Dynamic stabilization of plasmas by means of a high-frequency-modulated electron beam

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The stabilizing effects that a beam of longitudinally nonuniform density imposes on beam-plasma instabilities are investigated using a hydrodynamic model adapted for an infinite, one-dimensional beam-plasma system. The electron beam is characterized by either of two types of longitudinal inhomogeneity—square-wave or sinusoidal. The results, for the square-wave type of inhomogeneity, indicate a disappearance of the convective instabilities below a threshold frequency as well as a reduction in the growth rate for the remainder of the spectrum. Similar results are obtained for the case of a high-frequency sinusoidal pulsation of the beam density. Effects resulting from finite temperature plasma electrons are also discussed.

I. INTRODUCTION

Experimental studies[1,2] have shown that the excited oscillations in a plasma, which are due to the interaction of a beam of longitudinally nonuniform density with the plasma, will have considerably smaller growth rates than those due to the interaction of a uniform density beam with the plasma, so long as the spatial wavelength of the beam inhomogeneity, measured in the moving frame of reference attached to the beam, is considerably smaller than the wavelength of the excited oscillations. By a beam of longitudinally nonuniform density we mean an electron beam for which the density is a function (here periodic) of the spatial coordinate (along which the beam particles are drifting) measured in the frame of reference attached to the beam. Therefore, the space-time variation of the density of such a beam, written in the inertial frame of reference attached to the beam, is given in the form

$$n_b(\zeta, t) = n_{0b}(\zeta) + n_{1b}(\zeta)\exp(j\omega t), \tag{1}$$

where ζ is the longitudinal coordinate measured in the moving frame and $n_{0b}(\zeta)$ and $n_{1b}(\zeta)$ are the zeroth- and first-order perturbation terms of the total beam density, $n_b(\zeta, t)$, respectively. The suppressive effects that such a beam exhibits on beam-plasma instabilities have been qualitatively explained by the argument of phase destruction advanced by Fainberg.[3]

From a broader viewpoint, the rf excitations are also used to suppress the so-called drift-wave instabilities associated with particle drifts in an inhomogeneous plasma. Recently, there has been a growing interest in devising means of suppressing the abovementioned instabilities because of the resulting enhanced diffusion of the plasma across the magnetic field.[4–6] The present work deals with the effects of a beam-produced rf excitation in suppressing the convective instabilities that generally arise in a beam-plasma system.

II. MATHEMATICAL FORMULATION

The following assumptions are made in the course of formulating the problem:

1. The plasma is cold, stationary, and uniform with infinitely heavy ions which serve as a neutralizing background.
2. The electron beam is cold and drifting with a velocity $\mathbf{u}_0$ parallel to the applied dc magnetic field $\mathbf{B}_0 = \infty$.
3. The perturbations are assumed to be of small magnitude so as to make a linear analysis possible.
4. Nonrelativistic mechanics applies.
5. The quasistatic assumption is appropriate.

On the basis of the preceding assumptions, hydrodynamic equations of motion are written for both beam and plasma particles. The beam and plasma space-charge waves are coupled through an ac electric field and this coupling is expressed mathematically by Gauss' law.

The linearized equations of motion, the continuity equation, and Gauss' law, when written in an inertial frame of reference attached to the beam, have the following form:

$$\frac{\partial v_{1b}}{\partial t} = -\frac{e}{m} E_1, \tag{2}$$

$$\frac{\partial n_{1b}}{\partial t} + \frac{\partial}{\partial \zeta}(n_{0b} v_{1b}) = 0, \tag{3}$$

$$\frac{\partial v_{1p}}{\partial t} - u_0 \frac{\partial}{\partial \zeta} v_{1p} = -\frac{e}{m} E_1, \tag{4}$$

$$\frac{\partial n_{1p}}{\partial t} + \frac{\partial}{\partial \zeta}(n_0 v_{1p} - u_0 n_{1p}) = 0, \tag{5}$$

and

$$\frac{\partial}{\partial \zeta} E_1 = -\frac{e}{\epsilon_0}(n_{1b} + n_{1p}), \tag{6}$$

