The Radiation Pattern of an Electric Line Current Enclosed by an Axially Slotted Plasma Sheath-I

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ABSTRACT

A circular uniform plasma sheath of complex conductivity is enclosing a unit electric line source at the origin. The plasma sheath has an infinite axial slot of arbitrary cross section. The problem has been reduced to a Fredholm integral equation of the second kind in which the unknown function is the electric current in the plasma. An approximate solution of the integral equation has been obtained in the case of a locally partially transparent plasma sheath. On the basis of this solution a formula for the radiation pattern is derived. As an example, the radiation patterns are computed for a 36^{o} wedge slot in the plasma sheath of three different thicknesses. In addition, attenuation calculations have been performed on the unslotted plasma sheath.

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I. INTRODUCTION

A homogeneous plasma sheath of inside radii a and outside radii b is taken to enclose a unit electric line current at the origin of the coordinate system (r, ϕ, z) as shown in Fig. 1. The z-axis coincides with the line source. The plasma sheath may contain an axial slot of area A_s. The area of the plasma sheath we have indicated by A_p. The structure is uniform in the z-direction. The free space permittivity ϵ_o and the permeability μ_o is taken throughout, including the plasma. The electric conduction properties of the plasma (Rose and Clark, 1961) are accounted for by a complex conductivity

$$\sigma = \frac{\epsilon_o \omega_p^2}{\nu + j\omega} \tag{1}$$

where ω, ν and ω_p are the frequencies of field, collision and plasma respectively.

Since the primary current source as well as the plasma sheath does not depend on the z-coordinate, it follows that the electric and the magnetic fields depend only on the r and ϕ coordinates. Furthermore, the electric field is tangential to the line source, and the magnetic field is transverse to it. That is to say, we are dealing only with the electric field component $E_z(r, \phi)$ and the magnetic field components $H_r(r, \phi)$ and $H_\phi(r, \phi)$. The other field components are zero. If we let the plasma fill in the slot, then $E_z(r, \phi) \to E_z(r)$, $H_r(r, \phi) \to 0$, and $H_\phi(r, \phi) \to H_\phi(r)$. Thus we have only one component each of the electric and the magnetic fields, and they depend only on the radial variable r.

In this report we are concerned with the problem of finding the radiation pattern of the electric line source enclosed by a uniform plasma sheath that contains an infinite axial slot. That is to say, we wish to discover the ϕ variation of $E_z(r, \phi)$. This field may be found from the line current and the electric current induced in the plasma. The latter is the unknown quantity for which we derive a Fredholm

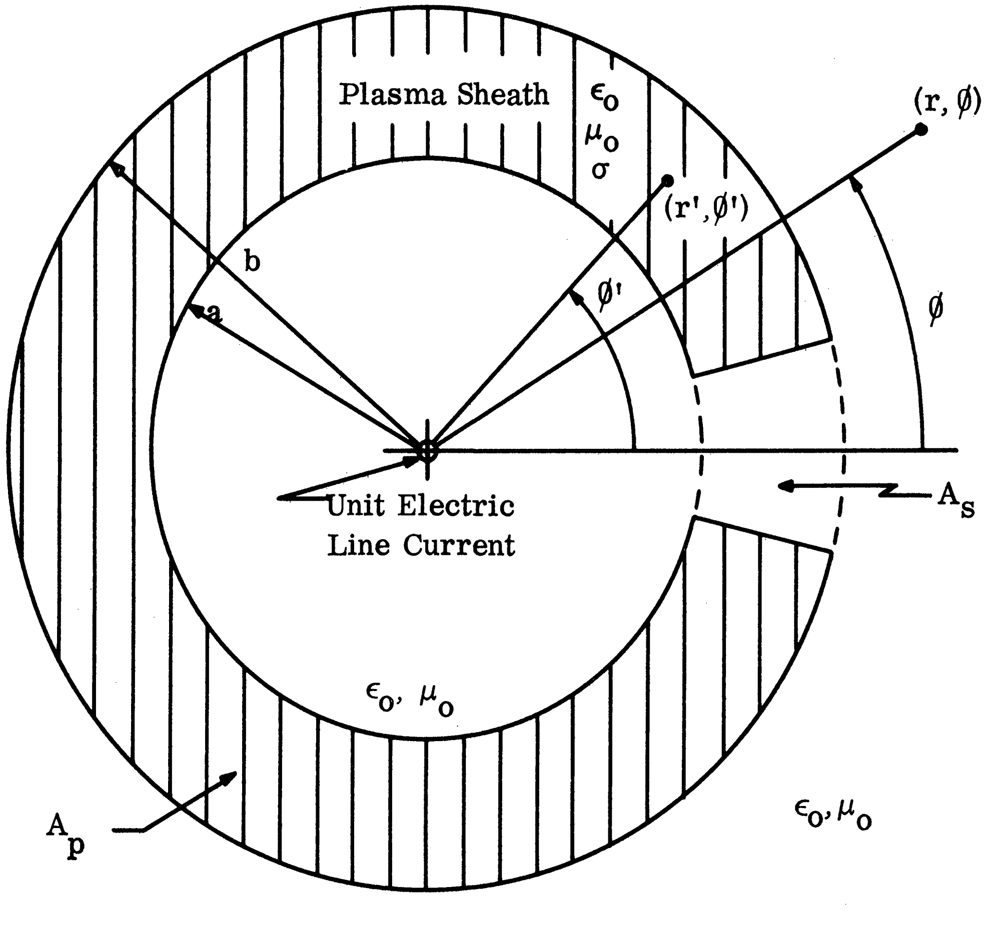

FIG. 1: COORDINATE SYSTEM AND SLOTTED PLASMA SHEATH.

integral equation of the second kind in Section II. We may solve this equation only under restrictions which make the solution of very limited practical value. Setting out in search of a better solution, we first solve the unslotted plasma sheath in Section III. Defining the sheath current with a slot as the unslotted sheath current plus a correction current, we are able, in Section IV, to derive an integral equation for the correction current. We use the unslotted sheath current as an approximate current from which we compute the radiation pattern for the slotted plasma sheath. An approximate solution for the correction current serves as a basis for checking the reliability of the radiation pattern calculations. The rationalized MKS system of units has been used with time dependence of $e^{j\omega t}$ understood. In section V we present numerical results and conclusions and indicate the direction of the future work.

II REDUCING THE PROBLEM TO A FREDHOLM INTEGRAL EQUATION OF THE SECOND KIND

The unit electric line source induces conduction currents $i_z(r', \phi')$ in the plasma sheath. The electric field (Harrington, 1961) of both currents is given by

$$E_z(r, \phi) = -\frac{1}{4}\omega\mu_0\left[H_0^{(2)}(k_0 r) + \iint_{A_p} i_z(r', \phi')H_0^{(2)}(k_0 |\bar{r}-\bar{r}'|)dA'\right] \qquad (2)$$

where $H_0^{(2)}$ is the Hankel function of the second kind, $k_0 = \omega\sqrt{\mu_0\epsilon_0}$, and $|\bar{r}-\bar{r}'|$ is the distance from the field point to the source point. We assume that Ohm's law holds in the plasma, i.e.

$$i_z(r, \phi) = \sigma E_z(r, \phi) \qquad (3)$$

We are neglecting thereby any space charge effects in the plasma. Eliminating

$E_z(r, \phi)$ between (2) and (3), we obtain an integral equation of the form

$$i_z(r,\phi) + \frac{1}{4}\omega\mu_o\sigma \iint_{A_p} i_z(r', \phi')H_o^{(2)}(k_o|\bar{r}-\bar{r}'|)dA' = -\frac{1}{4}\omega\mu_o\sigma H_o^{(2)}(k_o r) \quad . \tag{4}$$

We recognize that this is a Fredholm integral equation of the second kind with the kernel $H_o^{(2)}(k_o|\bar{r}-\bar{r}'|)$.

We put the integral equation in a more convenient form by defining

$$\tau \equiv \frac{\mu_o\omega\sigma}{4k_o^2} \tag{5}$$

and

$$k_o^2 i(\rho, \phi) \equiv i_z(r,\phi) \tag{6}$$

where $\rho = k_o r$, and hence

$$i(\rho,\phi) + \tau \iint_{A_p} i(\rho',\phi')H_o^{(2)}(|\bar{\rho}-\bar{\rho}'|)dA' = -\tau H_o^{(2)}(\rho) \tag{7}$$

where now $dA' = \rho' d\rho' d\phi'$. This is the fundamental equation of the problem.

The integral equation may be solved by the method of successive approximation (Mikhlin, 1964) provided that

$$|\tau| < \frac{1}{B} \tag{8}$$

where

$$B^2 = \iint_{A_p} \iint_{A_p} \left| H_o^{(2)}(|\bar{\rho}-\bar{\rho}'|)\right|^2 dA\, dA' \quad . \tag{9}$$

By considering unslotted plasma sheath and a Hankel asymptotic form for $H_o^{(2)}$, we obtain from (9) that

$$B \simeq 2\pi k_o(b-a) \quad , \tag{10}$$

and hence from (8) that

$$\frac{\omega_p^2}{|\nu + j\omega|} < \frac{\lambda_o}{\pi^2 (b-a)} \tag{11}$$

This estimate of the conditions of convergence is on the pessimistic side. Nevertheless, Eq. (11) gives us a good indication that the successive approximation method of solving the Fredholm integral equation is of value only for either very thin plasma sheath or very low plasma density.

III THE CASE OF UNIFORM PLASMA SHEATH

The situation is as in Fig. 1, with the slot filled in by the plasma. An exact solution to this problem can be obtained in an elementary manner. The electric field may be expressed in the form;

$$E_z(r) = -\frac{1}{4}\,\omega\,\mu_o H_o^{(2)}(k_o r) + A_1 J_o(k_o r)\ , \qquad 0 < r < a, \tag{12}$$

$$= A_2 J_o(kr) + B_2 N_o(kr), \qquad a < r < b\ , \tag{13}$$

$$= A_3 H_o^{(2)}(k_o r) \qquad b < r\ , \tag{14}$$

where J_o and N_o are Bessel and Neumann functions of order zero, respectively, and

$$k = k_o \sqrt{1 - j\sigma/(\omega\,\epsilon_o)}\ . \tag{15}$$

Then from the Maxwell's equations the corresponding magnetic field is given as

$$H_\phi(r) = -j\,\frac{1}{4} k_o H_1^{(2)}(k_o r) + j\,\frac{k_o}{\omega\mu_o} A_1 J_1(k_o r), \qquad 0 < r < a\ , \tag{16}$$

$$= j\,\frac{k}{\omega\mu_o}\Big[A_2 J_1(kr) + B_2 N_1(kr)\Big]\ , \qquad a < r < b\ , \tag{17}$$

$$= j\,\frac{k_o}{\omega\mu_o} A_3 H_1^{(2)}(k_o r)\ , \qquad b < r\ , \tag{18}$$

the cylindrical functions being of order 1.

