Transport of Noise at Microwave Frequencies through a Space-Charge-Limited Diode*

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Several analyses of the transport of cathode shot noise through a space-charge-limited diode at microwave frequencies have been published to date. Each of these analyses has been beset by inconsistencies arising from assumptions of monovelocity perturbation flow, direct or reflected. A new method of analysis of diode flow eliminating this problem has been developed. Numerical results based on this method are presented here. Attention is restricted to the now classical problem of one-dimensional longitudinal confined flow.

The magnitude and variation with distance of the so-called beam noise invariants is shown for a range of diode operating conditions. These calculated results, based for economy on an approximate static flow model, essentially substantiate the qualitative expectations suggested by prior analyses, and fit what little experimental data are available.

The method of analysis employed in the calculation of the numerical results comprises a linear multistream formulation, based on representation or approximation of the perturbation particle density for the noise flow as a composite of singular impulse streams, N in number, along characteristic trajectories in the velocity-distance phase space. The set of N coupled first-order linear differential equations resulting is solved by simultaneous numerical extrapolation.

I. INTRODUCTION

The transport of cathode noise through a space-charge-limited diode was first reported about twenty years ago in publications by several authors of articles on the suppression of noise in an idealized diode.[1,2] The diode model on which the analyses were based was a heterovelocity one-dimensional longitudinal flow model, which has become the classical model for this problem. Because these earliest analyses depended upon an assumption that the transit angles through the diode for essentially all electrons were negligible (that is, that the force fields remained stationary during the passage of an electron), the analyses were suitable only for determination of the transport of the low-frequency component of the cathode noise. The frequency range of applicability of the results obtained later was extended appreciably when it was recognized that the noise flow up to and in the vicinity of the minimum was little influenced by the flow beyond.[2] The flow beyond, a high-velocity flow, was found to be relatively well represented by recourse to a simple (perturbed) monovelocity model. This "pieced together" approach has proved quite productive for the midrange of frequency where, though the angle from minimum to anode may be large, the mean transit angle from cathode to minimum is small. For the higher-frequency range of concern here, where the mean transit angle from cathode to minimum is a significant portion of a cycle, neither the "pieced together" approach nor any monovelocity model has been shown adequate or applicable.

Efforts have been made during the past decade to find a general formulation and solution for the problem.[3–7] These investigations generally have been based on use of the linearized Liouville or Boltzmann transport equation to describe the flow process, with only continuum electric-field forces being retained. In the majority of these efforts, the potential minimum was considered a point current source, driven by the local perturbation voltage at the potential minimum as established by a cathode-emitted space-charge excitation current.[7] The excitation current was assumed to be decelerated by the static field only. Though widely accepted, this approach is not adequate because of the presence of infinite transit time for the potential-minimum-released current.[7]

In addition, a detailed simulation of the motion of discrete electrons in a mathematical diode of convenient parameters has been performed.[8] Unfortunately, such simulation, while providing some numerical results, adds little understanding of the physical process.

The method of analysis of the noise transport presented here permits resolution of the infinite transit time problem; provides a practicable, self-constent procedure for numerical calculations; and sheds light on the role of the linearization postulate and the relation of the postulate to frequency vs time domain resolution

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1 D. O. North, RCA Rev. 4, 441–472 (1940); 5, 106–124 (1940).

2 A. J. Rack, Bell System Tech. J. 17, 592–619 (1938).

3 J. K. Knipp, *Klystrons and Microwave Triodes* (M.I.T. Radiation Laboratory Series), Vol. 7; (McGraw-Hill Book Company, Inc., New York), Chap. 5.

4 J. R. Whinnery, IRE Trans. 1, 221–236 (1954). The same material is published in expanded form as TR Series No. 60, No. 132, Electronics Research Laboratory, University of California, University of California, Berkeley, California, February 28, 1955.

5 D. A. Watkins, J. Appl. Phys. 26, 622–624 (1955).

6 A. E. Siegman and D. A. Watkins, IRE Trans. on Electron Devices, ED-4, 82–86 (1957).

7 A. E. Siegman, "Microwave noise fluctuations in the potential-minimum region of an electron beam," Stanford Electronics Laboratories, Tech. Rept. No. 401-1, Stanford University, Stanford, California, April 22, 1957. (see Bibliography also).

