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APPLICATIONS OF NUCLEAR ENERGY TO TRANSPORTATION

Progress Report No. 5

INITIAL APPROACHES TO SYSTEMS ANALYSES OF NUCLEAR HEAT-POWER ENGINES

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1.0 ABSTRACT

Progress which is being achieved in analyzing nuclear-heat-power engine systems is presented.

Preliminary examinations and conclusions are reached for the various nuclear-heat-power reactors now under development, design and construction. Uses for nuclear-steam generators and steam power machinery for transportation purposes appear to be limited to application where size and weight are not critical and where shaft horsepower outputs are in excess of 10,000. Primary emphasis is directed to nuclear reactor concepts which can generate heat at high temperatures. Such high-temperature heat sources are analyzed in a preliminary way as a part of a heat engine cycle. Preliminary systems' analyses for a simple four-component cycle is presented. This approach suggests continuous correlation of nuclear parameters with heat transfer, fluid flow, thermal stress, and mechanical development. The progress which is being made indicates that certain guides with respect to component efficiencies will be helpful to outline needed areas of investigations. Planning suggests that as more detailed multi-group nuclear calculations are made, efforts be conducted to reduce such results to parameters which can be correlated with power plant engineering.

2.0 INTRODUCTION

Contract relationships between the Chrysler Corporation and the University of Michigan involve studies for determining applications of nuclear energy for transportation purposes. As partial fulfillment of the scopes of work outlined in task 2.5, title of the proposal "Applications of Nuclear Energy to Transportation", dated July 13, 1955, a portion of the work is in progress to evaluate the nuclear engine cycles and as a component of such heat engine cycle. To date this work has considered a nuclear power reactor as a heat source in an integrated heat engine cycle. A discussion is presented on the cycle and the nuclear reactor separately. This report endeavors to present the first preliminaries of a system analysis. To date it has been possible to evaluate theoretically the complete systems of most of the simple heat engine cycles and the components therein; namely, the four-component system comprised of:

2.1 A nuclear reactor as a heat source.

2.2 A turbine for the production of useful work.

2.3 A heat exchanger to serve as a heat sink.

2.4 A compressor to recover the pressure level necessary at turbine inlet.

It is believed that the system analyses conducted to date and presented herein, can serve as valuable guide-posts in evaluating specific nuclear heat generating devices, and the given power plant equipment associated therewith, when studying mobile propulsion over wide ranges of shaft horsepowers. It should be noted that the major portion of the work presented herein, involves systems analyses of working fluids which do not undergo phase changes, although a discussion is included of a phase change system mainly to serve as a case of comparison. The primary reason in delineating the work on phase change cycles is that the development currently being carried on by the United States Atomic Energy Commission in the field of nuclear technology has had as its general objective the achievement of large stationary power plants in which the nuclear reactor being substituted for fossil fuel fired steam boiler will undoubtedly result in the development of nuclear steam power plants.

Although mobile propulsion units permits consideration of steam cycles as a means of producing useful work, it is unlikely, that economical installations of nuclear steam generating heat engine devices can be achieved for applications where power outputs are under 10,000 horsepower. The first steps in nuclear steam power plant development and engineering appear to be the achievement of operating plants for installations where size or weight of power plant is not an important consideration, other than economy; where conservation of nuclear fuel is not a major consideration; and where reprocessing of nuclear fuels is somewhat dependent upon isotopic compositions of fissionable materials produced in power generation.

In the general field of nuclear development, relatively minor efforts are being conducted to achieve nuclear heat engines which can operate continuously at high temperatures, at optimum efficiencies, and where size and weight are of primary importance. If the nuclear energy sources are to have application for transportation devices such as merchant ships, aircrafts, land vehicles such as locomotives, it appears that research and development must be correlated continuously with programs of system analyses. Such a program must have as an ultimate aim the development of the entire propulsion equipment with the nuclear heat source as an integral part.

It is the object of this report to serve as the initial step in placing into proper perspective development programs concerned with the development of an integrated nuclear power plant which will permit a nuclear heat engine operation at optimum efficiencies over the necessary ranges of horsepower output required by transportation vehicles.

3.0 GENERAL OBJECTIVES AND ENGINEERING PHILOSOPHIES

Efforts are made to establish concepts for nuclear-heat-power engines, which have application as propulsion units for mobile transportation. Preliminary analyses of reactor development programs places nuclear-heat-power sources in several categories:

3.1 Nuclear Heat Sources Generating a High Pressure Saturated Steam

Nuclear heat power units of relatively proven design can be built and operated now to generate high pressure saturated steam. These systems have utilized high pressure water as a primary coolant to transfer heat to a steam generating heat exchanger boiler. Such a system has resulted in the production of steam at pressures between 500 and 600 psig as the optimum for the heat engine cycles. Such reactors produce steam at temperatures which are maximum and limited by saturated conditions. It does not appear likely that plants employing such systems in which water is basically the coolant and steam the medium can produce operating temperatures beyond 600°F., and pressures beyond 600 psig. The basic deterrent which prevents the development of higher operating temperatures for nuclear steam generators lies within the reactor design. It may be considered that nuclear steam types of systems have reached their ultimate in development. Further developments of power plants utilizing steam as the working fluid will require new concepts for the primary system involving the reactor. Utilizing a new concept in the primary system may result in higher operating temperatures and greater efficiencies and the possible application of such systems for certain types of propulsion units for transportation. Tentative conclusions are reached that this type of nuclear heat engine may find utilization as a propulsion unit where shaft horsepower requirement in excess of 10,000 are needed, and where size and weight of power generating equipment is not a primary factor for consideration. Nuclear reactors utilizing a primary pressurized aqueous system which appear to have promise as steam generating and propulsion devices in this general range of conditions are:

3.1.1 Pressurized water circulating reactor.

3.1.2 Boiling water reactors.

3.1.3 Aqueous homogeneous power reactors.

Since the major efforts of the Atomic Energy Commission, as well as many private industries, are endeavoring to undertake development of nuclear steam generators and steam propulsion units, the efforts in this project are limited to remaining abreast of those developments being undertaken aggressively by others.

3.2 High-Temperature Nuclear Heat Generation

A report entitled, "Engineering Applications of Nuclear Energy", dated April, 1955, by H. A. Ohlgren and Marx Weech have outlined specifically four types of high-temperature nuclear heat generating devices worthy of investigation. In this proposal there are outlined rather specifically the statements of problems in research and development which must be investigated before types of high-temperature nuclear-heat generators can be integrated with a given power cycle. In addition to a specific research and development program which will provide data for nuclear information, heat transfer and loop studies, working fluids for heat engine, basic materials of construction, shielding data, etc., programs of engineering studies, evaluations, and analyses are underway. These programs can be outlined into the following area:

3.2.1 Nuclear Reactor Calculations

In order to meet the program outlined in the proposal, a definite approach to nuclear reactor calculations has been taken. This approach consists mainly in the determination of constants required in the application of reactor theory for calculations. Upon the completion of the constants, a calculation utilizing two-group theory will be made. Results of such calculations will be plotted for various critical masses and sizes. The resulting curves will serve as a basis for further investigation of a particular type of reactor for a given application. Once the range of parameters has been established for the concepts to be evaluated, resort to one-group theory for investigating stability, reactor poisoning, controls, etc. will be made. Because of the type of program outlined, the initial effort being made is one which results in a large amount of work before any concrete results are indicated. Concurrently with the program, an effort is underway to utilize digital computers for the evaluation of a given reactor for a final design. Programing for a digital computer utilizing multi-group methods will also make use of the constants that will be developed. When methods and procedures for conducting calculations of this type are established, the following types of high-temperature reactor operation will be investigated.

3.2.1.1 Bismuth uranium, a homogeneous liquid metal reactor.

3.2.1.2 Fused salt, a homogeneous type of reactor.

3.2.1.3 Fluidized bed UO_2 high-temperature nuclear reactor.

3.2.1.4 Basic nuclear concepts for equilibrium isotope build-up under fast neutron energy conditions with U-238 as feed material to the reactor.

3.2.1.5 Investigations of a potential for equilibrium conditions of the fission process with thermonuclear fusion.

Although results on the above-mentioned nuclear energy source will not be forthcoming immediately, future investigations can be made in a reasonable period of time.

The last two items mentioned in the above list will probably require a different approach, and an entirely separate effort. Item 3.2.1.4 is now under a preliminary investigation and should result in answers that may be integrated with a given type of reactor.

3.2.2 Investigations of High-Temperature Material for a Nuclear Reactor

Calculations for a system analysis of nuclear heat engines are being made at high temperatures which are presently not under consideration and cannot be currently considered due to the limitation of the materials of construction. It is felt that future developments in materials will be able to resolve high-temperature problems and consequently the development of the reactor which may operate under these high temperatures. Concurrent with the development of optimum temperatures for nuclear heat generation, there must be a careful evaluation of the maximum temperature limits for a given power plant so that the materials-requirement for the power plant can be coordinated with the development of materials for nuclear heat generation.

3.2.3 Heat Transfer and Loop Study

Primary efforts in engineering studies and evaluations to date are specifically limited to the evaluation of a heat transfer loop for the liquid metal homogeneous reactor, employing a solution of bismuth-uranium in a reflected core. Basically, two approaches are being considered.

An external double loop employing pumping equipment so that working fluids for a heat engine will not become exposed to neutron, thus permitting use of air, nitrogen, CO_2, etc., for more-or-less conventional heat engine application. External loops and interchange liquid metal systems, however, require more massive shielding and also considerable mechanical complexity. Therefore, considerations are being given to conceptual designs of cartridge type of reactors, in which working fluids are passed directly through the active core by suitable heat exchange mechanisms.

In both of these cases the heat transfer studies have been limited to convective heat transfer. Basic considerations are given to high-temperature nuclear reactor heat generation in which case possibilities for radiant thermal heat transfer are possible.

3.2.4 Power Plant Devices

Giving consideration to the conversion of heat energy to mechanical energy, the prime consideration is to coordinate the maximum temperature limits of a nuclear reactor to the development of a heat engine which has a maximum power output for the minimum size and weight. In general, the efforts are to evaluate such power plants with shaft horsepower ranges from 100 to 25,000. To date, the major investigations have been for heat engine devices which employ gaseous working fluids, rather than systems which undergo phase change.

From a general consideration of possible heat engine cycle, it is observed that the choice of working fluid can be made either from those materials which exhibit a phase change during the cycle or from those which remain gaseous throughout, and possibilities exist for combinations of one or more fluids from either, or both, groups. If consideration is made of present practice with chemical engine for heat generation, it appears that gaseous fluids are more prominent in the small to medium range of horsepower output. This is exemplified by an internal combustion reciprocating engine and gas turbine power plants. Choices of heat engine working fluids cannot be based simply upon thermodynamics and mechanical design considerations, but the arrangements of the nuclear reactor must also be included therein.

Efforts are being made presently to evaluate the working fluids and cycles from thermodynamic viewpoint. It becomes rather obvious that research and development are needed before full consideration to the range of potential working fluids can be made. From a broad viewpoint, it appears that the working fluids for nuclear engines covers the following materials: water, air, helium, argon, nitrogen, carbon dioxide, sodium, potassium, and others. Basic nuclear considerations indicate that only helium can be exposed to neutrons at high energies and high fluxes without becoming radioactive. However, the possibility of impurities becoming contaminated in the helium does exist. The heat transfer properties of helium at high-temperature and high pressure are rather favorable for closed-cycle gas turbine power plants. A great number of mechanical, as well as economical, problems are apparent before helium can be given serious consideration. It is proposed, therefore, to evaluate carefully all the working fluids in conjunction with the optimum cooling system for a nuclear reactor so that each of the components of a heat cycle can be optimized without affecting or changing the design criteria for subsequent components.

3.2.5 Mechanical Developments

Present evaluations indicate that open and closed-cycle gas turbines in conjunction with high-temperature nuclear heat generating devices offer attractive potentialities as nuclear heat engines in transportation devices. In the case where shaft horsepower requirements are relatively high, and where it is desirable to consider working fluids that have become radioactive, the closed-cycle gas turbine system for generating mechanical power appears to have greater advantage. In the case of a nuclear reactor in which the working fluid for gas turbines does not become radioactive, and where the shaft horsepower requirements are less than about 8,000, it appears that open-cycle systems may offer advantages. With these as general objectives and engineering philosophies, the progress which is presented in this report endeavors to present an integrated system analyses of preliminary nature of the variables and parameters influencing the major components of a nuclear engine cycle, as well as detailed studies of specific cycle arrangements.

4.0 ENGINEERING ASPECTS OF NUCLEAR ENERGY POWER PRODUCTION

4.1 General Discussion

Extraction of useful energy in the form of heat from the fission reaction, which takes place in a nuclear reactor and its subsequent conversion to useful work, has been under constant development by national laboratories and recently by industries. More recent emphasis on the application of nuclear energy to mobile uses, particularly the transportation field, has led to the introduction of new problems imposed by the specific application. These problems are mainly ones set by the conditions of mobile operations which have to be satisfied. These are mainly rapid acceleration, deceleration and idling. The conditions become severe when land transportation vehicles are being considered. Primarily, the emphasis is placed on the reduction of power plant size through the optimization of such engineering parameters which will ultimately lead to such a reduction. The best approach is one which follows a coordinated and integrated analysis program of power plant cycles and nuclear reactors. This is done with the realization that operating parameters which affect the cycle also affect the reactor. In essence, the reactor may be considered the primary operating component with the power plant components acting as a feed-back loop of the reactor.

This approach allows plant cycle and nuclear reactor to be considered separately for the purpose of optimization of each. Having gone through such a preliminary analysis with definite conclusions on each, the cycle and reactor must then be considered as a single entity. Final analysis must consider all operating conditions which are imposed on the power plant as a whole. Once the power plant cycle and type of nuclear reactor are selected, the entire development program becomes an integrated effort.

This report is devoted to illustrating what has been accomplished in fulfilling the first phase, namely cycle and reactor optimization. It is felt that the approaches, methods, and procedures presented herein serve as a firm basis from which selection of a nuclear power plant and associated components for a particular application can be examined and developed. It should be understood that the simplified cases cited herein cannot be generalized to apply to the more complex physical cases encountered in a specific nuclear power analysis. This is in keeping with the approach outlined, which states that a second phase must be considered, namely the specific application.

4.2 Principles of Nuclear Power Plant Cycles

The evaluation of nuclear energy application to transportation devices prompts reviews of certain established heat engine criteria so that the proper perspective can be obtained on high-temperature operating conditions. Chemical-fueled heat engine development has resulted in improving efficiencies, with a reduction of power to weight ratio and an overall economic savings in fuels. A large percentage of the problems being currently considered in the development of chemical heat engines have application to nuclear-heat-power engines. Some of these problems will require more extensive investigations to determine the effects of operating within a radiation field. Induced activity can readily occur in working fluids. Radioactivity contributes additional problems such as damage effects on materials, requirements of health and safety shielding for auxiliary components, and added complications of maintenance.

In the numerous considerations which are being given to evaluation and relations of power plants in conjunction with various types of nuclear reactors, a portion of the studies are devoted to the development of thermodynamic expressions for power plant performance. Particular emphasis is given to the dependence of the power plant on the performance of a given nuclear reactor. So far, criteria used for such developments have been cycle efficiency and power output. Efforts are being directed to the correlation of these criteria with temperature distribution and system pressure losses.

