ANALYSIS OF A FALLING SPHERE EXPERIMENT FOR MEASUREMENT OF UPPER-ATMOSPHERE DENSITY AND WINDS

by

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1. General Discussion

1.1 Purpose of Study

The research program discussed in this report is being conducted under Contract Number AF 19(604)-5205 for the Geophysics Research Directorate of the Air Force Cambridge Research Center. The purpose of the program is to study methods of conducting an experiment in which measurements of air density and winds at high altitudes are carried out by means of a falling sphere ejected from a high-altitude rocket. The measurement of density and winds would be accomplished by measuring the drag acceleration of the sphere by means of accelerometers mounted on an inertial platform carried in the sphere. The study is intended to establish the range of altitudes and winds over which useful data can be obtained and to recommend the equipment and techniques to be used in conducting the experiment.

1.2 Significance of the Experiment

The purpose of the described experiment is to obtain data on density and wind structure of the upper atmosphere. Existing methods of high-altitude air density and wind determination have in the past provided useful data, but are limited in attainable accuracy and altitude range. The investigation described in this report indicates that the proposed experiment promises to yield relatively accurate data about atmospheric density up to at least 150 km and data about horizontal winds up to at least 120 km. (See Section 2.3 and 2.4.)

A variety of methods have been used by other investigators for obtaining information on air density. Density of the upper atmosphere has been studied by observing meteor trails (Whipple, 1952). A ground based technique of pulsed searchlighting has been used (Frieland et al., 1956). Rocket techniques have extended the range of measurements and provided a more direct method of attack.

One rocket technique which has been successfully used for determining air density consists of the measurement of the drag acceleration of a falling sphere. In the experiment, as being carried out presently (Jones, 1956 a,b; Jones et al., 1958), the acceleration of the sphere is measured by a transit time accelerometer, in which a bobbin is periodically clamped and released. When released, the bobbin is free to travel in any direction until it makes contact with the inside surface of the housing in which it is carried. The time taken for the bobbin to make contact is measured and telemetered to the ground. Since no drag acts on the bobbin, the transit time provides information about the magnitude of the drag acceleration. However, no information is obtained on the direction of acceleration. Thus, the telemetered signals do not provide direct information about the trajectory either of the missile or of the sphere after the ejection, and wind determination is not possible. The range of the experiment is 90 km (Jones et al., 1958).

In spite of the great advances in research techniques, the density of the upper atmosphere has not been sufficiently determined. Thus, at 100 km altitude, reports of different researchers show a variation by a factor of 4 (Nicolet, 1959).

Winds have been determined by observing the motions of meteor trails (Whipple, 1952; Robertson et al., 1953). At an altitude of about 80 km, wind information can be obtained from the movements of noctilucent clouds (Ludlam, 1957). This technique is limited to high geographic latitudes. Rocket techniques for wind measurement consist of observing an artificial sodium cloud

(Edwards, 1956), chaff (Smith, 1958) and propagation of sound from grenades at high altitudes (Stroud et al., 1958). Of the three above methods, only the sodium method extends somewhat above 100 km. At the present the wind information up to 100 km is rather incomplete, and above 100 km very scanty. The atmospheric motions above 100 km are of great current scientific interest, both theoretically, since their theory hinges upon the characteristics of the atmosphere as a whole, and more practically, since they affect radio communication and radio astronomy. Hines (1959) in his article on motions in the ionosphere lists almost 70 references, practically all recent, dealing with various aspects of this problem. The opening sentence of his conclusions reads "Perhaps the most immediate conclusion that can be drawn from all these remarks is that a very great deal has yet to be learned about motions in the ionosphere".

While the results sought in this experiment are of basic scientific interest, they have a direct application to the flights of ballistic missiles and to the study of effects of explosions at high altitudes.

[redacted]

1.3 Description of the Experiment

1.3.1 Method of Data Analysis

The falling sphere method for the determination of winds and density in the upper atmosphere consists of measuring the drag acceleration of a sphere ejected from a high altitude rocket (Jones and Bartman, 1956). The drag acceleration vector $\bar{a}$ is related to the drag force vector $\bar{D}$ by the equation

$$\bar{D} = m\bar{a} \qquad 1.1$$

where m is the mass accelerated by the drag force. The drag force is a function of the air density :

$$\bar{D} = -\rho \frac{AC_d |\bar{c}|}{2} \bar{c} \qquad 1.2$$

where A is the sphere cross-section area, C_d the coefficient of drag and $\bar{c}$ the sphere's velocity vector relative to the ambient air. The coefficient of drag C_d generally depends on the Mach number and Reynolds number, but ~~various~~ varies rather slowly in most of the range of interest. Thus, if m and A are known, and $\bar{a}$ is measured, the density, ρ, can be determined.

