Technical Change, Accumulation and the Rate of Profit

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ABSTRACT: The effect of technical change and accumulation on the profit rate depends on their effect on the real wage rate. Employing a standard one-sector "circulating-capital" model, it is assumed that the wage rate is at least non-decreasing in labor demand. Then, *ceteris paribus*, capital-using labor-saving technical change does not increase the real wage rate. Depending on parameters, there are then cases of viable capital-saving technical change, possibly labor-saving as well, in which the rate of profit falls, as well as other cases in which it rises. But, in the absence of sufficient accumulation, capital-using labor-saving technical change, and, further, technical change in which the organic composition of capital rises, induces a *rise* in the rate of profit.

Attempts to ground a tendency for the rate of profit to fall in other social processes long precede Marx's (1967) theoretical investigation of the effect of technical change and capital accumulation on the rate of profit. Okishio's (1961) demonstration that in certain economic models viable technical change raises the rate of profit unless the real wage rate increases has profoundly influenced subsequent study, notably that of Foley (1986) showing that, with a wage rate rising to maintain a constant wage share of output, capital-using technical change lowers the rate of profit.

These concerns are here investigated within a standard "circulating capital" model in which the focus is directed to circumstances under which the real wage rate may itself be influenced by technical change and accumulation. That is, the real wage rate is treated endogenously. It is argued that even the weak assumption that the real wage rate is non-decreasing in the demand for labor entails noteworthy consequences. In particular it is shown that, in the absence of sufficient capital accumulation, viable capital-using technical change raises the general rate of profit and, more

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generally, viable technical change which increases the "organic composition of capital" raises the general rate of profit. *Ceteris paribus,* viable capital-using technical change decreases the demand for labor, a point arguably insufficiently appreciated in a venerable tradition of economic thinking.

In the course of presenting these arguments a graphic representation is developed that facilitates understanding received positions in a comparative context and suggests lines of investigation heretofore neglected.

I

In a one-period, one-sector, fixed-proportions-technology model, i.e., the standard one-sector "circulating-capital" model, the fundamental price equation is

$$m(1 + r) + wl = 1$$

if wages are paid at the end of the period .

Here m is the material input/output coefficient (in material-units/material-units) and l is the labor input/output coefficient (in labor-units/material-units). That is, m is the quantity of material input used to produce a unit of output and l is the quantity of labor input. Then r is the rate of profit (a pure number), and w is the real wage rate (in material-units/labor-units).

That is, the (numeraire) price of output, 1, is equal to the (numeraire) price of material input per unit of output marked up by the rate of profit plus the wage rate times the labor input per unit of output.

Rearranging,

$$r = \frac{1 - wl}{m} - 1$$

is the rate of profit equation.[1, 2]

In the terminology of Okishio (1961), a technical change is *viable* just in case, at prevailing relative prices, the rate of profit would be higher with the new technology. If individual firms are price-takers and profit-maximizers, they will separately introduce available alternative technologies only if they are viable.

That is, an alternative technology m', l' is viable just in case[3]

$$\frac{1 - wl'}{m'} > \frac{1 - wl}{m}.$$

Or, reformulating the condition in terms of the proportional change in each coefficient, with $\mu = (m' - m)/m$ and $\lambda = (l' - l)/l$, and employing the abbreviation $q = (1 - wl)/wl$,[4] the viability condition *VIA* is

$$\lambda < -q\mu. \qquad VIA$$

Graphs in μ, λ space allow visualization of all possible technical changes from some initial technology m, l represented by the origin (O, O). (Of course only points to the right of $\mu = -1$ and above $\lambda = -1$ are feasible; neither the material nor labor inputs can be negative.) This space can be partitioned in various ways to represent technical changes meeting conditions of interest, for example viability and a lower equilibrium rate of profit.

For example, as graphed in μ, λ space in Figure 1, the viability condition *VIA* determines a straight line through the origin with downward slope $-q$. Every point to the left and below this line is viable.

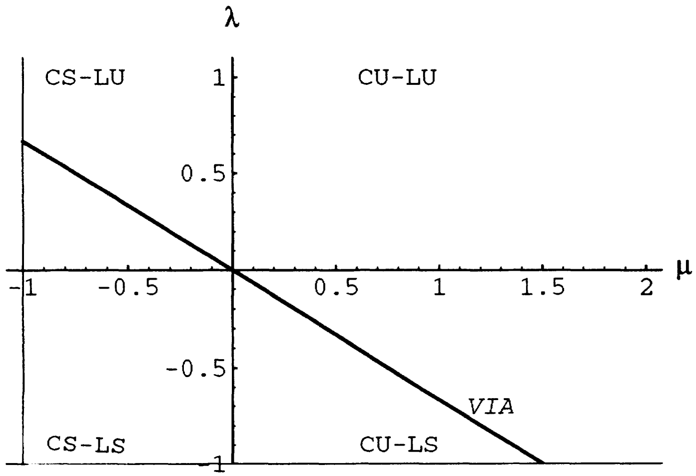

Figure 1

Thus any technical change which is neither labor-using (LU) nor capital-using (CU), i.e., any technical change in which both $\mu \leq 0$

and $\lambda \leq 0$, is viable, while any technical change which is neither capital-saving (CS) nor labor-saving (LS), i.e., both $\mu \geq 0$ and $\mu \geq 0$ and $\lambda \geq 0$, is not viable. In two classes of cases, viability depends on the value of q: cases of LS but CU technical change in which $\lambda < 0$ but $\mu > 0$, and cases of CS but LU technical change in which $\mu < 0$ but $\lambda > 0$.

Okishio then considers the effect of viable technical change on the equilibrium rate of profit after consequent changes in relative prices, if any. His celebrated theorem establishes, for a large class of fixed-proportions-technology models, that, given a constant real wage rate, the equilibrium rate of profit rises with the introduction of viable technical change. This result has since been extended to classes of models with less restrictive technologies and with fixed capital.[5]

Of course, in the one-sector "circulating-capital" model, the Okishio theorem is disappearingly trivial since the real wage rate is the only relative price. Viable technical change *per se* increases the equilibrium rate of profit.

Some have regarded the Okishio results as devastating to Marxian arguments to the effect that technical change introduced by profit maximizing individual capitalists could, or even would, result in a fall in the general rate of profit. Among attempts to counter this interpretation, an argument of Foley (1986) is of interest. Okishio shows that viable technical change raises the equilibrium rate of profit if the real wage rate is unchanged. But the real wage rate might well change in the course of technical change. Foley's alternative construal is to hold fixed instead what he calls "the value of labor-power," inventively defined as the product of the nominal wage rate and "the labor value of money," which in turn is specified as the ratio of the aggregate labor input to the nominal aggregate net product. That is, Foley's "value of labor power" is the wage share of net output.

For, where $\underline{w}$ is the nominal wage rate, p is the price of output (and the real wage rate is thus $w = \underline{w}/p$), Z is aggregate gross output, L is aggregate labor input (and thus $lZ = L$), and M is aggregate material input (and thus $mZ = M$), Foley proposes holding fixed

$$\frac{\underline{w}L}{p(Z - M)} = \frac{wl}{1 - m}.$$

When one notes that the labor embodied or "labor value" per unit of output in this model is $\Lambda = l/(1 - m)$ (since $\Lambda Z = \Lambda M + L$), one sees that Foley is thereby fixing Λw, the labor value of the real wage rate.

Foley's holding the value of labor-power constant is also equivalent to fixing the "rate of surplus value" or "rate of exploitation," i.e., the ratio s of "surplus value" to the value of labor power, i.e.,

$$s = \frac{L - \Lambda wL}{\Lambda wL} = \frac{1}{\Lambda w} - 1 = \frac{1 - m}{wl} - 1.$$

In terms of the rate of exploitation Foley's argument can be formulated immediately: Solving the last equation for the real wage rate

$$w = \frac{1 - m}{l(s + 1)}$$

and substituting into the rate of profit equation we obtain

$$r = \left(\frac{1}{m} - 1\right)\left(\frac{s}{s + 1}\right)$$

which, with the rate of exploitation given, is no longer a function of the labor coefficient at all and is moreover decreasing in the material coefficient if the rate of exploitation is positive.6 For a new technique to be viable, $\mu < 0$ and/or $\lambda < 0$, i.e., the technical change must be CS and/or LS. In the former case, the general rate of profit increases if s is fixed. In the latter case, either $\mu = 0$ and the general rate of profit is unchanged or, finally, and as Foley wishes to emphasize, $\mu > 0$ and the general rate of profit falls. That is, Foley shows that, given an unchanged (positive) rate of exploitation, CU-LS technical change lowers the equilibrium rate of profit.

