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LIGHT ABSORPTION BY ELECTRON BREMSSTRAHLUNG IN HIGHLY IONIZED GASES

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ABSTRACT

The effects of ion thermal motions and of electron collective behavior on electron bremsstrahlung in fully ionized plasmas are separately investigated. It is found that the influence of finite ion temperature enters only in terms and factors of the form $(1 + mT^I/MT^E)$, where m and M are electron and ion masses respectively. Thus, except in quite unusual cases, the neglect of ion thermal motions seems quite justified.

A formula for a net absorption coefficient for photons due to electron bremsstrahlung in a real electron gas (electrons and ions now at the same temperature) is derived and discussed. The formula presented here differs significantly from others that have been proposed earlier. However, the absolute effect of electron collective behavior appears to be small, and therefore the differences between our results and those obtained by others are probably not important.

INTRODUCTION

The purpose of this investigation is the examination in some detail of two aspects of environmental influence on bremsstrahlung rates in fully ionized plasmas. One is the effect of ion temperature and the other is the effect of collective electron behavior.

The first of these—the effect of ion motions—does not seem to have been given much, if any, attention before. This is probably because of the (generally correct) intuitive anticipation that it is a matter of little consequence—either quantitatively or qualitatively. Unless the average ion velocities become comparable to average electron velocities, an ion doppler effect is hardly to be expected. Such a situation might obtain in a fusion-sustained plasma, since the energy is being fed directly into the ion distributions whereas energy losses are largely taken from the electron distributions by radiation. But, except for this case, it is somewhat difficult to conceive of environments capable of supporting such improbable energy distributions.

Consequently, the primary motivations for the examination of ion temperature effects on bremsstrahlung were to provide a little logical support for intuition, and to fill in a small gap in the subject that seemed hitherto to have been ignored.

The motivations for examining the possible effect of collective electron behavior on bremsstrahlung were somewhat different. In this instance, formulas purportedly describing such effects have appeared here and there(1) in the literature—

hence there is no apparent attention gap to be filled. However, the derivations of these formulas generally followed calculational routes quite foreign to conventional bremsstrahlung theory,[2] and the question arose as to whether or not the latter approaches to the problem could be made to yield the same results. The attempt to answer this question provided the main motivation for this part of the present study.

In Section I, we briefly sketch the arguments leading to the "working formulas" from which, by diverse specializations, we may examine these two problems. Conventions and notations will be the same as employed in an earlier report in this series.[3]

In Section II, we consider an hypothetical ideal gas of electrons and ions characterized by electron and ion temperatures of arbitrary ratio.

In Section III, treating the ions as at rest, we examine the effects of collective electron behavior.

Section IV is comprised of a summary and concluding remarks.

SECTION I. A STATEMENT OF THE PROBLEM

Our main interest is in the absorption and emission of electromagnetic radiation by electrons undergoing transitions between continuum states of the ionic fields in plasmas. Consequently a nonrelativistic treatment of the particle system should suffice. However, because of the specific aspects of the problem to be considered here, we find ourselves forced to a perturbation treatment which is known to be erroneous[(2)] for such low energy emitters. Thus it is not expected that the results obtained here will be quantitatively reliable; though it is hoped that, with respect to the specific issues raised, they will be qualitatively meaningful.

The energy of the plasma plus radiation field may be displayed as

$$H = H^{R} + H^{E} + H^{I} + H^{ER} + H^{IR} + H^{EI} \quad , \tag{1}$$

where H^{R} is the energy of the free radiation field, H^{E} and H^{I} are the energies of the electrons and ions respectively—including electron-electron and ion-ion interactions—and H^{ER}, H^{IR}, and H^{EI} are interaction energies between electrons and the radiation field, between ions and the radiation field, and between the electrons and the ions. The eigenstates of the "noninteracting" system are

$$|\eta\alpha K> = |\eta>|\alpha>|K> \quad , \tag{2}$$

where

$$H^R|\eta> = E_\eta|\eta> \quad , \tag{3a}$$

$$H^E|\alpha> = E_\alpha|\alpha> \quad , \tag{3b}$$

$$H^I|K> = E_K|K> \quad . \tag{3c}$$

Since H^E and H^I include particle-particle interactions, the states $|\alpha>$ and $|K>$ are complicated and generally unknown many-particle eigenfunctions. Conversely, the states of the radiation field are conventional and familiar.[2,3] The formula to be used here to represent the transition probability per unit time corresponding to photon emission is

$$T_{\alpha'K',\alpha K}(\lambda k)$$

$$= \frac{2\pi}{\hbar}\left| \sum_{\alpha''K''\eta''} \frac{<\alpha'K'\eta_{\lambda k} + 1|H^{ER} + H^{EI}|\alpha''K''\eta''><\alpha''K''\eta''|H^{ER} + H^{EI}|\alpha K\eta_{\lambda k}>}{E_\alpha + E_K + E_\eta - E_{\alpha''} - E_{K''} - E_{\eta''}} \right|^2$$

$$\times \ \delta(E_{\alpha'} + E_{K'} + \hbar\omega - E_\alpha - E_K) \quad . \tag{4}$$

