SU(6) isoscalar factors for the product $405\times 56 \to 56$, 70*

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SU(6) isoscalar factors for the product $405\times 56 \to 56$, 70 are calculated. SU(3) isoscalar factors for the products $27\times 10 \to 10$, 8 and $\overline{10}\times 8 \to 8$ are also tabulated.

I. INTRODUCTION

The SU(6) symmetry group was first found useful for the classification of hadrons in the 1960's. It has recently been extended by Melosh[1] to apply to matrix elements of currents between hadron states. Following the ideas of Gell-Mann, the currents are postulated to belong to irreducible representations of an SU(6) of currents, while the particle states are classified by a different, constituent SU(6). These two different SU(6) symmetries are connected by a unitary transformation, the Melosh transformation. Melosh explicitly constructed this transformation for the free-quark model. The algebraic properties of currents transformed by the Melosh transformation have been extracted from this model and applied to physically relevant matrix elements. This method removed the inconsistencies which appeared in old SU(6) calculations of several axial coupling constants and the magnetic moments of the nucleons.[1] Gilman, Kugler, Meshkov and others,[2] used PCAC in addition to the algebraic properties of the Melosh transformed axial vector current to satisfactorily predict pionic emission amplitudes for the decays of mesons and baryons. Gilman, Karliner, and others,[3] also found that the application of the Melosh transformation technique to real photon emissions from baryons and mesons is consistent with experiment.

In each of the above applications, the basic technique is to use the Wigner—Eckart theorem to calculate a particular physically relevant matrix element. Thus, the matrix element of an operator between two hadron states is the product of appropriate SU(6) and angular momentum Clebsch—Gordan coefficients, times a reduced matrix element.[2] For each of the above applications, the Melosh transformed currents belong to 35 representations of SU(6). The baryons are classified in 56 and 70 representations, and the mesons form 35 representations of SU(6). The appropriate SU(6) Clebsch—Gordan coefficients for these applications have been calculated by Carter, Coyne, and Meshkov,[4] and by Cook and Murtaza.[5]

If one now wants to apply this technique to current-current matrix elements between baryon states which occur, for example, in nonleptonic weak decays, higher representations which originate from the product 35×35 must be considered. Explicitly, the product 35×35 is decomposed into the irreducible representations:

$$35\times 35 \to 1+35+35'+189+280+\overline{280}+405. \tag{1.1}$$

The only representations in (1.1) which will contribute to a matrix element between baryon states belonging to the 56 and the 56 or 70 representations are 35 and 405.[6] In this paper, the Clebsch—Gordan coefficients for the product $405\times 56 \to 56$, 70 are obtained so that such current—current processes may be treated in full. In Sec. II, the method of calculating the SU(6) isoscalar factors for the product $405\times 56 \to 56$ and 70 with appropriate choice of phase is explained. In Sec. III, the SU(6) isoscalar factors for $405\times 56 \to 56, 70$ are tabulated. SU(3) isoscalar factors for the products $27\times 10 \to 10, 8$ and $\overline{10}\times 8 \to 8$, which were used in the present calculation, are also given in Sec. III.

II. METHOD OF CONSTRUCTION

A given SU(6) representation may be reduced according to the subgroup $SU(3)\times SU(2)$. In terms of the spectroscopic notation A^{2S+1}, where A is the SU(3) representation label and $2S+1$ is the SU(2) spin multiplicity, the 35, 56, 70, and 405 representations have the following $SU(3)\times SU(2)$ contents:

$$\underline{35}=8^3, 8^1, 1^3, \tag{2.1}$$

$$\underline{56}=10^4, 8^2, \tag{2.2}$$

$$\underline{70}=8^4, 10^2, 8^2, 1^2, \tag{2.3}$$

$$\underline{405}=27^5, 27^3, 27^1, 10^3, \overline{10}^3, 8^5, 8^3_A, 8^3_B, 8^1, 1^5, 1^1. \tag{2.4}$$

Wavefunctions for these SU(6) representations are written using the 6 and $\bar{6}$ representations q_l and q^l, respectively defined in Table A1, Appendix A. A given wavefunction within an SU(6) multiplet may be classified according to its $SU(3)\times SU(2)$ quantum numbers,

$$|A; YII_3; S S_3\rangle, \tag{2.5}$$

where Y, I, I_3 are the hypercharge, I-spin, and third component of the I-spin, respectively, and S, S_3 are the spin and third component of the spin. The relative phases between wavefunctions within a given SU(3) multiplet are chosen to agree with the phase conventions of deSwart.[7] The relative phases of the wavefunctions within a given spin multiplet agree with the Condon—Shortley phase convention for SU(2).[8] The wavefunction of highest weight in successive $SU(3)\times SU(2)$ multiplets within a given SU(6) representation is determined by requiring orthogonality between states with the same additive quantum numbers, Y, I_3, and S_3, and that the traceless condition for each representation be satisfied. For example, the 405 wavefunctions must have the following form:

$$\begin{aligned}\xi^{kl}_{ij} \propto \{q_iq_j\}\{q^kq^l\} - \tfrac{1}{8}\sum_m \,[\delta^k_i\{q_mq_j\}\{q^mq^l\} + \delta^k_j\{q_iq_m\}\{q^mq^l\} \\ + \delta^l_i\{q_mq_j\}\{q^kq^m\} + \delta^l_j\{q_iq_m\}\{q^kq^m\}] \\ + \tfrac{1}{56}(\delta^k_i\delta^l_j + \delta^l_i\delta^k_j)\sum_{m,n}\{q_mq_n\}\{q^mq^n\},\end{aligned} \tag{2.6}$$