From the continuity of $E_z(r)$ and $H_z(r)$ at r=a and r=b, we obtain four algebraic equations. By solving them we obtain the complex amplitudes of the waves:

$$A_1 = \frac{\omega\mu_o}{4Dk_o^2}\Big\{ k_oJ_o(ka)H_1^{(2)}(k_oa)\left[k_oN_o(kb)H_1^{(2)}(k_ob)-kN_1(kb)H_o^{(2)}(k_ob)\right] + k_oN_o(ka)H_1^{(2)}(k_oa)\left[kJ_1(kb)H_o^{(2)}(k_ob)-k_oJ_o(kb)H_1^{(2)}(k_ob)\right] + kN_1(ka)H_o^{(2)}(k_oa)\left[k_oJ_o(kb)H_1^{(2)}(k_ob)-kJ_1(kb)H_o^{(2)}(k_ob)\right] + kJ_1(ka)H_o^{(2)}(k_oa)\left[kN_1(kb)H_o^{(2)}(k_ob)-k_oN_o(kb)H_1^{(2)}(k_ob)\right]\Big\}, \tag{19}$$

$$A_2 = \frac{j\omega\mu_o}{2\pi aDk_o^2}\left[k_oN_o(kb)H_1^{(2)}(k_ob)-kN_1(kb)H_o^{(2)}(k_ob)\right], \tag{20}$$

$$B_2 = \frac{j\omega\mu_o}{2\pi aDk_o^2}\left[kJ_1(kb)H_o^{(2)}(k_ob)-k_oJ_o(kb)H_1^{(2)}(k_ob)\right] \tag{21}$$

$$A_3 = \frac{j\omega\mu_o}{\pi^2 k_o^2 abD} \tag{22}$$

where

$$D = J_1(k_oa)H_1^{(2)}(k_ob)\left[J_o(ka)N_o(kb)-J_o(kb)N_o(ka)\right] + \frac{k}{k_o}\Big\{J_1(k_oa)H_o^{(2)}(k_ob)\left[J_1(kb)N_o(ka)\;\;J_o(ka)N_1(kb)\right] + J_o(k_oa)H_1^{(2)}(k_ob)\left[J_o(kb)N_1(ka)-J_1(ka)N_o(kb)\right]\Big\} + \left(\frac{k}{k_o}\right)^2 J_o(k_oa)H_o^{(2)}(k_ob)\left[J_1(ka)N_1(kb)-J_1(kb)N_1(ka)\right]. \tag{23}$$

The current induced in the uniform plasma sheath $i_z(r)$ is given by

$$i_z(r) = \sigma E_z(r) , \qquad a < r < b$$
$$= \sigma\left[A_2 J_o(kr) + B_2 N_o(kr)\right] . \tag{24}$$

By a similar definition as in (6) we introduce

$$i(\rho) = \frac{\sigma}{k_o^2}\left[A_2 J_o(kr) + B_2 N_o(kr)\right] . \tag{25}$$

The electric field outside the plasma sheath as given by (14) is rewritten in the form

$$E_z(r) = -\frac{1}{4}\,\omega\mu_o H_o^{(2)}(k_o r)\left[-\frac{4}{\omega\mu_o} A_3\right] . \tag{26}$$

The first factor gives the electric field of the unit electric line source in the free space, the second factor introduces the effect of the plasma sheath on the radiation field. We define the latter factor as the radiation pattern for the electric line source radiating through a concentric uniform plasma sheath, i. e.

$$P_o = -\frac{4}{\omega\mu_o} A_3 . \tag{27}$$

This pattern is a constant; it does not depend either on r or ϕ. However, it is rather difficult to calculate because of the involved denominator in A_3. However, in the case $k_o a \gg 1$ and $|ka| \gg 1$, P_o can be reduced with a good approximation to an elementary form by using the Hankel asymptotic form for the cylindrical functions. Taking two terms in the Hankel asymptotic series, we obtain

$$P_o \simeq \frac{2k e^{-j\left[k(b-a) - k_o b + (8k_o b)^{-1} + \frac{3}{4}\pi\right]}}{k_o\left[d_o + d_1(8k_o a)^{-1} + \ldots\right]} \tag{28}$$

where

$$d_o = \left[1 + \frac{k}{k_o} + (-1 + \frac{k}{k_o})e^{-j2k(b-a)}\right]\sin(k_o a - \frac{\pi}{4}) +$$
$$j\frac{k}{k_o}\left[-1 - \frac{k}{k_o} + (-1 + \frac{k}{k_o})e^{-j2k(b-a)}\right]\cos(k_o a - \frac{\pi}{4}) , \tag{29}$$

and

$$d_1=j\left(\frac{k}{k_o}-\frac{k_o(b-a)}{kb}\right)\left[-1-\frac{k}{k_o}+(-1+\frac{k}{k_o})e^{-j2k(b-a)}\right]\sin(k_oa-\frac{\pi}{4})+$$

$$\frac{a}{b}\left[-1-\frac{k}{k_o}-(-1+\frac{k}{k_o})e^{-j2k(b-a)}\right]. \tag{30}$$

From (1) and (15) we find that $k=k'-jk''$ where

$$k'=k_o\left[\frac{1}{2}A+\frac{1}{2}\sqrt{A^2+B^2}\right]^{1/2} \tag{31}$$

$$k''=k_o\left[-\frac{1}{2}A+\frac{1}{2}\sqrt{A^2+B^2}\right]^{1/2} \tag{32}$$

and

$$A\equiv 1-\frac{X^2}{1+Y^2}\,; \qquad B\equiv\frac{YX^2}{1+Y^2} \tag{33}$$

with

$$X\equiv\frac{\omega_p}{\omega}\,; \qquad Y\equiv\frac{\nu}{\omega}\,. \tag{34}$$

From (28) we observe that the amplitude of P_o is determined principally by the factor $\exp\left[k''(b-a)\right]$ as would also be the corresponding case of a normally incident plane wave radiation through a plasma slab.

The asymptotic current distribution in the plasma associated with (28) is given by

$$i(r)\simeq\frac{j\tau\sqrt{\frac{2}{\pi k_or}}}{d_o+d_1(8k_oa)^{-1}}\left\{\left[(1+\frac{k}{k_o})-j\frac{1}{8}(\frac{1}{k_ob}+\frac{1}{k_or})(1+\frac{k_o}{k})\right]e^{-jk(r-a)}+\right.$$

$$\left.\left[(\frac{k}{k_o}-1)+j\frac{1}{8}(\frac{1}{k_ob}-\frac{1}{k_or})(1-\frac{k_o}{k})\right]e^{jk\left[r-2(b-a)\right]}\right\}. \tag{35}$$

The first term in the $\{\,\}$ brackets represents outward propagating current wave and the second term the reflected current wave. Again the exponentials are the dominant factors in this formula.

The approximate forms of P_o and i(r) for the case $k_oa \gg 1$ and $|k|b \ll 1$ are given by

$$P_o \simeq \frac{e^{j(k_ob-\frac{3\pi}{4})}}{k_o\sqrt{ab}} \left\{ \left[\frac{1}{k_ob} + j\,\ell n\left(\frac{b}{a}\right) \right] \sin(k_oa-\frac{\pi}{4}) - j\frac{1}{k_oa}\cos\left(k_oa-\frac{\pi}{4}\right) \right\}^{-1} \tag{36}$$

and

$$i(r) \simeq \frac{X^2(1+jk_ob\,\ell n\frac{b}{r})}{(1-jY)2\sqrt{2\pi k_oa}} \left\{ \left[1+jk_ob\,\ell n\frac{b}{a} \right] \sin(k_oa-\frac{\pi}{4}) - j\frac{b}{a}\cos(k_oa-\frac{\pi}{4}) \right\}^{-1} . \tag{37}$$

The two formulas have been obtained using the small argument approximation for the cylindrical functions of argument kr and the first term in the Hankel asymptotic series for those of argument k_or. That is to say, these formulas are valid approximations for a plasma sheath of sufficiently large radius and $\omega \sim \omega_p$.

IV AN APPROXIMATE SOLUTION TO THE CURRENT INTEGRAL EQUATION AND THE RADIATION PATTERN

We recognize that the normalized current of the uniform plasma sheath , (25), is a solution of the integral equation (7) with $A_p \to (A_p+A_s)$, i. e.

$$i(\rho)+\tau \iint_{A_p+A_s} i(\rho')\,H_0^{(2)}(|\bar{\rho}-\bar{\rho}'|)dA' = -\tau H_0^{(2)}(\rho) . \tag{38}$$

We define a current $i_c(\rho, \phi)$ by

$$i(\rho, \phi) \equiv i(\rho)+i_c(\rho, \phi) \tag{39}$$

and thus from (7) obtain

$$i_c(\rho\ \phi)+\tau\iint_{A_p} i_c(\rho',\phi')H_o^{(2)}(|\bar{\rho}-\bar{\rho}'|)dA'=-i(\rho)-\tau H_o^{(2)}(\rho)\ -$$
$$-\tau\iint_{A_p} i(\rho)H_o^{(2)}(|\bar{\rho}-\bar{\rho}'|)dA' \quad . \tag{40}$$

However, from (38) we see that the right-hand side of (40) reduces to

$$\tau\iint_{A_s} i(\rho')H_o^{(2)}(|\bar{\rho}-\bar{\rho}'|)dA'$$

and thus

$$i_c(\rho,\phi)+\tau\iint_{A_p} i_c(\rho',\phi')H_o^{(2)}(|\bar{\rho}-\bar{\rho}'|)dA'=\tau\iint_{A_s} i(\rho')H_o^{(2)}(|\bar{\rho}-\bar{\rho}'|)dA' \ . \tag{41}$$

