

T H E U N I V E R S I T Y O F M I C H I G A N

COLLEGE OF ENGINEERING

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Technical Report No. 13

A STUDY OF DYNAMIC TIRE PROPERTIES OVER A RANGE OF TIRE CONSTRUCTIONS

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supported by:

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

GRANT NO. NGL 23-005-010

WASHINGTON, D.C.

administered through:

OFFICE OF RESEARCH ADMINISTRATION

ANN ARBOR

April 1972

TABLE OF CONTENTS

	Page
LIST OF ILLUSTRATIONS	iv
NOMENCLATURE	vi
I. INTRODUCTION	1
II. SUMMARY OF RESULTS	2
III. COMPARISON OF STRING THEORY BY MEANS OF PREDICTIONS AND MEASUREMENTS ON SCALE MODELS	3
IV. TWO-CONSTANT MODIFICATION OF FINITE CONTACT PATH STRING THEORY	18
V. REFERENCES	43
VI. ACKNOWLEDGMENTS	44
VII. DISTRIBUTION LIST	45

LIST OF ILLUSTRATIONS

Table		Page
I.	Standard Tire Operating Conditions and Static Tire Elastic Properties	4
Figure		
1.	Force and moment response to steer excitation of tire A using von Schlippe string theory.	6
2.	Force and moment response to steer excitation of tire B using von Schlippe string theory.	8
3.	Force and moment response to steer excitation of tire C using von Schlippe string theory.	10
4.	Force and moment response to steer excitation of tire D using von Schlippe string theory.	12
5.	Variation of λ in string theory, for steer excitation.	14
6.	Variation of h in string theory, for steer excitation.	16
7.	Geometry for tire contact patch at steady state yaw angle.	20
8.	Variation of λ in the two-constant modification of string theory.	22
9.	Variation of h in the two-constant modification of string theory.	24
10.	Steady state, yaw data at discrete yaw angles for tire D.	27
11.	Continuous yaw angle data for tire D.	28
12.	Dynamic force and moment coefficients using static h and slow rolling λ for four model tires.	29
13.	Static and slow rolling h and λ versus best fit values of h and λ for dynamic steer data for four tires.	32

LIST OF ILLUSTRATIONS (Concluded)

Figure	Page
14. Dynamic force and moment coefficients using best fit h and λ , for four model tires.	33
15. Comparison of force and moment response data to two-constant modified string theory using best fit values of λ and h for tire A.	34
16. Comparison of force and moment response data to two-constant modified string theory using best fit values of λ and h for tire B.	36
17. Comparison of force and moment response data to two-constant modified string theory using best fit values of λ and h for tire C.	38
18. Comparison of force and moment response data to two-constant modified string theory using best fit values of λ and h for tire D.	40

NOMENCLATURE

English Letters

C_F - dynamic lateral force coefficient

C_M - dynamic moment coefficient

D - tire diameter

F_ψ - force normal to wheel plane

F_z - tire vertical force

h - tire footprint half length

K_L - tire lateral stiffness

$K_{F\psi}$ - tire force yaw stiffness

K_{Mz} - tire moment yaw stiffness

M_z - moment about vertical axis

p_o - tire inflation pressure

t - time

v_o - road speed

z - distance from wheel plane to leading edge of tire contact patch

\bar{z} - distance from wheel plane to trailing edge of tire contact patch

Greek Letters

λ - tire relaxation length

ϕ_o - steer excitation amplitude

ψ - steady state yaw angle

Ω - tire excitation frequency

I. INTRODUCTION

Previous work [2] has shown that good agreement could be obtained between analytical predictions of dynamic tire behavior and experimental observations when the analysis was based on the so-called "string theory" description of the motion of the tire, and the experimental data was confined to tires of conventional bias ply construction. In that work, the tire elastic properties used in the analytical predictions were obtained from static and slow-rolling measurements. For at least this restricted class of tire constructions, there is good reason to feel that tire behavior in shimmy calculations can be closely approximated by such a string theory using readily measured tire elastic properties.

The present study extends this investigation to a much wider range of tire constructions, partly to determine the limits inside which string theory represents an adequate mathematical model for tire dynamic behavior, and partly to anticipate the need for broader tire mathematical models applicable to some of the more advanced aircraft tire constructions currently under development.

II. SUMMARY OF RESULTS

Four model tires of widely different constructions were used in this program. In the experimental portion of the work, these tires were run on The University of Michigan small-scale road wheel under conditions of sinusoidal steer at frequencies ranging from 0.1 to 10 Hz amplitudes of $\pm 2^\circ$, and at a range of surface velocities from 0.85 ft/sec to 19.6 ft/sec. Cornering force and self-aligning torque were measured during these tests and used as an indication of tire dynamic response.

The tire force and moment data was first compared to the finite contact patch string theory predictions, based on the theory given in previous work [2]. The effect of variation of the contact patch length and of relaxation length was examined. In general, agreement between finite contact patch theory and experimental observation was good.

A modified string theory is presented in which coefficients are separately determined for cornering force and self-aligning torque. Use of such a two-constant, or modified, string theory improves the correspondence between experimental observation and analytical predictions. The modified string theory seems to be most effective on tires with relatively high self-aligning torques. Two methods were used for determining the constants used in the modified string theory. In the first, direct measurement of these constants was carried out by static and slow-rolling tests. In the second, these constants were found by a best-fit process based on dynamic response data. There does not seem to be strong evidence at this time indicating the superiority of one method over the other.

III. COMPARISON OF STRING THEORY BY MEANS OF PREDICTIONS AND MEASUREMENTS ON SCALE MODELS

The present investigation is structured around comparison of analytical predictions of tire dynamic behavior with experimental measurements of such behavior. Previous work had indicated that the most sensitive single test which could be conducted easily involved sinusoidally oscillating steer angles applied to a tire, with consequent measurement of the tire cornering force and self-aligning torque on a continuously recorded basis. Examination of the force and moment amplitudes and phase angles with respect to the input steer displacement gives a sensitive measure of the adequacy of analytical predictions in describing dynamic tire response.

For this study, small-scale model tires of four different constructions were used to examine the influence of construction variations on the adequacy of existing theories. Two of the tires were scaled versions of existing aircraft tires and were of conventional bias construction. One tire was made without fabric, and was isotropic in its construction materials. The fourth tire was a fabric reinforced scaled tire using only radially-oriented fabric reinforcement, and as such represented a case of low-tire lateral stiffness characteristics. The dimensions and properties of these scaled model tires are given in Table I. During all subsequent tests reported here, the tires were operated under the conditions indicated in Table I.

The model tires described in Table I were run on The University of Michigan small-scale, road-wheel tire testing apparatus previously described

TABLE I

STANDARD TIRE OPERATING CONDITIONS AND STATIC TIRE ELASTIC PROPERTIES

Tire	Type	p_0 , psi	D, in.	F_z , lb	h, in.	λ , in.	K_L , lb/in.
A	Isotropic toroid	5	4.54	11.0	.825	1.61	18
B	2-ply bias-scaled 49 x 17-26 PR Type VII	25	4.08	41.0	.825	1.48	84
C	1-ply unbelted radial	10	4.68	22.7	1.000	1.53	24
D	2-ply bias-scaled 40 x 12-14 PR Type VII	21	4.58	44.0	.980	1.39	76

in [2]. The tires were moved in sinusoidal steer by means of a harmonic displacement generator operating through a linkage system. Cornering force, F_ψ , and self-aligning torque, M_z , were recorded. The resulting data were analyzed using the Fourier analysis program previously described in [2]. The tires were operated at several different road wheel velocities but with continuously varying steer excitation frequencies. This system allowed identification of resonant components associated with the natural frequencies of mechanical portions of the road wheel system, and these could then be filtered out of tire response data. Figures 1 through 4 show force and moment response data for the four tires tested under conditions described above. While tire A seems to show somewhat more scatter than the other three, in general the experimental observations seem to fall along well-defined bands.

Analytical predictions are also shown in Figures 1 through 4, based on formulations derived in [2] using both finite and point contact patch formulations. Comparison of self-aligning torque amplitudes for tires A and C show

that analytical predictions are significantly lower than observed data, although the force amplitudes and phase angles are in reasonably good agreement. All predictions were carried out using tire elastic properties determined from static and slow-rolling experiments as described in [2].

A short study was undertaken on the influence of errors in the measurement of contact patch length, h , and relaxation length, λ . This can best be seen by using different values of these two parameters in the analytical predictions of force and moment response. Figures 5 and 6 show the results of first varying relaxation length, λ , with a fixed h and subsequently varying contact patch length, h , with fixed λ , using tire D as an example. Examination of these two figures shows that, in general, contact patch length, h , is much more important in determining overall dynamic tire response, in terms of both amplitudes and phase angles, than is the relaxation length, λ .

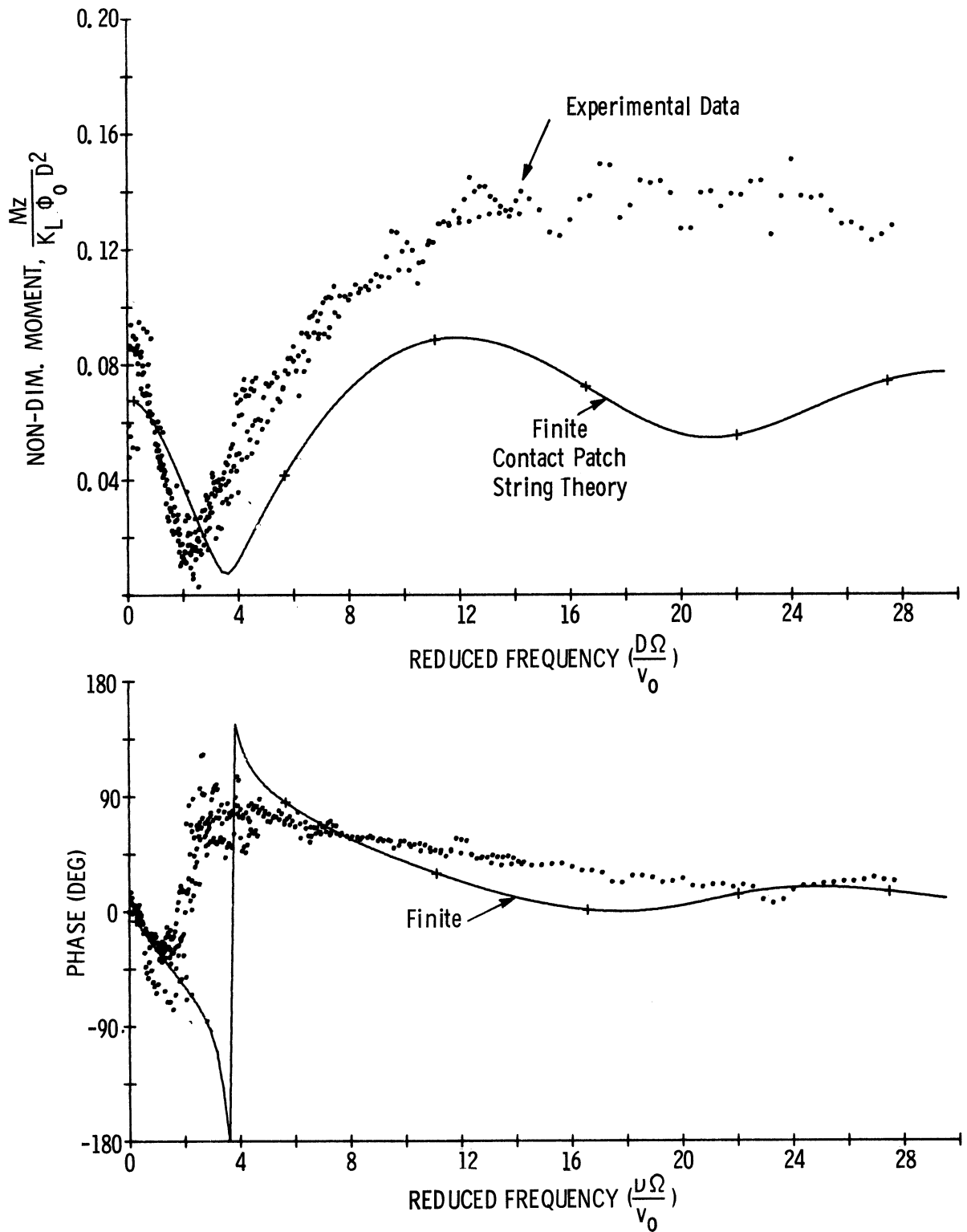


Figure 1. Force and moment response to steer excitation of tire A using von Schlippe string theory.

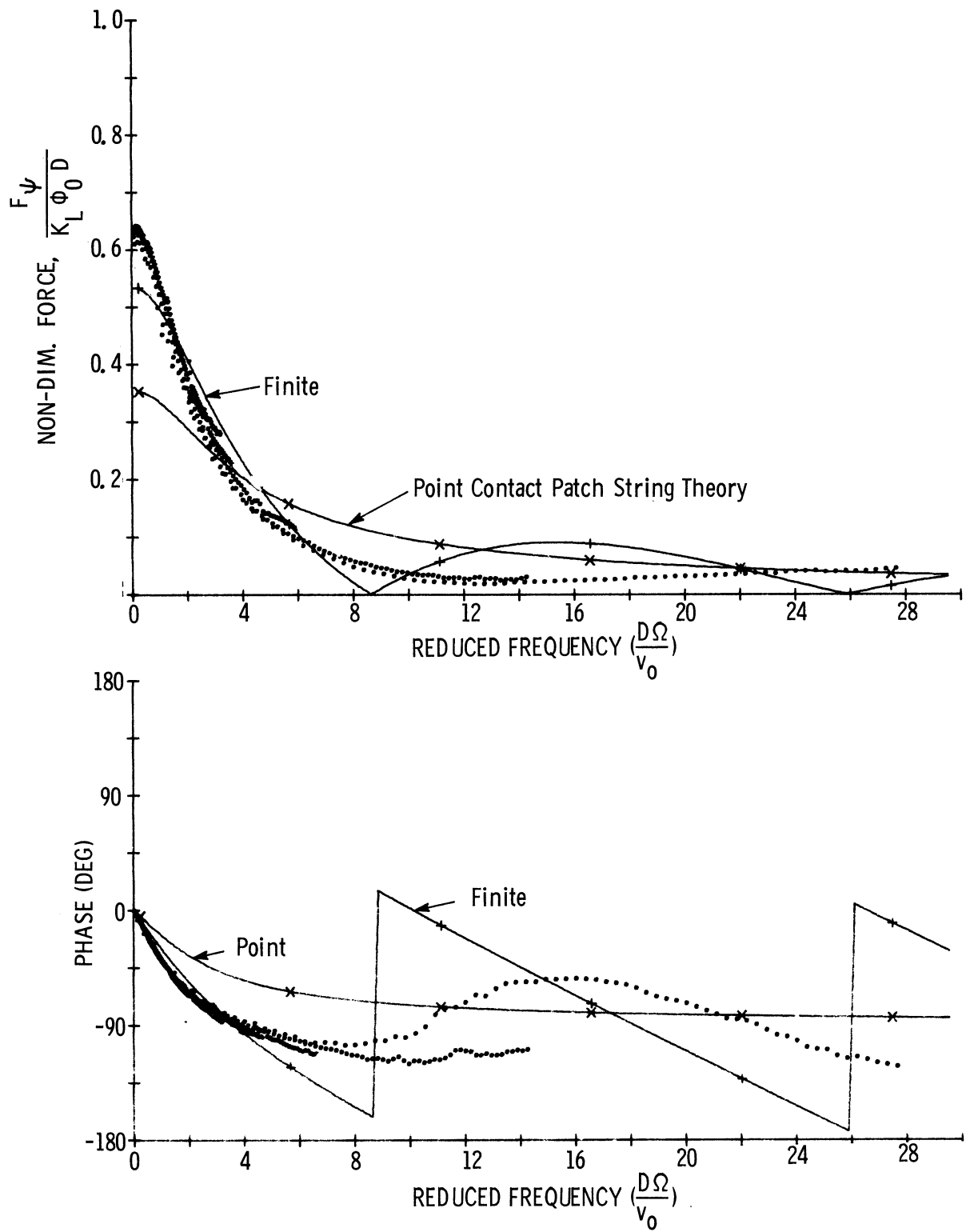


Figure 1. (Concluded)