TABLE 3.1. SU(6) isoscalar factors for $405\times 56\rightarrow 56,\ 70$

	$27^5\times 10^4$	$27^5\times 8^2$	$27^3\times 10^4$	$27^3\times 8^2$	$27^1\times 10^4$	$27^1\times 8^2$	$10^3\times 8^2$	$\overline{10}^3\times 10^4$	$\overline{10}^3\times 8^2$	$8^5\times 10^4$	$(8^5\times 8^2)_s$	$(8^5\times 8^2)_a$
$\underline{56}$												
10^4	$\frac{7}{15}$	$-2\sqrt{7}/15$	$\frac{7}{15}$	$-2\sqrt{7}/15\sqrt{5}$	$7\sqrt{2}/15\sqrt{5}$		$2\sqrt{14}/15\sqrt{3}$			$4\sqrt{14}/45$		$-2\sqrt{7}/45$
8^2	$\sqrt{14}/3\sqrt{5}$		$\sqrt{14}/15$	$\sqrt{7}/15$		$\sqrt{14}/30$	$\sqrt{7}/3\sqrt{15}$	$-2\sqrt{7}/3\sqrt{15}$	$\sqrt{7}/3\sqrt{15}$	$\sqrt{14}/9\sqrt{5}$		
$\underline{70}$												
8^4	$\sqrt{5}/3\sqrt{2}$	$-\sqrt{5}/6\sqrt{2}$	$\sqrt{5}/3\sqrt{2}$	$-1/6\sqrt{2}$	$\frac{1}{3}$		$\sqrt{5}/3\sqrt{6}$	0	0	$\sqrt{5}/9\sqrt{2}$	$\sqrt{5}/18\sqrt{2}$	$-5/18\sqrt{2}$
10^2	$\sqrt{7}/3\sqrt{2}$		$\sqrt{7}/3\sqrt{10}$	$2/3\sqrt{5}$		$2/3\sqrt{10}$	$2/3\sqrt{6}$			$\frac{4}{9}$		
8^2	$-\sqrt{5}/3\sqrt{2}$		$-1/3\sqrt{2}$	$\frac{-1}{6}$		$-1/6\sqrt{2}$	$-\sqrt{5}/6\sqrt{3}$	$-\sqrt{5}/3\sqrt{3}$	$\sqrt{5}/6\sqrt{3}$	$-\sqrt{5}/9\sqrt{2}$		
1^2								$-2\sqrt{5}/3\sqrt{3}$				

	$8^3_A\times 10^4$	$(8^3_A\times 8^2)_s$	$(8^3_A\times 8^2)_a$	$8^3_B\times 10^4$	$(8^3_B\times 8^2)_s$	$(8^3_B\times 8^2)_a$	$8^1\times 10^4$	$(8^1\times 8^2)_s$	$(8^1\times 8^2)_a$	$1^5\times 10^4$	$1^5\times 8^2$	$1^1\times 10^4$	$1^1\times 8^2$
$\underline{56}$													
10^4	0		$-2\sqrt{7}/15\sqrt{3}$	$-2\sqrt{7}/15\sqrt{3}$		$-2\sqrt{14}/15\sqrt{15}$	$2\sqrt{7}/45\sqrt{5}$			$\sqrt{7}/9\sqrt{5}$		$\sqrt{2}/45$	
8^2	$-2\sqrt{7}/3\sqrt{30}$	0	0	$2\sqrt{7}/15\sqrt{3}$	$-2\sqrt{7}/15\sqrt{6}$	0		$-4\sqrt{7}/45\sqrt{2}$	$-\sqrt{14}/9\sqrt{5}$				$-1/9\sqrt{2}$
$\underline{70}$													
8^4	0	$-\sqrt{5}/6\sqrt{6}$	$1/6\sqrt{6}$	$-\sqrt{5}/12\sqrt{3}$	$-1/4\sqrt{3}$	$\sqrt{5}/12\sqrt{3}$	$\frac{1}{36}$				$-\sqrt{5}/18\sqrt{2}$		
10^2	0		$-1/3\sqrt{3}$	$-2/3\sqrt{30}$		$-7/6\sqrt{30}$			$1/18\sqrt{10}$	$\sqrt{5}/9\sqrt{2}$			
8^2	$-\sqrt{5}/3\sqrt{6}$	$\sqrt{5}/6\sqrt{3}$	$-1/6\sqrt{3}$	$1/2\sqrt{3}$	$-1/12\sqrt{6}$	$\sqrt{5}/12\sqrt{6}$		$-7/36\sqrt{2}$	$-5\sqrt{5}/36\sqrt{2}$				$-\sqrt{7}/18\sqrt{2}$
1^2		$-\sqrt{2}/3\sqrt{3}$			$\sqrt{5}/6\sqrt{3}$			$-\sqrt{5}/6$					

