefly in this report.

Hydrogen concentration and pressure distributions were measured in the cold jet by means of a probe mounted on a traversing mechanism which allowed positioning of the probe at any desired location in the jet. Radial traverses were made of the composition at the nozzle exit and at an axial location at the Mach disc. For these experiments, the hydrogen-to-air ratio was such as to give an approximately stoichiometric mixture ratio on the axis at the Mach disc. The stagnation temperature of the mixture was approximately ambient. These measurements indicated a quite uniform mixture ratio over most of the area of the Mach disc. Of course, using the method of hydrogen injection, the mixture ratio on the centerline is much richer than the overall average ratio (by about a factor

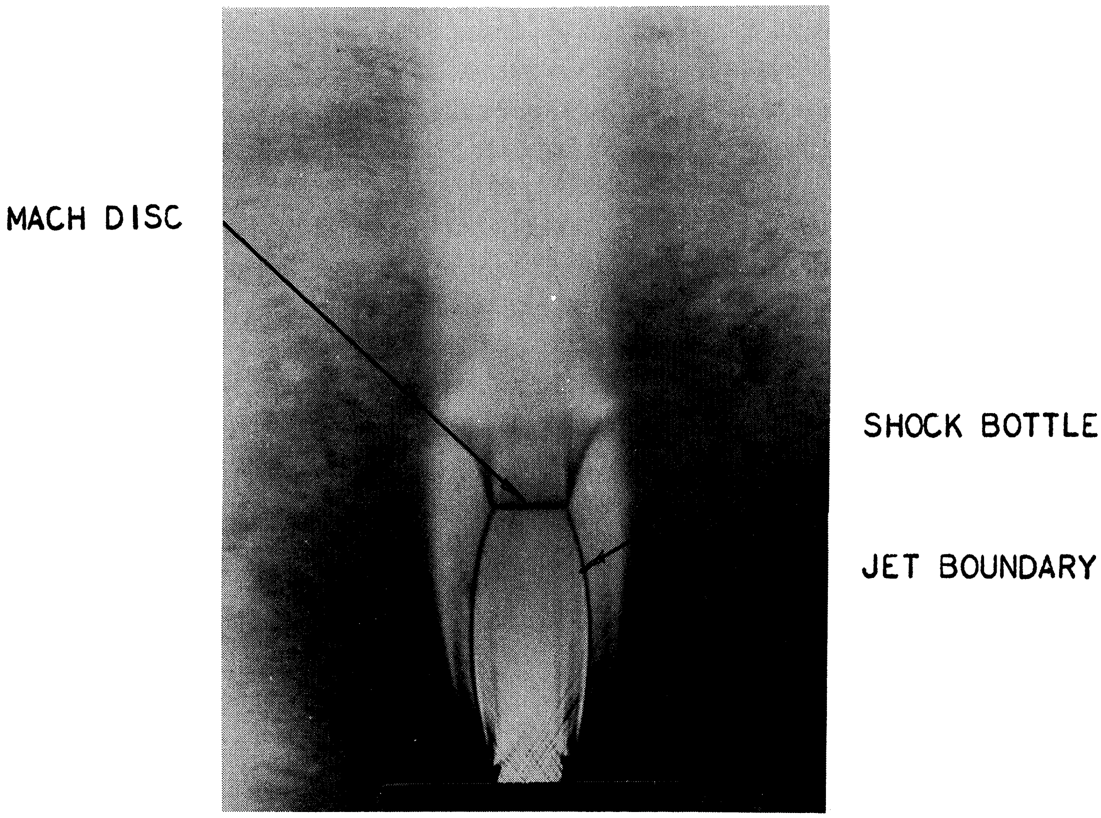

Fig. 4. Structure of the jet of a highly underexpanded nozzle.

of 3). While nearly stoichiometric in the center, the hydrogen concentration falls off to zero in the outer fringes of the jet.

Total pressure measurements were also made at the exit of the nozzle and immediately upstream of the Mach disc. These measurements, along with the nozzle-exit static-pressure measurements and the assumption of constant stagnation pressure up to the Mach disc, allowed determination of the Mach number at the disc location. Under usual operating conditions, this Mach number was on the order of 4.7. Other Mach numbers could readily be obtained by variation of stagnation pressure conditions. It was found that stagnation pressure downstream of the disc was approximately 1.4 atmospheres. Also, measurements obtained by traversing in the radial direction showed relatively uniform total pressure across the Mach disc area.