8 P. K. Tien and J. Moshman, J. Appl. Phys. 27, 1067–1078 (1956).

and to infinite transit delays. Numerical results based on this analysis are given here. Implications with regard to low-noise amplification are suggested, and the method of analysis is outlined.

II. STATEMENT OF PROBLEM

A. Field and Transport Equations

A planar diode having no lateral variations is studied. Only the longitudinal motion of the electrons is considered. The effects of lateral motion in creating longitudinal motion are ignored. (These effects are of second order when lateral variations are excluded.) Occurrence of collisions is assumed negligible, and any effects therefrom are ignored. The time-average potential in the diode is assumed to possess a potential minimum. All electrons *incident* on the cathode are assumed absorbed. A phase-space particle density $N_t(x,u,t)$, where $u=dx/dt$ for a particle is assumed to represent the flow of discrete particles comprising the space charge.

The second-order force arising from the nonlongitudinal magnetic field generated by the flow is ignored compared to the first-order force caused by the longitudinal electric field $E_t(x,t)$. The electric field will be assumed representable by an electric potential $\phi_t(x,t)$ (diode length assumed small compared to free-space wavelength). The flow of particles in the phase space is assumed to obey the Liouville condition, and thus satisfy the Boltzmann transport equation. Combining the electric field and transport equations, one arrives at two integro-differential equations coupling the density N_t and the longitudinal electric field E_t,

$$0=\frac{\partial N_t}{\partial t}+u\frac{\partial N_t}{\partial x}+\frac{q}{m}E_t\frac{\partial N_t}{\partial u}, \tag{1}$$

$$0=\int_{-\infty}^{\infty} quN_t du+\epsilon\frac{\partial E_t}{\partial t}+Y\int_{x_c}^{x_a} E_t dx, \tag{2}$$

where q is the charge of the electron; m its mass; ϵ is the free space dielectric constant; Y is the admittance per unit area of the external circuit connecting the anode and cathode; and x_a, x_c, and x_m are the locations of the anode, cathode, and potential minimum, respectively. Observe that this system is nonlinear.

The noise is represented as a perturbation $N_1(x,u,t)$ to the time-invariant or static solution $N_0(x,u)$ for a diode, which solution is known. Then N_1 and E_1 must satisfy

$$0=\frac{\partial N_1}{\partial t}+u\frac{\partial N_1}{\partial x}+\frac{q}{m}E_0\frac{\partial N_1}{\partial u}+\frac{q}{m}\frac{\partial N_0}{\partial u}E_1+\frac{q}{m}E_1\frac{\partial N_1}{\partial u}, \tag{3}$$

$$0=\int_{-\infty}^{\infty} quN_1 du+\epsilon\frac{\partial E_1}{\partial t}+Y\int_{x_c}^{x_a} E_1 dx. \tag{4}$$

It is assumed that the noise perturbations are sufficiently small so that the nonlinear cross term $E_1(\partial N_1/\partial u)$ can be dropped. Implications of this hypothesis will be considered subsequently. Solutions having the spectral form $N_1(x,u,t)=N(x,u)e^{j\omega t}$ will be sought; an appropriately weighted ensemble of solutions of this form will be employed to represent the noise process. The linearized equations then reduce to

$$0=j\omega N+u\frac{\partial N}{\partial x}+\frac{q}{m}E_0\frac{\partial N}{\partial u}+\frac{q}{m}\frac{\partial N_0}{\partial u}E, \tag{5}$$

$$0=\int_{-\infty}^{\infty} quNdu+j\omega\epsilon E+Y\int_{x_c}^{x_a} Edx. \tag{6}$$

B. Transformation to Dimensionless Variables

A transformation to dimensionless variables now will be made to simplify subsequent exposition. The distance and velocity variables (x,u) will be scaled to (ξ,λ), defined as

$$\xi=2L(x-x_m), \tag{7}$$

$$\lambda=2u(\alpha)^{\frac{1}{2}}, \tag{8}$$

where

$$\frac{\alpha(\pi\alpha)^{\frac{1}{2}}}{\epsilon}\frac{q}{m}I_0=L^2, \tag{9}$$

and

$$\alpha=\frac{m}{2kT_c}, \tag{10}$$

Where k is Boltzmann's constant, T_c the cathode temperature, and $I_0=J_0$ is the static current density.