where $\{q_iq_j\}\equiv q_iq_j+q_jq_i$, with the traceless condition

$$\sum_i \xi^{il}_{ij}=0. \tag{2.7}$$

As seen in (2.4), $\underline{405}$ contains 8^3 twice. The state $|8_A;011;11\rangle$ is chosen to be the simplest state consistent with the required orthogonality and traceless conditions. $|8_B;011;11\rangle$ is then determined by requiring, in addition, that it be orthogonal to $|8_A;011;11\rangle$. The relative phases among different SU(3)×SU(2) multiplets within a given SU(6) representation is arbitrary. The phases of the wavefunctions within the 35, 56, and 70 representations are chosen to conform to Meshkov's revised phase conventions[9] for the table of SU(6) isoscalar factors for $35\times 56\rightarrow 56, 70$. This table is given for reference in Appendix C. The highest weight wavefunctions, themselves, for each SU(3)×SU(2) multiplet in 35, 56, and 70 are listed in Appendix B. The present choice of relative phase for the SU(3)×SU(2) multiplets within 405 is also made explicit in Appendix B by listing the highest weight wavefunctions for each SU(3)×SU(2) multiplet within 405. The rest of the wavefunctions can easily be constructed by applying the generators $I_\pm$, $V_\pm$, and $S_\pm$.[7]

SU(6) Clebsch–Gordan coefficients can be written in terms of the product of an SU(6) isoscalar factor with SU(3) and SU(2) Clebsch–Gordan coefficients. For the product:

$$|R;A;YII_3;SS_3\rangle\times|R';A';Y'I'I'_3;S'S'_3\rangle \longrightarrow |R'';A'';Y''I''I''_3;S''S''_3\rangle,$$

where R, R', and R'' are SU(6) representation labels, and the others are SU(3)×SU(2) multiplet labels within each respective SU(6) representation, the Clebsch–Gordan coefficient is written:

$$\left(\begin{matrix} R & R' \\ A,S & A',S' \end{matrix}\middle|\begin{matrix} R'' \\ A'',S'' \end{matrix}\right)\begin{pmatrix} A & A' & A'' \\ YII_3 & Y'I'I'_3, & Y''I''I''_3 \end{pmatrix} \times (SS_3S'S'_3, S''S''_3). \tag{2.8}$$

The first factor in (2.8) is the SU(6) isoscalar factor to be determined. The second factor is the full SU(3) Clebsch–Gordan coefficient for $A\times A'\rightarrow A''$, many of which have been tabulated by McNamee and Chilton.[10] The third factor is the usual SU(2) Clebsch–Gordan coefficient.[8] For the product $405\times 56\rightarrow 56$, 70 the additional SU(3) Clebsch–Gordan coefficients for the products $27\times 10\rightarrow 10$, 8 and $\overline{10}\times 8\rightarrow 8$ are needed. These coefficients can also be expressed, in terms of isoscalar factors times an SU(2) I-spin Clebsch–Gordan coefficient, as

$$\begin{pmatrix} A & A' & A'' \\ YII_3 & Y'I'I'_3, & Y''I''I_3 \end{pmatrix} = \left(\begin{matrix} A & A' \\ YI & Y'I' \end{matrix}\middle|\begin{matrix} A'' \\ Y''I'' \end{matrix}\right)(II_3I'I'_3, I''I''_3). \tag{2.9}$$

The SU(3) isoscalar factors for $27\times 10\rightarrow 10$, 8 and $\overline{10}\times 8\rightarrow 8$ were calculated according to the method of deSwart.[7] They are listed in Tables 3.2 and 3.3 in Sec. 3.

