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On the Drag of a Sphere at Extremely High Speeds

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The formula for the pressure-drag coefficient of a sphere which moves at extremely high speeds is derived. In the derivation it is assumed that the nose of the shock contour follows exactly the frontal half of the spherical surface and the local pressure on the frontal spherical surface corresponds to the pressure behind the shock wave after the statistical equilibrium between the various degrees of freedom of the molecule has been reached. The chemical dissociation of the molecules behind the shock wave is taken into account in the analysis. The theoretical results of the drag coefficient of the spheres agree with the corresponding available measured value within the experimental error for the range of Mach number between 5 and 10.

1. INTRODUCTION

THE measurement of the supersonic drag of spheres has been the subject of some recent papers.[1,2] It has been generally assumed that the drag coefficient of a sphere moving at a supersonic speed is a function of the Reynolds number and the Mach number. At high Reynolds numbers the supersonic drag coefficient of the spheres depends almost exclusively on the Mach number.[3]

The complexity of the analysis usually makes it very unfruitful to develop the theory of sphere drag at supersonic speeds. It is observed, however, that the apparently close wrapping of the shock wave around the frontal portion of a sphere moving at an extremely high speed as shown in Fig. 5 of Hodges' paper[2] suggests the possibility of analyzing the pressure drag of the sphere by the use of oblique shock relations.

For the purpose of calculating the pressure drag of a very fast moving sphere, we assume that the nose of the shock follows the exact contour of the frontal half of the sphere and that the rear half of the sphere is exposed to a narrow vacuum half shell (see Fig. 1). It is further assumed that statistical equilibrium between the various degrees of freedom of the molecule has been attained when the molecules reach the sphere. The chemical dissociation of the molecules behind the shock wave has to be considered in the analysis in view of the extremely high temperatures. The radial pressure gradient resulting from finite curvature of the streamlines between the frontal shock and the sphere is neglected.

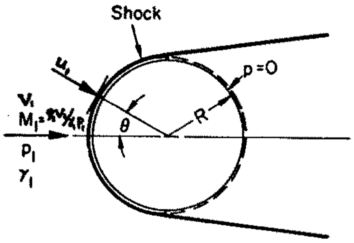

FIG. 1. Model assumed in calculations.

2. ANALYSIS

The pressure coefficient based on normal Mach number $M_1 \cos\theta$ (see Fig. 1) is, from the use of the oblique shock relations,[4]

$$\frac{[(p_2/p_1)-1]}{\gamma_1 M_1^2 \cos^2\theta}=1-\frac{u_2}{u_1}, \tag{1}$$

where u denotes the velocity component normal to the shock contour, and subscripts "1" and "2" designate conditions before and after the shock wave, respectively. The calculation of the ratio of normal velocity components across the shock u_2/u_1 is complicated by the chemical dissociation of the gas behind the shock. It was shown,[4] however, that when the air temperature behind the shock is extremely high, the dissociation of air becomes more and more significant, and the normal-velocity ratio approaches the curve

$$\frac{u_2}{u_1}=\frac{\gamma_e-1}{\gamma_e+1}\left[1-\frac{2}{(\gamma_e-1)M_1^2\cos^2\theta}\right], \tag{2}$$

where $\gamma_e=1.15$; (2) represents the normal-velocity ratio corresponding to a constant γ_e. In general, γ, the local isentropic exponent,[4] is given by

$$\gamma=(\partial \log p/\partial \log\rho), \tag{3}$$

differentiating along the isentrope. ρ denotes the density.

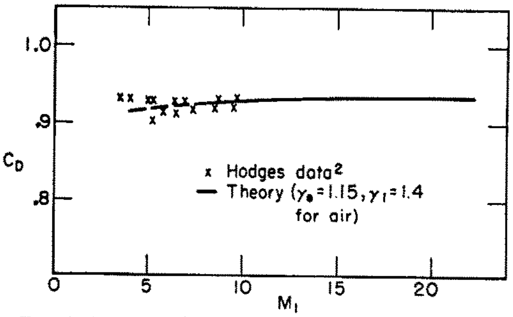

FIG. 2. Drag coefficient of spheres *vs* mach number (sphere velocity/sound velocity).