In the sections to follow, the discussion will center around the thermodynamic properties of a particular cycle and the effect on those properties when a nuclear reactor is considered as the heat source. This approach is taken to point out the ramifications due to nuclear considerations rather than present a thermodynamic analysis which can be obtained through straight-forward applications of known principles. Essentially, three cycles will be discussed, primarily because of the considerations that have been given to them by other nuclear study teams. In addition, it is felt that with present knowledge and the reasonable extension of such knowledge, the greatest return will be realized by the further development of one particular cycle. Also, the material presented will indicate which should receive the greatest development emphasis.

4.2.1 Steam Cycle

Analysis of the steam cycle has been divided into the two regions represented on a Mollier Diagram, namely, the saturated and super-heat regions. Operation of the power plant within the saturated steam region can be accomplished ideally only through use of an infinite number of extraction feed water heaters with the consequence of approaching the ideal Carnot cycle. This form of operation is shown aptly in Figure 1. However, certain limitations are imposed on the reactor operating temperature, namely, that maximum

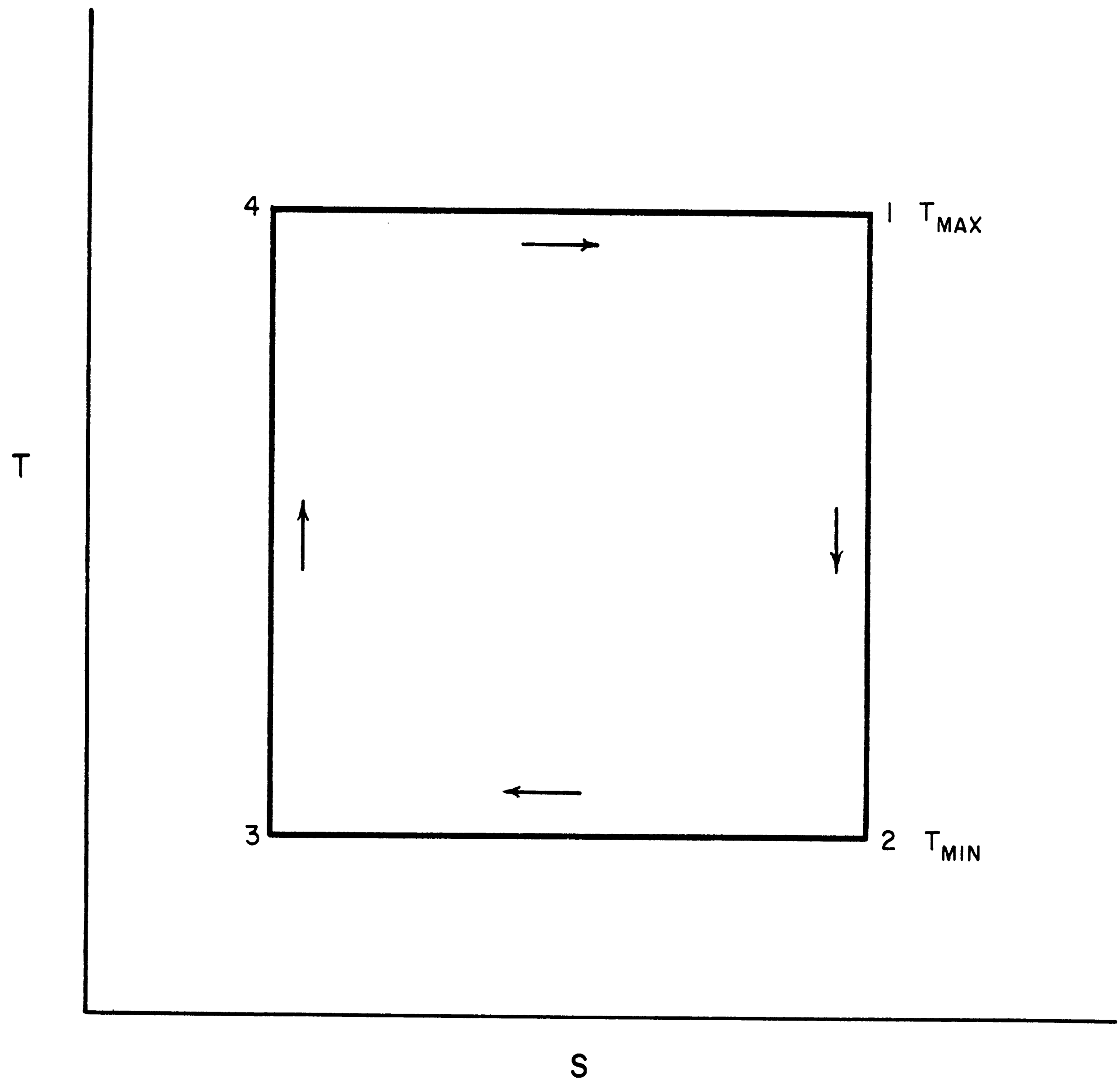

FIG. 1 CARNOT CYCLE

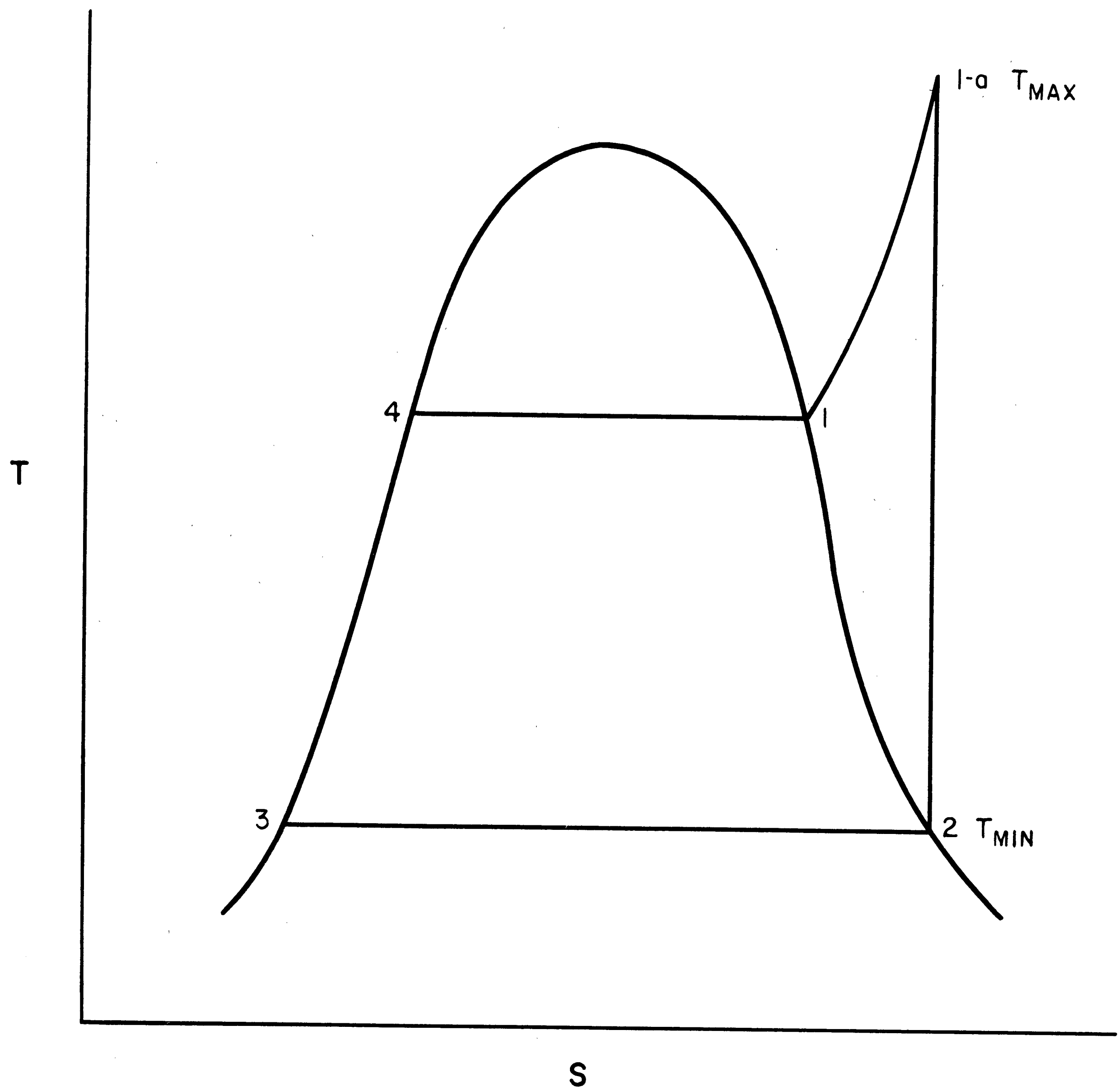

FIG. 2 SUPERHEAT STEAM CYCLE

temperatures are set by the critical temperature or the critical pressure of the working fluid. In the case of water, the critical temperature is 705°F, which immediately sets the maximum theoretical efficiency which can be obtained. Such considerations are independent of a nuclear reactor. When the effect of this temperature on a nuclear reactor is considered, a specific type of reactor must be considered. For example, a pressurized water reactor, utilizing a steam cycle operating within the saturated region, might have conditions as follows:

Inlet reactor temperature, °F	400
Outlet temperature, °F	500
Pressurized water, primary system, psia	1000
Condition at turbine throttle:	
Steam temperature, °F	370
Steam pressure, psia	175
Thermal efficiency, %	24

It is seen that a cycle efficiency of 24 percent is obtained with a steam temperature at 370°F, which is considerably lower than the 705°F critical. It becomes obvious that the critical temperature was not the determining factor in the attainment of maximum efficiency. In this case, construction of the reactor fuel elements combined with thermal and pressure stresses dictated the operating temperature.

The next step is the investigation of the effect of operating under super-heat conditions. As to be expected, overall efficiency is improved as indicated by Figure 2. Additional advantages are obtained, such as higher turbine efficiency, at essentially the same steam pressures. Unfortunately, disadvantages appear when the reactor is considered as a part of the cycle. The elimination is on the basis that superheat cannot be obtained on a practical design basis with this type of reactor, due to the limitation of operable pressures, imposed by materials consideration.

A third region may be considered for this cycle and that is the super-critical water region. Although further cycle improvements can be made, the attainable efficiency is considerably below the ideally possible. Practical attainment is forestalled by the development of equipment to withstand

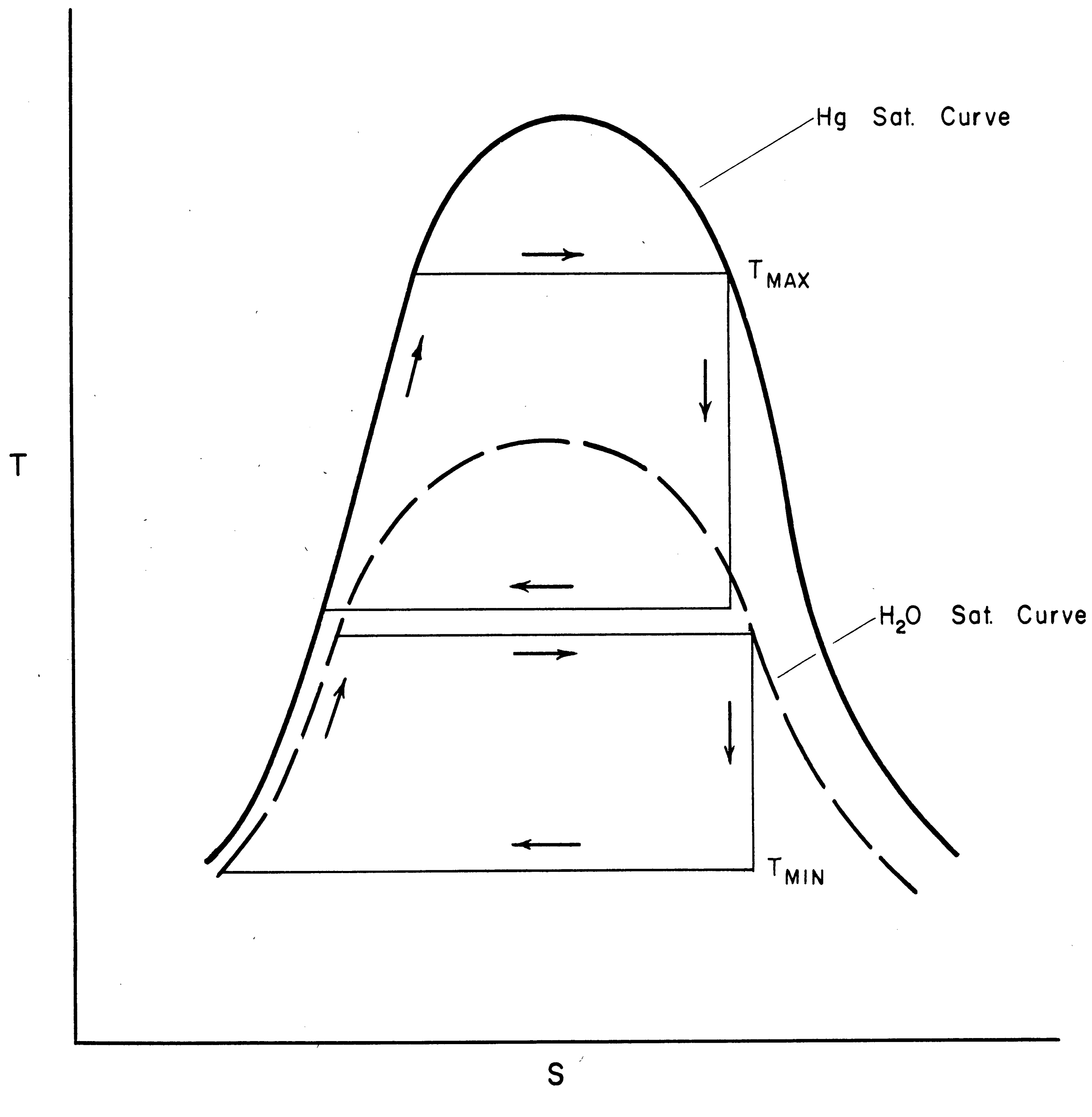

FIG. 3 MERCURY-STEAM BINARY CYCLE

the very high pressures involved (approximately between 5,000 and 10,000 psi). Assuming such hurdles would be overcome, consideration of the reactor to supply the cycle would require the development of a new concept which would not involve fuel in the form of elements or in some type of aqueous solutions. The latter type of fuel would involve a considerable development program on the reduction of corrosion rates of aqueous solutions at high pressures.

From the foregoing discussion, it becomes obvious that a fluid is required which would extend the saturation region beyond the presently available heat source temperature limitations imposed by material problems. In addition, the fluid must possess sufficient vapor pressure at the minimum temperature to allow feasible power extraction through a turbine. Besides possessing the mentioned characteristics, the fluid must be compatible to the nuclear reactor, namely low or zero activation cross-section, low capture-cross section, and stability under high neutron fluxes.

