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A SIMPLIFIED METHOD FOR COMPUTING UPPER-ATMOSPHERE TEMPERATURE AND WINDS IN THE ROCKET-GRENADE EXPERIMENT

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ABSTRACT

This report describes a simplified method for computing temperatures and winds at high altitudes from data obtained as results of the rocket-grenade experiment. The method is exemplified by applying it to the data from SM1.01 and SM1.02 rocket flights.

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1. INTRODUCTION

The rocket-grenade experiment for high-altitude temperatures and winds consists of exploding H.E. charges (hereafter referred to as grenades) carried aloft by a rocket, and recording information on the propagation of the explosion waves. This information consists of the coordinates of the explosion (coordinates of the rocket at the time of the explosion obtained by DOVAP, and sometimes the coordinates of the explosions obtained by ballistic cameras), the time of the explosion (obtained by a disturbance in the DOVAP record, and sometimes by ground-based flash detectors), and the times of arrival at a number of points (array of microphones) on the ground.

In the currently used method of the data reduction*,** the effects of temperature and winds are treated separately. More accurate results can be obtained by reiteration. The concept of virtual source is introduced in the calculations, which is the position at which an explosion at height z would have to occur if there is no wind below z so that its sound wave would arrive at the microphone array from the measured direction.

In this report a new approach is presented to the problem of the computation of winds and temperatures in the layers between the explosions from the recorded data on propagation of the explosion waves. The effects of temperature and winds are taken into account simultaneously. The method is simple both from the conceptional point of view and from the point of view of programming for a solution on a digital computer. As in the currently used method, zero vertical winds are assumed.

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*A. G. Weisner, "The Determination of Temperatures and Winds Above Thirty Kilometers", in Rocket Exploration of the Upper Atmosphere, Pergamon Press, 1954, p.133.

**W. G. Stroud, W. Nordberg, and J. R. Walsh, "Atmospheric Temperatures and Winds Between 30 and 80 Km," J. Geophys. Res., 61, 45 (March, 1956).

2. NOTATION

A_k Arrival time at the k-th microphone measured relative to the arrival time at the center microphone, and corrected for horizizontal and vertical departures of the microphone location from an ideal array of two horizontal lines.

$\tilde{A}_k$ Measured arrival time at the k-th microphone.

$A(x)$ Function relating the time of arrival at microphones nos. 6, 7, 8, and 9, as a function of distance x.

$A(y)$ Function relating the time of arrival at microphones nos. 5, 4, 2, and 1, as a function of distance y.

ΔA_h Correction in the time of arrival for horizontal displacement of a microphone.

ΔA_z Correction in the time of arrival for a vertical displacement of a microphone.

$a_1 \ldots a_k$ Coefficients of the powers of x in function A(x).

$b_1 \ldots b_k$ Coefficients of the powers of y in function A(y).

d_o Distance between the point (x_o, y_o, z_o) and the wave front passing through the origin of coordinates, measured parallel to the vector of propagation $\overline{P}$.

e East direction.

ϵt Short time interval.

ϵu Short displacement along u axis.

ϵx Short displacement along x axis.

ϵy Short displacement along y axis.

ϵz Short displacement along z axis.

f	Frequency.
h	Altitude above the sea level.
i	Index of the explosion.
k_ℓ	Infinite characteristic velocity.
k_m	Minimum characteristic velocity.
k_e, k_n, k_x, k_y	Characteristic velocity in certain horizontal directions.
$\tilde{k}_x$, $\tilde{k}_y$	Approximate characteristic velocity.
ℓ	Direction of infinite characteristic velocity.
m	Direction of minimum characteristic velocity.
n	North direction.
$\overline{P}$	Propagation velocity vector.
p	Magnitude of propagation velocity.
p_e, p_n, p_x, p_y, p_z	Components of the propagation vector in a Cartesian coordinate system.
T	Temperature arrived at by Eq. (9.2).
t_o	Travel time of the sound wave through distance d_o.
t_g	Correction in travel time of the wave for finite amplitude propagation.
t_i	Travel time of a sound wave from the explosion to the center microphone.

t_m Measured travel time of the explosion wave from the explosion to the center microphone.

Δt Time interval spent in a layer.

u Arbitrary horizontal direction.

$\overline{V}$ Velocity of sound vector.

v Magnitude of sound velocity.

v_e, v_n, v_x, v_y, v_z Components of the sound velocity vector in a Cartesian coordinate system.

v_{ph} Phase velocity.

v_g Group velocity.

$\overline{W}$ Wind vector.

w_e, w_n, w_x, w_y Horizontal wind components.

$\overline{X}$ Unit vector in x direction.

x Horizontal axis.

x_o Displacement of one of microphones nos. 1, 2, 4, or 5 in x direction.

x_k Distance from the center microphone along x axis of one of microphones nos. 6, 7, 8, or 9.

$\overline{Y}$ Unit vector in y direction.

y Horizontal axis.

y_o Displacement of one of microphones nos. 6, 7, 8, or 9 in y direction.

y_k Distance from the center microphone along y axis of one of microphones nos. 1, 2, 4, or 5.

$\overline{Z}$ Unit vector in z direction.

z Vertical axis.

z_o Displacement of a microphone in z direction.

α Angle between the velocity of sound $\overline{V}$ and x axis.

β Angle between the velocity of sound $\overline{V}$ and y axis.

γ Angle between the velocity of sound $\overline{V}$ and z axis.

τ Relaxation constant.

ω Angular frequency.

Θ Angle between m direction and u direction.

ψ Angle between x direction and u direction.

3. OUTLINE OF THE METHOD OF THE DATA REDUCTION

In the calculations, the physical quantities are expressed by their components in the Cartesian coordinate system and the computation of angles is avoided. The origin of the coordinate system is taken at the center point of the microphone array with two axes horizontal and the z axis up.

As a first step, arrival times at the microphones are used to compute characteristic velocities (trace velocities) at the center point of the microphone array. The calculation is presented in Chapter 4.

These characteristic velocities are computed for two directions, x and y, in a horizontal plane, along the arms of the array, which has the form of a cross. From these, the characteristic velocities in the north and east directions are computed, using formulas presented in Chapter 5.

The sound ray which arrived at the center point of the microphone array from the lowest grenade explosion (explosion no. 1) is then traced through layers with known meteorological parameters, i.e., layers in which the temperature and winds were obtained by balloon measurements shortly before or after the rocket flight.* The winds and the temperature are assumed constant in a layer. The appropriate formulas for tracing of the sound ray in the layers with constant, known temperature and winds are presented in Chapter 7. The formulas are based on the law of refraction, which is stated in terms of a Cartesian coordinate system in Chapter 6.

The tracing through layers with known meteorological parameters is continued up to the altitude of the balloon flight; but in the rare case that balloon data extend to the altitude of the lowest explosion, explosion no. 1, the tracing is stopped 3 or 4 km under the altitude of the lowest explosion. This altitude up to which the ray is traced through layers with known meteorological parameters can be conveniently referred to as the altitude of explosion no. 0. As a result of the tracing we know the east and north coordinates of the point at which the ray from explosion no. 1 to the center point of the array intersects the altitude of explosion no. 0, and we know likewise the time spent in propagation from the center point of the array up to the altitude of explosion no. 0. This time is subtracted from the determined time of sound propagation from explosion no. 1 to the center point of the microphone array,** to yield the time of propagation $(\Delta t)_1$ from explosion no. 1 to the altitude of explosion no. 0. The distances $(\Delta n)_1$ and $(\Delta e)_1$ that the sound ray travels in the north and east direction within the layer are likewise computed. Knowing the geometrical path through the layer between explosion no. 1 and the altitude of explosion no. 0, the meteorological parameters in the layer between explosion no. 0 and explosion no. 1 are computed using equations derived in Chapter 8.

Then the sound ray from explosion no. 2 is traced, starting from the center microphone, using equations of Chapter 7, up to the altitude of explosion no. 1. The coordinates north and east of this point and the time of travel from this point down to the center point of the microphone array become available. The difference $(\Delta n)_2$, $(\Delta e)_2$, and $(\Delta t)_2$ for propagation of the ray from explosion no. 2 to the altitude of explosion no. 1 are computed. Knowing

*It will be shown later in Chapter 12 that the end results, i.e., the computed temperatures and winds in the layers between the explosions are not at all sensitive to the accuracy of those balloon data.

**The measured time of the propagation of the explosion wave, i.e., the difference between the time of the explosion and the time of arrival at the center microphone, is incremented by the estimated theoretically finite-amplitude-propagation effect to yield the determined time of sound propagation. The finite-amplitude-propagation effect has been discussed and the amount of increment as a function of altitude computed for 2-lb and 4-lb grenade explosions in The Effect of Finite-Amplitude Propagation in the Rocket Grenade Experiment for Upper-Atmosphere Temperature and Winds, by Joseph Otterman, Univ. of Mich. Eng. Res. Inst. Report 2387-34-T, Ann Arbor, April, 1958.

the geometrical path through the layer between explosion no. 2 and the altitude of explosion no. 1, the meteorological parameters in the layer are computed using equations derived in Chapter 8.

The process is continued for successive explosions yielding each time the meteorological parameters of successive layers. Reiteration of the tracing is therefore unnecessary.

The method is outlined again in Chapter 9 as a Flow Chart for a computer program. The pertinent equations are included in the Flow Chart. The process can be visualized by considering Fig. 1, where the order of tracing through the layers is indicated by successive numbers. The solid lines indicate the layers in which tracing is done by means of the equations of Chapter 7, and the dotted lines indicate the layers in which the meteorological parameters are calculated by means of equations in Chapter 8.

The equations of both Chapter 7 and Chapter 8 are based on the assumption of constant temperature and constant winds in the layer. It will be shown in the discussion in Chapter 12 that this assumption does not lead to appreciable error in computing the average parameters in the layer. The equations are based on the assumption of zero vertical wind velocity. This is a very crucial assumption, and it can possibly be the source of significant errors. This is also discussed in Chapters 12 and 13.

It should be pointed out that the average meteorological parameters computed for the layer between explosion no. 0 and explosion no. 1 do not possess the accuracy of the rocket-grenade experiment. Actually, the accuracy of these parameters is worse by an order of magnitude than the accuracy of the data from balloon flights.

4. CALCULATION OF THE CHARACTERISTIC VELOCITIES FROM THE TIMES OF ARRIVAL AT THE MICROPHONES

The microphone array, which has been used for the IGY series of the rocket-grenade experiment in Fort Churchill, Manitoba Province, Canada, consists of nine microphones arranged in the form of a horizontal cross (Fig. 2). The purpose of this chapter is to describe the computational procedure for obtaining the characteristic velocities in the directions along the arms of the cross from the arrival times of the wave at the microphones.