Using the experimental arrangement described, many experiments were conducted which led to very stable shock-wave—combustion-zone configurations. In these cases, the desired air flow was established and hydrogen was then added until combustion was initiated behind the Mach disc. The resultant phenomenon was recorded by 35-mm schlieren photography as well as by 16-mm observation of the visible flame front. Data were taken during operation with air alone as well as during the combustion phase. It was found impossible to detect the combustion zone on the schlieren photographs, so that it was necessary to superimpose the 16-mm results on the schlieren to determine relative positions of the combustion front and shock zone. A reference object of known dimensions, photographed by both cameras, allowed the determination of absolute distances. A typical pair of photographs, magnified to the same scale, appear as Fig. 5. The flame shape and position is dotted in on the schlieren photograph. As is evident, there is a small distance between the position of the Mach disc and the initiation of combustion. This distance corresponds to about 25 μsec for the conditions of the run shown, and represents, we believe, a chemical ignition time delay. That is, it is the time between the instant of preparation of the gas for combustion by the shock wave and the time when appreciable chemical reaction takes place. The rather intense luminosity of the flame is attributed to the sodium added in the form of common salt. The occurrence of the ignition time delay along with a static pressure measurement at the exit of the nozzle gives almost conclusive proof that little or no combustion occurs within the nozzle. On the other hand, experiments have been performed wherein a small flame was detected at the exit of the nozzle, as shown in Fig. 6. In this case there appears to be no ignition-time-delay zone and combustion is initiated right at the shock front. This is readily explainable since premature combustion within the nozzle serves to increase the stagnation temperature of the gases and thus the static temperature immediately downstream of the shock. The chemical kinetics, depending exponentially on temperature, are then sufficiently rapid to yield time delays shorter than can be resolved experimentally. Of course, this is no longer a true hydrogen-air detonation. In experiments of this type, an appreciable increase in nozzle exit pressure (i.e., 115 to 175 lb/sq in.) was noted at the onset of combustion in the nozzle. Furthermore, a temperature increase was detected by a thermocouple immersed in the stream im-