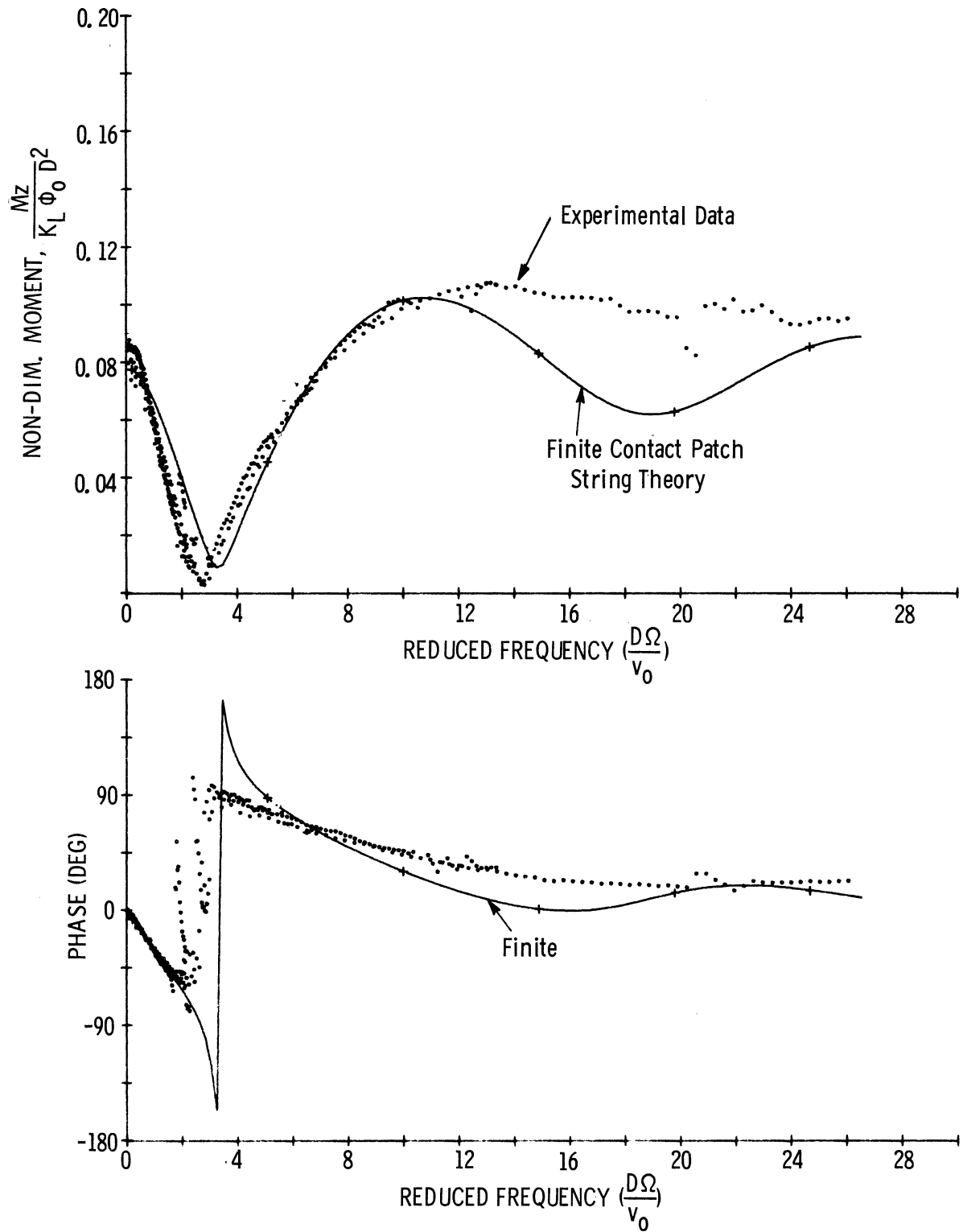


Figure 2. Force and moment response to steer excitation of tire B using von Schlippe string theory.

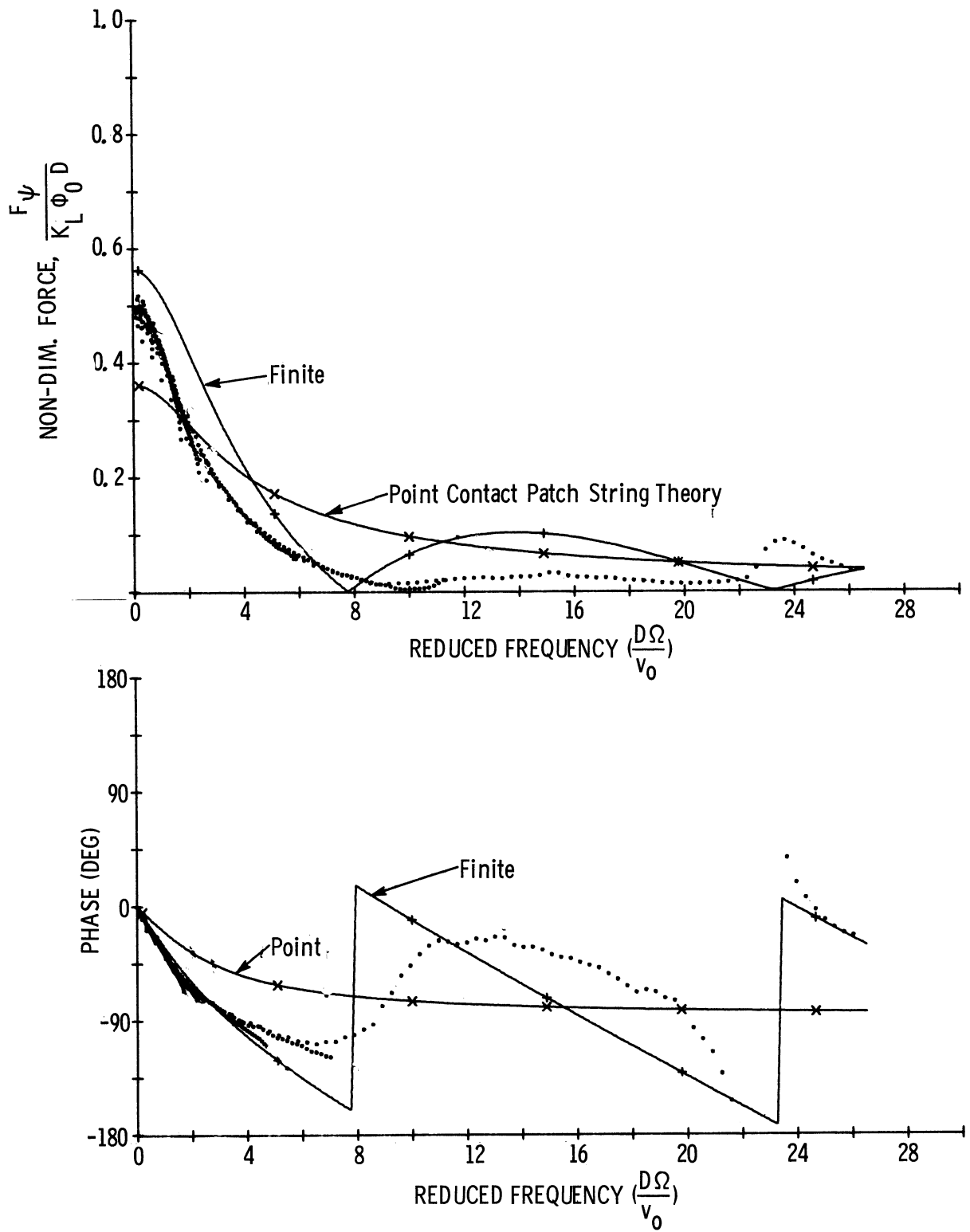


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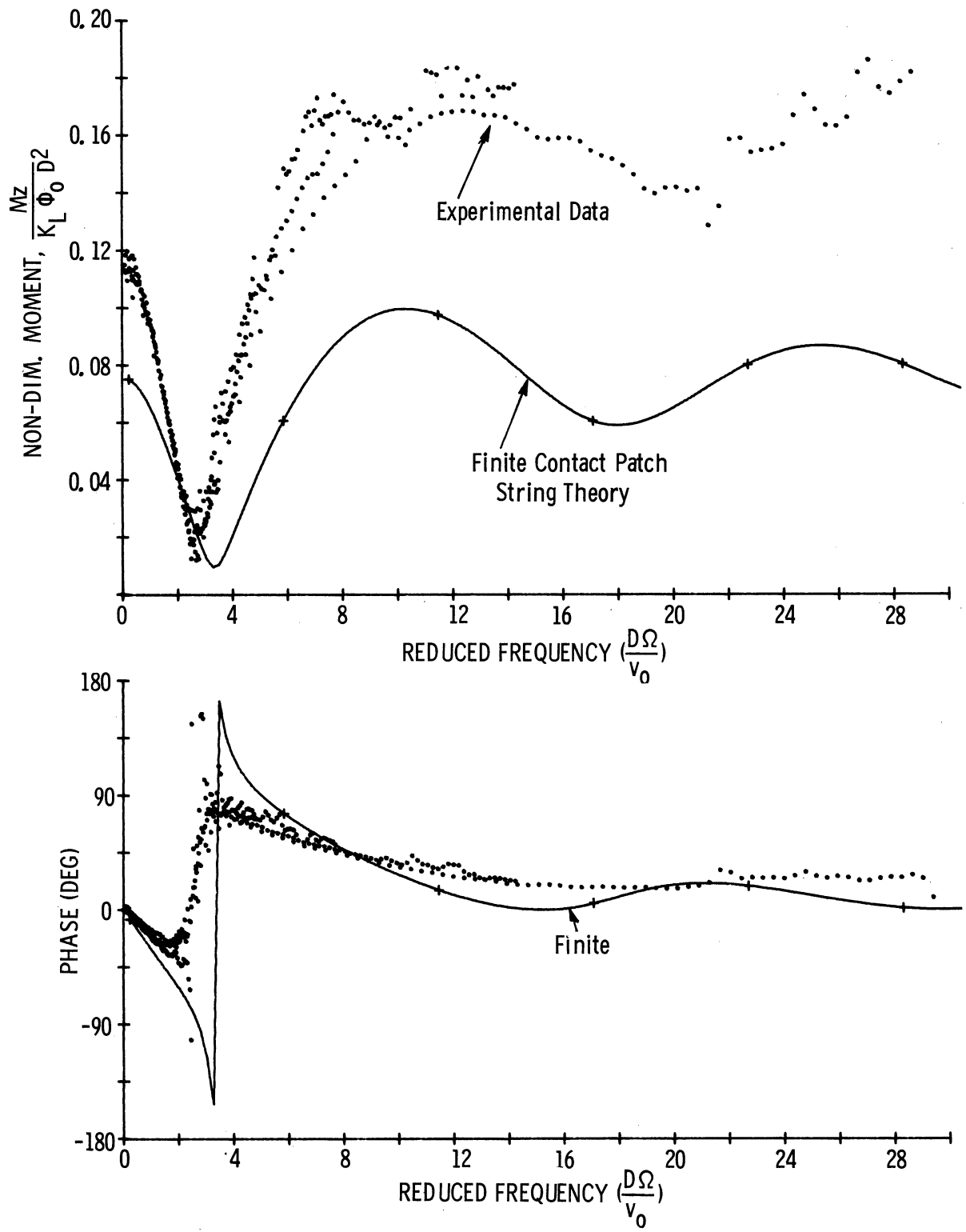


Figure 3. Force and moment response to steer excitation of tire C using von Schlippe string theory.

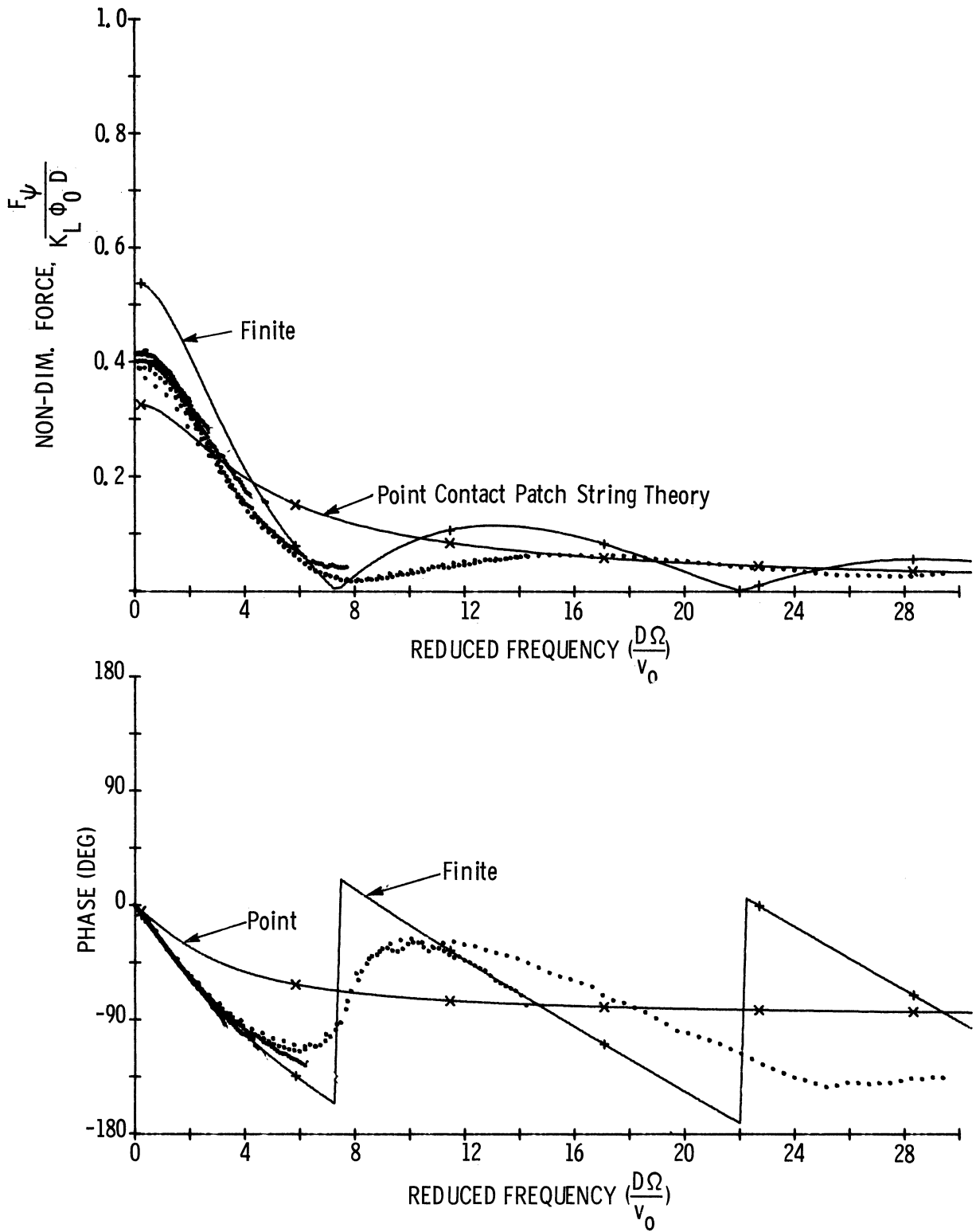


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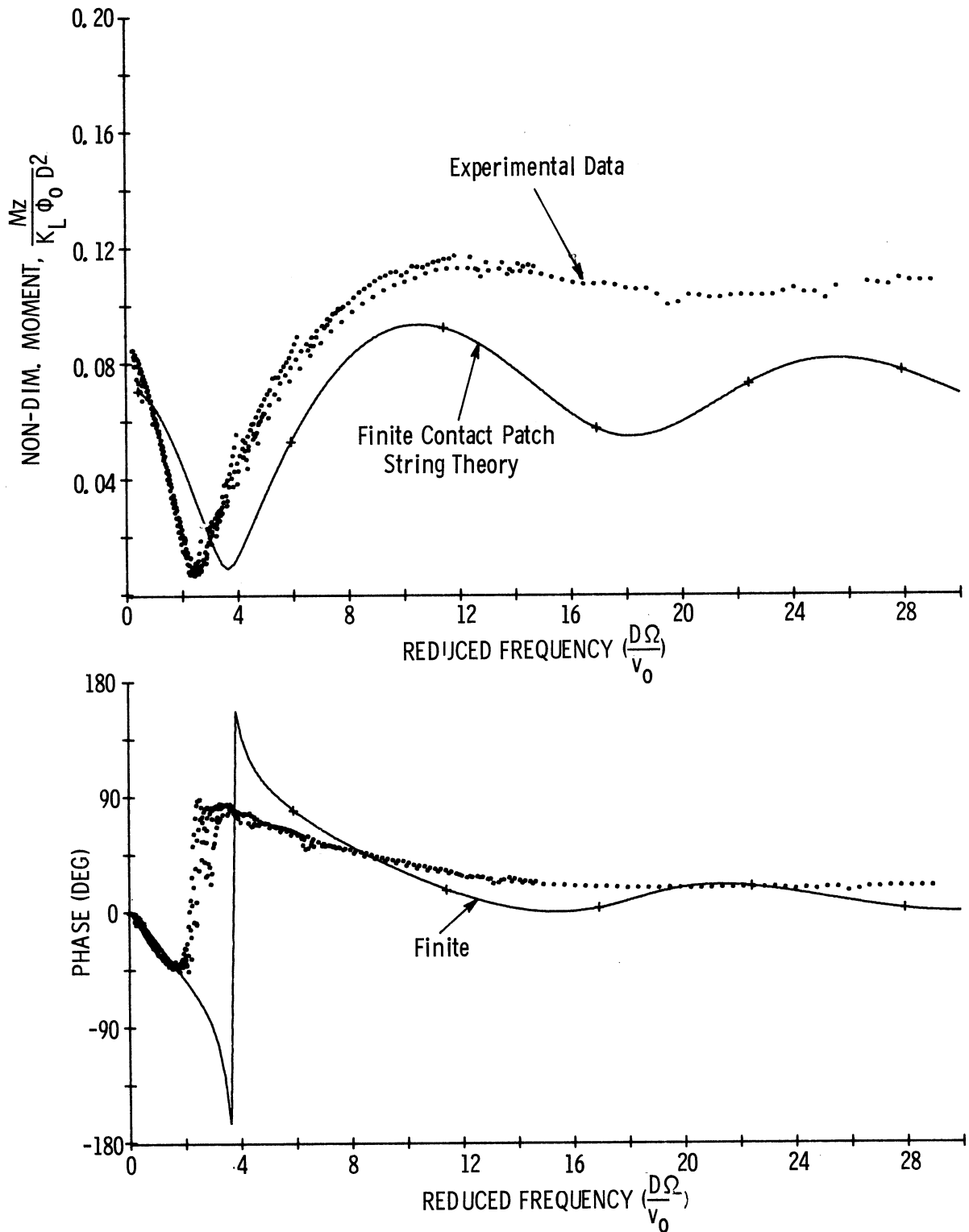


Figure 4. Force and moment response to steer excitation of tire D using von Schlippe string theory.

