Electromechanical Lead Networks for A.C. Servo Mechanisms

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The characteristics of conventional a.c. lead networks for a.c. servo mechanisms are compared to the characteristics of d.c. lead networks. Electromechanical lead networks for a.c. servo mechanisms having superior characteristics to those of the conventional a.c. lead networks are described and analyzed. A practical design for one of these electromechanical lead networks and its application to a servo system is shown.

ELECTRIC lead networks are used quite extensively to improve the response of servo mechanisms. The theoretical justification of the use of lead networks to improve the response of servo mechanisms and the general characteristics of basic lead networks has been considered thoroughly in several servo mechanisms texts.[1,2] In general, the notation used in this summary and in the following sections is that used in conventional servo mechanisms analysis. Such lead networks may operate on the error voltage, the input or output signal voltages separately, the feed-back voltage from an output tachometer, various internal loop voltages, and other signals. In many cases the input voltage to the lead network is a modulated, suppressed-carrier voltage, and it is necessary that the output voltage of the lead network also be a modulated, suppressed-carrier voltage of the same carrier frequency. Consequently, the basic d.c. lead network of Fig. 1, which is normally used where the output signal may be d.c. is not applicable by itself to these problems, and an a.c. lead network must be used.

The conventional a.c. lead networks do not operate on their input signals so effectively as does the network of Fig. 1 in a d.c. servo mechanism. Electromechanical a.c. lead networks as effective as the network of Fig. 1 can be designed and will be described following a brief summary of the characteristics of the convention a.c. lead networks.[3]

CONVENTIONAL A.C. LEAD NETWORKS

When the input and output voltages of a lead network must be modulated, suppressed-carrier voltages, two general types of a.c. lead networks are normally used. In the first type, the signal is demodulated, then the demodulated signal is passed through a d.c. lead network, like the one of Fig. 1, and lastly, the carrier is remodulated with the output voltage of the d.c. lead network. This type of a.c. lead network was probably the first to be used. The conventional tuned RLC and parallel-T networks are representative of the second type of a.c. lead network.[4]. Regardless of the type of lead network, it may be stated that the essential characteristic of an a.c. lead network is that it produces a pronounced positive phase shift of the modulating envelope over specific portions of the modulating frequency band. As shown below, the two types of conventional a.c. lead networks do not meet these requirements so well as does the basic network of Fig. 1 when operating on a d.c. voltage.

The demodulator and modulator circuits of the first type of a.c. lead network produce noise. Filter circuits added to the demodulator and modulator tend to reduce the magnitude of this noise. Unfortunately the reduction in noise by this method is accompanied by a time delay or an effective negative phase shift of the modulating envelope. Such a negative phase shift may be partially compensated for by an increase in the attenuation of the d.c. lead network. This increase in the attenuation will result in an increase of the over-all noise-to-signal ratio. Hence, in the first type of a.c. lead network, an engineering compromise must be

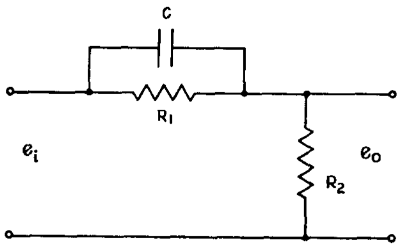

FIG. 1.

$r=\frac{R_1+R_2}{R_2}$

$T=R_1C$

$\phi=\left|\frac{e_0}{e_i}\right.$

if

$$e_i=e\sin w_s t \tag{1-1}$$

$$\frac{e_0}{e_i}=\frac{1}{r}\,\frac{1+jw_sT}{1+jw_s(T/r)}, \tag{1-2}$$

where maximum phase shift

$$\phi_{sm}=\sin^{-1}\left[\frac{r-1}{r+1}\right] \tag{1-3}$$

and occurs at

$$w_{sm}=\frac{(r)^{\frac{1}{2}}}{T} \tag{1-4}$$

$$\frac{e_0/e_i]_{w_s=\infty}}{e_0/e_i]_{w_s=0}}=r=\text{attenuation.} \tag{1-5}$$

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[1] G. S. Brown and D. P. Campbell, "Principles of servomechanisms" (John Wiley and Sons, Inc., New York, 1948), p. 210.

[2] Albert C. Hall, "The analysis and synthesis of linear servomechanisms" (The Technology Press, Massachusetts Institute of Technology, Cambridge, 1943), p. 122.

[3] University of Michigan External Memorandum No. 25, "Improvements in the characteristics of a.c. lead networks for servomechanisms." This paper was also presented as a conference paper at the AIEE Mid-Winter Convention of 1949.