The Foley result and the Okishio theorem are, of course, consistent because each follows from different assumptions about the real wage rate. Okishio treats the real wage rate as exogenous and examines the consequences of viable technical change with a fixed real wage rate. Foley in contrast treats the real wage rate as endogenous and focuses on cases of viable CU-LS technical change in which the real wage rate increases to maintain a fixed value of labor power, i.e., equivalently, a fixed rate of exploitation.[7] The prospect of a rising real wage in such circumstances might seem *prima facie* dubious (with less labor required per unit of output and more capital required per unit of labor), but any assumption of a real wage rate unaffected by technical change is not on evidently firmer ground.

II

Technical change lowers the equilibrium rate of profit if, at equilibrium prices, i.e., at the real wage rate w' prevailing after the technical change has been made general[8]

$$\frac{1 - w'l'}{m'} < \frac{1 - wl}{m}$$

or, equivalently, with $\omega = (w' - w)/w$, the falling rate of profit condition *FRP* is

$$\omega > - \frac{q\mu + \lambda}{\lambda + 1} \qquad FRP$$

Thus, since the viability condition *VIA* requires that the right hand side of the falling rate of profit condition *FRP* be positive, viable technical change lowers the profit rate only if the real wage rate sufficiently increases. Where $\rho = (r' - r)/r$ is the proportional change in the rate of profit, *FRP* is equivalently $\rho < 0$.

If the real wage rate and its change are not treated as functions of other parameters, that is, if w and w' (and thus ω) are treated (for the moment) as exogenous with respect to, for example, m and m' (and thus to μ) and to l and l' (and thus to λ) then the *FRP* condition can be regarded as determining a straight line with downward slope $-q/(\omega + 1)$ and vertical intercept at $-\omega/(\omega + 1)$.

If it is assumed *à la* Okishio that the real wage rate is unchanged, i.e., $\omega = 0$, *FRP* and *VIA* of course determine the same line (represented in Figure 1) to the left and below which technical change is viable and to the right and above which the equilibrium rate of profit falls.

If instead it is conjectured that the change in the real wage rate is positive, *à la* Foley, the situation is as shown in Figure 2 below (ignoring for the moment the lines labeled *URE* and *ROC*). At every point to the right and above the line determined by *FRP*, the equilibrium profit rate is lower with the technical change. Thus, if ω is positive, there is a cone of points between the viability condition

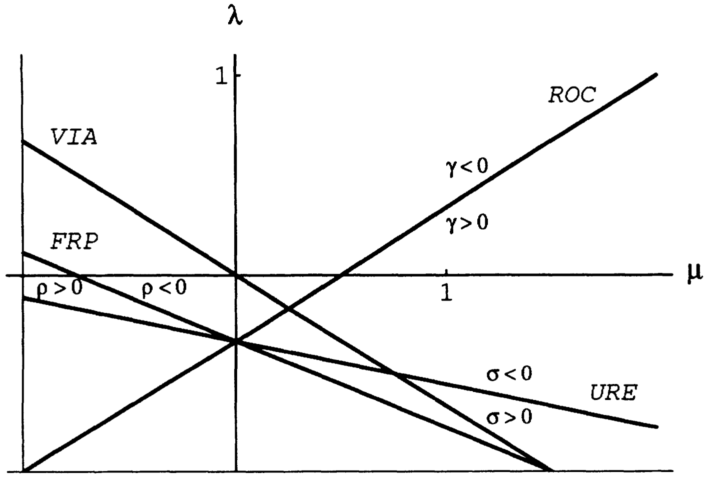

Figure 2

VIA and the line determined by *FRP* and above and to the left of their intersection (at $\mu = 1/q, \lambda = -1$). This cone is the realm in which viable technical change lowers the equilibrium profit rate and includes areas in all except the upper-right quadrant, of which only the area in the lower-right quadrant includes the region to which Foley draws attention.

That is, Foley's argument is focused on only a small locus of points within that portion of the cone in the lower-right (CU-LS) quadrant, those with an unchanged rate of exploitation *URE*, i.e., points also on the line determined by

$$\omega = -\frac{[m/(1 - m)]\mu + \lambda}{\lambda = 1} \qquad URE$$

which is the third downward sloped line in Figure 2.[9] Above the condition *URE* the rate of exploitation is lower; below, it is higher.

Where $\sigma = (s' - s)/s$ is the proportional change in the rate of exploitation, *URE* is just $\sigma = 0$. *URE* shares its vertical intercept at $-\omega/(\omega + 1)$ with *FRP* but is less steeply sloped (given a positive rate of profit).[10] The rate of profit evidently falls with any viable CU-LS technical change in which the rate of exploitation is unchanged or, for that matter, falls.

It can further be noted that Foley's cases are also a subset of those in which the "organic composition of capital" increases, where the organic composition of capital is taken as the ratio, *g*, of "constant capital," λM, to "variable capital," λwL. (This ratio might be referred to less reproachably but also less familiarly as the "value composition of capital." In the one-sector model these conceptions are the indistinguishable.)

There is a rise in the organic composition of capital *ROC* just in case

$$\omega < \frac{\mu - \lambda}{\lambda + 1} \qquad ROC$$

which determines the positively sloped line in Figure 2 (which also intercepts the vertical axis at $-\omega/(\omega + 1)$.[11] In cases below this line the organic composition has risen with technical change, including all cases of CU technical change in which the rate of exploitation is unchanged, that is, CU cases in which *URE* holds. Where $\gamma = (g' - g)/g$ is the proportional change in the organic composition, *ROC* is just $\gamma > 0$.

A curious analog to Foley's cited result may be noted from the diagram: given an unchanged (or, for that matter, even *lower*) organic composition, viable CU-LS technical change lowers the equilibrium rate of profit.

Of course, if the real wage rate does *not* rise, the Foley prospect is closed. For then all cases of CU-LS technical change in which the rate of exploitation is unchanged fail the viability test. That is, if $\omega \leq 0$ then the situation is not as depicted in Figure 2. Instead the vertical intercept shared by *FRP*, *URE* and *ROC* is non-negative. In particular, in the lower-right quadrant, *URE* is entirely beyond the viability boundary. The situation is rather as in Figure 3.

That is, even if the rate of exploitation were somehow maintained unchanged, the possibility of viable CU-LS technical change which would then lower the equilibrium rate of profit is *not* thereby established. For no such change will be *viable* unless the real wage rate increases. But an unchanged rate of exploitation and CU-LS technical change provide *per se* no grounds for expecting an increase in the real wage rate. For that an independent argument would be required.

