Abstract

*This study provides a model for the valuation of the benefits associated with government guarantee programs. The model generates the social value of a government loan guarantee under disequilibrium conditions that justify public investment as a second best decision rule. The social value of a loan is obtained by conditioning the benefits on the survival of the borrowing firm. An estimated hazard function captures the annual rate of benefit attrition resulting from the failure of borrowing firms. Benefit-cost indexes are generated by business class, thus providing ex ante asset allocation guidelines. Application of the model to a portfolio of loan guarantees administered by the Ontario Development Corporation indicates a positive social value under the assumed disequilibrium conditions.*

# THE VALUATION OF GOVERNMENT LOAN GUARANTEES: A THEORETICAL AND EMPIRICAL PERSPECTIVE

*SAMUEL MENSAH*
University of Michigan–Flint

The focus of this study is on government loan guarantees to private businesses. In both Canada and the United States, financing of small business through direct loans and guaranteed loans has become a major component of government disbursements to the private sector. Today, there are about 14 permanent agencies of government in Canada providing financing to private sector firms under about 76 programs. In the United States, both federal and state governments have programs that support the financing of small business.

A major gap exists in the finance and economics literature with respect to the valuation of the social benefits of government loan

AUTHOR'S NOTE: This article has benefited from extensive discussions with Myron Gordon, Ralph Winter, and Donald Brean. Jack Mintz commented on an earlier version. I am grateful to seminar participants at the University of Toronto Capital Markets Workshop for many helpful comments. The assistance of the Ontario Development Corporation in providing the data is gratefully acknowledged.

PUBLIC FINANCE QUARTERLY, Vol. 24 No. 2, April 1996 263-281

programs, with the existing tools for a cost-benefit analysis of government loans still at a rudimentary stage. This study develops a valuation model for government loan guarantees that measures the social benefits of government-sponsored loans to businesses in terms of the income generated by the borrowing firms. The model can be used for program evaluation where the primary interest is the effectiveness of a particular lending program in achieving the anticipated benefits. In addition, ex ante benefit-cost indexes by business class provide guidelines for asset allocation within a loan guarantee portfolio. The application of the model is illustrated through a valuation analysis of the small business loan programs of the Ontario Development Corporation (ODC).

The article is structured as follows. In the next section, I review the existing literature. The subsequent section presents the valuation model for loan guarantees, and the section that follows it applies to the loan guarantee program of the ODC. The final section concludes the study, identifies its policy implications, and suggests some future research possibilities.

## A REVIEW OF THE LITERATURE

Government lending for private investment has been rationalized on the basis of positive and normative considerations. The positive arguments justify public financing of private investment as an efficient response to the failure of private credit markets, whereas the normative arguments support a wide variety of public benefit arguments such as employment, economic diversification, and technological growth. With respect to credit market failure, the key theoretical and empirical questions relate to the causes of credit market failure, the extent to which it exists in the credit markets, and the effectiveness of government loan programs as market correction mechanisms.

Asymmetric information in credit markets and the moral hazard and adverse selection consequences have been well documented in the credit-rationing literature by Stiglitz and Weiss (1981), Chan and Kanatas (1985), and Besanko and Thakor (1987), among others; whereas studies of interest rate and other lending practices of banks such as that of Wynant and Hatch (1991) reveal practices that may cause firms that could borrow in an otherwise competitive market to be denied credit. Break (1965) argued that a government-sponsored loan program for market failure correction should offer loan terms and conditions that would prevail in an efficient and competitive market for that type of loan.

The argument that government lending resolves a market failure problem is, however, undermined by the large subsidy component and high rate of default among government borrowers. Subsidies in Canada on government loans to private enterprises for 1978-1979 were estimated at 1% to 41% of outstanding loans, depending on the program.[1] For the ODC, 1990 loan losses constituted 5.9% of outstanding loans and guarantees.[2] By contrast, Wynant and Hatch (1991) report that the actual average loss for Canadian-chartered banks for domestic loans during 1987-1989 was 0.39% of average outstanding loans.

The subsidies and loss rates in government loan programs make it necessary to look for alternative rationalizations for such programs. One promising approach is to analyze government financing of private investment as public investment decisions with private sector firms being the instruments of public investment. Stated in these terms, the financing of private sector investment can be justified as second best rules for public investment in a regime of disequilibrium in the labor and capital markets. Marchand, Mintz, and Pestieau (1985) have justified public investment in a regime under which both the real wage and the real rate of interest are set too high to clear the labor and capital markets.[3] Implicit shadow prices for labor and capital are shown to be less than market prices. This theory provides a welfare-theoretic justification for government loans.

