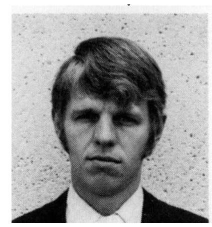
Some time-saving methods for the digital simulation of highway vehicles

by
James E. Bernard
Physical Factors Group
Highway Safety Research Institute
University of Michigan
Ann Arbor, Michigan 48105



JAMES BERNARD is an associate research engineer at the University of Michigan's Highway Safety Research Institute. He earned the PhD degree from the Engineering Mechanics Department of the University of Michigan, where he was an instructor from 1967 to 1970. His professional work has been mainly concerned with the mathematical modeling and simulation of dynamic systems. At General Motors he programmed the hybrid simulation and graphic display of vehicle ride, including a normal-mode approach to the automobile-shake problem. At the University of Michigan, in addition to teaching basic mechanics courses, he had responsibility for the engineering-mechanics analog computer laboratory. His previous work at the Institute has included development of an eleven-degrees-of-freedom hybrid model for studying the braking performance of articulated vehicles, and development of digital-computer-based mathematical models for the study of truck and tractor-trailer braking-and-handling characteristics. He presently is heading a simulation effort with several facets, including the simulation of a doubles (tractor-semitrailer-full-trailer) combination and the development of antiskid braking models. He is a member of ASME and SCS.

Dr. Bernard is devoted to his "beautiful wife and two precocious children." In his free time "such as it is," he enjoys duplicate bridge and handball.

ABSTRACT

Simulation has been used extensively as a tool for the solution of vehicle-dynamics problems. To handle nonlinear simulations of increasing size and complexity, both digital and hybrid methods have been used. As might be expected, purely digital simulation often proves to be more convenient, while hybrid proves to be more economical.

Methods have been developed to provide substantial economies in the digital simulations. Savings by roughly a factor of five may be realized by transforming the wheel-spin integrations into a solvable set of algebraic equations and by making use of some well-known mechanical characteristics of vehicles to simplify the integration procedure.

1 INTRODUCTION

The problems of vehicle handling appeared in the literature as long ago as 1925, when the pioneering analysis of Broulhiet[1] was published. Subsequent investigators developed linearized equations whose solution would yield the trajectory of a vehicle subject to time-varying steering or braking. More recently, efforts have been made to analyze various nonlinear aspects of the vehicle system, including, most notably, nonlinear tire properties. Perhaps the best source of an overview of this subject has been given by Ellis.[2]

Since the equations of vehicle motion can become quite difficult to handle in the general case, it is not surprising that simulation has been a tool frequently used by vehicle dynamicists. Perhaps the best-known early computer simulation was developed in 1961 by Ellis,[3] who developed a three-degrees-of-freedom analog-computer model for studying the lateral motion of an articulated vehicle. Since that time, the advent of more and more sophisticated computing equipment has led to the possibility of simulations of increasing complexity. Presently, many research facilities make use of highly nonlinear passenger-car simulations with at least fourteen degrees of freedom, including six degrees of freedom for the vehicle body (the so-called sprung mass), a vertical or "wheel hop" degree of freedom for each wheel (or unsprung mass), and a spin degree of freedom for each wheel. (See, for example, Speckhart.[4])

LIST OF SYMBOLS

FX	Longitudinal force at the tire-road interface
FY	Lateral force at the tire-road interface
JS	Spin moment of inertia of a wheel
N	Normal force at the tire-road interface
RR	Rolling radius
S	Longitudinal slip
uw	wheel-hub longitudinal velocity
uwd	$\frac{d}{dt}\,uw$
α	Tire sideslip angle
μ_o	Nominal coefficient of friction at the tire-road interface
Ω	Wheel-spin rate

The important sprung-mass motions tend to be in the lowest frequency range. A simulation which only includes sprung-mass motions may be programmed safely with a maximum expected frequency of 2 Hz; thus, when a vehicle model which is to include only sprung-mass motions is to be integrated digitally, a time step of .02 second or even larger is usually used. The inclusion of the vertical degree of freedom for the wheels is more demanding; one normally expects the tire wheel-hop frequency to be about 10 Hz. If the wheel-hop degree of freedom is included in the model, an integration time step of .005 second may be chosen. (See McHenry and Deley.[5] Note also that in some cases it may be possible to "get away" with larger intervals, especially in a smooth-road maneuver in which the wheel-hop mode of oscillation tends not to be excited.)