By opening a slot A_s in the uniform plasma sheath, we change the sheath current by $i_c(\rho,\phi)$ and the integral equation (41) defines this current. Integral equations (7)and (41) differ only in their right-hand sides. Thus the condition (8) on the convergence of the resolvent applies also to the latter. The zeroth order approximation to $i_c(\rho,\phi)$ is given by

$$i_c^{(o)}(\rho\ ,\phi)=\tau\iint_{A_s} i(\rho')H_o^{(2)}(|\bar{\rho}-\bar{\rho}'|)dA' \quad . \tag{42}$$

By opening the slot we reduce $i(\rho)=0$ in the slot area A_s and assuming that we may neglect $i_c(\rho,\phi)$ we obtain by a simple transformation from (2) and (14)

$$E_z(r,\phi)\simeq A_3H_o^{(2)}(k_or)+\frac{1}{4}\omega\mu_o\iint_{A_s} i(r')H_o^{(2)}(k_o|\bar{r}-\bar{r}'|)dA' \quad ,\ r>b\ . \tag{43}$$

Using the asymptotic forms for the Hankel functions, we obtain

$$E_z(r,\phi) \simeq -\frac{1}{4}\omega\mu_o \sqrt{\frac{2}{\pi k_o r}}\, e^{-j(k_o r-\frac{\pi}{4})}\left[P_o - \iint_{A_s} i(r')e^{jk_o r'\cos(\phi-\phi')} dA'\right], \qquad r >> b \tag{44}$$

and the radiation pattern $P_s(\phi)$, consistent with the earlier definition in (27), is then

$$P_s(\phi) \simeq P_o - \iint_{A_s} i(r')e^{jk_o r'\cos(\phi-\phi')} dA' \ . \tag{45}$$

It is clear that including the current $i_c^{(o)}(r,\phi)$ in the approximation we obtain for the radiation pattern

$$P_{sc}(\phi) \simeq P_o - \iint_{A_s} i(r')e^{jk_o r'\cos(\phi-\phi')} dA' + \iint_{A_p} i_c^{(o)}(r',\phi')e^{jk_o r'\cos(\phi-\phi')} dA' \ . \tag{46}$$

Equation (45) shows us that opening a slot amounts to adding to the existing current its negative, i.e. to make the total current in the slot area zero. For overdense plasmas $|P_o| << 1$ and the negative current in the slot area then dominates the radiation pattern. We believe that (45) gives a good approximation to the radiation pattern whenever i(r) penetrates into the sheath. The adding of $i_c^{(o)}(r,\phi)$ to the radiation pattern $P_s(\phi)$ in (46) is not so much to improve the approximation in (45) as to provide perhaps a means of feeling out its bounds of validity. Whenever

$$P_s(\phi) \simeq P_{sc}(\phi) \tag{47}$$

one can reasonably say that (45) gives a good approximation to the radiation pattern.

If the slot is a wedge slot of width $2\phi_o$, then the double integral in (45) may be transformed into a series, and we obtain for the radiation pattern

$$P_s(\phi) \simeq P_o + 2\phi_o \sum_{n=o}^{\infty} \epsilon_n \frac{\sin(n\phi_o)}{n\phi_o} R_n \cos n\phi \tag{48}$$

where

$$\epsilon_n = 1, \qquad n = 0$$
$$= 2, \qquad n = 1, 2, 3, \ldots$$

and

$$R_n \equiv -j^n \int_{k_o a}^{k_o b} i(\rho) J_n(\rho)\, \rho\, d\rho \,. \tag{49}$$

It is clear that the pattern is an even function in ϕ. The ϕ variable is measured from the center of the slot.

V NUMERICAL RESULTS AND CONCLUSIONS

In Figs. 2, 3 and 4 we present the radiation through a uniform plasma sheath for Y=0.01, 0.1 and 1.0 respectively. With Y=0.01 the plasma is only slightly lossy. With Y=1.0 the plasma is very lossy. The inside radius of the plasma sheath has been kept at one free space wavelength λ_o, and the sheath thickness has been taken up to $3\lambda_o$. The plasma parameter X is varied in steps from 0.25 to 2.0.

In Figs. 2 and 3 the calculations have been carried out using the exact formula (27) for all X values except X=2. For X=2 calculation we used the asymptotic form (28) with d_1 set to zero. The same asymptotic form was used for all calculations in Fig. 4. The asymptotic form for P_o gave excellent agreement with the exact results in Figs. 2 and 3 where it was applicable, and surprisingly little disagreement, even for cases (e.g. Y=0.01, X=1) where the arguments of the cylinder functions were far too small for the Hankel asymptotic expansion to be valid.

The processes that inhibit radiation through the sheath are reflections, absorptive attenuation, and reactive attenuation by the plasma. The attenuation of the radiation for $0 < X < (\sqrt{2}$ approx.) is due to reflections and absorptive attenuation, and for $X > (\sqrt{2}$ approx.) due mainly to both reactive and absorptive attenuation. In

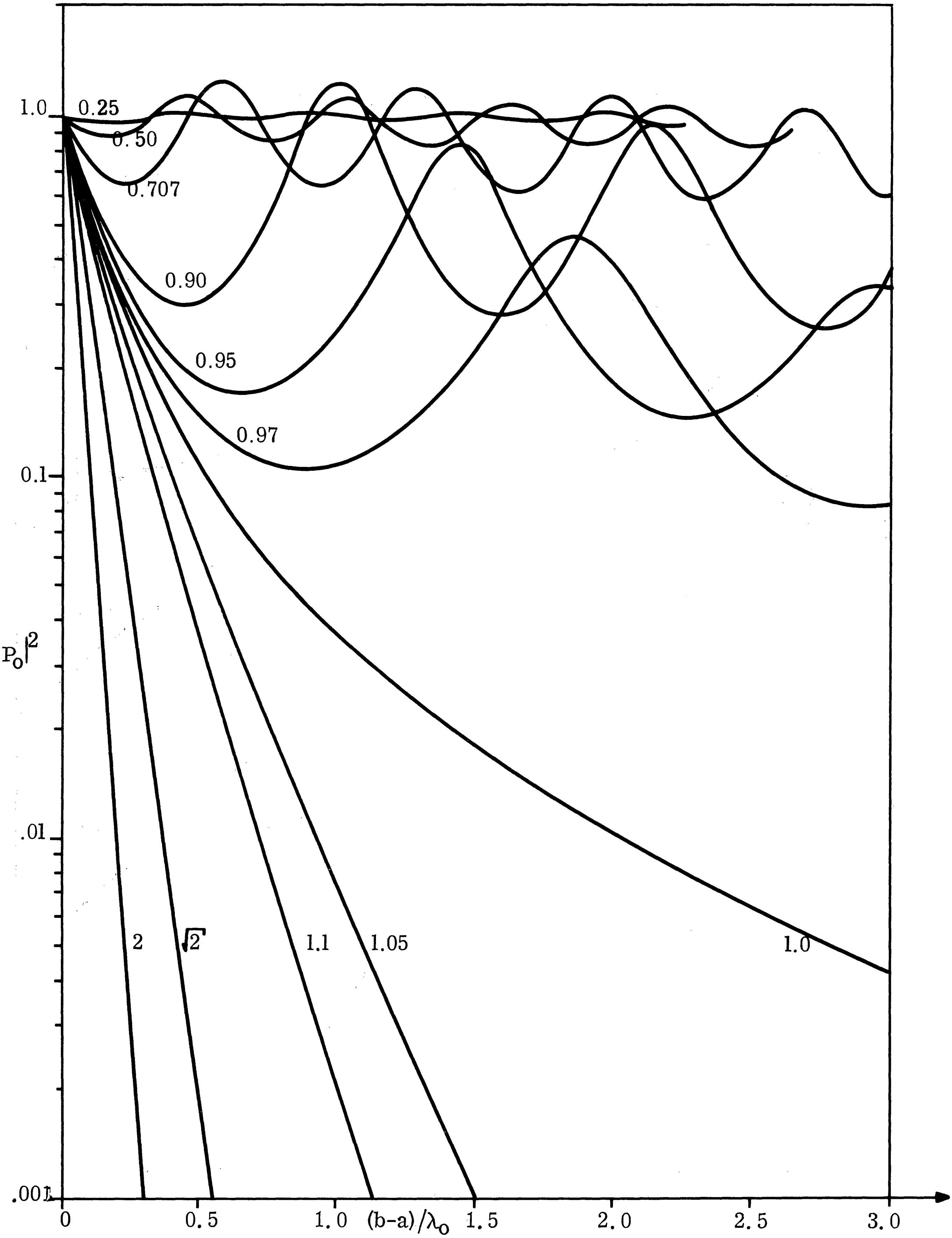

FIG. 2: RADIATION GRAPHS THROUGH UNIFORM PLASMA SHEATH VS SHEATH THICKNESS FOR $a=1\lambda_o$, Y=0.01 AND X AS A PARAMETER.