Here we have introduced $\hbar\omega = \hbar ck$ to represent the energy of a photon of wave-vector, $\underline{k}$. The formula describing the transition probability per unit time for absorption is similar. However, for the conditions that interest us here, it will be seen that absorption rates can be computed from a knowledge of emission rates. Hence initially we will concentrate on the emission process only.

The interactions may be represented as

$$H^{ER} = -\frac{e}{c}\int d^3x \ \underline{A}(\underline{x})\cdot\underline{J}^E(\underline{x}) \quad , \tag{5a}$$

and

$$H^{EI} = - ze^2 \int \frac{\rho^E(\underline{x})\, \rho^I(\underline{x}')}{|\underline{x} - \underline{x}'|}\, d^3x\, d^3x' \quad . \tag{5b}$$

In these formulas, $\underline{A}$ is the vector potential of the radiation field, $\underline{J}^E$ is the electron current, ρ^E is the electron density, and ρ^I is the density of ions of charge ze. The part of the electron-radiation interaction which is proportional to A^2 has not been included in Eq. (5a) because it makes no contribution to the matrix elements appearing in Eq. (4). For present purposes, the vector potential is conveniently displayed as

$$\underline{A}(\underline{x}) = \sqrt{\frac{2\pi\hbar c}{L^3 k}}\; e^{-i\underline{k}\cdot\underline{x}}\; \underline{\epsilon}_\lambda(\underline{k})\; a_\lambda^\dagger(\underline{k}) \quad , \tag{6}$$

where L^3 is the volume of the "quantization box",[4] $\underline{\epsilon}_1$ and $\underline{\epsilon}_2$ are the unit polarization vectors for the photon of wave-vector $\underline{k}$, and $a_\lambda^\dagger(\underline{k})$ is a creation operator. Entering (6) into Eq. (5a), we obtain for the electron-radiation interaction

$$H^{ER} = - \frac{e}{c}\sqrt{\frac{2\pi\hbar c}{L^3 k}}\; a_\lambda^\dagger\; \underline{\epsilon}_\lambda \cdot \underline{J}^E(-\underline{k}) \quad . \tag{7}$$

Inserting (5b) and (7) into Eq. (4), we find that

$$T^e_{\alpha'K',\alpha K}(\lambda k) = \left(\frac{2\pi z e^3}{c}\right)^2 \frac{c(1 + \eta_{\lambda k})}{L^3 k}$$

$$\times \left| \sum_{\alpha''K''} \left[\frac{\langle\alpha'K'|\underline{\epsilon}_\lambda\cdot\underline{J}^E(-\underline{k})|\alpha''K''\rangle\langle\alpha''K''|V^{EI}|\alpha K\rangle}{E_{\alpha'} + E_{K'} + \hbar\omega - E_{\alpha''} - E_{K''}} + \frac{\langle\alpha'K'|V^{EI}|\alpha''K''\rangle\langle\alpha''K''|\underline{\epsilon}_\lambda\cdot\underline{J}^E(-\underline{k})|\alpha K\rangle}{E_\alpha + E_K - E_{\alpha''} - E_{K''} - \hbar\omega} \right] \right|^2$$

$$\times\; \delta(E_{\alpha'} + E_{K'} + \hbar\omega - E_\alpha - E_K) \quad , \tag{8}$$

where we have introduced the notation

$$V^{EI} = \int \frac{\rho^E(\underline{x})\, \rho^I(\underline{x}')\, d^3x\, d^3x'}{|\underline{x} - \underline{x}'|} \quad . \tag{9}$$