To the first approximation, the characteristic velocities k_x and k_y at the center microphone, no. 3, in the x and y directions are given by:

$$\tilde{k}_x = \frac{x_9 - x_6}{\tilde{A}_9 - \tilde{A}_6} \approx \frac{x_7 - x_8}{\tilde{A}_7 - \tilde{A}_8} \quad \text{and} \qquad (4.01)$$

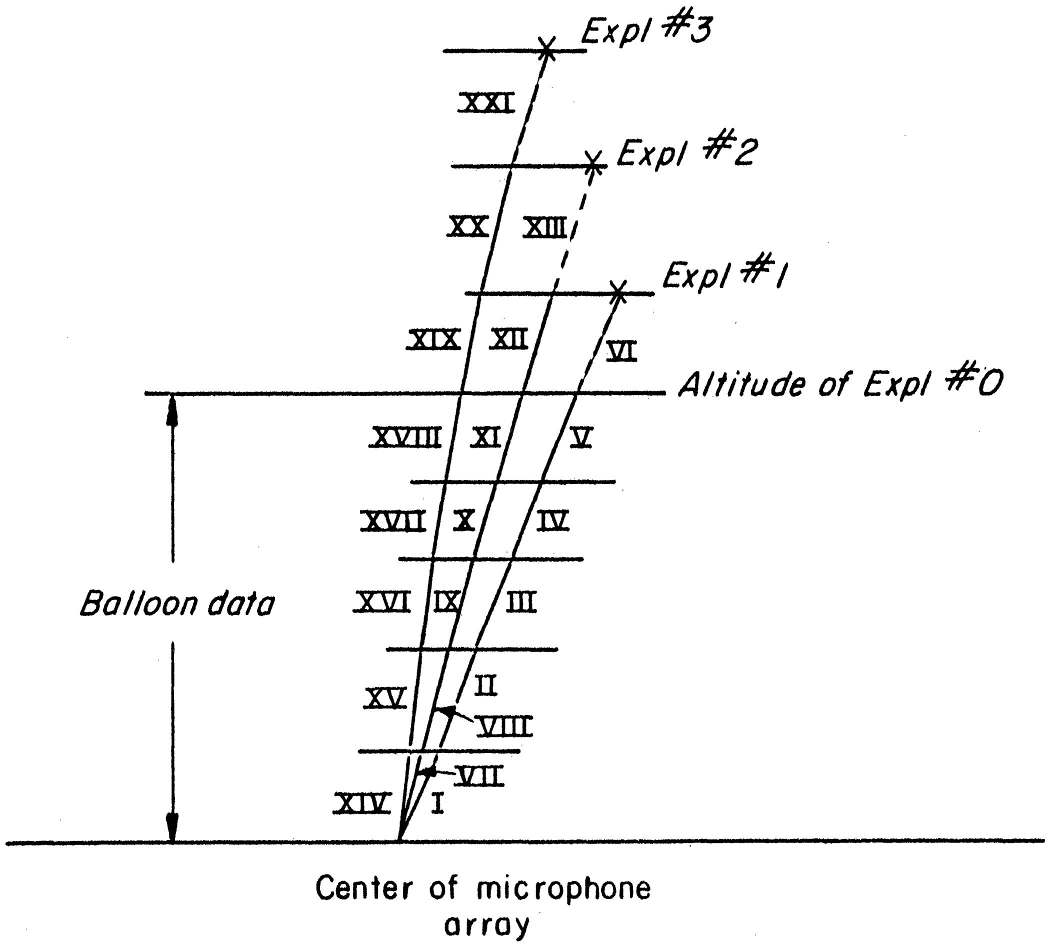

Fig. 1. Tracing of sound rays through successive layers.

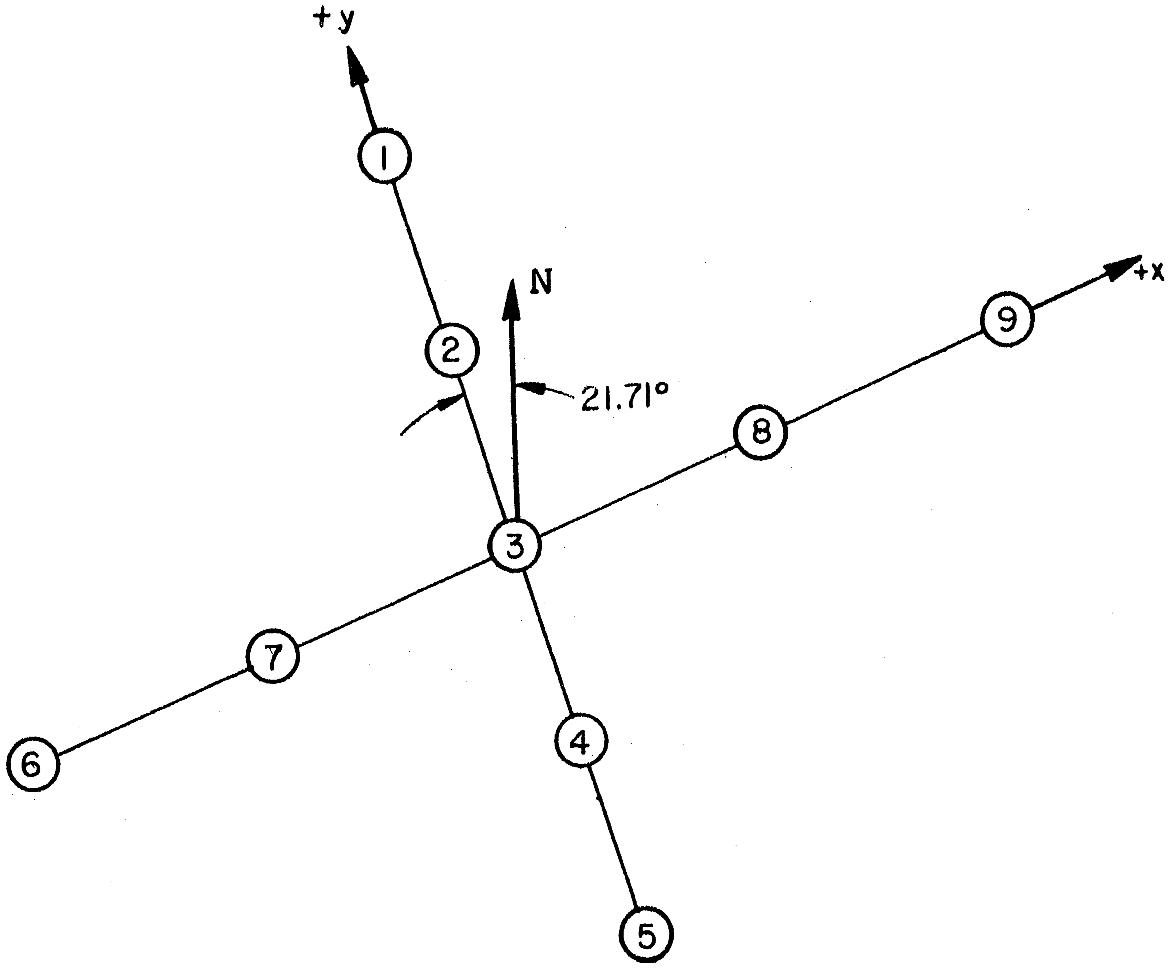

Fig. 2. The microphone array at Fort Churchill, Manitoba Province, Canada. This array has been used during the IGY series of the rocket-grenade experiment.

$$\tilde{k}_y = \frac{y_5 - y_1}{\tilde{A}_5 - \tilde{A}_1} \approx \frac{y_4 - y_2}{\tilde{A}_4 - \tilde{A}_2} \quad , \tag{4.02}$$

where $\tilde{A}_k$ denotes the recorded time of arrival at microphone no. k. The above equations would determine exactly the characteristic velocities if the waves were planar and if the microphones were located on two horizontal lines.

The actual locations of the microphones depart from an ideal array of two horizontal straight lines. The arrival times at each microphone have first to be corrected to allow for these departures, in both the horizontal and the vertical directions.

The horizontal correction ΔA_h is rather easy to compute. Let x_o be the horizontal displacement of one of the microphones nos. 6, 7, 8, or 9, and y_o be the horizontal displacement of one of the microphones nos. 1, 2, 4, or 5. We have then, from definition of the characteristic velocities, the following equations for the correction of arrival times:

$$\Delta A_h = \frac{x_o}{k_x} \cong \frac{x_o}{\tilde{k}_x} \tag{4.03}$$

for one of microphones nos. 6, 7, 8, or 9, and

$$\Delta A_h = \frac{y_o}{k_y} \cong \frac{y_o}{\tilde{k}_y} \tag{4.04}$$

for one of microphones nos. 1, 2, 4, or 5. These corrections, in the case of the Churchill array, involve only fractions of a millisecond and the use of approximate characteristic velocities [Eqs. (4.01) and (4.02)] is completely adequate.

The derivations of equations, to be presented now, for correcting the arrival times to allow for the vertical displacements of the microphones are much lengthier.

The basic equations expressing the propagation vector $\overline{P}$ and the velocity of sound in the medium $\overline{V}$ in terms of their components are:

$$\overline{P} = p_x \cdot \overline{X} + p_y \cdot \overline{Y} + p_z \cdot \overline{Z} \tag{4.11}$$

$$\overline{V} = v_x \cdot \overline{X} + v_y \cdot \overline{Y} + v_z \cdot \overline{Z} \tag{4.12}$$

$$\overline{P} = \overline{V} + \overline{W} \tag{4.13}$$

$$p_x = v_x + w_x \tag{4.14}$$

$$p_y = v_y + w_y \tag{4.15}$$

$$p_z = v_z \tag{4.16}$$

$$v^2 = |\overline{V}|^2 = v_x^2 + v_y^2 + v_z^2 \tag{4.17}$$

$$p^2 = |\overline{P}|^2 = p_x^2 + p_y^2 + p_z^2 \quad . \tag{4.18}$$

We develop first a formula for the distance d_o between a point x_o, y_o, z_o and the plane tangent to the wave front at the origin of the coordinates, where the distance is measured along, i.e., parallel to, the vector of propagation $\overline{P}$.

The equation for points in a plane passing through the origin of the coordinates and tangent to the wave front will be

$$(x \cos \alpha + y \cos \beta + z \cos \gamma) \text{ constant} = 0 \quad , \tag{4.19}$$

where $\cos \alpha$, $\cos \beta$, and $\cos \gamma$ are the directional cosines of the normal to the wave front. The validity of Eq. (4.19) can be seen from the following: components of a point in the plane are x, y, and z. The scalar product of the vector $x\overline{X} + y\overline{Y} + z\overline{Z}$ representing this point and of unit normal vector $\cos \alpha\overline{X} + \cos \beta\overline{Y} + \cos \gamma\overline{Z}$ must be zero.