Studies of the social benefits of government loans such as Break (1965) and Bosworth, Carron, and Rhyne (1987) evaluate a government loan by focusing on the magnitude of the government subsidy contained in government credit as a basis for establishing whether the social benefits are at least equal to the subsidy. In empirical work using U.S. Small Business Administration (SBA) data, Rhyne (1988) estimated that for SBA loans the subsidy value of a loan is 11% of the loan amount. Therefore, the criterion of benefit is that the incremental benefits attributed to a loan must equal or exceed 11% of the loan value. A somewhat more comprehensive measure of cost is used by the Economic Council of Canada (1982) and Brent (1991), making use of the theory of economic cost-benefit analysis. The loss of welfare, as measured by the benefits that society forgoes by investing funds in subsidized ventures rather than in the private sector, is the cost of government intervention. The Economic Council of Canada estimated, for example, that for fiscal 1978-1979, the social opportunity cost of a government loan was 14.26% of the loan amount.

The literature on the social costs and benefits of government loans suffers from the weakness that, although the subsidy costs of loans were estimated, no corresponding attempt was made to estimate benefits. In an obvious recognition of this deficiency, Rhyne (1988) observed in her SBA study, "With respect to externality goals, the information here cannot conclusively state whether SBA loans produce enough benefits to compensate for the subsidy" (p. 91). Brent (1991) has formulated a model that provides a cost-benefit analysis of a government loan by analyzing the loan as a public expenditure while evaluating the social benefits in terms of the redistribution implicit in the subsidized rate of interest charged to specified target groups of borrowers. The model, however, does not consider the consequences of the failure of firms for the anticipated benefits of the public investment. The contribution of this article is the presentation of a model that considers both social costs and benefits as well as the possibility of firms defaulting and/or failing after receiving a government loan.

## THE MODEL

From society's perspective, a loan guarantee achieves its welfare objectives if the project[4] undertaken with the loan guarantee generates income in excess of the social cost of funds. The social cost and benefits are modeled under the following assumptions:

1. Loans within the government portfolio represent a net increase in credit availability.[5]

2. There is involuntary unemployment as identified by Marchand et al (1985) such that public investment is justifiable as a second best investment rule.

## THE BENEFITS OF A LOAN GUARANTEE

The benefits of a loan guarantee are assumed to be given by the income generated and is captured by the net income of the project. When there is no economic slack, the real productive activity associated with the loan may lead to the displacement of the income of competing nonborrowing businesses. Our assumed involuntary unemployment implies that the income resulting from the loan is a net addition with the annual net income for a single project given by $Y$.

The present value at time 0 of the perpetual benefit on a default-free loan is

$$W(0) = \frac{Y}{r_g}, \quad (1)$$

where $r_g$ is the social discount rate.

Ex ante, the duration of the expected benefits of a loan depends on the survival of the borrowing firm. The uncertainty associated with the duration of benefits is captured by specifying a survival distribution of firms in the loan portfolio. Firms are thus assumed to be exposed to a loan spell at the initiation of the loan, with the spell lasting for the loan period. If the firm defaults, the spell ends and the firm is deemed to have failed. If the firm survives the loan spell, it is assumed to continue forever. The latter assumption, although strong, is increasingly inconsequential in an intertemporal setting the longer the loan term.

There are several equivalent methods of characterizing the probability distribution of the survival of firms over time.[6] Let $\tilde{T}$ = the survival time of a randomly selected firm, where survival time covers all states in which the firm is not in default including the period beyond the loan term. The survival time $\tilde{T}$ is a random variable with probability density function given by

$$f(t) = \lim_{\Delta t \to 0} \text{Prob} \frac{(t < \tilde{T} \le t + \Delta t)}{\Delta t} \quad 0 < t < \infty. \quad (2)$$

The cumulative distribution is $F(t) = \text{Prob}(\tilde{T} < t)$ or the probability that spell duration time $\tilde{T}$ is less than some fixed value $t$, whereas the survivor function $S(t) = \text{Prob}(\tilde{T} \geq t) = 1 - F(t)$ is the probability that survival time will equal or exceed a given value $t$.[7] For a loan with a payment term of $t^*$, the probability that the firm will not default during the loan term is $S(t^*)$.