III

What is needed is an account of the determination of the real wage rate more subtle than either Okishio's treating the real wage rate as

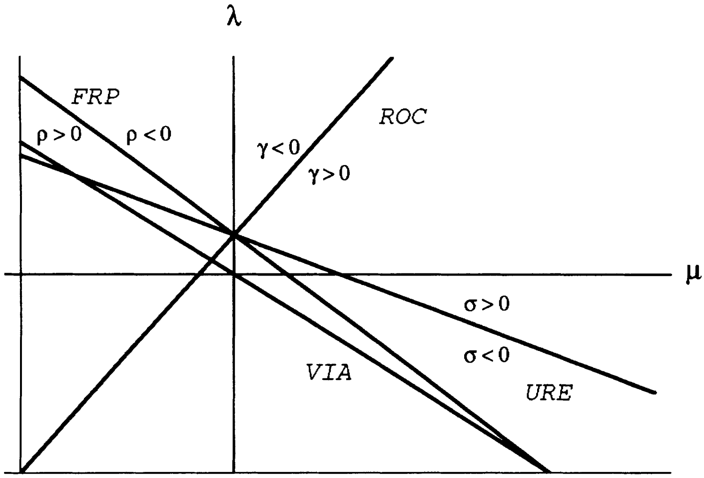

Figure 3

exogenous or the Foley device in which the rate of exploitation is taken as exogenous. In this section an account is offered of how the equilibrium real wage rate is affected by technical change and accumulation via changes in the aggregate demand for labor. Failure to take into account the effect of technical change and accumulation on the aggregate demand for labor and in turn the effect of changes in the demand for labor on the real wage rate is arguably the fundamental lacuna in the history of efforts to connect technical change and accumulation to the movement of the rate of profit. Introducing a minimal assumption that the real wage rate is nondecreasing in the aggregate demand for labor is sufficient to show that viable capital-using technical change does not *ceteris paribus* lower the equilibrium rate of profit. A perhaps more striking conclusion employing Marxian concepts is that this minimal assumption entails that *ceteris paribus* viable technical change which increases the organic composition of capital *increases* the rate of profit.

The demand for labor, $L = lZ$, is increasing in output, Z, and in the amount of labor technically requisite per unit of output, l. Equivalently, since $Z = M/m$, labor demand is increasing in the technical labor/capital coefficient, l/m, and in the material input, M, to which labor is applied.

In technical change a technique m', l' is chosen as well as a level of (gross) investment K' and thus a rate of accumulation $\kappa = (K - K')/K$, i.e., accumulation is net investment. (Excluding borrowing, the rate of accumulation cannot exceed the initial rate of profit, i.e., $\kappa \leq \rho$; capital available for (net) investment is just rK.) Thus labor demand is increasing in the technical labor/capital ratio and in the accumulation level.[12]

In particular, if wages are paid at the end of the period, technical changes resulting in unchanged labor demand *ULD* are those satisfying the condition

$$\lambda = \frac{-\kappa + \mu}{\kappa + 1}. \qquad ULD$$

Technical changes above *ULD* result in increased labor demand, that is, $\Delta > 0$ where $\Delta = (L' - L)/L$ is the proportional change in labor demand.[13] In those technical changes below *ULD*, labor demand is lower. If there is no accumulation, i.e., if $\kappa = 0$, *ULD* is simply the $\lambda = \mu$ diagonal. If rather there is positive accumulation, $\kappa > 0$, *ULD* is rotated downward (pivoting on (-1, -1)) as in Figure 4.

Among proposed accounts of the determination of the equilibrium real wage rate are those that make use of a notion of labor supply as well as of labor demand. Thus whether viable technical change induces a rise in the real wage rate sufficient to lower the equilibrium profit rate would depend not only on the aggregate labor demand of firms and thus on the technical change and accumulation (if any) undertaken, but as well on the aggregate labor supply by workers and thus, in some elaborations, on workers' preferences regarding income and labor time.

Purported Walrasian equilibria before and after the technical change would equate supply and demand in the labor market implicitly defining equilibrium wage rates w and w' and determining the proportional change in the real wage rate ω.

But theories in which wage rates are determined by clearing labor markets are vulnerable to devastating criticism and the central arguments here make no such assumption. Here use is made rather of the requirement, true of stable Walrasian labor market equilibria, but met as well by a very wide range of accounts of the determination of the real wage rate, that the equilibrium real wage rate is non-decreasing in labor demand.[14] In particular this requirement is met by a variety of accounts according to which the labor market does not clear at equilibrium and manifests instead equilibrium involuntary unemployment.

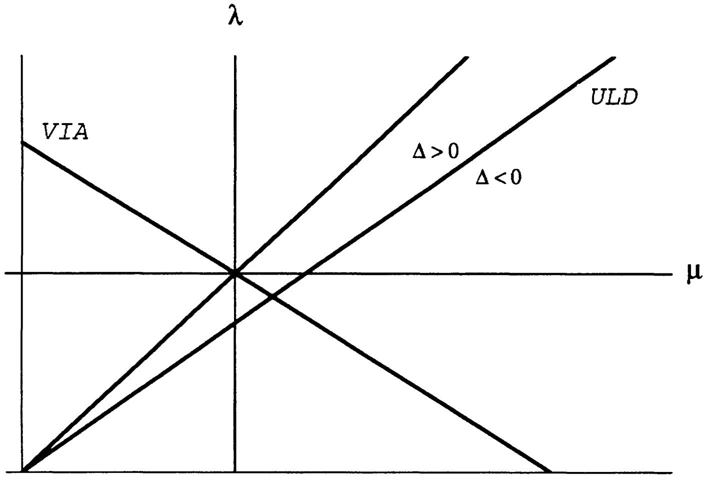

Figure 4

That labor demand is increasing in the technical labor/capital ratio and in the accumulation rate and that the equilibrium real wage rate is non-decreasing in labor demand together imply that condition *ULD* partitions technical changes into three classes of cases: technical changes precisely satisfying *ULD* leave the real wage rate unchanged; changes lying above *ULD* do not decrease the real wage rate; and changes located below *ULD* do not increase the real wage rate. This implication can be dubbed the *Minimal Assumption* on the effect of technical change and accumulation on the real wage rate, that is, that whatever processes determine the equilibrium real wage rate operate such that[15]

if $\Delta > 0$, then $\omega \geq 0$;
if $\Delta = 0$, then $\omega = 0$; and *Minimal Assumption*
if $\Delta > 0$, then $\omega \leq 0$.

The *Minimal Assumption* specifies that, on *ULD*, whatever its position as determined by the rate of accumulation, the real wage rate is unchanged and that above (below) *ULD* the real wage rate is no lower (higher).

Now the Okishio assumption of a real wage rate unchanged by technical change can be seen as holding on *ULD* (and to the left of *VIA* for viability). Foley's position can also be located here. Suppose first that there is viable CU-LS technical change but no accumulation, i.e., $\kappa = 0$. From the *Minimal Assumption* it follows that the real wage rate does not rise and thus that the resulting equilibrium rate of profit is higher rather than lower. At all points below both the viability condition and *ULD*, which in the absence of accumulation is the $\lambda = \mu$ diagonal, the equilibrium profit rate has risen. (Since the real wage rate has not increased, the uniform rate of exploitation condition *URE* now has a non-negative vertical intercept[16] while remaining less negatively sloped than *FRP*. Thus if the rate of exploitation were unchanged, no case of CU technical change would meet the viability condition.)

On the other hand, if viable CU-LS technical change is accompanied by accumulation, i.e., $\kappa > 0$, *ULD* runs below the diagonal and some portion of the CU-LS area lies above it. In this case Foley's cases of viable CU-LS technical change lowering the rate of profit are not excluded. But it should be noted that what creates the possibility of viable technical change lowering the rate of profit is *not* that technical change be capital-using—indeed the prospect of capital saving technical change leading to a lower rate of profit appears at least as saliently—but rather that accumulation is positive. Capital-using technical change is biased against increasing labor demand, a bias which may or may not be overcome by countervailing pressure from accumulation.

To summarize: Foley showed that if the rate of exploitation is unchanged, viable CU technical change lowers the equilibrium rate of profit.[17] But, the *Minimal Assumption* granted, viable CU technical change can leave the rate of exploitation unchanged only if accumulation is positive; absent accumulation, viable CU technical change raises the rate of exploitation.[18] Equivalently, with an unchanged rate of exploitation, CU technical change can be viable only with positive accumulation. Otherwise any viable CU technical change *raises* the equilibrium rate of profit. To be sure, there are circumstances under which the *Minimal Assumption* holds and viable technical change induces a fall in the equilibrium rate of profit even in the absence of accumulation; but these are not in the class of cases to which Foley (or Marx) drew attention.