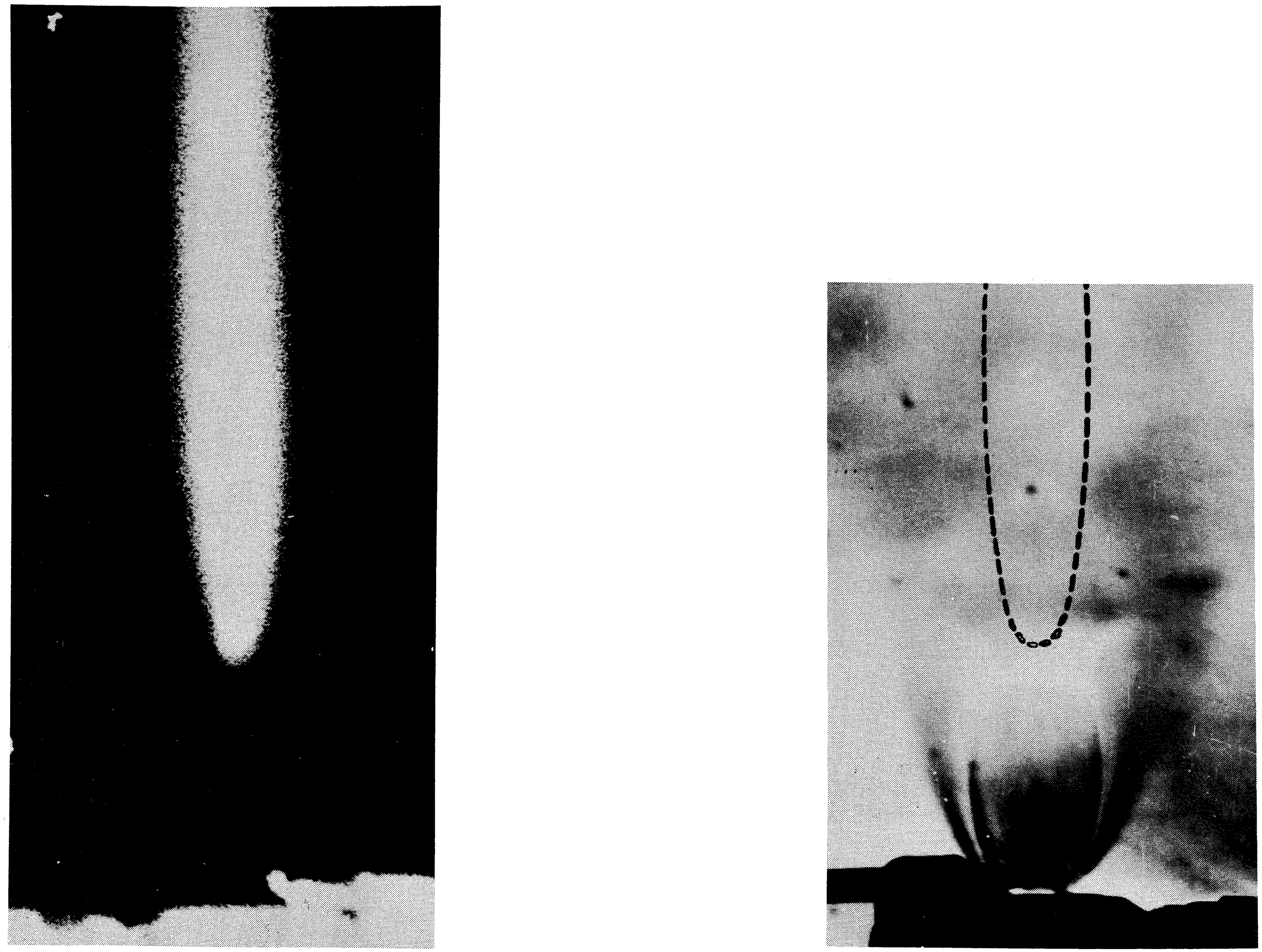

Fig. 5. Direct and schlieren photographs of jet during combustion (no apparent burning at nozzle exit).

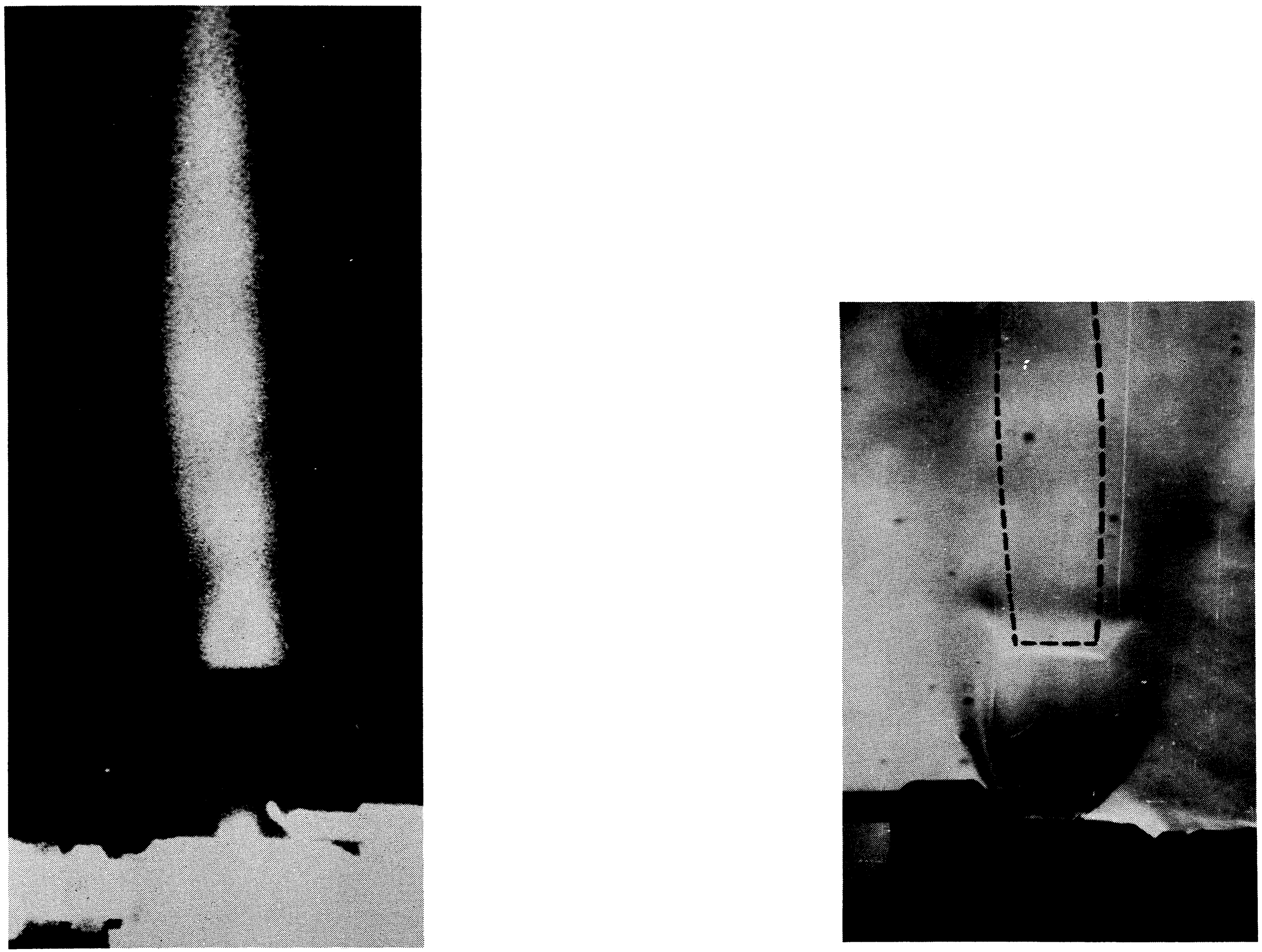

Fig. 6. Direct and schlieren photographs of jet during combustion (flame cone at nozzle exit).

mediately outside the nozzle. Approximate calculations on the above experimentally observed conditions showed that the pressure increase can be explained by a relatively small heat addition. In other words, only a small portion of the input hydrogen actually burned in the nozzle. Combustion of the remaining hydrogen presumably occurred downstream of the Mach disc.