The hazard rate ($h(t)$) is the conditional probability that a firm completes its spell at $t$ given that it has survived to $t$ and is related to the probability distribution and survival distribution through $h(t) = f(t)/S(t)$. The function $h(t)$ characterizes the immediate risk attached to firms known to have survived to a given age $t$ and is thus the transition rate from survival to failure. Over the lifetime of a firm, the rate at which society loses the benefits of the loan in each year is the transition rate from survival to failure during that year and is given by $h(t)$. Define $\tilde{Y}(t)$ as the "hazard-adjusted benefit" of a loan during period $t$. Then the expected benefit of the loan during $t$ after deducting the benefit loss resulting from failure during $t$ is $\tilde{Y}(t) = Y[1 - h(t)]$. The annual hazard-adjusted benefit over the loan term is given by the following set of equations:

$$
\begin{aligned}
\tilde{Y}(1) &= Y[1 - h(1)] \\
\tilde{Y}(2) &= Y\{[1 - h(1)][1 - h(2)]\} = \tilde{Y}(1)[1 - h(2)] \\
&\;\;\cdots \\
\tilde{Y}(t^*) &= Y\{[1 - h(1)][1 - h(2)]\} \ldots [1 - h(t)].
\end{aligned}
\qquad (3)
$$

Assuming that the hazard-adjusted benefit in the final year of the loan is sustained permanently, and letting

$$S'(t) = \prod_{i \leq t}^{t} [1 - h(i)]$$

we can more compactly specify $\tilde{Y}(t)$ as follows:

$$
\begin{aligned}
\tilde{Y}(t) &= YS'(t) \text{ for } t < t^* \\
\tilde{Y}(t) &= YS'(t^*) \text{ for } t \geq t^*.
\end{aligned}
\qquad (4)
$$

The present value of the hazard-adjusted benefits of a loan is

$$\tilde{Y}(0) = \sum_{t=1}^{t*-1} \frac{\tilde{Y}(t)}{(1+r_g)^t} + \frac{\tilde{Y}(t^*)}{r_g}\left(\frac{1}{1+r_g}\right)^t. \tag{5}$$

## THE SOCIAL COST OF A GOVERNMENT LOAN GUARANTEE

Mintz, Carrière, and McCaughey (1984) have analyzed the opportunity cost of a loan guarantee as being the sum of the cost of default and the administrative cost of processing loans less any fees charged by the government. The issue of whether to cost loan guarantees at their social opportunity cost or at the actual monetary outlay is resolved by assuming that the resources for the settlement of defaults and administrative costs are withdrawn from the private sector's productive activities rather from a new supply of resources. The social opportunity cost $C_s$ is thus given by

$$C_s = \theta C + (1 - \theta)C_*, \tag{6}$$

where $\theta$ = the portion of resources drawn from productive activity, $C$ is the private firm's opportunity cost of using the resource, and $C_*$ is the individual's evaluation of the last unit of foregone activity. With $\theta$ = 1, $C_s = C$ and the opportunity cost of the loan guarantee for a given project is

$$C(0) = L(0) + K(\delta - \alpha), \tag{7}$$

where $C(0)$ = the social cost at time 0 of the loan guarantee, $L(0)$ = the present value of losses paid by the government when the loan goes into default, $\alpha$ = the fee per dollar of loan disbursement charged by the government as a guarantee fee, $\delta$ = the administrative cost per dollar of loan disbursement, and $K$ = the amount of the loan.

With the exception of the administrative cost, a disbursement is made only if the loan goes into default and the guarantee is called, thus making resource cost of the loan guarantee a contingent obligation. The valuation of loan guarantees as contingent obligations has been addressed in the options valuation literature.[8] The basis of the valuation of a loan guarantee is that the value of a bank loan is the loan's risk-free value less the value of a put option written on the assets of the borrowing firm and sold to the shareholders of the borrowing firm by the bank. The bank's risk premium is reflected in the option premium of the put, whereas the put option allows the shareholders to walk away from their obligation to the bank when the company is insolvent. When the government puts in place a loan guarantee, the cost of the put option is assumed by the government, thus relieving the borrowing firm of the cost of the risk premium that it would have paid for bank financing. The cost of the loan guarantee to the government is, therefore, the option premium. For empirical work, however, valuation of the put option in a loan guarantee presents some intractable problems. When a loan is amortized (as is the case with many government loan schemes), the put option is a package of sequential options, each with an exercise price equal to the payment due on the loan for a particular year. A second problem that is especially severe for small firms is how to obtain the market values of the assets of the borrowing firms on which the put option is assumed to be written.

As an alternative to the options valuation model, we can specify the cost of a loan guarantee in terms of an actuarial reserve that represents the present value of the expected losses to the government under the guarantee. If the sequence of put options implicit in a loan guarantee reflects the risk of the borrowing firm, then the actuarial reserve would mimic the present value of acquiring a series of put options during the period of the guarantee.