For our purposes here, it suffices to treat the ion system as an ideal gas. Thus

$$<\alpha''k''|V^{EI}|\alpha K> = \frac{4\pi}{L^3} \sum_{\sigma}^{N^I} \frac{<\alpha''|\rho^E(\underline{K}^\sigma - \underline{K}''^\sigma)|\alpha>}{|\underline{K}^\sigma - \underline{K}''^\sigma|^2} \quad , \tag{10}$$

where $\underline{K}^\sigma$ is the wave-vector for the σth ion. Equation (8) then becomes

$$T^e_{\alpha'K',\alpha K}(\lambda k) = \left(\frac{8\pi^2 z e^3}{c}\right)^2 \frac{c(1 + \eta_k)}{L^9 k}$$

$$\times \left| \sum_{\alpha''} \sum_{\sigma}^{N^I} \frac{<\alpha'|\underline{\epsilon}_\lambda \cdot \underline{J}^E(-\underline{k})|\alpha''><\alpha''|\rho^E(\underline{K}^\sigma - \underline{K}'^\sigma)|\alpha>}{(E_{\alpha'} + \hbar\omega - E_{\alpha''})|\underline{K}^\sigma - \underline{K}'^\sigma|^2} \right.$$

$$\left. + \frac{<\alpha'|\rho^E(\underline{K}^\sigma - \underline{K}'^\sigma)|\alpha''><\alpha''|\underline{\epsilon}_\lambda \cdot \underline{J}^E(-\underline{k})|\alpha>}{(E_\alpha + E^\sigma_K - E_{\alpha''} - E^\sigma_{K'} - \hbar\omega)|\underline{K}^\sigma - \underline{K}'^\sigma|^2} \right|^2 \times \delta(E_{\alpha'} + E_{K'} + \hbar\omega - E_\alpha - E_K) \quad . \tag{11}$$

In this expression we have made explicit use of the fact that the electron-ion system states are factorable, i.e., $|K\alpha> = |K>|\alpha> = |K'>|K^2>\ldots|K^\sigma>\ldots|\alpha>$.

If, at this point, the electrons were also regarded as an ideal gas, we would find that

$$E_{\alpha'} - E_{\alpha''} + \hbar\omega = \hbar\omega + \Theta\left(\frac{\hbar\omega}{mc^2}\right) \quad ,$$

$$E_\alpha + E^\sigma_K - E_{\alpha''} - E^\sigma_{K'} - \hbar\omega = -\hbar\omega + \Theta\left(\frac{\hbar\omega}{mc^2}\right) \quad , \tag{12}$$

and that the cross-terms in the coherent sum over ions would vanish. Assuming that this situation is not significantly altered for the real gas of electrons, we find that

$$T^{e}_{\alpha'K',\alpha K}(\lambda k) = \left(\frac{8\pi^2 ze^3}{c}\right)^2 \frac{c(1 + \eta_k)N^I}{L^9 k}$$

$$\times \left| \frac{\langle\alpha' | [\underline{\epsilon}_\lambda \cdot \underline{J}^E(-\underline{k}),\ \rho^E(\underline{K} - \underline{K}')] | \alpha\rangle}{|\underline{K} - \underline{K}'|^2 \hbar\omega} \right|^2 \times \delta(E_{\alpha'} + E_{K'} + \hbar\omega - E_\alpha - E_K) \ , \qquad (13)$$

where now the label, $\underline{K}$, stands for the wave vector of a typical ion in the plasma. The commutator in Eq. (13) is readily shown to be

$$[\underline{\epsilon}_\lambda \cdot \underline{J}^E(-\underline{k}),\ \rho^E(\underline{K} - \underline{K}')] = \frac{\hbar}{m} \underline{\epsilon}_\lambda \cdot (\underline{K} - \underline{K}')\ \rho^E(\underline{K} - \underline{K}' - \underline{k}) \ , \qquad (14)$$

so that

$$T^{e}_{\alpha'K',\alpha K}(\lambda k) = \frac{64\pi^4 N^I (1 + \eta_k) z^2 e^6}{m^2 \omega^3 L^9} \frac{|\underline{\epsilon}_\lambda \cdot (K - K')|^2}{|K - K'|^4}$$

$$\times |\langle\alpha' | \rho^E(K - K' - k) | \alpha\rangle|^2 \times \delta(E_{\alpha'} + E_{K'} + \hbar\omega - E_\alpha - E_K) \ . \qquad (15)$$

At this point is is useful to note that a formula for the transition probability per unit time for the system to go from state $|\alpha K\rangle$ to state $|\alpha' K'\rangle$ while <u>absorbing</u> a photon is readily obtainable from Eq. (15), i.e.,

$$T^{a}_{\alpha'K',\alpha K}(\lambda k) = T^{e}_{\alpha K,\alpha' K'}(\lambda k) \frac{\eta_k}{1 + \eta_k}$$

$$= \frac{64\pi^4 N^I \eta_k z^2 e^6}{m^2 \omega^3 L^9} \frac{|\underline{\epsilon}_\lambda \cdot (\underline{K} - \underline{K}')|^2}{|\underline{K} - \underline{K}'|^4}$$

$$\times |\langle\alpha | \rho^E(K' - K - k) | \alpha'\rangle|^2 \times \delta(E_\alpha + E_K + \hbar\omega - E_{\alpha'} - E_{K'}) \ .$$