In May, 1958, we observed what we believe to be the first case of a stabilized gaseous detonation wave. In this experiment the same procedure was followed as outlined above but the stagnation temperature of the air was 2600°R, the highest we had obtained. The air flow was first established and the jet was photographed by both cameras. Hydrogen was then added and combustion initiated. Examination of the data revealed that the original Mach number into the shock, that is, with air alone, was 6.1. However, with the onset of combustion, the shock wave was driven upstream to a lower Mach number of approximately 5.7. Also, the ignition time delay was the shortest we had observed to date (about 8.5 μsec). Calculations reveal that the final wave corresponds very closely to the Chapman-Jouguet detonation wave. Accordingly, we believe this to be a case of a stabilized detonation wave. Whether this can be appropriately be called a Chapman-Jouguet detonation wave is somewhat open to question. In fact, considerable ambiguity surrounds this whole question of identification of waves and strength of waves. That is, are the different types of waves observed detonation waves or are they merely cases of shock ignition? It is our feeling that waves of the type shown in Fig. 4 can hardly be classified as detonation waves, as there was no apparent effect of the combustion on the shock-wave position. Certainly in a true detonation wave there is strong coupling between the two. For this reason, we choose to consider this a case of shock ignition. On the other hand, the case represented in Fig. 5 may be interpreted as a detonation wave but not of hydrogen-air. Partial combustion has occurred within the nozzle so that the free-stream condition of the wave is no longer a pure hydrogen-air mixture. The last case that is mentioned above corresponds to detonation as coupling is observed between the two phenomena. Other considerations are pertinent in each of these cases, which renders actual definition of strength extremely difficult if not impossible. Historically, detonation waves are classified on a one-dimensional constant-area basis. From this the types "weak detonation," "Chapman-Jouguet detonation," and "strong detonation" are derived. In the cases at hand, there are certainly two-dimensional effects downstream of the Mach disc which impose pressure fields on the combustion process and hence lead to different results from those normally considered in the classical case. Of course, such effects may become negligible by operation at higher temperatures wherein the ignition time delay is minimized and the combustion process is expedited. The classical description may then be approached quite closely.

THE STABILITY OF A NORMAL CHAPMAN-JOUGUET DETONATION WAVE

INTRODUCTION

The stability of a normal C-J detonation wave in a compressible nonviscous gas will be discussed here as follows.[9] Consider a plane C-J detonation wave initially at rest normal to the flow of the gas. Then introduce a plane sound wave into this uniform but discontinuous flow. This sound wave disturbs the detonation wave and the flow on both sides of it. Since the detonation is assumed to be initially Chapman-Jouguet, the Mach number of the flow downstream of the wave is equal to one. Hence we may restrict ourselves to sound waves incident from the upstream or supersonic side of the wave. To simplify the problem for the present, we will assume that the detonation wave remains Chapman-Jouguet at all times and treat only the one-dimensional case. The problem could be very easily extended to two dimensions, but under the more restrictive assumption of C-J detonation the one-dimensional approach is satisfactory. The incident sound wave causes the detonation wave to oscillate from its initial position and also gives rise to the appearance of a refracted sound wave and an entropy wave. Assuming an incident sound wave of given strength and frequency, the amplitudes of both the refracted sound wave and entropy wave may be computed. Moreover, the amplitude of the oscillation of the detonation may also be obtained. The detonation wave is then said to be stable if this amplitude of its oscillation is finite.

THE LINEARIZED DIFFERENTIAL EQUATIONS AND THE SHOCK CONDITIONS

The one-dimensional flow of a compressible, nonviscous gas is described by the following equations:

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + \rho\frac{\partial u}{\partial x} = 0 \tag{1}$$

$$\rho\frac{\partial u}{\partial t} + \rho u\frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} = 0 \tag{2}$$

$$\frac{\partial s}{\partial t} + u\frac{\partial s}{\partial x} = 0 \quad , \tag{3}$$

where p, u, ρ, s represent the pressure, velocity, density, and entropy of the gas, respectively. Assuming that the medium behaves as an ideal gas, we may write the following equation of state:

$$p = K\rho^{\gamma} e^{s/c_v} \quad , \tag{4}$$

where c_v is the specific heat at constant volume, γ is the ratio of specific heats, and K is an appropriate constant. c_v and γ are assumed to be constants. In particular, these equations are satisfied by the steady uniform flow of a gas through a plane normal C-J detonation wave. Let P, U, D, S, and C denote the ambient pressure, velocity, density, entropy, and sound speed of this uniform flow, on either side of the wave.