The SU(6) isoscalar factors are found by writing representative wavefunctions in each of the SU(3)×SU(2) multiplets of 56 and 70 in terms of the product wavefunctions of 405 and 56 and the Clebsch–Gordan coefficients given in (2.8). The unknown SU(6) isoscalar factors are determined by operating on these expressions with the SU(6) H_4 and H_5 operators defined in Appendix A. In particular, the expressions

$$H_4|10;1\tfrac{3}{2}\tfrac{3}{2};\tfrac{3}{2}\tfrac{3}{2}\rangle=3|10;1\tfrac{3}{2}\tfrac{3}{2};\tfrac{3}{2}\tfrac{3}{2}\rangle, \tag{2.10}$$

TABLE 3.2 SU(3) isoscalar factors for $27\times 10\rightarrow 10,\ 8$

	Y_1I_1	$(0,2)$	$(0,2)$	$(1,\frac{3}{2})$	$(1,\frac{3}{2})$	$(-1,\frac{3}{2})$	$(-1,\frac{3}{2})$	$(2,1)$	$(2,1)$	$(0,1)$	$(0,1)$	$(0,1)$
	YI / Y_2I_2	$(1,\frac{3}{2})$	$(0,1)$	$(0,1)$	$(-1,\frac{1}{2})$	$(1,\frac{3}{2})$	$(0,1)$	$(-1,\frac{1}{2})$	$(-2,0)$	$1,\frac{3}{2}$	$(0,1)$	$(-1,\frac{1}{2})$
$\underline{10}$												
	$(1,\frac{3}{2})$	$5/3\sqrt{7}$		$5\sqrt{2}/3\sqrt{21}$				$\sqrt{10}/3\sqrt{7}$		$-\sqrt{5}/3\sqrt{7}$		
	$(0,1)$		$5\sqrt{2}/9\sqrt{7}$		$4\sqrt{5}/9\sqrt{7}$	$10\sqrt{2}/9\sqrt{7}$			$\sqrt{10}/3\sqrt{7}$		$\sqrt{2}/\sqrt{21}$	
	$(-1,\frac{1}{2})$						$2\sqrt{10}/3\sqrt{21}$					$4/3\sqrt{7}$
	$(-2,0)$											
$\underline{8}$	$(0,1)$		$\sqrt{10}/9$		$\frac{4}{9}$	$-\sqrt{10}/9$			$\sqrt{2}/3$		$-\sqrt{10}/5\sqrt{3}$	
	$(1,\frac{1}{2})$	$-\sqrt{5}/3$		$-2\sqrt{2}/3\sqrt{3}$				$\frac{-1}{3}$		$1/3\sqrt{5}$		
	$(-1,\frac{1}{2})$						$\sqrt{2}/3\sqrt{3}$					$2/3\sqrt{5}$
	$(0,0)$					$-\sqrt{2}/\sqrt{3}$					$-2/\sqrt{15}$	

	Y_1I_1	$(-2,1)$	$(-2,1)$	$(1,\frac{1}{2})$	$(1,\frac{1}{2})$	$(1,\frac{1}{2})$	$(-1,\frac{1}{2})$	$(-1,\frac{1}{2})$	$(-1,\frac{1}{2})$	$(0,0)$	$(0,0)$	$(0,0)$	$(0,0)$
	YI / Y_2I_2	$(1,\frac{3}{2})$	$(0,1)$	$(0,1)$	$(-1,\frac{1}{2})$	$(-2,0)$	$(1,\frac{3}{2})$	$(0,1)$	$(-1,\frac{1}{2})$	$(1,\frac{3}{2})$	$(0,1)$	$(-1,\frac{1}{2})$	$(-2,0)$
$\underline{10}$													
	$(1,\frac{3}{2})$			$-4/3\sqrt{21}$						$1/3\sqrt{7}$			
	$(0,1)$				$2/9\sqrt{7}$		$-8/9\sqrt{7}$				$-5/9\sqrt{7}$		
	$(-1,\frac{1}{2})$	$2\sqrt{5}/3\sqrt{7}$				$2/\sqrt{21}$		$\sqrt{2}/3\sqrt{21}$				$-1/3\sqrt{7}$	
	$(-2,0)$		$\sqrt{10}/\sqrt{21}$						$2\sqrt{2}/\sqrt{21}$				$1/\sqrt{7}$
$\underline{8}$													
	$(0,1)$				$-7/9\sqrt{5}$		$4/9\sqrt{5}$				$4/9\sqrt{5}$		
	$(1,\frac{1}{2})$			$\sqrt{2}/3\sqrt{15}$									
	$(-1,\frac{1}{2})$	$\frac{-2}{3}$				$1/\sqrt{15}$		$-4\sqrt{2}/3\sqrt{15}$				$-2/3\sqrt{5}$	
	$(0,0)$				$-1/\sqrt{15}$								

$$H_5\,|8;1\tfrac{1}{2}\tfrac{1}{2};\tfrac{1}{2}\tfrac{1}{2}\rangle=|8;1\tfrac{1}{2}\tfrac{1}{2};\tfrac{1}{2}\tfrac{1}{2}\rangle, \tag{2.11}$$

$$H_4\,|8;1\tfrac{1}{2}\tfrac{1}{2};\tfrac{1}{2}\tfrac{1}{2}\rangle=(-4\sqrt{2}/3)\,|10;1\tfrac{3}{2}\tfrac{1}{2};\tfrac{3}{2}\tfrac{1}{2}\rangle + \tfrac{5}{3}\,|8;1\tfrac{1}{2}\tfrac{1}{2};\tfrac{1}{2}\tfrac{1}{2}\rangle, \tag{2.12}$$