where the subscript 1 denotes the first-order perturbation quantities, subscripts b and p refer to the beam and the plasma parameters, respectively, and ζ is the longitudinal coordinate in the moving frame of reference. A steady-state solution to these equations can be sought by making all the first-order quantities assume an $\exp(j\omega t)$ variation in time. If this consideration is made, a combination of (2)–(6) yields the following differential equation for the longitudinal ac electric field:

$$\left(j\omega-u_0\frac{d}{d\zeta}\right)^2[\epsilon_b(\omega,\zeta)E_1]+\omega_p^2E_1=0, \tag{7}$$

where $\epsilon_b(\omega, \zeta)$ is the beam dielectric constant defined as

$$\epsilon_b(\omega,\zeta)\equiv1-[e^2n_{0b}(\zeta)/m\epsilon_0\omega^2]. \tag{8}$$

A transformation of the dependent variable can be made to standardize the preceding differential equation. Hence, if $Z(\zeta)$ is defined as

$$Z(\zeta)\equiv\epsilon_b(\omega,\zeta)E_1(\zeta)\exp[-j(\omega/u_0)\zeta], \tag{9}$$

a direct substitution into (7) yields the following differential equation for $Z(\zeta)$:

$$\frac{d^2}{d\zeta^2}Z(\zeta)+\frac{\omega_p^2}{u_0^2\epsilon_b(\omega,\zeta)}Z(\zeta)=0. \tag{10}$$

A differential equation of this form has been discussed in detail by Cesari[7] who gave the exact solution of the equation for an arbitrary form of $\epsilon_b(\omega,\zeta)$. A special case of particular interest is when $\epsilon_b(\omega,\zeta)$ is periodic, in which case (10) is called Hill's equation. However, the exact methods of solving such equations suffer from a lack of applicability to equations of order higher than two.

On the other hand, the method of averaging, as demonstrated by Bogoliubov and Mitropolsky,[8] can be successfully used to determine the approximate solution to the differential equations of the same type as (10) as well as equations of higher order. The applicability of this method hinges upon the requirement that $\epsilon_b(\omega,\zeta)$ be periodic in ζ with a periodicity much smaller than the periodicity of the natural oscillations of the system. This condition is equivalent to the assumption that the wavelength of the excited oscillations is much longer than the wavelength of the beam inhomogeneity, measured in the moving frame of reference. In the following, the use of this method is illustrated by its direct application to (7). For purely analytical reasons, Eq. (7) is rewritten in terms of the electric flux density D_1:

$$\left(j\omega-u_0\frac{d}{d\zeta}\right)^2D_1+\frac{\omega_p^2}{\epsilon_b(\omega,\zeta)}D_1=0. \tag{11}$$

For an electron beam whose density profile is periodic with a spatial wavelength of $2\pi/k_0$, the beam density and the dielectric constant can be expanded in Fourier series. In particular, it is asserted that

$$[\epsilon_b(\omega,\zeta)]^{-1}=\sum_n\varphi_n(\omega)\exp(-jnk_0\zeta). \tag{12}$$

In accordance with the method of averaging, $D_1(\zeta)$ can be decomposed according to the following:

$$D_1(\zeta)=D_1^{(0)}(\zeta)+D_1^{(1)}(\zeta)=D_1^{(0)}(\zeta)+\sum_{n\neq0}{}'d_n(\omega)\exp(-jnk_0\zeta), \tag{13}$$

where $D_1^{(1)}(\zeta)$ is the fast oscillatory part of the solution $D_1(\zeta)$, whose amplitude is small compared with that of $D_1^{(0)}(\zeta)$. Direct substitution of (12) and (13) into (11), separating the zeroth- and nth-order terms and setting each equal to zero, yields the following results:

$$\left(j\omega-u_0\frac{d}{d\zeta}\right)^2D_1^{(0)}+\omega_p^2\varphi_0(\omega)D_1^{(0)}+\omega_p^2\sum_{n\neq0}{}'\varphi_n(\omega)d_{-n}(\omega)=0 \tag{14}$$

and

$$-(\omega+nk_0u_0)^2d_n(\omega)+\omega_p^2\varphi_n(\omega)D_1^{(0)}+\omega_p^2\sum_{m\neq0}{}'\varphi_{n-m}(\omega)d_m(\omega)=0;\quad(n=\pm1,\pm2,\cdots). \tag{15}$$