The spin degree of freedom for the wheels is more difficult to categorize. While the spin rate may be easily computed under some conditions,* the initiation or cycling of brake torque (as in an antiskid system) may lead to rapidly changing spin derivatives requiring a digital time-step that is orders of magnitude shorter than those required for the rest of the simulation. Such a system of equations, of course, lends itself to hybrid computation. The analog integration of the wheel-spin equations poses no special problems, and the complicated algebra required in the computation of the shear forces at the tire-road interface can be done digitally.

There are, however, conditions when the user is constrained to use digital facilities. For example, work was begun at the Highway Safety Research Institute (HSRI) in early 1971 with the goal of the creation of a digital simulation of the handling motions of articulated vehicles. Digital simulation was chosen for two reasons: (1) the final size of the program was expected to severely tax available hybrid facilities, and (2) the programs were to be delivered to several companies for their own in-house use. Since it was expected that the costs of exercising this simulation could easily become prohibitive, special methods were developed to make the digital integration more economical. These methods, which have increased our in-house computation speed[†] by a factor of nearly five, will be discussed in some detail below.

Initially, a new method for handling the spin degree of freedom of the wheels will be considered. As a prelude to this discussion, we shall consider briefly the mechanics of the tire-road interface and point out some methods previously used to deal with the wheel-spin problem.

2 THE SHEAR FORCES AT THE TIRE-ROAD INTERFACE

It seems obvious that, since the vehicle trajectory is almost entirely dependent on forces applied to the vehicle from the road, the representation of the tire-road interface is of primary interest in vehicle simulation. We will restrict our attention here to smooth road maneuvers in which the normal force on the tire may be expected to be a straightforward function of the vertical position of the tire, and examine briefly the character of the shear forces. A schematic diagram of the relevant kinematics and some experimental data are given in Figures 1, 2, and 3.

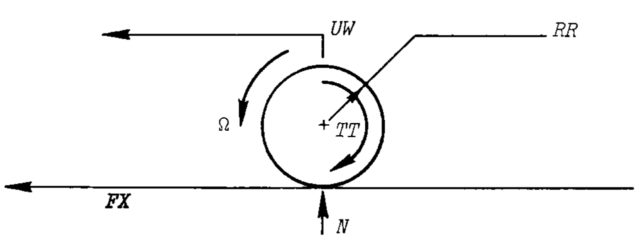


Figure 1a - A rolling tire with applied brake torque

In Figure 1a is shown a tire rolling in planar motion, while some curves relating the ratio of longitudinal force to normal force FX/N and longitudinal slip S are given in Figure 1b. The longitudinal slip S is defined as follows:

$$S = 1 - \frac{RR \cdot \Omega}{uw} \tag{2.1}$$

where RR is the rolling radius, Ω is the spin velocity, and uw is the longitudinal translational velocity of the wheel center.

The so-called μ-slip curves shown in Figure 1b are typical of measurements of braking forces produced by an automobile tire on a dry surface. In most cases, it is reasonable to expect that the part of the curve characterizing the very-low-slip region will be characterized by a nearly vertical linear characteristic which shows only nominal changes with normal load and vehicle speed. The rest of the curve, however, may be expected to be a function of both normal load N and hub speed uw.