The *Minimal Assumption* has consequences as well for any doctrine according to which a rising organic composition of capital might account for a falling rate of profit. In the absence of accumulation, viable CU technical change implies that there is no increase in the real wage rate. Thus viable CU technical change which, in particular, raises the organic composition of capital entails a rise rather than a fall in the rate of profit. On the other hand, viable CS technical change which raises the organic composition raises the rate of profit whether or not there is accumulation.[19]

That is, in the absence of accumulation and given the *Minimal Assumption*, any viable technical change which increases the organic composition of capital increases the rate of profit. Conversely, the rate of profit only falls if the organic composition of capital falls as well. In the context of more than a century of discussion of the purported consequences of a rising organic composition of capital, this result may be of some interest.

Of course, consistent with the *Minimal Assumption*, technical change into the area above condition *ULD* may raise the real wage rate. But it may not, and even if it does the effect on the equilibrium profit rate is not thereby determined. That is, accumulation with CU-LS technical change by no means entails a rise in the real wage rate; rather the net effect of such accumulation may lower labor demand. Furthermore, even if the real wage rate does rise in such circumstances, this by no means entails a falling rate of profit.

In fact, as we shall see, given plausible characterizations of the response of the wage rate to labor demand, there will in general be three non-empty classes of cases of viable technical change with a higher real wage rate: those in which the equilibrium profit rate rises, those in which the rate remains unchanged, and finally those in which the equilibrium rate of profit falls.

This last class of cases has been of course of prominent enduring interest. There, individual firms institute technical changes which would increase the rate of profit at current relative prices, but the effect of introduction of such changes is to lower the equilibrium rate of profit. The rate of profit falls as a consequence of profit maximization.

It should go without saying that none of these conclusions in any way entails that the formal arguments of Okishio or Foley are somehow unsound. They are not. But these conclusions do undermine superficial inferences from their results. Okishio and Foley have demonstrated that *ceteris paribus* viable technical change, respectively, raises the equilibrium rate of profit given a fixed real wage rate and lowers the equilibrium rate of profit given a fixed rate of exploitation. But Okishio thereby provides no positive reason to expect the equilibrium rate of profit to rise, and nor does Foley thereby provide any positive reason to expect it to fall. Constructions taking the real wage rate – the level of producers' subsistence – or, alternatively, taking the rate of exploitation – the class division of the net product – as exogenous fail to consider that each of these central variables is itself profoundly affected by the course of technical change and accumulation. The *Minimal Assumption* is proposed as the weakest plausible conjecture as to what this affect might be, and this assumption is sufficient to show that *ceteris paribus* viable capital-using technical change does not cause the equilibrium rate of profit to fall.

IV

The *Minimal Assumption* is a weak postulate on the influence, via labor demand, of technical change and accumulation on the real wage rate. It is trivially equivalent to the condition that the elasticity of the real wage rate with respect to labor demand (inverting the commonplace notion) be non-negative. By recasting the analysis in terms of the demand elasticity of the real wage rate, the consequences of stronger hypotheses than the *minimal assumption* can be examined.

That is, where Δ is the proportional change in labor demand, and ε is the labor demand elasticity of the real wage rate, then $\omega = \varepsilon\Delta$. If wages are paid at the end of the period and thus $\Delta = (\kappa + 1)(\lambda + 1)/(\mu + 1) - 1$,[20] we have then[21]

$$\omega = \varepsilon\left[\frac{(\kappa + 1)(\lambda + 1)}{\mu + 1} - 1\right],$$

which gives the real wage rate as an *explicit* function of technical change, the rate of accumulation and the elasticity parameter. The *Minimal Assumption* is precisely the requirement that $\varepsilon \geq 0$.

With this characterization of the dependence of the equilibrium real wage rate on technical change, the condition for technical change lowering the equilibrium rate of profit, $\rho < 0$, can be respecified as

$$\varepsilon\left[\frac{(\kappa + 1)(\lambda + 1)}{\mu + 1} - 1\right] > -\frac{q\mu + \lambda}{\lambda + 1} \qquad \textit{FRPa}$$

which picks out a set of points in μ, λ space for given κ and ε. Condition *FRPa* differs from *FRP* in treating ω as a(n explicit) function of μ and λ (as well as of κ and ε).[22, 23]

In the same manner, the condition for an unchanged rate of exploitation, $\sigma = 0$, can be represented as[24]

$$\varepsilon\left[\frac{(\kappa + 1)(\lambda + 1)}{\mu + 1} - 1\right] = -\frac{[m/(1 - m)]\mu + \lambda}{\lambda + 1} \qquad UREa$$

as can the condition for a rise in the organic composition, $\gamma > 0$, as[25]

$$\varepsilon\left[\frac{(\kappa + 1)(\lambda + 1)}{\mu + 1} - 1\right] < \frac{\mu - \lambda}{\lambda + 1}. \qquad ROCa$$

A range of particular assumptions, all satisfying but more restrictive than the *Minimal Assumption*, on the responsiveness of the wage rate to changes in labor demand now invite investigation. For example, at one extreme the wage rate might be regarded as entirely independent of changes in labor demand, i.e., $\varepsilon = 0$. (On one account this would be the consequence of perfectly elastic labor supply.) This case delivers the Okishio situation: technical change and accumulation leave the wage rate unchanged and thus viable technical change raises the profit rate. Here the falling profit rate condition *FRP*a simplifies to $\lambda > -q\mu$, i.e., precisely to the viability condition *VIA*. Further, the unchanged rate of exploitation condition *URE*a becomes $\lambda = -[m/(1 - m)]\mu$; and the rising organic composition condition *ROC*a is bounded by the $\lambda = \mu$ diagonal.

The opposite limit case would be a real wage rate infinitely elastic with respect to change in labor demand, i.e., $\varepsilon \to \infty$. In this limit, *FRP*a, *URE*a and *ROC*a all determine the same line, $\lambda = (-\kappa + \mu)/(\kappa + 1)$, i.e., *UDL*. Above this line the rates of profit and exploitation fall while the organic composition rises; below it the movements are reversed.

Thus in this case, if there is no accumulation, i.e., if $\kappa = 0$, technical change lowers the rate of profit only if $\lambda > \mu$, i.e., only if production becomes more labor intensive. On the other hand, if there is no accumulation, any viable CU technical change raises the rate of profit. If instead there is accumulation, $\kappa > 0$, viable CU technical change which lowers the general rate of profit is possible, but of course not guaranteed since below *ULD* viable CU technical change raises the rate of profit. That is, the situation is just as in Figure 4 above.

These limit cases (meeting the *Minimal Assumption*) of zero and infinite elasticity of the real wage rate with respect to labor demand are perhaps less plausible than intermediate cases of positive but finite elasticity. One intermediate case (and three of its subcases) will be pursued here because of its unique tractability which permits clear illustrations of the fundamental relationships among *VIA, FRP, URE* and *ROC*.

This amenable intermediate case is that of unit elasticity of the real wage rate with respect to labor demand. When $\varepsilon = 1$ the falling rate of profit condition becomes simply

$$\frac{(\kappa + 1)(\lambda + 1)}{\mu + 1} - 1 > -\frac{q\mu + \lambda}{\lambda + 1}$$

or equivalently

$$(\kappa + 1)(\lambda + 1)^2 + q\left(\mu - \frac{1}{2(q - 1)}\right)^2 > \frac{(q + 1)^2}{4q} \qquad FRPb$$

which specifies an (unrotated) ellipse translated to center at $\mu = 1/[2(q - 1)]$ and $\lambda = -1$. (Of course only the curve constituting the upper half of the ellipse above $\lambda = -1$ is within the range of feasibility.) Above, on, and below the feasible range of this curve the rate of profit is respectively lower, unchanged, and higher.

With $\varepsilon = 1$, the condition for an unchanged rate of exploitation simplifies to

$$(\kappa + 1)(\lambda + 1)^2 + [m/(1 - m)]\left(\mu + 1 - \frac{1}{2m}\right)^2 = \frac{1}{4m(1 - m)} \qquad UREb$$

specifying another (unrotated) ellipse translated to center at $\mu = 1/2m - 1$ and $\lambda = -1$. Above, on, and below the curve constituting the upper half of this ellipse the rate of exploitation is respectively lower, unchanged, and higher.