Let $a(t)$ = the annual contracted payment on the loan for year $t$. The annual attrition of the payments on the loan is given by the transition rate of firms from survival to failure for each year. The expected annual loss is, therefore, the difference between the contracted payment on the loan $a(t)$ and an expected payment $\tilde{a}(t)$, which is the hazard-adjusted annual payment on the loan based on the hazard function $h(t)$. Let $\tilde{L}(t)$ denote the expected loss in period $t$. The annual loan loss for the given hazard function $h(t)$ is given by the following set of equations:

$$\begin{aligned} \tilde{L}(1) &= a(t) - a(t)[1 - h(1)] = a(t) - \tilde{a}(1) \\ \tilde{L}(2) &= a(t) - a(t)\{[1 - h(1)][1 - h(2)]\} = a(t) - \tilde{a}(2) \\ &\;\;\ldots \\ \tilde{L}(t^*) &= a(t) - a(t)\{[1 - h(1)][1 - h(2)] \ldots [1 - h(t^*)]\} = a(t) - \tilde{a}(t^*) \end{aligned} \tag{8}$$

or more compactly as

$$\tilde{L}(t) = a(t)\{[1 - \prod_{i \le t}^{t} [1 - h(i)]\} = a(t)[1 - S'(t)]. \tag{9}$$

The present value of the hazard-adjusted loan loss represents the actuarial reserve that is needed to satisfy the guarantee and is given by

$$\tilde{L}(0) = \sum_{t=t}^{t^*} \frac{\tilde{L}(t)}{(1 + r_g)^t}, \tag{10}$$

whereas the hazard-adjusted social cost is now

$$\tilde{C}(0) = \tilde{L}(0) + K(\delta - \alpha). \tag{11}$$

The ex ante or expected net present social value of a loan guarantee $V(0)$ is the difference between the hazard-adjusted benefits and costs and is given by

$$V(0) = \tilde{Y}(0) - \tilde{C}(0). \tag{12}$$

## The Benefit-Cost Index of a Loan Guarantee

The benefit-cost ratio is defined as the ratio of the present value of gross benefits to cost as follows:

$$I = \frac{V(0) + \tilde{C}(0)}{\tilde{C}(0)}. \tag{13}$$

In the presence of a government budget constraint for funding loan losses, decision rules based on the net present value of social benefits will be suboptimal because funds may be directed toward individual projects with large values, whereas in fact several small loans with a high ratio of benefits to cost may in fact provide a higher aggregate social benefit. The benefit-cost criterion provides a basis for selecting projects for financing. For example, ex ante benefit-cost indexes for specific business classes could provide project selection guidelines for lending agencies under loan guarantee programs.

**The Minimum Required Duration**

A government agency might be interested in the minimum survival time that is necessary for a loan to break even in social terms. The minimum required duration of the loan $t_{min}$ is obtained by solving for the number of years over which the hazard-adjusted income equals the social cost of the loan:

$$\sum_{t=t}^{t_{min}} \frac{\tilde{Y}(t)}{(1+r_g)^t} - \tilde{C}(0) = 0. \quad (14)$$

Only projects that are expected to survive beyond the minimum required duration would have positive social net present values. However, project selection using the minimum required duration would be biased in favor of projects with high initial incomes and may not be consistent with a criterion that selects projects with high positive social net present values. Thus the minimum required duration rule may be more appropriate when there are budgetary constraints on the funding of loan losses.

## ANALYSIS OF THE NEW VENTURES LOAN PROGRAM OF ONTARIO

The loan guarantee program of the ODC is the New Ventures Program (NVP), which is designed to assist small business start-ups by guaranteeing personal loans of up to $15,000 for a 5-year term from a participating financial institution. Borrowers are required to provide a matching equity contribution from their own resources. Repayments are interest-only for the first year with principal repayments spread over the subsequent 4 years. When a borrower defaults, the lending institution submits a claim to the ODC, which then pays off the loan balance and pursues recovery of the outstanding amount.

The empirical objective is to establish whether a sample of NVP loans has a positive social value. Values are calculated for individual loans and used to obtain portfolio totals and averages for social values, benefit-cost indexes, and minimum required durations. The social values are calculated for the business classes represented in the loan portfolio. Benefit-cost indexes are calculated for each business class and tested for homogeneity across business classes with both parametric and nonparametric tests.

The NVP valuation analysis is based on a sample of 467 loans drawn sequentially from the starting date of the program, which was September 1986. Loan data were supplied by the ODC and are classified into six industrial classes: retail, food, manufacturing, service/tourism, construction, and wholesale/distribution. The raw loan data are summarized in Table 1. (In this and other tables, no Class 3 is identified.) The total amount of loans disbursed is $6.3 million with the average loan size being $13,343. Both in terms of number of loans and amount, the retail and service/tourism sectors dominate the portfolio, each accounting for about one third of the program.