In addition, the potential and density functions (ϕ,N) will be rescaled to (η,P), defined as

$$\eta=\eta(\xi)=-q/kT[\phi_0(x)-\phi_0(x_m)], \tag{11}$$

$$\eta_{c0}=-q/kT[\phi_0(x_c)-\phi_0(x_m)], \tag{12}$$

$$P=P(\xi,\lambda)=2\alpha(I_0/q)^{-1}N(x,u). \tag{13}$$

In addition, the radian frequency ω will be rescaled to μ defined as $\mu=\omega/\omega_{pm}$, where $\omega_{pm}=L/(\alpha)^{\frac{1}{2}}$ is the plasma frequency at the potential minimum. It can be shown that $(1/\omega_{pm})$ is of the order of magnitude of the transit time for motion from the cathode to the minimum plane of a particle moving at the mean forward stream velocity.[7] With the above variables, the transport equation and field Eq. (5) and (6) can be rewritten, after elimination of $E(x)$ as

$$0=j\mu P+\lambda\frac{\partial P}{\partial \xi}+2\frac{d\eta}{d\xi}\frac{\partial P}{\partial \lambda} -\frac{1}{(\pi)^{\frac{1}{2}}j\mu}\frac{\partial P_0}{\partial \lambda}\left[\int_{-\infty}^{\infty}\lambda Pd\lambda-\frac{2I}{I_0}\right]. \tag{14}$$

C. Physical Static Flow

The static density of particles in a single-cathode physical diode is known to be

$$P_{ob}(\lambda,\xi)=S(\lambda-\lambda_1)\exp[\eta-(\lambda^2/4)]=S(\lambda-\lambda_1)e^{-w}, \quad (15)$$

where

$$\lambda_1=\mp 2\eta^{\frac{1}{2}}=\text{cutoff velocity:} \quad (16)$$

$$w=\frac{\lambda^2}{4}-\eta=\text{the normalized total energy,} \quad (17)$$

$$S(\lambda-\lambda_1)=\begin{cases}0 \text{ for } \lambda<\lambda_1\\ 1 \text{ for } \lambda_1\leq\lambda,\end{cases} \quad (18)$$

where the upper sign pertains to $\xi<0$ and the lower $\xi>0$. All electrons of the static flow having velocities $\lambda>|\lambda_1|$, or $w>0$, have sufficient energy to be transported past the potential minimum on to the anode. This velocity range is designated as the transmitted flow. The range $\lambda_1<\lambda<|\lambda_2|$, or $w<0$, is designated as the reflected flow.

The potential $\eta(\xi)$ is not simply expressible in closed form. It has been calculated and tabulated.[9] At the potential minimum, $\eta=0$. In the region near $\xi=0$, one can approximate as follows:

$$\frac{d\eta}{d\xi}\cong\mp\eta^{\frac{1}{2}}-\frac{2}{3(\pi)^{\frac{1}{2}}}\eta\mp\frac{1}{4}\eta^{3/2}\cdots. \quad (19)$$

In the immediate vicinity of the potential minimum, the potential η has a square-law dependence on the distance, $\eta\cong(\xi/2)^2$.

D. Mathematical Model Static Flow

Solution of the transport equation has been accomplished for a static density not identical to the static density $P_{ob}(\lambda,\xi)$ as described above for a physical diode, but instead for a synthetic mathematical model diode having a static density $P_{os}(\lambda,\xi)$ so defined that it can be made to approximate P_{ob} arbitrarily well. The resulting solution, an exact solution for the mathematical model, then can approximate the solution for the physical diode arbitrarily well. This approach has proved advantageous because it successfully uncovers many of the singular properties the perturbation solutions possess, because it affords a relatively easy procedure for obtaining numerical solutions, because it provides a good qualitative understanding of the frequency character of the noise suppression process, and because it sheds light on the consequences of the linearization postulated earlier.

The solution is generated by superposition of impulse flows along discontinuities in the model static flow, these discontinuities occurring along constant-energy characteristic trajectories of the static flow. The approximating static density $P_{os}(\lambda,\xi)$ used is

$$P_{os}(\lambda,\xi)=\sum_{i=1}^{N_{os}}\Delta_i S(\lambda-\lambda_i), \quad (20)$$

where

$$\lambda_i^2=4\eta_s+w_i=\lambda_i^2(\xi), \quad (21)$$

$$0=\sum_{i=1}^{N_{os}}\Delta_i, \quad (22)$$

$N_{os}=N_{os}(\xi)$ such that all $\lambda_i^2<0$ are excluded, and w_i, Δ_i are numerical constants independent of λ, ξ.