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[1] A. May, J. Appl. Phys. 28, 910 (1957).

[2] A. J. Hodges, J. Aeronaut. Sci. 24, 755 (1957).

[3] A. C. Charters and R. N. Thomas, J. Aeronaut. Sci. 12, 468 (1945).

[4] W. E. Moeckel, Natl. Advisory Comm. Aeronaut., Tech. Notes, Washington, D. C., No. 3895 (1957).

Assuming that at an extremely high Mach number u_2/u_1 can be approximated by (2), we can evaluate the pressure-component integral,

$$\int_0^{\pi/2} 2\pi R^2(p_2-p_1)\sin\theta\cos\theta d\theta, \quad (4)$$

from which the pressure-drag coefficient of a sphere moving at an extremely high speed in the air can be obtained,

$$C_D=[2/(\gamma_e+1)]-[4/(\gamma_e+1)M_1^2]+(2/\gamma_1 M_1^2). \quad (5)$$

(Note that C_D is defined as the total pressure-drag force divided by $\frac{1}{2}\pi R^2\rho_1 V_1^2$.) The third term on the right-hand side of (5) represents the base drag coefficient.

It is of interest to note from (5) that, at high Mach numbers and Reynolds numbers, C_D depends on γ_e, γ_1 in addition to M_1. The theoretical values of C_D calculated from (5) with the assumption of chemical dissociation of air after the shock wave such as at $M=10$ agrees with Hodges' measured value within his experimental error [see Fig. 2 and note that $K_D=(\pi/8)C_D$]. This close agreement actually extends to $M_1=5$, although it is doubtful that the assumption concerning chemical dissociation of air still holds. The significance of the surprisingly close agreement between the results of measurement and the theory must be taken with guarded optimism until more experimental data are available, especially with different gases, considering the crudeness of the model assumed and the neglect of the skin friction.

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X-Ray Study of Cold Work in Molybdenum*

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Filings from a molybdenum rod, pressed into a briquet, were used as the cold worked sample. Satisfactory peak shapes for 110, 200, 211, 220, and 310 were obtained with filtered CuKα. Peak shapes for 400 and 422 obtained with filtered MoKα were not satisfactory, partly because of fluorescence radiation. Peaks were recorded with a Norelco diffractometer, and instrumental broadening corrected by the Stokes method. The Fourier coefficients indicate approximately isotropic incoherent domains $L=260$ A, and strains inversely as Young's modulus. Molybdenum is similar to tungsten in that there is no evidence for faulting on (211) as a result of cold work.

I. INTRODUCTION

It has been shown by Guentert and Warren[1] that if cold work in a body-centered cubic metal produces deformation or twin faulting on the (211) planes, there should be a particle size type of broadening of the x-ray reflections with effective sizes in the ratio $L(110):L(100):L(211)=2.83:1:1.63$, whereas incoherent boundaries should give a particle size broadening which is nearly isotropic. Measurements on β brass filings[1] indicated a particle size broadening which is largely due to faulting on the (211) planes, while measurements on tungsten filings[2] indicated an isotropic particle size of around $L=200$ A. Since tungsten is elastically isotropic, it is of interest to make similar measurements on molybdenum. For this material the slight anisotropy is unusual, Young's modulus for the 100 direction is greater than that for the 110 direction, $E(100)=1.17E(110)$.

II. EXPERIMENTAL PROCEDURE

Filings obtained from a molybdenum rod were put through a sieve of 325 mesh per inch. The cold worked sample was a briquet pressed at 60 000 lb per square inch. Other samples were prepared by annealing briquets of filings in vacuum for one hour at 415°, 660°, and 908°C. A briquet of tungsten filings annealed one hour at 1200°C was used as a standard for correcting the instrumental broadening. The powder pattern reflections were recorded with a Norelco diffractometer using filtered CuKα and MoKα radiation. Attempts to record reflections such as 400 and 422 with filtered Mo radiation were unsatisfactory, partly because of the high background due to fluorescence Mo radiation. It was concluded that only the reflections 110, 200, 211,

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[1] O. J. Guentert and B. E. Warren, J. Appl. Phys. 29, 40 (1958).

[2] M. McKeehan and B. E. Warren, J. Appl. Phys. 24, 52 (1953).