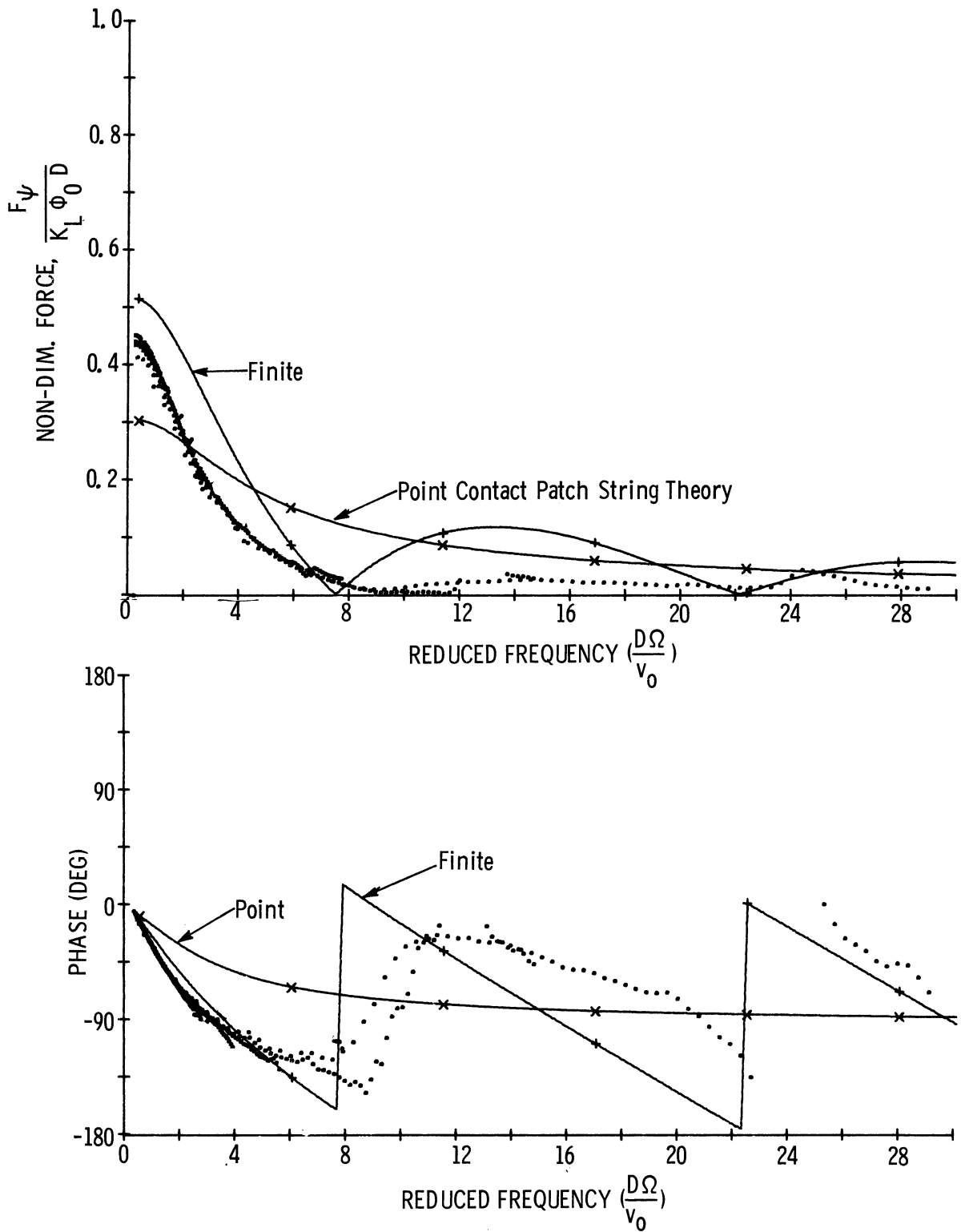


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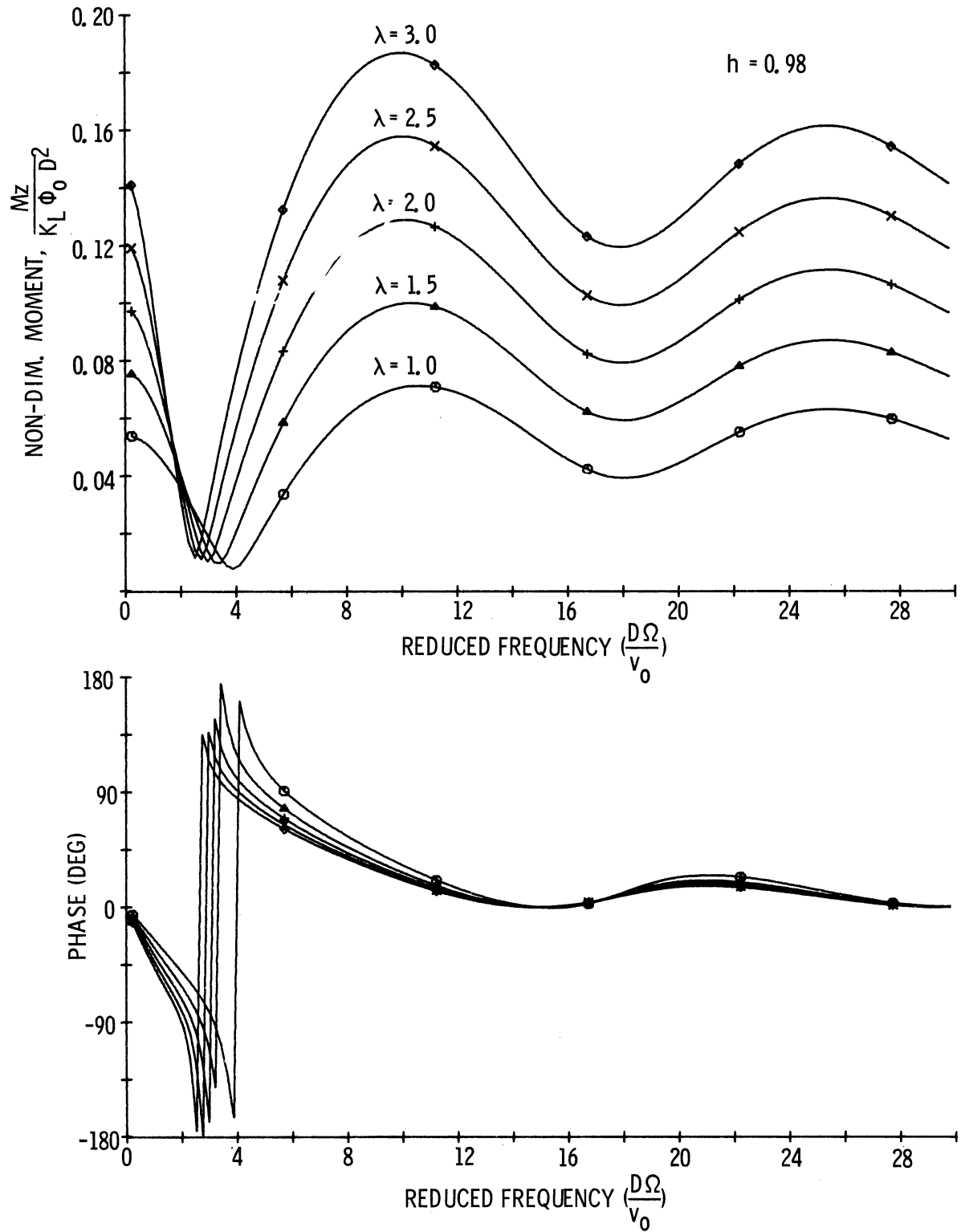


Figure 5. Variation of λ in string theory, for steer excitation.

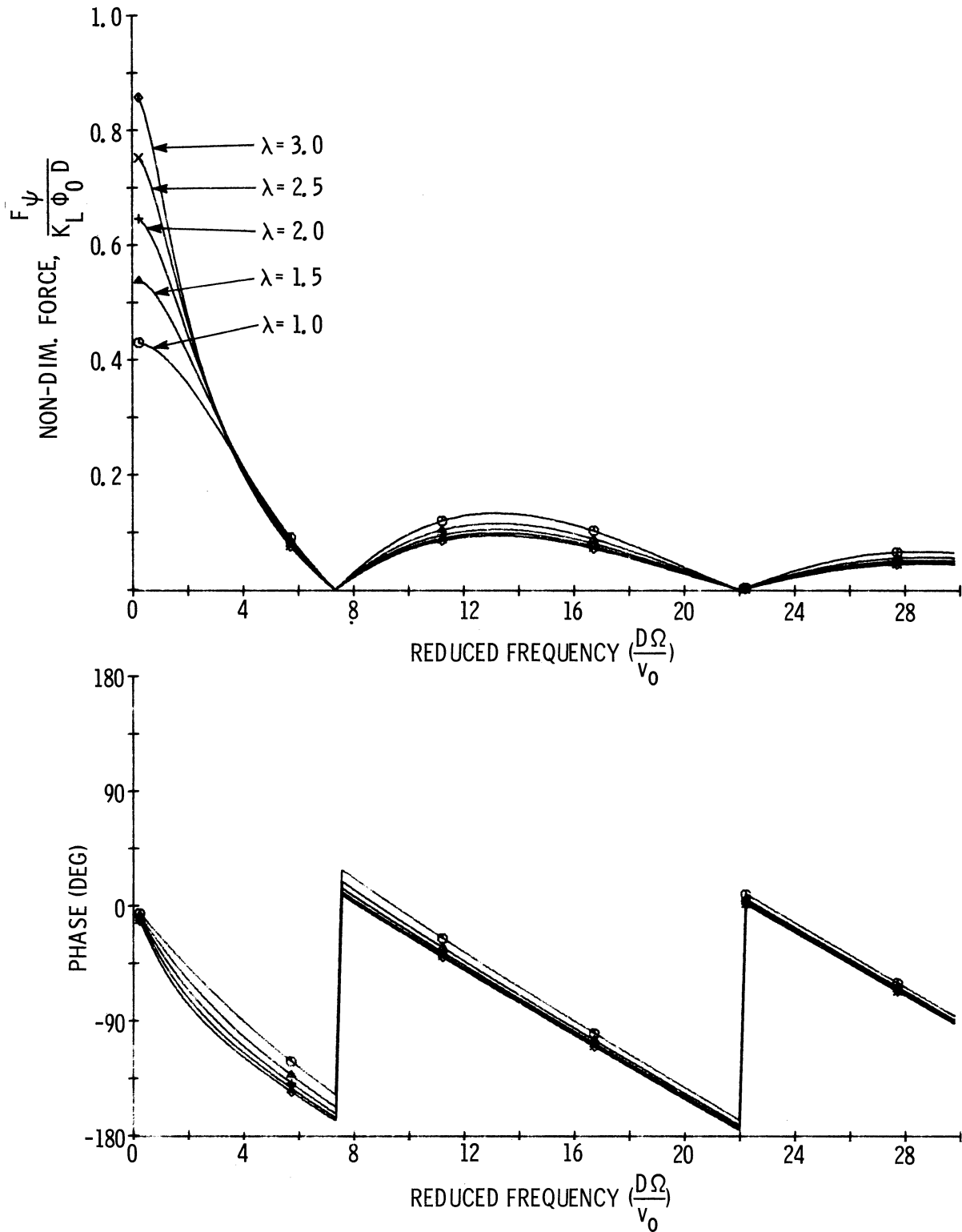


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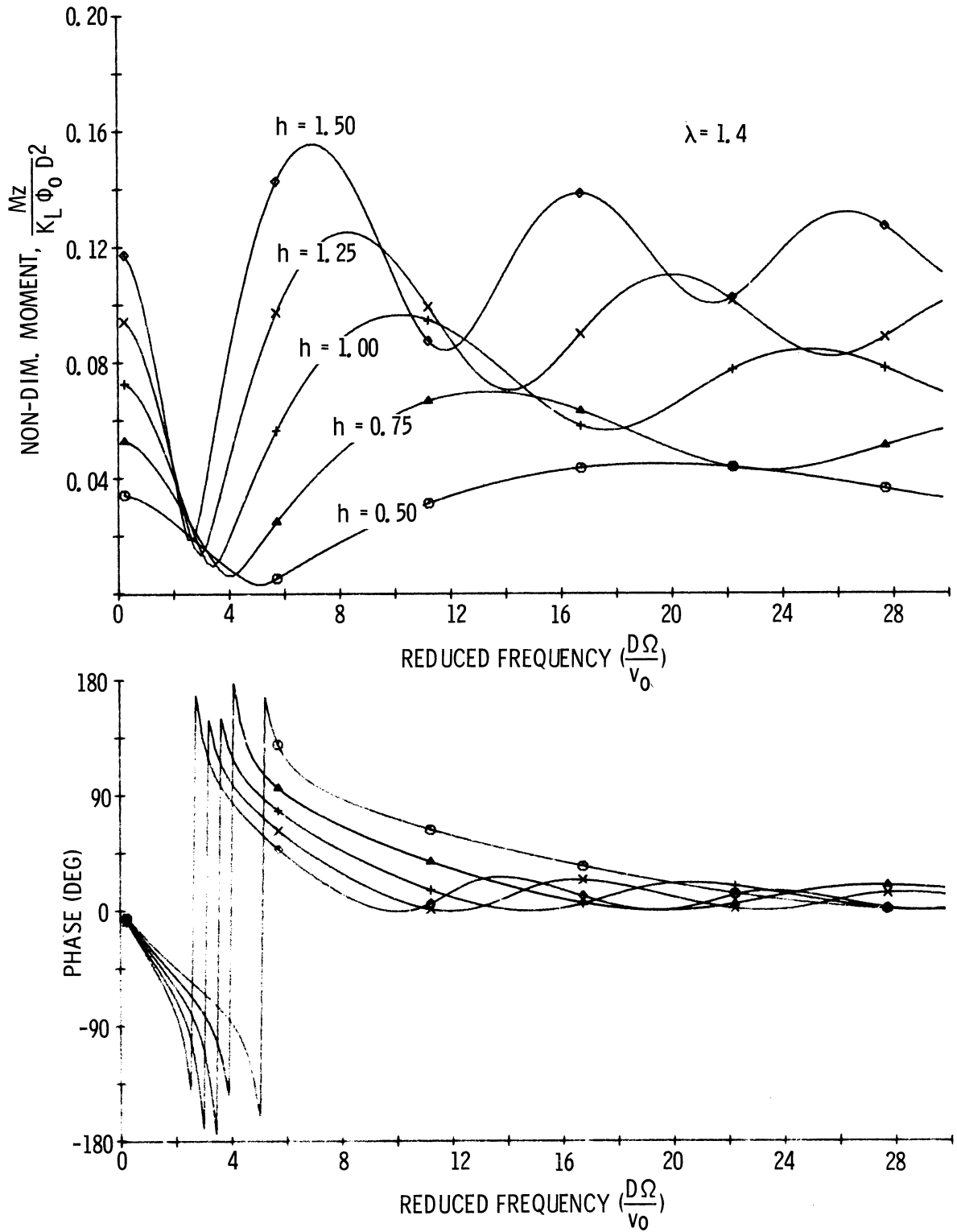


Figure 6. Variation of h in string theory, for steer excitation.