(16)

Emission and absorption rates may now be computed from Eqs. (15) and (16) according to

$$e(\lambda\underline{k}) = \sum_{\alpha' K' \alpha K} T^{e}_{\alpha' K', \alpha K}(\lambda k)\, P^{E}_{\alpha} P^{I}_{K} = \frac{8 n^{I} z^{2} e^{6} \pi}{L^{3} m^{2} \omega^{3}} \sum_{\alpha\alpha'} (1 + \eta_{\lambda k})\, P^{E}_{\alpha}$$

$$\times \int d^{3}K' d^{3}K\, P^{I}(\underline{K}) \frac{|\underline{\epsilon}_{\lambda} \cdot (\underline{K} - \underline{K}')|^{2}}{|\underline{K} - \underline{K}'|^{4}}$$

$$\times |\langle\alpha'|\rho^{E}(\underline{K} - \underline{K}' - \underline{k})|\alpha\rangle|^{2}\, \delta(E_{\alpha'} + E_{K'} + \hbar\omega - E_{\alpha} - E_{K}) \quad , \tag{17}$$

and

$$a(\lambda\underline{k}) = \sum_{\alpha' K' \alpha K} T^{a}_{\alpha' K', \alpha K}(\lambda k)\, P^{E}_{\alpha} P^{I}_{K} = \frac{8 n^{I} z^{2} e^{6} \pi}{L^{3} m^{2} \omega^{3}} \sum_{\alpha'\alpha} \eta_{\lambda k}\, P^{E}_{\alpha}$$

$$\times \int d^{3}K' d^{3}K\, P^{I}(\underline{K}) \frac{|\underline{\epsilon}_{\lambda} \cdot (\underline{K} - \underline{K}')|^{2}}{|\underline{K} - \underline{K}'|^{4}}$$

$$\times |\langle\alpha|\rho^{E}(K' - K - k)|\alpha'\rangle|^{2}\, \delta(E_{\alpha} + E_{K} + \hbar\omega - E_{\alpha'} - E_{K'}) \quad . \tag{18}$$

In these equations we have introduced the ion density according to $n^{I} = N^{I}/L^{3}$, and have approximated sums over wave vectors by integrals in the usual way. The quantities P^{E}_{α} and P^{I}_{K} are the probabilities of finding the electron system in the initial state $|\alpha\rangle$ and an ion with initial momentum $\hbar\underline{K}$, respectively. The ion distribution in the continuum is defined by

$$\sum_{K} P^{I}_{K} = \int d^{3}K\, P^{I}(\underline{K}) \quad . \tag{19}$$

If the electron and ion distributions are thermodynamic, i.e.,

$$P_\alpha^E = z_\alpha^{-1} e^{-E_\alpha/\Theta}$$

$$P^I(\underline{K}) = z_K^{-1} e^{-E_K/\Theta} , \qquad (20)$$

then it is easily seen from Eqs. (17) and (18) that

$$a(\underline{k}) = e^{\hbar\omega/\Theta} e(\underline{k}) . \qquad (21)$$

However, these formulas have a validity which extends beyond this case. In particular we will be interested in the case in which the electron and ion distributions are of Boltzmann type, but characterized by different temperatures.

Before leaving this section, we construct from Eqs. (17) and (18) a net absorption probability per unit time per photon of wave-vector $\underline{k}$ and polarization λ. Note that the emission rate given by Eq. (17) includes both spontaneous and induced emission. Calling e^s the stimulated emission, the net absorption rate that we desire is obtained from

$$\bar{\alpha}(\omega,\underline{\Omega},\lambda) = [a(\omega,\underline{\Omega},\lambda) - e^s(\omega,\underline{\Omega},\lambda)]/\eta_{\lambda k} = \frac{8\pi m^I z^2 r_e^2 \hbar c^5}{L^3\omega^3} \left(\frac{e^2}{\hbar c}\right)$$

$$\times \sum_{\alpha\alpha'} P_\alpha^E \int d^3K d^3K' \; P^I(\underline{K}) \frac{|\underline{\epsilon}_\lambda \cdot (\underline{K} - \underline{K}')|^2}{|\underline{K} - \underline{K}'|^4}$$

$$\times [|\langle\alpha|\rho^E(\underline{K}' - \underline{K} - \underline{k})|\alpha'\rangle|^2 \delta(E_\alpha + E_K + \hbar\omega - E_{\alpha'} - E_{K'})$$

$$- |\langle\alpha'|\rho^E(\underline{K} - \underline{K}' - \underline{k})|\alpha\rangle|^2 \delta(E_{\alpha'} + E_{K'} + \hbar\omega - E_\alpha - E_K) . \qquad (22)$$