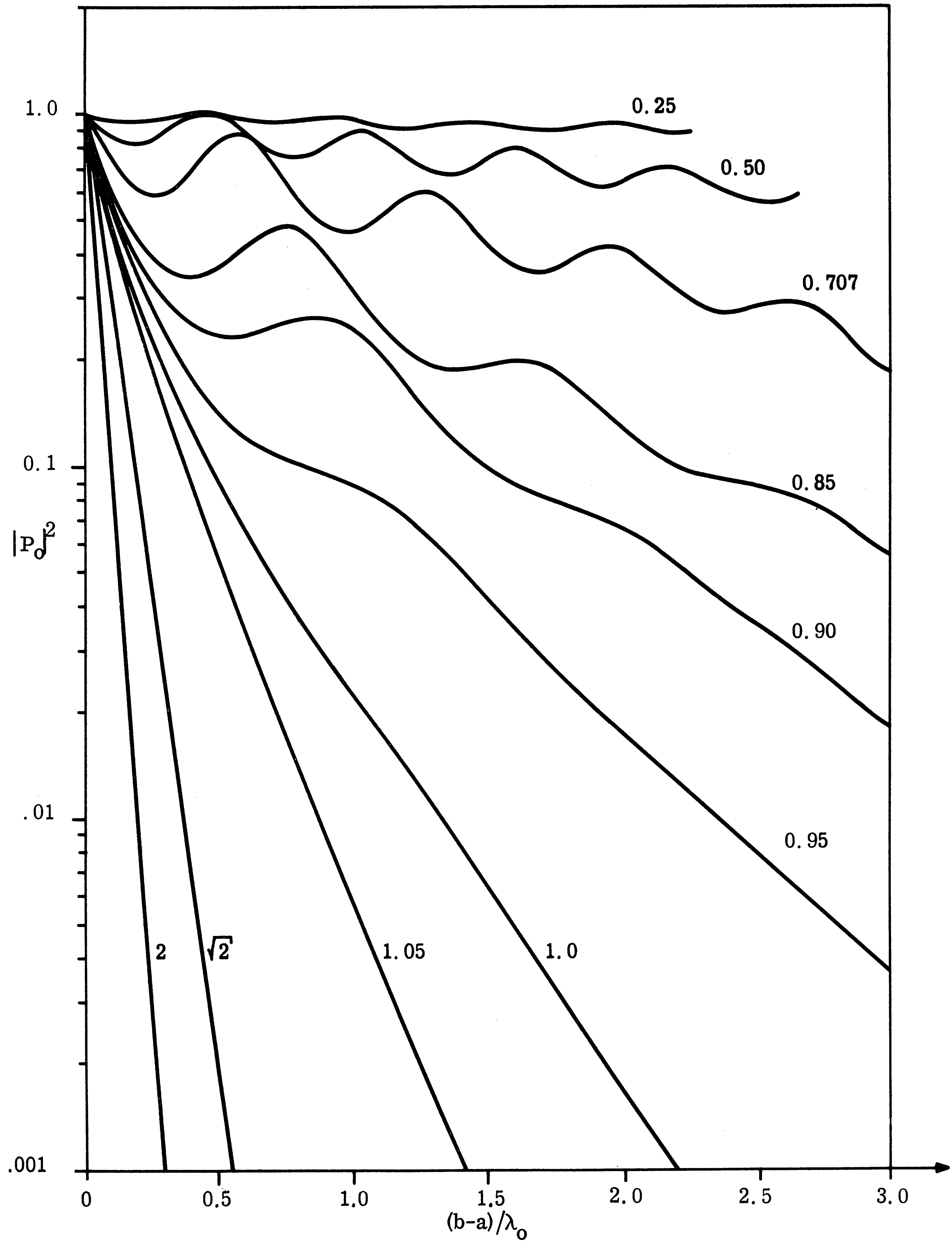

FIG. 3: RADIATION GRAPHS THROUGH UNIFORM CIRCULAR PLASMA SHEATH VS SHEATH THICKNESS FOR $a=1\lambda_o$, Y=0.1, AND X AS A PARAMETER

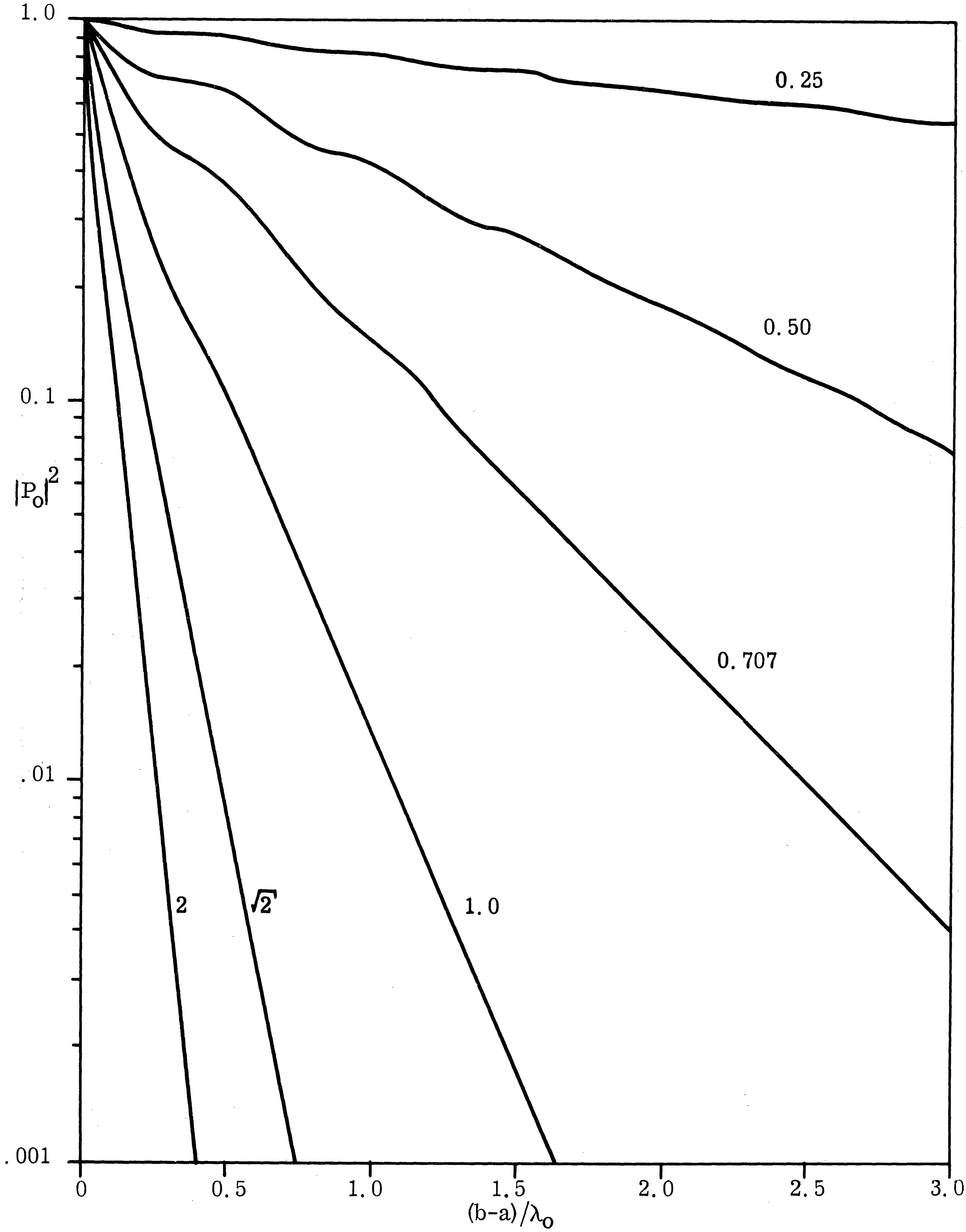

FIG. 4: RADIATION GRAPHS THROUGH UNIFORM CIRCULAR PLASMA SHEATH VS SHEATH THICKNESS FOR $a=1\lambda_o$, $Y=1.0$ AND X AS A PARAMETER

each case we may identify conditions where only one process is primarily causing the attenuation. Thus for $X < \sqrt{2}$ and Y=0.01 we see, in Fig. 2, the radiation graphs oscillating because of the reflections. As Y increases to 0.1 in Fig. 3, the amplitude of the oscillations decreases, and for Y=1 in Fig. 4, the oscillations are practically gone; but now the graphs have assumed a downward slope. This is caused by the absorptive attenuation in the plasma taking over and eliminating the reflections. Also, we see that we get quite a bit more attenuation for very lossy plasma (Y=1) than for very low loss plasma (Y=0.01). For the case of $X > \sqrt{2}$ the opposite is true: low loss plasma causes more attenuation than a very lossy plasma. This is due to the effectiveness of the reactive attenuation.

In Fig. 5, we show a 36^o wedge slot in the plasma sheath, and in Figs. 6 to 14 we present the radiation power pattern $|P_S(\phi)|^2$ calculated from (48) for the sheath thickness of $0.5\lambda_o$, $1.0\lambda_o$ and $2.0\lambda_o$, respectively. The inside sheath radius is kept at $1.0\lambda_o$. The graphs have been arranged in sets of three: Y=1.0, 0.1 and 0.01, and X is a parameter in each set. Since the power patterns are even functions in ϕ, we have plotted them only from 0^o to 180^o. By the constant lines we show the attenuation in the power radiated before the plasma slot is introduced, i.e. the radiation power pattern for the unslotted plasma sheath. These lines are obtained from Figs. 2 to 4. In some cases the attenuation is so large that it is off scale. These values we will give in a note. As a way of calibration we mention that the radiation power pattern of unit amplitude would correspond to the electric line source radiating in the free space.

In Figs. 6, 7 and 8 we present the set Y=1. By opening a slot in this very lossy plasma we have, for all practical purposes, re-established radiation in the forward direction, i.e. ϕ=0, independent of the sheath thickness. Outside of the front lobe there is no significant recovery of the signal strength. Increased slot width, the calculations will show, narrows the front lobe with some additional gain