[4] Arthur H. Benner, "Phase lead for a.c. servomechanisms," J. App. Phys., 268–273 (1949).

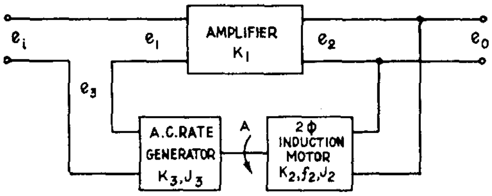


FIG. 2.

$$e_0 = e_2 \tag{2-1}$$

$$e_1 = e_i - e_3, \quad e_1' = e_i' - e_3' \tag{2-2}$$

$$e_2 = K_1 e_1 \tag{2-3}$$

$$K_2 e_2' \cong [(J_2 + J_3)p^2 + f_2 p]A, \quad \text{where } p = d/dt \tag{2-4}$$

$$e_3' = K_3 p A \tag{2-5}$$

$$J = J_2 + J_3, \quad f = f_2 \quad w_s \ll w_c \tag{2-6}$$

$$\frac{e_0'}{e_i'} \cong \frac{K_1 f}{f + K_1 K_2 K_3} \times \frac{1 + p(J/f)}{1 + p[J/(f + K_1 K_2 K_3)]} \tag{2-7}$$

(in general a primed variable denotes the envelope of the variable); let

$$\frac{f + K_1 K_2 K_3}{f} = r \quad T = J/f \tag{2-8}$$

$$\frac{e_0'}{e_i'} \cong \frac{K_1}{r} \, \frac{1 + pT}{1 + p(T/r)}, \tag{2-9}$$

if

$$e_i = e \sin w_s t \sin w_c t \tag{2-10}$$

$$\frac{e_0'}{e_i'} \cong \frac{K_1}{r} \, \frac{1 + jw_s T}{1 + jw_s(T/r)} \tag{2-11}$$

$$\phi_{sm} = \sin^{-1}\left[\frac{r-1}{r+1}\right] \tag{2-12}$$

$$w_{sm} = \frac{(r)^{\frac{1}{2}}}{T} \tag{2-13}$$

$$\frac{e_0'/e_i']_{w_s=\infty}}{e_0'/e_i']_{w_s=0}} = r. \tag{2-14}$$

reached between (1) the amount of noise suppressed by filters in the demodulator and modulator and (2) the increase in noise-to-signal ratio resulting from the increase in attenuation required to offset the effect of the filters. These several factors cause this type of network to produce a smaller maximum positive phase shift than the d.c. lead network can produce by itself.

The a.c. lead networks of the second type are quite sensitive to changes in the carrier frequency which can reduce the effective lead developed by these networks and can, in some cases, produce lag instead of lead. This effect is more noticeable at 400 c.p.s. than at 60 c.p.s., because it is the absolute change in carrier frequency which affects the network, and, in general, 60 c.p.s. sources are, and can be, better frequency-regulated on an absolute basis than 400 c.p.s. sources. Also, the maximum value of the lead time constant T of the second type of a.c. lead network is limited, and this limits the minimum value of w_{sm}, the frequency at which the maximum positive phase shift of the modulating envelope occurs.[1,2] In some cases when the minimum value of w_{sm} is too large, it is possible to increase the phase shift at lower frequencies by increasing the attenuation. Unfortunately this increase in attenua-

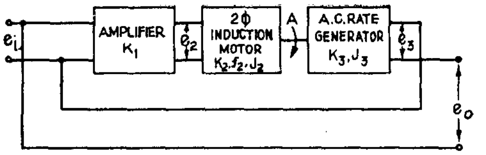


FIG. 3.

$$e_2 = K_1 e_i \tag{3-1}$$

$$K_2 e_2' \cong [(J_2 + J_3)p^2 + f_2 p]A, \quad \text{where } p = d/dt \tag{3-2}$$

$$e_3' = K_3 p A \tag{3-3}$$

$$e_0 = e_i - e_3, \quad e_0' = e_i' - e_3' \quad w_s \ll w_c, \tag{3-4}$$

$$J = J_2 + J_3, \quad f = f_2 \tag{3-5}$$

$$\frac{e_0'}{e_i'} \cong \frac{f - K_3 K_2 K_1}{f} \times \frac{1 + p[J/(f - K_1 K_2 K_3)]}{1 + p(J/f)} \tag{3-6}$$