Data on winds can also be determined from the same measurements, by making use of the fact that the drag acceleration and the velocity of the sphere relative to the ambient air are colinear and in opposite directions. It is shown in Section 2.2 that a solution of the equations representing conditions existing at any instant during the flight can be performed to determine horizontal winds if the assumption is made of zero vertical winds. This is the same assumption that is currently being used in the data reduction of the rocket grenade experiment for upper atmosphere temperature and winds (Stroud et al., 1958).

The vertical winds can be determined if the data reduction on both the upleg and downleg of the sphere trajectory for a given altitude is carried out simultaneously. If the density and wind conditions are assumed constant in the time interval between the upleg and downleg (this time interval will be of the order of 10 minutes), an overdeterminate solution is obtained and can be solved for the best fit of the data for the density and the wind components. Alternatively, a determinate solution can be obtained for a given altitude by assuming that the horizontal winds do not remain constant.

1.3.2 Experimental Procedure

In order to measure drag acceleration, an inertial platform, similar to those used for the guidance of [redacted] ballistic missiles, is mounted in the sphere. The inertial platform contains three accelerometers measuring acceleration along three mutually-perpendicular axes, which are maintained fixed or rotated at a constant rate with respect to inertial space.

The experiment thus consists of shooting a rocket carrying an inflatable sphere inside which is located an inertial platform. At an altitude of approximately 50 km this instrumentation system is ejected from the rocket and the folded container is inflated to form a sphere with the inertial platform at its center, the total system weighing in the neighborhood of 120 lbs. The sphere continues on a trajectory which is, except for drag, a free-fall trajectory. Its peak should be of the order of 250 km.

The accelerometer data of thrust and drag accelerations are [redacted] telemetered to the ground throughout the flight, where they are [redacted] converted possibly by means of an electronic computer, into data on the velocity and position of the rocket and, subsequent to the ejection, of the sphere. The computation involves integration of rocket and sphere measured acceleration due to thrust and drag, and computed acceleration due to gravity, and conversion from an inertial system of coordinates to an earth system of coordinates. Seven coordinate systems have been studied; however, no choice has yet been made on which system to use. Values of the components of acceleration, velocity, and position can then be used to determine density and winds as a function of altitude. The basic equations are derived in Section 2.1.

If radar or optical tracking systems are available at the site of the experiment, they may be used to supplement the accelerometer data in determining sphere velocity and position as a function of time. They cannot, however, provide data of sufficient accuracy to determine drag acceleration, since the drag is so small at the altitudes of greatest interest that the deviation of the sphere from a trajectory without drag would be too small to detect from the ground.

1.4 Threshold Altitudes for Density and Wind Determination

The analysis given in Section 2.3 provides an indication of the greatest altitude at which satisfactory data can be obtained on air density. In this analysis a 12-ft. (3.66 m.) diameter, 120 lb (54.4 kg.) sphere is assumed to fall vertically through the atmosphere. The vertical velocity which it must have as it falls through a given altitude in order to produce a drag acceleration of 1×10^{-4}g is determined. (This acceleration value is 5 times the assumed accelerometer threshold error of 2×10^{-5}g.) This velocity can then be used to compute the required peak of the trajectory.

Assuming the Model A atmosphere of Kallmann (1958), the analysis shows that the density-measuring threshold occurs at an altitude of about 166 km if the sphere has a velocity of 1500 m/sec, which is equivalent to a trajectory peak of 166 + 120 = 286 km. Since the accelerometer error is the dominant one at high altitudes in the determination of density, the density can thus be determined within about 20%. Assuming the 1956 ARDC standard atmosphere (Minzner and Ripley, 1956) the density threshold occurs at about 147 km.

The analysis given in Section 2.4 provides an indication of the greatest altitude at which satisfactory data can be obtained on winds. It is assumed arbitrarily that the wind threshold will occur when the error in the horizontal wind component is 10 m/sec. The crucial factor in determining the altitude limit to wind measurement is the instrument error caused by the accelerometers. As assumed previously, for small accelerations the accelerometer error is approximately 2×10^{-5}g. For the Model A atmosphere, the wind measurement threshold then will occur at about 120 km, where the density is 6.9×10^{-8} kg/m^3. For the 1956 ARDC standard atmosphere the wind threshold will occur at about 116 km.