Now let the flow on either side of this plane normal C-J detonation wave be slightly disturbed and introduce the dimensionless perturbation variables p', u', ρ', s' by means of the following relations:

$$\left.\begin{aligned} p &= P + p'DCU \\ u &= U + u'U \\ \rho &= D + \rho' D \\ s &= S + s'c_p \end{aligned}\right\} \tag{5}$$

Upon substituting (2) and neglecting the squares of the small perturbation variables, Eqs. (1) - (4) become:

$$\frac{\partial \rho'}{\partial t} + U\frac{\partial \rho'}{\partial x} + U\frac{\partial u'}{\partial x} = 0 \tag{6}$$

$$\frac{\partial u'}{\partial t} + U\frac{\partial u'}{\partial x} + C\frac{\partial p'}{\partial x} = 0 \tag{7}$$

$$\frac{\partial s'}{\partial t} + U\frac{\partial s'}{\partial x} = 0 \tag{8}$$

$$Up' = C(\rho'+s') \quad , \tag{9}$$

where we have made use of the relation $C^2 = \gamma P/D$. Eliminating ρ' from the continuity equation (6) by means of (8) and (9), we may rewrite the system as three linear differential equations containing p', u', and s', and one linear algebraic equation defining ρ' in terms of p' and s'. The resulting equations are:

$$\frac{1}{C}\left(\frac{\partial p'}{\partial t} + U\frac{\partial p'}{\partial x}\right) + \frac{\partial u'}{\partial x} = 0 \tag{10}$$

$$\frac{1}{C}\left(\frac{\partial u'}{\partial t} + U\frac{\partial u'}{\partial x}\right) + \frac{\partial p'}{\partial x} = 0 \tag{11}$$

$$\frac{\partial s'}{\partial t} + U\frac{\partial s'}{\partial x} = 0 \tag{12}$$

$$\rho' = \frac{U}{C}p' - s' \tag{13}$$

In the following only the three equations (10), (11), and (12) of the system will be used. The fourth equation (13) enables us to find the density after the pressure and entropy have been obtained from the first three equations.

Assuming that the detonation wave is disturbed from its initial state of rest at $x = 0$, let this disturbance be described by $x = f(t)$ and let U_w denote the velocity of the detonation wave. The relative velocities of the flow to the shock are

$$\begin{aligned} u_{R_1} &= u_1 - U_w = U_1 + u_1' U_1 - f_t \\ u_{R_2} &= u_2 - U_w = U_2 + u_2' U_2 - f_t \ , \end{aligned} \tag{14}$$

where the subscripts 1 and 2 denote the states of the gas in front and behind the wave, respectively. Across a C-J detonation wave we can write the following relations:

$$\left.\begin{aligned} \rho_1 u_{R_1} &= \rho_2 u_{R_2} \\ p_1 + \rho_1 u_{R_1}^2 &= p_2 + \rho_2 u_{R_2}^2 \\ Q + \frac{\gamma}{\gamma-1}\frac{p_1}{\rho_1} + \frac{1}{2}u_{R_1}^2 &= \frac{\gamma}{\gamma-1}\frac{p_2}{\rho_2} + \frac{1}{2}u_{R_2}^2 \ , \end{aligned}\right\} \tag{15}$$

Where the heat release Q is given by the following relation:[10]

$$Q = \frac{c_p \, \tau_1 (\mathcal{M}_1^2 - 1)}{2(\gamma+1)\mathcal{M}_1^2} \tag{16}$$

where

τ_1 = temperature of the gas in front of the wave, and
$\mathcal{M}_1$ = Mach number of the gas in front of the wave.

Observe that we may also write

$$\tau_1 = \frac{u_{R_1}^2}{\gamma \, R_1 \mathcal{M}_1^2} \tag{17}$$

where

R_1 = the gas constant associated with the gas in front of the wave.