The equation of the line passing through x_o, y_o, z_o and parallel to $\overline{P}$ is

$$\frac{x - x_o}{p_x} = \frac{y - y_o}{p_y} = \frac{z - z_o}{p_z} = \lambda \quad , \tag{4.20}$$

where λ is a parameter. The point of intersection of this line with the plane tangent to the wave front shall be designated by x_1, y_1, and z_1 (parameter λ_1). We have, therefore,

$$\frac{x_1 - x_o}{p_x} = \frac{y_1 - y_o}{p_y} = \frac{z_1 - z_o}{p_z} = \lambda_1 \quad , \tag{4.21}$$

and since the point lies in the plane defined by Eq. (4.19),

$$\cos \alpha x_1 + \cos \beta y_1 + \cos \gamma z_1 =$$

$$\cos \alpha(\lambda_1 p_x + x_o) + \cos \beta(\lambda_1 p_y + y_o) + \cos \gamma(\lambda_1 p_z + z_o) = 0 \tag{4.22}$$

$$\lambda_1 \cdot (\cos \alpha p_x + \cos \beta p_y + \cos \gamma p_z) = -(\cos \alpha x_o + \cos \beta y_o + \cos \gamma z_o) \qquad (4.23)$$

$$\lambda_1 = - \frac{\cos \alpha x_o + \cos \beta y_o + \cos \gamma z_o}{\cos \alpha p_x + \cos \beta p_y + \cos \gamma p_z} . \qquad (4.24)$$

We have for the distance d_o:

$$d_o = [(x_1-x_o)^2 + (y_1-y_o)^2 + (z_1-z_o)^2]^{1/2} . \qquad (4.25)$$

From Eq. (4.21):

$$x_1 - x_o = p_x \lambda_1 \qquad (4.26)$$

$$y_1 - y_o = p_y \lambda_1 \qquad (4.27)$$

$$z_1 - z_o = p_z \lambda_1 \qquad (4.28)$$

and therefore (if the negative sign root is chosen)

$$d_o = - (p_x^2 + p_y^2 + p_z^2)^{1/2} \lambda_1 = p \frac{\cos \alpha x_o + \cos \beta y_o + \cos \gamma z_o}{\cos \alpha p_x + \cos \beta p_y + \cos \gamma p_z} , \qquad (4.29)$$

and if both numerator and denominator are multiplied by v, we have

$$d_o = p \frac{v_x \cdot x_o + v_y \cdot y_o + v_z \cdot z_o}{v_x \cdot p_x + v_y \cdot p_y + v_z \cdot p_z} . \qquad (4.30)$$

The difference in arrival times to between the origin of coordinates and the point x_o, y_o, z_o will be (the two points are assumed to be close together, so that the direction of arrival does not vary):

$$t_o = \frac{d_o}{p} = \frac{\cos \alpha x_o + \cos \beta y_o + \cos \gamma z_o}{\cos \alpha p_x + \cos \beta p_y + \cos \gamma p_z} = \frac{v_x \cdot x_o + v_y \cdot y_o + v_z \cdot z_o}{v_x \cdot p_x + v_y \cdot p_y + v_z \cdot p_z} . \qquad (4.31)$$

This rather cumbersome formula is used in a simplified form, to correct for departures from the ideal array in the vertical direction. This correction is denoted by ΔA_z:

$$\Delta A_z = \frac{v_z \cdot z_o}{v_x \cdot p_x + v_y \cdot p_y + v_z \cdot p_z} = \frac{v_z \cdot z_o}{v_x(v_x+w_x) + v_y(v_y+w_y) + v_z^2} . \qquad (4.32)$$

The corrections ΔA_z in the case of the Churchill array amount to as much as 10 msec; it is thus desirable to compute the correction with an accuracy of about 1%. It has been found that the terms $v_x \cdot w_x$ and $v_y \cdot w_y$ contribute insignificantly to the denominator. Thus, unless the ground winds are strong and the direction of arrival departs considerably from the vertical, Eq. (4.32) can be replaced by a simpler relation:

$$\Delta A_z \cong \frac{v_z \cdot z_o}{v^2} \quad . \tag{4.33}$$

The distances z_o are known from the survey of the microphone array, and v^2 is determined by the average temperature at the array. The vertical component v_z of velocity still remains to be determined separately for each arrival. This vertical component is determined from:

$$v_z = (v^2 - v_x^2 - v_y^2)^{1/2} \quad . \tag{4.34}$$

The horizontal components are determined from the following two equations:

$$v_x = \frac{v^2}{k_x - w_x - (k_x/k_y)\,w_y} \cong \frac{v^2}{\tilde{k}_x} \tag{4.35}$$

$$v_y = \frac{v^2}{k_y - w_y - (k_y/k_x)\,w_x} \cong \frac{v^2}{\tilde{k}_y} \quad . \tag{4.36}$$

The above equations are based on Eqs. (7.4) and (7.5), which will be derived later. It should be pointed out that components v_x and v_y do not need to be determined to an accuracy of 1%, and still v_z can be known with 1% accuracy from (4.34).

The corrected time of arrivals, which are given by

$$A_k = \tilde{A}_k - \Delta A_h - \Delta A_z \quad , \tag{4.37}$$

could be used to determine exactly the characteristic velocities k_x and k_y at the center microphone in accordance with Eqs. (4.01) and (4.02) if the waves were planar. However, since in reality the waves are not planar, a more sophisticated approach is necessary.

Regard the arrival time A_k to be a function of x, or of y. Then the characteristic velocities k_x and k_y at the center microphone will be

$$\frac{1}{k_x} = \left.\frac{\partial A(x)}{\partial x}\right|_{x=0} \tag{4.38}$$

and

$$\frac{1}{k_y} = \left.\frac{\partial A(y)}{\partial y}\right|_{y=0} \quad . \tag{4.39}$$

Each of the functions $A(x)$ and $A(y)$ is known at five points. And obviously many approaches can be used for determination of a reasonably fitting function. Three approaches which were tried by the writer are outlined below. In the following, arrivals A_k are measured relative to the arrival at the center microphone, no. 3.

A function of the type $A(x) = a_1x + a_2x^2$ with least-squares fit has been rejected, since the value of the function at the microphones differed by more than 1 msec from the actual values A_k. A function of the type $A(x) = a_1x + a_2x^2 + a_3x^3$ with the least-square fit and a function of the type $A(x) = a_1x + a_2x^2 + a_3x^3 + a_4x^4$ gave virtually identical results in two cases. This last type of function, which leads to computationally rather simple determination of the characteristic velocities, has been used.

Only coefficients a_1 and b_1 in the functions

$$A(x) = a_1x + a_2x^2 + a_3x^3 + a_4x^4 \tag{4.40}$$

and

$$A(y) = b_1y + b_2y^2 + b_3y^3 + b_4y^4 \tag{4.41}$$

need to be determined for our purposes. To fit the corrected arrival times, the functions of Eqs. (4.40) and (4.41) are rewritten as follows:

$$\begin{aligned} A(x) = {} & A_6 \frac{(x-x_7)(x-x_8)(x-x_9)(x-0)}{(x_6-x_7)(x_6-x_8)(x_6-x_9)(x_6-0)} \\ & + A_7 \frac{(x-x_6)(x-x_8)(x-x_9)(x-0)}{(x_7-x_6)(x_7-x_8)(x_7-x_9)(x_7-0)} \\ & + A_8 \frac{(x-x_6)(x-x_7)(x-x_9)(x-0)}{(x_8-x_6)(x_8-x_7)(x_8-x_9)(x_8-0)} \\ & + A_9 \frac{(x-x_6)(x-x_7)(x-x_8)(x-0)}{(x_9-x_6)(x_9-x_7)(x_9-x_8)(x_9-0)} \end{aligned} \tag{4.42}$$

and

$$A(y) = A_1 \frac{(y-y_2)(y-y_4)(y-y_5)(y-0)}{(y_1-y_2)(y_1-y_4)(y_1-y_5)(y_1-0)}$$

$$+ A_2 \frac{(y-y_1)(y-y_4)(y-y_5)(y-0)}{(y_2-y_1)(y_2-y_4)(y_2-y_5)(y_2-0)}$$

$$+ A_4 \frac{(y-y_1)(y-y_2)(y-y_5)(y-0)}{(y_4-y_1)(y_4-y_2)(y_4-y_5)(y-0)}$$

$$+ A_5 \frac{(y-y_1)(y-y_2)(y-y_4)(y-0)}{(y_5-y_1)(y_5-y_2)(y_5-y_4)(y_5-0)} \quad . \qquad (4.43)$$

The characteristic velocities for each explosion wave are given by:

$$\frac{1}{k_x} = a_1 = -\left(A_6 \frac{x_7\ x_8\ x_9}{(x_6-x_7)(x_6-x_8)(x_6-x_9)x_6} + A_7 \frac{x_6\ x_8\ x_9}{(x_7-x_6)(x_7-x_8)(x_7-x_9)x_7} + A_8 \frac{x_6\ x_7\ x_9}{(x_8-x_6)(x_8-x_7)(x_8-x_9)x_8} + A_9 \frac{x_6\ x_7\ x_8}{(x_9-x_6)(x_9-x_7)(x_9-x_8)x_9}\right) \qquad (4.44)$$

and

$$\frac{1}{k_y} = b_1 = -\left(A_1 \frac{y_2\ y_4\ y_5}{(y_1-y_2)(y_1-y_4)(y_1-y_5)y_1} + A_2 \frac{y_1\ y_4\ y_5}{(y_2-y_1)(y_2-y_4)(y_2-y_5)y_2} + A_4 \frac{y_1\ y_2\ y_5}{(y_4-y_1)(y_4-y_2)(y_4-y_5)y_4} + A_5 \frac{y_1\ y_2\ y_4}{(y_5-y_1)(y_5-y_2)(y_5-y_4)y_5}\right) \qquad (4.45)$$

Thus, the evaluation of one characteristic velocity consists of multiplying four corrected arrival times A_k by coefficients of the type

$$\frac{x_7\ x_8\ x_9}{(x_6-x_7)(x_6-x_8)(x_6-x_9)x_6} \ ,$$

which are determined by the geometry of the array, and adding the results.*

*The resulting coefficients are such that the arrival times at the interior microphones are weighted by a factor of about ten more heavily than the exterior microphones. It has since been demonstrated to the writer by Captain William R. Bandeen that this method of computing the characteristic velocities is

The presentation in this chapter has been aimed at describing the computational procedures for determining the characteristic velocities for the IGY series of experiments in Fort Churchill. However, the method is readily applicable in any array in the form of a cross (or a hexagon), provided the departures of the microphone locations from straight horizontal lines are not large.

The approach outlined here brings out the fact that, for a truly horizontal array, it is not necessary to record the temperature and winds at the array during the arrivals. And this holds true for winds determination in the case of an array with departures from the horizontal, unless the winds are very strong and the waves arrive at a relatively low elevation angle.

5. ROTATION OF THE CHARACTERISTIC VELOCITIES

The quantities k_x and k_y that are obtained from the arrival times at the microphones are the characteristic velocities in the horizontal directions along the lines of the microphones, as explained previously. For purposes of ray tracing, it becomes necessary to determine the characteristic velocities along different directions (in the case of the Churchill array, in the north and east directions). Therefore, the way of computing the characteristic velocity in an arbitrary direction in a horizontal plane, when k_x and k_y are given, will be presented now.