A pictorial illustration depicting the character of the physical flow vs the mathematical model is shown in Fig. 1 (see also Fig. 3). Observe that the mathematical model density P_{os} has discontinuous jumps of magnitude Δ_i at a set of corresponding velocities λ_i. These velocities λ_i vary with distance as required by conservation of energy in the static flow.

(Note that N_{os} here is a sum limit, not a density function as were N_t, N_1 and N in II. *A* in the foregoing.)

The foregoing static flow will satisfy the time-invariant transport equation provided the time-invariant potential η is modified to η_s as defined by the differential equation (Poisson's law)

$$\frac{d^2\eta_s}{d\eta^2}=\frac{\int_{-\infty}^{\infty}P_{os}d\lambda}{(\pi)^{\frac{1}{2}}\int_{-\infty}^{\infty}\lambda P_{os}d\lambda}. \quad (23)$$

By selecting the constants Δ_i, w_i appropriately, and letting N_{os} become large, η_s can be made to approach η, and simultaneously $P_{os}(\lambda,\xi)$ to approach $P_0(\lambda,\xi)$, arbitrarily closely.

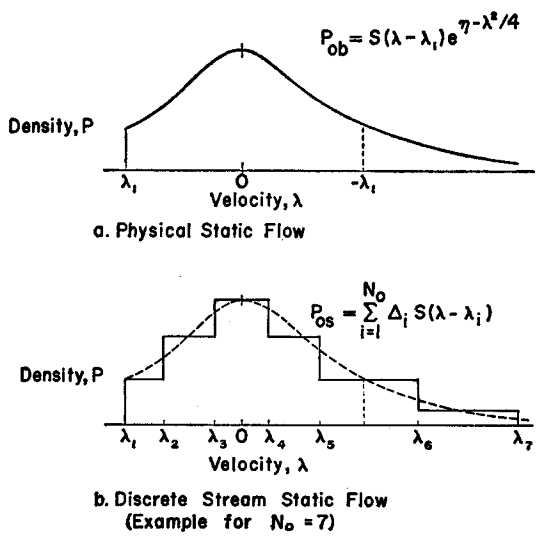

FIG. 1. Static flow velocity dependence (illustrated for an arbitrary point $\xi<0$).

[9] I. Langmuir, Phys. Rev. 21, 419 (1923).

E. Representation of Perturbation Density

The perturbation solution P_s, which satisfies the transport equation based on P_{0s} will be generable as a linear superposition of elements of the type comprising $\partial P_{0s}/\partial\lambda$, plus additional purely kinematic flows, so that

$$P_s=\frac{1}{\lambda}\sum_{i=1}^{N_s} Z_i S'(\lambda-\lambda_i)=\sum_{i=1}^{N_s}\frac{Z_i}{\lambda_i}S'(\lambda-\lambda_i); \qquad (24)$$

that is, the perturbation flow degenerates to a finite set of singular flows, each along a velocity trajectory of the static flow.[10]

On introducing the set of singular flows into the transport equation, and then utilizing the orthogonality properties of impulse functions, one obtains a set of separated, but coupled, ordinary differential equations,

$$0=\frac{j\mu}{\lambda_i}Z_i+\frac{dZ_i}{d\xi}-\Delta_i\frac{1}{j\mu(\pi)^{\frac{1}{2}}}\left(\sum_{\nu}^{N_s} Z_\nu-\frac{2I}{I_0}\right). \qquad (25)$$

This set or matrix of coupled first-order ordinary differential equations having nonconstant coefficients can be solved by straightforward numerical procedures, even for N_{0s} large. The solution of the physical continuum problem is thus replaced by solution of an approximating discrete set problem. Though it may appear awkward, computer solution of the problem in this manner proves quite practical.

Examination of the local behavior of the flow near the potential minimum, $\xi=0$, further shows that in order for a unique flow solution to exist, the current Z_1, the cutoff edge current, must vanish at the minimum, and be proportional to distance nearby. The earlier-met problems arising from infinite transit delays are found to be eliminated when this constraint is recognized.

This condition is the only tactic permitting the linearization and spectral decomposition of the perturbation equations.[7] Had the diode not possessed a potential minimum, such a condition would not have been needed. In essence, this condition states that the net current associated with the infinite-time-delayed perturbation flows at zero velocity will vanish when all time-delayed components are taken into account.