Since no single-known fluid possesses suitable characteristics at high and low temperatures, it may be desirable to achieve results through the combination of several fluids. The next section will treat this as a separate cycle consideration.

4.2.2. Binary Cycles

Past consideration of the binary cycle has dealt mainly with the mercury-vapor-steam cycle, with virtually all the development having been conducted by the General Electric Company. Although this particular combination of fluids in a power cycle, Figure 3, has resulted in a marked increase in power plant efficiency, about 46 percent*; nuclear properties of mercury makes it an undesirable fluid. In order to overcome the objectionable nuclear properties of mercury, a third fluid, such as sodium, potassium, or sodium-potassium eutectic can be used in the primary loop. Although this type of cycle may have some merits, the added cost in additional equipment may offset any gain to be realized.

A true binary cycle is one that would involve a fluid with high-temperature properties, no phase change preferable, transferring the heat of the reactor to a secondary fluid which may or may not have phase change properties. The secondary fluid leaves little to choice since only two media, namely, air and water, have been utilized in heat-work conversion equipment. Therefore, the primary fluid becomes the one which is investigated for its nuclear and heat transfer properties. Once it has been determined that a particular fluid is compatible to the required properties, a complete analysis of the system can be made.

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* Kent's Mechanical Engineers' Handbook, Power, 12th edition, pp. 8-93. - This value is based on the assumption of 100% "boiler efficiency" for the 1020°F cycle.

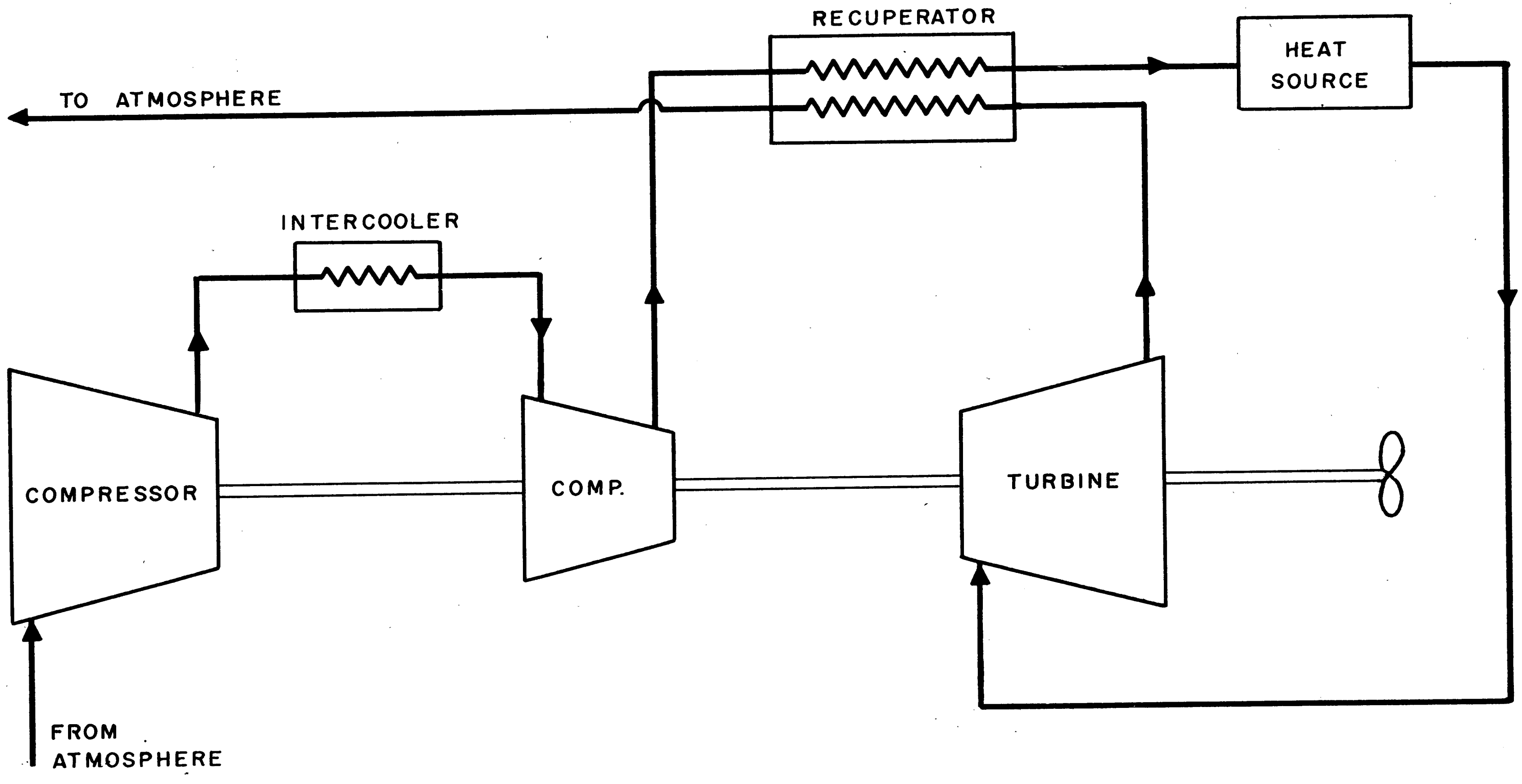


FIG. 4 SIMPLE GAS TURBINE CYCLE

At the present time, calculations are underway to investigate quantitatively the possibilities discussed in the foregoing. Considerations of machinery feasibility, applicable power levels, and costs, as well as thermal efficiency will be included.

4.2.3 Open and Closed Gas Cycles

Evaluation of aforementioned cycles has resulted in a final consideration as to what type of coolant may be considered. Although all the possible ramifications have not been discussed, mainly, because progress in the analysis have not reached that stage; it has become obvious that what is required is a fluid that results in a simple cycle without undue complications.

From this point it is quite natural to consider the simple open gas cycle (Figure 4), with its four component system. Complexity can be introduced to this system by the introduction of such artifices as reheat stages, intercooler and regenerator for the purpose of approaching the ideal cycle.

In order to have a basis for comparison in future analysis, where the devices mentioned will be introduced, the basic four component system has been evaluated on a component basis with the following assumptions pertaining to the components:

4.2.3.1 Compressor in which the pressure level of the working fluid is raised above ambient.

4.2.3.2 The nuclear reactor in which the working fluid is heated to some top temperature.

4.2.3.3 A turbine by which shaft horsepower is generated and work done.

4.2.3.4 A heat exchanger which cools the working fluid again to ambient temperature for entrance to the compressor.

It will be noticed that assumption number 4.2.3.4 provides for investigating a closed-cycle system. This is done with the realization that the open-cycle, using air as the working medium, results in the discharge of radioactive air. In essence this is the simplest thermodynamic system capable of producing work which will permit reaching definite conclusions in future considerations. It is interesting to note that a gas-cooled reactor considered for aircraft propulsion closely parallels the consideration presented herein. The method of approach and analysis of the simple gas cycle is presented in complete detail in Appendix 1.

The analysis presented in appendix, has resulted in a set of curves (Figure 5), which shows that the efficiency reaches a region of vanishing return as temperature is increased indefinitely. Within the limits of the assumptions made to date, it is seen that the power output appears to rise continually. It may not be possible to increase temperature along a line of constant components efficiency, since it is quite possible for the component efficiency to decrease with the increasing temperature. Consequently, design of the turbine, compressors, heat exchanger, and nuclear reactor must be constantly balanced and matched in the complete system. Review of the calculations for validity of assumptions can be justified by noting that if the parameter $\underline{a}$ is as low as 0.75, the temperature ratio required for the system to have a work output greater than zero is about 11.5, which means that with an ambient temperature To, of 500°R the top temperature would have to be 5750°R. Thus component efficiencies for any practical design must be greater than 75%. Present day gas turbine practice use temperature ratios of 3 to 9, due to the limiting conditions imposed by materials of construction, thus the lowest average component efficiency cannot be less than 85 percent. Citing current jet engine values for efficiencies are as follows:

μ_C = 0.80 (Compressor)

μ_R = 0.95 (Combustion chamber)

μ_T = 0.85 (Turbine)

μ_H = 1.00 (Open-cycle, discharge to atmosphere)

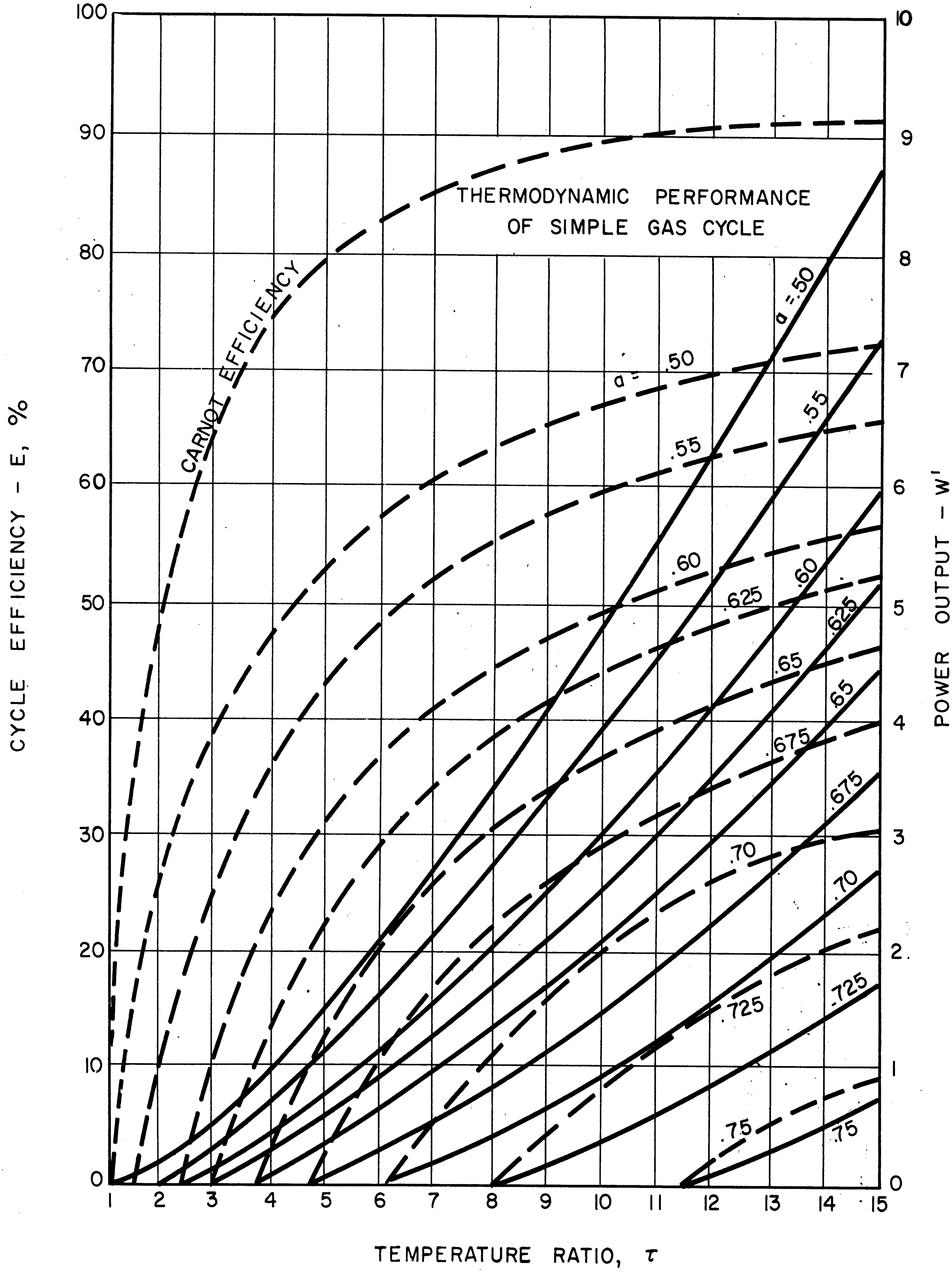
μ $\overline{0.90}$

It can be seen by referring to Figure 5 that the average efficiency of 90 percent corresponds to a minimum temperature ratio of 2.4. With an ambient temperature of 70°F (530°R), the temperature at the inlet of the turbine is 1270°R or 810°F.

In actual design practice, it is more useful to consider power system from "design point operation". For system design, where the "design point" is 1200°F, component efficiencies are arranged to have maximum values as near as possible to the "design point". At all other conditions the component efficiencies will be less than the maximum value. For propulsion devices, such as nuclear powered aircraft - the "off-design considerations" must be considered for changes in

FIG. 5

PLOT OF POWER OUTPUT & CYCLE EFFICIENCIES
VS.
TEMPERATURE RATIO



speed, altitude, ambient temperatures, cargo loads, etc. Marginal designs are subject to two limitations:

4.2.3.5 If the "design point", which is assumed is not achieved because a given component does not perform adequately, marginal design may not operate at all.

4.2.3.6 Marginal designs in cases of gas turbines will have poor off-design performances particularly starting conditions.

Having dealt with the general design performance characteristics of the simple gas cycle it is well to mention some of the difficulties to be encountered in the nuclear considerations. Although, the topic has been mentioned only briefly, it is necessary to give considerations to the selection of the proper working fluid. This decision is closely interrelated to the reactor design, since the gas may conceiveably be used as the reactor coolant. Thus the nuclear properties of the gas, the question of closed versus open-cycle where the possibility of discharge of radioactive gas to the atmosphere exists, the operating pressure of the gas, the type of application (proximity to inhabited areas for example), etc., all factors closely involving both the reactor and heat engine cycle. At first glance it is obvious that a large number of gases, including air, helium, hydrogen, nitrogen, oxygen, carbon-dioxide and others might be suitable. Interpretation of data* is premature at this time, other than to state from a nuclear consideration helium will give the least amount of trouble nuclear wise. Final interpretation must be given in conjunction with a given cycle.

Subsequent cycle analysis will be more specific in that actual fluids will be considered in the evaluation of practical cycles.

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* Reactor Handbook, Vol. 2, Engineering, USAEC; AECD-3646, Chapters 3.2 to 3.8.

4.3 Nuclear Reactor Heat Source Considerations

It can be readily established that there is no clear theoretical upper limit to the rates of energy released of power production, due to fission. In practice, however, the maximum operating power level of a reactor is frequently determined by the rate at which convection heat can be removed. Therefore, when considering the generation of power from the fission process, certain general conclusions can be thus established for the basis of evaluating methods for removing power generated. Some of these are:

4.3.1 Power density is proportional to neutron flux, and density of fissionable materials.

4.3.2 90 percent of the energy is produced at the point of fission and about 5 to 10 percent is absorbed in reflectors, shielding, control rods, and other materials.

4.3.3 Most of the power is produced at the time of fission, with approximately 6 percent of the energy being delayed which decays exponentially after shutdown in the nuclear fission reactor.

4.3.4 The critical mass of fissionable materials is a function of the critical radius of the reactor. For a given critical radius, the critical mass is less than for either or smaller critical radii. The critical mass and critical radii can be expressed in terms of a dimensionless, physical, geometrical parameter.