for 56 and

$$H_4\,|8;1\tfrac{1}{2}\tfrac{1}{2};\tfrac{3}{2}\tfrac{3}{2}\rangle=|8;1\tfrac{1}{2}\tfrac{1}{2};\tfrac{3}{2}\tfrac{3}{2}\rangle, \tag{2.13}$$

$$H_4\,|10;1\tfrac{3}{2}\tfrac{3}{2};\tfrac{1}{2}\tfrac{1}{2}\rangle=|10;1\tfrac{3}{2}\tfrac{3}{2};\tfrac{1}{2}\tfrac{1}{2}\rangle, \tag{2.14}$$

$$H_5\,|8;1\tfrac{1}{2}\tfrac{1}{2};\tfrac{1}{2}\tfrac{1}{2}\rangle=|8;1\tfrac{1}{2}\tfrac{1}{2};\tfrac{1}{2}\tfrac{1}{2}\rangle, \tag{2.15}$$

$$H_4\,|1;000;\tfrac{1}{2}\tfrac{1}{2}\rangle=(-2/\sqrt{3})(|8;010;\tfrac{3}{2}\tfrac{1}{2}\rangle+|8;010;\tfrac{1}{2}\tfrac{1}{2}\rangle), \tag{2.16}$$

$$H_4\,|10;011;\tfrac{1}{2}\tfrac{1}{2}\rangle=H_4\,|8;011;\tfrac{3}{2}\tfrac{1}{2}\rangle=-H_4\,|8;011;\tfrac{1}{2}\tfrac{1}{2}\rangle, \tag{2.17}$$

for 70 are sufficient to determine all the isoscalar factors. These factors are tabulated in Table 3.1 in Sec. III. Each row of isoscalar factors is normalized separately. The leftmost isoscalar factor for the 56, 10^4 multiplet and the 70, 8^4 multiplet are chosen to be

TABLE A1. Basic representations 6 and $\bar{6}$ with eigenvalue assignments for H_4 and H_5.

Name	$\|YII_3;SS_3\rangle$	H_4	H_5
$q_1=p_\uparrow$	$\|\frac{1}{3}\frac{1}{2}\frac{1}{2};\ \frac{1}{2}\frac{1}{2}\rangle$	1	1
$q_2=n_\uparrow$	$\|\frac{1}{3}\frac{1}{2}-\frac{1}{2};\ \frac{1}{2}\frac{1}{2}\rangle$	-1	1
$q_3=\lambda_\uparrow$	$\|-\frac{2}{3}\ 00;\ \frac{1}{2}\frac{1}{2}\rangle$	0	-2
$q_4=p_\downarrow$	$\|\frac{1}{3}\frac{1}{2}\frac{1}{2};\ \frac{1}{2}-\frac{1}{2}\rangle$	-1	-1
$q_5=n_\downarrow$	$\|\frac{1}{3}\frac{1}{2}-\frac{1}{2};\ \frac{1}{2}-\frac{1}{2}\rangle$	1	-1
$q_6=\lambda_\downarrow$	$\|-\frac{2}{3}\ 00;\ \frac{1}{2}-\frac{1}{2}\rangle$	0	2
$q^1=\bar{p}_\downarrow$	$\|-\frac{1}{3}\frac{1}{2}-\frac{1}{2};\frac{1}{2}-\frac{1}{2}\rangle$	-1	-1
$q^2=\bar{n}_\downarrow$	$-\|-\frac{1}{3}\frac{1}{2}\frac{1}{2};\ \frac{1}{2}-\frac{1}{2}\rangle$	1	-1
$q^3=\bar{\lambda}_\downarrow$	$-\|\frac{2}{3}\ 00;\ \frac{1}{2}-\frac{1}{2}\rangle$	0	2
$q^4=\bar{p}_\uparrow$	$-\|-\frac{1}{3}\frac{1}{2}-\frac{1}{2};\ \frac{1}{2}\frac{1}{2}\rangle$	1	1
$q^5=\bar{n}_\uparrow$	$\|-\frac{1}{3}\frac{1}{2}\frac{1}{2};\ \frac{1}{2}\frac{1}{2}\rangle$	-1	1
$q^6=\bar{\lambda}_\uparrow$	$\|\frac{2}{3}\ 00;\ \frac{1}{2}\frac{1}{2}\rangle$	0	-2