And the condition for a rise in organic composition becomes

$$\lambda < \frac{\mu + 1}{\sqrt{\kappa + 1}} - 1. \qquad ROCb$$

The figures specified by *FRP*b, *URE*b and *ROC*b intersect at $\lambda = \mu = -1$ and, in the feasible range, at $\mu = 0$, $\lambda = 1/\sqrt{\kappa + 1} - 1$. Further, since a positive initial rate of profit requires that

$q > m/(1 - m)$, the rate of exploitation curve is centered to the right of the rate of profit curve.

Further specification of these formulas with different values the parameters q, m and κ, illustrates interestingly dissimilar classes of cases. Suppose first that q is unity (and thus workers' initial share of gross output is 1/2[26]) and thus that the viability condition is simply

$$\lambda < -\mu. \qquad \textit{VIAc}$$

Suppose further that the initial capital coefficient is 1/3 (and thus the initial rate of profit is 1/2).

The remaining exogenous parameter is the rate of accumulation, κ. Suppose first that there is no net investment, i.e., $\kappa = 0$. Then the falling equilibrium profit rate simplifies to

$$(\lambda + 1)^2 + \mu^2 > 1, \qquad \textit{FRPc}$$

the unchanged rate of exploitation condition to

$$2(\lambda + 1)^2 + (\mu - 1/2)^2 = \left(\frac{3}{2}\right)^2 \qquad \textit{UREc}$$

and the rising organic composition condition to

$$\lambda < \mu \qquad \textit{ROCc}$$

and the circumstances are those diagramed in Figure 5.

In this case note first that every viable CS-LU technical change lowers the profit rate, as do a portion of technical changes which are CS-LS. In each of these cases, labor demand has increased, thus raising the wage rate sufficiently to lower the profit rate.

On the other hand, every viable CU-LS technical change *raises* the profit rate, as do a portion of technical changes which are CS-LS. Here one can note that for a given degree of LS technical change, e.g., $\lambda = -1/2$, there are CU technical changes, e.g., $\mu = 1/3$,, such that the combination is viable and raises the equilibrium profit rate as, for that matter, would leaving the capital coefficient unchanged ($\mu = 0$) or lowering it somewhat, e.g., to $\mu = -1/2$. But if that degree of LS technical change is combined with sufficiently more CS technical change, e.g., $\mu = -4/5$, even though the combined change is viable, the equilibrium profit rate

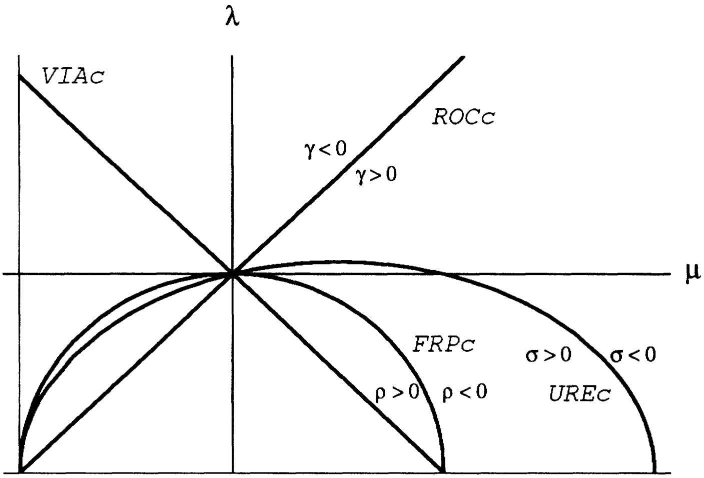

Figure 5

falls. Employing the capital freed by such sweeping capital-saving technical change raises labor demand and thus the real wage rate so much that the equilibrium rate of profit is driven down.

In particular one can note how under these conditions, any viable CU technical change raises the rate of exploitation as well as the rate of profit. Without accumulation there are no cases of viable technical change, an unchanged rate of exploitation and a falling rate of profit. Further, in every case of viable technical change in which the organic composition of capital rises, the rate of profit rises as well. A fall in the rate of profit requires a *fall* in the organic composition of capital.

We can also take parenthetical note of the perhaps surprising existence here of a realm of technical changes, in every case CU-LS, which are *not* viable (they are to the right of condition *VIA*c) but which nevertheless would raise the equilibrium profit rate (they are below *FRP*c). If such technologies are available, no individual firm will move to introduce them. But if such technologies are generally used, every firm reaps increased profits. The invisible hand is here (for capitalists!) paralyzed.[27]

Suppose on the other hand that accumulation is positive. Maintaining the example initial parameters of $q = 1$ and $m = 1/3$ and thus $r = 1/2$, let us suppose that $\kappa = 1/2$. The viability condition remains as in *VIA*c above, but the falling profit rate, unchanged exploitation rate and rising organic composition conditions are now respectively as follows

$$\left(\frac{3}{2}\right)(\lambda + 1)^2 + \mu^2 > 1 \qquad FRPd$$

$$3(\lambda + 1)^2 + \left(\mu - \frac{1}{2}\right)^2 = \left(\frac{3}{2}\right)^2 \qquad UREd$$

$$\lambda < \sqrt{\frac{2}{3}} - 1 + \sqrt{\frac{3}{2}}\,\mu \qquad ROCd$$

and the graphic situation is as in Figure 6, with salient detail shown in enlarged scale in Figure 6a. Given accumulation viable CU-LS technical change can lead to a fall in the general rate of profit. In particular it is now possible for viable CU-LS technical change with an unchanged rate of exploitation *à la* Foley (or for that matter a rising rate of exploitation) to lead to a fall in the profit rate, Or, here encompassing the Foley case, it is now possible for viable CU-LS technical change with a rising organic composition to lead to a fall in the profit rate.

Of course, as is evident in Figure 6, other cases of viable CU-LS technical change remain possible even when accumulation is assumed. Viable CU-LS technical change increasing the organic composition of capital and/or the rate of exploitation may still raise the rate of profit, and viable CU-LS technical change decreasing the organic composition may lower the rate of profit.

For a second class of cases with unit labor demand elasticity of the wage rate, $\varepsilon = 1$, consider the situation in which q exceeds unity; for example let it be 4 (and thus workers' share of revenue be only 1/5). Now the viability condition is

$$\lambda < -4\mu.$$

Suppose further that the initial capital coefficient is $m = 8/15$ (and thus the initial rate of profit is 1/2 as before), and consider first the case in with no accumulation, $\kappa = 0$. Then the falling profit rate and unchanged exploitation rate conditions are respectively

$$(\lambda + 1)^2 + 4\left(\mu + \frac{3}{8}\right)^2 > \left(\frac{5}{4}\right)^2 \qquad FRPe$$

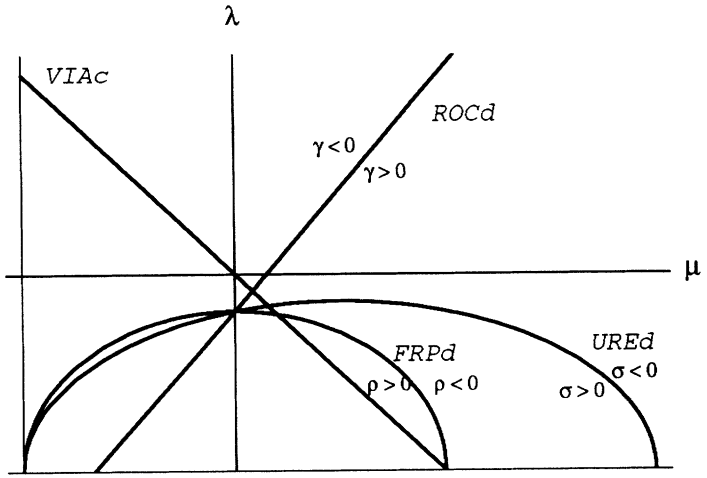

Figure 6

$$14(\lambda + 1)^2 + 16\left(\mu + \frac{1}{16}\right)^2 = \left(\frac{15}{4}\right)^2 \qquad \textit{UREe}$$

while the rising organic composition is $\lambda < \mu$, i.e., *ROC*c as before. The situation is as graphed in Figure 7.