Consider Fig. 3. At $t = 0$ the wave front arrives at the origin (0;0). After ϵt the wave front arrives at points $(\epsilon x;0)$ and $(0;\epsilon y)$ on the x and y axis, respectively. Assuming that the wave front is smooth, in the limit for $\epsilon t \rightarrow 0$, the small portion of the wave can be assumed planar; and therefore at ϵt the wave will intersect the horizontal plane along a straight line passing through $(\epsilon x;0)$ and $(0;\epsilon y)$. The characteristic velocities along the x axis and the y axis are, respectively, $k_x = \epsilon x/\epsilon t$ and $k_y = \epsilon y/\epsilon t$. Along an arbitrary direction u the characteristic velocity k_u is given by $\epsilon u/\epsilon t$, where ϵu is the distance from the origin to the line of intersection at ϵt measured along the direction u.

The characteristic velocity in an arbitrary horizontal direction u is thus given by the distance along the direction u from the origin to the line joining the end points of k_x and k_y. It should be pointed out that x and y need not be perpendicular; but for greater accuracy of measurements, the angle should be close to 90°.

The maximum characteristic velocity is always infinite and in the direction parallel to the line joining k_x and k_y. The minimum characteristic velocity,

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inferior to the approach through which the arrival times at different microphones influence the determination of the characteristic velocity more equally. The differences can be significant.

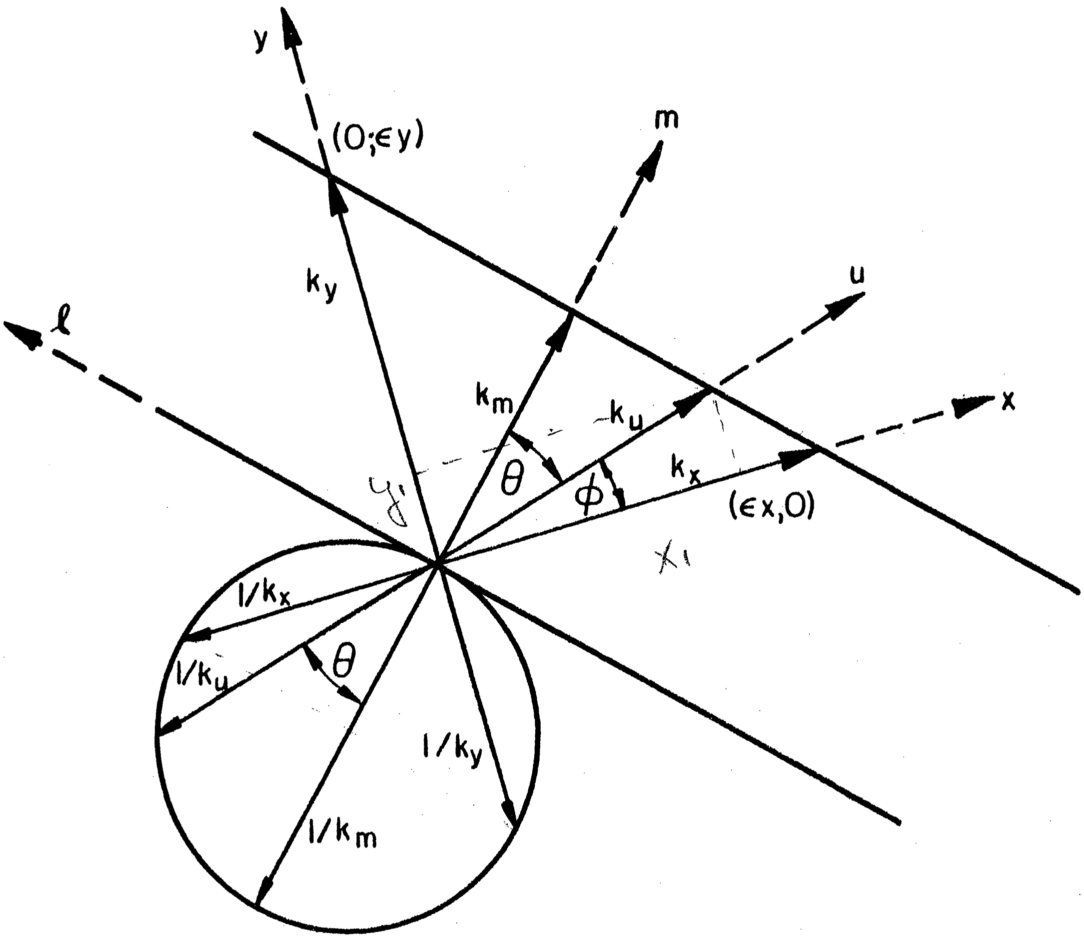


Fig. 3. Rotation of the characteristic velocities.

k_m, is in the direction perpendicular to the line joining end points k_x and k_y.

The geometrical relationships inherent in Fig. 3 are grasped more easily if it is realized that the end points of inverses of the characteristic velocities map on a circle. This is because

$$k_u = k_m/\cos\Theta \quad , \tag{5.1}$$

where Θ is the angle between the direction m of the minimum characteristic velocity and the direction u. Therefore the end points of $1/k_u$

$$1/k_u = \cos\Theta/k_m \tag{5.2}$$

are located on a circle with a diameter of $1/k_m$. It follows easily that, when x and y are perpendicular, the minimum characteristic velocity k_m is given by

$$1/k_m = (1/k_x)^2 + (1/k_y)^2 \tag{5.3}$$

or

$$k_m = \frac{k_x\, k_y}{\sqrt{k_x^2 + k_y^2}} \quad . \tag{5.4}$$

To find the magnitude of the characteristic velocity k_u in a direction u which is ϕ degrees counterclockwise from x, when k_x and k_y are given (x and y perpendicular), we note that the equation for the line joining the end points of k_x and k_y is:

$$y = -\frac{k_y}{k_x}\, x + k_y \tag{5.5}$$

and the equation for the line in the direction u is:

$$y = \tan\phi \cdot x \quad . \tag{5.6}$$

The intersection of the two lines occurs at a point with the coordinates x_1 and y_1 given by:

$$x_1 = \frac{k_y}{k_y/k_x + \tan\phi} = \frac{k_y\, k_x}{k_y + k_x \tan\phi} \tag{5.7}$$

and

$$y_1 = \frac{\tan\phi\; k_y\, k_x}{k_y + k_x \tan\phi} \quad . \tag{5.8}$$

The characteristic velocity k_u is given by

$$k_u = \sqrt{x_1^2 + y_1^2}$$

$$= \frac{k_y\ k_x}{k_y + k_x \tan \phi} \sqrt{1 + \tan^2\phi} = \frac{k_y\ k_x}{k_y \cos \phi + k_x \sin \phi} . \qquad (5.9)$$

This is the formula that is applied in the case of the Churchill array to yield the north and east characteristic velocities through a rotation by -21.71° (see Fig. 2).

6. THE LAW OF REFRACTION EXPRESSED IN COMPONENTS OF VELOCITIES IN A CARTESIAN COORDINATE SYSTEM

The refraction of the sound waves is caused by the changes in the velocity of propagation, i.e., temperature, and changes in the velocity of the medium, i.e., winds. The law will be stated in components of velocities in a Cartesian coordinate system. The different media of propagation consist of uniform horizontal layers. The vertical component of the wind is assumed zero.

The law of refraction, sometimes called Snell's law, is based on the premise that the characteristic velocity of a ray (a small portion of the wave front) in any direction parallel to the plane separating two media is the same in both media. The law can be derived from the Huygens principle from which it follows that a plane wave, after undergoing refraction in the plane separating two media, remains a plane wave.

The equations for the characteristic velocities in terms of the components of the velocity of sound and wind velocities can be arrived at from the basic Eq. (4.31) in the following manner:

$$1/k_x = \partial t_o/\partial x_o = \frac{v_x}{v_x\ p_x + v_y\ p_y + v_z\ p_z} = \frac{v_x}{v_x\ p_x + v_y\ p_y + v_z^2} \qquad (6.1)$$

and

$$1/k_y = \partial t_o/\partial y_o = \frac{v_y}{v_x\ p_x + v_y\ p_y + v_z\ p_z} = \frac{v_y}{v_x\ p_x + v_y\ p_y + v_z^2} . \qquad (6.2)$$

However, the above equations for the characteristic velocities will be derived here again from geometrical considerations depicted in Fig. 4.

Let A, B, and C be the intersections of the wave front with the x, y, and z axes, respectively, at t = 0. At 1 sec the wave intersects the x and y axes at points A* and B*, where A*B*||AB. The distance $\overline{DO}$ represents $\overline{V}$ the vector of sound velocity. The following relations hold:

$$\overline{AO} = \frac{v}{\cos \alpha} \qquad (6.30)$$

$$\overline{BO} = \frac{v}{\cos \beta} \qquad (6.31)$$

$$\overline{CO} = \frac{v}{\cos \gamma} \qquad (6.32)$$

$$\overline{OA^*} = w_x + w_y \frac{\overline{AO}}{\overline{BO}} = w_x + w_y \frac{\cos \beta}{\cos \alpha} \qquad (6.40)$$

$$\overline{OB^*} = w_y + w_x \frac{\overline{BO}}{\overline{AO}} = w_y + w_x \frac{\cos \alpha}{\cos \beta} \quad . \qquad (6.41)$$

The characteristic velocities are defined by:

$$k_x = \overline{AA^*} = \overline{AO} + \overline{OA^*}$$

$$= \frac{v}{\cos \alpha} + w_x + w_y \frac{\cos \beta}{\cos \alpha}$$

$$= \frac{v^2}{v_x} + w_x + w_y \frac{v_y}{v_x} = \frac{v^2 + w_x v_x + w_y v_y}{v_x}$$

$$= \frac{v_z^2 + (v_x + w_x) v_x + (v_y + w_y) v_y}{v_x}$$

$$= \frac{v_z^2 + p_x v_x + p_y v_y}{v_x} \qquad (6.5)$$

and similarly

$$k_y = \overline{BB^*} = \overline{BO} + \overline{OB^*}$$

$$= \frac{v}{\cos \beta} + w_y + w_x \frac{\cos \alpha}{\cos \beta} = \frac{v^2}{v_y} + w_y + w_x \frac{v_x}{v_y}$$

$$= \frac{v^2 + w_x v_x + w_y v_y}{v_y} = \frac{v_z^2 + p_x v_x + p_y v_y}{v_y} \quad . \qquad (6.6)$$

