Consistency Theory Is Alive and Well

Frank Harary
University of Michigan

Abelson asks the question in his title, "Whatever became of consistency theory?" In this discussion of his article, the above title gives my answer, "Consistency theory is alive and well." Actually, that is a substantial understatement. More accurately, consistency theory is thriving. Abelson comments on his observation of this phenomenon with puzzlement and wonder. Plausibility considerations will be presented here to serve as a provisional explanation.

Abelson notes that the Cartwright and Harary (1956) article was the stimulus for his article with Rosenberg (1958), which they called "symbolic psycho-logic." It does not appear to be generally known in the psychological literature that as a mathematician working at the Research Center for Group Dynamics (RCGD) I discovered the structure theorem for signed graphs, which was published in a journal (Harary, 1953) that is less well known than *Psychological Review*. When I walked into Doc Cartwright's office (he was then the Director of the RCGD) and showed him the theorem characterizing the balance in signed graphs, he exclaimed, "Well I'll be darned. You have independently rediscovered Heider's theory of balance, and you have generalized it to signed structures of any size." This took place near the beginning of our long collaboration, during which we met every Tuesday morning for a quarter of a century, and which led to our 1956 article, which (surprisingly) is still widely cited. This I attribute to Doc's remarkably clear writing style and to his endowing with psychological verisimilitude my simple, straightforward mathematical observations.

In their addition and multiplication tables for psycho-logic, Abelson and Rosenberg (1958) presented an algebraic system with letters i, p, n, and a as the initials for indifference, positivity, negativity, and ambivalence. It is precisely isomorphic to the four-element boolean algebra (when a mathematician become immortal, his name is converted to a lowercase adjective—as euclidean or cartesian or boolean), in which the two atoms are p and n, their union (addition or disjunction) is the symbol a, and their intersection (multiplication or conjunction) is i. Thus, this A-algebra is really a special case of the very well-known B-algebra.

Abelson also mentions that the dissonance theory (which was not actually a theory, as it lacked consistency) of Festinger (1957) is no longer regarded as an active subject of study. The same is also somewhat true of the congruity theory of Osgood and Tannenbaum (1955), to which I owe a personal debt as it suggested that a graph can have signed points (positive or negative people or

Personality and Social Psychology Bulletin, Vol. 9 No. 1, March 1983 60-64

issues) as well as signed lines (standing for positive or negative affect relationships). Calling a marked graph one whose points (but not its lines) are signed, Beineke and I defined consistency in marked graphs and digraphs (1978a, 1978b) analogously to balance in signed graphs. This approach enabled Kabell and me (1980, 1981) to solve simply what had been regarded as a formidable open question: to determine efficiently, using an easy algorithm based on a theorem (mathematical of course—there are no theorems in any other subject), whether or not a given signed graph is balanced, even though it might be rather large and complex.

A major reason for the vitality of some versions of consistency theory is that balance theory is at their core. Why is this the source of their vitality? The answer is that balanced and clusterable (Cartwright & Harary, 1968) signed graphs provide a logically sound, empirically applicable, mathematical model for diverse phenomena. It is this wide applicability of balanced signed graphs extending throughout not only the social but also the physicial sciences that accounts for its citation index. Most recently, balance theory has been generalized to other mathematical models, including matroids (Harary & Lindstrom, 1981) and partially ordered sets (Harary & Sagan, in press).

Here are a few examples of the diverse applications of balance theory. An early effort in applying balanced signed graphs was made (Cartwright & Harary 1959) when we analyzed the classic paper by Freud on developmental psychology in terms of signed graphs and observed a tendency toward balance. In another article (Harary, 1961) international relations were modeled in terms of signed graphs, taking nations as points, with a positive line joining two points standing for allies and a negative line for enemies. The chronological sequence of signed graphs during the Suez Canal war of 1956 clearly displayed a tendency toward balance. The limitations of using a tendency toward balance for predicting which changes would occur in signed graph structures was discussed, and suggestions for such predictions were proposed. In a detailed computerized study of the many such changes that took place during thirty years of warfare in Europe during the time of Bismarck, Healy and Stein (1973) verified that these hypotheses did indeed serve as reasonably accurate predictors. Several other international signed graphs were also observed to follow this pattern, as noted in a later article (Harary, 1977). It appears quite reasonable to assume that the rationale for changes in international relationships can be directly coordinated with changes in interpersonal and other affective-cognitive relationships.

Cartwright and Harary (1977) proposed a mathematical model in which a specified portion of a structure is regarded as the system at hand and the remainder of the structure as its environment. This approach leads to the analysis in terms of structural balance (Harary, 1982) of variations on the Golden Rule, which were applied directly (Hage and Harary, 1982) to kinship systems in anthropology.

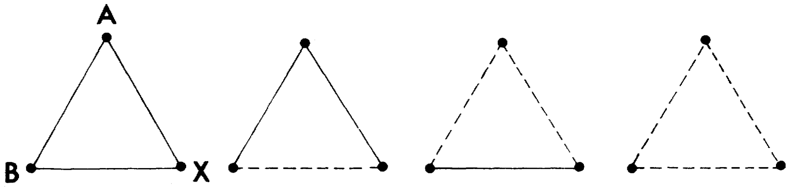

Figure 1: The four signed triangles.