There are some parametric families of survival distributions such as the Weibull and lognormal. However, in the estimation of parametric distributions, it is assumed that the functional form of the distribution in the absence of censoring[9] is known. However, the data for this study are censored, with a 5-year observation period starting in September 1986 and ending in September 1991. Thus nonparametric techniques are used to estimate the survival distribution. The estimation method is the life table method that splits the survival times into equal time intervals.[10] Failure is assumed to occur on the date the bank calls the guarantee. Table 2 summarizes the raw failure data by business class. Out of the total sample of 467 loans, there were 141 failures with the remaining 326 loans being censored.

Hazard-adjusted benefits and costs based on the total sample hazard function would be inaccurate if the underlying survival function differs by business class. To determine the homogeneity of survival functions across business classes, I applied two tests for homogeneity. The log-rank test places more weight on larger survival times, whereas the Wilcoxon test places more weight on early survival times. Table 3

**TABLE 1: New Ventures Loans: Summary Data**

| *Class* | *Description* | *Number of Loans* | *Percentage* | *Loan Amount (in dollars)* | *Percentage* |
|---|---|---|---|---|---|
| 1 | Retail | 163 | 34.90 | 2,244,820 | 36.02 |
| 2 | Food | 48 | 10.28 | 662,860 | 10.64 |
| 4 | Manufacturing | 64 | 13.70 | 860,800 | 13.81 |
| 5 | Service/tourism | 150 | 32.12 | 1,904,925 | 30.57 |
| 6 | Construction | 12 | 2.57 | 158,500 | 2.55 |
| 7 | Wholesale/ distribution | 30 | 6.42 | 399,600 | 6.41 |
| Total | | 467 | 100.00 | 6,231,505 | 100.00 |

**TABLE 2: Summary of Number of Censored and Uncensored Values**

| *Class* | *Total* | *Failed* | *Censored* | *Percentage Censored* |
|---|---|---|---|---|
| 1 | 163 | 53 | 110 | 67.48 |
| 2 | 48 | 15 | 33 | 68.75 |
| 4 | 64 | 14 | 50 | 78.13 |
| 5 | 150 | 49 | 101 | 67.33 |
| 6 | 12 | 2 | 10 | 83.33 |
| 7 | 30 | 8 | 22 | 73.33 |
| Total | 467 | 141 | 326 | 69.81 |

presents hazard function estimates for all six business classes. All the estimated hazard rates satisfy a 95% confidence interval for a normal distribution. Both the log-rank and Wilcoxon tests are unable to reject the null hypothesis of homogeneity, indicating that the survival estimates are homogeneous across all survival times. Based on this result, I applied the total sample hazard function estimates to the determination of hazard-adjusted benefits and costs.

The survival estimates for the total sample are presented in Table 4. The probability estimates for the survivor function and the hazard function all satisfy a 95% confidence interval for the normal distribution. The cumulative survival rate is .7281, indicating that 27% of firms in the program fail in the long run. The hazard function rises and falls, peaking during the second year of the loan with the probability of a surviving firm failing at that point in time being .13. A slight

**TABLE 3: Hazard Estimates, by Business Class**

| *Interval (years)* | *Class 1* | *Class 2* | *Class 4* | *Class 5* | *Class 6* | *Class 7* |
|---|---|---|---|---|---|---|
| 0-1 | .05031 | .04255 | .03175 | .02020 | .00000 | .00000 |
| 1-2 | .13793 | .06742 | .06667 | .16176 | .18182 | .18182 |
| 2-3 | .09302 | .12346 | .05310 | .10084 | .00000 | .04082 |
| 3-4 | .07595 | .05405 | .07547 | .07339 | .00000 | .00000 |
| 4-5 | .07018 | .16667 | .03922 | .07619 | .00000 | .16667 |

NOTE: All hazard estimates satisfy a 5% confidence interval under the assumption of a normal distribution. Test of equality over business classes: log-rank—$\chi^2 = 4.0716$, $df = 5$, $p > \chi^2 = .5392$; Wilcoxon—$\chi^2 = 4.0225$, $df = 5$, $p > \chi^2 = .5462$.