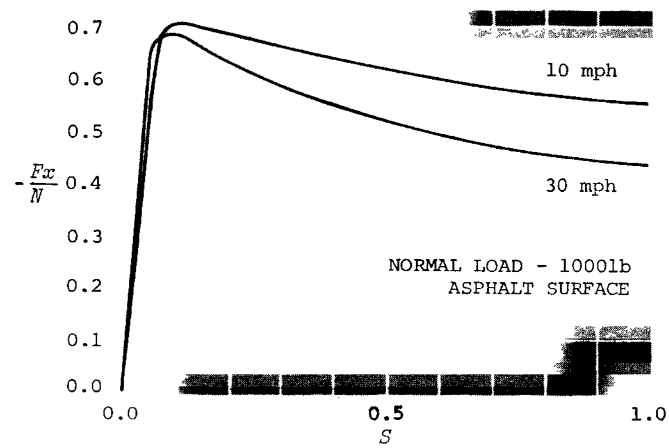


Figure 1b - Empirical "μ-slip" curves for a passenger car tire

In Figure 2a a sketch of a tire rolling with angular velocity Ω and sideslip angle α is given. Under the free-rolling condition

$$RR \cdot \Omega \doteq uw \tag{2.2}$$

the lateral force FY may readily be measured. These measurements may be given in carpet-plot form as shown for a new 10.00-20/F truck tire in Figure 2b. Again, the lower portions of the curves tend to be linear, although in this case, the slope changes quite markedly with normal load.

If, as in any maneuver involving braking and turning, longitudinal slip and sideslip angle are simultaneously nonzero, complex interrelationships govern the generation of FX and FY. A sample of such an interrelationship is given in Figure 3. It is clear from this figure, in which empirical data measured for a passenger-car tire is given, that the generation of braking forces seriously impedes the generation of lateral forces. This fact is at the heart of many vehicle stability problems.

3 SOME MODELING TECHNIQUES AND THEIR RESULTS

All vehicle-handling simulations must make an attempt to model the phenomena discussed above. Simplifications may, of course, be made; certain problems may be considered even though the relationships depicted in the figures may be modeled in a quite elementary fashion. Consider, for example, the diagram of the rolling tire given in Figure 1a. Neglecting any yaw-plane dynamics, the equations of motion of the wheel may be written

$$JS \cdot \dot{\Omega} = -FX \cdot RR - TT \tag{3.1}$$

*A trivial but not uncommon example occurs when the wheels remain locked due to excessive brake torque, resulting in identically zero wheel-spin rate.

†On an IBM 360/67 through the Michigan Terminal System.

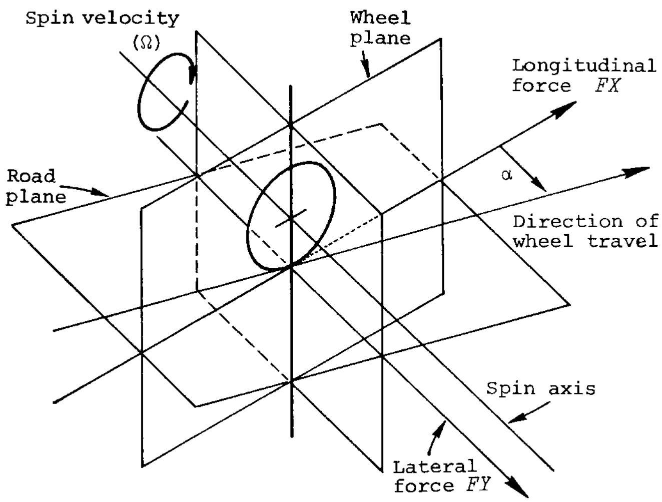


Figure 2a - Longitudinal and lateral forces at the tire-road interface

where

JS is the polar moment of inertia,

TT is the applied brake torque.

The dot indicates differentiation with respect to time.