The values used for the weight of the sphere and the accuracy capabilities of the inertial package represent a realistic estimate of capabilities of present day systems. The investigation thus indicates that the proposed experiment promises to yield relatively accurate data about atmospheric density up to at least 150 km, and data about winds up to at least 120 km.

1.5 Equipment Considerations

In this section, the major features of the instrumentation system to be used in the experiment are discussed. Although final decisions have not yet been reached regarding the selection of components and operating methods, certain assumptions have been made in this discussion to permit the presentation of at least a first order design of the system. Further theoretical and experimental analysis will be necessary to arrive at the most suitable design.

1.5.1 Inertial Platform System

The inertial platform system consists of a 4-gimbal stabilized platform, a group of electronic circuits for controlling the platform and operating the gyros and accelerometers, and the necessary power supply.

Accuracy of the accelerometers and the gyros of the inertial platform determine the obtainable accuracy of the experiment. By taking all possible precautions before take-off to eliminate bias errors, it is believed that accelerometer errors can be held to values as low as 2×10^{-5}g. Gyro drift during the experiment, which will continue for about twenty minutes, is expected to be about .002 radians.

The over-all dimensions and weight of the inertial platform equipment influence the size of the rocket required to send the sphere into the upper atmosphere. In addition, size and weight affect the area-to-mass ratio of the falling sphere and hence the sensitivity of the experiment. The weight of the inertial platform system, including the batteries and the transmitter, is of the order of 80 lbs. The dimensions of the platform are approximately 10 inches in diameter and 15 inches long.

1.5.2 Inflatable Sphere

An inflatable mylar sphere of 12-ft. diameter will be used in the experiment. It will be packed into a section of the missile with all of the experimental equipment inside the folded sphere. Details of ejection mechanism and equipment attachment remain to be worked out. However, the techniques have been used successfully by various groups (Jones, 1952; Kehlet and Patterson, 1959; Arthur D. Little Inc., 1958).

Particular attention must be given to the problem of limiting the rates of angular velocity and angular acceleration of the sphere throughout the flight so that they do not exceed the capabilities of the inertial platform control system. Adequate roll control of the missile, and avoidance of sphere ejection transients are approaches which can be used to avoid this difficulty.

The inflated sphere will have an internal pressure of the order of 2 inches of water. It will therefore begin to collapse at a point in the trajectory where the total pressure exceeds this value. This will not occur until the sphere altitude is below the region of interest, and actually under altitudes which will provide a check point for accuracy of density determination.

In view of the high cost of the inertial platform, it would be very desirable to avoid destruction of the platform at impact. This could be accomplished if a parachute could be incorporated into the assembly which would open automatically at some prescribed altitude. Inertial platforms are reasonably rugged devices some of which can withstand accelerations as high as 20g without damage and accelerations up to 50g with only minor damage. The possibility of successfully recovering the platform by the use of a parachute therefore deserves further consideration.

[redacted]

1.5.3 Telemetering Equipment

The form and content of the signal output of the accelerometers affects the method of conditioning and telemetering the signal and hence the accuracy of the data received ~~[illegible]~~ on the ground.

The method of signal transmission assumed in this discussion is based on the use of an accelerometer which produces an output signal consisting of a precision current which periodically reverses direction at a frequency of several hundred cycles per second. The difference in the time duration of current in each direction is an indication of the acceleration. The instants at which zero crossover occurs can be transmitted to the ground by means of a 30 kc signal channel with a 30 db signal-to-noise ratio. The received signal can be converted to a measurement of acceleration by referring it to a 1 mcs clock pulse system. It is believed that this technique can also be adapted to an accelerometer having a non-pulsating d-c output.

Thus, the telemetry system must provide for three 30 kc signal channels with a 30 db signal-to-noise ratio. This can be provided by means of a conventional FM/FM telemetry system. A turnstile antenna mounted inside the sphere can provide a non-directional pattern of radiation, so that ground reception will not be dependent on the attitude of the sphere.

1.5.4 Auxiliary Equipment

The sphere must carry a primary source of power for the inertial platform and communication equipment. Nickel-cadmium batteries, which may be conservatively assumed to have a rating of 10 watt-hrs per lb., can be used for

[redacted]

this purpose. Additional equipment may also be required to convert this d-c power to a form compatible with the equipment using it.