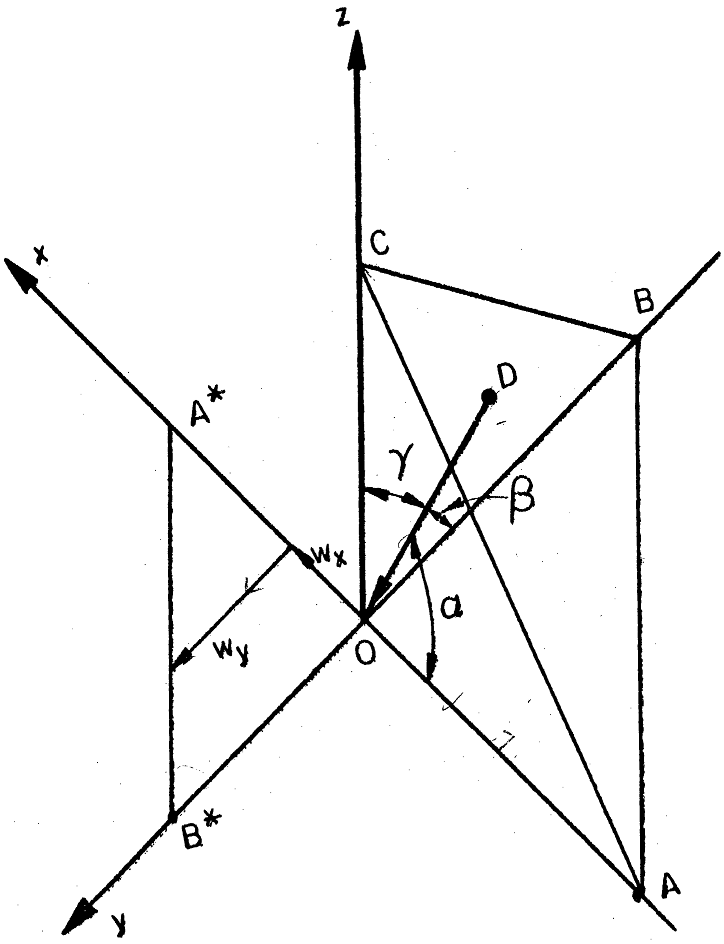


Fig. 4. Characteristic velocities in a Cartesian coordinate system.

The same results have been obtained in Eqs. (6.1) and (6.2). The above two equations constitute the basic equations of refraction we will use.

7. EQUATIONS FOR TRACING THE SOUND RAY THROUGH A LAYER WITH KNOWN METEOROLOGICAL PARAMETERS

In this chapter the equations are presented for tracing the sound ray through a layer with known meteorological parameters. The horizontal axes are taken to be in the north and east directions.

The knowledge of meteorological parameters implies the knowledge of the wind components w_n and w_e and the knowledge of the velocity of sound v. (The formula by which v is computed as a function of temperature is given in Chapter 9.) The characteristic velocities of the ray, k_n and k_e, are known. The basic equations for finding the components of v, i.e., v_n, v_e, and v_z are Eqs. (4.17), (6.5), and (6.6). The equations are presented here again for convenience, in the north and east horizontal coordinates.

$$v^2 = v_n^2 + v_e^2 + v_z^2 \tag{4.17}$$

$$k_n = \frac{v_z^2 + p_n v_n + p_e v_e}{v_n} = \frac{v^2 + v_n w_n + v_e w_e}{v_n} \tag{6.5}$$

$$k_e = \frac{v_z^2 + p_n v_n + p_e v_e}{v_e} = \frac{v^2 + v_n w_n + v_e w_e}{v_e} . \tag{6.6}$$

It follows that

$$\frac{v_n}{v_e} = \frac{k_e}{k_n} . \tag{7.0}$$

Equations (6.5) and (6.6) can be rewritten

$$v_n (k_n - w_n) - v_e w_e = v^2 \tag{7.1}$$

$$v_e (k_e - w_e) - v_n w_n = v^2 . \tag{7.2}$$

Introducing the value of v_e from Eq. (7.0) into Eq. (7.1), we obtain

$$v_n (k_n - w_n) - v_n \frac{k_n}{k_e} w_e = v^2 , \tag{7.3}$$

or

$$v_n = \frac{v^2}{k_n - w_n - (k_n/k_e) w_e} , \tag{7.4}$$

and in a similar manner, introducing the value of v_n from Eq. (7.0) into Eq. (7.2), we obtain

$$v_e = \frac{v^2}{k_e - w_e - (k_e/k_n)w_n} \quad . \tag{7.5}$$

Equations (7.4) and (7.5) are used for the determination of the components v_n and v_e. Subsequently v_z is determined by means of Eq. (4.17).

The following can be noted here. Two perpendicular directions, ℓ and m, can be chosen so that k_ℓ is infinite and k_m is minimum. This is shown graphically in Fig. 3. If a set of equations equivalent to (7.4), (7.5), and (4.17) is written for directions m, ℓ, and z, we have:

$$v_m = \frac{v^2}{k_m - w_m} \tag{7.60}$$

$$v_\ell = 0 \tag{7.61}$$

$$v_z^2 = v^2 - v_m^2 \quad . \tag{7.62}$$

These equations are simpler. However, this approach, although theoretically attractive, is rather cumbersome, since the rotation has to be effected through a different angle (and the angle has to be computed first) for each grenade explosion. Therefore the north and east coordinates were used.

Once the velocity components are determined, the path of the ray is traced by the following equations.

The time Δt in the layer is

$$\Delta t = \frac{\Delta z}{v_z} \quad . \tag{7.7}$$

The north-south direction traveled, Δe, is

$$\Delta e = \Delta t \cdot (v_e + w_e) = \Delta t \cdot p_e \tag{7.8}$$

and the east-west direction traveled, Δn, is

$$\Delta n = \Delta t \cdot (v_n + w_n) = \Delta t \cdot p_n \quad . \tag{7.9}$$

In the above, p_n and p_e denote the components of the vector of propagation p in the north and east directions.

8. EQUATIONS FOR METEOROLOGICAL PARAMETERS IN A LAYER WITH A KNOWN SOUND-RAY PATH

Once the ray originating from the i-th explosion has been traced from the microphone array up to the altitude of the lower, i-1, explosion, the path of the ray between the upper and lower explosion becomes known.

In the following, n_i and e_i denote, respectively, the north and east coordinates of the explosion relative to the center microphone and t_i denotes the determined time of sound propagation from the explosion to the center microphone. The quantities Δt, Δn, and Δe, when reference is made to layers with unknown meteorological parameters, are indexed as follows: $(\Delta t)_i$ $(\Delta n)_i$, and $(\Delta e)_i$, where the index refers to the upper explosion of the layer.

$$(\Delta t)_i = t_i - \sum^{i-1} \Delta t \tag{8.1}$$

$$(\Delta n)_i = n_i - \sum^{i-1} \Delta n \tag{8.2}$$

$$(\Delta e)_i = e_i - \sum^{i-1} \Delta e \tag{8.3}$$

The summation in each case extends for layers from the ground (microphone array) up to the altitude of the lower, i-1, explosion. The thickness of the layer $(\Delta z)_i$ is known. We have for the layer between the explosions

$$v_z = \frac{(\Delta z)_i}{(\Delta t)_i} \tag{8.4}$$

$$p_n = v_n + w_n = \frac{(\Delta n)_i}{(\Delta t)_i} \tag{8.5}$$

$$p_e = v_e + w_e = \frac{(\Delta e)_i}{(\Delta t)_i} \quad . \tag{8.6}$$

The quantities p_n and p_e become known, but the quantities v_n, w_n, and v_e are determined first on the basis of Eqs. (6.5) and (6.6), which are restated here.

$$v_n = \frac{v_z^2 + v_n p_n + v_e p_e}{k_n} \tag{6.5}$$

and

$$v_e = \frac{v_z^2 + v_n p_n + v_e p_e}{k_e} \; . \qquad (6.6)$$

It follows that

$$v_n (k_n - p_n) - v_e p_e = v_z^2 \qquad (8.1)$$

$$v_e (k_e - p_e) - v_n p_n = v_z^2 \; . \qquad (8.2)$$

Substituting the value of v_e from Eq. (7.0) into Eq. (8.1), we obtain

$$v_n (k_n - p_n) - v_n \frac{k_n}{k_e} p_e = v_z^2 \; , \qquad (8.3)$$

and therefore

$$v_n = \frac{v_z^2}{k_n - p_n - p_e (k_n/k_e)} \; . \qquad (8.4)$$

In a similar manner, introducing the value of v_n from Eq. (7.0) into Eq. (8.2), we obtain

$$v_e = \frac{v_z^2}{k_e - p_e - p_n (k_e/k_n)} \; . \qquad (8.5)$$

Since the velocity components v_n, v_e, and v_z are known, the velocity v can be determined, Eq. (4.17), and hence the temperature, Eq. (9.2).

The wind components can be now determined from the following equations:

$$w_n = p_n - v_n \qquad (8.6)$$

and

$$w_e = p_e - v_e \; . \qquad (8.7)$$

9. THE DETERMINATION OF TEMPERATURE FROM THE VELOCITY OF SOUND

The temperature of the layer between two successive explosions is determined by means of the formula*

*B. Gutenberg, "Sound Propagation in the Atmosphere," Compendium of Meteorology, edited by Thomas F. Malone, American Meteorological Society, Boston, 1951, pp. 366-375.

$$v = 20.06 \sqrt{T} \qquad (9.1)$$

or

$$T = \frac{v^2}{402.4} , \qquad (9.2)$$

where temperature is measured in °K and velocity in meters per second. Considerable departures from this formula occur at both low and high temperatures. Since temperatures as low as 165°K are involved in the data reduction, consideration was given to the formula suggested by Quigley:*

$$v^2 = 387.62\, T + 180430\, T^{-1} - 20364000\, T^{-2} + 806 + 0.03007\, T^2 . \qquad (9.3)$$

If use is made of Eq. (9.3), the determined temperatures would be higher by about 1.5°. Formula (9.3) is possibly more accurate in the range of temperatures involved.

It appears that the velocity of sound changes by a negligible amount only because of the change of density. This is gathered first of all from the following calculation.

Use is made of a formula developed by R. B. Lindsay for the phase velocity v_{ph} of sound at low pressures.** The formula, in a much simplified approximate form, reads as follows:

$$v_{ph} = v\,(1 + 3/8\, \omega^2 \tau^2) , \qquad (9.3)$$

where ω is the angular frequency and τ is the relaxation constant.

The group velocity, v_{group}, can be obtained from the phase velocity by the formula

$$\frac{1}{v_{group}} = \frac{d\left(\frac{\omega}{v_{ph}}\right)}{d\omega} , \qquad (9.4)$$

and therefore

$$\frac{1}{v_{group}} = \frac{d\left[\frac{\omega}{v\,(1 + 3/8 \cdot \omega^2\tau^2)}\right]}{d\omega} \cong \frac{1}{v}\, \frac{d(\omega - 3/8\, \omega^3\tau^2)}{d\omega} = \frac{1}{v}\,(1 - 9/8\, \omega^2\tau^2) \qquad (9.5)$$

*T. H. Quigley, "An Experimental Determination of the Velocity of Sound in Dry CO_2-Free Air and Methane at Temperatures Below the Ice Point," Phys. Rev., 67, 298-303 (1945).