4.3.5 Excess neutrons over those required for criticality can be used for converting certain unfissionable materials into fissionable materials. However, for transportation devices such as compact heat engines, it appears desirable to reduce the critical mass and geometry of a given power producing reactor to minimum size and provide means for reflection and optimum geometry so that the maximum number of neutrons are absorbed in the fission process. When removing power generated by the fission process in a nuclear heat power engine, it is necessary to correlate methods by which such power is removed with the distribution of energy throughout the reactor. In general, such energy is removed by heat transfer. If the reactor is operated at high power density, there will generally be large temperature gradients and large thermal stresses to be considered. In actual design of the heat engine utilizing nuclear power, the thermal efficiency will depend largely upon the top temperature available at the exit of the reactor and the optimization of heat transfer means so that maximum energy levels are available for conversion of the fission energy to ultimate mechanical energy.

Thus, it is desirable to evaluate the net gains which can be achieved by operating a reactor at the limits of creep resistance for high-temperature materials. Since the removal of thermal energy involves, in most cases, circulating a cooling fluid, it becomes necessary to evaluate the proportion of pumping power to the total power delivered. Evaluation of this effect requires the establishment of fluid flow, and thermodynamic properties of fluids to be considered and the subsequent correlation of the data with the nuclear properties of the fluid. In giving consideration to heat transfer, evaluations depending upon temperature limits considered, radiant heat transfer, conductive heat transfer, and convective heat transfer have to be made. Since the immediate applications of nuclear energy for transportation devices are in temperature ranges below the point at which radiant heat transfer has significant influence, present work conducted in heat engine studies has neglected this effect. Further to date, engineering calculations have been made at or near the limits of known or proven materials of construction for reactor design as well as heat engine design. Efforts are being made to correlate convective heat transfer with the limitations of power densities in a given reactor with the attendant pressure losses due to mass flow of fluids optimizing heat transfer coefficients. Since relationships of fluid flow to film coefficients are such that heat conduction plays an important part in overall heat transfer mechanisms, continual attention must be given to heat conduction problems.

4.3.6 Heat Transfer of Nuclear Reactors

In giving consideration to nuclear heat power systems which have as general objectives the achievement of small packaged heat sources and minimum size of heat engines for maximum horsepower, nuclear reactors must of necessity be ones with high specific power. The heat transfer must be optimized and the heat engine size reduced so that the nuclear heat engine natural system is compatible in proportion for transportation devices under consideration. Such reactors of necessity are of high specific power, operating at maximum temperatures, and near the ultimate construction materials. In the case of heterogeneous fuel reactors with high specific power, special considerations must be given to the design of fuel rods in which the energy source is contained in the fuel elements. The heat must be transferred by conduction so that some suitable fluid or circulating coolant can absorb such heat. In the case of homogeneous liquid metal reactors, the heat conduction problem is basically associated with the design of vessels to contain a critical geometry, relationships of film coefficients for convective heat transfer which heat conduction in heat exchange equipment, and liquid metal to gas or working fluid transfer systems. In the event, temperatures in excess of 1800°F are achieved as an operating temperature in a nuclear reactor, heat transfer by radiation must also be given consideration. A portion of the work and studies being conducted in evaluating nuclear heat engines involves long-range concepts of high-temperature reactors.

4.3.7 Heat Distribution

Problems which have been outlined are basically the general ones pertaining to the investigation of heat generation in the reactor. A more detailed examination of the heat sources and their distribution in the reactor core and structure is one that cannot be neglected. Complications arise due to the fact that the heat is generated in the reactor by a number of ways and possesses a distribution which is not uniform. The major portions arise from the kinetic energy of the fission fragments which usually manifest itself as heat produced entirely within a fuel element or within a fuel solution. In addition, heat is also produced from the slowing down of the neutrons and beta particles, and in the absorption of various gamma radiations. Since the neutrons and gamma rays are not uniformly distributed throughout the reactor core and structure, the associated heat source distribution will also be non-uniform. An accurate knowledge of neutron and gamma ray fluxes is therefore necessary for a complete solution of the reactor cooling problem. Consequently, the investigation of these fluxes becomes quite a difficult problem, one almost entirely without theoretical approach. Consequently, the reactor designer must rely on data obtained from experiments performed on operating reactors.

Another factor which adds to the problem of heat removal from a reactor is that the volumetric heat-release rates may be higher than for any thermal system designed for continuous operation. There are two reasons for this. One is the necessity to keep the size of the reactor small, and two, the desire to increase the core output per unit mass of fuel. The combination of high operating temperatures for maximum thermal output of the reactor and maximum thermodynamic efficiency for ultimate conversion of power and large volumetric heat sources make special demands upon the design of the reactor thermal system. Not only do the large source densities cause stresses due to temperature differences within core components to increase rapidly as their thicknesses increase, but the nuclear radiations of high power densities, high neutron fluxes, may have adverse effects on thermal conductivity and other properties of reactor materials. Even though the major advances in reactor design have come from materials fabrication sources, it is only by optimizing the reactor systems from the heat transfer standpoint that their full advantage can be realized.

4.3.8 Convective Heat Transfer

When considering conductive heat transfer, efforts are being made to evaluate two quantities, heat transfer efficiency and power density. Each of these is dependent on the geometry of the physical and chemical properties of the cooling fluid, and the pressure drop or pressure losses encountered in the heat transfer system. The relationship between heat transfer and pressure losses determines to some extent a limitation on the power density available in the nuclear reactor and the efficiency of operation of the system of which the nuclear reactor is the heat source.

It should be noted that the limitation placed upon power density as well as upon thermal stress is under the control of the designer and the nuclear engineer. Calculations are being made for various types of loop systems in which the pressure losses for various temperature levels of a fluid media extracting heat are being examined. In principle, the approaches are system-analysis in which pressure drop correlations are evaluated with respect to the flow of fluid for various types of geometry. The fluid flow characteristics are evaluated in terms of heat transfer coefficients in which Reynolds numbers for various heat transfer geometries must be considered. Such fluid flow equations in turn are correlated with the selections of pumping equipment and heat exchanger surfaces so that optimum selection of systems can be made in view of specific components which must be developed for the specific nuclear heat-power system under consideration. As temperatures in nuclear reactors are developed to higher levels, it becomes apparent that more data in fluid flow, heat transfer, and heat engine development are required. As the progress of work proceeds, efforts will be made to point up and outline those areas of investigations which are needed for effective engineering design of nuclear heat engines.

5.0 NUCLEAR REACTOR PHYSICS

The methods and approach for the determination of a given critical size of a reactor for a specific application has been under consideration with the ultimate aim of setting up a firm foundation from which future reactor concepts can be evaluated. In order to satisfy the scope of work outlined in the section 2.3 of the proposal submitted to Chrysler, a program analysis has been established which will serve as a basis for reference. Normally the analysis and calculations for the nuclear reactor is based upon previous studies. This, of course, holds only for reactors tried and proved, whereas, in the case of new concepts, the entire range of parameters will be evaluated in order to establish optimum points. With the above in mind, a program similar to that followed on the nuclear energy propulsion of aircraft study has been initiated.

A numerical evaluation of the uranium-bismuth type of reactor has been performed utilizing one velocity, spherical, homogeneous reactor theory. Calculations were performed for the determination of critical mass and radii for various U-235 concentrations and U-235 to graphite ratios. Simultaneously for this initial program of calculations, the mathematical model has been established for a cylindrical internally moderated, externally reflected, reactor, and also for a cylindrical, externally reflected with a central core reflector. This analysis is now nearing completion and will be submitted for numerical evaluation. In order to arrive at a complete set of curves which will serve as the basis for the establishment of a specific design for the LMFR type of reactor, the following parameters have been considered: U-235 concentration, operating temperature, carbon to fuel ratio. Prior to the performance of the calculations, it is necessary to establish group constants, which will predict accurately the state of the neutrons, at their various stages in their slowing down process. This determination is by far the most critical one in any reactor calculation, since the accuracy of the final results is dependent upon the accuracy of the group constants. The state of these determinations will be given in the next section along with the assumptions made. One additional step is required before the calculations can be performed and that is the estimation of the neutron fission spectrum across the reactor core. Inasmuch as this is an estimation, it will be necessary, after performing preliminary calculations by the two-group method, to go back and correct estimated fission spectrum. The neutron spectrum is used to evaluate the heat distribution within the core, and also to get a measure of the neutron economy, which in turn is a function of critical mass requirement.

Although the above program involves a considerable amount of background work, once having accomplished it, it will be possible to evaluate the nuclear properties of any specific LMFR design. More specifically, it will be possible to optimize a reactor design in light of the thermal effects imposed upon it by the overall heat cycle.

5.1 Constants of the Theory

The nuclear constants which are independent of the neutron spectrum are computed for pure bismuth and pure carbon at various operating temperatures. In addition, a very rough approximation for the Fermi age is given.

It is assumed that

5.1.1 σ_a follows the $1/v$ law.

5.1.2 $\sigma_T = (\sigma_a + \sigma_s)$ only.

5.1.3 Scattering is isotropic in the center of mass system so that $\bar{\mu}_o = 2/3A$.

5.1.4 A is sufficiently large so that $\xi = 2/(A + 2/3)$.

5.1.5 D_{th} and Σ_a are constant up to 2 Mev in order to approximate the age.

In computing the constants for pure carbon, assumption (5.1.4) is not applicable and $\xi = 1 + [(A - 1)^2/2A] \ln [(A - 1)/(A + 1)]$ is used.

The following equations are used.

$$\sigma_a (T) = \sigma_{a_o} \left(\frac{T_o}{T}\right)^{1/2}$$

$$\Sigma_a (T) = \frac{N_o \rho}{A} \sigma_a$$

$$\sigma_{tr} (T) = (\sigma_T - \sigma_a)_T (1 - \bar{\mu}_o)$$

$$\Sigma_{tr} (T) = \frac{N_o \rho}{A} \sigma_{tr}$$

$$D_{th} = \frac{1}{3\Sigma_{tr}}$$

$$L^2 = D_{th}/\Sigma_a$$

$$\tau = \frac{D_{th}}{\xi} \frac{u_{th}}{\Sigma_{tr}}$$

All equations are from Glasstone and Edlund. The values of $\sigma_T (T)$ and σ_{a_o} are from BNL 325. The values for ρ are from a graph drawn from points in the "Liquid Metal Handbook" (1952).

TABLE I

NUCLEAR CONSTANTS FOR BISMUTH AT OPERATING TEMPERATURES

E (v)	T (^{o}C)	ρ (gr/cm^3)	σ_T (barns)	σ_a (barns)	Σ_a (cm^{-1})	σ_{tr} (barns)	Σ_{tr} (cm^{-1})	D_{th} (cm)	L^2 (cm^2)	u_{th}	τ (cm^2)
.062	450	9.86	8.9	.0203	$.0536 \times 10^{-4}$	8.88	.2521	1.319	2.278×10^5	17.46	95.9×10^2
.07	541	9.74	8.9	.0191	$.0536 \times 10^{-4}$	8.88	.2495	1.334	2.489×10^5	17.34	07.5×10^2
.08	655	9.59	8.9	.01786	$.0493 \times 10^{-4}$	8.88	.02453	1.358	2.755×10^5	17.21	100.2×10^2
.09	812	9.43	8.9	.01685	$.0458 \times 10^{-4}$	8.88	.2411	1.381	2.951×10^5	17.09	102.7×10^2
.095	827	9.33	8.9	.0164	$.0441 \times 10^{-4}$	8.88	.2383	1.395	3.160×10^5	17.06	104.5×10^2
A = 209			ξ = .009539		$(1 - \mu_o)$ = .9968						

TABLE III

NUCLEAR CONSTANT FOR CARBON ($\rho_{T=0}$ = 1.9 gr/cm^3) AT OPERATING TEMPERATURES

$A = 12, \xi = .158, (1 - \bar{\mu}_o) = .9444$

E (ev)	T (^{o}C)	ρ (gr/cm^3)	σ_t (barns)	σ_a (barns)	Σ_a (cm^{-1})	σ_{tr} (barns)	Σ_{tr} (cm^{-1})	D_{th} (cm)	L^2 (cm^2)	u_{th}	τ (cm^2)
0.062	450	1.830	5.00	.00203	1.862×10^{-4}	4.72	.434	0.768	41.2×10^2	17.46	195.1
0.07	541	1.820	4.70	.00191	1.741×10^{-4}	4.44	.405	0.822	47.2×10^2	17.34	222.5
0.08	655	1.800	4.75	.001786	1.611×10^{-4}	4.48	.405	0.822	51.0×10^2	17.21	221.0
0.09	812	1.776	4.70	.001685	1.501×10^{-4}	4.44	.395	0.843	57.0×10^2	17.09	230.1
0.095	827	1.762	4.60	.00164	1.451×10^{-4}	4.33	.383	0.870	59.8×10^2	17.06	244.5

6.0 BIOLOGICAL SHIELDING

6.1 Introduction

The required shielding for any reactor must be designed for operation at the maximum operating power level. Under normal working conditions, the design dose rate should be kept below tenth tolerance (to be defined in following section), while in more inaccessible areas the dose rate is kept at tolerance. The maximum permissible exposure levels accepted by the National Radiation Protection Committee will be used for calculating tolerance dose rates. It should be understood that this is subject to modification, depending on the type of service to be considered.

There are four sources of radiation:

6.1.1 The fission process.

6.1.2 Fission products in the fuel.

6.1.3 Neutron activated components of the power plant.

6.1.4 Induced radioactivity in the primary coolant. The radioactivity in the coolant comes both from the activation of the fuel itself, and the release of delayed neutrons.

These radiations can be dealt with in four ways. Radiation from the reactor can be attenuated by a combination of light and heavy density materials, such as carbon, hydrogeneous materials, steel, and lead. Neutrons from the reactor are attenuated to the point when induced radioactivity in components becomes negligible. The primary circulation loop can be completely shielded. A chemical separation system can be provided to remove fission products continually.

6.2 Tolerance Dose Rates

The maximum permissible exposure level for civilians accepted by the National Radiation Protection Committee is given as 0.3 rems per week or for a 40-hour week, this corresponds to 7.5 mr per hour of gamma radiation. These quoted levels are naturally on the safe side, but for the purpose of evaluating nuclear power vehicles or transportation devices which may have military application, some new acceptable standard must be established.

A few basic facts have to be established before a reasonable evaluation can be made. First, it should be pointed out that gamma ray doses limited to the hand and forearm can exceed the normal dose by a factor of five, or 0.0375 r per hour. Assuming that the entire body can receive this without detectable effects, this immediately sets an upper limit on the gamma dosage.

The neutron situation is not quite so simple, since there are many ways in which the energy can be deposited in the flesh. Dr. Walter S. Snyder of ORNL has calculated the tolerance dose of fast and thermal neutrons. Although the calculations are probably accurate, they apply to a collimated beam of neutrons incident normally on a 30 centimeter thick slab of meat. This particular point is of interest in specifying the dose inside a crew compartment for neutrons arriving from all directions. The dose for this case is much more difficult to specify.

For laboratory shielding work, Snyder's results serve admirably and are reported in the following table.

It is surprising that fewer thermal neutrons can be tolerated than 5 KEV. The reason is that the latter have a higher probability for being reflected from the tissue, doing relatively little harm.