The next step is to ignore the term

$$\sum_{m\neq0,n}{}''\varphi_{n-m}(\omega)d_m(\omega)$$

in (14) and (15). Neglect of this term is equivalent to stating that the spectrum of the fast oscillatory part of the excited field has nonzero components only at those wavelengths for which the spectrum of the beam pulse has nonzero components. In this fashion the possibility of getting a cascade in wavenumber space and obtaining a Kolmogoroff spectrum is excluded and a limit is put on the number of degrees of freedom for the system. Both of these are in accordance with the dynamical features of weakly turbulent plasmas[9,10] and therefore neglect of the sum

$$\sum_{m\neq0,n}{}''\varphi_{n-m}(\omega)d_m(\omega)$$

can appropriately be termed "the weak turbulence approximation."

The remaining is straightforward and a combination of (14) and (15) resolves into a dispersion relation which, after a Doppler transformation $\omega\rightarrow\omega_D\equiv\omega-ku_0$, is reduced to

$$\varphi_0(\omega_D)-\frac{\omega^2}{\omega_p^2}+2\omega_p^2\sum_{n=1}^{\infty}\frac{|\varphi_n(\omega_D)|^2}{n^2\omega_0^2-\omega_p^2\varphi_0(\omega_D)}=0, \tag{16}$$

where $\omega_0\equiv k_0u_0$. It is noted that in the limit when $\varphi_n\rightarrow0$, i.e., when the beam is homogeneous, (16) reduces to the Bohm–Gross dispersion relation.[11]

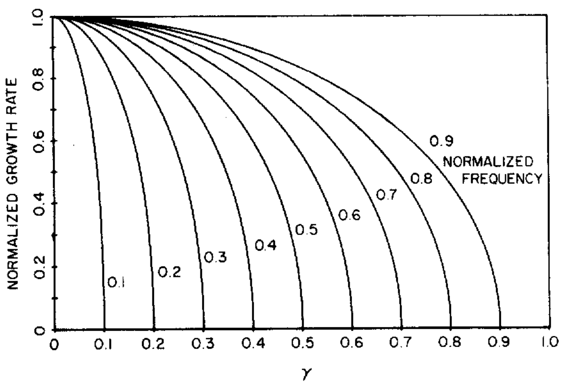

FIG. 1. Normalized growth rate δ_γ/δ_0 vs γ for square-wave inhomogeneity.

III. RESULTS (COLD PLASMA CASE)

A. Square-Wave Inhomogeneity

For this case (16) will lead directly to

$$k=\frac{\omega}{u_0}\pm\frac{\omega_b}{u_0}\left(\frac{1-\gamma^2\omega_p^2/\omega^2}{1-\omega_p^2/\omega^2}\right)^{1/2}, \tag{17}$$

where terms of order (ω_p^2/ω_0^2) and higher were neglected and ω_b is the beam plasma frequency defined by $\omega_b^2=e^2n_{0b}/m\epsilon_0$. In (17), k is the propagation constant and γ is a measure of the fraction of time that the beam remains turned off [$\gamma\equiv(1-\Delta)^{1/2}$, Δ is the duty cycle]. From (17) the region of instability can easily be found as

$$\gamma<\omega/\omega_p<1. \tag{18}$$

It is noted that square-wave pulsation places a lower limit on the extent of the instability region measured on the ω scale. Throughout the region of instability, the growth rate is diminished by a factor of $[1-\gamma^2(\omega_p^2/\omega^2)]^{1/2}$. The effectiveness of the suppression of convective instabilities by means of a high-frequency, square-wave pulsation is shown in Fig. 1, where variations of the normalized growth rate δ_γ/δ_0 are shown as a function of γ, with the normalized frequency ω/ω_p taken as a constant, with δ_γ defined as

$$\delta_\gamma\equiv\frac{\omega_b}{u_0}\left(\frac{1-\gamma^2\omega_p^2/\omega^2}{\omega_p^2/\omega^2-1}\right)^{1/2},$$

$$\gamma<\omega/\omega_p<1,\qquad 0\leq\gamma<1. \tag{19}$$

The quantity δ_γ corresponds to the imaginary part of the wavenumber or propagation constant k [Eq. (17)]. δ_γ/δ_0 is a measure of the inhomogeneous beam growth rate relative to that for a homogeneous beam. It is seen that the suppression is more effective for the lower-frequency modes. This is in agreement with the argument of phase destruction which can be summarized as follows.