**R. B. Lindsay, "Transmission of Sound Through Air at Low Pressure," Am. J. Phys., 16, 371 (October, 1948). The formula referred to is Eq. (16), p. 372.

$$v_{group} \cong v\,(1 + 9/8\,\omega^2\tau^2) \quad . \qquad (9.6)$$

Taking $\tau = 10^{-10}$ second for air at normal atmospheric pressure and room temperature, and assuming that τ changes inversely with pressure (temperature change and a possible change in γ are neglected), we observe that for 95-km altitude

$$\tau = 10^{-10}/10^{-6} = 10^{-4} \text{ second} \quad .$$

For frequencies of under 50 cps, the group velocity will depart by less than

$$9/8\,(2\pi\, f \cdot \tau)^2 = 9/8\,(2\pi\, 50 \cdot 10^{-4})^2 \cong 10^{-3} = 0.1\%$$

from the velocity of sound under normal atmospheric pressure and under the same temperature.

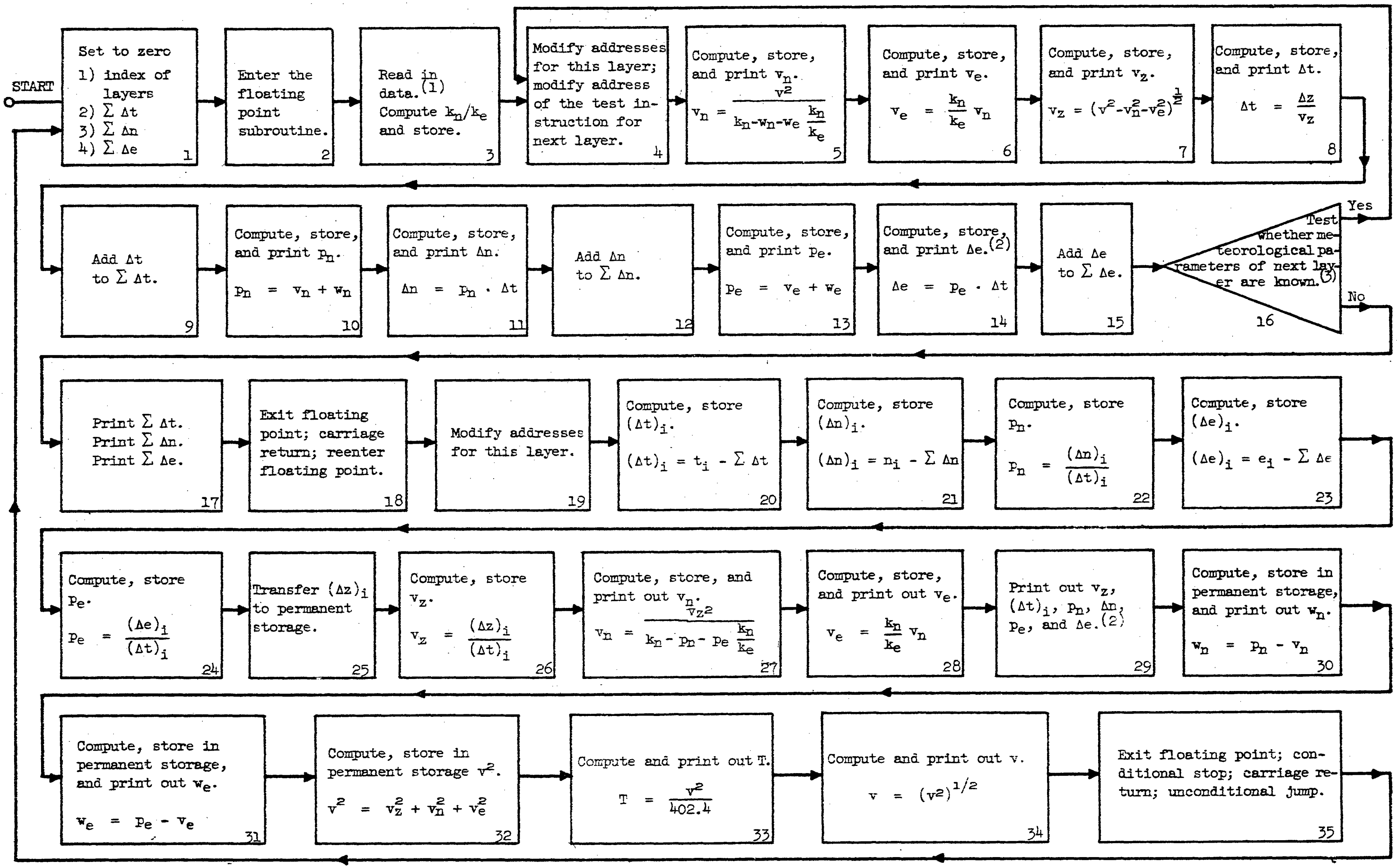
This conclusion, that rarefication by a factor of 10^6 has very little effect on sound velocity for $f < 50$ cps, is to a certain extent borne out by experimental evidence from measurements of propagation of very-high-frequency sound under the conditions of normal pressure. Such a propagation resembles, in the ratio of the wavelength to the mean free path of molecules in the medium, the conditions of high-altitude propagation with which we are concerned. Experiments indicate that the velocity is practically independent of frequency.

10. DIGITAL COMPUTER PROGRAM

The computation of winds and temperatures has been carried out on the LGP-30 computer. This is a desk-size general-purpose digital computer with 4096-words magnetic-drum storage. Floating-point subroutine has been used. The program consists of 170 instructions. The computer-solution time for a rocket flight has been from 2 to 3-1/2 hours, depending on the number of grenades and the number of balloon layers.

The flow diagram of the program, including the appropriate equations, is presented in Fig. 5. The flow diagram is presented in detail, since it is relatively easy to rewrite such a program for any general-purpose computer. It should be pointed out that the program will be somewhat simpler for computers with index registers.

A backward-propagation convention has been followed, i.e., the velocities and distances are taken positive in propagating from the microphone array to the explosion. By this convention the meteorological notation of winds results, i.e., if the north and east are positive directions, the winds components will be positive if blowing from east and north, respectively.



(1) The quantities k_n, k_e, $(\Delta z)_i$, n_i, e_i, and t_i are read each time. Moreover, in the first reading, data about balloon layers are read.

(2) An automatic carriage return occurs after the printout.

(3) Negative numbers are initially stored in the block containing v^2 of the layers. The test instruction consists of testing whether the number is positive or negative.

Fig. 5. The flow diagram for the computer program.

11. CALCULATIONS AND RESULTS FOR SM1.01 AND SM1.02 ROCKET FLIGHTS

Aerobee SM1.01, carrying eighteen 4-lb grenades, was fired at 0548 CST on November 12, 1956. Grenades nos. 8 and 18 failed to explode. Grenade no. 15 provided very weak arrival signals, making accurate determination of arrival times very difficult. The weak arrivals could be interpreted as caused by a subnormal detonation of the explosive. In such a case, the finite-amplitude propagation could introduce considerable errors. For these two reasons it has been decided to omit the data on grenade no. 15 and to consider a single layer between explosions nos. 14 and 16.

Aerobee SM1.02, carrying nineteen grenades (of which the first twelve were 2-lb and seven were 4-lb), was fired at 2216 CST on July 21, 1957. Grenade no. 14 failed to explode, and the arrivals from grenade no. 19 were masked by the rocket re-entry wave.

The input data to the computer, consisting of north and east coordinates n_i and e_i of the explosion relative to the center microphone, layer thickness $(\Delta z)_i$ between the explosions, time of sound propagation t_i,* and characteristic velocities k_n and k_e are presented in Table I for SM1.01 and in Table II for SM1.02. The balloon data for SM1.01, where 21 balloon layers have been used, are omitted, since they are thought to be almost completely immaterial to the solution (see Chapter 12). The data on five balloon layers for SM1.02 are presented in Table III to facilitate understanding of the computer tracing of the sound rays, which is presented for the first three explosions of SM1.02 in Table IV.

The computed temperatures and winds are presented in Tables V and VI and in Figs. 6 and 7 for SM1.01 and SM1.02, respectively.

12. COMMENTS ON THE EXPECTED SOURCES OF ERRORS

The discussion in this chapter on systematic errors is kept short relative to the importance of the question since there already is an ample analysis** of the accuracy of the experiment. A few remarks are presented here on points which the writer has considered in more detail or points which are especially significant in the suggested method of data reduction.

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*The time of sound propagation is computed from $t_i = t_m + t_g$. See the footnote on page 6.

**W. Nordberg, A Method of Analysis for the Rocket-Grenade Experiment, Tech. Memo NR. M-1856, U.S. Army Signal Engineering Laboratories, Forth Monmouth, N. J., February, 1957 (now the U.S. Army Research and Development Laboratory).

TABLE I

SM1.01 DATA

i	z_i meters	n_i meters	e_i meters	$(\Delta z)_i$ meters	t_m sec	t_g sec	t_i sec	k_n m/sec	k_e m/sec
1	25178.8	7721.4	-2165.1	5721.1	86.582	.064	86.646	1281.7	-23840.6
2	28375.9	6973.1	-2503.7	3197.1	96.828	.074	96.902	1630.7	-23200.5
3	31421.5	6233.4	-2840.2	3045.6	106.589	.084	106.673	2091.5	-24603.7
4	34384.3	5485.3	-3180.5	2962.8	116.184	.094	116.278	2726.3	-28426.7
5	37163.3	4750.0	-3512.6	2779.0	125.080	.105	125.185	3619.6	-23239.1
6	39792.7	4028.0	-3840.8	2629.4	133.534	.116	133.650	5059.0	-24907.8
7	42337.4	3296.9	-4174.1	2544.8	141.509	.127	141.636	7902.4	-26743.2
9	47068.3	1835.8	-4841.1	4730.9	156.415	.150	156.565	37088.4	-73549.8
10	49213.4	1119.7	-5170.3	2145.1	162.963	.161	163.124	-52494.1	184457.1
11	51300.5	386.0	-5507.2	2087.1	169.390	.173	169.563	-29594.5	-114412.8
12	53252.2	-343.0	-5842.6	1951.7	175.499	.186	175.685	-24151.7	26916.8
13	55044.6	-1054.9	-6171.6	1792.4	181.022	.200	181.222	-10400.2	-152779.7
14	56762.7	-1782.2	-6506.6	1718.1	186.277	.215	186.492	-8081.5	-179810.3
16	59820.3	-3226.4	-7175.1	3057.6	195.939	.248	196.187	-4752.1	19175.0
17	61181.9	-3950.5	-7511.1	1361.7	200.346	.264	200.610	-3952.9	19613.0