TABLE III

NEUTRON FLUXES EQUIVALENT TO 0.3 REP OF GAMMA RAYS PER 40-HOUR WEEK

Neutron Energy	Flux, n/cm^2/sec
Thermal	1800
5 KEV	2250
0.5 MEV	94
2.5 MEV	39
5 MEV	25
10 MEV	26

At the Chalk River Conference, it was also agreed that "no manifest permanent injury is to be expected from a single exposure of the whole body to 25 r or less." This dose has been used so consistently since then that it has required variability, if not authenticity. Results will have a large bearing on the proper tolerance to be selected for any transportation device, if not actually set the tolerance for all military applications. All of the above facts have to be taken into consideration on tolerance selection and with this in mind, a dose rate of approximately 0.07 r per hour is a basis for a reasonable selected tolerance. This figure was arrived at by considering what is a practical shield for a transportation device.

For the purpose of evaluating shield design, the gamma flux required to give a dose rate of 1 r per hour has been determined for various energies where they are tabulated in Table IV and plotted in Figure 6 . The surface and volume sources have been tabulated as indicated.

TABLE IV
GAMMA FLUXES FOR DOSE RATE OF 1 R/HR

E	$\left(\frac{\mu - \sigma s}{\rho}\right)a$	Io
0.7 - Mev	0.029	7.08×10^5
1.0	0.07	5.33×10^5
1.5	0.0245	3.93×10^5
2.0	0.023	3.13×10^5
3.0	0.0205	2.34×10^5
5.0	0.018	1.60×10^5
7.0	0.0163	1.268×10^5

TABLE V
VOLUME SOURCES WHICH GIVE A DOSE RATE OF
1 R/HR AT THE SURFACE OF A SEMI-INFINITE VOLUME

E	μ(air)	E Qv	Qv (photons/cc/sec)
0.7 - Mev	0.029	7.08×10^5	4.11×10^4
1.0	0.027	5.33×10^5	2.88×10^4
1.5	0.0245	3.93×10^5	1.925×10^4
2.0	0.023	3.13×10^5	1.44×10^4
3.0	0.0205	2.34×10^5	$.96 \times 10^4$
5.0	0.018	1.60×10^5	$.576 \times 10^4$
7.0	0.0163	1.268×10^5	$.413 \times 10^4$

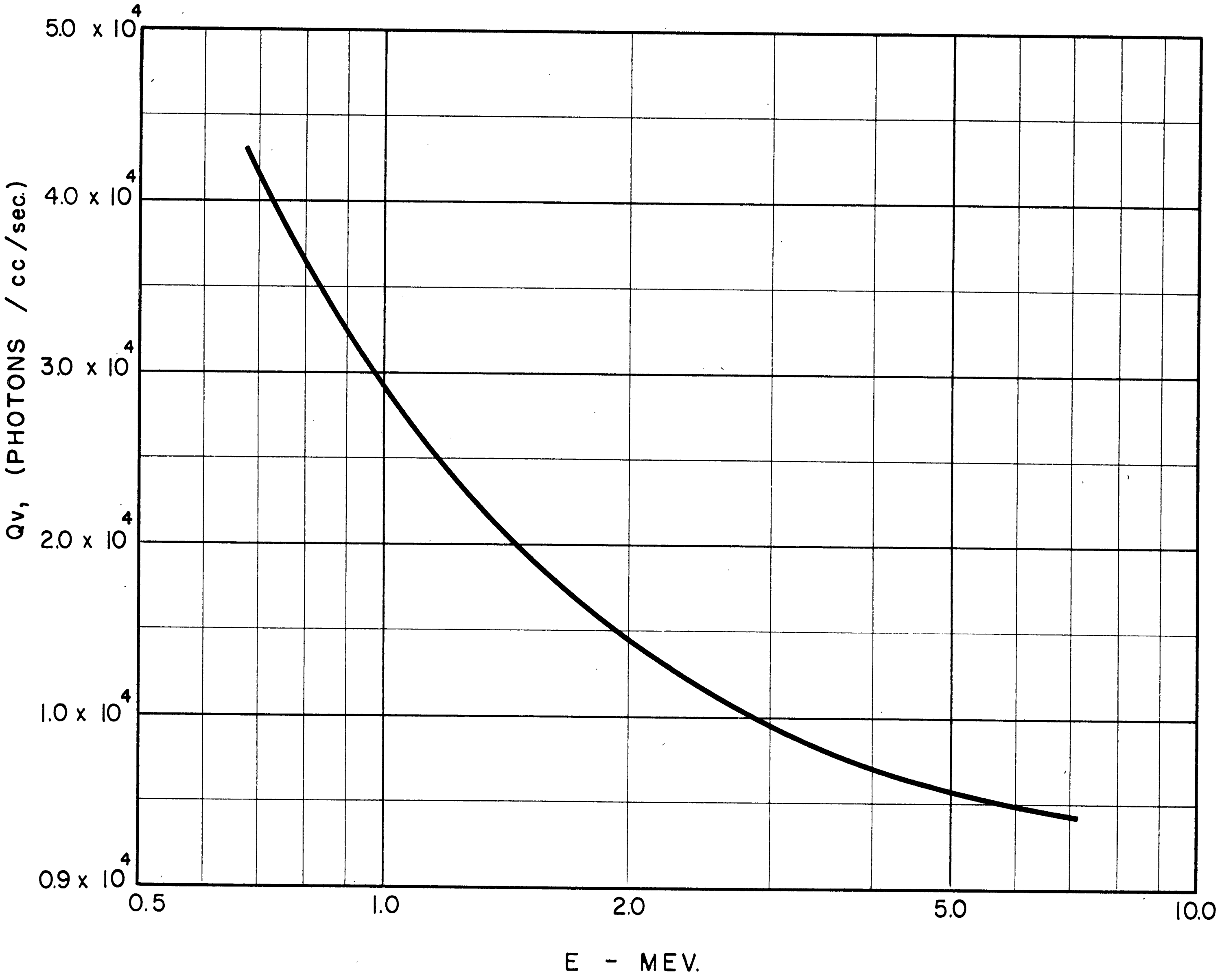


FIG. 6 VOLUME DISTRIBUTED SOURCE STRENGTHS WHICH PRODUCES A DOSE RATE OF 1 R/HR. ON THE SURFACE OF A SEMI - INFINITE VOLUME

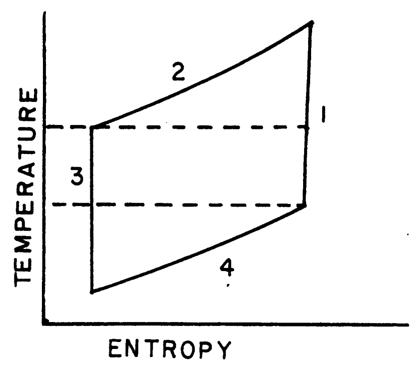
APPENDIX I

OPEN AND CLOSED GAS CYCLES ANALYSIS

It is assumed that the physical states of the working fluids are determined completely by the assignments of values of two variables, temperature and entropy.

A typical plot of a cycle relating temperature and entropy is shown below.

TYPICAL TEMPERATURE-ENTROPY RELATION



(1) Compression

(2) Heating

(3) Expansion

(4) Heat Rejection

If a cycle is to operate continuously, it is obvious that the curve in this diagram must be closed. Analytically, this can be stated that the requirements for the existence of a thermodynamic cycle are:

$$\oint dT = 0 \tag{1}$$

$$\oint \frac{d\,S(T)}{dT}\,dT = 0 \tag{2}$$

Equations (1) and (2) establish the relation $S = S(T)$. These expressions involve such parameters as friction losses in components which are used to define component efficiency. The cycle efficiency and power output can be expressed in terms of component efficiency and temperature ratios across components.

COMPONENT PERFORMANCE

For any thermodynamic process the conservation of energy can be written as

$$dq - dw = dh + dk + de \tag{3}$$

when

dq = heat allowed to unit weight of fluid.

dw = external work done by fluid.

dh = change in enthalpy of fluid.

dk = change in kinetic energy.

de = change in potential energy.

The conservation of momentum requires

$$-dw = dk + de + \frac{1}{\gamma} dp + df \tag{4}$$

where

γ = weight density of fluid

p = pressure of fluid

df = frictional energy dissipated per unit weight of fluid.

Enthalpy by definition is

$$dh = du = + \frac{1}{\gamma} dp + p\, d\left(\frac{1}{\gamma}\right) \tag{5}$$

where

du = change in internal energy

Equation (3), the energy equation can be written as:

$$dq - dw = du + p\, d\left(\frac{1}{\gamma}\right) + \frac{1}{\gamma} dp + dk + de \tag{6}$$

Subtract equation (4) from (6),

$$dq + df = du + p\, d\left(\frac{1}{\gamma}\right) \tag{7}$$

The change in entropy can be defined as

$$ds = \frac{du + p\, d\left(\frac{1}{\gamma}\right)}{T} \tag{8}$$

Combining equation (7) and (8)

$$ds = \frac{dq + df}{T} \tag{9}$$

In equation (9) it is necessary to express the frictional loss df in terms of known energy changes in the cycle. Such known energy changes must be expressed in terms of temperatures in the cycle. The thermal efficiency of a cycle or the "mechanical" efficiency of a component can be defined as the energy output E_o from the system to the energy input to the system E_o , thus

$$\mu = -\frac{E_o}{E_m} \tag{10}$$

For example, in a compressor, the shaft work is the input E and an enthalpy change in the gas is the output E_o. In a turbine the reverse is true, the enthalpy change in the gas flowing through the turbine is the input and the shaft work is the output. For any given heat cycle, the energy balance involves the differences between energy input and output, and for most cases this difference can be defined by the heat rejected to the environment, here called E_f. For a component, this difference may be the friction. Thus the efficiency and the energy balance equation can be combined to eliminate either energy input or output. For a process in a system where the input is from an external source, such as a nuclear reactor, the relationship between rejected energy and energy input is obtained from $E_o + E_m + E_f = 0$, (11)

as,

$$E_f = -(1 - \mu)\, E_m \tag{12}$$

For the process in a system where the output is external such as a turbine or heat exchanger, energy losses in terms of energy output is expressed by

$$E_f = -(1 - \mu^{-1})\, E_o \tag{13}$$

Frictional energy dissipated per unit weight of fluid (df) can be defined for a process involving work as

$$df_1 = -(1 - \mu_1^{\pm 1})\, dW \tag{14}$$

where the positive exponent indicates compression and the negative exponent expansion. For a process involving heat from a nuclear reactor

$$df_2 = (1 - \mu_2^{\pm 1})\, dq \tag{15}$$

where the positive sign indicates heat added to the reactor and the negative sign indicates heat removed from the reactor. The efficiency referred to in equation (14)4 is sometimes referred to by gas turbine engineers as "small stage efficiency", and is actually the limit of efficiency for a given stage as the number of stages required to either expand or compress a working fluid approaches infinity.

For a process which involves both heat and work at the same time, the expression in terms of frictional energy losses per unit weight of working fluid is:

$$df = -(1-\mu_1^{\pm 1})\, dw + (1-\mu_2^{\pm 1})\, dq \qquad (16)$$

Substituting equation (16) into equation (9),

$$ds = \frac{-(1-\mu_1^{\pm 1})\, dw + (2 - \mu_2^{\pm 1})dq}{T} \qquad (17)$$

In terms of power media, S = S(T), changes in work and heat must be expressed as functions of temperature.

$$dw = -\psi_1 (T)\, dT \qquad (18)$$

and

$$dq = \psi_2 (T)\, dT \qquad (19)$$

If over small intervals ψ_1, and ψ_2 are constant, work and heat changes can be expressed as:

$$dw = -C_1\, dT \qquad (20)$$

$$dq = C_2\, dT \qquad (21)$$

If $K = [(1-\mu_1^{\pm 1})\, C_1 + (2-\mu_2^{\pm 1})\, C_2]$, equation (17) for entropy change can be expressed as

$$ds = K \frac{\partial T}{T} \qquad (22)$$

For any process, equation (22) can be integrated to give the entropy difference between process m and process (m-1)

$$S_m - S_{m-1} = \log\left(\frac{T_m}{T_{m-1}}\right)^K \qquad (23)$$

or

$$\left(\frac{T_m}{T_{m-1}}\right) = \tau_m \quad \text{(temperature ratio)}$$

and

$$S_m - S_{m-1} = \log\, (\tau_m)^K \qquad (24)$$

where T_m = the temperature at the end of the m $\frac{th}{}$ process.

(20)

In the simplified system, there are four discrete processes.

Equation (1) in terms of the four components of compression, heating, expansion and heat rejection can be expressed in terms of temperature difference between the m^{th} and $m-1^{th}$ process as

$$\sum^{4} (T_m - T_{m-1}) = 0 \tag{25}$$

Equation (25) can be rewritten as

$$\sum_{m=1}^{4} T_m = \sum_{m=0}^{3} T_{(m)} \tag{26}$$

Cancelling identities

$$\frac{T_4}{T_o} = 1 \tag{27}$$

In terms of temperature ratios

$$\frac{T_4}{T_o} = \sum_{m=1}^{4} \tau_m \tag{28}$$

or

$$\sum_{m=1}^{4} (\tau_m) = 1 \quad \text{(for a four process cycle)}$$

For a four process cycle equation (2) can be written in terms of discrete processes as:

$$\sum_{m=1}^{4} (S_m - S_{m-1}) = 0 \tag{29}$$

By substituting the value derived in equation (24)

$$\sum_{m=1}^{4} \log (\tau_m)^K = 0 \tag{30}$$

or

$$\log \sum_{m=1}^{4} (\tau_m)^K = 0 \tag{31}$$

or

$$\sum_{m=1}^{4} (\tau_m)^K = 1 \tag{32}$$

Equation (32) and (28) are necessary conditions for the existence of a power cycle in terms of temperature ratios and process efficiencies which determine the factor K. Where changes of phase occur (liquid-vapor power system) in which temperature remains constant during a given process (such as evaporation or condensation), a different approach is required. Therefore, the derivations are applicable to cycles which employ gases as working fluids. The derivation can be considered for a simple gas turbine cycle.

For equations (20) and (21) C_1 and C_2 are considered constants and can be evaluated for each of the four processes in the simple cycle. For compression or expansion in which no heat is added and changes in kinetic and potential energy are small. Equation (3) for energy conservation can be written as

$$dw = -dh \tag{33}$$

If the working fluid can be considered a perfect gas, then,

$$dh = C_p \, dT \tag{34}$$

and

$$dw = -C_p \, dT \tag{35}$$

Thus, for compression and expansion under ideal conditions the constant C_1 , equation (20) for evaluating change in work is an identity with specific heat a constant pressure.

$$C_1 = C_p \tag{36}$$

In the case of heating and cooling in which there is no external work

$$dq = dh = CpdT \tag{37}$$

so the constant C_2 in equation (21) is

$$C_2 = C_p \quad \text{also} \tag{38}$$

The values for C_1 and C_2 can be used to evaluate the K in equation (23) and as a result the work done and heat added can be determined.

Consider as examples, an adiabatic compressor, a nuclear heat generator, an adiabatic turbine, and a heat exchanger. This particular combination of process is ideal for a closed-cycle gas turbine in which the working fluid for the gas turbine flows and exchanges heat with the nuclear-heat power unit.