Abelson asserts, "My position is that *emotion arises whenever alternative construals of events with sharply different hedonic import for an individual are concurrently mentally exercised.*" This sentence is in complete accord with the viewpoint that imbalance is stressful, when interpreted and illustrated by the four signed triangles of Figure 1, in which a dashed line (being built from minus signs) is negative and the solid lines are positive. But first it seems that the essence of Abelson's assertion can be restated in smaller words as follows: *Emotion arises when both positive and negative feelings are simultaneously present.*

The triangles of Figure 1 are the building blocks for the cognitive theory of the eminent philosopher-psychologist Fritz Heider (1958). When the top point of each triangle is labeled A and the two base points B and X, the results are typical ABX triangles of Newcomb (1953). There are two paths in each triangle from A to X: the direct, single-step path, containing only the line AX and the two-step path from A to B to X. In the first and third triangles, which are balanced, these paths have the same sign—positive and negative, respectively. In the other two (unbalanced triangles) one of these paths is positive and the other is negative, which suggests that Abelson's assertion can be restated in this form: *Emotion arises in states of imbalance.* This statement is quite similar to the approach of Heider (1958).

In his Figure 2, Abelson lists the affective intensity associated with congruity and with various states of incongruity and suggests that the intensity values are 0, +, ++, and +++. These values correspond to those proposed by Cartwright and Harary (1970) for various states of ambivalence in which the positive and negative components need not be the same.

In summary, consistency theory is alive, well, and thriving—a sentiment echoed by my fellow discussant, Robert Zajonc. An early survey of balance in signed graphs is given in Chapter 9 of the book "Structural Models," which is clear because the final draft (Harary, Norman & Cartwright, 1965) was written by Doc Cartwright. More recently, a review of balance and clusterability as utilized in the psychological literature was presented in Cartwright and Harary (1979). The best mathematical survey is due to Zaslawski (1982), who has developed a logical tour de force with his comprehensive survey of and far-reaching contribution to the theory of signed graphs.

REFERENCES

Abelson, R. P., & Rosenberg, M. J. Symbolic psycho-logic: A model of attitudinal cognition. *Behavioral Science,* 1958, 3, 1-13.

Beineke, L. W., & Harary, F. Consistent graphs with signed points. *Rivista di matematica per le scienze economiche e sociale,* 1978, 1, 81-88. (a)

Beineke, L. W., & Harary, F. Consistency in marked digraphs. *Journal of Mathematical Psychology,* 1978, 18, 260-269. (b)

Cartwright, D., & Harary, F. Structural balance: A generalization of Heider's theory. *Psychological Review,* 1956, 63, 277-293.

Cartwright, D., & Harary, F. A note on Freud's "Instincts and their vicissitudes." *International Journal of Psychoanalysis,* 1959, 40, 287-290.

Cartwright, D., & Harary, F. On colorings of signed graphs. *Elemente der Mathematik,* 1968, 23, 58-59.

Cartwright, D., & Harary, F. Ambivalence and indifference in generalizations of structural balance. *Behavioral Science,* 1970, 15, 497-513.

Cartwright, D., & Harary, F. A graph theoretic approach to the investigation of system-environment relationships. *Journal of Mathematical Sociology,* 1977, 5, 87-111.

Cartwright, D., & Harary, F. Balance and clusterability: An overview. In P. Holland and S. Leinhardt (eds.) *Perspectives on Social Network Research.* New York: Academic Press, 1979.

Festinger, L. *A Theory of Cognitive Dissonance.* Evanston, IL: Row Peterson, 1957.

Hage, F., & Harary, F. Reciprocity in kinship nets. *Cambridge Anthropology,* 1982, 7, 39-43.

Harary, F. On the notion of balance of a signed graph. *Michigan Mathematical Journal,* 1953, 2, 143-146.

Harary, F. A structural analysis of the situation in the Middle East in 1956. *Journal of Conflict Resolution,* 1961, 5, 167-178.

Harary, F. Graphing conflict in international relations. *Papers Peace Science Society (International),* 1977, 27, 1-9.

Harary, F. Variations on the Golden Rule, *Behavioral Science,* 1982, 27, 155-161.

Harary, F., & Kabell, J. A. A simple algorithm to detect balance in signed graphs. *Mathematical Social Sciences,* 1980, 1, 131-136.

Harary, F., & Kabell, J. A. Counting balanced signed graphs using marked graphs. *Proceedings of the Edinburgh Mathematical Society,* 1981, 24, 99-104.

Harary, F., & Lindström, B. On balance in signed matroids. *Journal of Combinatorics, Information and System Sciences,* 1981, 6, 123-128.

Harary, F., Norman, R. Z., & Cartwright, D. *Structural models: An introduction to the theory of directed graphs.* New York: Wiley, 1965.

Harary, F., & Sagan, B. Balanced posets. *Discrete Applied Mathematics,* in press.

Healy, B., & Stein, A. The balance of power in international history: Theory and reality. *Journal of Conflict Resolution,* 1973, 17, 33-61.

Heider, F. *The Psychology of Interpersonal Relations.* New York: Wiley, 1958.

Newcomb, T. M. An approach to the study of communicative acts. *Psychological Review,* 1953, 60, 393-404.

Osgood, C. E., & Tannenbaum, P. H. The principle of congruity in the prediction of attitude change. *Psychological Review,* 1955, 62, 42-55.

Zaslawski, T. Signed graphs. *Discrete Applied Mathematics,* 1982, 4, 47-74.

Frank Harary is Professor of Mathematics at the University of Michigan. He also served as Faculty Associate from 1950-1982 at the Research Center for Group Dynamics of the Institute for Social Research at the University of Michigan.