**TABLE 4: Life Table Survival Estimates for New Ventures Loans**

| *Interval* | | | | | | |
|---|---|---|---|---|---|---|
| *Lower Bound* | *Upper Bound* | *Number Failed* | *Number Censored* | *S(t) Survival* | *F(t) Failure* | *h(t) Hazard*[a] |
| 0.00000 | 1.00000 | 15 | 0 | 1.0000 | 0 | .03264 |
| 1.00000 | 2.00000 | 56 | 0 | .9679 | .0321 | .13208 |
| 2.00000 | 3.00000 | 33 | 0 | .8480 | .1520 | .08696 |
| 3.00000 | 4.00000 | 23 | 0 | .7773 | .2227 | .06543 |
| 4.00000 | 5.00000 | 14 | 326 | .7281 | .2719 | .08235 |

a. All hazard estimates satisfy a 5% confidence level under the assumption of a normal distribution.

increase in the hazard function is observed for the fifth year of the loan, which is the terminal year of the loan.

To calculate the social net present value and benefit-cost indexes, the following parameter values are used.

*Administrative costs.* Participating commercial banks are allowed to charge an administrative fee of $150, which is deducted from the loan amount as reimbursement for loan processing costs. In addition, the ODC incurs expenditures for administering the NVP. However, the ODC does not charge borrowers a fee for the guarantee. As at March 31, 1989, the program had incurred total administrative expenditures (including the administration fee) of $2,983,347 on total loan guarantees of $139,996,434, making for an administrative cost of $0.0213 per dollar of loan guarantee.[11] Thus the parameter values for equation (7) are $\delta = 0.0213$ and $\alpha = 0$.

*The discount rate.* The social discount rate is assumed to be the borrowing rate of the government of Ontario. Musgrave and Musgrave

(1984) justify the borrowing rate as being appropriate for state and provincial governments that are primarily concerned with the allocation of resources within their specific regions and not on a nationwide basis. The discount rate is the yield to maturity on the 5-year Ontario Bond, which was 9.45% on September 1, 1986[12]; therefore, $r_g = .0945$.

*Loan interest rate*. A loan interest rate is required for the calculation of the required annual payment on a loan. The NVP interest rate is either the lender's prime commercial rate plus 1% or the lender's fixed interest rate for similar projects. Our analysis is based on the prime commercial lending rate as reported by the Bank of Canada[13] for the end of September 1986. On this basis, prime plus 1% is 10.75%.

*Income*. The income of the project is based on the first-year projected net income reported by the business in its loan application. The reported income figure may be upwardly biased if loan applicants generally tend to exaggerate the expected income of the project. However, by using only first-year reported net income and thus excluding income growth, there is some offset against the excessively optimistic projections.

Equations (3) and (4) are based on the assumption that the income benefits of the loan are sustained permanently if the loan is paid off by the maturity date. The calculated social value of the loan based on this assumption is thus an upper bound. For purposes of comparison, I also calculated a lower bound that assumes that social benefits terminate at the end of the loan term. One insight that is gained from this comparison is that we are able to establish whether the lower bound is positive.

## VALUATION RESULTS

The calculated social values and minimum required durations are presented in Table 5, both by business class and in total. The lower bound and upper bound values for all business classes are positive. For the entire portfolio, we have a lower bound of $56.6 million and an upper bound of $131.3 million. The average social values per loan are $0.121 million (lower bound) and $0.281 million (upper bound). The average minimum required duration for the entire portfolio is 0.1952 years or 2.34 months. The minimum required duration varies

**TABLE 5: Social Value of New Ventures Loan Guarantees (thousands of dollars)**

| | *Total Social Value* | | *Average Social Value* | | |
|---|---|---|---|---|---|
| *Class* | *Lower Bound* | *Upper Bound* | *Lower Bound* | *Upper Bound* | *Average Minimum Required Duration 5-Year Survival* |
| 1 | 15,030 | 34,967 | 92 | 215 | .2539 |
| 2 | 7,323 | 16,959 | 153 | 353 | .0991 |
| 4 | 8,529 | 19,767 | 133 | 309 | .1390 |
| 5 | 15,027 | 34,898 | 100 | 233 | .2051 |
| 6 | 2,465 | 5,696 | 205 | 475 | .0582 |
| 7 | 8,250 | 19,042 | 275 | 635 | .1439 |
| Total | 56,624 | 131,329 | 121 | 281 | .1952 |

significantly across business classes, being as high as 0.2539 (3.04 months) for Class 1 (retail) and as low as 0.0582 (0.69 months) for Class 6.[14]

In Table 6, I present the benefit-cost index calculations. For the entire portfolio, the benefit-cost index ranges from 68.95 (lower bound) to 159 (upper bound). To test for the homogeneity of benefit-cost indexes across business classes, I employed two tests: a one-way analysis of variance (based on a normality assumption) and the non-parametric Kruskall-Wallis test (based on the ranks in the data). The null hypothesis of homogeneity is rejected by both tests at probability levels of .0001 and .0308, respectively. This result indicates that ex ante rules for the selection of small business financing projects may be made on the basis of ex ante benefit-cost indexes. For example, a project selection rule that discriminates in favor of Class 7 projects would generate higher aggregate social values.