Here we have introduced $r_e = e^2/mc^2$ to represent the classical radius of the electron. This formula represents the starting point for the applications of the next two sections. A useful, alternative way of writing Eq. (22) is

$$\bar{\alpha}(\omega,\underline{\Omega},\lambda) = \frac{8\pi n^I z^2 r_e^2 \hbar c^5}{L^3\omega^3} \left(\frac{e^2}{\hbar c}\right)$$

$$\times \sum_{\alpha\alpha'} \int d^3K d^3K' [P^E_{\alpha'} P^I(\underline{K}') - P^E_{\alpha} P^I(\underline{K})] \left\{ \frac{|\underline{\epsilon}_\lambda \cdot (\underline{K} - \underline{K}')|^2}{|\underline{K} - \underline{K}'|^4} \right\}$$

$$\times |\langle\alpha'|\rho^E(\underline{K} - \underline{K}' - \underline{k})|\alpha\rangle|^2 \, \delta(E_{\alpha'} + E_{K'} + \hbar\omega - E_\alpha - E_K) \quad . \qquad (22a)$$

SECTION II. THE TWO-TEMPERATURE PLASMA

In this section, we examine in some detail the case alluded to just above, i.e., the mixture of two ideal gases each characterized by its own temperature. In such an instance, the electron system matrix elements in Eq. (22a) reduce to single electron matrix elements of the form

$$|\langle\alpha'|\ ^{E}(\underline{K} - \underline{K}' - \underline{k})|\alpha\rangle| \rightarrow N^{E}\ \delta_{KR}(\underline{K} + \underline{q} - \underline{K}' - \underline{q}' - \underline{k}) \quad , \qquad (23)$$

where we have employed $\underline{q}$ to represent a free electron wave vector. Entering (23) into Eq. (22a) we find that

$$\bar{\alpha}\ (\omega,\underline{\Omega},\lambda) \ = \ \frac{8\pi m^{I} n^{E} z^{2} r_{e}^{2} \hbar c^{5}}{\omega^{3}} \left(\frac{e^{2}}{\hbar c}\right)$$

$$\times \int d^{3}q d^{3}K d^{3}q' d^{3}K' [P^{E}(\underline{q}') \ P^{I}(\underline{K}') - P^{E}(\underline{q}) \ P^{I}(\underline{K})]$$

$$\times \frac{|\underline{\epsilon}_{\lambda}\cdot(\underline{K} - \underline{K}')|}{|\underline{K} - \underline{K}'|^{4}} \ \delta(\underline{K} + \underline{q} - \underline{K}' - \underline{q}' - \underline{k}) \ \ \delta(E_{K} + E_{q} - E_{K'} - E_{q'} - \hbar\omega) \ . \qquad (24)$$

The Dirac delta function in Eq. (24) is obtained from the Kronecker delta function introduced in Eq. (23) by, e.g.,

$$\sum_{q'} \delta_{KR}(\underline{K} + \underline{q} - \underline{K}' - \underline{q}' - \underline{k}) \ = \ \int d^{3}q' \ \delta(\underline{K} + \underline{q} - \underline{K}' - \underline{q}' - \underline{k}) \quad . \qquad (25)$$

To illustrate the effect of different electron and ion temperatures, it is sufficient to consider the integral in the emission term of Eq. (24), i.e.,

$$I^e \equiv \int d^3q d^3q' d^3K d^3K' \; P^E(\underline{q}) \; P^I(\underline{K}) \frac{|\underline{\epsilon}_\lambda \cdot (\underline{K} - \underline{K}')|^2}{|\underline{K} - \underline{K}'|^4}$$

$$\times \; \delta(\underline{K} + \underline{q} - \underline{K}' - \underline{q}' - \underline{k}) \; \delta(E_K + E_q - E_{K'} - E_{q'} - \hbar\omega) \quad . \tag{26}$$