Power dissipation capabilities of the sphere should also be investigated in relation to the power produced by the enclosed electronic equipment. Coolant bottles can be used, if necessary, to absorb the excess heat.

1.5.5 Missile Characteristics

No detailed attention has yet been given to the selection of a missile for the specific application discussed in this report. The primary requirements which must be met include the following:

1. The payload will weigh approximately 120 lbs.
2. The payload will have a diameter of approximately 12 inches.
3. The payload must be carried to an altitude of 50 km. and ejected into a trajectory reaching an altitude of about 250 km.
4. Peak acceleration during powered flight should preferably not exceed 15g.
5. An antenna must be provided to transmit drag acceleration data to the ground.

1.5.6 Ground Based Equipment

As in the case of the missile characteristics, no detailed attention has been given to the selection of the ground-based equipment required for this experiment. The functions which must be performed by the ground-based equipment include the following:

1. Handling and launching of the missile.

2. Checkout of the instrumentation equipment, and in particular, alignment of the inertial platform system.

3. Receipt and conversion of telemetering signals to proper form for entry into data-processing system.

4. Digital computation. (This function might be performed at a centralized computing facility, if more convenient. In this case, magnetic tape recording equipment would be required at the site of the experiment).

5. Radar or optical tracking, if desired, to provide supplementary trajectory information.

1.6. Cost of the Experiment

The cost of the inertial stable platform package with accelerometers ranges [redacted] from $80,000 to $200,000 per unit in small batches. It is suggested that an initial series of experiments should provide for three to five rocket shots. A very rough cost estimate for such an initial series puts the cost at 1.2 to 2.5 million dollars.

2. Detailed Discussion

In the first section of this report, the most important results of an investigation into methods of conducting a falling-sphere experiment to determine high-altitude density and winds have been discussed in a general manner. The detailed analysis of available equipment and techniques and the derivation of mathematical relationships on which the previous discussion was based are presented in this section.

The basic equations which describe the motion of a falling sphere subsequent to ejection and inflation are described in Section 2.1. An explanation of the method of calculating density and winds from these basic equations is included.

The primary item of instrumentation for obtaining the data from which density and winds are to be computed is the inertial navigation equipment. The factor of major importance in the selection of this equipment concerns the accuracy with which acceleration data can be determined; however, other factors, such as size, weight, cost, and adaptability to environment must also receive due consideration. All of the above factors are discussed in Section 2.2.

The potential value of the experiment depends on the altitude range over which useful information can be obtained. A mathematical analysis is presented in Section 2.3 which results in the computation of threshold of maximum altitude at which air density can be determined. A corresponding analysis for wind threshold is presented in Section 2.4.

2.1 BASIC EQUATIONS

A set of equations can be written describing the forces and motion of the falling sphere subsequent to the ejection and inflation. From the equations, density and winds can be calculated, as explained hereunder.

a_e; a_n; a_z	East, North, and Up components of (drag and thrust) acceleration in the earth-coordinate system.
v_e; v_n; v_z	East, North, and Up components of sphere velocity relative to the earth-coordinate system.
c_e; c_n; c_z	East, North, and Up components of sphere velocity relative to the ambient air in the earth-coordinate system.
w_e; w_n; w_z	East, North, and Up components of wind.

The basic drag equation is:

$$(a_e^2 + a_n^2 + a_z^2)^{\frac{1}{2}} = \left| a \right| = \frac{C_d \rho A}{2m} (c_e^2 + c_n^2 + c_z^2) \qquad 2.1$$

where C_d is the coefficient of drag, ρ the density, A the cross-section of the sphere and m its mass. If the aerodynamic forces caused by the spinning of the sphere are neglected, the drag acceleration and the velocity of the sphere relative to the ambient air are colinear (and in the opposite direction). This results in the following equations:

$$c_e = c_z \frac{a_e}{a_z} \qquad 2.2$$

$$c_n = c_z \frac{a_n}{a_z} \qquad 2.3$$

By combination of Equations (1), (2), and (3)

$$\rho = \frac{2m\, a_z}{C_d\, A\, c_z^2\left[1 + (a_e/a_z)^2 + (a_n/a_z)^2\right]} \qquad 2.4$$