Although the simple gas turbine with the four simplified processes may be of little practical value to an actual system, the principles for applications to processes, expressions of efficiency, and power output are identical to actual systems which will be studied in future work. Consider first the case of the adiabatic compressor:

The value for K_c for compressor (39) becomes:

$$K_c = (1-\mu_c)\, C_p$$

The work done in driving the compressor is

$$\Delta Wc = -Cp\,(T_1 - T_o)$$

$$= -Cp\, T_o\, (\tau_c - 1) \qquad (40)$$

For the case of the nuclear reactor the constant K_R can be written from equation (25) as

$$K_R = (2 - \mu_R)\, Cp \qquad (41)$$

Thus, the heat provided by the nuclear reactor is

$$\Delta g_R = Cp\,(T_2 - T_1)$$

$$= Cp\, T_o\, \tau_c\, (\tau_R - 1) \qquad (42)$$

For the case of a simple adiabatic gas turbine; the K_T from equation (23) is:

$$K_T = (1 - \frac{1}{\mu_T})\, C_p \qquad (43)$$

The work done by the turbine is

$$\Delta W_T = -C_p\, (T_3 - T_2) \qquad (44)$$

$$= -C_p\, \tau_c\, \tau_R\, T_o\, (\tau_T - 1)$$

For the case of the heat exchanger the K_H from equation (23) is

$$K_H = (2 - \frac{1}{\mu_H})\, C_p \qquad (45)$$

The heat absorbed by the heat exchange is therefore;

$$\Delta q = C_p\, (T_4 - T_3) \qquad (46)$$

$$= C_p\, T_o\, (\tau_c\, \tau_R\, \tau_T)\, (\tau_H - 1)$$

The temperature ratio τ can be defined as the ratio of the top temperature at entrance to the turbine to the ambient temperature T_o then

$$\tau = \tau_c \; \tau_R \tag{47}$$

Then the heat added by the nuclear heat generation from equation (42) can be written as

$$Q = \Delta q_R = C_p \; T_o \; (\tau - \tau_c) \tag{48}$$

The net work done by the cycle is the sum of the turbine work and the compressor work or

$$W = -C_p \, T_o \left\{ \tau_c - 1 + \tau \; (\tau_T - 1) \right\} \tag{49}$$

Since the efficiency is defined as the rate of output energy to input energy

$$(\text{Eff}) = \frac{W}{Q} = \frac{\tau_c - 1 + \tau \; (\tau_T - 1)}{\tau - \tau_c} \tag{50}$$

This equation is subject to the condition of equations (28) and (32) or

$$\tau \; \tau_T \; \tau_H = 1 \tag{51}$$

$$\tau_c^{K_c} \; \tau_R^{K_R} \; \tau_T^{K_T} \; \tau_H^{K_H} = 1 \tag{52}$$

Eliminating τ_H from (51) and (52)

$$\tau_T^{(K_T - K_H)} \; \tau_c^{(K_c - K_R)} \; \tau^{(K_R - K_H)} = 1 \tag{53}$$

Solving for τ_T,

$$\tau_T = \tau_c^{\left\{\frac{-(K_c - K_R)}{K_T - K_H}\right\}} \; \tau^{\left\{\frac{-(K_R - K_H)}{K_T - K_H}\right\}} \tag{54}$$

Substituting equation (54) into equation (49) then expression for work is obtained:

$$W = -C_p \, T_o \left[\tau_c - 1 + \tau \left\{ \tau_c^{\left[\frac{-(K_c - K_R)}{K_T - K_H}\right]} \; \tau^{\left[\frac{-(K_R - K_H)}{K_T - K_H}\right]} - 1 \right\} \right] \tag{55}$$

The work by the cycle will be maximum when the differential of change of work with respect to τ_c is equal to zero.

$$\left(\frac{\partial W}{\partial \tau_c}\right) = 0 \tag{56}$$

or

$$\frac{K_C - K_R}{K_T - K_H} \, \tau^{\frac{K_T - K_R}{K_T - K_H}} \, \tau_c^{\frac{-K_C - K_R + K_T - K_H}{K_T - K_H}} = 1 \tag{57}$$

Solving equation (57) for τ_c

$$\tau_c = \left(\frac{K_T - K_H}{K_C - K_R}\right)^{\frac{K_T - K_H}{-K_C - K_R + K_T - K_H}} \tau^{\frac{-(K_T - K_R)}{-K_C - K_R + K_T - K_H}} \tag{58}$$

Let

$$a = \frac{K_T - K_R}{-K_C - K_R + K_T - K_H} \quad \text{and} \quad b = \frac{K_T - K_H}{-K_C - K_R + K_T - K_H} \tag{59 and 60}$$

Equation (58) is then

$$\tau_c = \left(\frac{1 - b}{b}\right)^b \tau^a \tag{61}$$

Equation (54) is

$$\tau_T = \tau_c^{-\frac{(1 - b)}{b}} \, \tau^{\frac{(a - b)}{b}} \tag{62}$$

Substitute equation (61) in equation (62)

$$\tau_T = \left(\frac{1 - b}{b}\right)^{-(1 - b)} \tau^{(a - 1)} \tag{63}$$

Substituting (61), (59) and (60) into the expression for work

$$W = -C_p T_o \left\{ \left(\frac{1 - b}{b}\right)^b \tau^{(a - 1)} + \tau \left(\frac{1 - b}{b}\right)^{-(1 - b)} \tau^{(a-1)} - 1 \right\} \tag{64}$$

Equation (64) can be simplified by adding and subtracting the quantity $2\,\tau^{1/2}$

$$W = C_p T_o \left\{ \left[\tau^{1/2} - 1\right]^2 - \left[\frac{1}{b^b (1-b)^{1-b}} \tau^a - 2\tau^{1/2}\right] \right\} \tag{65}$$

From equation (50) the efficiency of the cycle is now

$$(\text{Eff}) = \frac{(\tau^{1/2} - 1)^2 - \left[\frac{1}{b^b(1-b)^{1-b}} \tau^a - 2\tau^{1/2}\right]}{\tau - \left(\frac{1 - b}{b}\right)^b \tau^a} \tag{66}$$

Values for [a] and [b] are given by equation (59) and (60). If the efficiencies of all components were the same and if the specific heat were constant throughout the cycle certain simplifications result.

COMPONENT PERFORMANCE WHEN C_p IS CONSTANT AND COMPONENT EFFICIENCIES ARE ALL THE SAME.

For such a case, equation (60) is

$$b = 1/2 \tag{67}$$

The temperature ratio across the compressor is

$$\tau_c = \tau^a \tag{68}$$

The temperature ratio across the turbine is

$$\tau_T = \tau^{a-1} \tag{69}$$

The work output from the cycle is:

$$W = C_p T_o \left\{ [\tau^{1/2} - 1]^2 - 2 [\tau^a - \tau^{1/2}] \right\} \tag{70}$$

The cycle efficiency is:

$$Eff = \frac{(\tau^{1/2} - 1)^2 - 2 [\tau^a - \tau^{1/2}]}{\tau - \tau^a} \tag{71}$$

From equation (59)

$$a = \frac{1 + \frac{1}{\mu} - \mu}{2} \tag{72}$$

Solving this for efficiency

$$\mu = 1/2 \left(1 - 2a + \sqrt{5 - 4a + 4a^2} \right) \tag{73}$$

If a is near unity

$$\mu = 1.5 - a \tag{74}$$

The physical interpretation that can be obtained from equation (74) is that, in the case of a cycle where change in specific heat are not appreciable and where the efficiency of each component is the same as other components, the overall efficiency of the system is about 1.5 less the average component efficiency where component efficiencies are about unity.

It is expected that this case may serve as a guide-post and indicator to specific cycle performances. The value μ to be used is very close to the numerical average of the actual component efficiencies for the cycle. It appears that results using b = 1/2 may be good checks even though b may be quite different from 1/2.

Figure 7 is a plot of the dimensionless power output from a simple gas cycle.

Temperature ratios are plotted against cycle efficiencies and power output W^1 where

$$W^1 = \frac{W}{C_p T_o}$$

For comparison the Carnot efficiency is also shown where the Carnot efficiency is

$$E = \frac{\tau - 1}{\tau}$$

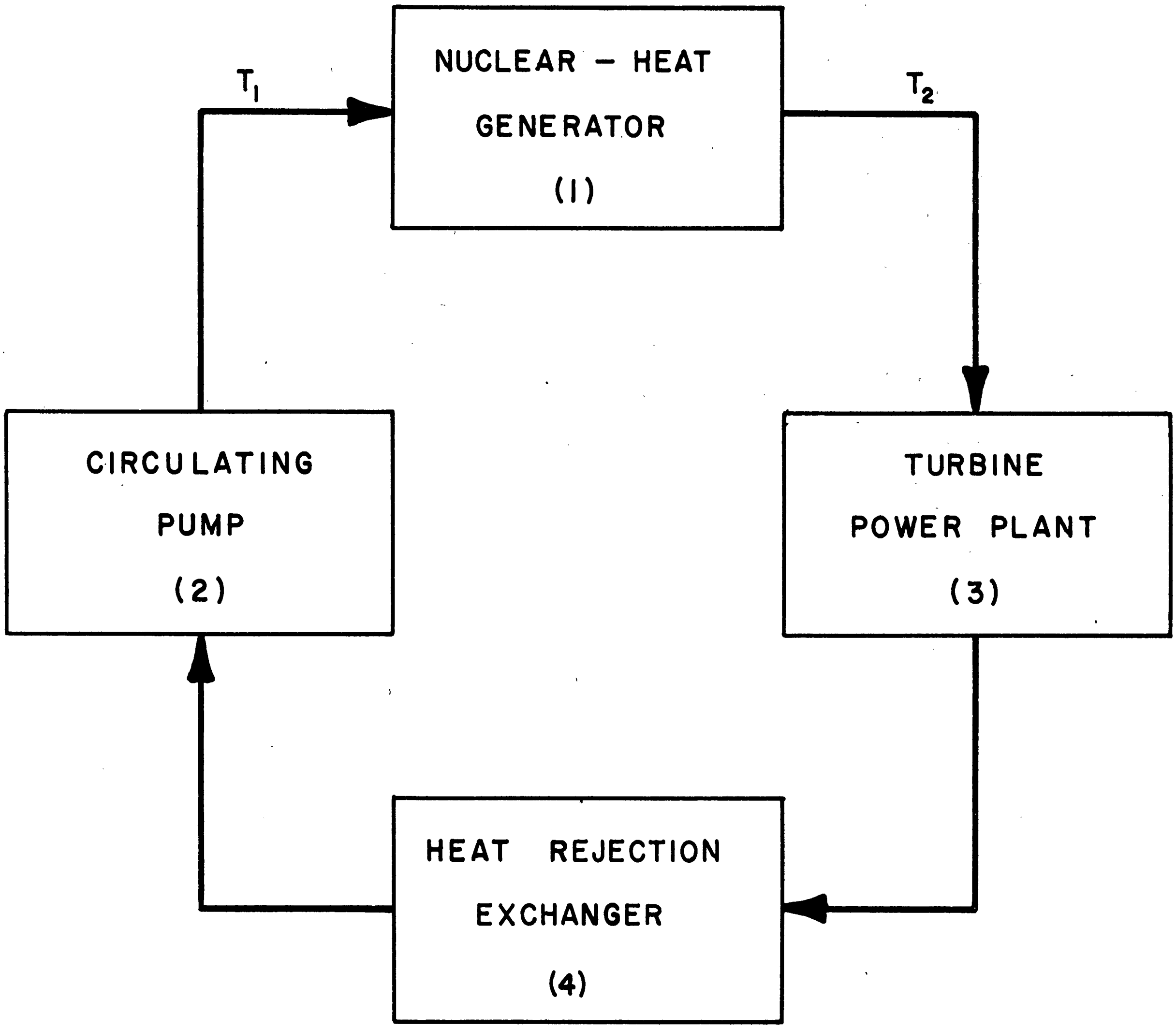
In the simple gas cycle, equation (71) for efficiency in the case where a = 1/2 and the efficiency of each component is 100 percent would be:

$$\text{Eff.} = \frac{\tau^{\frac{1}{2}} - 1}{\tau^{\frac{1}{2}}}$$

The addition of intercooling and reheating features without introducing pressure losses of large magnitudes permits the efficiency of a gas cycle to approach more closely the Carnot efficiency.

FIG. 7

ILLUSTRATIVE FOUR-COMPONENT CONCEPTION
OF
NUCLEAR HEAT - POWER SYSTEM



APPENDIX II

A SIMPLIFIED RESOLUTION OF REACTOR EQUATION TO DIMENSIONLESS PARAMETER

To illustrate the possibilities for reducing mathematical aspects of nuclear engineering to parameters which might prove useful to ultimate design and comparison of nuclear power systems; the work conducted to date has been cursory and somewhat superficial investigation relationship which determine neutron flux distribution and density of fissionable material. Evaluation of these nuclear parameter requires consideration of conditions for criticality.

From reactor theory, there can be considered three equations:

$$\nabla^2_q = \frac{\partial q}{\partial \tau} \qquad (1)$$

This equation represents the slowing down of neutrons from energies at fission to thermal energy limits.

$$\frac{1}{3\Sigma_t} \nabla^2 (nv) - \Sigma_a(nv) + pq_{th} = 0 \qquad (2)$$

This equation defines the diffusion of neutrons and the losses of thermal neutrons due to capture mechanisms.

$$qf = f\eta\, \xi\Sigma_a\, (nv) \qquad (3)$$

This equation expresses the production of neutrons in the fission process which results from the capture of neutrons by the fissionable material. The boundary conditions for the slowing down range of neutrons are provided by equation (2) and (3).

To describe the three equations in present by accepted nomenclature the following definitions should be borne in mind:

TABLE VI

NOMENCLATURE USED IN SAMPLE ANALYSES AND NUMERICAL METHODS

$q = \frac{nv\xi\Sigma_s}{p}$ = slowing down density neutrons /cm^3/sec. passing thru energy E.

n = density of neutrons

v = velocity of neutrons

ξ = average logarithmic energy loss per collision.

TABLE VII (continued)

$$\tau = \exp\left\{\int_0^1 \frac{1}{3\Sigma_t\,(\xi\,\Sigma_s + \gamma\,\Sigma_a)}\,du\right\} = \text{"neutron age" in slowing down.}$$

$$p = \exp\left\{-\int_0^u \frac{\Sigma_a}{\xi\,\Sigma_s + \gamma\Sigma_a}\,du\right\} = \text{resonance escape probability.}$$

Σ_t = macroscopic transport cross-section.

Σ_s = macroscopic scattering cross-section.

Σ_a = macroscopic absorption cross-section.

u = lethargy = $\log\left(\frac{E_o}{E}\right)$

$f = \frac{\Sigma u}{\Sigma a}$ = thermal utilization

$\eta = \nu\,\frac{\Sigma b}{\Sigma u}$ = neutrons per capture in fissionable material.

ξ = fast effect, fissions from fast neutrons.

ν = neutrons per fission

The limitations upon the use of the equations is not of concern presently as the effort is basically to review possible general methods for treatment of problems and to predict certain general limitations of nuclear reactor characteristics.