(in general a primed variable denotes the envelope of the variable); let

$$\frac{f}{f - K_1 K_2 K_3} = r \quad T' = J/f \tag{3-7}$$

$$\frac{e_0'}{e_i'} \cong \frac{1}{r} \, \frac{1 + prT'}{1 + pT'} \tag{3-8}$$

or

$$\frac{e_0'}{e_i'} \cong \frac{1}{r} \, \frac{1 + jw_s rT'}{1 + jw_s T'} = \frac{1}{r} \, \frac{1 + jw_s T}{1 + jw_s(T/r)}, \tag{3-9}$$

where

$$rT' = T \tag{3-10}$$

$$\phi_{sm} = \sin^{-1}\left[\frac{r-1}{r+1}\right] \tag{3-11}$$

$$w_{sm} = \frac{(r)^{\frac{1}{2}}}{T}. \tag{3-12}$$

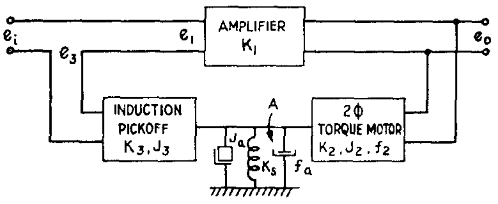


FIG. 4.

$$e_0 = K_1 e_1 \tag{4-1}$$

$$e_1 = e_i - e_3, \quad e_1' = e_i' - e_3' \tag{4-2}$$

$$e_3' = K_3 A \tag{4-3}$$

$$K_2 e_0' \cong [(J_2 + J_3 + J_a)p^2 + (f_2 + f_a)p + K_s]A, \quad \text{where } p = d/dt, \tag{4-4}$$

$$J = J_2 + J_3 + J_a, \quad f = f_2 + f_a \tag{4-5}$$

$$\frac{e_0'}{e_i'} \cong K_1 \frac{Jp^2 + fp + K_s}{Jp^2 + fp + (K_s + K_1 K_2 K_3)} \quad w_s \ll w_c; \tag{4-6}$$

(in general a primed variable denotes the envelope of the variable); let

$$e_0 = e \sin w_s t \sin w_c t \tag{4-7}$$

$$w_n^2 = K_s/J \tag{4-8}$$

$$c = \frac{f}{2(K_s J)^{\frac{1}{2}}} \tag{4-9}$$

$$h^2 = \frac{K_s + K_1 K_2 K_3}{K_s} \tag{4-10}$$

$$u = \frac{w_s}{w_n} \tag{4-11}$$

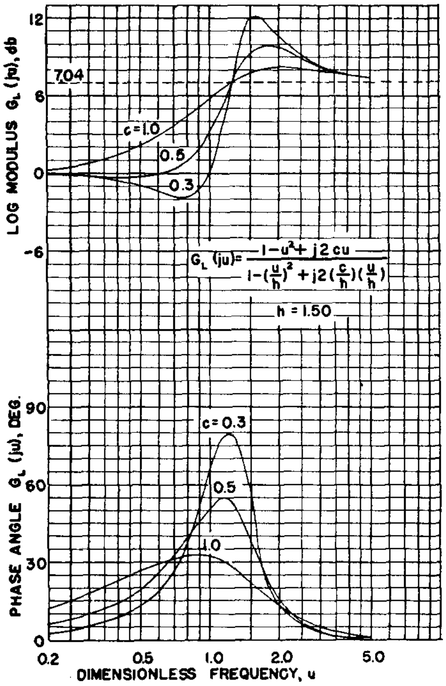
then

$$\frac{e_0'}{e_i'} \cong \frac{K_1}{h^2} \, \frac{1 - u^2 + j2cu}{1 - (u/h)^2 + j2(c/h)(u/h)} = K_L G_L(ju). \tag{4-12}$$

tion may increase the noise-to-signal ratio of the lead network sufficiently to offset any improvement derived from an increase in positive phase shift at lower frequencies.