A convenient way of rewriting this integral for the purpose of integration is

$$I^e = \frac{(z^E z^I)^{-1}}{(2\pi)^4} \int_{-\infty}^{\infty} dy \int d^3x d^3\kappa d^3q d^3q' d^3K \times \frac{|\underline{\epsilon}_\lambda \cdot \underline{\kappa}|^2}{\kappa^4} e^{-E_q/\Theta_e} e^{-E_K/\Theta_i}$$
$$\times \; e^{i\underline{x}\cdot(\underline{q} - \underline{\kappa} - \underline{q}' - \underline{k}) + iy(E_K + E_q - E_{K+\kappa} - E_{q'} - \hbar\omega)} \tag{27}$$

where z^E and z^I are the electron and ion ideal gas partition functions evaluated at the respective temperatures, and where we have introduced the coordinate transformation $\underline{\kappa} \equiv \underline{K}' - \underline{K}$. Making only the approximations of neglecting (m/M) and $(\hbar\omega/mc^2)$ compared to unity, the integral in Eq. (27) reduces to

$$I^e = \frac{1}{16\pi\hbar}\sqrt{\frac{2\pi m}{\Theta_e}} \int du \int_0^\infty d\Omega_\kappa \; |\underline{\epsilon}_\lambda \cdot \underline{\Omega}_\kappa|^2 \frac{e^{-\frac{\hbar\omega[1+u^2+\sqrt{2\eta}\, u\mu]^2}{4\Theta_e[(1+\xi)u^2+\sqrt{2\eta}\, u\mu+\eta 2]}}}{\sqrt{(1+\xi)u^2 + 2\eta\, u\mu + \eta 2}} \quad , \tag{28}$$

where $\mu = \underline{k}\cdot\underline{\Omega}_\kappa$, $\underline{\Omega}_\kappa \equiv \underline{\kappa}/\kappa$, $\xi \equiv mT^I/MT^E$, and $\eta \equiv \hbar\omega/mc^2$. Evidently, unless the ion temperature is of the order of 10^3 times the electron temperature, the effect of ion motion on electron bremsstrahlung is truly negligible. Thus, we complete the evaluation of the integral in (28) by setting $T^I = T^E$, and hence $1 + \xi \simeq 1$; by neglecting the terms in the integrand containing η; and by summing over photon polarization states. We obtain[5]

$$\sum_{\lambda} I^e = \frac{1}{6h}\sqrt{\frac{2\pi m}{\Theta_e}}\, e^{-\beta/2}\left[\frac{1}{2}\int_0^\infty \frac{dy}{y}\, e^{-y-(\beta/2)^2/4y}\right] = \frac{1}{6h}\sqrt{\frac{2\pi m}{\Theta_e}}\, e^{-\beta/2}\, K_o(\frac{\beta}{2}) \quad , \tag{29}$$

where we have introduced $\beta \equiv \hbar\omega/\Theta_e$. This result, together with the relation displayed in Eq. (21) enables us to express the net absorption coefficient of Eq. (24) (summed over photon polarizations) as[1,6]

$$\bar{\alpha}(\omega,\underline{\Omega}) = \frac{8\pi}{3}\sqrt{\frac{2\pi m}{\Theta_e}}\,\frac{e^2}{hc}\,\frac{n^I n^E z^2 r_e^2 c^5}{\omega^3} \boxtimes \sinh\frac{\beta}{2}\, K_o(\frac{\beta}{2}) \quad . \tag{30}$$

SECTION III. ELECTRON COLLECTIVE EFFECTS

In this section we examine some of the effects on free-free absorption accrueing from real gas behavior of the electrons in the plasma. Since we do not anticipate that ion temperature effects will be significant, we initiate the analysis on the assumption that the electron and ion temperatures are the same. In this instance, Eq. (22a) may be rewritten as

$$\bar{\alpha}(\omega,\underline{\Omega},\lambda) = \frac{8\pi n^{I} z^{2} r_{e}^{2} \hbar c^{5}}{L^{3}\omega^{3}} \left(\frac{e^{2}}{\hbar c}\right) \times (e^{\beta} - 1) \sum_{\alpha\alpha'} \int d^{3}K d^{3}\kappa \, P_{\alpha}^{E} P^{I}(\underline{K})$$

$$\times \frac{|\underline{\epsilon}_{\lambda} \cdot \underline{\Omega}_{\kappa}|^{2}}{\kappa^{2}} |\langle\alpha'|\rho^{E}(-\underline{\kappa} - \underline{k})|\alpha\rangle|^{2} \times \delta(E_{\alpha'} + E_{K+\kappa} + \hbar\omega - E_{\alpha} - E_{K}) \quad , \tag{31}$$

where here, as before, we have introduced $\underline{\kappa} \equiv \underline{K}' - \underline{K}$ and $\beta \equiv \hbar\omega/\Theta$. Employing conventional arguments, the net absorption coefficient is readily displayed as

$$\bar{\alpha}(\omega,\underline{\Omega},\lambda) = \frac{8\pi n^{I} n^{E} z^{2} r_{e}^{2} c^{5}}{\omega^{3}} \left(\frac{e^{2}}{\hbar c}\right) \times (e^{\beta} - 1) \int d^{3}K d^{3}\kappa \, P^{I}(\underline{K}) \frac{|\underline{\epsilon}_{\lambda} \cdot \underline{\Omega}_{\kappa}|^{2}}{\kappa^{2}} \times S(q,\omega') \quad , \tag{32}$$