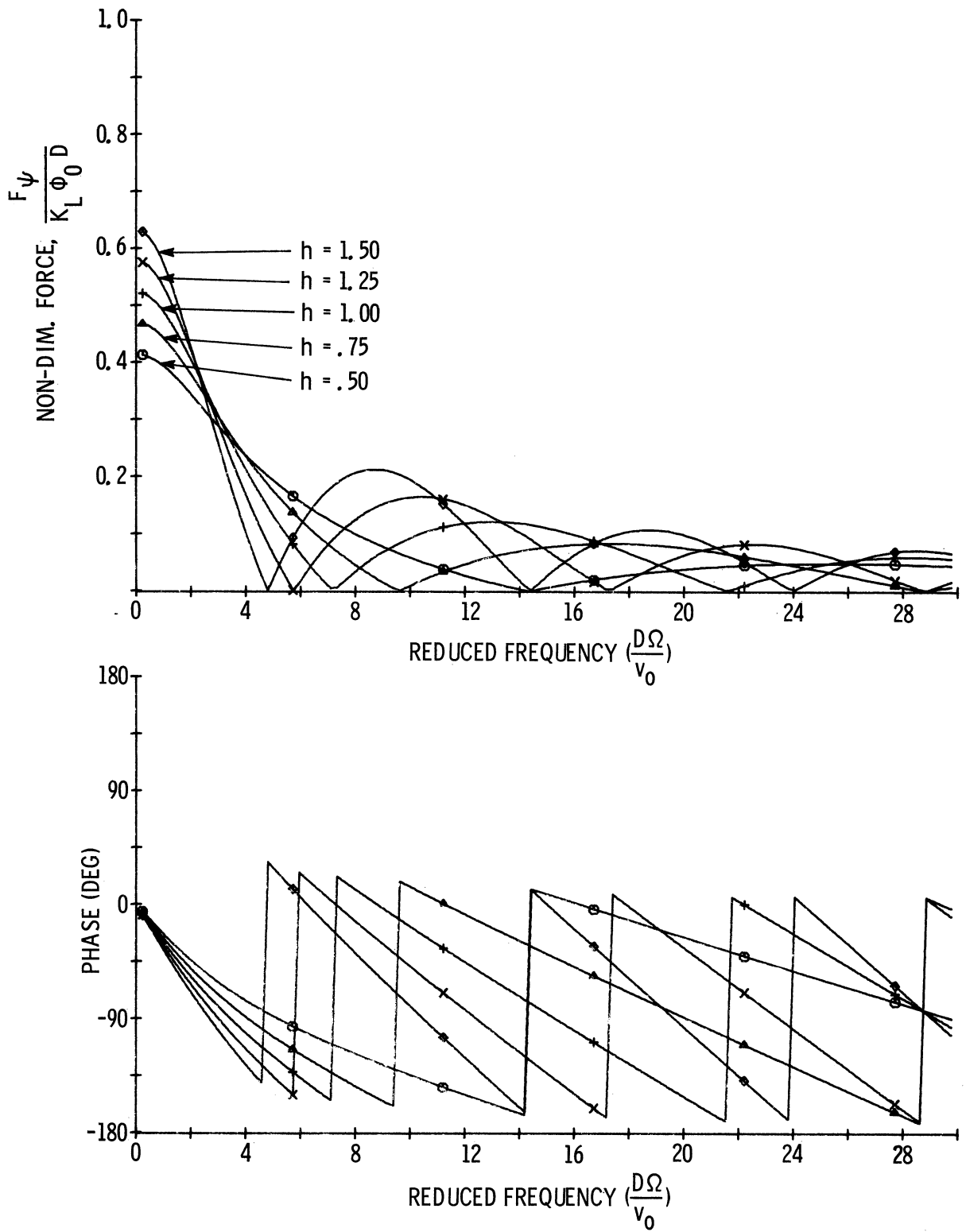


Figure 6. (Concluded)

IV. TWO-CONSTANT MODIFICATION OF FINITE CONTACT PATCH STRING THEORY

Agreement between experiment and string-theory predictions based on finite contact patch length can be improved by introduction of a modified theory using separate force and moment coefficients which are determined experimentally for each type of tire and each set of operating conditions. Normal string-theory predictions, as shown in [2], yield the equations for concerning force and self-aligning torque as follows:

$$F_{\psi} = K_L \left(\frac{\bar{z}+z}{2} \right) \quad (1)$$

and

$$M_z = K_L \left[\lambda + \frac{h^2}{3(h+\lambda)} \right] \left(\frac{\bar{z}-z}{2} \right) \quad (2)$$

These equations are obtained by integrating an assumed unit stiffness constant around the periphery of the tire. This idealization can be avoided by assuming more general relationships of the form

$$F_{\psi} = C_F \left(\frac{\bar{z}+z}{2} \right) \quad (3)$$

and

$$M_z = C_M \left(\frac{\bar{z}-z}{2} \right) \quad (4)$$

where, as before, the displacements z and \bar{z} are gotten from the tire tracking equation. C_F and C_M are, respectively, a dynamic lateral force coefficient and a dynamic self-aligning torque coefficient. The relationships between the

tire lateral stiffness, K_L , and the force and moment coefficients are given by Eqs. (5) and (6).

$$C_F = K_L \quad (5)$$

and

$$C_M = K_L \left[\lambda + \frac{h^2}{3(h+\lambda)} \right]. \quad (6)$$

These constants can be related directly to the tire cornering power.

Running a tire at steady state yaw angle produces a side force, F_ψ , and a self-aligning torque, M_z . For small yaw angles there is a linear relationship between this force and couple and the corresponding steer angle in the form

$$F_\psi = K_{F\psi} \psi \quad (7)$$

and

$$M_z = K_{Mz} \psi \quad (8)$$

The geometry of the contact patch is shown in Figure 7, from which it may be seen that the relationship between contact patch displacements, steer angle, relaxation length, and the contact patch length are given by

$$\frac{z+\bar{z}}{2} = (\lambda+h) \tan \psi \quad (9)$$

which holds for a tire at a steady state yaw angle. Substituting Eq. (9) into Eq. (3) yields

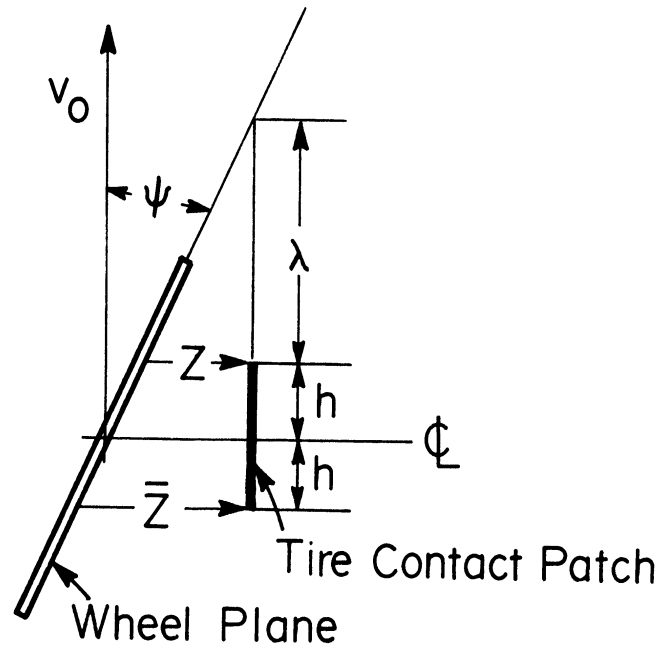


Figure 7. Geometry for tire contact patch at steady state yaw angle.

$$F = C_F \left(\frac{z + \bar{z}}{2} \right) = C_F (\lambda + h) \tan \psi \quad (10)$$

For small angles, $\tan \psi = \psi$, so that the above equation becomes

$$F_\psi = C_F (\lambda + h) \psi = K_{F\psi} \psi \quad (11)$$

In addition, the relation

$$C_F = \frac{K_{F\psi}}{(\lambda + h)} \quad (12)$$

defines a relationship between the yaw force spring constant and the dynamic force coefficient.

For the self-aligning torque, the geometric relation

$$\psi = \frac{\bar{z} - z}{2h} \quad (13)$$

is used. Then the moment is given, using Eq. (4), as

$$M_z = C_M \left(\frac{\bar{z} - z}{2} \right) = K_{Mz} \left(\frac{\bar{z} - z}{2h} \right) \quad (14)$$

which yields the relation

$$C_M = \frac{K_{Mz}}{h} \quad (15)$$

This is the relationship between the "spring rate" of the self-aligning torque and the dynamic moment coefficient.

Variations in the contact patch length, h , and the relaxation length, λ , in these formulations affect the predictions of dynamic moment response, although this variation has no effect on the side force response. Figures 8 and 9 show the effects of varying relaxation length, λ , and the contact patch length, h , in Eqs. (3) and (4), using tire D again as an example, and using arbitrary C_F and C_M . Again, variations in contact patch length, h , are much more important in changing tire moment response than is variation in relaxation length, λ . Force response, as previously pointed out, is independent of variations in these two parameters.

The tire constants C_F and C_M can now be varied independently to raise or lower the predicted force and moment amplitudes, and in this sense they give an extra degree of freedom to allow theoretical predictions to agree more closely with observed data. This represents an alternate theoretical framework to that of conventional string theory, and yet one which is closely based on string theory. Such a formulation appears to be most valuable for tires of widely varying construction features such as encountered in this study.

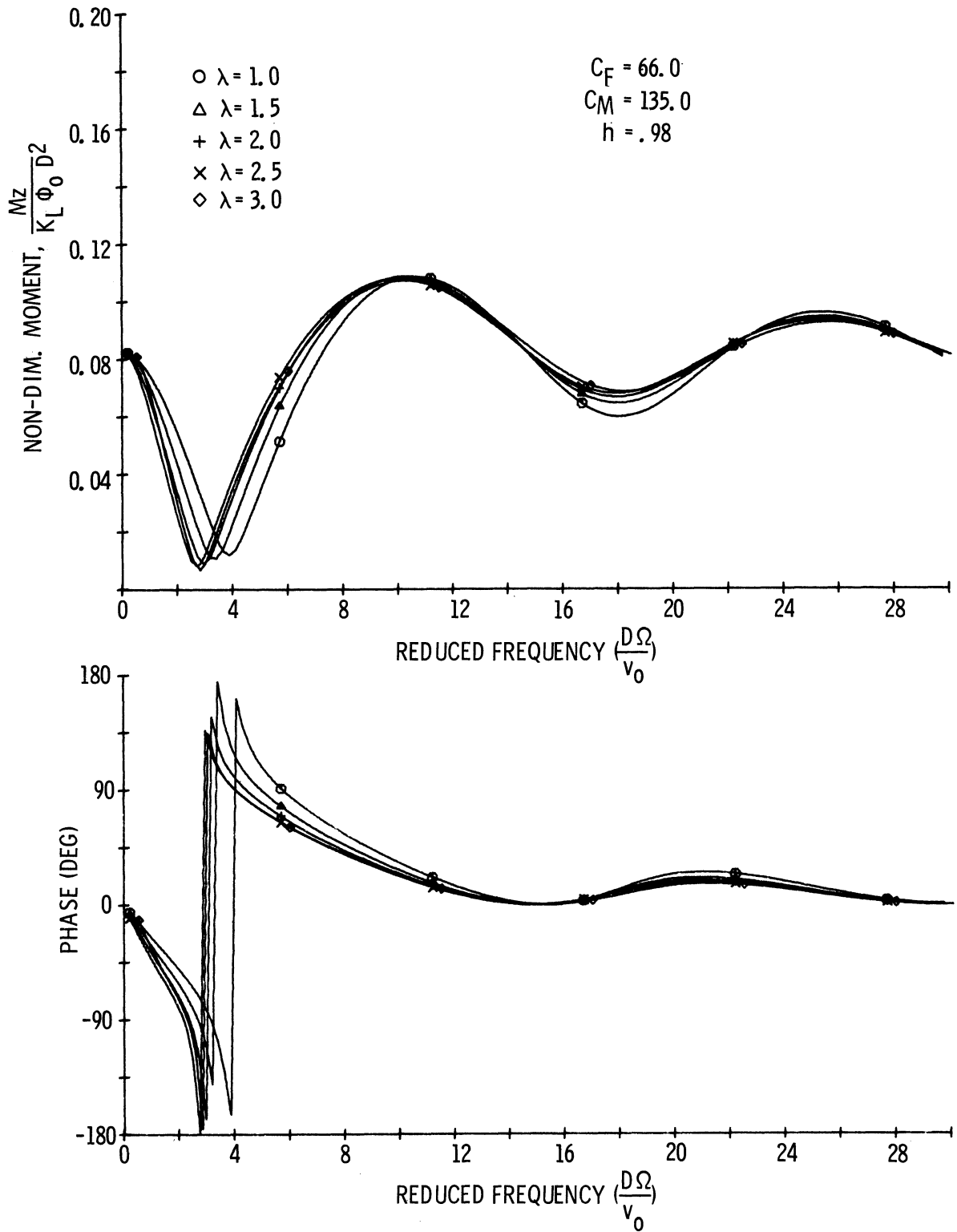


Figure 8. Variation of λ in the two-constant modification of string theory.

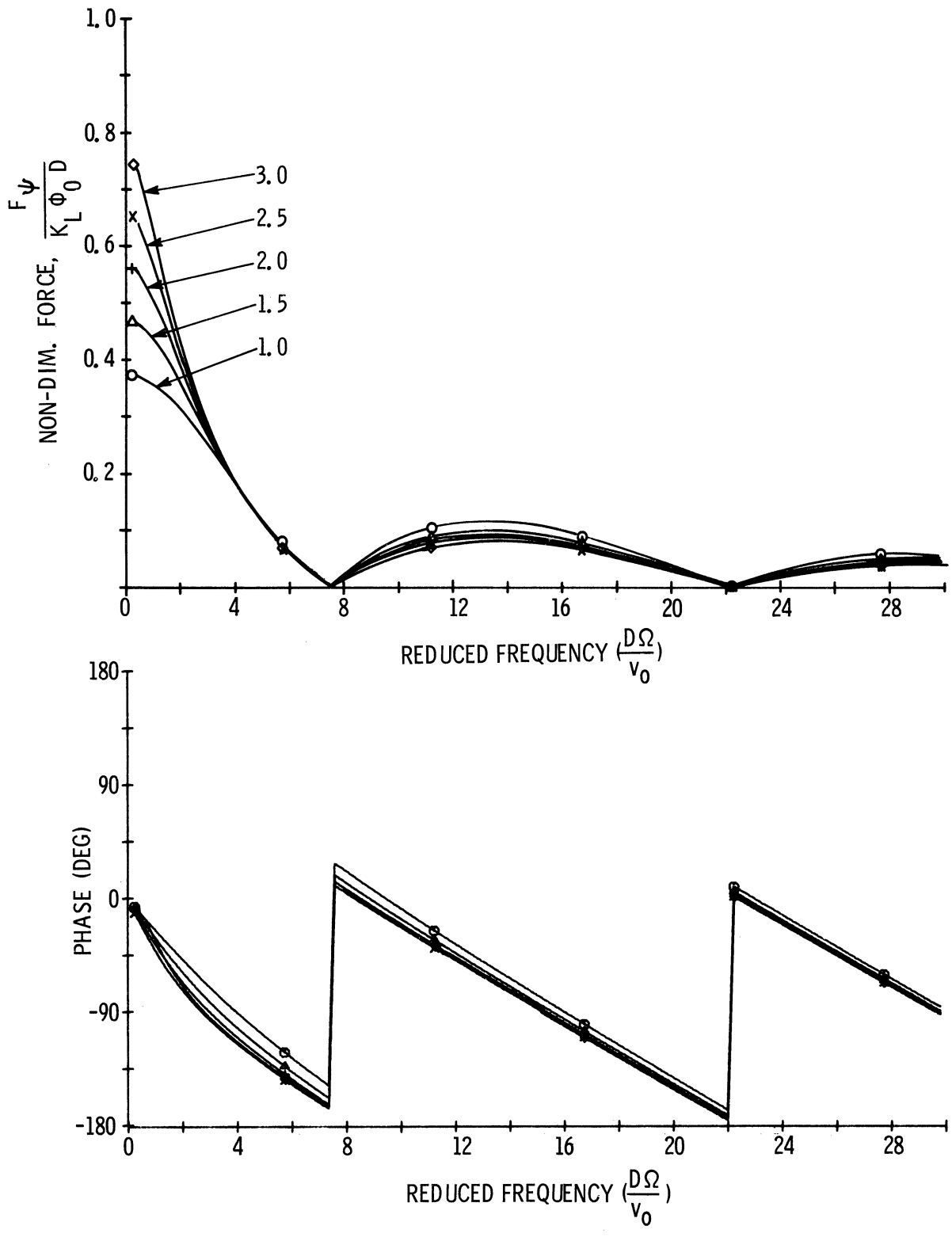


Figure 8. (Concluded)