It should be borne in mind that equations which express diffusion approximations to Boltzman distributions are not accurate in ranges where rapid changes in neutron flux occur. The extrapolation length depends on the transport mean free path. If in turn, the mean free path is a function of lethargy or energy ratios, effective reactor sizes and geometries can be deduced to be functions of energy. If the energies of the neutrons are not thermal the energy must be defined in the form of an integral over the limits of fast fission energies to thermal range known more commonly as the "slowing down" range. If the transport cross-section is strictly a function of space variables, the Laplacian for diffusion should be expressed as:

$$\nabla\left\{\frac{1}{\Sigma_t}\,\nabla(nv)\right\}$$

rather than

$$\frac{1}{\Sigma_t}\,\nabla^2(nv)$$

Progress to date in the systems analyses of nuclear heat-power engines have not permitted examinations of all such possibilities on actual fuel and geometrical requirements for specific reactors. As such data, information, nuclear constants, etc. become available and known influences upon heat cycles are apparent, future progress reports will endeavor to cover this material.

To evaluate the physical quantities associated with nuclear heat engines, continuing efforts will be made to separate functions with physical dimensions from numerical constants which can be calculated or determined through experiment or research.

Equation (1) which represents "slowing down" of neutrons can be expressed in dimensionless parameters by multiplying through by dimensions L^5T. Since the Laplacian $\nabla^2 q$ has dimensions of L^{-2} it can be expressed as the ratio of a parameter α and the square of crital length R^2 for a particular nuclear reactor.

$$\nabla^2 q = -\frac{\alpha^2}{R^2} \qquad (4)$$

If both sides of equation (4)are multiplied by $\frac{\tau th}{q}$, it becomes:

$$\tau th \frac{\nabla^2 q}{q} = \frac{\tau th}{q} \frac{\partial q}{\partial \tau} \qquad (5)$$

which is dimensionless.

Rearranging equation (5)

$$-\frac{\tau th\, \alpha^2}{R^2} = \frac{\partial q}{q} \frac{1}{\frac{\partial \tau}{\tau th}} = \frac{\partial (\ln q)}{\partial \frac{\tau}{\tau th}} \qquad (6)$$

Equation (6) can be integrated over the range of neutron energies

$$-\frac{\tau th}{R^2} \int_0^1 \alpha^2 \, \partial\left(\frac{\tau}{\tau_{th}}\right) = \int_{qf}^{qth} \partial(\ln q) \qquad (7)$$

$$= \ln\left(\frac{q\ th}{q\ f}\right)$$

or the ratio of slowing down densities at fission energies to thermal energies can be expressed as

$$\frac{q\ f}{q\ th} = \exp\left\{\frac{\tau th}{R^2} \int_0^1 \alpha^2 \, \partial\left(\frac{\tau}{\tau th}\right)\right\} \qquad (8)$$

Equation (2) and (3) can be made dimensionless by dividing each by $\Sigma_a (nv)$ resulting in:

$$\frac{\nabla^2 (nv)}{3\Sigma_t \Sigma_a (nv)} - 1 + \frac{p\, q\, th}{\Sigma_a (nv)} = 0 \qquad (9)$$

and

$$\frac{p\, q\, f}{q\, th} \quad \frac{q\, th}{\Sigma_a\, nv} = K \qquad (10)$$

The expression $\frac{p}{\Sigma_a} \frac{q\, th}{nv}$ can be eliminated from equations (9) and (10)

giving:

$$\frac{\nabla^2 (nv)}{3\Sigma_t \Sigma_a\, nv} - 1 + pf\eta\, \xi \, \frac{q\, th}{q\, f} = 0 \qquad (11)$$

The Laplacian $\frac{\nabla^2 (nv)}{(nv)}$ has dimensions of L^{-2}, and can be expressed

in terms of a parameter β and a crital length R or:

$$\frac{\nabla^2 \eta\, v}{nv} = -\frac{\beta^2}{R^2} \qquad (12)$$

Substitution can be made into equation (12) so that

$$-\frac{\beta^2}{3\Sigma_t \Sigma_a R^2} - 1 + p\, f\, \eta\, \xi \, \frac{q\, th}{q\, f} = 0 \qquad (13)$$

The ratio $\frac{q\, th}{q\, f}$ can be eliminated by recalling equation (8)

$$\left(\frac{\beta^2}{3\Sigma_t \Sigma_a R^2} + 1 \right) \exp \left\{ \frac{\tau_{th}}{R^2} \int_0^1 \alpha^2 \, \partial \left(\frac{\tau}{\tau_{th}} \right) \right\} = p\, f\, \eta\, \xi \qquad (14)$$

It should be noticed that equation (14) is essentially the same as the Fermi age equation in which

$$\frac{\beta^2}{3\Sigma_t \Sigma_a R^2} = B^2 L^2 \qquad (15)$$

$$\frac{\tau_{th}}{R^2} \int_0^1 \alpha^2 \, \partial \left(\frac{\tau}{\tau_{th}} \right) = B^2 \tau_{th} \qquad (16)$$

and

$$p\, f\, \eta\, \xi = k \qquad (17)$$

Equation (14) can be used to approximate roughly the mass of fissionable material in a nuclear reactor which is needed for criticality.

For purposes of illustration, the use of equation (14), the cross-section for absorption can be considered as the sum of absorption in fissionable material Σ_u and absorption in all other materials Σ_c or

$$\Sigma_a = \Sigma_u + \Sigma_c \tag{18}$$

Since by definition thermal utilization is the ratio of $\frac{\Sigma_u}{\Sigma_a}$ it can be written as

$$f = \frac{\Sigma_u}{\Sigma_u + \Sigma_c} \tag{19}$$

Since the macroscopic cross-section for absorption in fissionable material is

$$\Sigma_u = \sigma_u \frac{M_u}{V_R} \frac{a}{A_u} \tag{20}$$

where

σ_u = microscopic cross-section for absorption in fissionable material

M_u = critical mass

a = Avagadro's number

V_R = reactor volume

A_u = mass number of fissionable material

If the reactor volume can be expressed as $V_R = \gamma R^3$, then

$$\Sigma_u = \frac{\sigma_u M_u a}{\gamma R^3 A_u} \tag{21}$$

By substituting the values for f in equation (14) and rearranging terms, it can be written as:

$$\left(\frac{\beta^2}{3\Sigma_t \Sigma_c R^2} + 1\right) = \frac{\Sigma_u}{\Sigma_c} p\, n\, \epsilon\, e^{-I} - 1 \tag{22}$$

where

$$I = \frac{\tau_{th}}{R^2} \int_0^1 \alpha^2 \partial\left(\frac{\tau}{\tau_{th}}\right)$$

or the dimensionless equation which involves the critical mass of a nuclear heat power reactor is

$$\frac{\Sigma_u}{\Sigma_c} = \frac{\sigma_u M_u \quad a}{\Sigma_c \gamma R^3 \quad A_u} = \frac{\dfrac{\beta^2}{3\Sigma_t \Sigma_c R^2} + 1}{p\eta \epsilon e^{-I} - 1} \qquad (23)$$

In equation (23) note that α , β , γ are geometrical terms and the following expressions are physical

$$\frac{\sigma_u \quad M_u \quad a}{\Sigma_c \quad R^3 \quad A_u} ;$$

$$\Sigma_t \; \Sigma_c \quad R^2 ;$$

$$\frac{\tau_{th}}{R^2} ;$$

$$p ;$$

$$\eta ;$$

$$\xi ;$$

Accurate values for geometric factors are determined normally by numerical methods of integrations. In cases of unreflected, spherical reactors the geometrical factors approach α. In cases of systems of nuclear reactors where dimensionless parameters are the same one group of integrations will give solutions for the entire group. Equation (23) can be expressed also by referring each of the dimensionless ratios to the critical dimension R. It is desirable to demonstrate that a minimum value of fissionable mass exists for one value of critical dimension. The ratios can be expressed in terms of this minimum. Let it be defined thus:

$$\mu = \frac{M_u}{M_{min}} ; \qquad \rho = \frac{R}{R_{min}}$$

$$A = \frac{A_u \gamma \Sigma_c R^3_{min}}{a \quad _{min}} ; \qquad C = p \eta \epsilon$$

$$B = \frac{A_u \gamma \quad R_{min} \beta^2}{3\sigma_u \quad a \, M_{min} \Sigma_t}$$

$$D = \frac{\tau_{th}}{R^2_{min}} \int_0^1 \alpha^2 \, \partial \left(\frac{\tau}{\tau_{th}}\right)$$

Equation (23) can be written as

$$\mu = \frac{A\rho^3 + \beta\rho}{Ce^{-\frac{P}{\rho^2}} - 1} \tag{24}$$

or

$$\ln\mu = \ln(A\rho^3 + B\rho) - \ln(Ce^{-\frac{P}{\rho^2}} - 1) \tag{25}$$

A minimum would occur when the derivative is equal to zero or

$$\frac{d \ln\mu}{\partial\rho} = 0 = \frac{3A\rho^2 + B}{A\rho^3 + B\rho} - \frac{C(-2D\rho^{-3}e^{-\left(\frac{D}{\rho^2}\right)})}{Ce^{-(P\rho^2}-1)} \tag{26}$$

At this point μ and ρ are equal to 1. Thus a minimun occur when

$$\frac{3A + B}{A + B} - \frac{2D\,Ce^{-D}}{Ce^{-(D}-1)} = 0 \tag{27}$$

rewriting

$$\frac{\frac{3A}{B} + 1}{\frac{A}{B} + 1} = \frac{2\,DC}{C - e^D} \tag{28}$$

at the minimum point equation (24) is:

$$\frac{A + B}{Ce^{-(D}-1)} = \tag{29}$$

Equation (28) can be solved by numerical approximation for the ratio $\frac{A}{B}$ and thus obtain the critical radius. Equation (29) can be solved for critical mass.

Note that critical mass approaches infinity when ρ approaches infinity for which equation (24) becomes

$$\mu = \frac{A\rho^3}{C - 1} \quad \text{for } \rho$$

If Σ_c is relatively large and

$$\mu = \frac{B\rho}{C - 1} \quad \text{for large } \rho$$

The second case in which the critical mass approaches infinity is if Σc is relatively small.

and $\mu \to \infty$ and

$$C\, e^{\frac{P}{\rho^2}} = 1$$

or

$$\rho^2 = \frac{P}{\ln C} .$$

APPENDIX III

SOME NUMERICAL METHODS AS AN APPROACH TO RESOLVING REACTOR PROBLEMS

In reactor engineering calculations, numerical methods are used for resolution of complex mathematical expressions. In general, the many types of numerical methods can be separated into two classes.

1) Methods using approximate differentiations.

2) Methods of approximate integrations.

Since analyses and evaluations of nuclear-heat power-cycles involve many parameters and variables other than solution to nuclear equations, efforts are being made to achieve a rapid method of approximate solutions for particular reactor configuration and heat cycles. So that each case does not require complicated calculation techniques, efforts are being made to use multi-group methods for the complex cases. It is hoped that framework will be established and constants achieved to permit the following examples of the two methods for approximating solutions within given boundary conditions.

To illustrate the numerical method for approximate differentiation, equation (1), (2) and (3) can be written in spherical coordinates for a spherical reactor.

$$\frac{1}{r}\frac{\partial^2(rq)}{\partial r^2} = \frac{\partial(rq)}{\partial \tau} \qquad (1)$$

where $\nabla^2 q = \frac{1}{r}\frac{\partial^2}{\partial r^2}(rq)$

$$\frac{1}{3\xi_t}\frac{\partial^2}{\partial r^2}(r\eta v)_{th} - \xi_a(r\eta v)_{th} + p\,r\,q_{th} = 0 \qquad (2)$$

$$r\,q\,f = f\eta\xi \quad (r\eta v)_{th} \quad \xi_a \qquad (3)$$

Considering the above in terms of dimensionless terms; set

$$(r\eta v)_{th} = R\,A\,\varphi$$

$$(rq) = A\,\psi$$

Where A is the parameter which makes φ and ψ dimensionless; let

$$r = \xi R$$

$$\tau = \theta\tau_{th}$$

Substitution of these values give

$$\frac{\partial^2 \psi}{\partial \xi^2} = \frac{R^2}{\tau_{th}} \frac{\partial \psi}{\partial \theta} \tag{4}$$

$$\frac{1}{3\,\xi_t\, R} \frac{\partial^2 \varphi}{\partial \xi^2} - \xi_a R \varphi + p\, \psi_{th} = 0 \tag{5}$$

$$\psi_f = f\,\eta\,\xi\,\Sigma_a R\,\varphi \tag{6}$$

The above equations are now in suitable form to apply approximate differentiations to analyze the physical significance of the "slowing down" equation.

Consider the first derivative as

$$\frac{\partial \psi}{\partial \xi} = \lim_{\Delta\xi \to 0} \left(\frac{\psi(\xi + \Delta\xi, \theta) - \psi(\xi, \theta)}{\Delta\xi} \right) \tag{7}$$

Then the second derivative is defined as, (8)

$$\frac{\partial^2 \psi}{\partial \xi^2} = \lim_{\Delta\xi \to 0} \frac{\lim_{\Delta\xi \to 0} \frac{\psi(\xi + \Delta\xi, \theta) - }{\Delta\xi} - \lim_{\Delta\xi \to 0} \left[\frac{\psi(\xi, \theta) - \psi(\xi - b\,\xi\ \theta)}{\Delta\xi} \right]}{\Delta\xi}$$

If only small intervals of $\Delta\xi$ are considered, then approximately

$$\frac{\partial \psi}{\partial \xi} = \frac{\psi(\xi + \Delta\xi, \theta) - \psi(\xi, \theta)}{\Delta\xi} \tag{9}$$

and

$$\frac{\partial^2 \psi}{\partial \xi^2} = \frac{\psi(\xi + \Delta\xi, \theta) - 2\,\psi(\xi, \theta) + \psi(\xi + \Delta\xi, \theta)}{(\Delta\xi)^2} \tag{10}$$

In a similar manner:

$$\frac{\partial \psi}{\partial \theta} = \frac{\psi(\xi, (\theta + \Delta\theta) - \psi(\xi, \theta)}{\Delta\theta} \tag{11}$$

By substituting equations (10) and (11) into equation (4) and assume that

$$\frac{2}{(\Delta\xi)^2} = \frac{R^2}{\tau_{th}} \frac{1}{\Delta\theta} \tag{12}$$

Equation (4), becomes:

$$\frac{\psi(\xi + \Delta\xi, \theta) + \psi(\xi - \Delta\xi, \theta)}{2} = \psi(\xi\ \theta + \Delta\theta) \qquad (13)$$

Note that equation (13), expresses that the value of ψ at a point ξ and an age of $\theta + \Delta\theta$ is given by the average of its values at $\xi + \Delta\xi$ and $\xi - \Delta\xi$ at the previous neutron age θ. The intervals $\Delta\theta$ and $\Delta\xi$ are related by the assumption made in equation (12). The plot given in Figure 8 illustrates schematically. This plot is known as the "Schmidt Plot". Note that if a given initial condition is known, a later condition can be determined by graphical means. It is possible to consider two and three dimensional problems. Thus, the neutron flux equation would be

$$\varphi_{@\,\xi=0.6} = 0.6A + C\int_0^{0.6\xi}\int_0^{1} 3\Sigma_t \Sigma_a R^2\left(\frac{\varphi}{\varepsilon}\right) d\xi - \int_0^{0.6} d\xi_1 \int_0^{\xi_1} 3\Sigma_t R\rho\, \psi_{th}\, d\xi_2 \qquad (14)$$

For a beryllium oxide moderator

$\tau_{th} = 110\ cm^2$

$R^2 = 4400\ cm^2$

$\xi_t = 0.67\ cm^{-1}$

$R = 66.3\ cm$

$C\ \xi_u = .00452$

$3\xi_t\ \xi_u\ C\ R^2 = 40.0$

$3\xi_t\ R_e\rho = 133$

$A = 1.28$

$\varphi_{0.60} = 0.70$

$C = 0.74$

$\xi_u = .0061\ cm^{-1}$

Critical mass = 1.0 kg.