In many early simulations, Equation 3.1 has been simplified by the assumption that the $JS \cdot \dot{\Omega}$ term is negligible. This results in

$$FX = -\frac{TT}{RR} \qquad (3.2a)$$

for $\frac{TT}{RR} \leq \mu_o N$

and $FX = -\mu_o N \qquad (3.2b)$

when $\frac{TT}{RR} > \mu_o N$

where the nominal friction coefficient μ_o is chosen with appropriate empirical data in mind.

Since the rotational terms are in fact negligible under some conditions, this approximation may in some cases lead to meaningful results. Further, the fact that the wheel-spin degree of freedom has been neglected does not preclude the study of vehicle handling. Possibly the best known method for the inclusion of the effect of longitudinal slip on the generation of lateral force without explicitly considering wheel rotation is the "friction circle"* concept. This method will be summarized briefly here:

a) The brake force $FX(t)$ is an input function to the simulation, limited only by the product of the nominal coefficient of friction and the normal force as in Equation 3.2b.

b) In the absence of brake forces, the lateral-force characteristics are computed as a cubic function of sideslip angle α.

$$FY = -\frac{3}{2}\frac{\mu_o N}{\alpha_m}\left(\alpha - \frac{\alpha^3}{3\alpha_m^2}\right); \quad |\alpha| < \alpha_m \qquad (3.3a)$$

*The friction circle was later modified to be a "friction ellipse." (See, for example, Ellis.[2])

$$FY = -\mu_o N \frac{\alpha}{|\alpha|}; \quad |\alpha| \geq \alpha_m \qquad (3.3b)$$

where

α_m is the sideslip angle of "saturation" of the side force (taken by Ellis[3] to be 12°).

To approximate the effect of longitudinal slip on side-force generation, *FY* is limited by the boundary of the friction circle.

$$|FY| < [(\mu_o N)^2 - FX^2]^{1/2} \qquad (3.4)$$

Thus, the effects of sideslip angle on the brake forces are neglected, and the effects of longitudinal slip on side force are simulated only when the side force as computed in (3.3) violates inequality (3.4). In spite of these approximations, this tire model may predict the trends of vehicle motion quite successfully. A recent development, however, has necessitated the detailed consideration of the wheel-rotation rate. Certain sections of Motor Vehicle Safety Standard 121, effective in September 1974, will require that:

a) Commercial vehicles have the capability to produce a steady-state deceleration of more than 17 feet/second^2 from 60 mph to a stop without prolonged wheel lockup on a surface characterized by skid number 75. (The skid number is equal to one hundred times the ratio of locked wheel brake force to normal load of a standard passenger car tire when tested at a specified load and velocity. See Reference 6 for details.) During this stop the vehicle must be kept in a twelve-foot lane.

b) Commercial vehicles have the capability to produce a steady-state deceleration of more than 8 feet/second^2 from 20 mph to a stop without prolonged wheel lockup on a surface characterized by skid number 30. During this stop the vehicle must be kept in a twelve-foot lane.

Since commercial vehicles are commonly subject to wide extremes of payload, and since substantial utilization of available friction is required on both wet and dry surfaces, vehicle manufacturers are now giving serious consideration to the use of antiskid brakes to prevent prolonged wheel lockup. There is a variety of antiskid designs which will prevent wheel lockup during braking by automatic