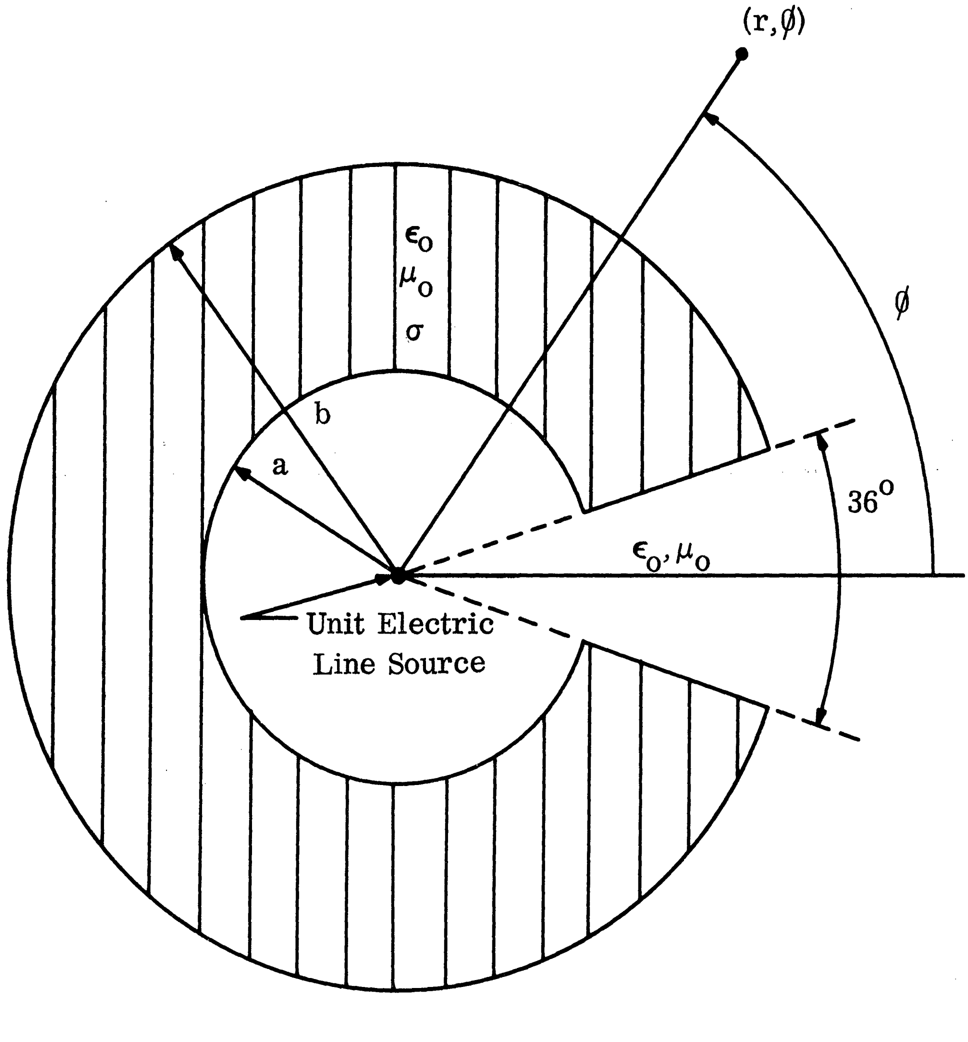

FIG. 5: PLASMA SHEATH WITH $a=1\lambda_o$, $b=2\lambda_o$, AND A 36^o WEDGE SLOT

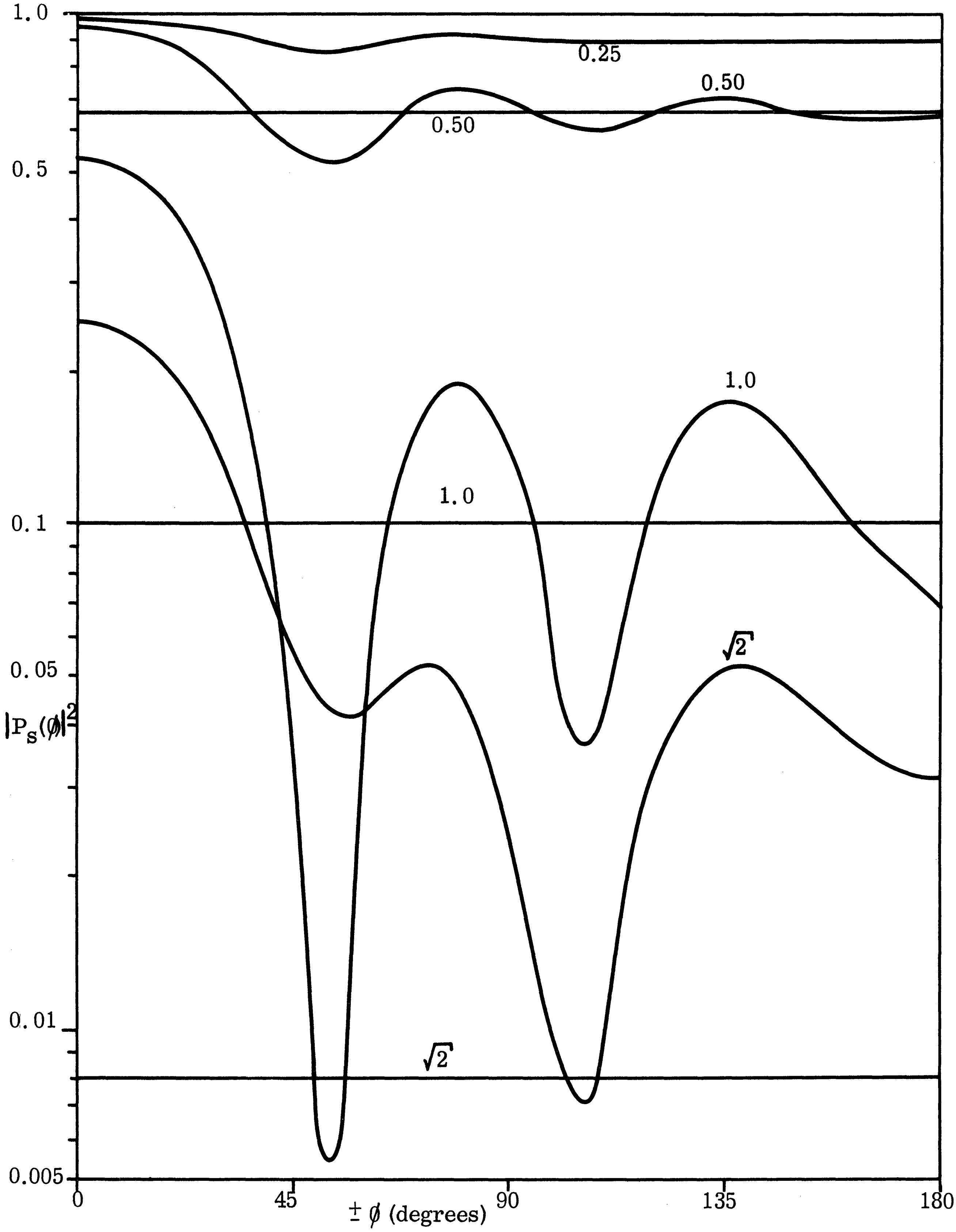

FIG. 6: RADIATION POWER PATTERNS FOR $0.5\lambda_0$ THICK PLASMA SHEATH WITH 36^o WEDGE SLOT, Y=1, AND X AS A PARAMETER. The parallel lines are the corresponding patterns for the unslotted case.

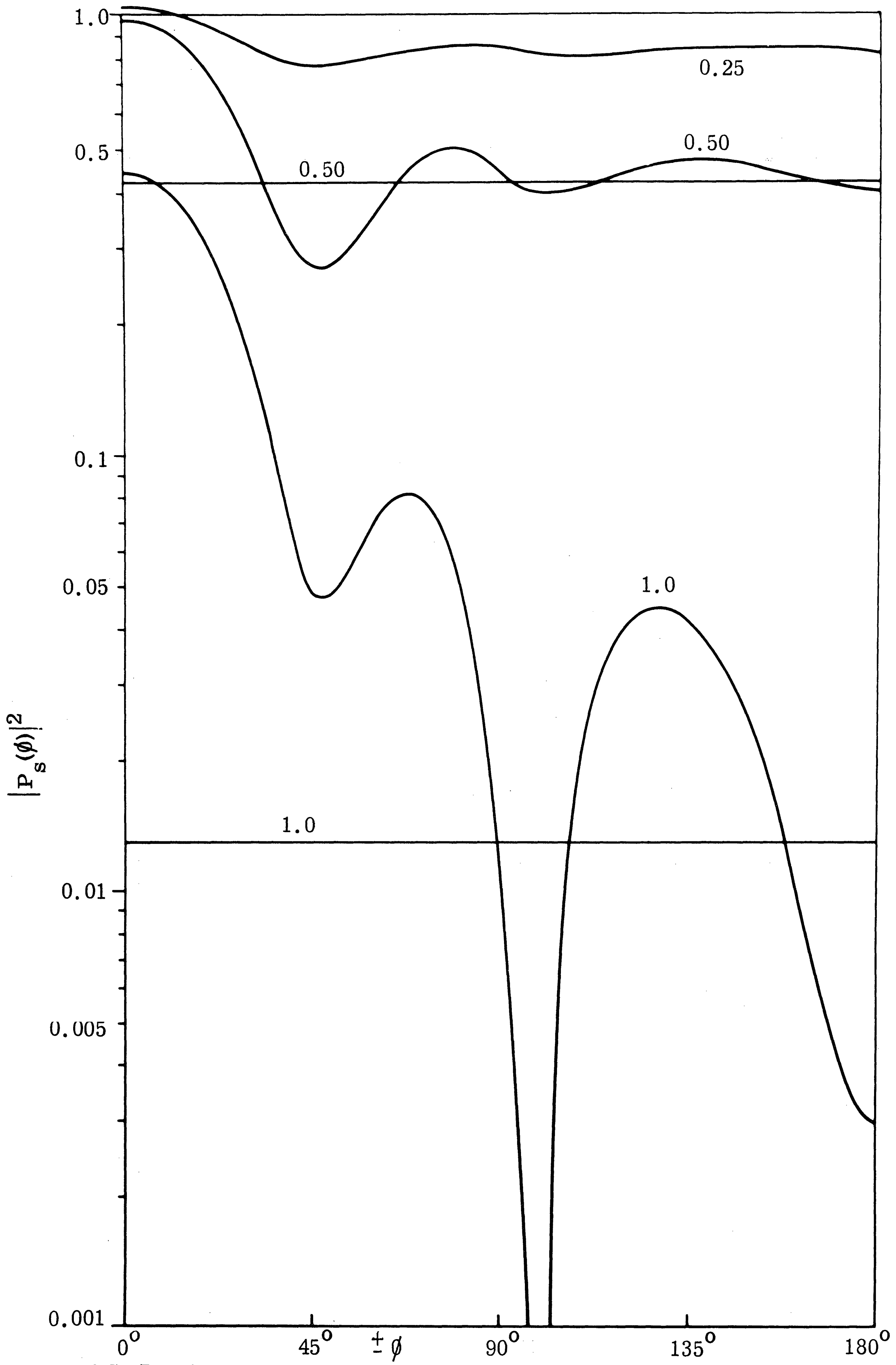

FIG. 7: RADIATION POWER PATTERN FOR 1.0λ THICK PLASMA SHEATH WITH A 36° WEDGE SLOT; Y = 1, AND X AS A PARAMETER. The parallel lines are the corresponding patterns for the unslotted case.