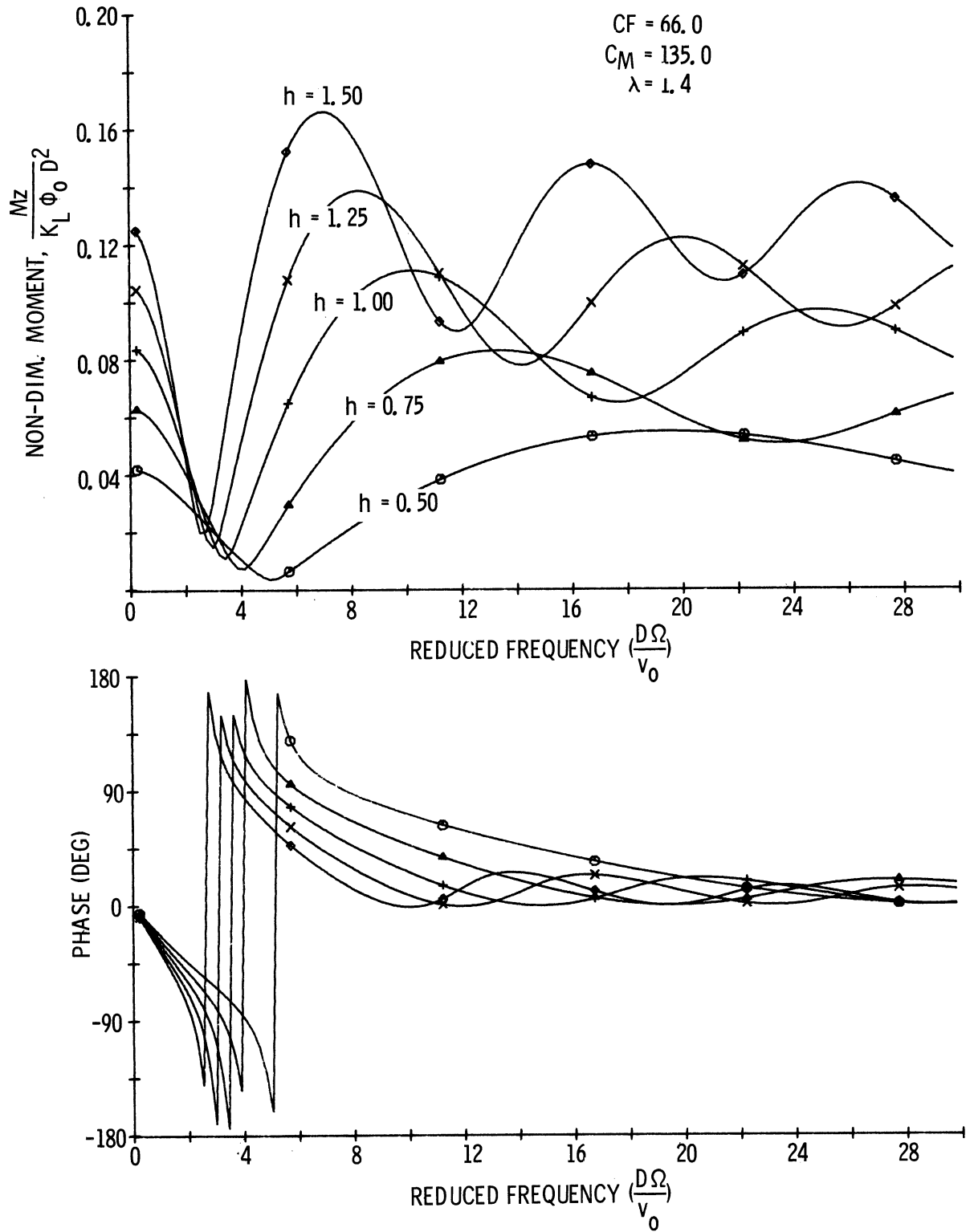


Figure 9. Variation of h in the two-constant modification of string theory.

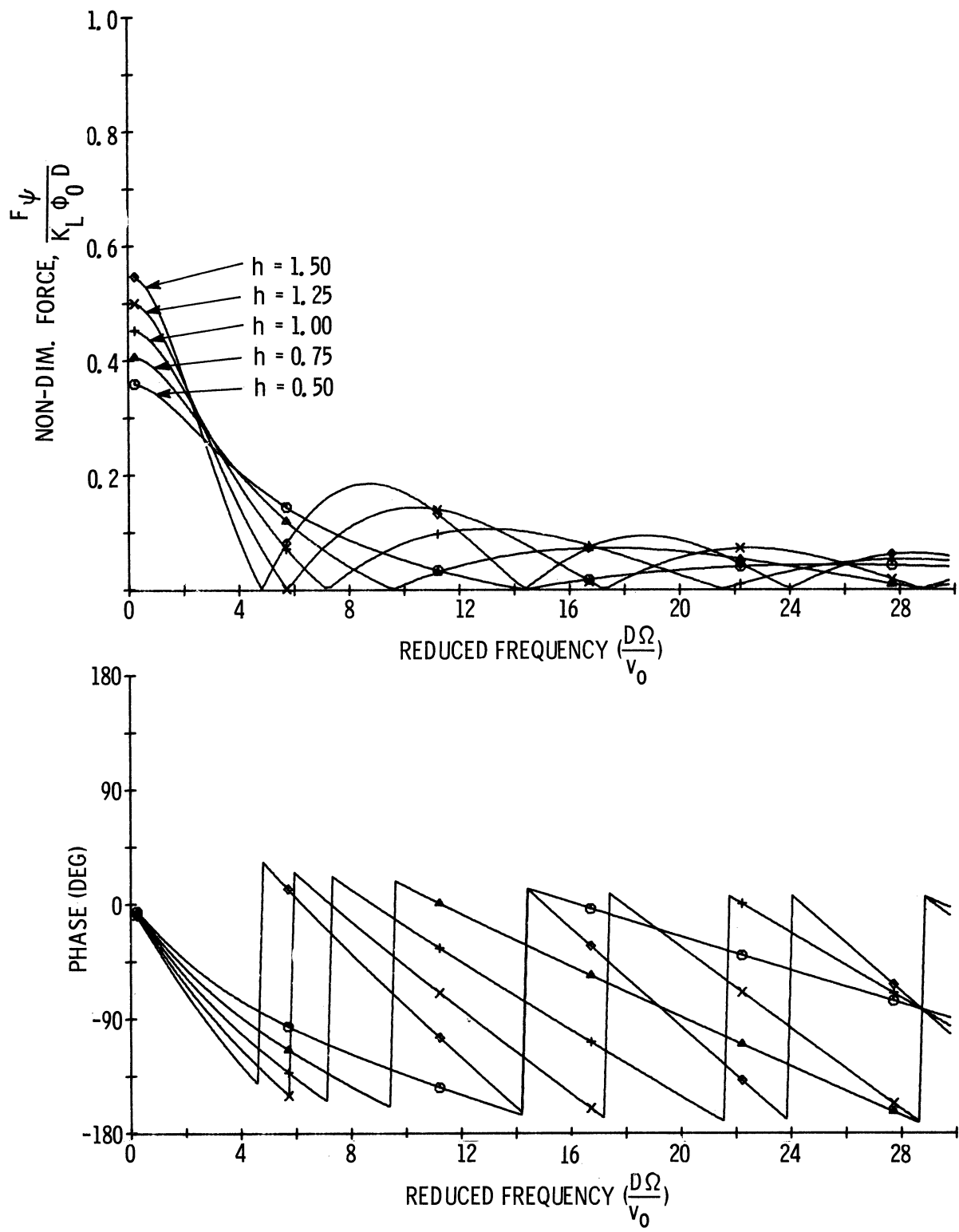


Figure 9. (Concluded)

In practice, C_F and C_M are not difficult to obtain since they are based on direct measurements of side force and self-aligning torque, usually taken in slow-rolling, steady-state conditions. Figure 10 shows typical data taken on tire D of this study, where measured self-aligning torque and side force are used to determine the appropriate side force and self-aligning torque spring rates, from which the coefficients C_F and C_M may be obtained directly. An alternate method of obtaining such measurements is suggested by noting that the side force and moment response may be measured at low frequencies under dynamic steer conditions. Data taken under these conditions should approach steady-state data. Figure 11 shows data measured using slow continuously varying yaw angles $D\Omega/v_o = .009$, which generate a continuous sinusoidal record of force and moment change. The force response from the data shown in Figure 11 is quite linear for the entire range of swept angles and agrees well with steady-state data shown in Figure 10. However, self-aligning torque response under continuously varying steer angles is linear only to within a very small range of yaw angles centered about zero. This implies slip in the contact patch at relatively small values of steer. However, the resulting stiffness characteristics associated with self-aligning torque, K_{Mz} is much more accurate for dynamic work than that gotten from steady-state data. Figure 12 shows a comparison of values of the force and moment coefficients, C_F and C_M , for all four model tires. These coefficients are arrived at using four methods. In the first, the von Schlippe string theory predicts C_F and C_M coefficients directly using Eqs. (5) and (6). Secondly, the best fit values of C_F and C_M are determined by using the two-constant modification to string theory and by varying C_F and

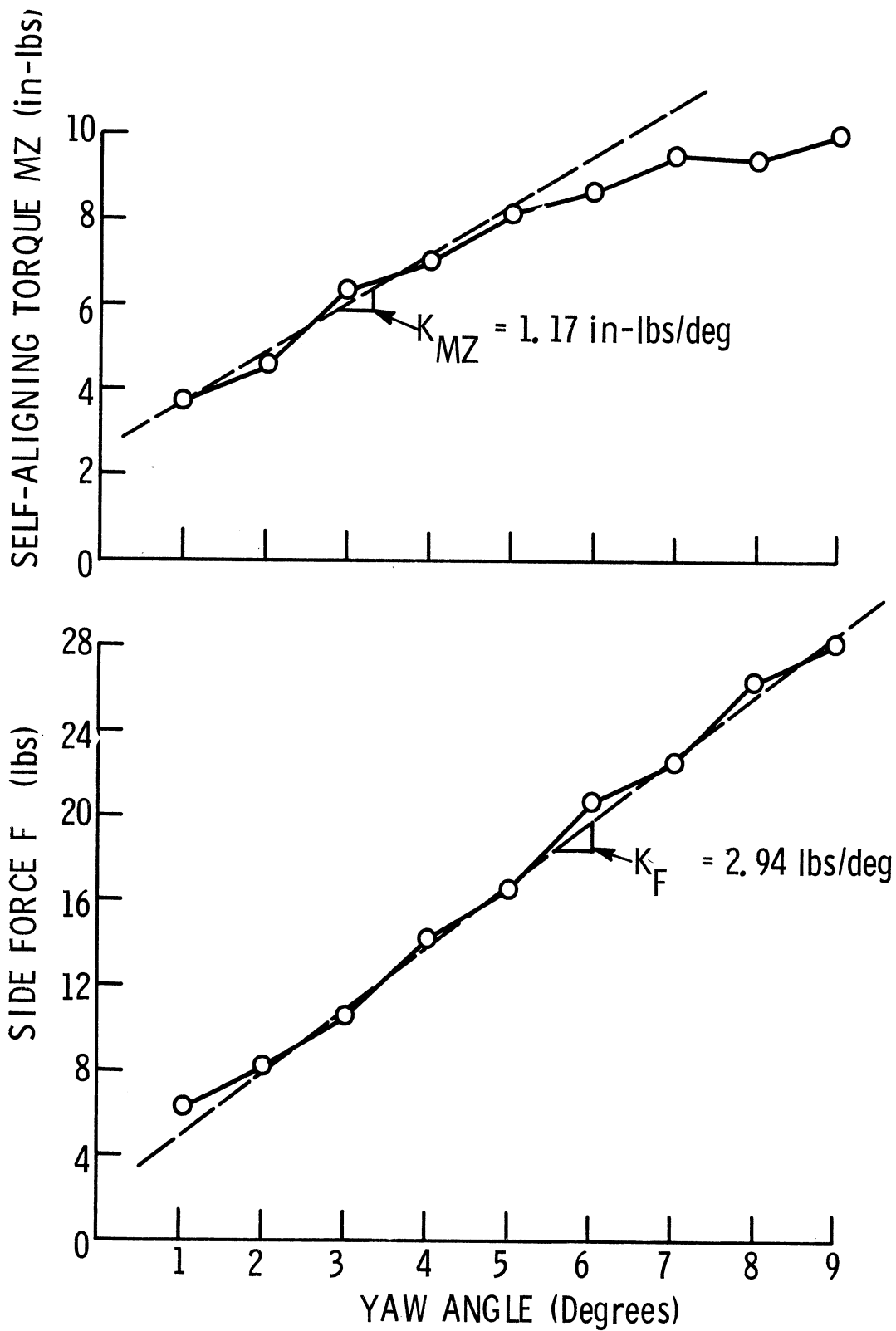


Figure 10. Steady state, yaw data at discrete yaw angles for tire D.

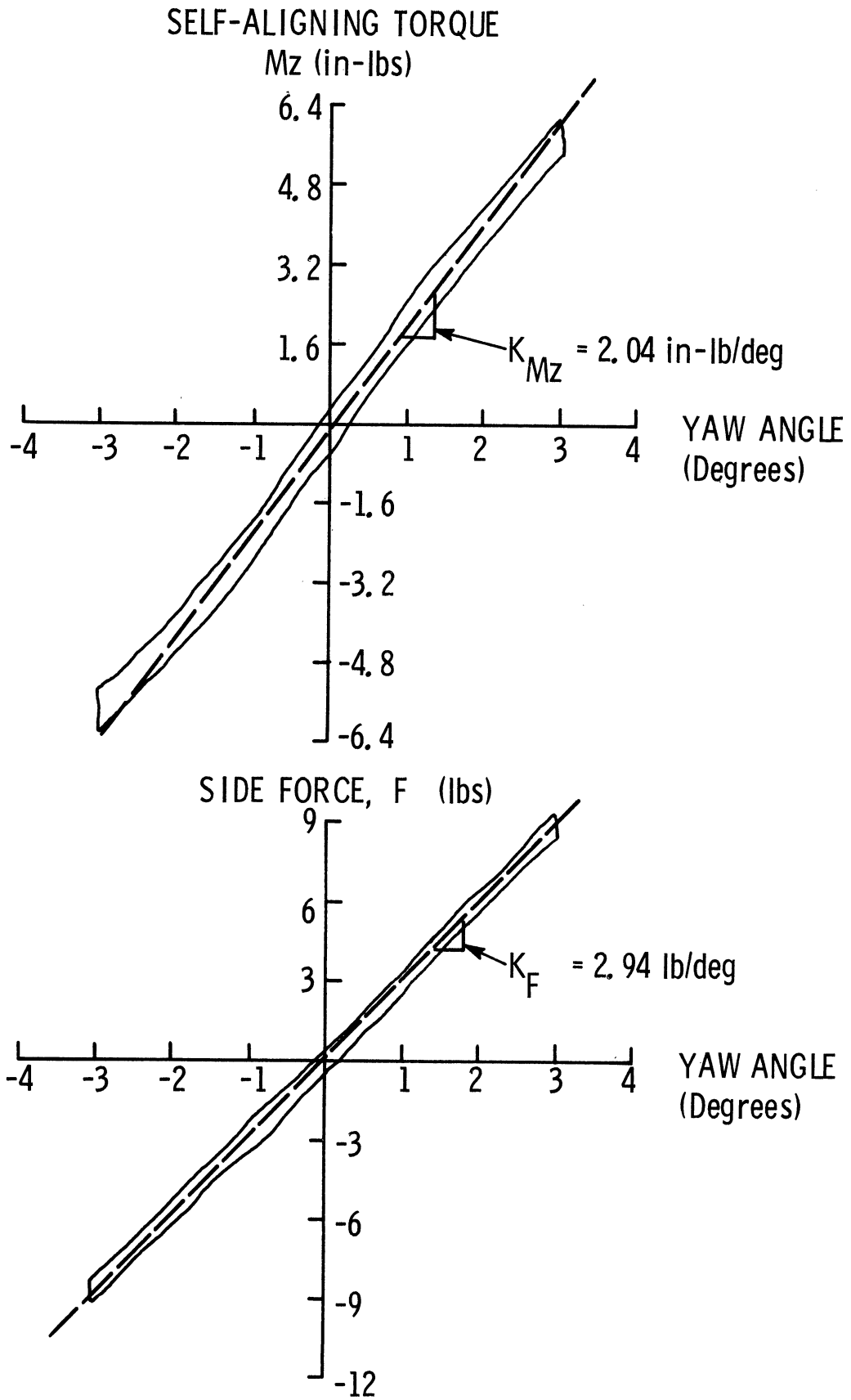


Figure 11. Continuous yaw angle data for tire D.