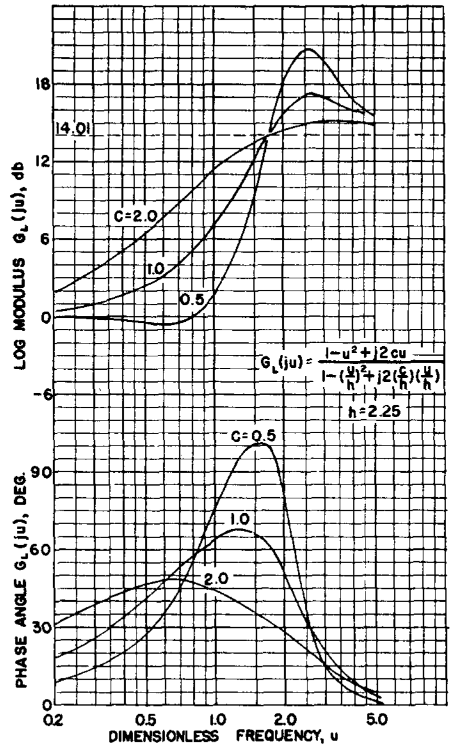
ELECTROMECHANICAL A.C. LEAD NETWORKS

By rearrangement of the elements of the lead network and by the choice of demodulators and modulators with excellent noise-to-signal ratios, it is possible to design an a.c. lead network which has performance comparable to the d.c. lead network in a d.c. servo mechanism. Such a lead network is shown in Fig. 2, and, because it contains electromechanical components, shall be referred to as an electromechanical lead network.[1] In the most general form, electromechanical lead networks are equally applicable for d.c. and a.c. operation, but in this article, only the a.c. form is discussed. (The network of Fig. 2 was described previously by the author in Electronics, November 1948. The meaning of the rearrangement of elements and the choice of the electromechanical components has been considered quite extensively in reference 3.)

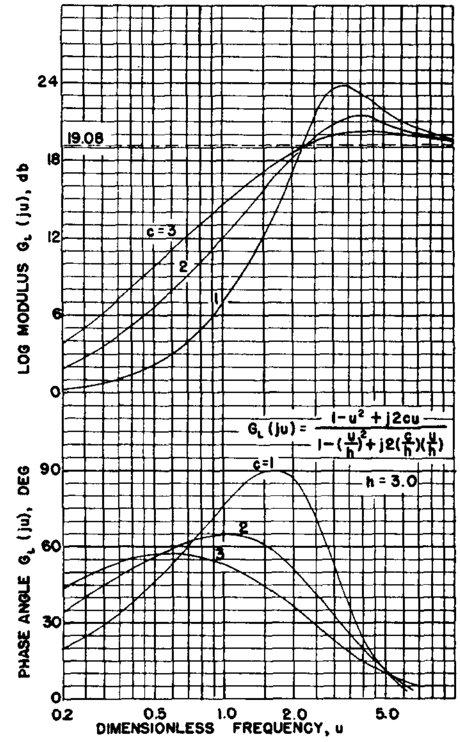


GRAPH 1.

In the system of Fig. 2 the motor functions as a demodulator, and the generator as a modulator. Both of these functions are approximate, with the limitations that w_s and the motor-generator speed be small compared to synchronous. The analytical expressions of (2–11), (2–12), and (2–13) of Fig. 2 show that the characteristics of this system are the same as for the conventional a.c. lead network.[2] By proper adjustment of K_1, K_2, and K_3, it is possible to establish any value of attenuation r, and the lead time constant T can be made as large as desired by the addition of inertia to the common motor-generator shaft. Unfortunately the minimum value of T is $(J_2+J_3)/f$, and this value may be too high for some servo applications.



GRAPH 2.



GRAPH 3.

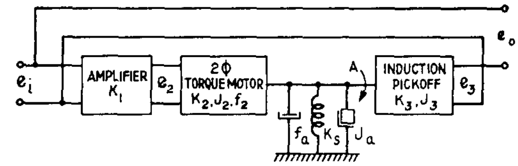


Fig. 5.

$$e_0 = e_i - e_3, \quad e_0' = e_i' - e_3' \tag{5-1}$$

$$e_3' = K_3 A \tag{5-2}$$

$$K_2 e_2' \cong [(J_2 + J_3 + J_a)p^2 + (f_2 + f_a)p + K_s]A, \quad \text{where } p = d/dt \tag{5-3}$$

$$e_2 = K_1 e_i \tag{5-4}$$

$$J = (J_2 + J_3 + J_a), \quad f = f_2 + f_a \tag{5-5}$$

$$\frac{e_0'}{e_i'} \cong \frac{Jp^2 + fp + (K_s - K_1 K_2 K_3)}{Jp^2 + fp + K_s} \quad w_s \ll w_c \tag{5-6}$$