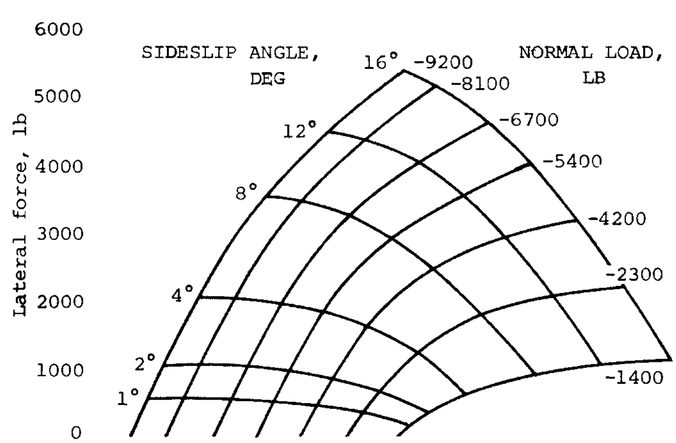


Figure 2b - Lateral force versus sideslip angle at various normal loads

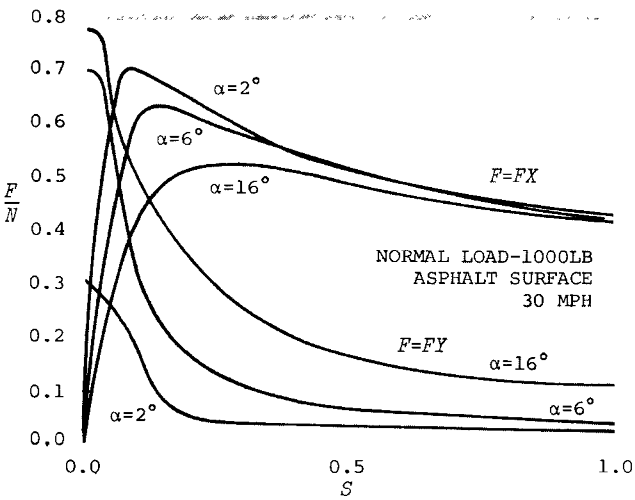


Figure 3 - Lateral force and longitudinal force vs. slip at various sideslip angles

regulation of pressure within the braking system. However, the device controlling the pressure invariably uses the wheel speed Ω or its time derivatives as control variables; thus the simulation of vehicles with antiskid brakes requires a spin degree of freedom for the wheels. This requirement poses severe problems in digital simulation. While a reasonable time-step for digital integration of the vehicle motion might be about 0.005 second (thus accommodating the 10-Hz wheel-hop frequency), the step required for the wheel rotation is many times smaller, with time steps of 0.0001 second not unusual. Thus, to make computations as economical as possible, many investigators[5,7,8] were led to hold all kinematic variables (except wheel-spin rate) as well as the input brake torque and sideslip angle α constant during a short wheel-rotation integration time-step, and update them on a schedule determined by the integration time-step of the main body of the simulation. While such a method certainly leads to reasonable results, the increased computational expense over models with nonrotational wheels is obviously very significant, in some cases by as much as a factor of four.

The methods discussed below use the previously developed idea of updating certain variables for use in the wheel-spin calculations only on completion of the interval of integration appropriate for the vehicle motion. However, it will be shown that this may be done in a manner which allows generation of solvable differential equations of wheel rotation. In this way, the added costs of integration of the wheel rotation are virtually eliminated.

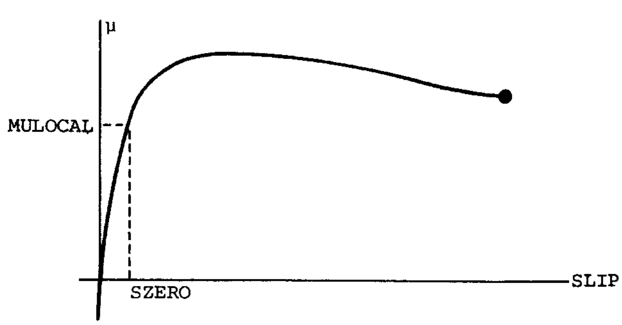


Figure 4 - A μ-slip curve

4 THE ROTATIONAL DEGREE OF FREEDOM

It is our purpose in this section to consider the spin motion of the wheel without allowing variation in kinematic variables such as the normal force N and the brake torque T. Although the existence of a vehicle longitudinal acceleration uwd is admitted, the time range of validity of the following analysis must be short enough to allow uw to be reasonably approximated by a constant. Thus the differentiation of the longitudinal slip S with respect to time yields

$$\dot{S} = \frac{RR \cdot \dot{\Omega} \cdot uw - RR \cdot \Omega \cdot uwd}{uw^2} \qquad (4.1)$$