TABLE II

SM1.02 DATA

i	z_i meters	n_i meters	e_i meters	$(\Delta z)_i$ meters	t_m sec	t_g sec	t_i sec	k_n m/sec	k_e m/sec
1	26203.2	10570.0	-1794.8	5503.2	91.300	57	91.357	857.4	-5613.1
2	29658.8	10238.8	-2020.8	3461.8	101.625	65	101.690	982.1	-5446.9
3	33141.3	9896.7	-2254.7	3482.6	112.019	75	112.094	1122.4	-5046.6
4	36654.6	9543.2	-2499.1	3513.3	122.468	85	122.553	1280.5	-4542.4
5	40104.8	9185.2	-2747.7	3450.2	132.656	96	132.752	1447.1	-4180.9
6	43629.5	8806.8	-3011.8	3524.7	142.871	108	142.979	1656.4	-3975.6
7	47101.1	8421.1	-3281.8	3471.6	152.906	121	153.027	1908.3	-3951.4
8	50457.5	8033.5	-3554.0	3356.4	162.604	136	162.740	2125.7	-3790.9
9	53877.1	7625.0	-3844.5	3419.7	172.629	155	172.784	2391.3	-3689.7
10	57322.0	7194.2	-4149.7	3445.0	182.893	179	183.072	2718.5	-3442.4
11	60766.0	6742.1	-4473.4	3444.0	193.333	210	193.543	3130.8	-3268.7
12	64241.5	6260.4	-4815.1	3475.5	204.262	247	204.509	3531.1	-2986.7
13	67661.1	5756.9	-5175.2	3419.6	215.304	360	215.664	4326.7	-2895.9
15	74579.6	4627.2	-5996.0	6918.5	239.370	530	239.900	6135.1	-2487.7
16	78029.6	3983.4	-6458.1	3449.9	251.825	625	252.450	7747.2	-2432.4
17	81529.7	3248.4	-6995.3	3500.1	265.307	752	266.059	11493.2	-2228.0
18	84937.3	2418.3	-7592.9	3407.6	278.814	900	279.714	35515.4	-2080.4

TABLE III

SM1.02 BALLOON-LAYERS DATA

Balloon Layer	Layer Thickness m	Velocity of Sound, Squared $(m/sec)^2$	North-Wind Component, w_n, m/sec	South-Wind Component, w_e, m/sec
I	700	115000	- 12.0	- 2.0
II	6000	108000	6.0	3.0
III	7000	100000	9.0	2.0
IV	7000	90000	2.0	- 8.0

TABLE IV

COMPUTER OUTPUT: TRACING OF THE SOUND RAY*

v_n		v_e		v_z		Δt		P_n		Δn		P_e		Δe	
.1323254	03	.2021201-	02	.3115789	03	.2246622	01	.1203254	03	.2703257	03	.2221201-	02	.4990198-	02
.1267854	03	.1936579-	02	.3025730	03	.1982992	02	.1327854	03	.2633124	04	.1636579-	02	.3245324-	03
.1178300	03	.1799790-	02	.2929030	03	.2389870	02	.1268300	03	.3031071	04	.1599790-	02	.3823290-	03
.1053675	03	.1609433-	02	.2804258	03	.2496203	02	.1073675	03	.2680112	04	.2409433-	02	.6014436-	03
$\Sigma\Delta t$		$\Sigma\Delta n$		$\Sigma\Delta e$											
.7093728	02	.8614632	04	.1358207-	04										
v_n		v_e		v_z		$(\Delta t)_1$		P_n		$(\Delta n)_1$		P_e		$(\Delta e)_1$	
.9577752	02	.1462951-	02	.2695049	03	.2041966	02	.9575908	02	.1955368	04	.2138105-	02	.4365937-	03
w_n		w_e		T		v									
.1843262	01-	.6751542-	01	.2038284	03	.2863917	03								
v_n		v_e		v_z		Δt		P_n		Δn		P_e		Δe	
.1157216	03	.2086574-	02	.3180772	03	.2200723	01	.1037216	03	.2282625	03	.2286574-	02	.5032117-	02
.1105803	03	.1993871-	02	.3088275	03	.1942832	02	.1165803	03	.2264959	04	.1693871-	02	.3290908-	03
.1027237	03	.1852208-	02	.2985042	03	.2345025	02	.1117237	03	.2619948	04	.1652208-	02	.3874470-	03
.9196037	02	.1658136-	02	.2850760	03	.2455485	02	.9396037	02	.2307183	04	.2458136-	02	.6035917-	03
.8361510	02	.1507662-	02	.2734985	03	.2012150	02	.8359667	02	.1682090	04	.2182817-	02	.4392154-	03
$\Sigma\Delta t$		$\Sigma\Delta n$		$\Sigma\Delta e$											
.8975564	02	.9102441	04	.1809666-	04										
v_n		v_e		v_z		$(\Delta t)_2$		P_n		$(\Delta n)_2$		P_e		$(\Delta e)_2$	
.9521309	02	.1716786-	02	.2900714	03	.1193430	02	.9521791	02	.1136359	04	.1769141-	02	.2111346-	03
w_n		w_e		T		v									
.4814148	02-	.5235538-	00	.2323610	03	.3057804	03								
v_n		v_e		v_z		Δt		P_n		Δn		P_e		Δe	
.1014172	03	.2255529-	02	.3228092	03	.2168463	01	.8941718	02	.1938978	03	.2455529-	02	.5324725-	02
.9668391	02	.2150261-	02	.3133526	03	.1914776	02	.1026839	03	.1966167	04	.1850261-	02	.3542835-	03
.8978114	02	.1996742-	02	.3025568	03	.2313615	02	.9878114	02	.2285415	04	.1796742-	02	.4156970-	03
.8045803	02	.1789395-	02	.2884550	03	.2426721	02	.8245803	02	.2001026	04	.2589395-	02	.6283742-	03
.7317408	02	.1627400-	02	.2764072	03	.1990975	02	.7315564	02	.1456511	04	.2302554-	02	.4584328-	03
.8331601	02	.1852957-	02	.2936269	03	.1178979	02	.8332083	02	.9823352	03	.1905313-	02	.2246324-	03
$\Sigma\Delta t$		$\Sigma\Delta n$		$\Sigma\Delta e$											
.1004191	03	.8885350	04	.2134667-	04										
v_n		v_e		v_z		$(\Delta t)_3$		P_n		$(\Delta n)_3$		P_e		$(\Delta e)_3$	
.8610088	02	.1914893-	02	.2982985	03	.1167488	02	.8662603	02	.1011349	04	.1028128-	02	.1200327-	03
w_n		w_e		T		v									
.5251541	00	.8867650	01	.2404633	03	.3110660	03								

*The explanatory notation has been added in this table; it is not printed by the computer. Floating-point notation is used in such a way that .1323254 03 designates 132.3254. The units are m/sec for velocities, sec for time, m for distance, and °K for temperature.

TABLE V

SM1.01 RESULTS

Layer	Mean Layer Altitude Above MSL, meters	Layer Thickness $(\Delta z)_i$, meters	Temperature, °K	North-Wind Component, w_n, m/sec	East-Wind Component, w_e, m/sec	Total Wind, m/sec	Counterclockwise Wind Azimuth, deg
1-2	26821	3197.1	207.9	6.7	- 28.0	28.8	256.4
2-3	29942	3045.6	215.7	5.1	- 32.4	32.8	261.1
3-4	32946	2962.8	217.8	1.9	- 37.1	37.2	267.1
4-5	35817	2779.0	227.8	- 1.6	- 24.2	24.3	273.9
5-6	38521	2629.4	229.1	0.8	- 38.6	38.6	268.7
6-7	41106	2544.8	244.1	2.7	- 41.9	42.0	266.3
7-9	44746	4730.9	246.1	-15.5	- 63.4	65.2	283.7
9-10	48184	2145.1	265.5	- 7.7	- 91.8	92.2	274.8
10-11	50300	2087.1	261.6	-76.4	- 18.7	78.7	346.3
11-12	52320	1951.7	253.9	-58.2	- 81.0	99.7	305.7
12-13	54192	1792.4	265.5	0.1	- 35.3	35.3	269.3
13-14	55947	1718.1	269.8	-37.2	- 66.4	76.1	299.3
14-16	58335	3057.6	263.0	27.9	-181.5	183.6	261.3
16-17	60544	1361.7	257.7	39.7	- 77.7	87.3	243.0

TABLE VI

SM1.02 RESULTS

Layer	Mean Layer Altitude Above MSL, meters	Layer Thickness $(\Delta z)_i$, meters	Temperature, °K	North-Wind Component, w_n, m/sec	East-Wind Component, w_e, m/sec	Total Wind, m/sec	Counterclockwise Wind Azimuth, deg
1-2	27971	3461.8	232.4	0.0	- 0.5	0.5	90.5
2-3	31443	3482.6	240.5	0.5	8.9	8.9	93.4
3-4	34941	3513.3	249.6	1.0	19.3	19.3	92.9
4-5	38423	3450.2	260.1	- 2.9	21.0	21.2	82.1
5-6	41910	3524.7	270.2	6.3	16.2	17.4	111.2
6-7	44408	3471.6	272.1	13.5	3.3	13.9	166.2
7-8	48823	3356.4	284.6	- 9.8	18.0	20.5	61.4
8-9	52210	3419.7	276.4	-27.6	12.5	12.8	77.5
9-10	55643	3445.0	271.8	2.8	33.8	33.9	94.7
10-11	59087	3444.0	263.0	8.2	27.8	29.0	106.7
11-12	62547	3475.5	252.8	- 7.7	53.2	53.7	81.8
12-13	65994	3419.6	228.8	29.2	18.5	34.6	147.6
13-15	71163	6918.5	208.5	0.7	49.7	49.7	90.9
15-16	76348	3449.9	189.5	2.7	11.4	11.7	103.1
16-17	79823	3500.1	174.0	16.1	59.8	61.9	105.1
17-18	83277	3407.6	164.0	48.0	47.0	67.1	135.6