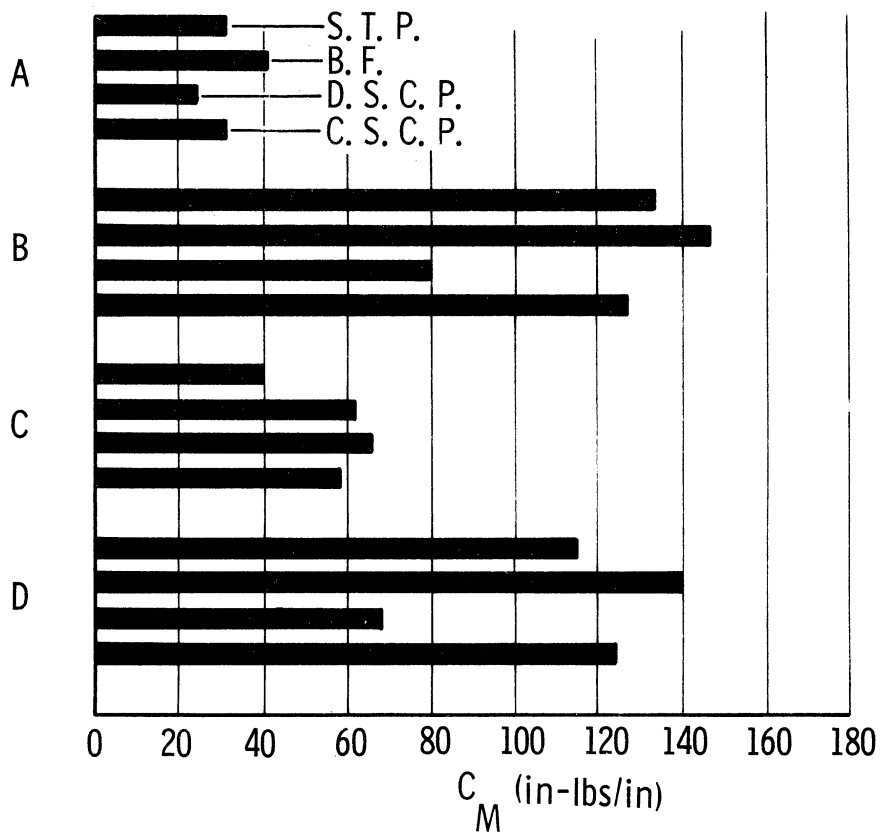
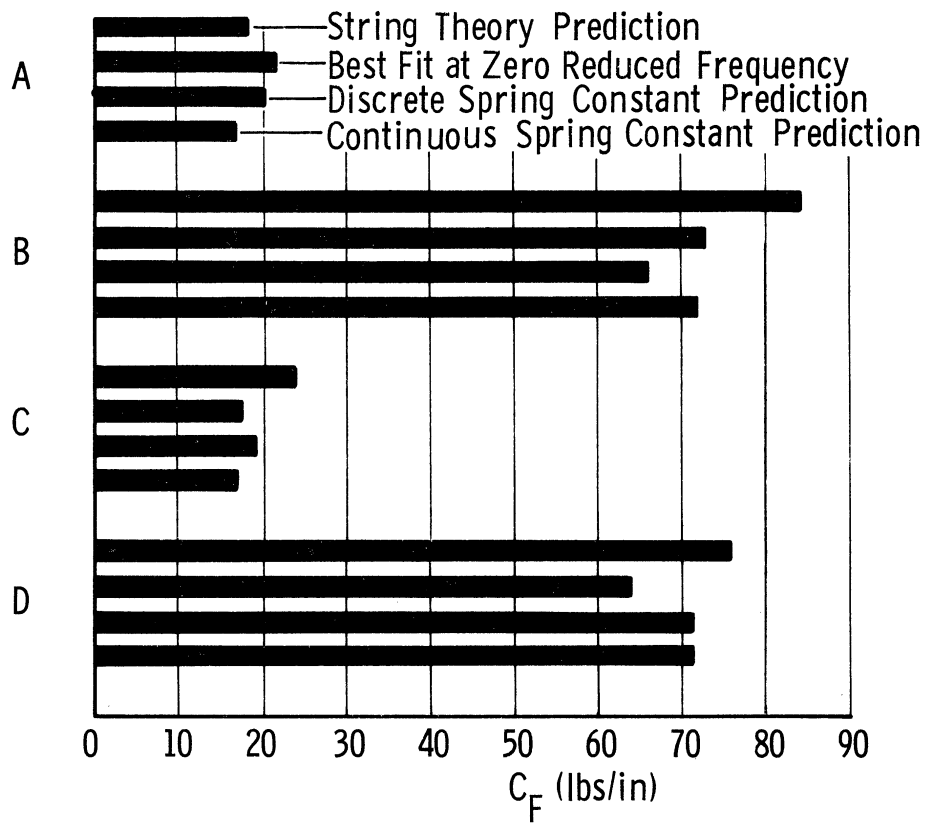


Figure 12. Dynamic force and moment coefficients using static h and slow rolling λ for four model tires.

C_M until the theoretical predictions at zero reduced frequency agree with the experimental data at zero reduced frequency in Figures 1 through 4. Finally, the discrete and continuous spring constant values are derived from Eqs. (12) and (15), using the two sets of slow-rolling experimental values of $K_{F\psi}$ and K_{Mz} .

The values of C_F are in relatively good agreement for all methods. On the other hand, the moment coefficient C_M is much more dependent upon the experimental method used to determine self-aligning torque. Subsequent comparison indicates, as noted previously, that the better method for determining C_M is to use data generated by slowly varying steer angle operation of the tire rather than steady-state steer conditions. This again points up the extreme sensitivity of the self-aligning torque property of a tire; it is apparently quite dependent upon local slip in the contact patch and on the nature of the friction surface. Basically, it is a small quantity whose origin lies in the difference between two forces, and hence might be expected to show sensitivity to such factors.

This two-constant modification of string theory can be used to predict more closely the dynamic response of tires by variation of the relaxation length property, λ , and the contact patch length, h , in order to match existing data. For example, a best fit for contact patch length, h , can be obtained by matching the phase jump in the force-amplitude response. Relaxation length, λ , may be adjusted to match the minimum point of the self-aligning torque data. Thus, a choice of values of λ and h arbitrarily allows the major features of the force and moment response to be well matched. Finally, C_F and C_M are

chosen to match the predicted force and moment amplitudes of the data at zero reduced frequency. Figure 13 shows a comparison of the static and slow-rolling values of h and λ with the best-fit values of h and λ gotten by using the procedure. These best-fit values of h and λ can then be used, in conjunction with slow-rolling steer and moment data, to obtain best-fit values for C_F and C_M . Figure 14 shows the comparison of C_F and C_M values for the four tires used in this set of experiments, using the best-fit values of h and λ . These values of C_F and C_M are determined using the same basic procedures used in determining the values in Figure 12, except, instead of using the static h and slow-rolling λ we use the values of h and λ gotten by matching the basic features of the data to string theory. The most notable feature of this comparison is that string-theory predictions using the best-fit values of h and λ give values of C_F and C_M which are much too high, while use of the spring rate concept involving coefficients $K_{F\psi}$ and K_{Mz} gives values of C_F and C_M which are much closer to the direct best-fit values obtained for these coefficients.

Figures 15 through 18 show a comparison of the two-constant modified string theory using the best-fit values of λ and h with corresponding values of C_F and C_M , to the experimental data for the four model tires. Although use of the two-constant modified string theory suggests that optimum values of h and λ are somewhat different from those obtained statically, it is also possible that better agreement between theory and experiment may come from complex tire force theories. In other words, the types of constants actually used here are determined entirely by the theoretical framework from which they are derived. Nevertheless, the best-fit values of h and λ found here suggest that

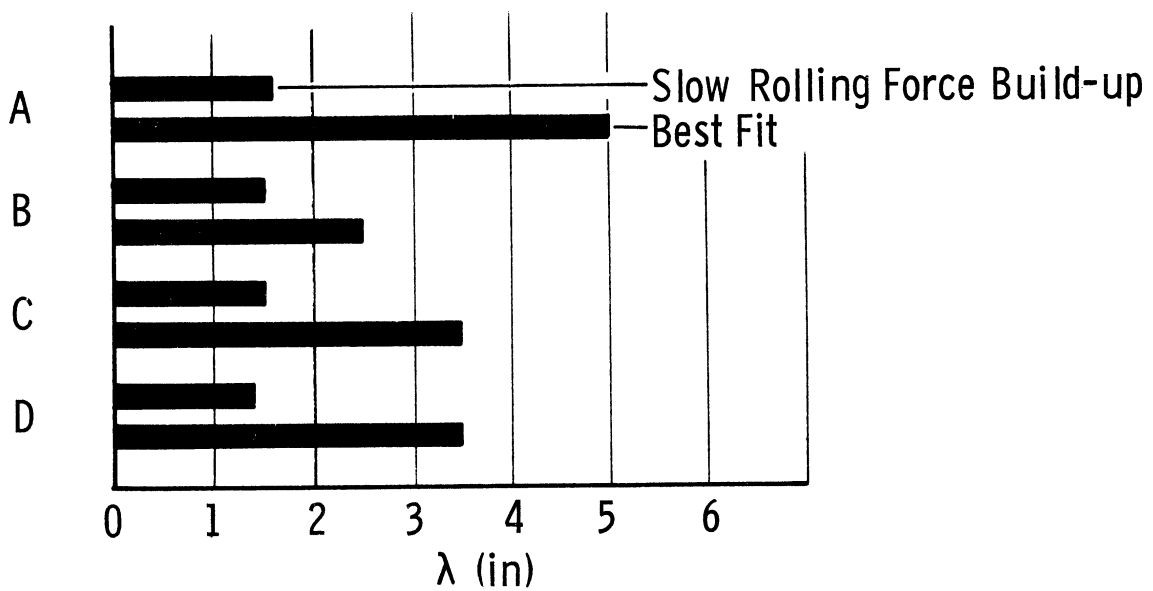
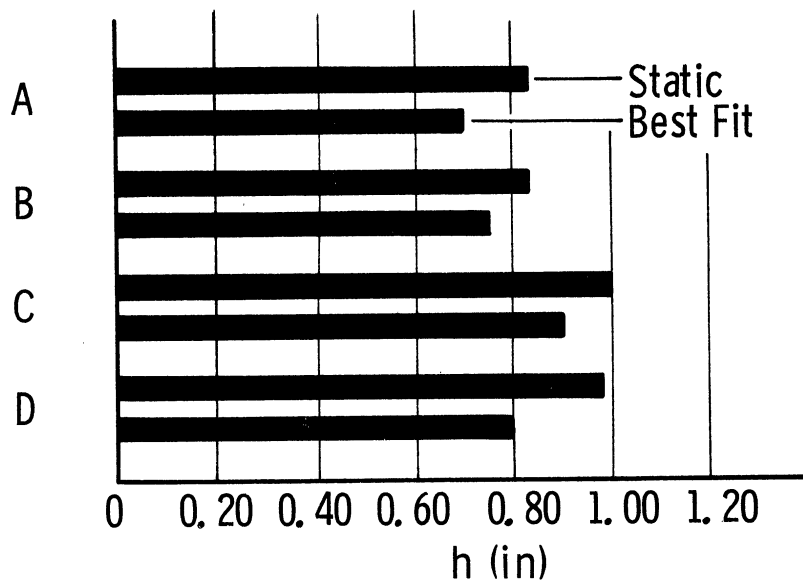


Figure 13. Static and slow rolling h and λ versus best fit values of h and λ for dynamic steer data for four tires.

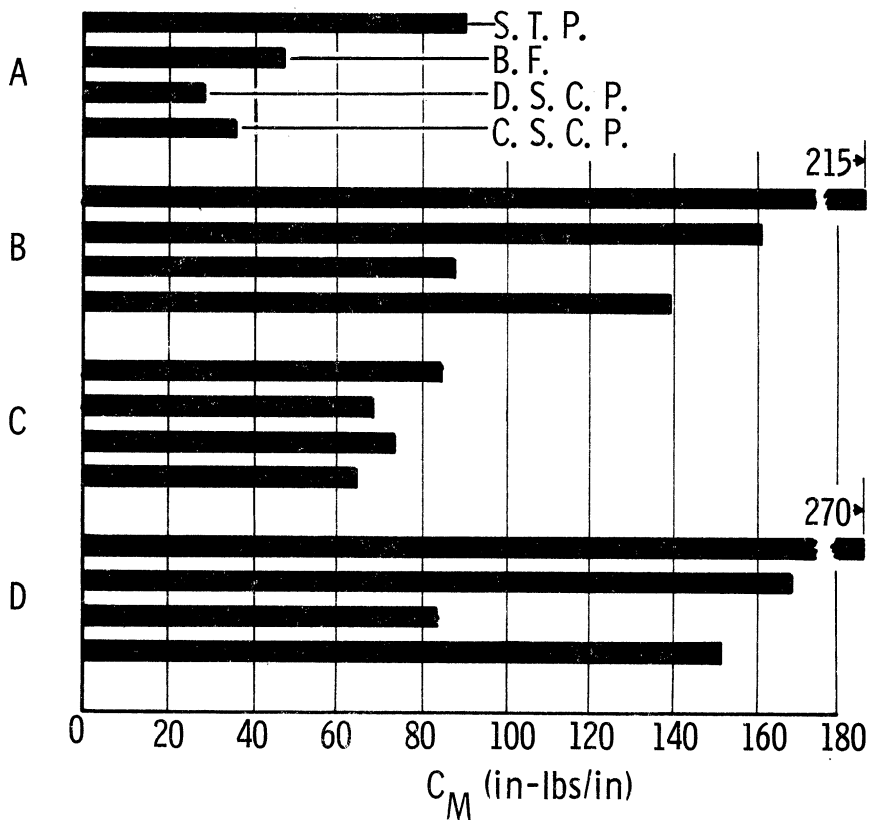
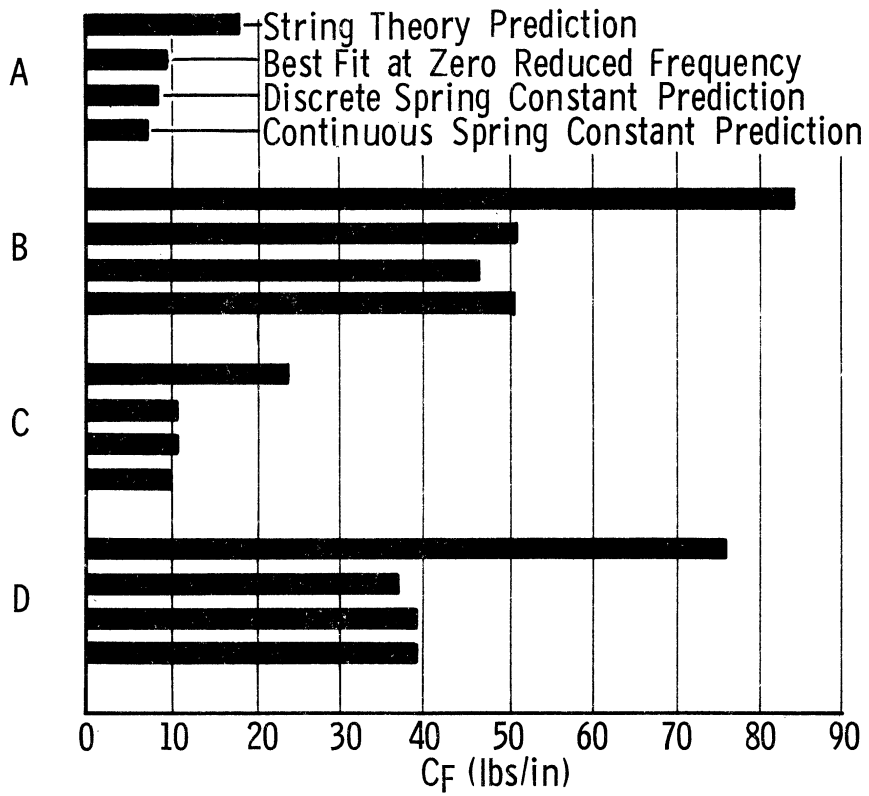


Figure 14. Dynamic force and moment coefficients using best fit h and λ , for four model tires.