The combination of (4.1) and (3.1) leads to

$$\dot{S} = \frac{-RR}{uw \cdot JS}[-TT - FX \cdot RR] + uwd\frac{(1-S)}{uw} \qquad (4.2)$$

Let $SZERO$ be the value of S at $t=t_0$ for a μ-slip curve in the form shown in Figure 4. At $S = SZERO$,

$$FX = -MULOCAL \cdot N \qquad (4.3)$$

Expanding the μ-slip relationship in a Taylor series about $S = SZERO$,

$$\mu = MULOCAL + \frac{\partial\mu}{\partial S}(S-SZERO) \qquad (4.4)$$
$$+ \text{ higher-order terms}$$

Neglecting the higher-order terms, FX may be written

$$FX = \bar{\eta} + \bar{\beta}(S) \qquad (4.5)$$

where

$$\bar{\eta} = -N\left(MULOCAL - \frac{\partial\mu}{\partial S} \cdot SZERO\right) \qquad (4.6)$$

$$\bar{\beta} = -\frac{\partial\mu}{\partial S}N \qquad (4.7)$$

Combining Equations 4.5 and 4.2 yields

$$\dot{S} + Q \cdot S = F \qquad (4.8)$$

where

$$\eta = -TT/JS \qquad (4.9)$$

$$\beta = -RR/JS \qquad (4.10)$$

$$F = \frac{-RR}{uw}(\eta + \beta\bar{\eta}) + \frac{uwd}{uw} \qquad (4.11)$$

$$Q = \frac{RR}{uw}\beta\bar{\beta} + \frac{uwd}{uw} \qquad (4.12)$$

The solution to Equation 4.7 is

$$S = \left(SZERO - \frac{F}{Q}\right)e^{-Q(t-t_0)} + F/Q \qquad (4.13)$$

Equation 4.13 may be solved for S by updating $SZERO$, F, and Q at the beginning of each integration time step, and the brake forces may then be found from the μ-slip relationship.

This method is general in the sense that any continuously differentiable μ-slip relationship is admissible. Further, it should be noted that the initial assumptions of constant normal force and brake torque[†] during the time step must be reasonable if the integration time step is reasonable; these variables are also used in the calculation of the sprung- and unsprung-mass derivatives, which are assumed constant during the time step.

5 ECONOMICS BASED ON INTEGRATION TECHNIQUE

With the wheel-rotation differential equations written in the form suggested in the previous section, no integration of the spin degree of freedom is required, and the solution for the brake force becomes an algebraic calculation only. Thus the vehicle dynamics problem may again be considered in two separate frequency bands--the motions of the sprung mass, which tend to be in the range of 1 Hz, and the wheel-hop motions, which tend to be in the range of 10 Hz.

As one might expect, digital integration of equations of the type considered here requires that CPU time will largely be spent in calculating time derivatives rather than in the integration process. This happens because the forces are not straightforward functions of the state variables. Further, the calculation of the wheel-hop derivatives tends to be a much more straightforward proposition than the calculation of the sprung-mass derivatives. Thus, the bulk of the computation time at each step is spent dealing with the slowly varying sprung-mass motions, while the time step must be chosen to accommodate the more rapid wheel-hop motions.

One solution to this seeming paradox is to consider the wheel-hop equations of motion and sprung-mass equations of motion separately, each with its own time step. This, of course, requires extensive bookkeeping, much in the manner of the usual methods of numerically integrating the spin degree of freedom separately from the rest of the vehicle motions. A much simpler, but nevertheless effective, technique based on a modification of Hamming's Predictor-Corrector method as found in HPCG, an IBM-produced digital integration subroutine,[9] is given below. The following methodology is used in HPCG:

a) Given the dependent variables $Y(t)$ and their time derivatives $DERY(t)$, the $Y(t+\Delta t)$ are computed.

b) $DERY(t+\Delta t)$ are calculated based on the $Y(t+\Delta t)$.

c) Based on $Y(t)$, $Y(t+\Delta t)$, and $DERY(t+\Delta t)$, the $Y(t+\Delta t)$ are "corrected."

d) The corrected $Y(t+\Delta t)$ are used to recalculate $DERY(t+\Delta t)$.