Fainberg[3] has pointed out that, if an electron beam is injected into a plasma, self-modulation of the beam particles will occur as a result of synchronization between the beam particles and the restoring electric field, maintaining plasma oscillations. This will eventually lead to a coherent oscillation of the particles in the beam, in phase with the restoring field, and thus contributes to the effectiveness of the interaction through enhancement of the field. Hence, on the basis of the above argument, it would seem natural that if the beam particles are somehow prohibited from falling into phase with the restoring field, there will be a disruption of the instabilities throughout the spectrum. Previous bunching of the beam density at a frequency other than that of the restoring field is an effective means by which a phase destruction between the beam particles and the restoring field of plasma oscillations is possible.

Consideration should be made of the parametric resonances that a previously bunched beam excites. These are resonances which occur at frequencies near the bunching frequency. Experimental studies,[1,2] however, have shown that parametric instabilities have a narrow width, and so they may be weakened or even disappear due to the inhomogeneities and collisions that occur in actual systems.

The present theoretical investigation supports this argument in two ways: in shrinking the region over which instabilities exist and in reducing the growth rate for the remainder of the spectrum.

B. Sinusoidal Inhomogeneity

The profile of the beam density in this case is given by

$$n_{0b}(\zeta)=n_{0b}(1+\alpha\cos k_0\zeta). \tag{20}$$

It is straightforward to show that the dispersion relation (16) can be expressed as follows for the case of a sinusoidal density profile:

$$\left[\left(1-\frac{\omega_b^2}{\omega_D^2}\right)^2-\alpha^2\frac{\omega_b^4}{\omega_D^4}\right]^{-1/2}-\frac{\omega^2}{\omega_p^2}+\frac{2}{\pi^2}\frac{\omega_p^2}{\omega_0^2}\times\sum_{n=1}^{\infty}\frac{1}{n^2}C_n^2\left(1-\frac{\omega_b^2}{\omega_D^2},-\alpha\frac{\omega_b^2}{\omega_D^2}\right)=0, \tag{21}$$

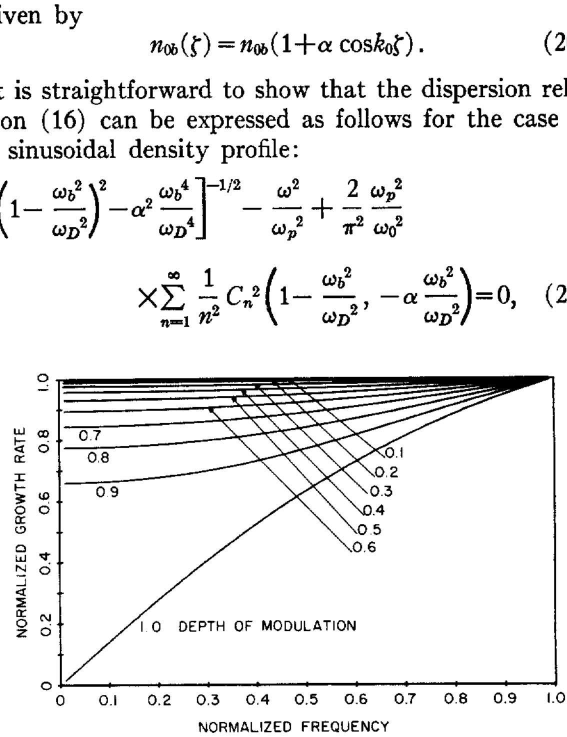

FIG. 2. Normalized growth rate δ_α/δ_0 vs normalized frequency ω/ω_p for sinusoidal inhomogeneity.