## SUMMARY, CONCLUSION, AND FURTHER RESEARCH ISSUES

This study has presented a model for the valuation in social terms of government loan guarantees for private investment. The structure of the model reflects the notion that, under a government loan, a public investment is undertaken using a private firm as the medium. In contrast to a direct public investment, the social benefits of an investment that is undertaken through a private firm are conditional on the

**TABLE 6: Average Benefit-Cost Indexes**

| *Class* | *Lower Bound* | *Upper Bound* |
|---|---|---|
| 1 | 48.03 | 110.43 |
| 2 | 82.25 | 189.11 |
| 4 | 70.63 | 162.38 |
| 5 | 66.48 | 152.85 |
| 6 | 129.41 | 297.54 |
| 7 | 145.89 | 335.42 |
| Total | 68.95 | 158.52 |

NOTE: ANOVA: $F$ value = 5.143, $p > F$ = .0001; Kruskall-Wallis test: $\chi^2 = 12.31$, $df = 5$, $p > \chi^2 = .0308$.

survival of the private firm. The use of the hazard function has provided a means of capturing the attrition of benefits due to firm failure. The hazard rate model also shows that it is not necessary for the recipients of a government loan to survive permanently for the loan program to have a positive social value. What is critical is that the firm survives for the minimum required duration ($t_{min}$). This perspective suggests that a binary evaluation that views the contribution of government loans in terms of whether the borrowing firm failed or survived may be simplistic.[15]

Within the context of the limited literature on government-sponsored loan programs, this article's contribution is twofold. First, it contributes to the study of the benefit side of government loan programs, which most of the existing literature has defined in terms of implicit subsidies rather than direct benefits. Second, it introduces a method for recognizing the attrition of benefits resulting from the failure of firms. This methodological contribution provides opportunities for enriching the few direct-benefit-oriented studies such as that of Brent (1991).

The estimated benefits indicate net positive social values and high benefit-cost ratios under the assumed disequilibrium conditions. The assumption of fixed real wages and real interest rates under a labor and capital markets disequilibrium is strong. However, Marchand et al. (1985) note that the specified conditions may exist under price flexibility if such flexibility is inadequate to clear markets in the short run. The widespread use of minimum wages, unemployment benefits, and restrictive employment protection laws suggests that such inflexibility exists in many economies. However, the magnitude of the benefits and the low minimum required duration also reflect the significant savings that accrue to loan guarantee programs because of the deferral of government disbursements to the date on which the lending institution calls the guarantee. The intertemporal framework of the model used in this study makes it possible to capture fully the value of this deferral, thus providing a partial justification for the findings.

Policy formulation and implementation should recognize, first, that departures from disequilibrium could imply social values that may be much less or even negative. Second, to the extent that the social benefits of the loan guarantee differ from the profitability of the loan to a lending institution participating in the loan guarantee program, the selection of borrowers by banks may lead to socially suboptimal loans being made. In the NVP portfolio, there seems to be little difference in the survival profiles of firms in different business classes so that the differences in benefit-cost indexes are induced primarily by differences in the rates of profitability of each business class. To the extent that the participating banks use expected profitability as a selection criterion, their project selection will be consistent with the social optimum.

The article points to several directions for additional research. First, the additionality of a loan is based on the assumption that there are no private lenders willing to make the loan in the absence of the government guarantee. To the extent that government loan guarantees crowd out private lenders, private output is lost. Thus further research is needed to establish the magnitude of such crowding out, if any. Second, the survival distribution may be sensitive to the loan terms and conditions specified within the government program. In the particular program studied in this article, there were minimal rules set by the government. However, programs with distributional objectives with interest rates and payment terms based on means tests could endogenize the survival distribution.[16] Further theoretical and empirical research would establish the extent to which the survival distribution under a government loan program is endogenous to the terms and conditions of the program. Finally, the availability of comparable data on loans administered directly would provide an interesting comparative evaluation that would reflect key differences such as the value of the deferral of disbursements, the differential screening ability of private lending institutions compared to government-sponsored lending agencies, and differences in administrative cost.