TABLE 3.3 SU(3) isoscalar factors for $\overline{10}\times 8 \to 8$

		Y_1I_1	$\left(-1,\frac{3}{2}\right)$	$\left(-1,\frac{3}{2}\right)$	$(0,1)$	$(0,1)$	$(0,1)$	$(0,1)$	$\left(1,\frac{1}{2}\right)$	$\left(1,\frac{1}{2}\right)$	$\left(1,\frac{1}{2}\right)$	$(2,0)$
	YI	Y_2I_2	$(0,1)$	$\left(1,\frac{1}{2}\right)$	$(0,1)$	$\left(1,\frac{1}{2}\right)$	$\left(-1,\frac{1}{2}\right)$	$(0,0)$	$(0,1)$	$\left(-1,\frac{1}{2}\right)$	$(0,0)$	$\left(-1,\frac{1}{2}\right)$
$\underline{8}$	$(0,1)$			$2\sqrt{2}/\sqrt{15}$	$-\sqrt{2}/\sqrt{15}$			$1/\sqrt{5}$		$-\sqrt{2}/\sqrt{15}$		
	$\left(1,\frac{1}{2}\right)$					$1/\sqrt{5}$			$-1/\sqrt{5}$		$1/\sqrt{5}$	$-\sqrt{2}/\sqrt{5}$
	$\left(-1,\frac{1}{2}\right)$		$2/\sqrt{5}$				$1/\sqrt{5}$					
	$(0,0)$				$\sqrt{3}/\sqrt{5}$					$\sqrt{2}/\sqrt{5}$		

positive. Expressions (2.12), (2.16), and (2.17), then determine the relative phases of the remaining rows. These expressions also provide an internal check on the normalization of each row.

III. RESULTS

Table 3.1 lists the SU(6) isoscalar factors for the product $405\times 56 \to 56$, 70. This table has been constructed to agree with the revised phase conventions for the isoscalar factors of the product $35\times 56\to 56$, 70.[4,9] When necessary, this revised table for $35\times 56\to 56$, 70, given in Appendix C, should be used with Table 3.1, rather than the table given in Ref. 4. Tables 3.2 and 3.3 list the SU(3) isoscalar factors for the products $27\times 10\to 10$, 8 and $\overline{10}\times 8\to 8$, needed in the construction of the full SU(6) Clebsch–Gordan coefficients.

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APPENDIX A: GENERATORS AND BASIC REPRESENTATIONS OF SU(6) GROUP

The 35 generators of SU(6) can be written[11] as X_i^j, $i, j = 1, 2, \ldots, 6$, with the condition

$$\sum_{i=1}^{6} X_j^i = 0, \tag{A1}$$

and the commutation relations

$$[X_i^j, X_k^l] = \delta_k^j X_i^l - \delta_i^l X_k^j. \tag{A2}$$

The familiar SU(3) hypercharge, I-spin, V-spin, and U-spin operators are written in terms of these generators in the following way:

TABLE B1. Representative wavefunctions for 35, 56 and 70 representations. For 70, $\varphi_{ij,k} \equiv q_iq_jq_k + q_jq_iq_k - q_kq_jq_i - q_jq_kq_i$.

$\lvert A;\ YII_3;\ SS_3\rangle$
$\underline{35}$
$\lvert 8;\ 011;\ 11\rangle = q_1q^5$
$\lvert 8;\ 011;\ 00\rangle = -(1/\sqrt{2})[q_1q^2 + q_4q^5]$
$\lvert 1;\ 000;\ 11\rangle = (-1/\sqrt{3})[q_1q^4 + q_2q^5 + q_3q^6]$
$\underline{56}$
$\lvert 10;\ 1\tfrac{3}{2}\tfrac{3}{2};\ \tfrac{3}{2}\tfrac{3}{2}\rangle = q_1q_1q_1$
$\lvert 8;\ 011;\ \tfrac{1}{2}\tfrac{1}{2}\rangle = (1/3\sqrt{2})[2\ (q_6q_1q_1 + q_1q_6q_1 + q_1q_1q_6) - (q_4q_3q_1 + q_4q_1q_3 + q_3q_4q_1 + q_1q_4q_3 + q_3q_1q_4 + q_1q_3q_4)]$
$\underline{70}$
$\lvert 8;\ 011;\ \tfrac{3}{2}\tfrac{3}{2}\rangle = (1/\sqrt{6})\ \varphi_{11,3}$
$\lvert 10;\ 1\tfrac{3}{2}\tfrac{3}{2};\ \tfrac{1}{2}\tfrac{1}{2}\rangle = (1/\sqrt{6})\ \varphi_{11,4}$
$\lvert 8;\ 011;\ \tfrac{1}{2}\tfrac{1}{2}\rangle = -(1/3\sqrt{2})[\varphi_{11,6} + \varphi_{34,1} + \varphi_{43,1}]$
$\lvert 1;\ 000;\ \tfrac{1}{2}\tfrac{1}{2}\rangle = -(1/6\sqrt{3})[2\varphi_{26,1} + \varphi_{62,1} + \varphi_{21,6} + 2\varphi_{15,3} + \varphi_{53,1} + \varphi_{13,5} + 2\varphi_{34,2} + \varphi_{43,2} + \varphi_{32,4}]$

