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TIME AVERAGE HOLOGRAPHY EXTENDED*

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It is demonstrated that the usefulness of time-average holography in analyzing vibration can be extended by modulating the reference beam.

Powell and Stetson have shown that time-average holography can be used to analyze vibrations.[1] This letter will describe how time-average holography can be extended in its range of usefulness by modulating the reference beam.

A basic concept in time-average holography is that a sinusoidally vibrating object will phase modulate light reflected from it with a modulation depth proportional to the amplitude of vibration. Consider a point, with coordinate vector **x**, on the vibrating object. Light scattered from this point to a point on the photographic plate will be phase modulated by the function $a(\mathbf{x})\cos[\omega t + \alpha(\mathbf{x})]$, where $a(\mathbf{x})$ is the modulation depth, ω the angular frequency of vibration, and $\alpha(\mathbf{x})$ the phase of modulation. Powell and Stetson demonstrated that the brightness of the reconstructed point is proportional to $J_0^2(a)$, where $J_0(a)$ is the zero-order Bessel function. Thus fringes are formed on the reconstructed object from which the vibration amplitudes of the object can be analyzed. However, note that as the amplitude of vibration increases, the fringe peaks decrease, and it becomes difficult to analyze the vibrations. Further note that the phase information $\alpha(\mathbf{x})$ is lost.

Both of these deficiencies can be corrected by phase modulating the reference beam with the function $b\cos(\omega t + \beta)$. It can then be shown[2] that the object point will reconstruct with a brightness proportional to $J_0^2(c)$, where c is given by the cosine law

$$c^2 = a^2 + b^2 - 2ab\cos(\alpha - \beta).$$

Thus the fringes are no longer formed about zero vibration amplitude, but about a vibration amplitude determined by the modulation depth and phase of the reference beam.

These results have been experimentally demonstrated by observing the mode structure of a resonant crystal. A 1-in.-cube ADP crystal (0° cut) was electrically excited into standing shear-wave modes. Such shear modes have been studied by crossed polaroid techniques.[3] Figure 1(a) shows the fifth-order shear mode in a crossed polaroid system.

For the hologram exposure collimated light is directed along the z axis of the crystal and is plane polarized at 45° to the crystal's x and y axes. Thus the output light remains plane polarized, and is phase modulated by the changing index of refraction. The output light is then incident onto a diffusing plate, and a hologram is made of the diffusing plate. Figure 1(b) shows the virtual image reconstructed from a normal time-average hologram of the fifth-order shear mode. Note that the nulls reconstruct brightest. Figure 1(c) is the reconstruction when the reference beam is modulated with a modulation depth and phase approximately equal to the peak of the central lobe of the fifth-order shear mode. Note that the central lobe is brightest, while the adjacent lobes, vibrating 180° out of phase with the central lobe, are less bright. A 180° phase shift in the reference beam modulation inverts the role of the lobes as is shown in Fig. 1(d).

The reference beam modulation for Figs. 1(c) and 1(d) was obtained by sending a narrow reference beam through the same crystal used as the object. Thus any modulation depth and phase that the object can produce can also be used as the reference bias. In fact the entire light distribution of the end face of the crystal has been imaged

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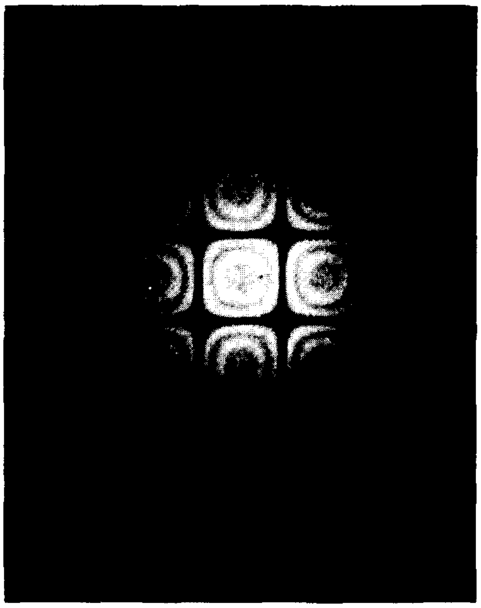
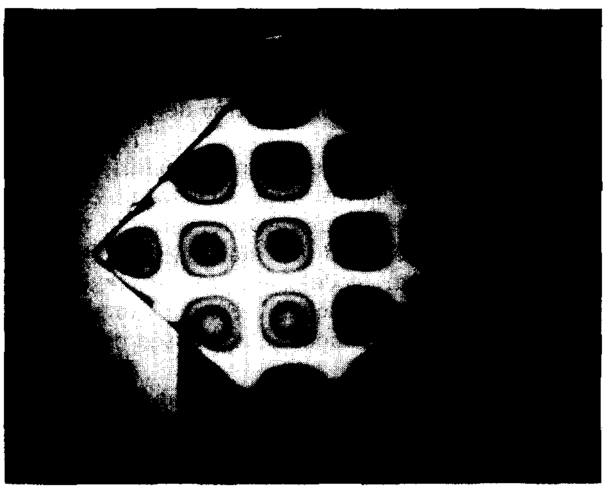


a



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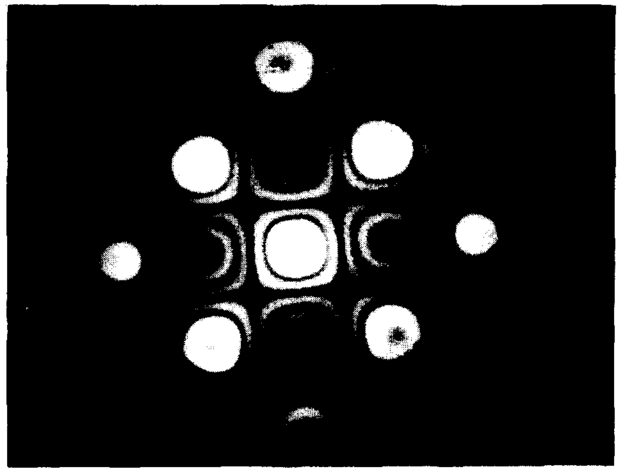

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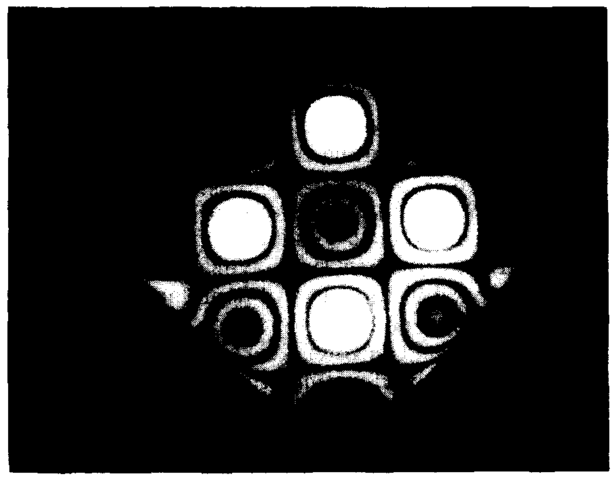

d

Fig. 1. Observations of the fifth-order shear mode of an electro-optic crystal: (a) with crossed polaroids, (b) with an unmodulated reference beam hologram, (c) with reference beam modulation depth and phase nearly equal to that of the central lobe peak, (d) with reference beam modulation nearly equal to that of the peak of a lobe adjacent to the central lobe.

onto the photographic plate and used as the reference beam. The reference bias is then determined by which section of the hologram is used to reconstruct the object.

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