## NOTES

1. See Economic Council of Canada (1982).
2. Source: Development Corporations of Ontario (1991).
3. The regime corresponds to Malinvaud's (1977) "classical unemployment."
4. The terms "firm" and "project" are used interchangeably. Benefits and costs are defined only for the project for which the loan was made, and termination of the project means termination of the firm.
5. Government lending agencies usually enforce this requirement by requiring prospective borrowers to show evidence of having been turned down by a financial institution.
6. The discussion follows that of Cox and Oakes (1984). For a review of economic applications, see Kiefer (1988).
7. The survival analysis literature follows the convention that $F(t) = \text{Prob}(T < t)$ rather than $\text{Prob}(T \leq t)$ (Cox and Oakes 1984).
8. See, for example, Merton (1977) and Selby, Franks, and Karki (1988).
9. In survival analysis, subjects that are still alive at the end of the observation period are said to be censored.
10. The alternative estimator is the Kaplan-Meier or product limit estimator. This estimator has the disadvantage that the intervals for which the hazards are computed depend on the data.
11. See Ministry of Industry, Trade and Technology (1989).
12. Source: Financial Post Information Service (1986).
13. Source: Bank of Canada (1987).
14. Although the average minimum required duration is generally inversely related to the social value of the loan, the ranking by business class may not be consistent. This occurs because social costs are recovered sooner when initial income is high. For business classes with cross-sectional skewness toward high income, the average minimum required duration would have a more favorable ranking than would the net social present value.
15. This approach is widespread in the popular media. See, for example, "Loan Guarantees" (1991).
16. I am grateful to an anonymous referee for this observation.

## REFERENCES

Bank of Canada. 1987. *Bank of Canada Review* (Ottawa), April.

Besanko, D., and A. Thakor. 1987. Competition, collateral and rationing: Sorting equilibria in the credit market. *International Economic Review* 98:671-90.

Bosworth, Barry, Andrew S. Carron, and Elisabeth Rhyne. 1987. *The economics of federal credit programs*. Washington, DC: Brookings Institution.

Break, George F. 1965. *Federal lending and economic stability*. Washington, DC: Brookings Institution.

Brent, Robert J. 1991. The cost-benefit analysis of government loans. *Public Finance Quarterly* 19:43-66.

Chan, Y. S., and G. Kanatas. 1985. Asymmetric valuations and the role of collateral in loan agreements. *Journal of Money, Credit and Banking* 17:84-95.

Cox, D. R., and D. Oakes. 1984. *Analysis of survival data*. London: Chapman & Hall.

Development Corporations of Ontario. 1991. *Development Corporations of Ontario: Annual review, 1989/90*. Toronto: Development Corporations of Ontario.

Economic Council of Canada. 1982. *Intervention and efficiency: A study of government credit and credit guarantees to the private sector*. Ottawa: Economic Council of Canada.

Financial Post Information Service. 1986. *1986 Canadian Bond Prices*. Toronto: Financial Post Information Service.

Kiefer, Nicholas M. 1988. Economic duration data and hazard functions. *Journal of Economic Literature* 26:646-79.

Loan guarantees haunting Alberta. 1991. *Globe and Mail*, 22 May.

Malinvaud, E. 1977. *The theory of economic unemployment revisited*. Oxford, England: Blackwell.

Marchand, Maurice, Jack Mintz, and Pierre Pestieau. 1985. Public production and shadow pricing in a model of disequilibrium in labour and capital markets. *Journal of Economic Theory* 36:237-56.

Merton, R. C. 1977. An analytic derivation of the cost of deposit insurance and loan guarantees. *Journal of Banking and Finance* 1:3-11.

Ministry of Industry, Trade and Technology. 1989. *Report on the evaluation of the New Ventures Program*, Small Business Advocacy Report No. 33. Ontario: Ministry of Industry, Trade and Technology.

Mintz, J. M., J. Carrière, and C. McCaughey. 1984. Determining the Subsidy Involved with Government Credit Programs: An Application to a Selected Group of Programs. Discussion Paper No. 256. Ottawa: Economic Council of Canada.

Musgrave, R. A., and P. Musgrave. 1984. *Public finance in theory and practice*. 4th ed. New York: McGraw-Hill.

Rhyne, Elisabeth. 1988. *Small business, banks and SBA loan guarantees: Subsidizing the weak or bridging a credit gap. New York: Quorum Books.*

Selby, M.J.P., J. R. Franks, and J. P. Karki. 1988. Loan guarantees, wealth transfers and incentives to invest. *Journal of Industrial Economics* 37:47-65.

Stiglitz, J. E., and A. Weiss. 1981. Credit rationing in markets with imperfect information. *American Economic Review* 71:393-410.

Wynant, Larry, and James Hatch. 1991. *Banks and small business borrowers*. London, Canada: Western Business School, University of Western Ontario.

*Samuel Mensah is Assistant Professor of Finance at the School of Management, University of Michigan–Flint. His main research interests are small business lending, venture capital, and corporate governance. He has published in the* Journal of Futures Markets *and has written several working papers on these subject areas.*