Now, since there is a region in the CS-LU quadrant which lies below *FRP*e, it is no longer true that any viable CS-LU technical change would lower the rate of profit. Workers' share of revenue is here so low that some viable proposals for technical change will use more labor per unit of output, and less capital as well (requiring

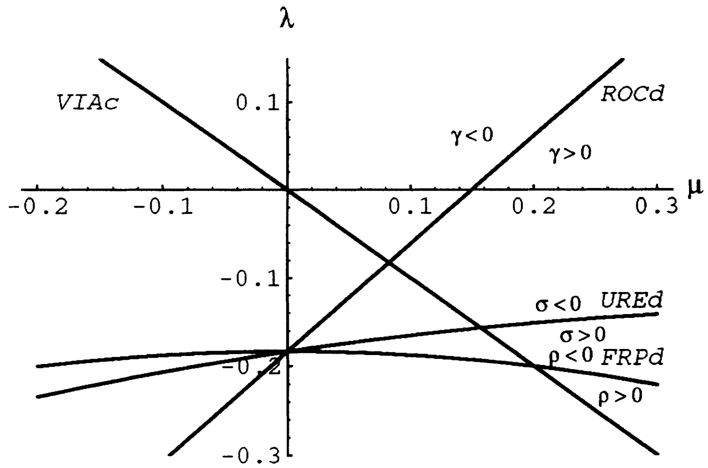

Figure 6a

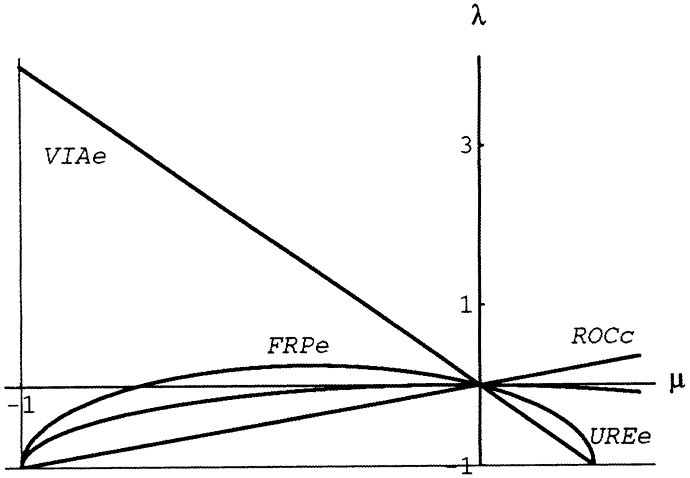

Figure 7

more labor to work freed capital), without the increase in the wage bill lowering the profit rate. It remains true here that some technical changes which would increase equilibrium profit are not viable. If, however, accumulation is positive, say at $\kappa = 1/2$, the falling profit rate, unchanged exploitation rate and rising organic composition conditions are instead respectively

$$\frac{3}{2}(\lambda + 1)^2 + 4\left(\mu + \frac{3}{8}\right)^2 > \left(\frac{5}{4}\right)^2 \qquad \textit{FRPf}$$

$$21(\lambda + 1)^2 + 16\left(\mu + \frac{1}{16}\right)^2 = \left(\frac{15}{4}\right)^2 \qquad \textit{UREf}$$

$$\lambda = \sqrt{\frac{2}{3}}\,(\mu + 1) - 1 \qquad \textit{ROCf}$$

and the situation is as graphed in Figure 8 and (in enlargement) 8a.

And finally let us consider an aspect of one situation in which q is less than unity, for example let it be 1/4 (and thus workers' revenue share be 1/4). Now the viability condition is

$$\lambda < -\frac{\mu}{4}. \qquad \textit{VIAg}$$

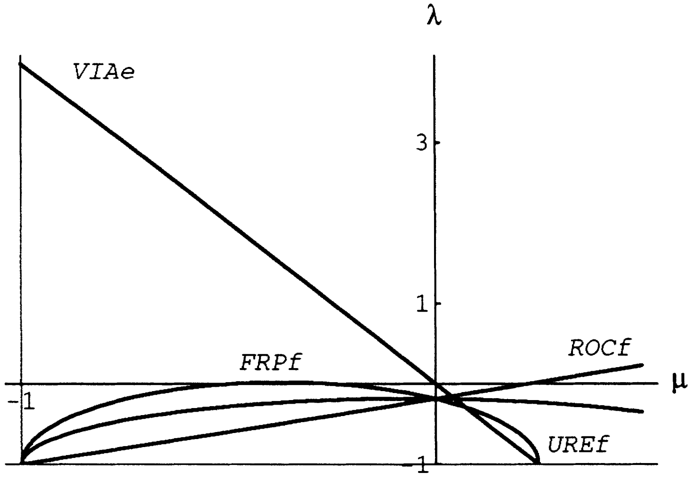

Figure 8

Suppose now that the initial capital coefficient is m = 2/15 (and thus the initial rate of profit is again 1/2), and consider only the case with no accumulation, $\kappa = 0$. Then the falling profit rate and unchanged exploitation rate conditions are respectively

$$(\lambda + 1)^2 + \left(\frac{1}{4}\right)\left(\mu - \frac{3}{2}\right)^2 > \left(\frac{5}{4}\right)^2 \qquad FRPg$$

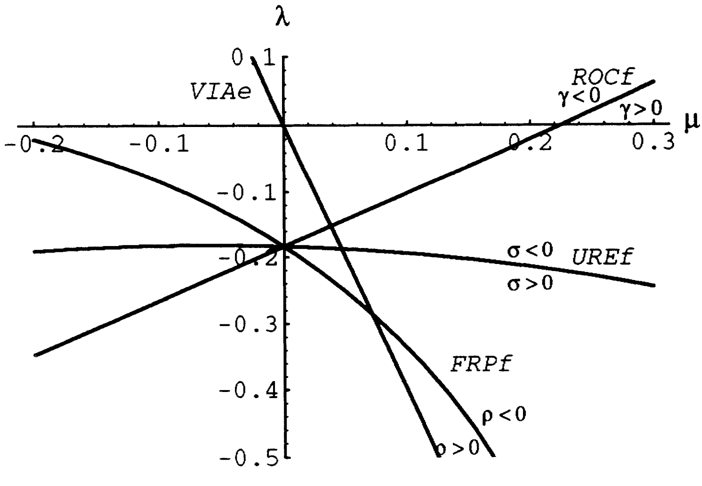

Figure 8a

$$(\lambda + 1)^2 + \left(\frac{2}{13}\right)\left(\mu - \frac{11}{4}\right)^2 = \frac{(15)^2}{104} \qquad UREg$$

while the rising organic composition is again $\lambda < \mu$, i.e., *ROCc*. The situation is as graphed in Figure 9.

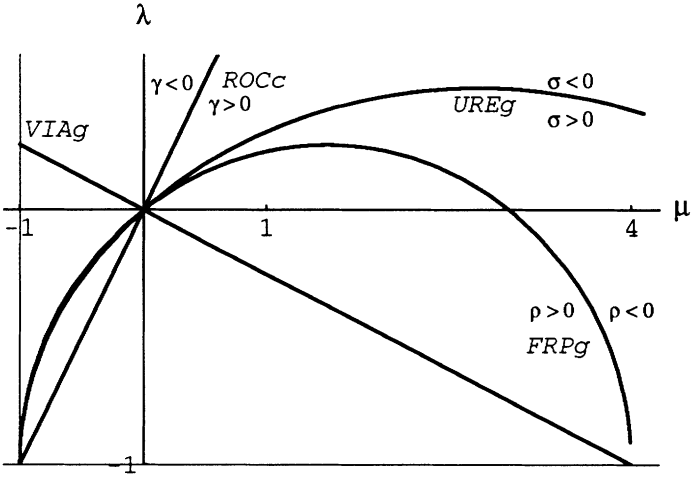

Figure 9

With workers' higher share of revenue, the possibility of CS-LU viable technical change which does not lower the rate of profit has disappeared (the CS-LU quadrant contains no points below *FRPg*). Yet the scope for technical changes which are not viable but which would increase the profit rate has grown. And a new and peculiar set of possibilities has emerged: technical changes which employ both more labor and more capital per unit of output (and which thus are decidedly not viable) but which would *raise* the rate of profit. Moving into this region uses an unambiguously less efficient production technology but is nevertheless profit increasing.[28, 29]