TABLE B2. Representative 405 wavefunctions $\{q_iq_j\} \equiv q_iq_j + q_jq_i$

$\lvert A;\ YII_3;\ SS_3\rangle$
$\lvert 27;\ 022;\ 22\rangle = q_1q_1q^5q^5$
$\lvert 8;\ 011;\ 22\rangle = (1/\sqrt{20})[2\{q_1q_2\}q^5q^5 + 2q_1q_1\{q^4q^5\} + \{q_1q_3\}\{q^5q^6\}]$
$\lvert 1;\ 000;\ 22\rangle = (1/\sqrt{24})[2q_1q_1q^4q^4 + 2q_2q_2q^5q^5 + 2q_3q_3q^6q^6 + \{q_1q_2\}\{q^4q^5\} + \{q_2q_3\}\{q^5q^6\} + \{q_1q_3\}\{q^4q^6\}]$
$\lvert 27;\ 022;\ 11\rangle = (1/2)[\{q_1q_4\}q^5q^5 + q_1q_1\{q^2q^5\}]$
$\lvert 10;\ 1\frac{3}{2}\frac{3}{2};\ 11\rangle = (1/2)[q_1q_1\{q^2q^6\} - q_1q_1\{q^3q^5\}]$
$\lvert \overline{10};\ -1\frac{3}{2}\frac{3}{2};\ 11\rangle = (1/2)[\{q_3q_4\}q^5q^5 - \{q_1q_6\}q^5q^5]$
$\lvert 8_A;\ 011;\ 11\rangle = (1/\sqrt{48})[2q_1q_1\{q^1q^5\} - 2q_1q_1\{q^2q^4\} + 2\{q_2q_4\}q^5q^5 - 2\{q_1q_5\}q^5q^5 + \{q_3q_4\}\{q^5q^6\} - \{q_1q_6\}\{q^5q^6\} - \{q_1q_3\}\{q^2q^6\} + \{q_1q_3\}\{q^3q^5\}]$
$\lvert 8_B;\ 011;\ 11\rangle = (1/\sqrt{480})[2q_1q_1\{q^1q^5\} - 8q_1q_1\{q^2q^4\} - 8\{q_2q_4\}q^5q^5 + 2\{q_1q_5\}q^5q^5 - 3\{q_1q_2\}\{q^2q^5\} - 3\{q_1q_4\}\{q^4q^5\} - 4\{q_3q_4\}\{q^5q^6\} + \{q_1q_6\}\{q^5q^6\} - 4\{q_1q_3\}\{q^2q^6\} + \{q_1q_3\}\{q^3q^5\}]$
$\lvert 27;\ 022;\ 00\rangle = (1/\sqrt{12})[2q_1q_1q^2q^2 + 2q_4q_4q^5q^5 + \{q_1q_4\}\{q^2q^5\}]$
$\lvert 8;\ 011;\ 00\rangle = (1/\sqrt{960})[2q_1q_1\{q^1q^2\} + 2\{q_1q_2\}q^2q^2 + 2q_4q_4\{q^4q^5\} + 2\{q_4q_5\}q^5q^5 + \{q_1q_3\}\{q^2q^3\} + \{q_1q_4\}\{q^1q^5\} + \{q_1q_4\}\{q^2q^4\} + \{q_1q_5\}\{q^2q^5\} + \{q_2q_4\}\{q^2q^5\} + \{q_4q_6\}\{q^5q^6\} - 7\{q_1q_6\}\{q^2q^6\} - 7\{q_3q_4\}\{q^3q^5\} + 8\{q_1q_6\}\{q^3q^5\} + 8\{q_3q_4\}\{q^2q^6\}]$
$\lvert 1;\ 000;00\rangle = (1/12\sqrt{14})[4q_1q_1q^1q^1 + 4q_2q_2q^2q^2 + 4q_3q_3q^3q^3 + 4q_4q_4q^4q^4 + 4q_5q_5q^5q^5 + 4q_6q_6q^6q^6 + 2\{q_1q_2\}\{q^1q^2\} + 2\{q_1q_3\}\{q^1q^3\} + 2\{q_1q_4\}\{q^1q^4\} - 5\{q_1q_5\}\{q^1q^5\} - 5\{q_1q_6\}\{q^1q^6\} + 2\{q_2q_3\}\{q^2q^3\} - 5\{q_2q_4\}\{q^2q^4\} + 2\{q_2q_5\}\{q^2q^5\} - 5\{q_2q_6\}\{q^2q^6\} - 5\{q_3q_4\}\{q^3q^4\} - 5\{q_3q_5\}\{q^3q^5\} + 2\{q_3q_6\}\{q^3q^6\} + 2\{q_4q_5\}\{q^4q^5\} + 2\{q_4q_6\}\{q^4q^6\} + 2\{q_5q_6\}\{q^5q^6\} + 7\{q_1q_5\}\{q^2q^4\} + 7\{q_1q_6\}\{q^3q^4\} + 7\{q_2q_4\}\{q^1q^5\} + 7\{q_2q_6\}\{q^3q^5\} + 7\{q_3q_4\}\{q^1q^6\} + 7\{q_3q_5\}\{q^2q^6\}]$