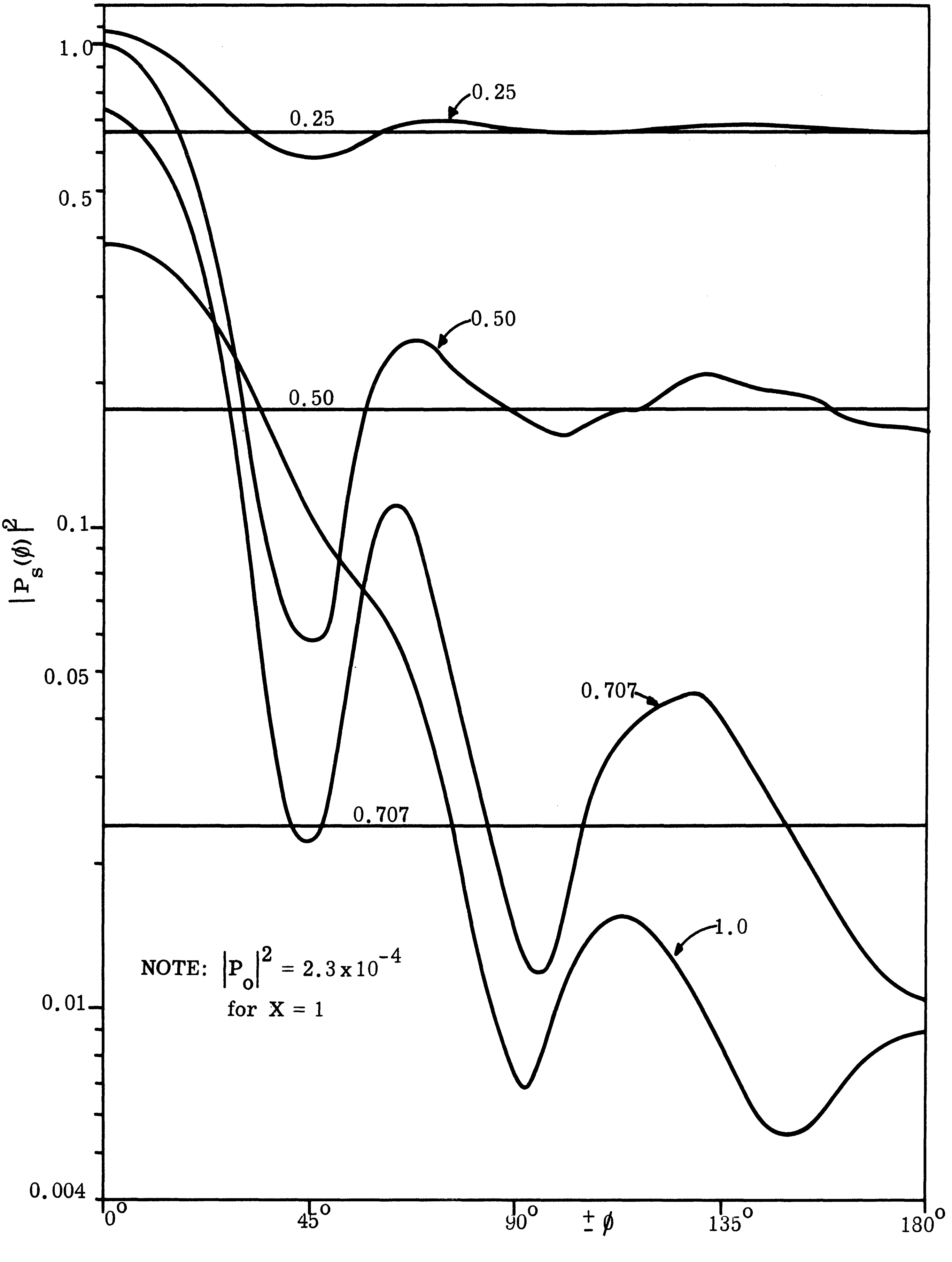

FIG. 8: RADIATION POWER PATTERN FOR 2.0λ_o THICK PLASMA SHEATH WITH A 36° WEDGE SLOT, Y = 1, AND X AS A PARAMETER. The parallel lines are the corresponding patterns for the unslotted case.

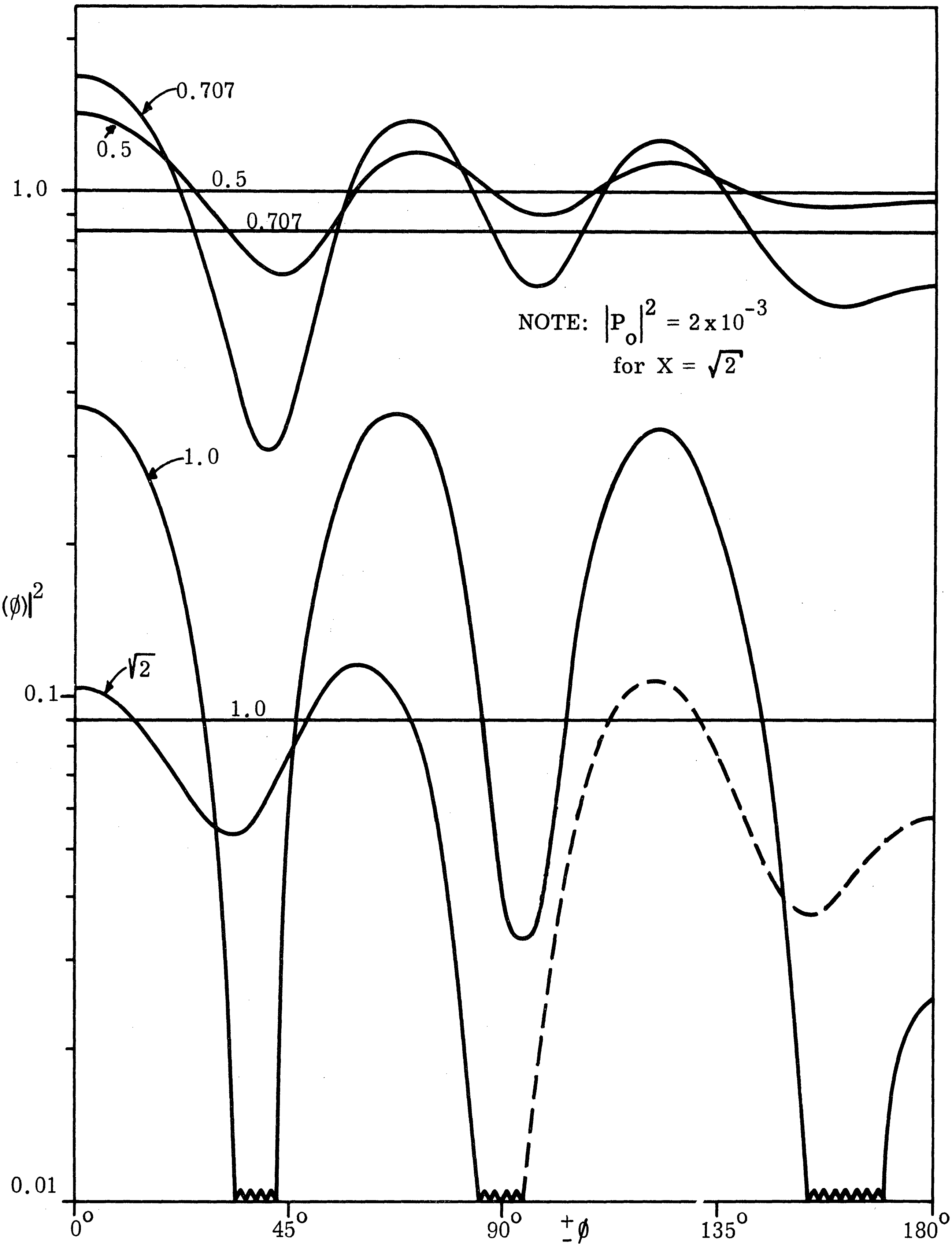

FIG. 9: RADIATION POWER PATTERN FOR $0.5\lambda_o$ THICK PLASMA SHEATH WITH A 36^o WEDGE SLOT, Y=0.1, AND X AS A PARAMETER. The parallel lines are the corresponding patterns for the unslotted case.

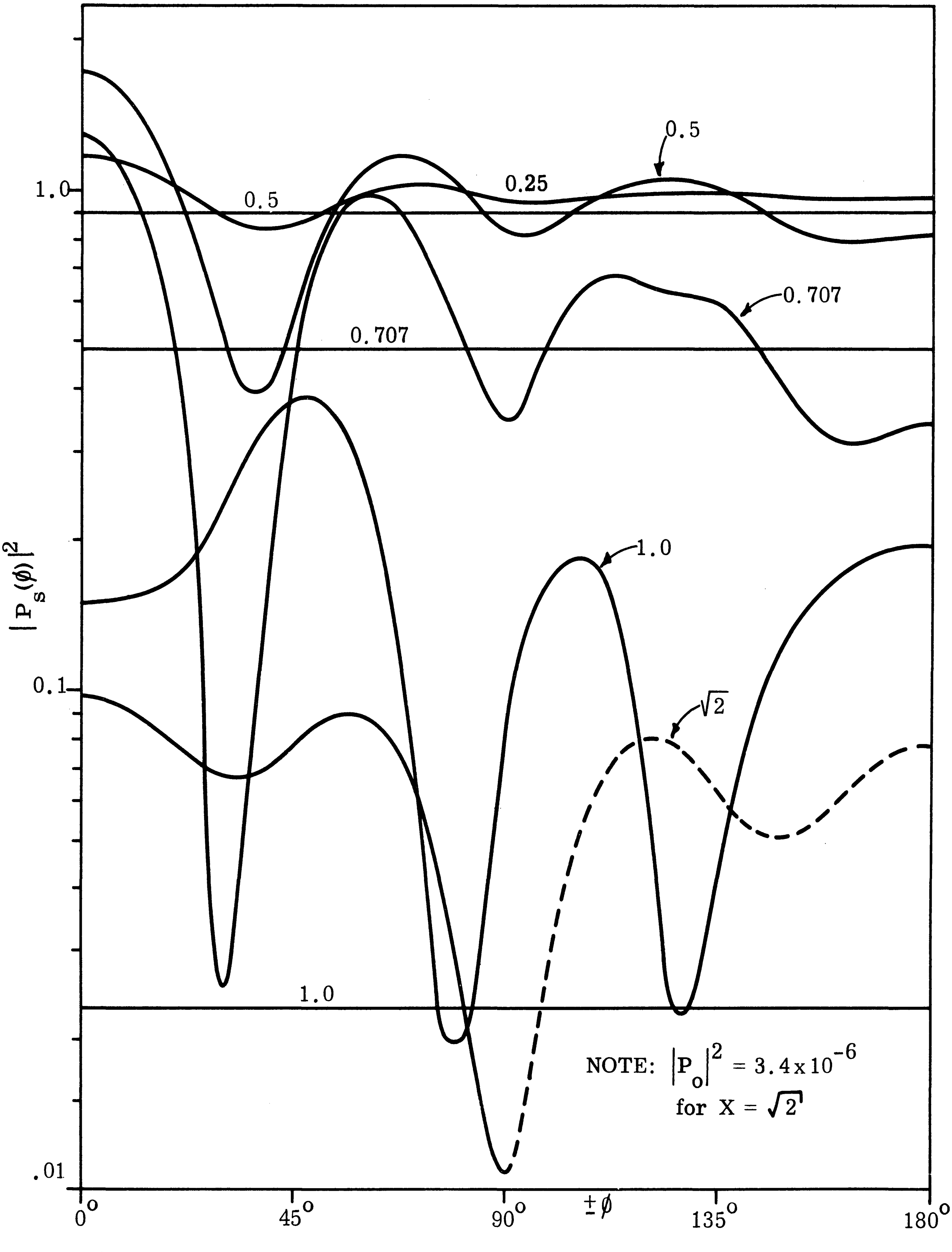

FIG. 10: RADIATION POWER PATTERN FOR $1.0\lambda_o$ THICK PLASMA SHEATH WITH A 36^o WEDGE SLOT, Y = 0.1, AND X AS A PARAMETER. The parallel lines are the corresponding patterns for the unslotted case.