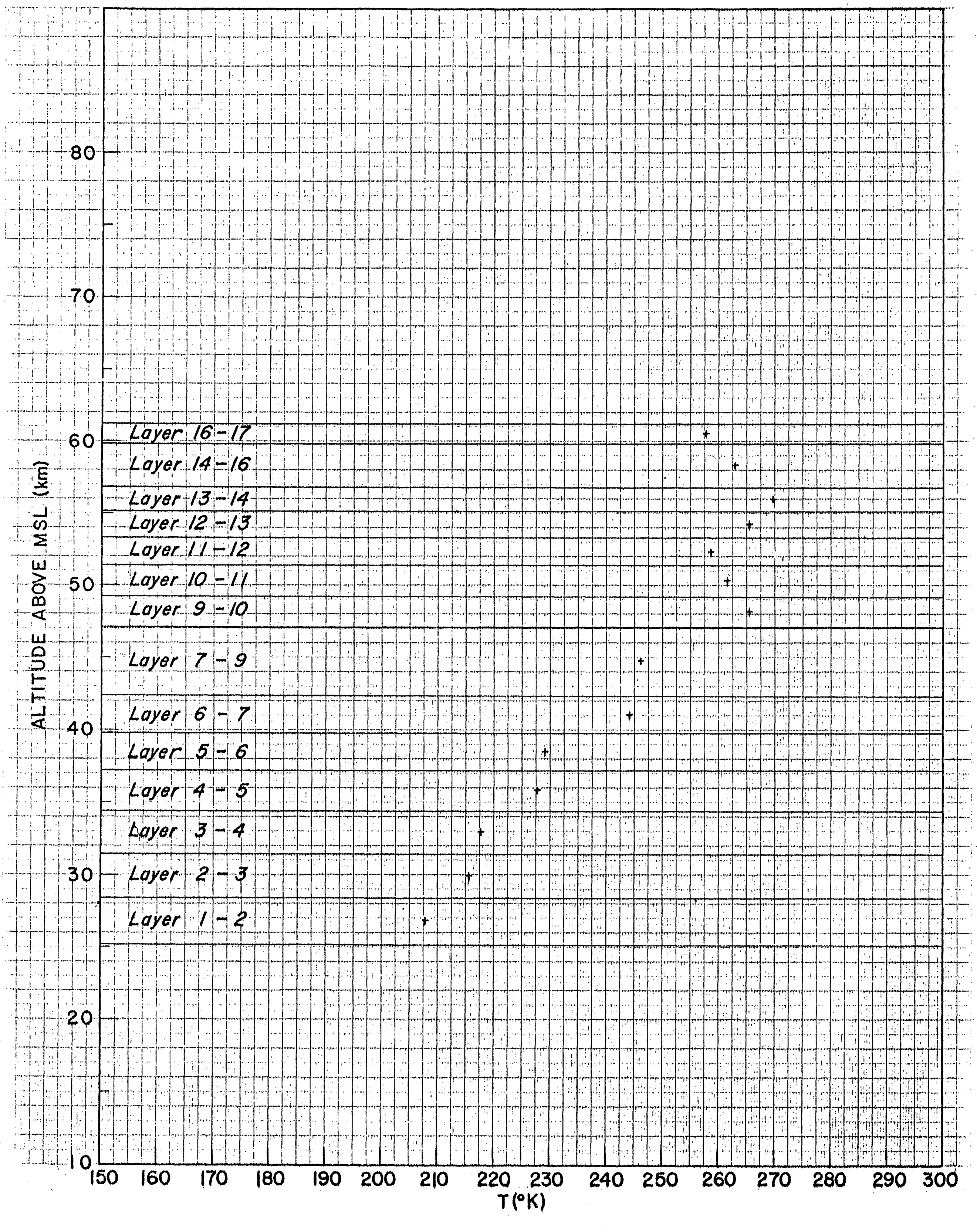

Fig. 6. SM1.01 results.

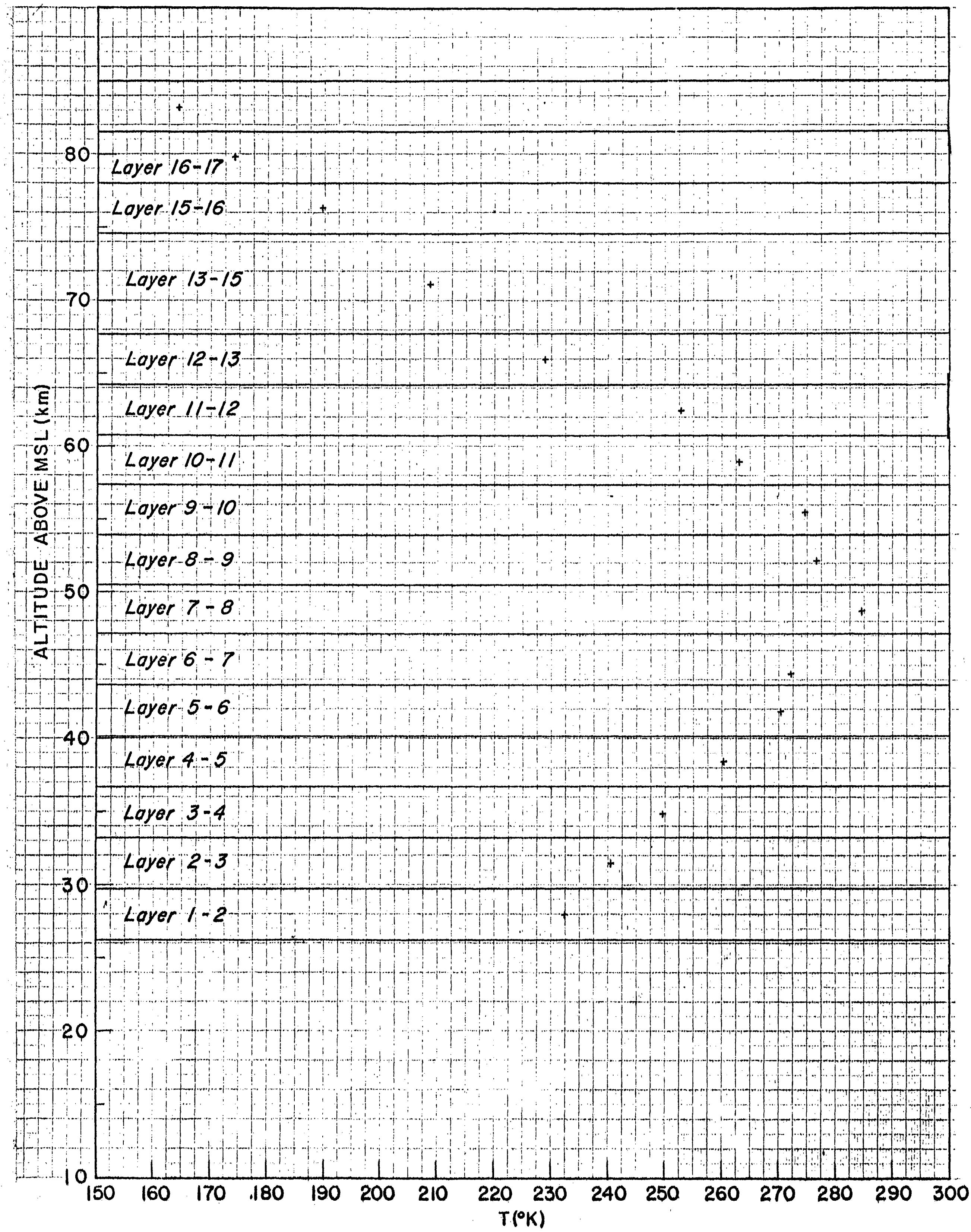

Fig. 7. SM1.02 results.

In the method described it is assumed that the temperature and horizontal winds are constant in a layer. In this connection the following can be said: if (a) an atmosphere finely divided into relatively thin uniform layers with parameters changing slowly from layer to layer is assumed, or if (b) a uniform atmosphere is assumed of the same thickness and of average parameters that represent the same travel time and the same horizontal travel distance for a sound ray with characteristic velocities $\overset{*}{k}_n$ and $\overset{*}{k}_e$, then a ray with characteristic velocities $k_n \cong \overset{*}{k}_n$ and $k_e \cong \overset{*}{k}_e$ will spend practically the same interval of time to traverse atmosphere (a) and to traverse atmosphere (b); and the horizontal distances traveled will be practically the same.

This statement has been investigated by means of computer programs. For instance, in one case (SM1.01), the tracing up to the zero grenade explosion was carried out through 21 thin balloon layers. Subsequently, the zero explosion has been taken at the center microphone level; i.e., a uniform layer extending from the ground up to the first explosion has been assumed. The temperatures and winds obtained on the basis of those two assumptions differed by less than a fraction of a degree and less than 0.5 m/sec. Similar results were obtained in SM1.02, where only five balloon layers were used in the first place. These computations indicate that no loss of accuracy occurs if balloon data are inaccurate or are unavailable. Layers of standard atmosphere can be conveniently used.

Moreover, it is the writer's contention that the above statement will still hold true if (a) is an atmosphere with linearly varying parameters. Thus data reduction based on linearly varying temperature and winds in a layer will not improve the accuracy of the results.

The question of vertical winds, discussed by Nordberg, seems to be of special interest in view of the fact that Groves* in England proposes to compute vertical winds by means of the rocket-grenade experiment. In his method, data from four widely spaced microphones are used to determine the three-velocity components and the speed of sound. The microphones must not lie in a straight line, and the times of travel to different microphones must not be all equal, i.e., the microphones must not be placed in a circle centered below the explosion. Groves plans to use more than four microphones to increase the accuracy of the experiment by deriving a least-square solution.

A point which has not been discussed by Nordberg is the assumption that the conditions remain constant through the experiment. The difference between consecutive arrivals is of the order of 15-20 sec. It seems possible that conditions of propagation, especially winds, might change somewhat in the low atmosphere. No real estimate of the possible errors is currently thought feasible.

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*G. V. Groves, "Theory of the Rocket-Grenade Method of Determining Upper-Atmosphere Properties by Sound Propagation," J. Atmos. Terres. Phys., 8, 189-203 (1956).

The question of finite-amplitude propagation (FAP for short) has been discussed by the writer in a technical report.* It will be pointed out here that the suggested correction, which can amount to about 5° for high-altitude layers, is thought to be known with an accuracy of possibly 30%. Thus an error of about ± 1.5° (again, only for high-altitude layers) is probable even after the correction has been carried out.

The analysis of the effect of random measurement errors on determination of the temperatures and winds based on the simplified method has not been concluded and may be presented in another technical report. One point will be made here relative to the importance of measuring the ground temperature.

In the currently used method,** the angle of elevation at the ground is computed, using the times of arrival and the velocity of sound at the ground. Both the arrival-times measurements and the temperature measurements contribute to the error in the elevation angle. (The errors in the determination of the array dimensions are negligible.) The error in the elevation angle contributes to the errors in temperature and winds.

Using the concept of characteristic velocities, the ground temperature needs to be determined only to correct for departures of the array from the horizontal (see Chapter 4). These corrections are of the order of 10 msec at most; thus, the error in the determination of the velocity of sound at the ground, which is of the order of 0.3%, does not contribute to the error in temperature and winds.

13. CONCLUSIONS

The suggested method seems to offer the advantage of being simpler, both conceptually and computationally, than the current method of data reduction: the refraction due to changes in temperature and winds is taken into account simultaneously. However, it does not offer the advantage of determining the vertical winds; and in this respect it is lacking when compared to the method suggested by Groves. It is the writer's opinion that Groves' method, even though it has disadvantages in that the computations are much more elaborate and the possible variations in the horizontal direction penalize the accuracy of the results, is an extremely interesting development. The first results of the British experiment, in which Groves' method of data reduction will be used, will be available in the near future; and the method can be assessed more conveniently then.

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*See footnote on page 6.

**Nordberg, op. cit., p. 28.

14. ACKNOWLEDGMENT

The rocket-grenade experiment has been conducted jointly by the U. S. Army Research and Development Laboratory, Fort Monmouth, New Jersey, and The University of Michigan under Contract No. DA-36-039 SC-64659. The temperatures and winds from SM1.01 and SM1.02 rocket flights were obtained as the result of the joint effort, and should by no means be regarded as obtained solely by the author of this report. The University of Michigan personnel participating in the experiment are listed on page v. The U. S. Army Research and Development Laboratory group has been headed by Mr. William G. Stroud and included Dr. Wilhelm Nordberg and Captain William R. Bandeen. All of them participated in all the stages of the experiment.