The wind component equations are as follows:

$$w_e = v_e - c_e = v_e - c_z\frac{a_e}{a_z} \qquad 2.5$$

$$w_n = v_n - c_n = v_n - c_z\frac{a_n}{a_z} \qquad 2.6$$

$$w_z = v_z - c_z \qquad 2.7$$

In Equations 2.2 through 2.7, [redacted] v_e, v_n, v_z and a_e, a_n, a_z are known trajectory data. The equations contain seven unknowns; c_e, c_n, c_z, w_e, w_n, w_z, and ρ, thus, the solution is impossible solely on the upleg or solely on the downleg without some additional assumption. It is proposed to use the assumption of zero vertical winds, the same assumption that is currently being used in the data reduction ~~XXXXXXXXX~~ of the rocket grenade experiment for upper atmosphere temperature and winds (Stroud, ~~XXXXXXXXX~~ 1958).

$$w_z = 0 \qquad 2.8$$

The errors introduced by this assumption both in density and wind determination are discussed later.

If the data reduction on both the upleg and downleg is carried out simultaneously for the same altitude, the vertical winds can be determined on the basis of certain assumptions. This, of course, is possible only at altitudes higher than the ejection of the sphere.

If the density and wind conditions are assumed constant in the time interval between the upleg and downleg (this time interval will be of the order of 10 minutes) we have at each altitude point 12 equations (each of the Eqs. ~~equations~~ 2.2-2.7 / [redacted] twice) with 10 unknowns; c_e, c_n, c_z, w_e, w_n, w_z, and c_e^*, c_n^*; c_z^*, where c_e, c_n, c_z refer to upleg and c_e^*, c_n^*, and c_z^* to downleg only. This set of equations is overdeterminate, and can be solved for the best fit in w_e, w_n, w_z and ρ. Alternately, it can be assumed that the horizontal winds w_e, w_n do not remain constant. Then two additional unknowns, i.e., w_e^* and w_n^*, appear and we have a system of twelve equations with twelve unknowns.

2.2 [redacted] INERTIAL NAVIGATION EQUIPMENT

In order to compute the information required for air density and wind determination, it is necessary to obtain ~~for the data reduction process~~ the components of sphere acceleration, velocity, and altitude ~~position as a function of time~~ with respect to a known coordinate system. The primary source of this information is an inertial platform system mounted in the sphere. It would be possible to obtain from the accelerometers of this system either the components of acceleration or, if integrating accelerometers (and a gravity computer) are used, the components of velocity. From the accelerometer outputs, all data on acceleration, velocity, and position could be computed in the sphere and transmitted to the ground. Alternatively, only the acceleration data or the velocity data could be transmitted, from which a ground-based computer could compute the remaining information.

The alternative of computing all information within the sphere requires that the inertial navigation system contain accurate computing equipment but minimizes the accuracy required of the telemetering system. The alternative of transmitting only acceleration data to the ground requires a telemetering system handling only three channels of information but requires high accuracy (in the neighborhood of 0.1 percent of the transmitted quantity) in the telemetering operation. The latter alternative is considered preferable, because of the importance of minimizing size and complexity of the airborne equipment. The following discussion of inertial navigation equipment is based on the assumed use of this latter alternative.

In order to determine which inertial navigation system would be most suitable for the purpose, a number of systems have been investigated. The following manufacturers have been contacted during this investigation:

Arma Division, American Bosch Arma Corporation

Reeves Instrument Corporation

Ford Instrument Company

Litton Industries

Honeywell Corporation

Autonetics Division, North American Aviation

Nortronics Division, Northrop Corporation

AC Spark Plug Division, General Motors Corporation

Kearfott Company, Inc.

The following paragraphs summarize the information [redacted] obtained on the characteristics of inertial navigation systems which are important to the application under consideration.

2.2.1 Accuracy

Accuracy of the accelerometers and the gyros of the inertial platform determine the obtainable accuracy of the experiment. Accelerometer accuracies of 0.1% of the measured acceleration are sufficient to permit computation of good velocity and position data. No difficulty is anticipated in meeting this requirement with any of the systems investigated. In addition, threshold acceleration errors should be kept as low as possible in order to maximize the altitude at which useable drag data can be obtained. Accelerometer errors are in the form of bias (zero uncertainty), threshold, and scale-factor and nonlinearity errors.