Hence we have the following relation:

$$Q = \frac{c_p \, u_{R_1}^2 (\mathcal{M}_1^2 - 1)}{2(\gamma H)\gamma \, R_1 \mathcal{M}_1^4} \tag{18}$$

By means of a straightforward algebraic manipulation, we may solve for p_2, u_{R_2}, ρ_2, s_2 in terms of the variables ahead of the wave and obtain the following relations:

$$\begin{aligned} p_2 &= p_1 \, F(\mathcal{M}_1) \\ u_{R_2} &= u_{R_1} \, G(\mathcal{M}_1) \\ \rho_2 &= \rho_1 \, G(\mathcal{M}_1)^{-1} \\ \frac{s_2 - s_1}{c_v} &= \ln F(\mathcal{M}_1) + \gamma \ln G(\mathcal{M}_1) \quad , \end{aligned} \tag{19}$$

where for brevity we use $F(\mathcal{M}_1)$ and $G(\mathcal{M}_1)$ which are defined by

$$F(\mathcal{M}_1) = \frac{1 + \gamma \mathcal{M}_1^2}{1+\gamma} \tag{20}$$

$$G(M_1) = \frac{1 + \gamma \mathcal{M}_1^2}{(1+\gamma)\mathcal{M}_1^2} = \frac{1}{\mathcal{M}_1^2} F(\mathcal{M}_1) \; . \tag{21}$$

We now linearize the shock conditions expressed by (13) and obtain the relations between the first-order disturbances. First we let

$$\mathcal{M}_1 = M_1 + m_1' \quad (i = 1,2) \tag{22}$$

where

M_1 = ambient Mach number of the gas relative to the wave in region (1), and
m_1' = the perturbation Mach number of the gas in region (1).

Expressing the first relation of (19) in differential form, we get

$$\delta p_2 = \delta p_1 \, F(\mathcal{M}_1) + p_1 \, F'(\mathcal{M}_1)\delta\mathcal{M}_1 \quad ,$$

where, up to first order,

$$\begin{aligned} \delta p_2 &= p_2' \, C_2 D_2 U_2 \\ \delta p_1 &= p_1' \, C_1 D_1 U_1 \\ \delta\mathcal{M}_1 &= m_1' \; . \end{aligned}$$

Neglecting terms of higher order than one in the perturbation variables, the first relation of (19) becomes

$$p_2'M_2 = p_1'M_1 + \frac{1}{\gamma}\frac{F'(M_1)}{F(M_1)}\, m_1' \quad . \tag{23}$$

In a similar way from the rest of (19) we may obtain the following first-order relations:

$$u_2' = u_1' + \frac{G'(M_1)}{G(M_1)} m_1' + \frac{f_t}{U_1}\left[\frac{1 - G(M_1)}{G(M_1)}\right] \tag{24}$$

$$s_2' = s_1' + \left[\frac{1}{\gamma}\frac{F'(M_1)}{F(M_1)} + \frac{G'(M_1)}{G(M_1)}\right] m_1' \quad . \tag{25}$$

To simplify (23), (24), and (25) we must first calculate m_1' in terms of u_1', v_1', p_1', and s_1'. Since

$$\mathcal{M}_1 = \frac{u_{R_1}}{C_1} = \frac{u_{R_1}\,\rho_1^{1/2}}{(\gamma\, p_1)^{1/2}} \quad ,$$

we have

$$\ln \mathcal{M}_1 = \ln u_{R_1} - \frac{1}{2}\ln p_1 + \frac{1}{2}\ln \rho_1 - \frac{1}{2}\ln\gamma \quad .$$

Differentiating this expression we have, up to first order,

$$\frac{m_1'}{M_1} = \frac{u_1'\,U_1 - f_t}{U_1} - \frac{1}{2}\,p_1'\,\frac{D_1C_1U_1}{P_1} + \frac{1}{2}\,\rho_1' \quad ,$$

which reduces, by virtue of $\rho' = (U/C)p' - s'$, to the following relation:

$$m_1' = M_1u_1' - \frac{2(M_1^2-1)}{\gamma - 1}\,p_1' - \frac{M_1}{2}\,s_1' - \frac{f_t}{C_1} \tag{26}$$

Substituting (26) into (23), (24), (25) and rearranging terms we finally obtain the following linearized shock conditions.

$$p_2'M_2 = p_1'M_1 + \frac{1}{\gamma}\frac{F'(M_1)}{F(M_1)}\left[M_1U_1' - \frac{2(M_1^2-1)}{\gamma - 1}\,p_1' - \frac{M_1}{2}\,s_1' - \frac{f_t}{C_1}\right] \tag{27}$$

$$u_2' = u_1' + \frac{G'(M_1)}{G(M_1)}\left[M_1u_1' - \frac{2(M_1^2-1)}{\gamma - 1}\,p_1' - \frac{M_1}{2}\,s_1' - \frac{f_t}{C_1}\right] + \frac{f_t}{U_1}\left[\frac{1-G(M_1)}{G(M_1)}\right] \tag{28}$$

$$s_2' = s_1' + \left[\frac{1}{\gamma}\frac{F'(M_1)}{F(M_1)} + \frac{G'(M_1)}{G(M_1)}\right]\left[M_1u_1' - \frac{2(M_1^2-1)}{\gamma - 1}\,p_1' - \frac{M_1}{2}\,s_1' - \frac{f_t}{C_1}\right] \quad . \tag{29}$$