where $C_n(a, b)$ is defined as

$$C_n(a,b) \equiv \int_0^{\pi} \frac{\cos nx\, dx}{a+b\cos x}. \qquad (22)$$

A three-term recurrence formula can be found[12] for the evaluation of C_n. The results are $C_{n+2}=-(2a/b)C_{n+1}-C_n$, $C_1=(\pi-aC_0)/b$, and $C_0=\pi(a^2-b^2)^{-1/2}\,\mathrm{sgn}(a)$ for $|a|>|b|$. An approximate solution to (21) can be expressed as

$$k=\frac{\omega}{u_0}\pm j\frac{\omega_b}{u_0}\left(\frac{1-\alpha^2}{[(\omega_p^4/\omega^4)-\alpha^2(\omega_p^4/\omega^4-1)]^{1/2}-1}\right)^{1/2}. \qquad (23)$$

The term $\omega_p^4/\omega^4-\alpha^2[(\omega_p^4/\omega^4)-1]$ is always positive ($\alpha<1$) and its value is greater than unity throughout the range $\omega/\omega_p<1$. Therefore, unlike the case of a rectangular inhomogeneity, the range of instability in this case is independent of the depth of modulation. Throughout the region $\omega/\omega_p<1$ the growth rate of the convective instability is diminished by a factor given as

$$\frac{\delta_\alpha}{\delta_0}=\frac{\mu(1-\Omega^2)^{1/2}}{\{[\mu^2(1-\Omega^4)]^{1/2}+\Omega^4-\Omega^2\}^{1/2}}, \qquad (24)$$

where δ_α is defined as the imaginary part of k as expressed in (23), and μ is defined as $(1-\alpha^2)^{1/2}$ and Ω is defined as ω/ω_p. Figure 2 shows the variations of the normalized growth rate δ_α/δ_0 as a function of the normalized frequency ω/ω_p, with the depth of modulation α taken as a parameter.

The results of both cases indicate diminished growth rates with a higher degree of suppression predicted for modes of lower frequencies. The width of the unstable region is narrowed for the square-wave modulation but is unchanged for the sinusoidal pulsation and exhibits no dependence on the depth of modulation.

IV. FINITE-TEMPERATURE EFFECTS

In this section the analysis is extended to include plasma-electron temperature effects as well as plasma-ion motion. The hydrodynamic model is again adopted for the beam and the plasma. It should be noted that by adopting a fluid model for plasmas the effects of such micromechanisms as Landau and cyclotron damping are inadvertently ignored. The equations of motion for plasma electrons and ions, as well as beam electrons, are written in the same manner as before with an additional term in the force equation for plasma electrons which is introduced to take into account the effects of temperate electrons. Beam ions are assumed to be infinitely heavy. In an inertial frame of reference attached to the beam, the equations of motion and Gauss' law are expressed as follows:

$$\frac{\partial v_{1be}}{\partial t}=-\frac{e}{m_e}E_1, \qquad (25)$$

$$\frac{\partial n_{1be}}{\partial t}+\frac{\partial}{\partial \zeta}(n_{0b}v_{1be})=0, \qquad (26)$$

$$\frac{\partial v_{1pe}}{\partial t}-u_0\frac{\partial v_{1pe}}{\partial \zeta}=-\frac{e}{m_e}E_1-\frac{\partial}{\partial \zeta}\left(\frac{p_{1e}}{n_{0p}m_e}\right), \qquad (27)$$

$$\frac{\partial v_{1pi}}{\partial t}-u_0\frac{\partial v_{1pi}}{\partial \zeta}=\frac{e}{m_i}E_1, \qquad (28)$$

$$\frac{\partial n_{1pe,i}}{\partial t}-u_0\frac{\partial n_{1pe,i}}{\partial \zeta}+n_{0p}\frac{\partial v_{1pe,i}}{\partial \zeta}=0, \qquad (29)$$

and

$$\frac{\partial}{\partial \zeta}E_1=\frac{e}{e_0}(n_{1pi}-n_{1pe}-n_{1be}), \qquad (30)$$

where the additional subscripts e and i denote the electronic and ionic quantities, respectively. The first-order perturbation in the plasma electron pressure is denoted by p_{1e} whose definition, in terms of electronic temperature, is

$$p_{1e}\equiv\gamma k_B T_e n_{1pe}, \qquad (31)$$

where γ is the compression constant of the electron gas. Under adiabatic conditions, γ will tend to a value equal to three at high frequencies (one-dimensional compression), and $\frac{5}{3}$ at somewhat lower frequencies (three-dimensional compression). At very low frequencies the compression will be isothermal and γ tends to unity.