[†]It is assumed here that the antiskid system cycles at 10 Hz or less. This is entirely reasonable for air brake systems and for vacuum-assisted hydraulic systems.

It is in steps (b) and (d) that the vast majority of the computations are performed, since in each of these steps the time derivatives must be calculated. Substantial savings may be made by observing that the sprung-mass derivatives, since their frequency band is far below those for which the time step Δt was chosen, will not require correction in step (d).

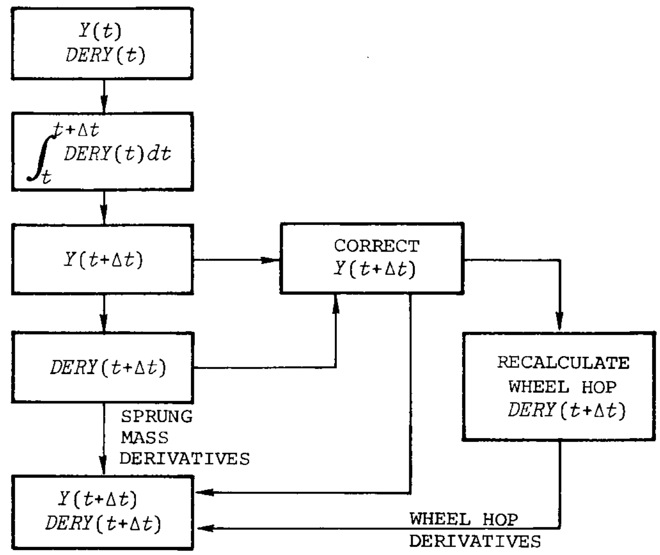


Figure 5 - A flow chart for modified *HPCG*

This technique, which is illustrated in Figure 5, has proven very satisfactory in the simulation of even the most violent maneuvers, showing virtually no difference from calculations in which HPCG is used in its original form. This has led to savings in the HSRI vehicle simulation of the order of 80%.

6 CONCLUSIONS

Savings of approximately a factor of five in CPU time have been realized in a large-scale passenger-car simulation by casting the wheel spin calculations into a form subject to algebraic solution rather than digital integration and by using Hamming's Predictor Corrector Method to modify the integration subroutine.

ACKNOWLEDGEMENT

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REFERENCES

1 BROULHEIT G
Suspension of Automobile Steering Mechanism: Shimmy and Tramp
Bulletin 78 Société des Ingénieurs Civils de France 1925

2 ELLIS J R
Vehicle Dynamics
Business Books Ltd London 1969

3 ELLIS J
The Dynamics of Vehicles During Braking
Symposium Control of Vehicles Automobile Division London:Inst Mech Engrs 1963 pp 20-29

4 SPECKHART F H
A Computer Simulation for Three-Dimensional Vehicle Dynamics
SAE Paper Number 730526 May 1973

5 McHENRY R DELEYS N
Vehicle Dynamics in Single Vehicle Accidents - Validation and Extensions of a Computer Simulation
Cornell Aeronautical Laboratory, Inc
Report No VJ-2251-V-3 December 1968

6 ASTM Standard E 274-70

7 KRAUTER A WILSON R
Simulation of Tractor-Semitrailer Handling
SAE 720922 1972

8 NICHOLAS N COMSTOCK T
Predicting Directional Behavior of Tractor-Semitrailers When Wheel Anti-Skid Brake Systems Are Used
ASME 72-WA/ANT-16 1972

9 System/360 Scientific Subroutine Package Version III Programmers Manual