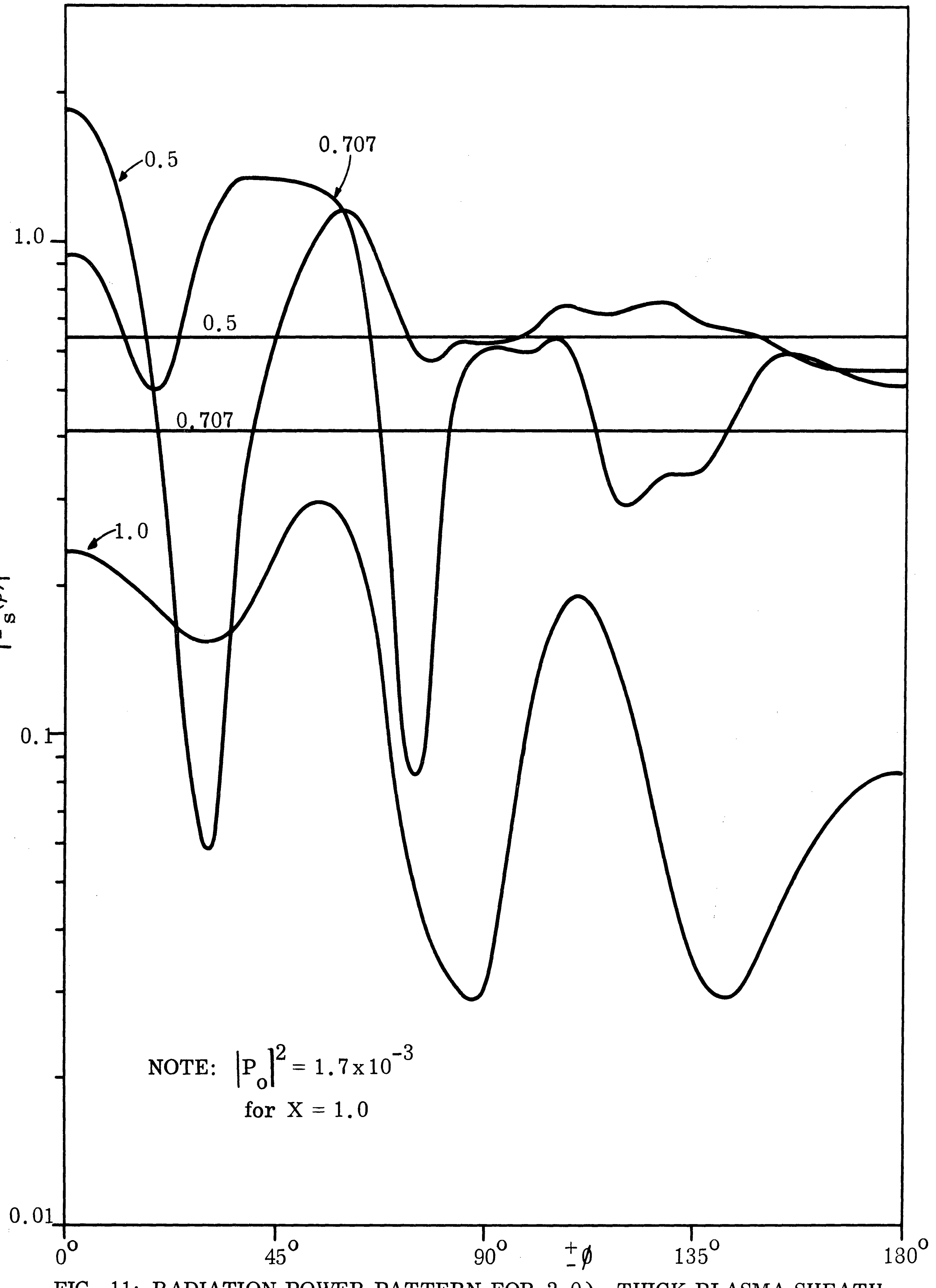

FIG. 11: RADIATION POWER PATTERN FOR $2.0\lambda_o$ THICK PLASMA SHEATH WITH A 36° WEDGE SLOT, Y = 0.1, AND X AS A PARAMETER. The parallel lines are the corresponding patterns for the unslotted case.

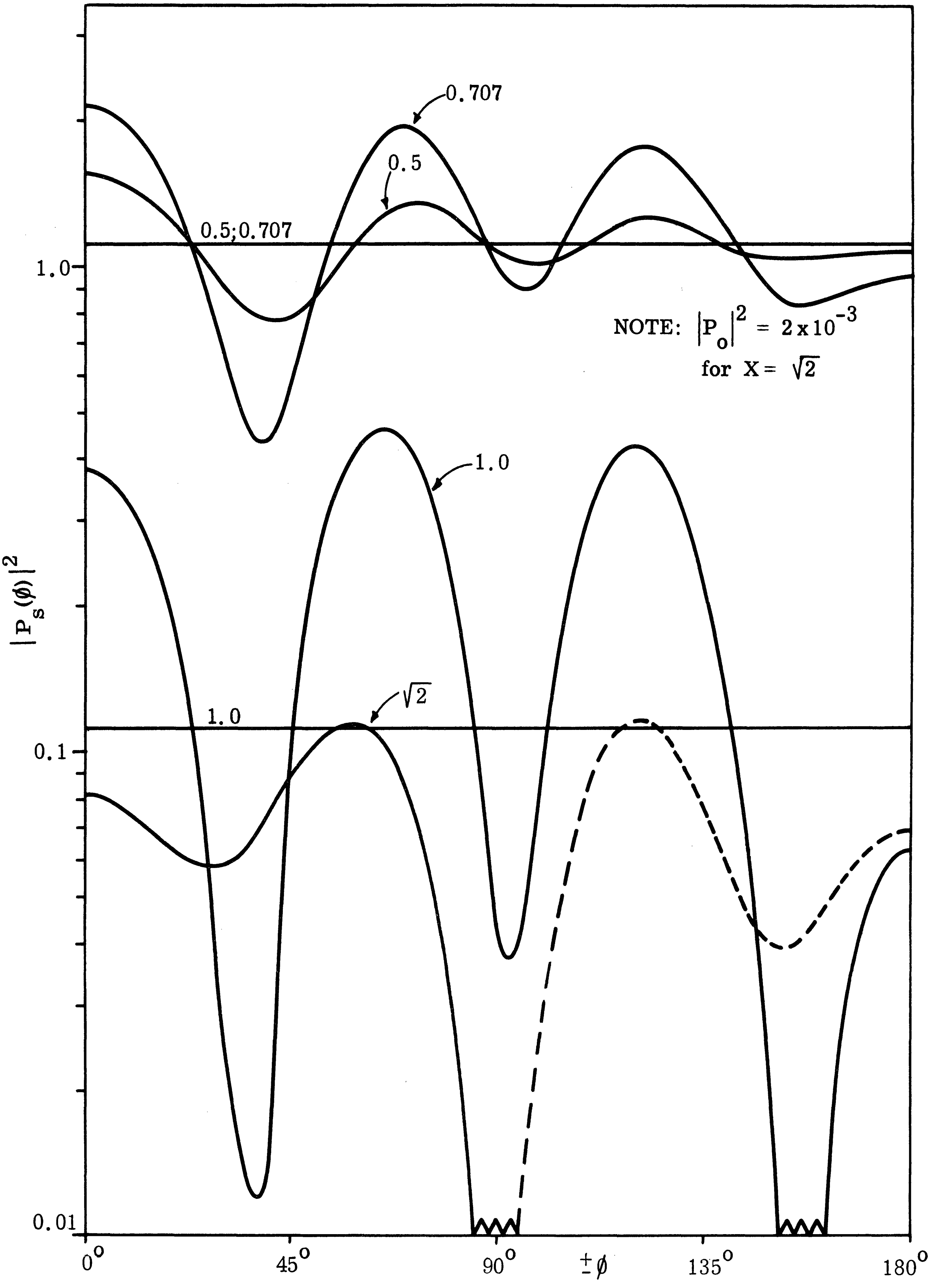

FIG. 12: RADIATION POWER PATTERN FOR $0.5\lambda_o$ THICK PLASMA SHEATH WITH A 36° WEDGE SLOT, Y = 0.01, AND X AS A PARAMETER. The parallel lines are the corresponding patterns for the unslotted case.