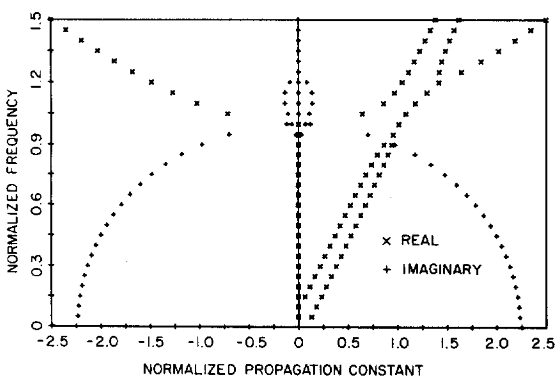

FIG. 3. Dispersion diagram for $\Delta=0.3$ (square-wave inhomogeneity) ($\Omega_b=0.1$, $r=0.2$).

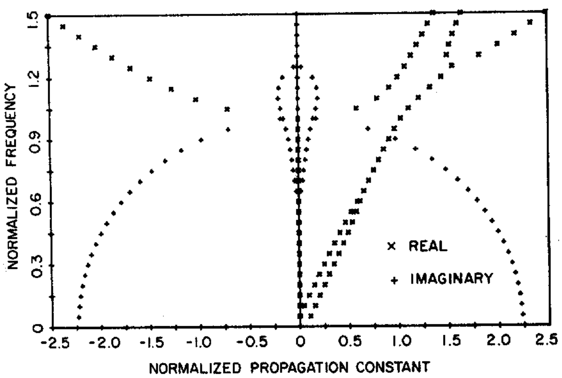

FIG. 4. Dispersion diagram for $\Delta=0.7$ (square-wave inhomogeneity) ($\Omega_b=0.1$, $r=0.2$).

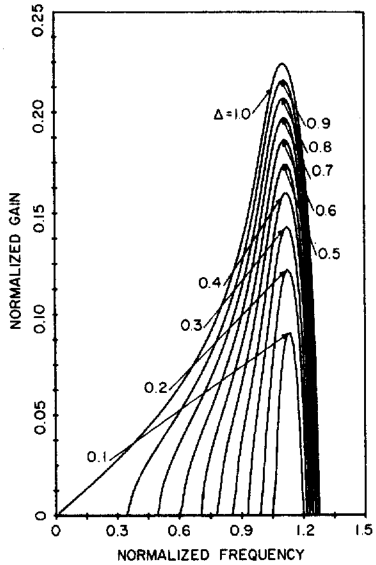

FIG. 5. Normalized gain $k_i u_0/\omega_p$ vs normalized frequency ω/ω_p with the duty cycle Δ taken as a parameter (square-wave inhomogeneity) ($r=0.2$).

Combination of the above equations, in a steady-state situation, yields the following differential equation for the ac electric flux density D_1:

$$\left[\left(j\omega-u_0\frac{d}{d\zeta}\right)^2-v_{th}^2\frac{d^2}{d\zeta^2}\right]\left(j\omega-u_0\frac{d}{d\zeta}\right)^2 D_1 +\left[(\omega_{pe}^2+\omega_{pi}^2)\left(j\omega-u_0\frac{d}{d\zeta}\right)^2-\omega_{pi}^2v_{th}^2\frac{d^2}{d\zeta^2}\right] \times\left(\frac{D_1}{\epsilon_b(\omega,\zeta)}\right)=0. \quad (32)$$