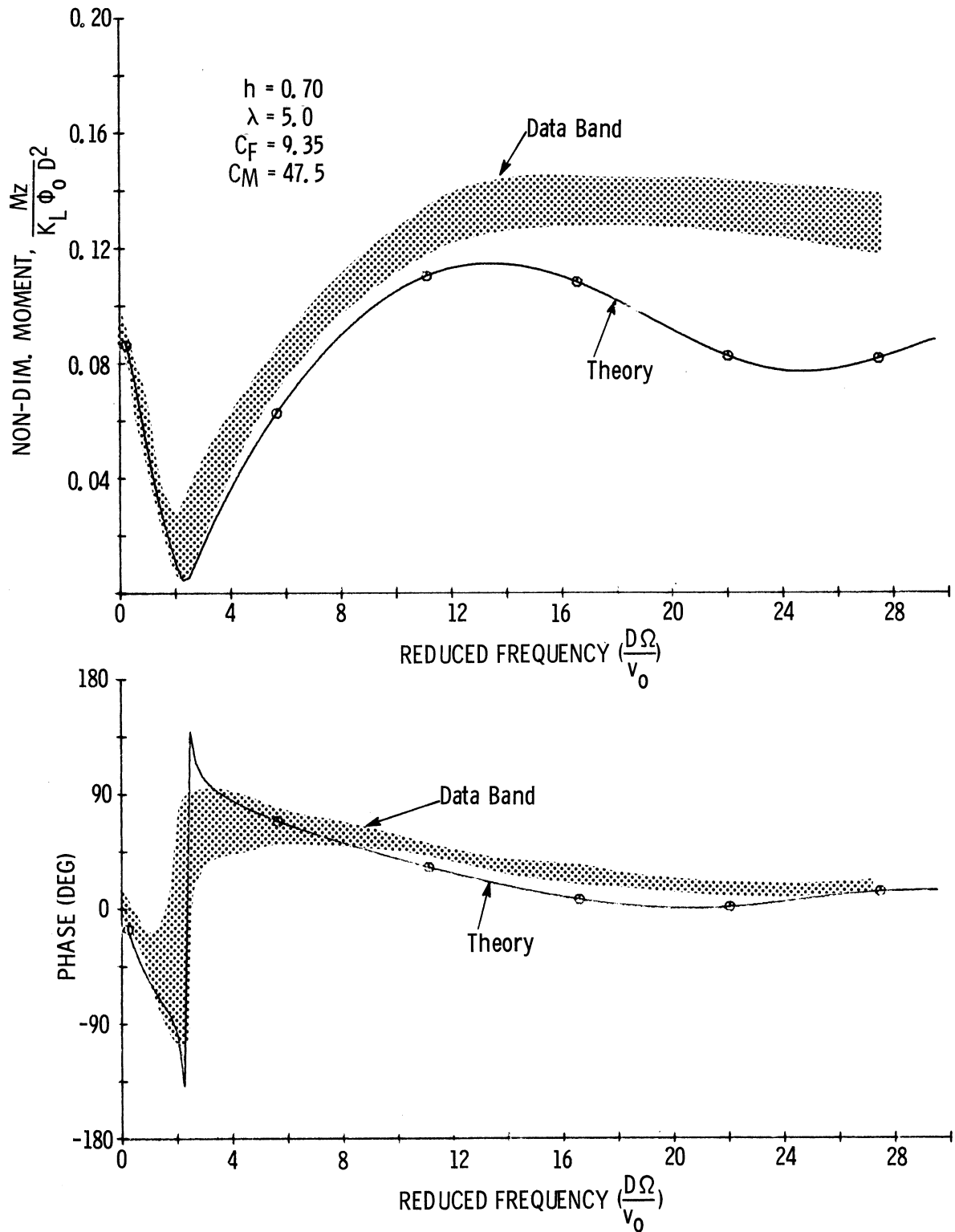


Figure 15. Comparison of force and moment response data to two-constant modified string theory using best fit values of λ and h for tire A.

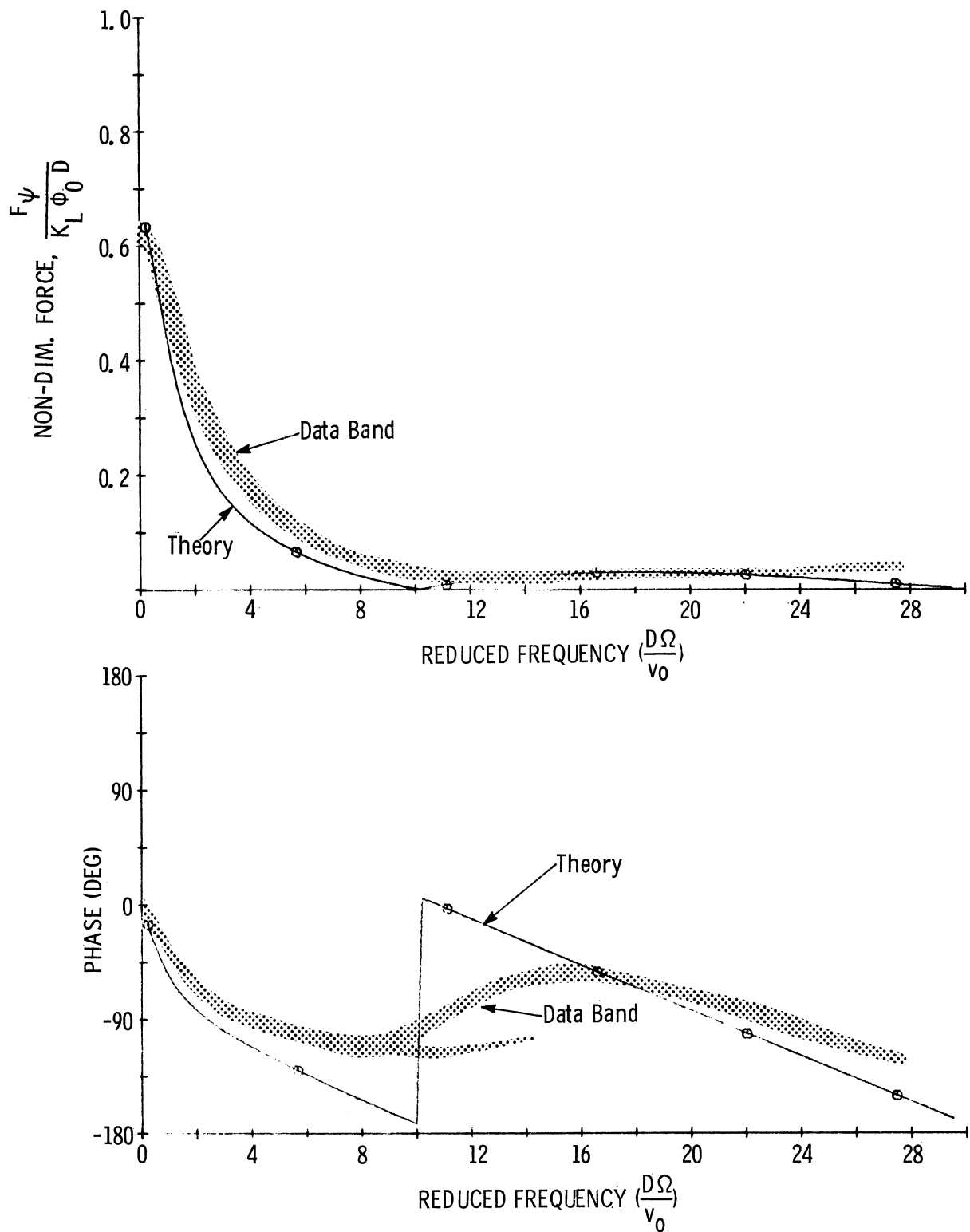


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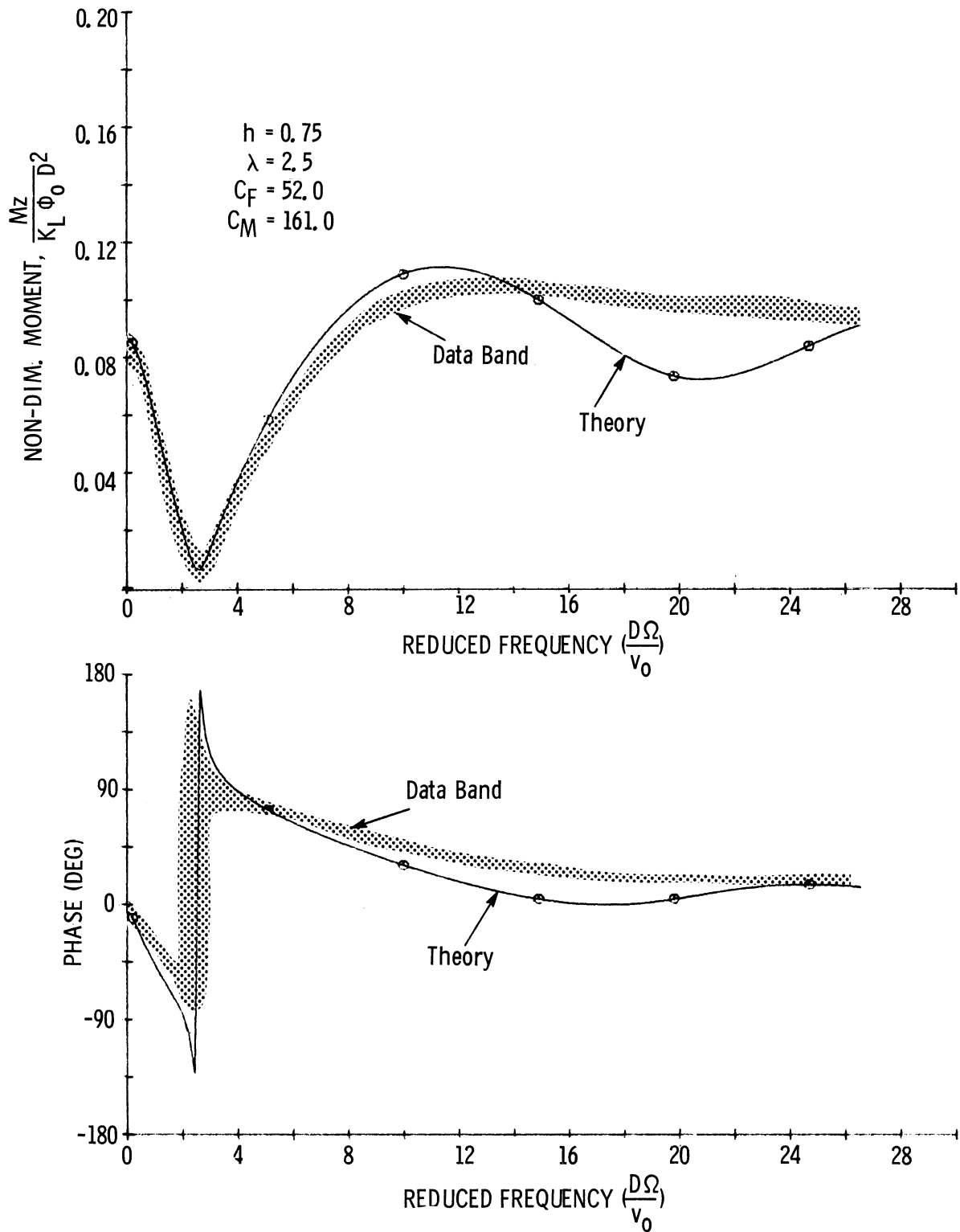


Figure 16. Comparison of force and moment response data to two-constant modified string theory using best fit values of λ and h for tire B.

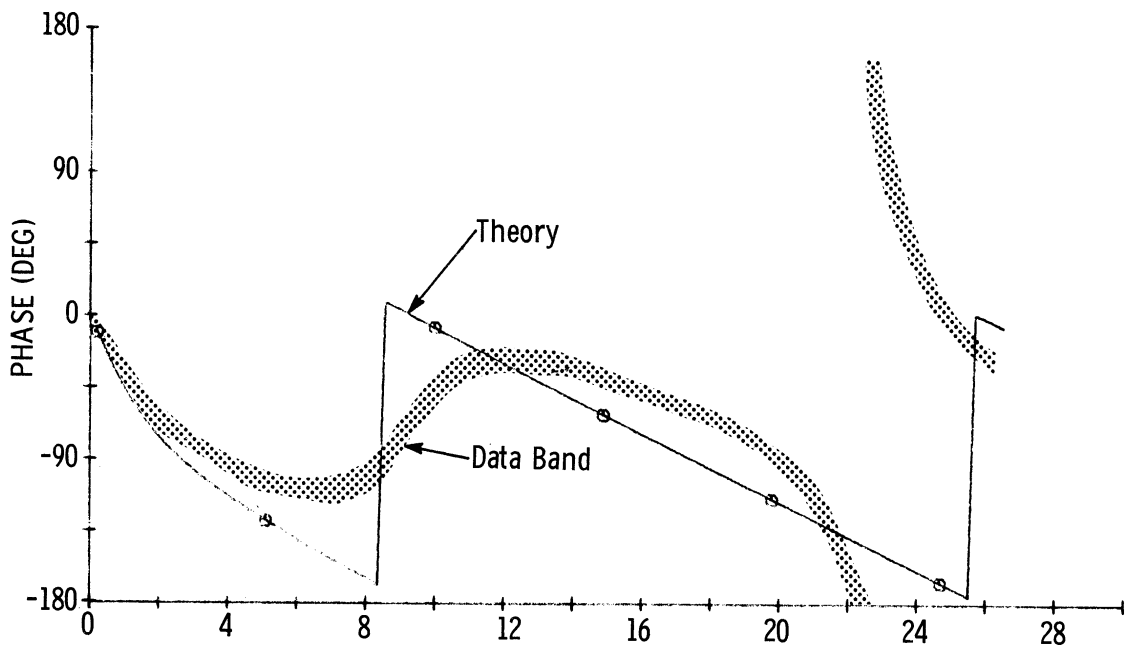
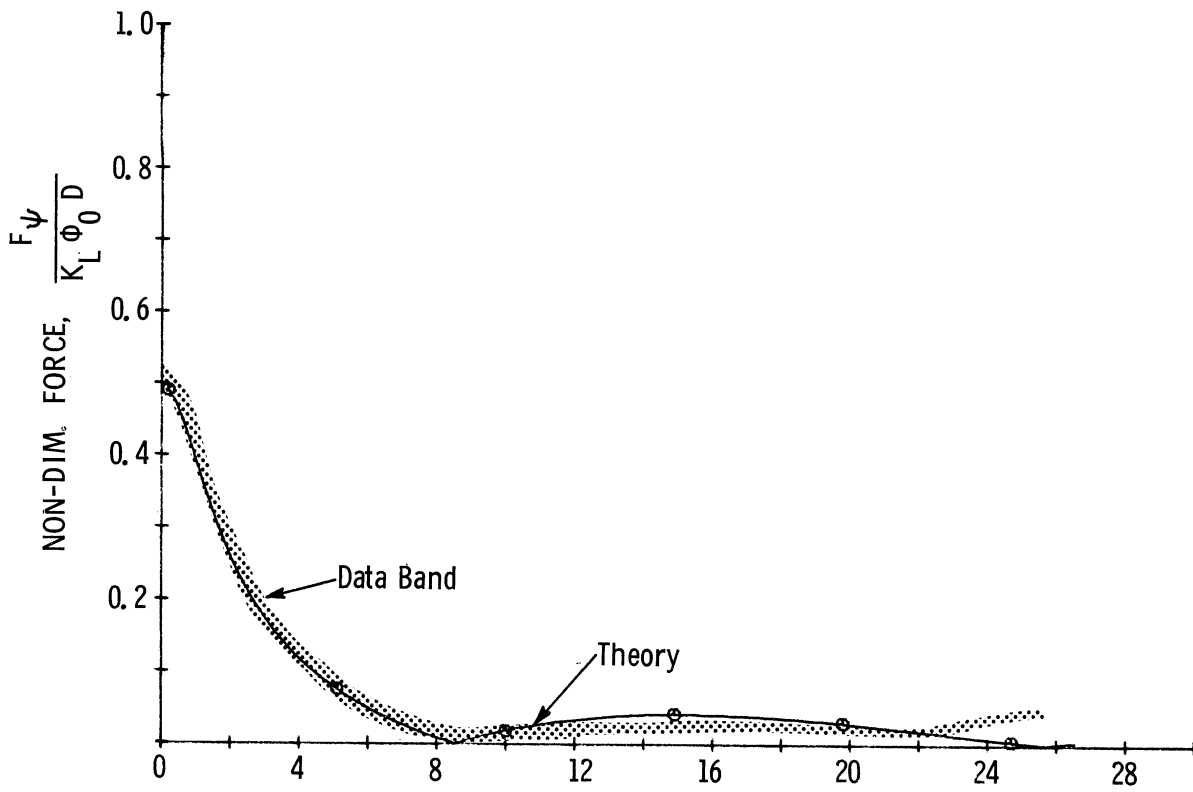