By taking all possible precautions before take-off, bias errors can be minimized. Moreover, it is believed that the bias error can be eliminated through an appropriate correction of after-flight data, as the following consideration shows. An analysis of the trajectory indicates that close to the peak, the drag in the z direction should be, under ideal conditions, approximately as given in Figure 1. The time origin in this figure corresponds to the peak, as determined by $v_z = 0$.

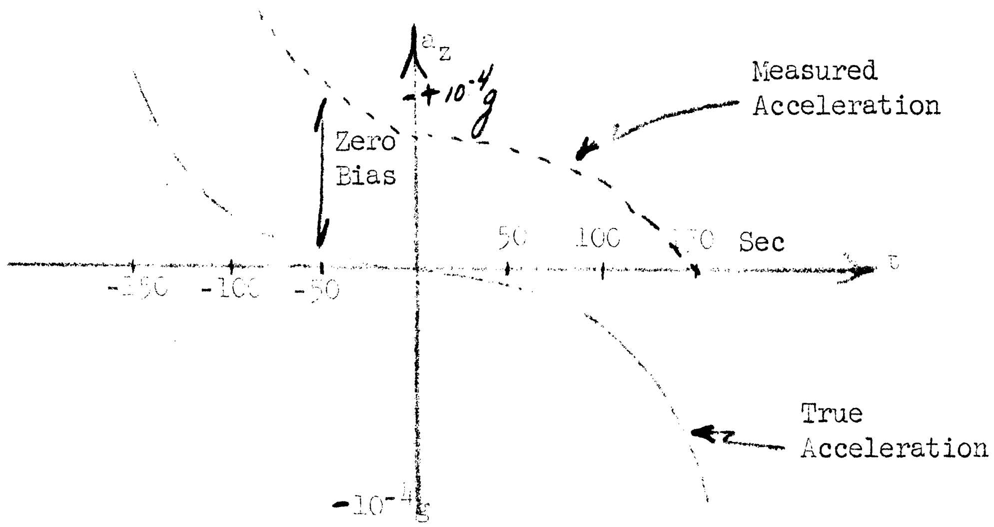

Figure 1

Alternatively, ~~(This~~ this peak time can be determined ~~alternately~~ by minimum-slope point of the drag data.) However, in case a zero bias exists, the curve will be lifted up or down by the amount of the bias, as indicated in the figure. Thus, the bias can be determined, and subsequently eliminated by observing the z accelerometer readings at the trajectory peak.

It is thought that the accelerometer data close to the threshold can contain spurious readings due to the sphere spin. If the center of rotation of the outer body around the stable platform does not correspond to the center of gravity of the outer body, the stable platform will be subject to sinusoidal accelerations at the spin frequency. These accelerations can seriously mask the drag accelerations at the threshold, and care must be exercised in balancing the sphere and minimizing the spin. Even so, it is thought that these sinusoidal accelerations will be present to some degree, but it is believed that their relatively steady pattern can be discerned and largely filtered out by data inspection.

These two considerations, of moving the zero bias and spin effects, present disadvantages at on-line data reduction by computers during flight at altitudes of density threshold and wind threshold determination.

Threshold errors of accelerometers as low as 2×10^{-5}g are claimed by more than one manufacturer, and this value is used in the analysis of errors. Scale-factor and nonlinearity errors can be neglected, insofar as drag and wind measurements are concerned.

Gyro accuracies are generally given as the sum of several components, some of which can be eliminated by a preflight check or by acceleration compensation. The random error which cannot be eliminated tends to be in the neighborhood of 5×10^{-4} radians per hour or less. Without acceleration compensation, gyro drift during the experiment, which will be about 20 minutes long, is expected to be about .002 radians. It is this value for gyro drift that is used in Sections 2.3 and 2.4 on density and wind measurement errors.

2.2.2 Size and Weight

The over-all dimensions and weight of the inertial navigation equipment influence the size of the rocket required to send the sphere into the upper atmosphere. In addition, size and weight affect the area-to-mass ratio of the falling sphere and hence the sensitivity of the experiment. The weight of the inertial platform system, including the batteries and transmitter, can be of the order of 80 lbs. This estimate is based on information concerning two of the lightest platform systems of those investigated. One of the smallest platforms is 9.5 inches in diameter and 14.5 inches long. Smaller systems are under development which would further reduce this space requirement.