Here, the expression $v_{th}\equiv(\gamma k_B T_e/m_e)^{1/2}$ is the mean thermal velocity of plasma electrons. The method of averaging can again be used to solve the above differential equation. The final form of the dispersion relation, having made the Doppler-shift transformation, is

$$\frac{\omega^2-k^2v_{th}^2}{\omega_p^2}-\left(1-\frac{k^2v_{th}^2}{\omega^2}\frac{\omega_{pi}^2}{\omega_p^2}\right)\varphi_0(\omega_D) -\frac{\omega_p^2}{\omega_0^2}\frac{1+(\omega_{pi}^2/\omega_p^2)(v_{th}^2/u_0^2)}{1-v_{th}^2/u_0^2}\sum_{n\neq0}{}'\frac{1}{n^2}|\varphi_n(\omega_D)|^2=0, \quad (33)$$

where $\omega_p^2\equiv\omega_{pe}^2+\omega_{pi}^2$.

It is noted from the above equation that for a realistic electron-to-proton mass ratio the effect of plasma ion motion is negligible in the context of the approximations employed throughout. The coefficient of $\varphi_0(\omega_D)$ in (33) can alternatively be expressed as

$$1-\frac{k^2v_{th}^2}{\omega^2}\frac{\omega_{pi}^2}{\omega_p^2}=1-\gamma\left(\frac{v_s}{\omega/k}\right)^2, \quad (34)$$

where the terms of order $(\omega_{pi}/\omega_{pe})^4$ and higher were neglected and the expression $v_s\equiv(k_BT_e/m_i)^{1/2}$ is the velocity of ion-acoustic oscillations in the plasma. For oscillations of wavelengths long enough such that $\omega/k\sim u_0$, which is in accordance with the previous assumptions of $k\ll k_0$ and $\omega\ll\omega_0$, the right-hand side of (34) tends to unity. This is equivalent to neglecting the motion of plasma ions. For oscillations at wavelengths short enough such that $\omega/k\sim v_s$, the right-hand side of (34) tends to zero. However the basic assumption underlying the method of averaging, namely, that $k\ll k_0$, does not allow consideration of short-wavelength oscillations. Other mathematical techniques for solving (32) must therefore be developed if the coupling to the ion-acoustic oscillations is of interest.

Neglecting the motion of plasma ions gives the following dispersion relation:

$$\frac{\omega^2-k^2v_{th}^2}{\omega_p^2}-\varphi_0(\omega_D)-\frac{\omega_p^2}{\omega_0^2}\left(1-\frac{v_{th}^2}{u_0^2}\right)^{-1} \times\sum_{n\neq0}{}'\frac{1}{n^2}|\varphi_n(\omega_D)|^2=0. \quad (35)$$

It is noted that in the limit when $\varphi_n\to0$, i.e., when the beam is homogeneous, (35) appropriately reduces to the Bohm–Gross dispersion relation[11] for the one-dimensional interaction of an electron beam with a temperate plasma.

V. RESULTS (TEMPERATE-PLASMA CASE)

A. Square-Wave Inhomogeneity

For this case the dispersion relation (35) yields a fourth-degree polynomial in the normalized propagation constant $x\equiv ku_0/\omega_p$ as follows:

$$rx^4-2r\Omega x^3+[1-\Omega^2+r(\Omega^2-\Omega_b^2)]x^2-2\Omega(1-\Omega^2)x +(1-\Omega^2)(\Omega^2-\Omega_b^2)+\Delta\Omega_b^2=0, \quad (36)$$

where $\Omega\equiv\omega/\omega_p$, $\Omega_b\equiv\omega_b/\omega_p$, $r\equiv\gamma k_BT_e/2eV_0$ (V_0 is the dc voltage of the beam), and Δ is the duty cycle. Figures 3 and 4 show the dispersion diagram for two values of the duty cycle $\Delta=0.3$ and 0.7, respectively.