THE INTERACTION OF PLANE SOUND WAVES WITH A NORMAL CHAPMAN-JOUGUET DETONATION WAVE

Consider solutions of the differential equations (10), (11), and (12) which are harmonic in time, i.e., assume solutions of (10) - (12) in the form:

$$\left.\begin{aligned} p' &= \pi\, e^{i\lambda x - i\omega t} \\ u' &= \mu\, e^{i\lambda x - i\omega t} \\ s' &= \sigma\, e^{i\lambda x - i\omega t} \quad . \end{aligned}\right\} \tag{30}$$

Substituting this into (10) - (12), we get the following equations:

$$\left.\begin{aligned} (-k + \lambda M)\pi + \lambda\mu &= 0 \\ \lambda\pi + (-k + \lambda M)\mu &= 0 \\ (-k + \lambda M)\sigma &= 0 \end{aligned}\right\} \tag{31}$$

where $k = \omega/C$. These equations have a nontrivial solution if the coefficient determinant vanishes. This yields the following relation for λ:

$$(-k + \lambda M)\ [(-k+\lambda M)^2 - \lambda^2] = 0 \quad .$$

The roots of this equation are given by

$$\left.\begin{aligned} \lambda_1 &= k/M \\ \lambda_2 &= k/(1+M) \\ \lambda_3 &= -k/(1-M) . \end{aligned}\right\} \tag{32}$$

The solutions to (31) corresponding to the first eigenvalue $\lambda = \lambda_1$ are:

$$\left.\begin{aligned} p' &= 0 \\ u' &= 0 \\ s' &= B\, e^{i(kx/M) - i\omega t} \quad , \end{aligned}\right\} \tag{33}$$

where B is an arbitrary constant. This wave corresponds to an entropy wave and it may be easily shown that they move with the fluid.

The solutions to (31) corresponding to the second eigenvalue $\lambda = \lambda_2$ are:

$$\left.\begin{aligned} p' &= A\, e^{i[kx/(1+M)] - i\omega t} \\ u' &= A\, e^{i[kx/(1+M)] - i\omega t} \\ s' &= 0 \end{aligned}\right\} \tag{34}$$

with A an arbitrary constant. This wave corresponds to a sound wave, i.e., it moves with the speed of sound relative to the fluid.

Finally, the third eigenvalue $\lambda = \lambda_3$ leads to the following solutions to (31):

$$\left.\begin{aligned} p' &= A\, e^{-i[kx/(1-M)]-i\omega t} \\ u' &= A\, e^{-i[kx/(1-M)]-i\omega t} \\ s' &= 0 \end{aligned}\right\} \tag{35}$$

This wave is identical with wave (35) except that it moves in exactly the opposite direction. Hence it corresponds also to a sound wave.

Plane waves incident from the supersonic side of the C-J detonation wave interact as follows. The incident plane wave is a sound wave moving in the supersonic flow and is described by (34), namely:

$$\left.\begin{aligned} p_1' &= \epsilon\, e^{i[kx/(1+M_1)]-i\omega t} \\ u_1' &= \epsilon\, e^{i[kx/(1+M_1)]-i\omega t} \\ s_1' &= 0 \;, \end{aligned}\right\} \tag{36}$$

where ϵ is the known amplitude of the incident waves and is supposed to be much less than 1 to justify use of the linear equations. Since the flow in front of the detonation wave is supersonic, no reflected waves are present. Thus we consider only a refracted sound wave and a refracted entropy wave on the sonic side of the detonation wave. These waves are described as follows:

Refracted Sound Wave:

$$\left.\begin{aligned} p_{2s}' &= A\, e^{i[kx/(1+M_2)]-i\omega t} \\ u_{2s}' &= A\, e^{i[kx/(1+M_2)]-i\omega t} \\ s_{2s}' &= 0 \end{aligned}\right\} \tag{37}$$

Refracted Entropy Wave:

$$\left.\begin{aligned} p_{2e}' &= 0 \\ u_{2e}' &= 0 \\ s_{2e}' &= B\, e^{i(kx/M_2)-i\omega t} \end{aligned}\right\} \tag{38}$$

Combining (37) and (38), the perturbation pressure, velocity, and entropy on the sonic side of the detonation wave are given by:

$$\left.\begin{aligned} p_2 &= A\, e^{i[kx/(1+M_2)]}\, e^{-i\omega t} \\ u_2' &= A\, e^{i[kx/(1+M_2)]}\, e^{-i\omega t} \\ s_2' &= B\, e^{i(kx/M_2)}\, e^{-i\omega t} \end{aligned}\right\} \tag{39}$$