where

$$S(\underline{q},\omega') \equiv \frac{1}{2\pi N^{E}} \int dt e^{-i\omega' t} \times \sum_{\alpha} P_{\alpha}^{E} \langle\alpha|\rho^{E\dagger}(\underline{q}) \, \rho^{E}(q,-t)|\alpha\rangle \quad , \tag{33}$$

and

$$\omega' = \omega + (E_{K+\kappa} - E_{K})/\hbar \quad ,$$

$$\underline{q} = -\underline{k} - \underline{\kappa} ,$$

and

$$\rho^E(\underline{q},-t) \equiv e^{-itH^E/\hbar} \rho^E(\underline{q}) \, e^{itH^E/\hbar} . \qquad (34)$$

The formula in Eq. (32) is particularly convenient for present purposes, since the function, $S(\underline{q},\omega')$, has been extensively studied elsewhere.[7] However, most of the studies of this density-density correlation function for plasma applications have been in the sense of some sort of classical limit. Thus, to use the results of such studies here, we first need to manipulate Eq. (33) into a form suitable for classical computation.

This problem is a knotty one, but the results of some recent investigations suggest a way out suitable to our purposes. In an examination of the same question in the context of slow neutron scattering, Aamodt, et al.,[8] have shown (at least for the so-called "self" part of S) that

$$S(\underline{q},\omega') = e^{-\frac{\hbar\omega'}{2\Theta} - \frac{\hbar^2 q^2}{8m\Theta}} S_c(\underline{q},\omega') , \qquad (35)$$

where S_c is to be computed from Eq. (33) with the reinterpretation of all operators as classical c-numbers. Subsequently, Schaibly[9] has shown that the relation in Eq. (35) holds for both the "self" and "distinct" parts of S, provided S_c is computed in the sense of a certain approximation scheme which is, in fact, consistent with the calculations of others[8] whose results we wish to use here. Thus we now display the net absorption coefficient as (summing over photon polarizations),

$$\bar{\alpha}(\omega,\underline{\Omega}) = \frac{8\pi n^I n^E z^2 r_e^2 c^5}{\omega^3} \left(\frac{e^2}{\hbar c}\right)\left(2 \sinh \frac{\beta}{2}\right) \int d^3K d^3\kappa \, e^{-(E_{K+\kappa} - E_K)/\Theta} \, P^I(\underline{K}) \, e^{-\hbar^2 q^2/8m\Theta}$$

$$\times \frac{1 - (\underline{\Omega}_\kappa \cdot \Omega_k)^2}{\kappa^2} S_c(\underline{q},\omega') \quad , \tag{36}$$

and borrow the function S_c from the classical calculations of others.[8] Since here, S_c represents the effects of density fluctuations in a pure electron gas, we find that

$$S_c(\underline{q},\omega') \simeq \frac{2}{q} \frac{1}{|\Delta|^2} M^E\left(\frac{\omega'}{q}\right) \quad , \tag{37}$$

where

$$\Delta = 1 + \frac{4\pi n^E e^2}{\Theta q^2} \int_{-\infty}^{\infty} \frac{du \; u \, M^E(u)}{u - \dfrac{\omega' - i\nu}{q}} \quad , \tag{38}$$

and M^E is a one-dimensional Maxwellian normalized to unity. The integral in Eq. (38) is to be evaluated in the sense of a limit as the positive quantity ν tends to zero. The effect of electron-electron interactions is, of course, contained entirely in Δ. In fact, if we set $\Delta = 1$ and then evaluate the integrals in Eq. (36) setting $P^I(\underline{K}) = \delta(\underline{K})$, (zero ion temperature limit), and then letting $M \to \infty$ and $q \sim \kappa$; we recapture the result presented in Eq. (30) above. It is interesting to note the importance of the momentum transfer factor in the relation between S and S_c, Eq. (35), to the process of carrying out these integrations.

Finally, keeping the electron-electron interactions, but still setting $P^I(\underline{K}) = \delta(\underline{K})$, $q \sim \kappa$, and taking the limit of infinite ion mass; we find for the net absorption coefficient,

$$\bar{\alpha}(\omega,\underline{\Omega}) = \frac{8\pi^2 n^I n^E z^2 r_e^2 c^5}{3\omega^3} \left(\frac{e^2}{\hbar c}\right) \sinh\frac{\beta}{2} \int_0^\infty \frac{d\kappa}{\kappa}\, \frac{e^{-\hbar^2\kappa^2/8m\Theta}\, M^E(\omega/\kappa)}{|\Delta(\kappa,\omega)|^2} \quad . \tag{39}$$

SECTION IV. DISCUSSION

The results of the present study are summarized in Eqs. (28) and (39) for ion thermal effects and electron collective effects, respectively. Actually, we should probably have reduced both the emission and absorption terms of Eq. (24) to integrals of the form displayed in Eq. (28), since our interest here is in a net absorption coefficient. But, as ion temperature enters only in terms like $1 + mT^I/MT^E$, it did not seem that the extra effort required for completeness was warranted.