(in general a primed variable denotes the envelope of the variable); let

$$e_0 = e \sin w_s t \sin w_c t \tag{5-7}$$

$$w_n{}^2 = K_s/J \tag{5-8}$$

$$c = \frac{f}{2(K_s J)^{\frac{1}{2}}} \tag{5-9}$$

$$q^2 = \frac{K_s}{K_s - K_1 K_2 K_3} \tag{5-10}$$

$$u = w_s/w_n \tag{5-11}$$

then

$$\frac{e_0'}{e_i'} \cong \frac{1}{q^2} \frac{1 - (qu)^2 + j2(qc)(qu)}{1 - u^2 + j2cu}. \tag{5-12}$$

The form of the electromechanical network of Fig. 2 will be referred to as a feed-back form because of the arrangement of the components in the circuit. Another arrangement of the components of Fig. 2 is shown in Fig. 3 and will be referred to as an electromechanical a.c. lead network of the parallel form. This parallel form of electromechanical network has characteristics similar to those of the feed-back form except that its lead time constant T is greater by a factor equal to the attenuation r. Consequently, in many cases the parallel form is not so useful as the feed-back form of electromechanical a.c. lead network. It is of interest to note that the amplifier K_1 can be omitted in the parallel form.

Figure 4 is a schematic of another electromechanical a.c. lead network. It is of the feed-back form but, as indicated by Eq. (4–12), does not have the phase and amplitude characteristics of the basic a.c. network. This network is a resonant lead controller.[2] Graphs 1, 2, and 3 show the phase and amplitude characteristics of this electromechanical system.

Figure 5 shows the parallel form counterpart of the electromechanical a.c. lead network of Fig. 4. Its characteristics are similar to those of the feed-back form.

Of the several types of electromechanical a.c. lead networks discussed, that of Fig. 4 appears to have the most desirable characteristics and also to be the simplest to manufacture. Figure 6 is a cut-away view of a pilot design of this electromechanical system. The main components of the system are (1) an induction torque motor, (2) a combination induction pickoff and electric spring, (3) an eddy-current damper, and (4) inertia weights. The induction torque motor and the combination induction pickoff and electric spring are made of simple, punched laminations and bobbin-wound coils and are called Microsyn elements.[5]

The induction torque motor is excited by a fixed a.c. reference voltage and by the signal voltage. It develops a torque proportional to the signal voltage which is virtually independent of the angular displacement of its rotor. The coils of the combination induction pickoff and electric spring are so arranged that there is a separate circuit for both the pickoff and electric spring. The pickoff circuit is excited with a.c. voltage and the circuit of the electric spring is excited with d.c. voltage. This combination then produces a signal voltage and a torque, both of which are proportional to the angular displacement of the rotor from neutral. By changing the d.c. excitation of the electric spring circuit, the torque gradient of the electric spring may be adjusted.

The eddy-current damper is made up of an aluminum damping disk and a multi-pole, Alnico permanent magnet with its poles parallel to the axis of rotation. This magnet is threaded axially into the housing; and therefore it is possible to adjust the spacing between the magnet and damping disk, thus changing the damping coefficient f.

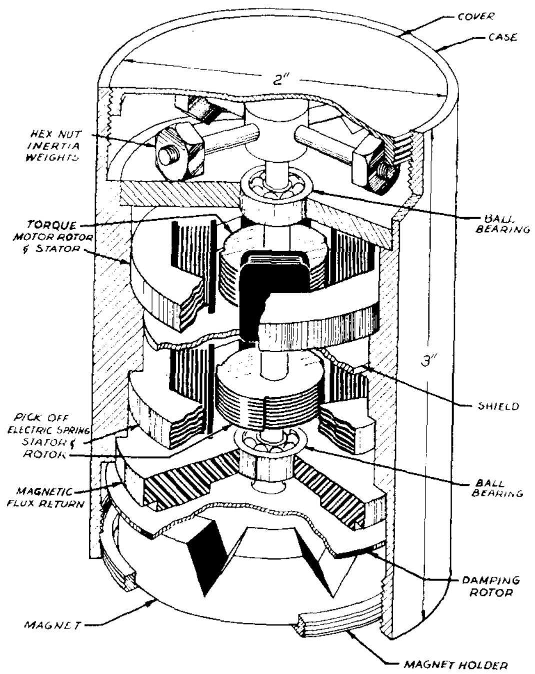


Fig. 6.