$$Y = -(X_3^3 + X_6^6), \tag{A3}$$

$$I_+ = X_1^2 + X_4^5, \tag{A4a}$$

$$I_- = X_2^1 + X_5^4, \tag{A4b}$$

$$I_3 = \tfrac{1}{2}(X_1^1 + X_4^4 - X_2^2 - X_5^5), \tag{A4c}$$

$$V_+ = X_1^3 + X_4^6, \tag{A5a}$$

$$V_- = X_3^1 + X_6^4, \tag{A5b}$$

$$V_3 = \tfrac{1}{2}(X_1^1 + X_4^4 - X_3^3 - X_6^6), \tag{A5c}$$

$$U_+ = X_3^2 + X_6^5, \tag{A6a}$$

$$U_- = X_2^3 + X_5^6, \tag{A6b}$$

$$U_3 = \tfrac{1}{2}(X_3^3 + X_6^6 - X_2^2 - X_5^5). \tag{A6c}$$

The generators of spin are given by

$$S_+ = X_1^4 + X_2^5 + X_3^6, \tag{A7a}$$

$$S_- = X_4^1 + X_5^2 + X_6^3, \tag{A7b}$$

$$S_3 = X_1^1 + X_2^2 + X_3^3. \tag{A7c}$$

The basic 6 and $\bar{6}$ representations, expressed as q_l, q^l, $l = 1, 2, \ldots, 6$, respectively, are given in Table A1 along with their eigenvalue assignments. The commutation relations of the generators with these basic representations are:

$$[X_i^j, q_l] = \delta_l^j q_i - \tfrac{1}{6}\delta_i^j q_l, \tag{A8a}$$

$$[X_i^j, q^l] = -\delta_i^l q^j + \tfrac{1}{6}\delta_i^j q^l. \tag{A8b}$$

A complete set of commuting operators, linear in the generators X_i^j, is given by Y, I_3, S_3, H_4, and H_5 where H_4 and H_5 are chosen such that,[12]

$$H_4 = \pm 4 I_3 S_3, \tag{A9a}$$

$$H_5 = \pm 6 Y S_3, \tag{A9b}$$

for the basic 6 and $\bar{6}$ representations. The positive sign is used for the 6 representation and the negative sign for the $\bar{6}$ representation. In terms of the generators X_i^j, H_4 and H_5 are written:

$$H_4 = X_1^1 - X_2^2 - X_4^4 + X_5^5, \tag{A10a}$$

$$H_5 = X_1^1 + X_2^2 - 2X_3^3 - X_4^4 - X_5^5 + 2X_6^6. \tag{A10b}$$

APPENDIX B: REPRESENTATIVE WAVEFUNCTIONS OF HIGHEST WEIGHT

Table B1 lists the highest weight wavefunctions, written in terms of the basic representations q_l and q^l, $l = 1, 2, \ldots, 6$, for each of the SU(3)×SU(2) multiplets in the 35, 56, and 70 SU(6) representations, respectively. The relative phases of these wavefunctions are chosen to agree with the table of Carter, Coyne, and Meshkov[4] with revised phase conventions.[9] (See Appendix C.) Table B2 lists the highest weight wavefunctions for the 405 representation.

APPENDIX C SU(6) ISOSCALAR FACTORS FOR THE PRODUCT 35×56→56, 70 WITH REVISED PHASE CONVENTIONS[4,9]

	$8^3\times10^4$	$8^1\times10^4$	$1^3\times10^4$	$(8^3\times8^2)_s$	$(8^3\times8^2)_a$	$(8^1\times8^2)_s$	$(8^1\times8^2)_a$	$1^3\times8^2$
<u>56</u>								
10^4	$2/3$	$-2/\sqrt{15}$	$-1/3$		$-2\sqrt{2}/3\sqrt{5}$			
8^2	$2/3$			$-\sqrt{2}/3$	$2\sqrt{2}/3\sqrt{5}$	0	$-\sqrt{2}/\sqrt{15}$	$-1/3\sqrt{5}$
<u>70</u>								
8^4	$5/4\sqrt{3}$	$-\sqrt{5}/4$		$-\sqrt{5}/4\sqrt{3}$	$-1/4\sqrt{3}$			$1/2\sqrt{6}$
10^2	$\sqrt{2}/\sqrt{3}$		$-1/\sqrt{6}$		$-1/2\sqrt{6}$		$-1/2\sqrt{2}$	
8^2	$\sqrt{5}/2\sqrt{3}$			$\sqrt{5}/4\sqrt{6}$	$-5/4\sqrt{6}$	$-\sqrt{5}/4\sqrt{2}$	$1/4\sqrt{2}$	$1/2\sqrt{3}$
1^2				$\sqrt{3}/2$		$-1/2$		

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