We now assume that f(t) depends on its variables in the same way as p', u', s':

$$f(t) = a\, e^{-i\omega t} . \tag{40}$$

Therefore,

$$f_t = -i\omega a\, e^{-i\omega t} . \tag{41}$$

Substituting (36), (39), and (41) into the linearized shock conditions (27) - (29), dropping the common exponential factors, and evaluating at x = 0, we get the following relations:

$$\left.\begin{aligned} A &= \left\{ M_1 + \frac{1}{\gamma}\frac{F'(M_1)}{F(M_1)}\left[M_1 - \frac{2(M_1^2-1)}{\gamma-1}\right]\right\}\epsilon + \frac{1}{\gamma C_1}\frac{F'(M_1)}{F(M_1)}\, i\omega a \\ A &= \left\{1 + \frac{G'(M_1)}{G(M_1)}\left[M_1 - \frac{2(M_1^2-1)}{\gamma-1}\right]\right\}\epsilon + (i\omega a)\left\{\frac{G'(M_1)}{G(M)C_1} - \frac{1}{U}\left[\frac{1-G(M)}{G(M)}\right]\right\} \\ B &= \left[\frac{1}{\gamma}\frac{F'(M_1)}{F(M_1)} + \frac{G'(M_1)}{G(M_1)}\right]\left[M_1 - \frac{2(M_1^2-1)}{\gamma-1}\right]\epsilon + \frac{1}{C_1}\left[\frac{1}{\gamma}\frac{F'(M_1)}{F(M_1)} + \frac{G'(M_1)}{G(M_1)}\right] i\omega a \end{aligned}\right\} \tag{42}$$

This may be regarded as a linear algebraic system of three equations in the three unknowns A, B, a. To obtain finite solutions we must investigate the coefficient determinant. This is given by

$$\Delta = \frac{G'(M_1)}{G(M_1)C_1} - \frac{1}{U_1}\frac{1-G(M_1)}{G(M_1)} - \frac{1}{\gamma C_1}\frac{F'(M_1)}{F(M_1)} \tag{43}$$

Using the definitions of $F(M_1)$, $G(M_1)$ as given in (20), (21), we calculate Δ to be

$$\Delta = -\frac{1}{M_1 C_1(1+\gamma M_1^2)}\,(3M_1^2 + 1) \tag{44}$$

Hence it may be seen that Δ is never equal to zero. Thus we arrive at a finite value of "a" which depends on γ, M_1, C_1, i.e., all known initial variables of the gas in front of the detonation wave. Therefore the C-J detonation wave is stable with respect to incident sound waves from the supersonic side, provided it stays Chapman-Jouguet at all times. Although this assumption is rather severe, it gives us a valuable insight in the stability analysis. Our next task will be to relax the assumption that the wave remains Chapman-Jouguet at all times, i.e., we will allow it to become a strong or weak detonation upon the interaction with the incident sound waves.

CONCLUSIONS

1. Experiments on the supersonic mixing of hydrogen and air in a cold jet indicate incomplete mixing within the diverging section of the nozzle. However, measurements at the Mach disc location indicate fairly uniform hydrogen concentration over most of the Mach disc area.

2. Cold hydrogen was injected at sonic velocity into a surrounding sonic hot air stream and then mixed in a divergent nozzle without the occurrence of combustion. The stagnation temperature of the air was as high as 2600°R and the pressure level at the injection point was about 280 lb/sq in.

3. A high-stagnation-temperature, stagnation-pressure combustible mixture has been accelerated to a high Mach number and ignition effected by a strong normal shock wave in the open jet. The resulting configuration, a shock wave separated by a small distance from the combustion zone, has proved to be extremely stable.

4. The distance between the shock wave and combustion zone corresponds to an ignition time delay and consequently is strongly dependent on temperature. Many runs have been made with time delays observed between 8.5 and 35 μsec over the approximate stagnation temperature range of 2000°R - 2600°R. Conceivably, higher temperatures would continuously shorten this delay.

5. A stable hydrogen-air detonation has been successfully generated. Other waves have been observed which appear to be cases of shock ignition. Actual description of the waves as to strength is difficult because of two-dimensional effects in the combustion zone. Further clarification is required of the interaction between shock wave and combustion.

6. It is felt that the stabilization of a shock-wave—combustion zone in a supersonic jet is a potentially powerful tool in the experimental study of combustion. The steady-state flow field will allow more detailed observations of temperature and pressure effects, chemical kinetics, ignition time delays, mixing, and many other aspects. Furthermore, such a wave could be used in a hypersonic ramjet to avoid serious diffuser losses and to allow for a much shorter combustion chamber.

7. The analytical study of the normal C-J detonation wave shows it to be stable with respect to incident sound waves from the supersonic side provided the wave remains C-J at all times.

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