# TECHNICAL COMMENT

## Backcasting

Dear Editors:

I read with interest the McLeods' column in the November issue of *Simulation* (pp.vii-ix). The discussion dealt with "backcasting"; that is, judging the validity of a model by simulating in reverse time.

In order to illustrate my own views on backcasting and to underscore the statements of others, I have prepared a simple example. In order to focus attention on the mathematics of the situation, the system to be simulated is an electrical network for which the validity of the model is incontrovertible.

Consider a resistance $R$ and a capacitor $C$ placed in series with a constant voltage source $E_o$. The charge $q$ on the capacitor is given by the equation

$$\dot{q} = -\frac{1}{CR}q + \frac{E_o}{R} \qquad (1)$$

It is easily shown that

$$q(t) = CE_o[1 - \exp(-\frac{t}{RC})] + q(0)\exp(-\frac{t}{RC}) \qquad (2)$$

Now consider backcasting with the model of this network (i.e., with Equation 1). Suppose we start with the charge at time $t = 0$ and investigate what happens when we proceed in reverse time. Denote the reverse time variable

$$\tau = -t \qquad (3)$$

It follows that

$$q(\tau) = CE_o[1 - \exp(\frac{\tau}{RC})] + q(0)\exp(\frac{\tau}{RC}) \qquad (4)$$

Obviously, the charge becomes unbounded as reverse time $\tau$ is allowed to grow indefinitely. The existence of an infinite charge is not easy to accept. Should we therefore conclude that Ohm's law and Kirchoff's laws should be rejected as a basis for simulation of electrical networks? I think not.

The explanation is that in order to achieve the charge $q(0)$ at $t = 0$ it is necessary to have a very large charge in past time. This charge is absorbed in the ideal voltage source.

No real circuit had infinite charge in past time because the initial condition $q(0)$ was achieved in another way. A switch was closed at some previous time; that is, *a structural change occurred in the system in past time.* Anyone who would backcast with my model and draw conclusions about its validity by comparing results with intuitive notions *is required to add this structural change to the model.* An example of an important structural change that must be accounted for by those who would backcast with socioeconomic models is the *disappearance* of medical technology.

It is not true that all physical laws remain valid when proceeding in reverse time. (If this statement were true, what meaning could be attached to the direction of time?) The second law of thermodynamics is violated in reverse time. The implication for the above example is that the resistor will convert low-grade thermal energy from the environment to useful electrical energy (stored on the capacitor). The diminishing of entropy in reverse time also has implications for the metabolism of the socioeconomic world in reverse time.

Alas, it seems that the conclusion to be drawn is that to predict the past, one simply cannot reverse time in the simulation used to predict the future. Backcasting *is* a valuable heuristic procedure but it is a nontrivial task.

Finally, I will demonstrate the numerical sensitivity problems of backcasting. Suppose there is some uncertainty in the value of the initial condition $q_o = q(0)$. A measure of the effect on the simulation is

$$s = \frac{\partial q}{\partial q_o} \qquad (5)$$

It follows from Equations 2 and 4 that

$$s(t) = \exp(-\frac{t}{RC}) \qquad (6)$$

and

$$s(\tau) = \exp(\frac{\tau}{RC}) \qquad (7)$$

Clearly, the numerical difficulties in backcasting are greater than for forward simulation.

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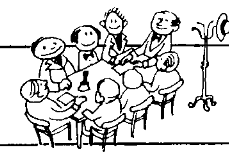
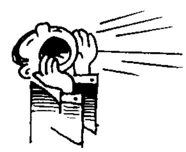


# MEETINGS AND CONFERERENCES



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