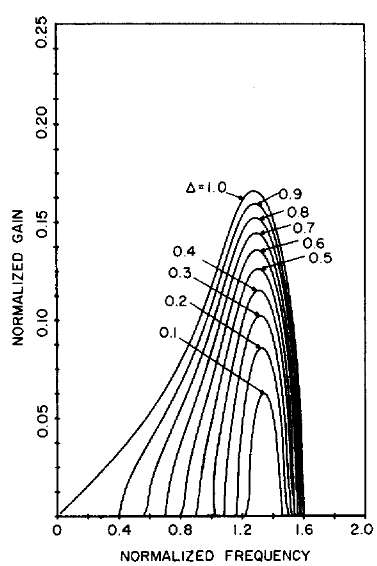

FIG. 6. Normalized gain $k_i u_0/\omega_p$ vs normalized frequency ω/ω_p with the duty cycle Δ taken as a parameter (square-wave inhomogeneity) ($r=0.4$).

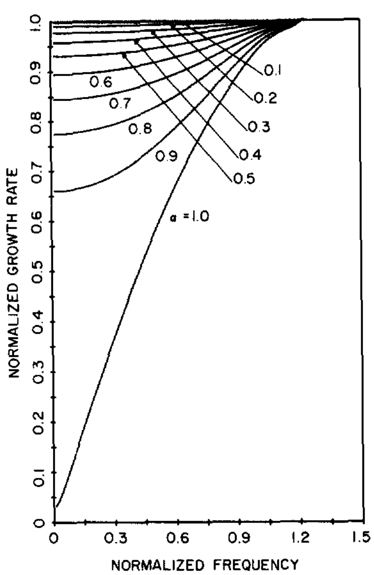

FIG. 7. Normalized growth rate $k_{i\alpha}/k_{i0}$ vs normalized frequency ω/ω_p with depth of modulation α taken as a parameter (sinusoidal inhomogeneity) ($r=0.2$).

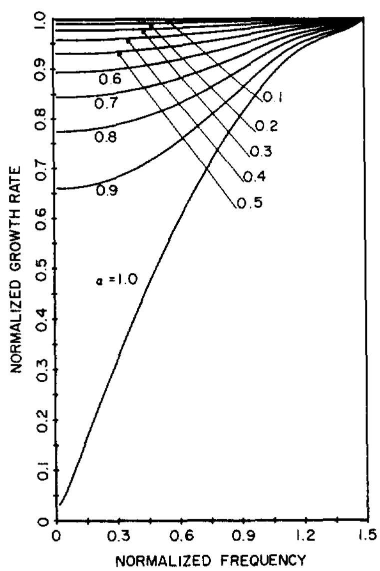

FIG. 8. Normalized growth rate $k_{i\alpha}/k_{i0}$ vs normalized frequency ω/ω_p with depth of modulation α taken as a parameter (sinusoidal inhomogeneity) ($r=0.4$).

The reduction in the growth rate as well as a narrowing of the region of instability is observed. The effects of an increase in the electronic temperature are shown in Figs. 5 and 6.

B. Sinusoidal Inhomogeneity

An eighth-degree dispersion relation is obtained of which only four roots correspond to the physically real modes of the system (slow and fast space-charge waves of the beam and symmetrical temperate plasma modes). The effect of a sinusoidal inhomogeneity is shown in Figs. 7 and 8 where the normalized growth rate, $k_{i\alpha}/k_{i0}$ ($k_{i\alpha}$ is the imaginary part of k for a depth of modulation given by α), is plotted vs normalized frequency ω/ω_p with the depth of modulation α taken as a parameter.

VI. CONCLUSIONS

The results of the temperate plasma case have indicated suppression of convective instabilities as a result of a periodic beam inhomogeneity in a manner similar to that previously obtained for the cold plasma. The effect of temperature is simply to remove the singularity of the propagation constant at the plasma frequency. A narrowing of both the ω and k spectra is observed for the case of a rectangular inhomogeneity, whereas the results of sinusoidal inhomogeneity have indicated a narrowing of the k spectrum only. Narrowing of both spectra is more effectively achieved by means of rectangular modulation. A higher degree of suppression is reported for the lower-frequency modes for the cold as well as the temperate plasma case.

An imposed static magnetic field, which is assumed to be infinite in the present case, is known to play a major role in the act of suppression. The effects of the static magnetic field have been investigated in some detail and will be discussed in another paper where two-dimensional effects are considered.

ACKNOWLEDGMENT

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