F. Boundary Conditions

The electron current expelled by a physical cathode comprises a sequence of individual electrons emitted at independent times with independent velocities. Thus at the surface of the cathode the noise excitation, interpreted in the frequency domain, is pure shot noise. That is, for an emission velocity interval $d\lambda$, generating a static emission current dI_0, the corresponding emission noise-current spectral self-density $dW_{JJ}(\omega/2\pi)$ is

$$dW_{JJ}=2qdI_0=qI_0\lambda P_0 d\lambda, \quad (\lambda>0 \text{ only}). \qquad (26)$$

Because the pure shot noise at the cathode surface is uncorrelated in time and velocity, the postulate of linearity assumed earlier permits the total noise flow to be expressed as the linear superposition of (arbitrary) individual sets of independent flow solutions (satisfying the transport and field equations). The solutions are so weighted that at the cathode the aggregation satisfies the spectral-noise density requirement above. Calculation and superposition of the necessary flow solutions, though tedious, are straightforward.

G. Information Sought

For amplifier design applications, the key parameters presently of interest in design of sensitive amplifiers are the transmitted-flow spectral self- and cross-densities, W_{JJ}, W_{JV}, and W_{VV}[11,12] and certain noise temperatures described below deduced from these parameters, where J indicates convection current and V indicates a potential commonly called the kinetic potential, and defined as[7]

$$V_1=(m/2q)J_1/J_0(\langle u_1^2\rangle-\langle u_0^2\rangle), \qquad (27)$$

where $\langle u_1^2\rangle$ and $\langle u_0^2\rangle$ are convection-current-normalized mean-square velocities defined as

$$\langle u^2\rangle=\int u^2(uNdu)\Big/\int(uNdu). \qquad (28)$$

Each of these spectral densities W are expectations taken over the total, aggregated transmitted flow; each is a function of distance, frequency, and diode operating conditions.

In the limit condition of frequency approaching infinity, that is $\mu\rightarrow\infty$, it can be shown that these densities degenerate to constants invariant with distance, attaining the following values: $W_{JJ}=2qI_0$ (full shot noise); $W_{VV}=(2kT_c)^2/2qI_0$ (full velocity noise); and Re $W_{JV}=$ Im $W_{JV}=0$ (zero correlation noise). Conversely, for $\mu\lesssim 1$ it is found that these densities vary radically with distance near the potential minimum.

The theory which has been developed over the past few years for optimization of the sensitivity of a linear stream amplifier (theory based on a simpler monovelocity flow model) states that the minimum achievable amplifier excess noise temperature T_n is[11]

$$\frac{T_n}{T_c}=\left(\frac{T_n}{T_c}\right)_S-\left(\frac{T_n}{T_c}\right)_\pi, \qquad (29)$$

where the total temperature is composed of two

[10] A. E. Siegman, "Signal propagation on a drifting electron beam with a square velocity distribution," unpublished memorandum, 1957.

[11] H. A. Haus, J. Appl. Phys. 26, 560–571 (1955).
[12] S. Bloom, RCA Rev. 16, 179–196 (1955).

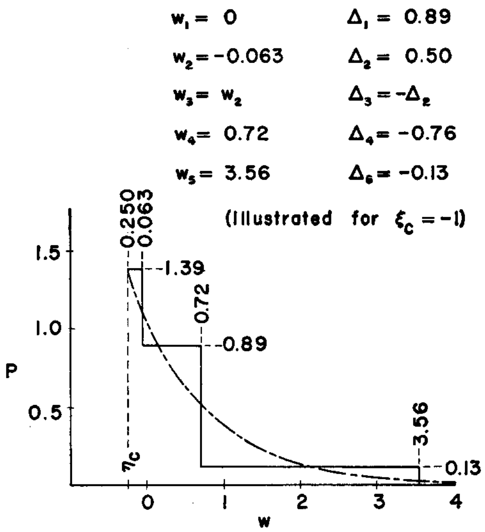

FIG. 2. Static flow approximation (numerical problem).