In this section the effect of technical change and accumulation on the equilibrium profit rate has been investigated under assumptions of zero, infinite and (at some length) unit elasticity of the real wage rate in labor demand. These assumptions provide examples which are especially tractable algebraically and graphically but are nevertheless fertile. In fact, investigations with other assumptions seem to provide no additional noteworthy classes of cases. It may nevertheless be of some interest to close the section with Figure 10 showing the effect on the falling rate of profit condition *FRP* of

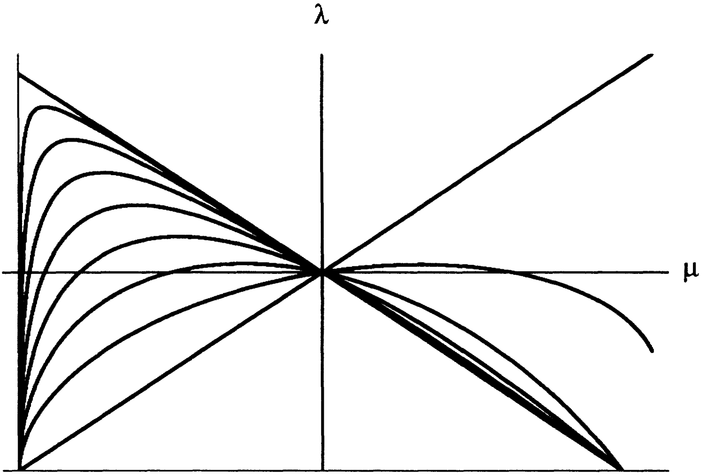

Figure 10

varying the elasticity parameter from zero (in which it coincides with *VIA*) to infinity (in which it coincides with *ULD*) while holding accumulation fixed (at zero).

V

The body of this paper has been devoted to exploring equilibrium consequences of choices of technology and investment. One might go on to ask which technologies and levels of investment would in fact be feasible and which, among those available, would be chosen. The apparatus developed here suggests little in answer to these questions.

To be sure, among technical changes some would certainly appear more attractive than others, i.e., those which offer larger increases in profit. Different technical changes which offer the same increase in profit for a given investment might be regarded as determining iso-viability loci fanning out lower to the left of the viability condition *VIA*.

Further, of equally viable technical changes, some may increase and others decrease the equilibrium rate of profit. That one proposed technical change is more viable than another, that it lies on a more attractive iso-viability line, is by itself no evidence that it will lead to a higher equilibrium rate of profit. It may rather lower, while the less viable alternative would have raised, the general profit rate.

Thus we might also generalize the falling equilibrium profit rate condition to consider the set of iso-profit-rate curves of which the zero change curve *FRP* is a baseline member. Connecting the points at which the profit rate expected on an iso-viability line equals the profit realized on an iso-profit-rate would provide a locus of profit expectations serendipitously met.[30]

Of course technologies can only be chosen from those available. The question thus arises as to which set of points in μ, λ space constitutes the technological possibilities faced by firms. The answer evidently depends on historical and institutional matters remote from the present inquiry. It can nonetheless be noted that, if attention is restricted to constant-returns-to-scale technologies, the possibilities sets will be convex sets, which include the origin. This is true since, for any given technical change, (μ, λ), running that technology in some proportion with the initial technology, (0, 0), will constitute a mixed technology on the straight line between (μ, λ) and the origin.

VI

In summary, the main and most general point of this paper is that the effect of technical change and accumulation on the equilibrium rate of profit depends in part on their effect on the real wage rate. Within the model here examined, if there were no such effect, any viable technical change would increase the equilibrium rate of profit (Okishio). But the real wage rate is surely not generally unaffected. If, for example, the wage rate were somehow to vary to maintain the share of labor income in net output constant, viable capital-using technical change would decrease the equilibrium profit rate by raising the real wage rate (Foley).

Yet a deeper account of the endogeneity of the real wage rate tempers expectations that viable capital-using technical change would lower the equilibrium rate of profit. Even under the plausible weak assumption that the real wage rate is non-decreasing in labor demand, in the absence of sufficient positive accumulation, viable capital-using technical change raises the equilibrium profit rate.

If the real wage rate is non-decreasing in labor demand, the effect of viable capital-using technical change on the equilibrium profit rate depends more complexly on several parameters: the initial rate of profit, the technical coefficients before and after the technical change, the elasticity of labor demand with respect to the real wage rate and the rate of accumulation. Unless there is sufficient positive accumulation, in all cases of viable technical change in which the organic composition of capital increases, the rate of profit also rises.

Of course there are classes of cases of viable technical change in which, after the dust has settled, the rate of profit has fallen. Given requisite values for relevant parameters, any form of viable technical change can lead to a fall in the rate of profit. But the analysis here provides no grounds for looking to capital-using labor-saving technical change, or, for that matter, technical change in which the organic composition of capital rises, as an especially likely impetus to a falling rate of profit. Rather, if anything, the analysis suggests that the traditional preoccupation with this form of technical change may be misplaced.

Finally, skeptical questions might reasonably be asked as to the relevance of the analysis developed here to research on technical change which proceeds in other theoretical frameworks which attempt to take into account, for example, fixed capital, multiple sectors, various market "imperfections", dynamics, strategic behavior, etc. But no simple answers to these questions are available. It might however be noted that the framework within which the effect of technical change on the rate of profit is here investigated is extraordinarily simple, but it is neither *ad hoc* nor at all esoteric. There is a reasonable presumption, arguably supported by the history of economics, that results developed in one-period, one-sector, fixed-proportions-technology models may well hold more generally. Further, the here pivotal assumption, that *ceteris paribus* the real wage rate is non-decreasing in labor demand, is not easily impugned. If, in models which differ in this respect from those here investigated, the rate of profit would automatically fall with viable capital-using technical change, or a rising organic composition of capital is rendered consistent with a falling rate of profit even without positive accumulation, a central point of interest will be which models facilitate better explanations of economic reality.

NOTES

1. For a derivation of the equation suppose each of N_F identical profit maximizing firms begin the period with an equal share of an aggregate K units of material. Suppose further that each firm has access to the fixed-proportion-production technology where M_F is the material input and L_F is the labor input of the firm and that the firm is a price taker in the labor market.

Assuming wages are paid at the end of the period, each firm, constrained by $M_F \leq K/N_F$ chooses M_F and L_F to maximize profit

$$\min\left\{\frac{M_F}{m}, \frac{L_F}{l}\right\} - M_F - wL_F$$

unless profit would thereby fail to be positive. Thus the firm chooses

$$M_F = \frac{K}{N_F}$$

$$L_F = \frac{(K/N_F)}{m}\, l$$

if it operates. (If $m + wl > 1$ the firm chooses $M_F = L_F = 0$.)
Thus profit (if positive) is

$$\left(\frac{K}{N_F}\right)\left(\frac{1 - wl}{m} - 1\right)$$

and the rate of profit (if positive) is

$$r = \frac{1 - wl}{m} - 1.$$

2. The arguments which follow do not depend on whether wages are regarded as pre-paid or instead as paid at the end of the period. While the latter assumption facilitates some exposition and is made in the body of the text, the alternative formulation (which is perhaps more common in the literature) is usually noted. If wages are pre-paid the fundamental equation is $(m + wl)(1 + r) = 1$ and thus the rate of profit is

$$r = \frac{1}{m + wl} - 1.$$

3. Alternatively, $m' + wl' < m + wl$ if wages are pre-paid.
4. $q = (1 - wl)/wl$ is the ratio (a constant determined by the initial technology and wage rate) of the share of gross output not received as wages to the share that is so received. Equivalently, $q = m/[1/(1 + r) - m]$. If wages are pre-paid it is rather $q = m/wl = m/[1/(1 + r) - m]$, which may be regarded as the "organic composition of capital" in this model.
5. Roemer (1981) provides a comprehensive review.
6. Alternatively, with pre-paid wages,

$$r = \frac{s + 1}{ms + 1} - 1.$$

7. The wage rate changes from

$$w = (1 - m)/l(s + 1) \text{ to } w' = (1 - m')/l'(s + 1)$$

and thus $w' > w$ just in case

$$\frac{1 - m'}{l'} > \frac{1 - m}{l},$$

i.e., just in case

$$\lambda < -\left(\frac{m}{l - m}\right)\mu,$$

which follows from viability and $\mu < 0$ if the initial rate of profit is positive, i.e., if

$$q < \frac{m}{1 - m}.$$

8. Or, if wages are pre-paid, $m' + w'l' > m + wl$.

9. *URE* follows from $s' = s$, i.e., $w'l'/(1 - m') = wl/(1 - m)$.