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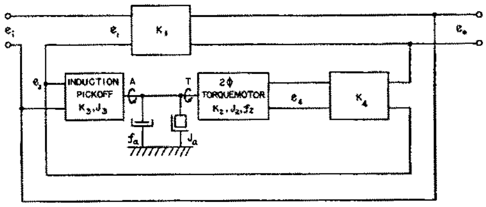
[5] The Microsyn elements were developed by the M. I. T. Instrumentation Laboratory and are more fully described in *Electronic Instruments* (McGraw-Hill Book Company, Inc., 1948), pp. 365–366. The combination of the induction pickoff and electric spring into one Microsyn element was developed by Mr. K. C. Mathews of the University of Michigan Aeronautical Research Center while he was a member of the Doelcam Corporation, Newton, Massachusetts.

FIG 7.



$$\frac{e_0'}{e_i'} \cong K_1 \frac{Jp^2+fp+K_2K_3K_4}{Jp^2+fp+K_2K_3K_4(1+K_1)}. \tag{7-1}$$

Let

$$K_2K_3K_4=K_s. \tag{7-2}$$

Then

$$\frac{e_0'}{e_i'} \cong K_1 \frac{Jp^2+fp+K_s}{Jp^2+fp+K_s(1+K_1)}. \tag{7-3}$$

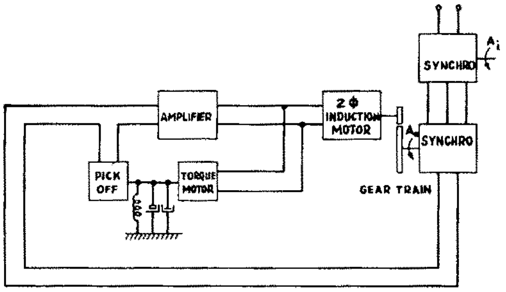
The total moment of inertia of the system may be changed by adjusting the position of the inertia weights.

Thus as there are three continuously adjustable parameters—the spring constant, the damping, and the moment of inertia—it is possible to adjust the two variables w_n and c independently. The attenuation factor h^2 can also be adjusted independently by changing the pickoff sensitivity or the amplifier gain.

As planned, w_n and c are to be continuously adjustable from 1 c.p.s. to 6 c.p.s. and 0.5 to 1, respectively. Over these ranges of network parameters, the maximum power consumption of the torque motor should not exceed one-third of a watt.

If the electromechanical system of Fig. 6 is used in the circuit of Fig. 7, its natural frequency w_n can be increased and its design can be simplified. In Fig. 7 the spring K_s of Fig. 4 has been replaced by the amplifier K_4. It can be shown that if h^2 is set equal to $1+K_1$, Eq. (7-3) can be non-dimensionalized and will have the same form as Eq. (4-12) of Fig. 4. Hence the response of the circuit of Fig. 7 is the same as the response of the circuit of Fig. 4 and Eq. (4–12) and Graphs 1, 2, and 3 apply for both circuits.

FIG. 8.



Changes in carrier frequency which will not prevent proper operation of any electromechanical components in the complete servo loop (such as motors and indicators) will not affect the operation of the electromechanical a.c. lead networks described. Graphs 1, 2, and 3 show that w_{sm} occurs near w_n, and, as noted above, w_n and hence w_{sm} may be adjusted to cover the region of frequencies required for almost every type of servo mechanism problem. Also the maximum value of phase shift ϕ_{sm} may be made larger for the network of Fig. 4 than for a single lead network having the same attenuation or noise-to-signal ratio.

An example of the application of the electromechanical lead network of Fig. 4 is shown in Fig. 8. Here a two-phase induction servo is being stabilized by the use of this electro mechanical a.c. lead network. As will be noted, the amplifier driving the servo motor is also used to drive the torque motor of the lead network. This is possible because of the very low power consumption of the torque motor.

In closing, if the feed-back paths of Figs. 2 and 4 are connected regeneratively, instead of degeneratively, and the parallel paths of Figs. 3 and 5 are connected so as to add, instead of subtract, these systems will become lag or integral networks instead of lead networks.

ACKNOWLEDGMENTS

I should like to thank Messrs. K. C. Mathews and P. E. Theobald of the University of Michigan Aeronautical Research Center for their assistance in the mechanical design of the system of Fig. 6.

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A High Power Attenuator for Microwaves

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A high power variable attenuator has been developed for use in 10-cm wave guide transmission lines. The attenuating material is a liquid which serves also to transfer the absorbed energy. The attenuator can handle power levels up to 100 watts, has a power standing wave ratio less than 1.1 to 1, and an attenuation which is variable from 1.5 db to 40 db.

THIS paper describes a high power attenuator developed to vary the main power flow in a 10-cm wave guide. It was used in experiments on microwave breakdown potentials in gases, recently carried out at Westinghouse Research Laboratories. In these experiments it was necessary to vary the power level in a