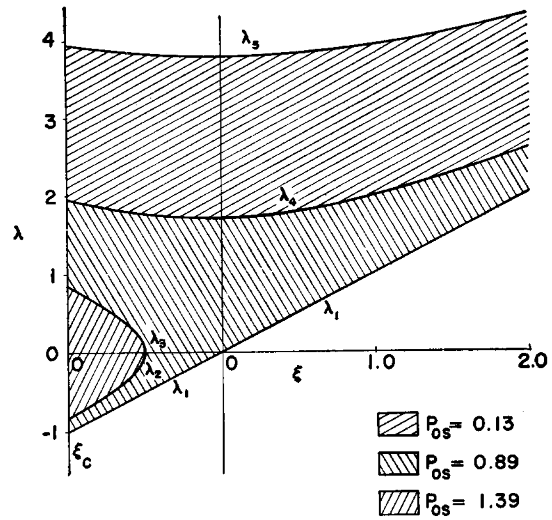

FIG. 3. Static flow approximation (phase plane representation).

components,

$$\left(\frac{T_n}{T_c}\right)_S=\left\{W_{UU}W_{KK}-\mathrm{Im}^2(W_{KU})\right\}^{\frac{1}{2}} \quad (30)$$

$$\left(\frac{T_n}{T_c}\right)_\pi=\mathrm{Re}(W_{KU}); \quad (31)$$

the foregoing statements are conveniently expressed in terms of the normalized spectral densities,

$$W_{KK}=[2qI_0]^{-1}W_{JJ}$$

$$W_{KU}=[2kT_c]^{-1}W_{JV}$$

$$W_{UU}=\left[\frac{(2kT_c)^2}{2qI_0}\right]^{-1}W_{VV}.$$

Subject to the restrictive assumptions of this earlier monovelocity theory, it can be shown that each of the two temperature components remain invariant along the stream. However, in heterovelocity flow at finite frequency, the two components do vary with distance, particularly near the potential minimum. Yet at moderate distance downstream from the minimum, evaluation of the components using either the hetero- or monovelocity theories must lead quantitatively to essentially the same results, because the velocity dispersion becomes small compared to the mean velocity. Provided that on evaluation for finite frequencies for heterovelocity flow at large distances downstream these components do approach stationary values (as is the case), the calculated temperature T_n will be the minimum achievable amplifier noise temperature. The variation of this temperature and of the spectral densities with frequency and diode operating conditions is graphed below.

III. NUMERICAL SOLUTIONS

Numerical results have been obtained by calculations performed on electronic computing equipment (IBM 704 computer). Corroborative checks were obtained by hand and analogue calculation.

A. Range of Parameters

Because of the limited time and funds available, only simple models could be examined. The numerical calculations covered only the limited range $-2.0\leq\xi\leq2.0$; the diode was assumed open-circuited at the frequency of calculation.

The model static flow P_{os} was selected so that the zero, first, third, and fifth velocity moments of $P_{ob}(0,\lambda)$ and $P_{os}(0,\lambda)$ were made identical. The minimal number (three) of transmitted streams permitting this end to be attained was employed. One reflected stream was included to permit a good fit to the static potential close to the cathode. The concomitant static flow is graphed in Fig. 2 and Fig. 3, for $\xi_c=-1$.

Five values of frequency were examined, $\mu=0.3$, 0.6, 1.0, 2.0, and 3.0. The lowest value for μ corresponds to a "low-frequency" condition (cathode-to-minimum transit angle for average energy transmitted flow electron being $0.1\pi/2$). The largest value for μ corresponds to a "high-frequency" condition (comparable cathode-to-minimum transit angle being $\pi/2$). Three cathode locations were examined: $\xi_c=-1$; -1.5; -2.0; the corresponding emitted-current-to-anode-current static saturation ratios I_{em}/I_0 are 1.3:1, 2.3:1, and 7.0:1, respectively.

B. Calculated Noise Transport

In Fig. 4, the transport of the self- and cross-power spectral densities W_{KK}, W_{UU}, $\mathrm{Im}(W_{KU})$, $\mathrm{Re}(W_{KU})$, also (T_n/T_c), $(T_n/T_c)_S$, and $(T_n/T_c)_\pi$ is graphed vs

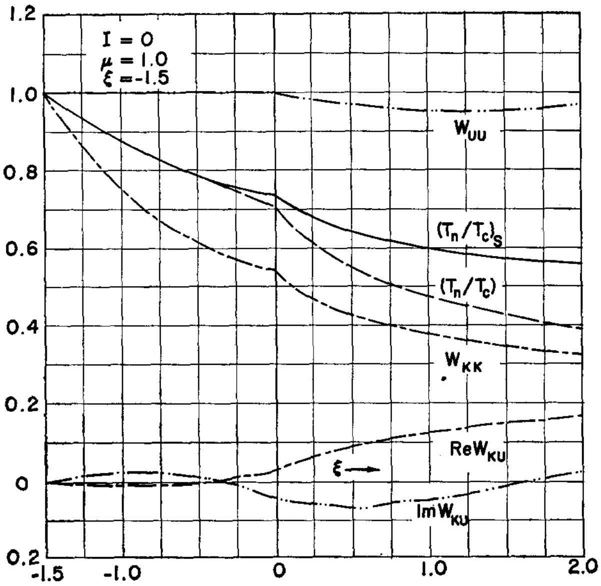

FIG. 4. Transport of noise parameters.