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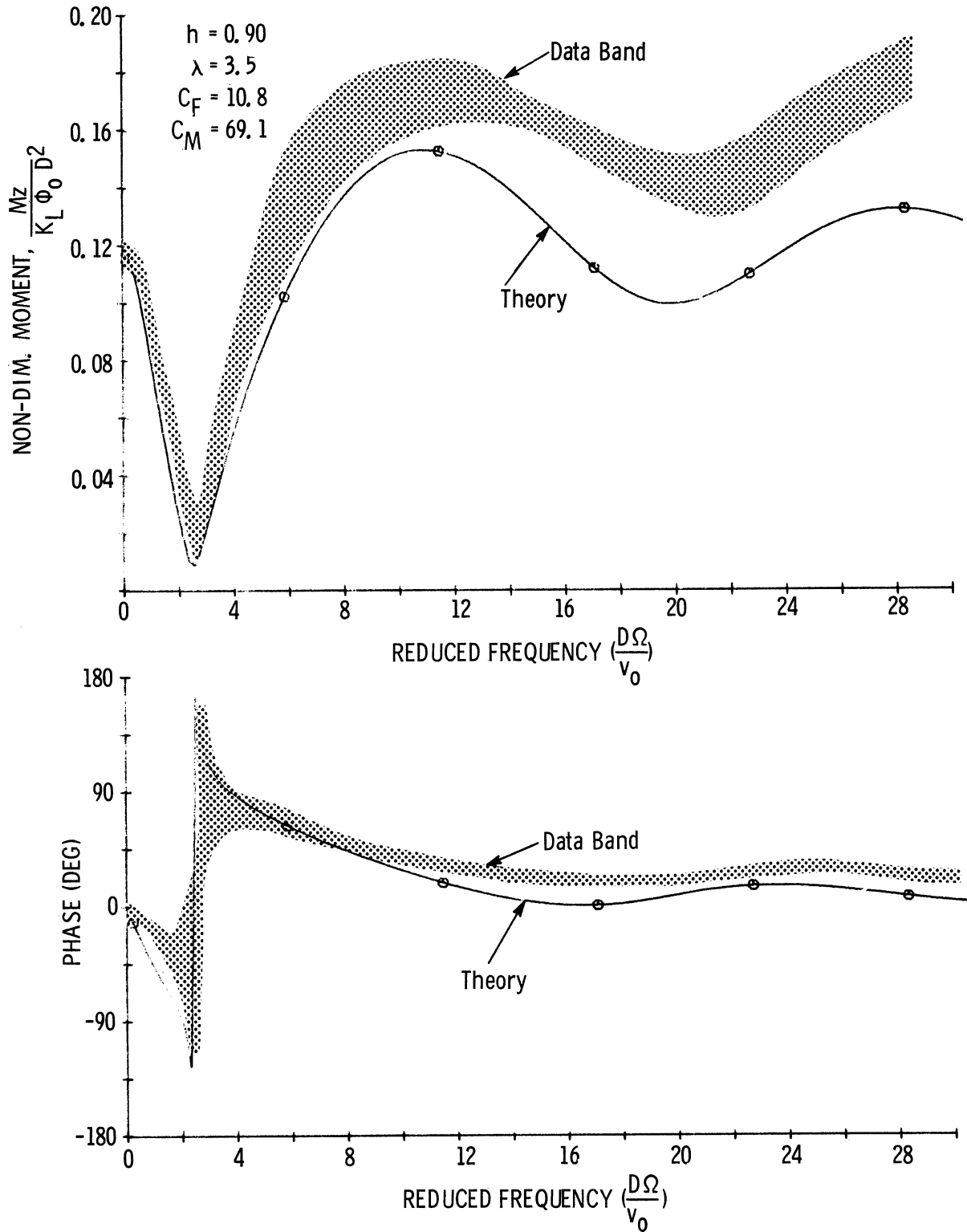


Figure 17. Comparison of force and moment response data to two-constant modified string theory using best fit values of λ and h for tire C.

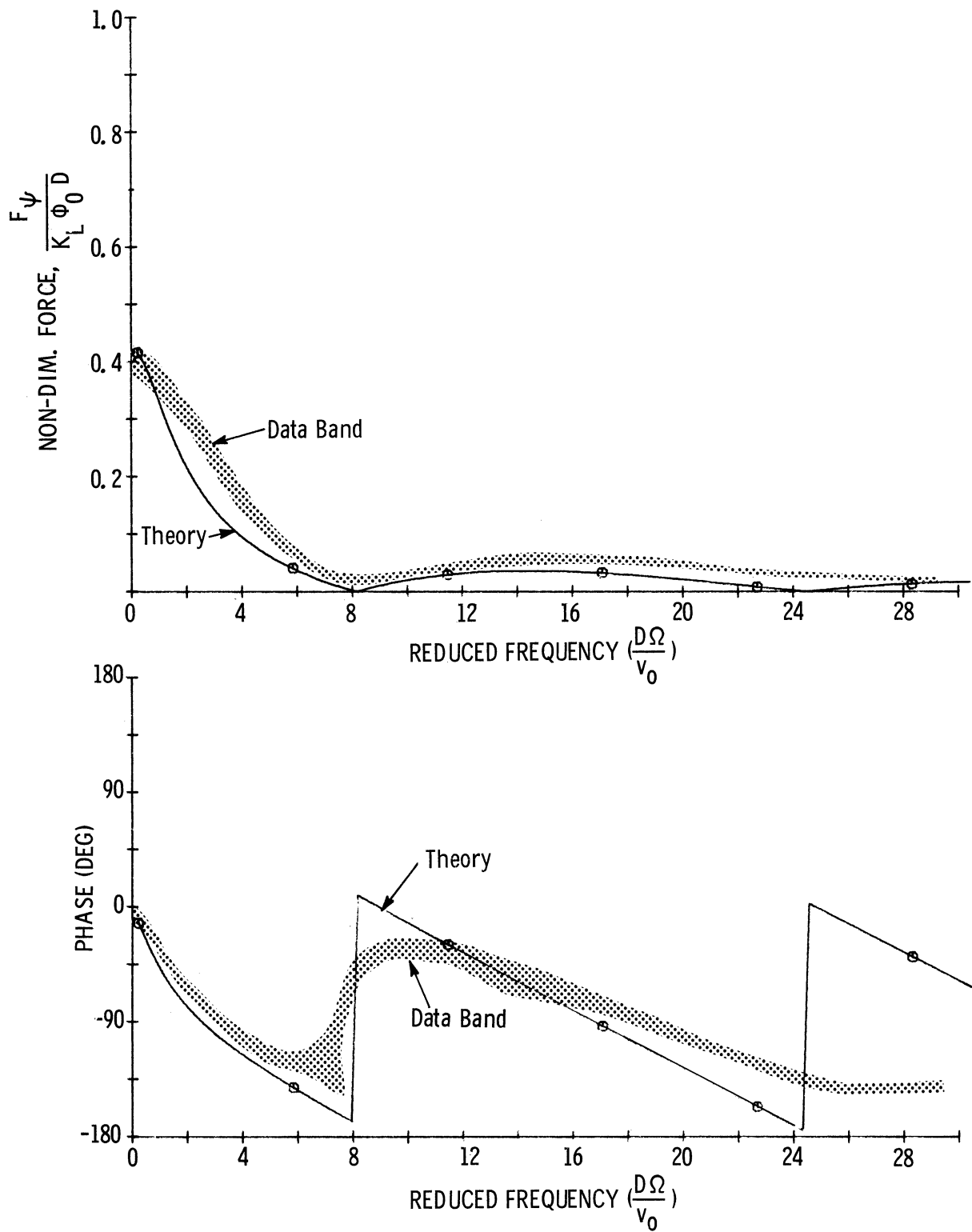


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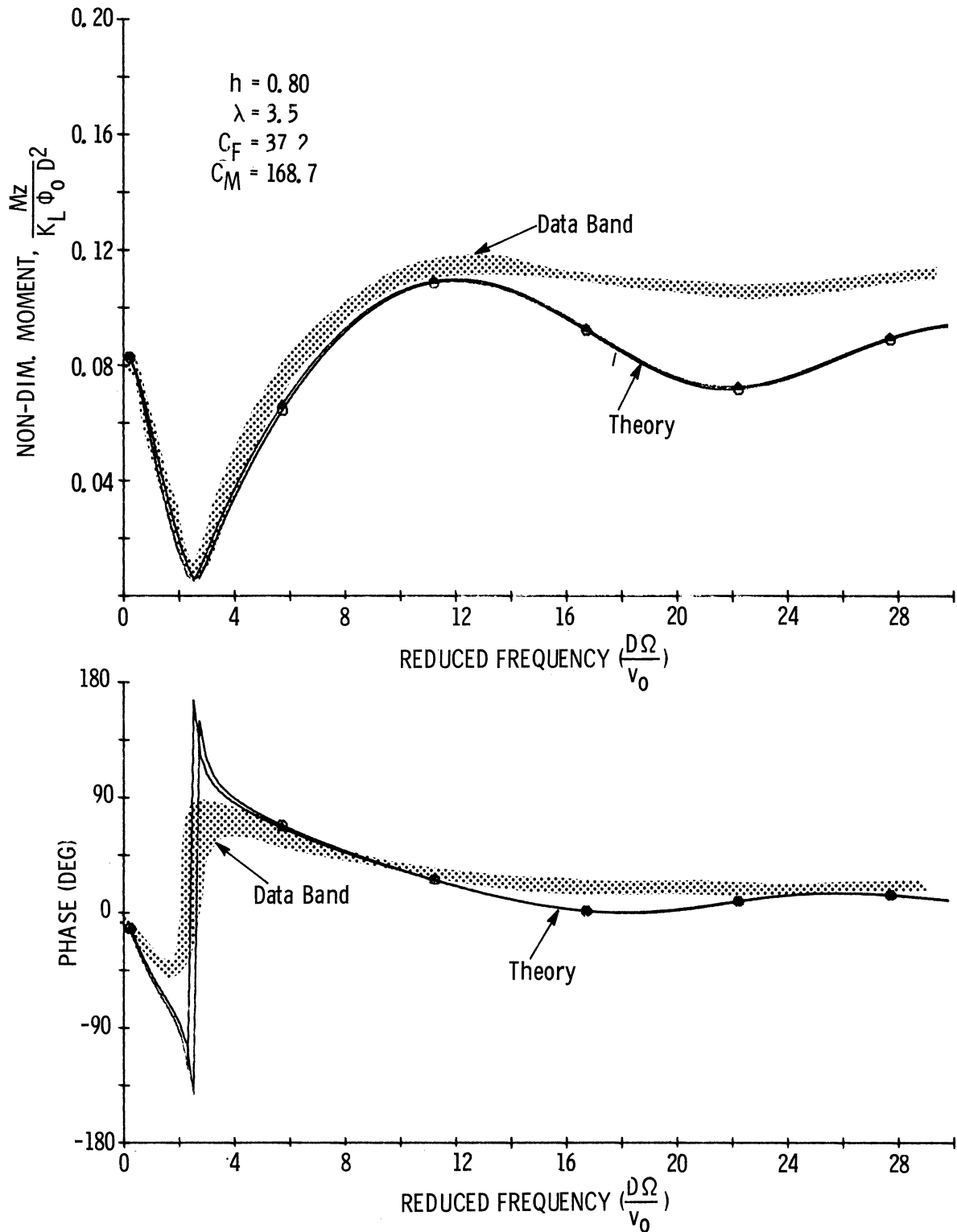


Figure 18. Comparison of force and moment response data to two-constant modified string theory using best fit values of λ and h for tire D.

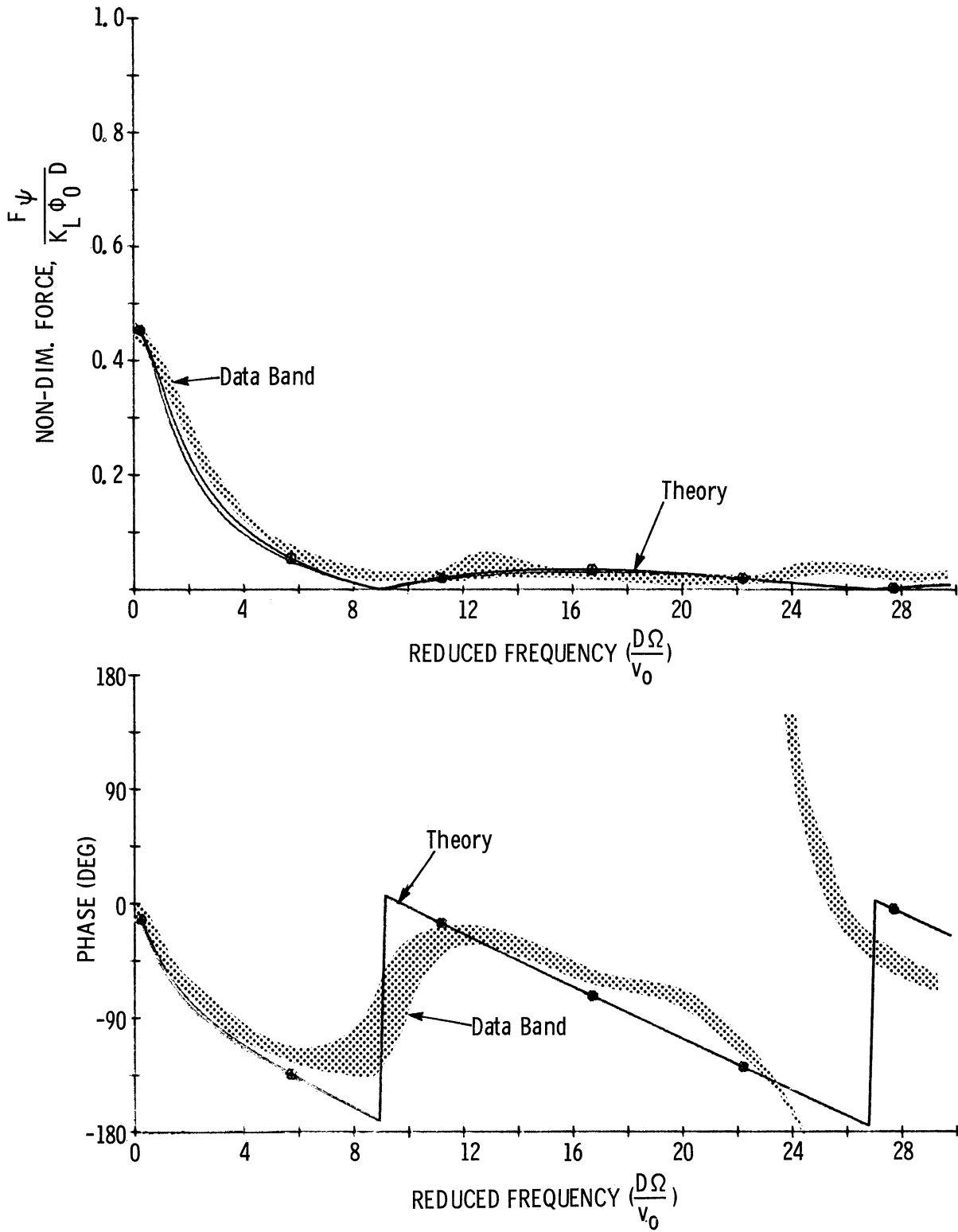


Figure 18. (Concluded)

under dynamic conditions a tire may have a somewhat different relaxation length than under static conditions, and may have a slightly different contact patch length as well. Direct measurement of the quantities λ and h under dynamic conditions should resolve this question.

In general, the two-constant theory seems to give the possibility of accurate prediction of dynamic tire characteristics over a wide range of tire constructions, a range so wide as to probably include most current or projected aircraft tire designs.

V. REFERENCES

1. von Schlippe and Dietrich, "Papers on Shimmy and Rolling Behavior of Landing Gears, presented at Stuttgart Conference, October 16-17, 1941," NACA Technical Note 1365, Washington, D.C., 1954.
2. S. K. Clark, R. N. Dodge, and G. H. Nybakken, "An Evaluation of String Theory for the Prediction of Dynamic Tire Properties Using Scale Model Aircraft Tires," The University of Michigan, Office of Research Administration, Report 056080-18-T, August 1971, Ann Arbor, Michigan.
3. H. B. Pacejka, "The Wheel Shimmy Phenomenon," Ph.D. Thesis, University of Delft, 1966, The Netherlands.
4. S. K. Clark, R. N. Dodge, J. I. Lackey, and G. H. Nybakken, "The Structural Modeling of Aircraft Tires," AIAA Paper No. 71-346, AIAA, N.Y. 1971.

VI. ACKNOWLEDGMENTS

The authors would like to thank Mr. John Gesink for aiding in the data acquisition, and the Bioelectrical Sciences Laboratory at The University of Michigan for lending the computer program used in the data reduction. Thanks should also go to the B. F. Goodrich Research Laboratories, Brecksville, Ohio, which furnished the materials used for the model tires.

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