10. The slope of *URE* is $-[m/(1 - m)]/(\omega + 1)$ while the slope of *FRP* is $-q/(\omega + 1)$, i.e., $-(1/wl - 1)/(\omega + 1)$ if wages are paid at the end, $-(m/wl)/(\omega + 1)$ if wages are pre-paid. In either case *FRP* is more steeply sloped than *URE* just in case $m = wl < 1$, i.e., just in case the initial rate of profit is positive.

11. Condition *ROC* follows from $\lambda'M'/\lambda'w'L' > \lambda M/\lambda wL$, i.e., $m'/w'l' > m/wl$.

12. If wages are paid at the end of the period, the material input *M* is the only investment: $K = M$. If wages are pre-paid we have rather $K = M + wL$. In either case *K* is the capital needed at the beginning of the period in order for production to take place. In this circulating capital model, capital entirely depreciates in the course of the period.

13. Condition *ULD* follows from $L' = L$, i.e., $l'K'/m' = lK/m$. If instead wages are pre-paid, then the condition corresponding to *ULD* is rather

$$\lambda = \frac{-\kappa(q + 1) + q\mu}{\kappa(q + 1) + q},$$

which also reduces to the $\lambda = \mu$ diagonal if $\kappa = 0$.

14. That this requirement is met by stable Walrasian equilibria is obvious: With labor demand downwardly sloped, stability of the equilibrium requires that labor supply be, if not positively sloped, at least more steeply sloped than labor demand. If labor demand is vertical, stability requires that labor supply be positively sloped. Each implies that the equilibrium real wage rate is non-decreasing in labor demand.

15. The *Minimal Assumption* should of course be taken in a *ceteris paribus* sense. Labor supply, labor solidarity, labor effort, etc., may well be affected by technical change and accumulation and in turn affect the wage rate.

16. As in the depiction of *URE* in Figure 3, as opposed to that in Figure 2 where $\omega > 0$ is assumed.

17. In terms of this formalism, Foley showed (without invoking the viability condition) that *URE* and $\mu > 0$ together imply *FRP* (if $r > 0$).

18. That is, *VIA*, $\kappa \leq 0$, $\mu > 0$., and the *Minimal Assumption* together imply $\sigma > 0$.

19. By *FRP* and *ROC*, a sufficient condition for an increase in the organic composition of capital implying a rise in the rate of profit is that

$$\frac{\mu - \lambda}{\lambda + 1} < -\frac{q\mu + 1}{\lambda + 1},$$

which holds if $\mu < 0$. For if the technical change is CS, i.e., $\mu < 0$, then $\mu < -q\mu$ since q is assumed positive (on either payment procedure) and thus $\hat{m} - 1 < -(q\hat{m} + \hat{l})$.

20. Since $\frac{L'}{L} = \frac{K'}{K}\frac{l'/m'}{l/m}$.

21. If wages are pre-paid the proportional demand for labor is rather

$$\Delta = \frac{(\kappa + 1)(\lambda + 1)(q + 1)}{q(\mu + 1) + (\omega + 1)(\lambda + 1)} - 1.$$

22. One might attempt to further theorize ε as itself being a function of λ and μ, but it will not be done here; ε is here treated as an exogenous constant.

23. The equation corresponding to the inequality *FRP*a is a quadratic equation. That is *FRPa* is equivalent to

$$\varepsilon(\kappa + 1)(\lambda + 1)^2 + (1 - \varepsilon)(\lambda + 1)\left(\mu - \frac{1}{2(q - 1)}\right) + q\left(\mu - \frac{1}{2(q - 1)}\right)^2$$

$$+ \left(\frac{1 - \varepsilon}{2(q + 1)}\right)(\lambda + 1) > \frac{(q + 1)^2}{4q},$$

which is quadratic in $(\lambda + 1)$ and $\left(\mu - \frac{1}{2(q - 1)}\right)$.

24. *URE*a is equivalent to

$$\varepsilon(\kappa + 1)(\lambda + 1)^2 + (1 - \varepsilon)(\lambda + 1)\left(\mu + 1 - \frac{1}{2m}\right) + \left(\frac{m}{1 - m}\right)\left(\mu + 1 - \frac{1}{2m}\right)^2$$

$$+ \left(\frac{1 - \varepsilon}{2m}\right)(\lambda + 1) = \frac{1}{4m(1 - m)},$$

which is quadratic in $(\lambda + 1)$ and $\left(\mu + 1 - \frac{1}{2m}\right)$.

25. *ROC*a is equivalent to

$$\varepsilon(\kappa + 1)(\lambda + 1)^2 + (1 - \varepsilon)(\lambda + 1)(\mu + 1) < (\mu + 1)^2,$$

which is quadratic in $(\lambda + 1)$ and $(\mu + 1)$.

26. Since $q = 1/wl - 1$ in these examples is constructed assuming labor demand with wages paid at the end of the period, $wl = 1/(q + 1)$ is workers' share of revenue.

27. Other such cases in which technical change which would increase the equilibrium rate of profit is not viable will be noted shortly. All such cases exemplify a perverse coordination problem in which it is individually irrational

for firms to institute technical change which if generally instituted would raise the profit rate for all firms. An especially salient example is that, if firms choose viable technical change which then lowers the equilibrium rate of profit, "turning back the clock," that is, simply returning to the initial technology would raise the rate of profit. But "turning back the clock" will not be viable.

28. To be sure, reaching and remaining in this region would require that capitalists solve two daunting coordination problems. Individually no firm wants to move to the non-viable but profit-rate-increasing technology, and were all firms to somehow be so moved, every individual firm would anticipate increased profit by returning to the old technology if not prevented from doing so. An executive committee of the bourgeoisie would have its hands full.

29. It is easy to construct such a case. Suppose $l = 5/6$, $m = 1/10$, and $K/N = (6/25)^2$. Thus $w = 0.48$, $q = 1/4$, and the rate of profit is, r, is 100%. From Figure 8 it is clear that if $\mu = 3/2$ and $0 < \lambda < 1/4$ we move to a less efficient technology which should nevertheless be more profitable. Suppose for example that $\mu = 3/2$ and $\lambda = 1/5$, i.e., that the non-labor coefficient m', is increased 150% to 1/4 and the labor coefficient is increased 20% to $l' = 1$.

When the new technology is used by all firms, the new equilibrium wage rate, w', falls by over half to 0.23. Gross output falls 60% and net output by 2/3. But the general rate of profit has *risen* over 108%. The new technology uses more of each input per unit of output but profits per firm and the general rate of profit are up. Capitalists are better off but workers are worse off.

30. Iso-viability loci would be straight lines pivoting around (-1, q), each specified by

$$\rho^{\epsilon} = \left(\frac{\lambda = q\mu}{(\mu + 1)(m - q + mq)}\right)$$

for an expected rate of profit $\rho^{\epsilon} = \rho^{\epsilon}$. Iso-profit-rate curves are for the form

$$\rho = \left(\frac{\lambda = q\mu + (\lambda + 1)\omega}{(\mu + 1)(m - q + mq)}\right)$$

for a realized rate of profit ρ. Thus expectations are met, i.e., $\rho^{\epsilon} = \rho$, only if the real wage rate does not change, i.e., $\omega = 0$. That is, if the *Minimal Assumption* is met and the labor demand elasticity of the real wage rate is positive, the locus of expectations met is just *ULD*.

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