The result presented in Eq. (39) disagrees with some of the formulas presented elsewhere purportedly describing the effect of collective electron behavior on photon absorption in fully ionized plasmas. Furthermore, the reason for the disagreement is readily ascertained; since, if instead of the identification displayed in Eq. (35) we had simply taken $S(\underline{q},\omega') = S_c(\underline{q},\omega')$, the disagreement disappears. Evidently, in this particular instance, a certain amount of care is required in the use of classical estimates of electron density correlation functions.

The denominator in the integrand of Eq. (39) may be written somewhat more explicitly as

$$|\Delta(\kappa,\omega)|^2 = \left[1 + (\kappa\lambda_D)^{-2}\left(1 + \frac{\omega}{\kappa} P \int_{-\infty}^{\infty} \frac{du\ M^E(u)}{u - \omega/\kappa}\right)\right]^2 + \left[\frac{\pi\omega}{\kappa(\kappa\lambda_D)^2} M^E\left(\frac{\omega}{\kappa}\right)\right]^2 , \quad (40)$$

where $\lambda_D = \left[\Theta/4\pi n^E e^2\right]^{1/2}$ is the electron Debye length. For those values of κ for which (ω/κ) is large compared to the thermal speed of the electrons, Eq. (40) may be approximated by

$$|\Delta(\kappa,\omega)|^2 \sim \left[1 - (\frac{\omega_e}{\omega})^2\right]^2 + \left[\frac{\pi\omega}{\kappa(\kappa\lambda_D)^2} M^E(\frac{\omega}{\kappa})\right]^2 , \qquad (41)$$

thus suggesting the possibility of unusually strong absorption for light frequencies of the order of the plasma frequency, i.e., $\omega \sim \omega_e = (4\pi n^E e^2/\theta)^{1/2}$. But this is already to be anticipated on other grounds. The absorption coefficient given in Eq. (39) is to be used in a radiation transport equation of the form

$$\frac{\partial f}{\partial t} + c\, \underline{\nabla} \cdot \eta \underline{\Omega} f = -\bar{\alpha} f + \epsilon k^2/4\pi^3 , \qquad (42)$$

where η is the index of refraction for the plasma. Consequently, the net absorption coefficient <u>per</u> <u>unit</u> <u>distance</u> is given by

$$\bar{\alpha}_d = \bar{\alpha}/c\eta . \qquad (43)$$

For the fully ionized plasma, in lowest approximation, we have therefore

$$\bar{\alpha}_d = \frac{\bar{\alpha}}{c}\left[1 + (\frac{\omega_e}{\omega})^2\right]^{1/2} . \qquad (44)$$

Thus, as the light frequency approaches the plasma frequency, the absorption begins to increase independently of collective effects on bremsstrahlung. Hence it is unclear whether or not the collective effects described in Eq. (39) play an observable role in this mode of light absorption.

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REFERENCES

1. "Radiation Processes in Plasmas," G. Bekefi, John Wiley and Sons, Inc., New York, 1966.
2. "The Quantum Theory of Radiation," W. Heitler, Clarendon Press, Oxford, 1954.
3. "An Elementary Review of the Scattering of Photons by Highly Ionized Plasmas," R. K. Osborn, Univ. of Mich. Tech. Rep. No. 07599-3-T, 1966.
4. "Quantum Mechanics," Leonard I. Schiff, McGraw-Hill Book Co., Inc., New York, 1949.
5. "A Treatise on the Theory of Bessel Functions," Univ. Press, Cambridge, 1962.
6. "Plasma Physics," James E. Drummond, McGraw-Hill Book Co., Inc., New York, 1961.
7. E. E. Salpeter, Phys. Rev., 120, 1528 (1960).
8. R. Aamodt, K. M. Case, M. Rosenbaum, and P. F. Zweifel, Phys. Rev., 126, 1165 (1962). See also M. Rosenbaum and P. F. Zweifel, Phys. Rev., 137, B271 (1965).
9. "Photon Scattering from a Quantum Plasma," John Henry Schaibly, Thesis, Univ. of Mich., 1968 (unpublished).