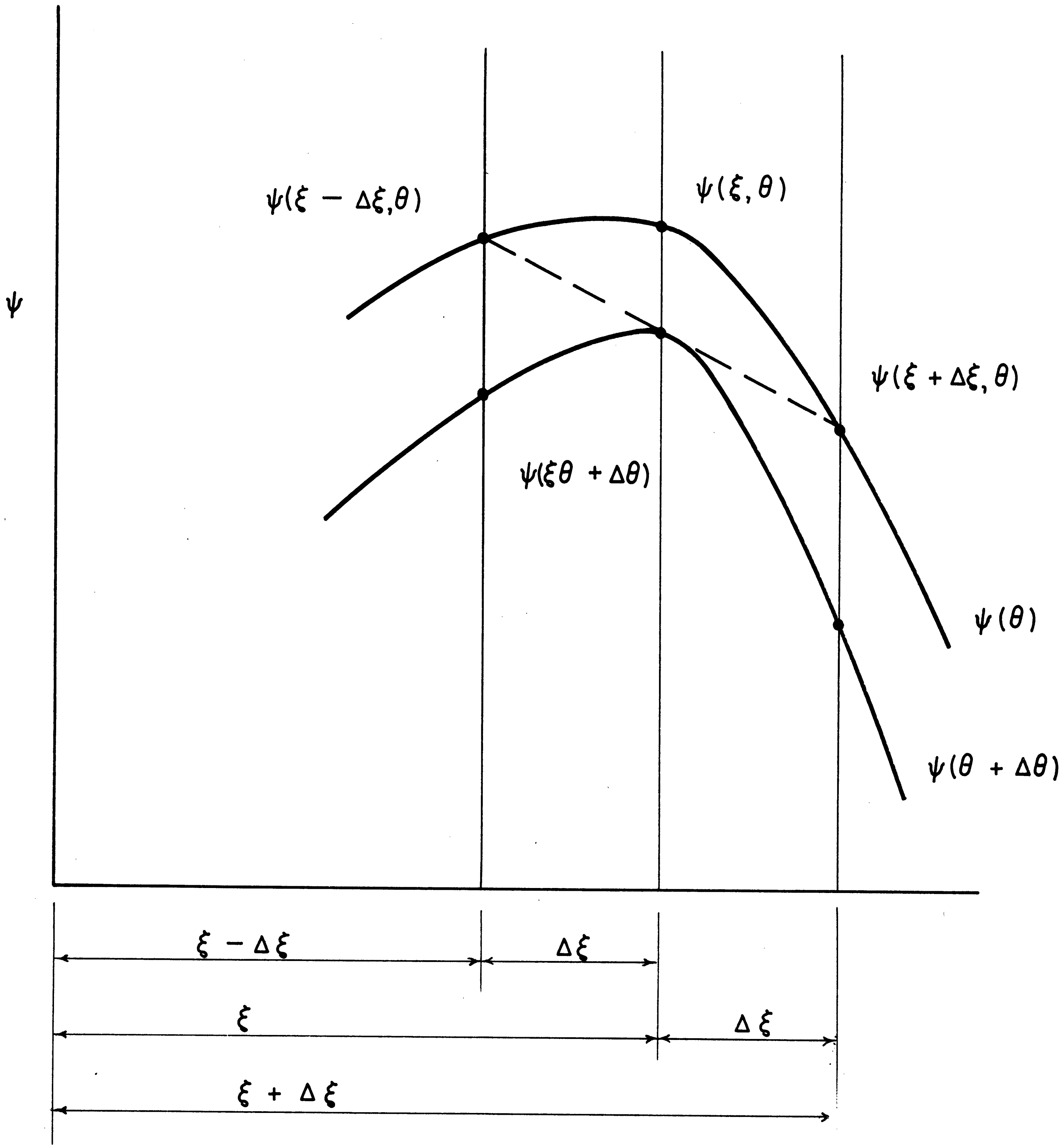
$\psi_f = 0.87\ \varphi$

The solution of the example is forced to steady state operation by

FIG. 8

EXAMPLE OF SCHMIDT PLOT

$$\frac{\Delta\xi}{2} = \frac{\tau\, th}{R^2}\,\Delta\theta$$



setting ψ_f obtained in equation,

$$(\psi_f = f \eta \xi \Sigma_a r \varphi)$$

Coincide with the ψ_f from the Schmidt plot. At some reference point say $\xi = 0.6$ so that equation,

$$\psi_f (\xi = 0.6) = C \eta \Sigma_\mu R \frac{\varphi}{C} (\xi = 0.6)^{= 0.6} \qquad (15)$$

Convergence of equation (15) can be obtained by the undetermined multiplier C so that values of calculated and assumed values for φ C $\xi = 0.6$ agree.

The integrations for one trial and subsequent trial and error solutions depend upon imposing certain boundary conditions.

Values for ψ_{th} are obtained from the Schmidt plot. Integrations are by the trapezoidal rule. For treat values of neutron flux φ, a sine wave distribution is assumed. The amplitude of the sine curve can be adjusted by adjusting values of an undetermined multiplier.

For numerical solution, it is assumed that a highly enriched U-235 reactor is moderated with beryllium oxide. For simplicity, it is assumed that the capture cross-section for absorbing materials is zero, so that

$\xi_a = \xi_u$ in the fuel

$\xi_a = 0$ in the reflector.

To illustrate the method of approximate integrations, an extension of the general problem is presented. The example considers the solutions of the equations for thermal neutron flux and to determine check points of slowing down densities formedfrom the Schmidt Plot. Equation (5) can be rearranged for numerical integration,

$$\frac{\partial^2 \varphi}{\partial \xi^2} = 3 \Sigma_t \xi_a R^2 \varphi - 3 \Sigma_t R p \psi_{th} \qquad (16)$$

An approximate solution to the first integration can be expressed as follows:

$$\frac{d \varphi}{d \xi} = A + \int_0^{\xi} \left\{ 3 \xi_t \xi_a R^2 \varphi - 3 \xi_t R \varphi \psi_{th}) \right\} d \xi_1 \qquad (17)$$

Second integration gives

$$\varphi = A \xi + B + \int_0^{\xi} d\xi_1 \int_0^{\xi_1} \frac{1}{3} \Sigma_t \xi_a R^2 \varphi - 3 \Sigma_t R \psi_{th}) d \xi_2 \qquad (18)$$

Since the neutron flux at $\xi = 0$ must be finite, i.e.,

$$\varphi_o = \frac{v(\eta v)}{A R} \quad @r = 0 \tag{19}$$

$$B = \varphi_o = 0$$

At $\xi = 1$, the flux is zero, and the constant A is

$$A = -\int_0^1 d\xi_1 \int_0^1 \left\{ 3 \Sigma_t \xi_a R^2 \varphi - 3 \Sigma_t R p \psi_{th} \right\} d\xi_2 \tag{20}$$

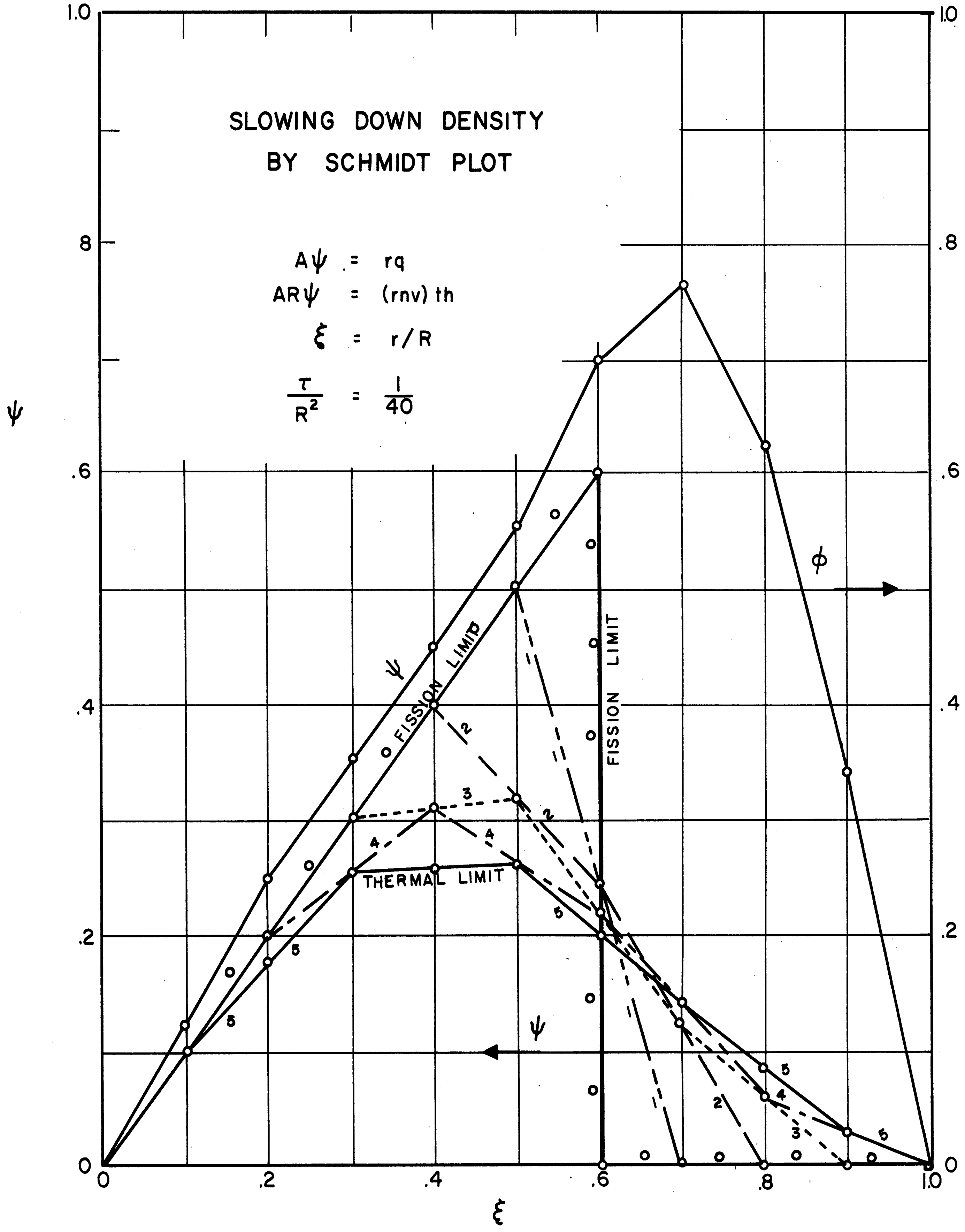
The problem is to find a numerical solution for equation (18). A common method is iterations, in which treat values for the φ function are substituted until they match the φ calculated on the left hand side and satisfy boundary conditions.

Figure 9 gives the Schmidt Plot for "Slowing Down Density." (See Page 32 Thompson lecture notes).

Table VIII summarizes the calculated data for this case.

By continual application of equation similar to those presented to introduce factors of changing material properties, variable extrapolation lengths, energy changes, etc., so that introductions of new systems, new geometries, new masses etc., can be evaluated.

FIG. 9



SLOWING DOWN DENSITY
BY SCHMIDT PLOT
Aψ = rq
ARψ = (rnv) th
ξ = r/R
τ/R² = 1/40
ψ
φ
FISSION LIMIT
FISSION LIMIT
THERMAL LIMIT
0
.2
.4
.6
.8
1.0
ξ

TABLE VII

FIRST TRIAL OF APPROXIMATE INTEGRATIONS

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
ξ	$\partial\xi$	ψ_{th}	$(3\ \partial\xi)$	$\int 4$	$(5 \times \partial\xi)$	$\int 6$	$\left(\frac{133\ x}{7}\right)$	ρ/ρ	$\left(\frac{9\ \times}{\partial\xi}\right)$	$\int 10$	$\left(\frac{11\ \times}{\partial\xi}\right)$	$\int 12$	$\left(\frac{40\ x}{13}\right)$	$A\xi$	$\left(\frac{15+14}{-8}\right)$	$0.87\ x\ 16\ \psi F$
0		0		0		0	0	0		0		0	0	0	0	0
-	.1	-	.005	-	.0003	-	-	-	.016	-	.0008	-	-	-	-	-
.1	-	.10	-	.005	-	.003	.04	.31	-	.016	-	.0006	.03	.13	.12	.10
-	.1	-	.014	-	.0012	-	-	-	.045	-	.0039	-	-	-	-	-
.2	-	.18	-	.019	-	.0015	.20	.59	-	.061	-	.0047	.19	.26	.25	.22
-	.1	-	.022	-	.0030	-	-	-	.070	-	.0096	-	-	-	-	-
.3	-	.25	-	.041	-	.0045	.60	.81	-	.131	-	.0143	0.57	.38	.35	.30
-	.1	-	.027	-	.0055	-	-	-	.088	-	.0175	-	-	-	-	-
.4	-	.27	-	.068	-	.0100	1.33	.95	-	.219	-	.0138	1.27	.51	.45	.39
-	.1	-	.026	-	.0082	-	-	-	.098	-	.0268	-	-	-	-	-
.5	-	.28	-	.096	-	.0182	2.42	1.00	-	.317	-	.0580	2.34	.64	.56	.49
-	.1	-	.025	-	.0109	-	-	-	.098	-	.0366	-	-	-	-	.061
.6	-	.21	-	.121	-	.0231	3.87	.95	-	.415	-	.0952	3.90	.77	.70	0
-	.1	-	.016	-	.0130	-	-	-	0	-	.0415	-	-	-	-	-
.7	-	.15	-	.139	-	.0421	5.60	.81	-	0.415	-	.1367	5.46	.90	.75	0
-	.1	-	.012	-	.0145	-	-	-	0 ξ=20	-	.415	-	-	-	-	- } 3=0
.8	-	.09	-	.151	-	.0566	7.53	.59	-	.415	-	.1782	7.13	1.02	.62	0
-	.1	-	.006	-	.0154	-	-	-	0	-	.0415	-	-	-	-	-
.9	-	.03	-	.157	-	.0720	9.57	.31	-	.415	-	.2197	8.76	1.15	.34	0
-	.1	-	.002	-	.0158	-	-	-	0	-	.0415	-				
1.0	-	0	-	.159	-	.0875	11.68	0	-	.415		.2612	10.40	1.23	0	0