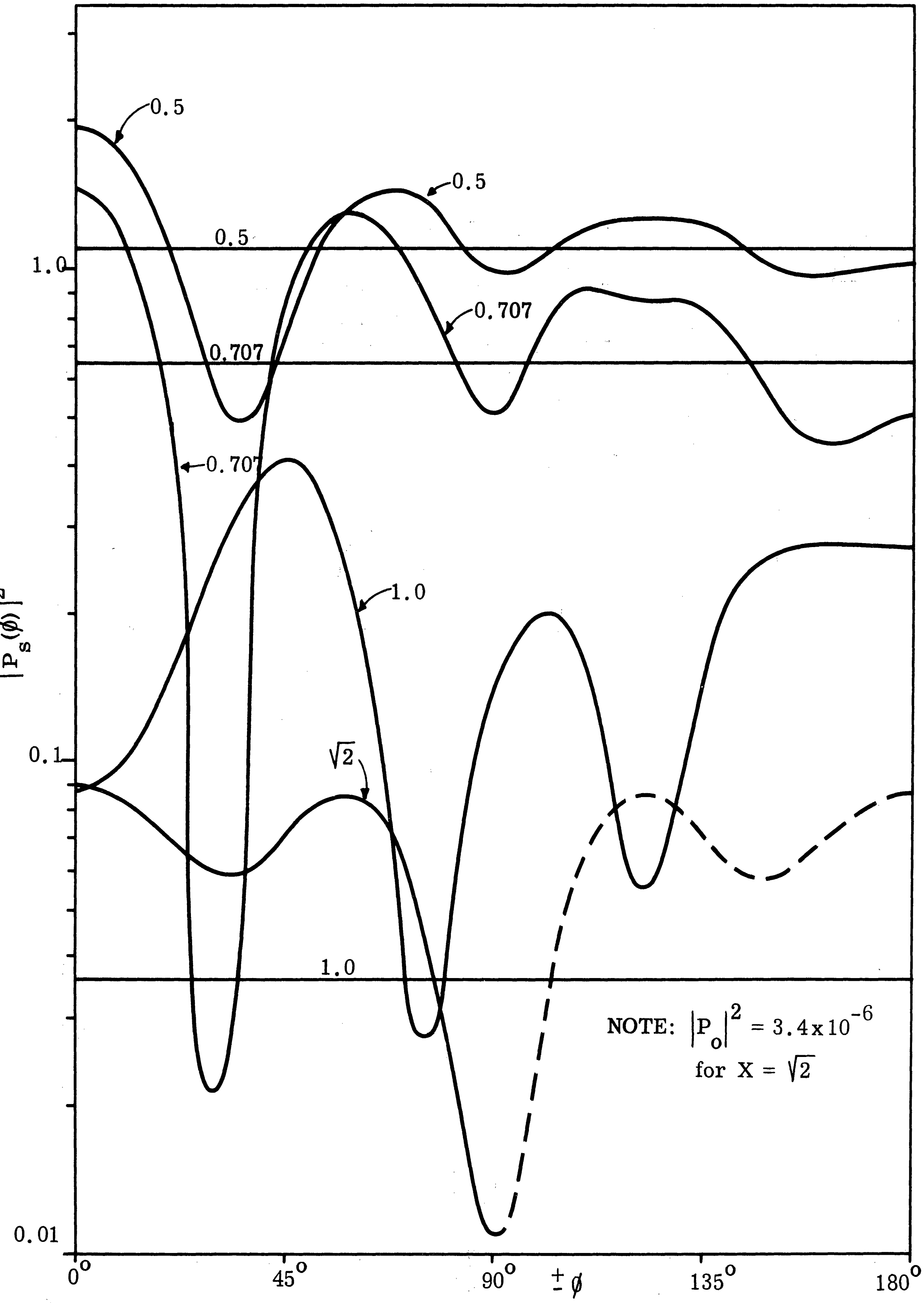

FIG. 13: RADIATION POWER PATTERN FOR $1.0\lambda_o$ THICK PLASMA SHEATH WITH A 36^o WEDGE SLOT, Y = 0.01, AND X AS A PARAMETER. The parallel lines are the corresponding patterns for the unslotted case.

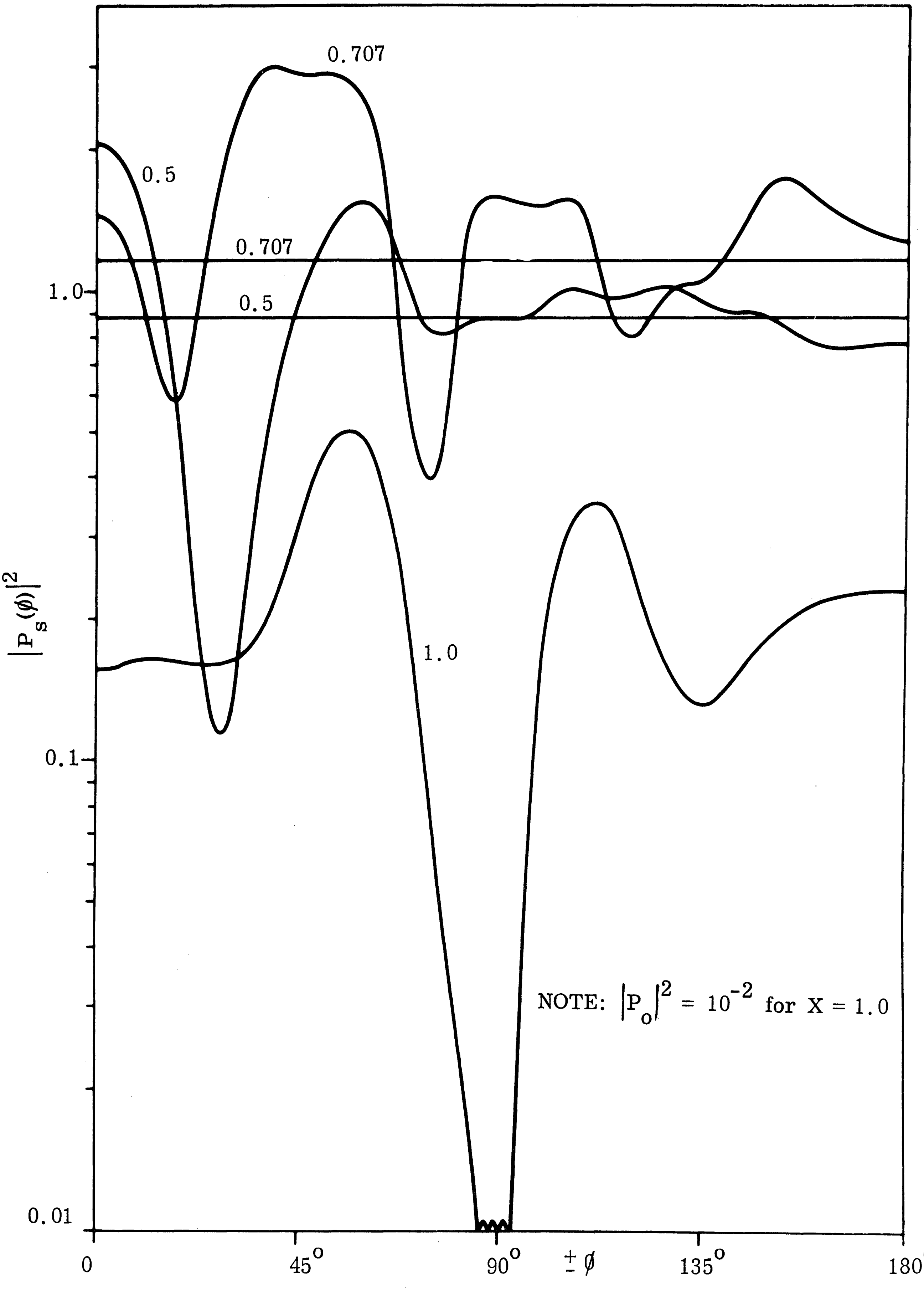

FIG. 14: RADIATION POWER PATTERN FOR 2.0λ$_o$ THICK PLASMA SHEATH WITH A 36^o WEDGE SLOT, Y = 0.01, AND X AS A PARAMETER. The parallel lines are the corresponding patterns for the unslotted case.

in the signal strength for ϕ=0. If we halve the slot width, the forward lobe is considerably reduced with no benefit for the rest of the pattern.

In Figs. 9, 10 and 11 we present the set Y=0.1. By opening a slot in this sheath of "medium" lossy plasma we observe that for $X < 0.707$ the formerly uniform pattern breaks into oscillations. The radiation is enhanced slightly in the forward direction. The oscillating character for the pattern persists as X is increased, i.e. a dominant forward mode is not formed in the range of the validity of approximation, in contrast to the case of the very lossy plasma sheath. Sections of graphs for which the approximations are expected to break down are shown in the dotted line. For $X > 0.707$ the slot improves the radiation very substantially as compared to the sheath without a slot.

In Figs. 12, 13 and 14 we present the set Y=0.01, i.e. a very "low" loss plasma sheath. The comments for this one are substantially the same as for the previous case.

The sets of data have been presented as they were calculated. A dominating forward lobe formed for the Y=1 set. It came as a surprise that for Y=0.1 and 0.01 sets no such dominance was found. However, there is no doubt that a dominant forward lobe would develop as X is increased beyond $\sqrt{2}$. Only (48) is not then a valid approximation in the case of our thick sheath.

The method we have presented for calculating the radiation pattern for the slotted plasma sheath makes use of the electric currents in the plasma before the slot is opened. It is an approximate method which is expected to give the radiation pattern with good accuracy only if the electric currents sufficiently penetrate the plasma sheath. If the attenuation of the unslotted plasma sheath is more than 20 db, i.e. $|P_0|^2 < 0.01$, then we are approaching the bounds of validity of the approximation for the pattern as a whole. However, one expects that the forward lobe will be given to a good order of approximation even for somewhat larger attenuation than 20 db, as was shown by calculations performed in (46).

If the attenuation of the unslotted plasma sheath is such that the radiation through it is practically insignificant, we then find that the electric currents penetrate the plasma only a small fraction of the free space wavelength. If we open a slot in the plasma under these conditions, the slot in fact becomes a waveguide which connects the source with the outside space. For suitable slot boundaries one can show that a finite number of modal fields can propagate in the slot waveguide. These modes may be used as a basis for calculating the radiation pattern. Work is in progress to exploit this waveguide approach for these cases where the integral equation method becomes too cumbersome. This work will be reported as Part II of this study.

The two significant conclusions indicated by this study are:

1) Opening a slot in the plasma sheath re-establishes radiation in the forward direction to a substantial level.

2) The plasma slot should be at least one-half wavelength wide at its narrowest cross section when the electric field is polarized tangential to the slot walls. Increasing the slot width beyond one-half wavelength brings only diminishing returns as far as the radiation enhancement in the forward direction is concerned.

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