distance for illustration for $\mu=1$, $\xi_c=-1.5$. The transport of these densities at lower and higher frequencies, though not shown here, is similar. Observe that T_n/T_c becomes relatively stationary with distance downstream, as anticipated.

Finally, in Fig. 5, T_n/T_c as evaluated at $\xi=2.0$ (furthest downstream point calculated) is graphed vs frequency. The weakening of the noise suppression process with increasing frequency is clearly evident. The weakening of the noise suppression with decreasing static current saturation is also evident.

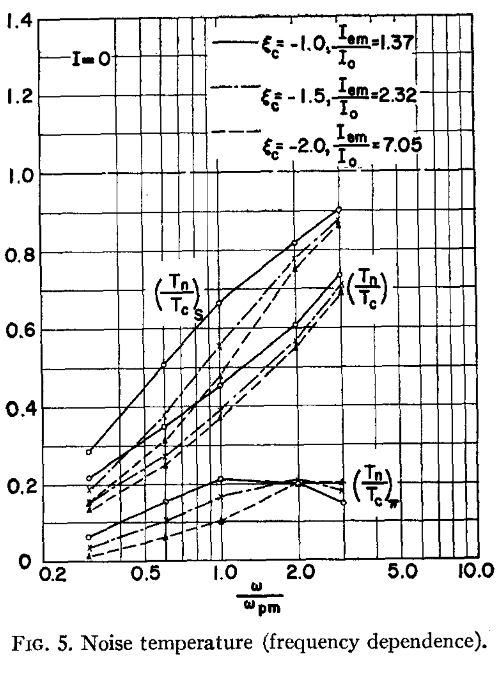

FIG. 5. Noise temperature (frequency dependence).

It is interesting to note that the cross powers remain virtually zero until past the potential minimum. Observe also that the cross-powers are at a maximum near the plasma frequency.

C. Comparison with Prior Evidence

As mentioned earlier, a detailed set of calculations has been performed by another investigator utilizing simulation in the time domain.[8] The results obtained here qualitatively are comparable to those obtained by his time-simulation method. The noise dip observed by him at $\mu=0.6$ did not occur in these results. However, direct quantitative comparison of the results is not possible because here $I=0$ was assumed, whereas in the other $I\neq 0$ was permitted ($V_{ca}=0$ assumed).[7]

Some experimental data are available and pertinent for comparison. Tests performed recently[13] on sample low-noise electron guns adjusted to provide space-charge-limited flow consistently indicated $(T_n/T_c)_\pi/(T_n/T_c)_S \simeq 0.2$ to 0.3, and $(T_n/T_c)_S \simeq 0.6$ to 0.8. Insufficient experimental data were provided to permit accurate calculation of μ and (I_{em}/I_0) for these tests. However, from examination of the experimental data provided, it seems reasonable to estimate $\mu \cong 2$, and $(I_{em}/I_0) \simeq 2$ or more. On inspection of Fig. 5, it is evident that there is excellent qualitative agreement between the calculated results and the published test data.

D. Remarks

It is important to realize that the noise suppression process examined above occurs very close to the cathode. For example, for a typical diode with $T_c=1000°$K, $I_0=0.5$ amp cm^{-2}, the normalized distance interval treated in Fig. 4 (from $\xi=-1.5$ to $+2.0$) represents a physical distance of 10 μ, or 4×10^{-4} in. This distance is of the magnitude of surface irregularity tolerances commonly permitted in cathode processing. Certainly it is small compared to distances over which control over a given potential profile is readily achieved by introduction of ungridded, nonintercepting electrodes.

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[13] S. Saito, "New method of measuring the noise parameters of the electron beam especially the correlation between its velocity and current fluctuations," Mass. Inst. Technol., Research Lab. Electronics, Tech. Rept. No. 333, August, 1957.