2.23 Spin Capability

The inertial platforms investigated were in all cases four-gimballed systems. Although the four-gimballed system is heavier and costlier, it avoids the possibility of gimbal-lock. In addition to avoiding gimbal lock, the system must be able to withstand continuous tumbling of the sphere without serious degradation of system accuracy. Allowable spin velocities ranged from 3 to 7 radians per second up to 20, and perhaps 30, radians per second. Information on angular acceleration capabilities were obtained for one of the systems. Allowable accelerations ranged from 8 radians/sec.^{2} in pitch to 35 radians/sec.^{2} in azimuth. It is expected that it will be possible to maintain spin rates within the magnitudes mentioned, provided sufficient care is taken during the rocket ascent and sphere ejection periods.

2.24 Signal Output

The form and content of the signal output of the accelerometers will affect the method of conditioning and telemetering the signal and hence the accuracy of the data received by the ground-based computer.

Certain mechanizations of the signal conversion and transmission process appear to be capable of achieving the required accuracy. One method, which has been completely developed and is in current use, is based on the use of vibrating-string accelerometers. The output of this type of accelerometer consists of two a-c voltages, whose difference in frequency is proportional to the acceleration. It is possible to determine the acceleration during a short sampling period by a method which counts the integral and fractional number of cycles of each string to a high accuracy and determines the difference in these numbers for the two strings. Essentially, this system is capable of obtaining high resolution by virtue of the fact that it can determine small fractions of a cycle, that is, fractions of a single increment of velocity change. It has the advantage that a conventional FM/FM telemetering system of moderate accuracy can be used.

Another method is based on the use of an accelerometer which produces an output signal consisting of a precision current which periodically reverses direction at a frequency of several hundred cycles per second. The difference in the time duration of current in each direction is an indication of the acceleration. The instants at which zero crossover occurs can be transmitted to the ground by means of a 30 kc signal channel with a 30 db signal-to-noise ratio. The received signal can be converted to a measurement of acceleration by referring it to a high-frequency clock pulse system.

2.2.5 Shock Resistance

The resistance of the system to shock and vibration will affect the possibility of salvaging and reusing the equipment after each flight. For most systems investigated, available information consisted of estimates of allowable accelerations. Generally speaking, these systems will all withstand at least 20g, and will sustain minor damage at 30g to 50g.

2.2.6 Cost

The cost of an inertial platform system, complete with platform electronics, ranges from $80,000 to $200,000 per unit.

2.3 DENSITY DETERMINATION

2.3.1 Threshold Altitude

Assuming a particular sphere size and mass falling through a given altitude-density distribution, the drag Equation 2.1 can be used to determine the necessary sphere velocity to give a minimum measurable drag acceleration at a given altitude. The object is to find the maximum altitude at which the necessary velocity is reasonably low.

Neglecting winds, and assuming a vertically falling sphere, equation 2.1 with velocity as the independent variable is

$$v_z^2 = \frac{2a_z m}{C_d A \rho} \qquad 2.9$$

Table I is a tabulation of velocity vs. altitude using a 12 ft. (3.66m) diameter, 120 pound (54.4 kg) sphere and assuming the minimum determinable drag acceleration a_z to be 1×10^{-4}g. It is believed that the value of a_z used represents an acceleration that can be determined with an error of about 20% (section 2.2). The drag coefficient C_d is assumed to be 2 for the altitudes and velocities considered. The equivalent free fall distance tabulated in Table I is the distance the sphere would have to fall to obtain the required velocity (assuming that the only force acting on the sphere is gravity). The equation for the free fall distance is

$$h = \frac{v^2}{2g} \qquad 2.10$$

where h is the free fall distance and g is the average gravitational acceleration at the altitudes considered.

Two altitude-density distributions are used for comparison in Table I; the ARDC model atmosphere (Minzner and Ripley, 1956) and the Model A atmosphere of Kallmann(1959). According to Kallmann the Model A atmosphere is a density curve which represents a recent [redacted] smooth fit of rocket and satellite data.

Assuming the Model A atmosphere, Table I shows that the density measuring threshold occurs at an altitude of [redacted] 166 km if the sphere has a velocity of 1500 m/sec, which is equivalent to a trajectory peak of 166 + 120 = 286 km.

2.3.2 Density Error Analysis

To consider errors in density measurement, notice that equation (4) gives density as a function of C_d, c_z, a_z, a_e, and a_n. On a probabilistic basis, the percent error in density measurement is of the form

$$\left(\frac{\epsilon\rho}{\rho}\right)\% = \frac{100}{\rho}\left[\left(\frac{\partial\rho}{\partial C_d}\epsilon C_d\right)^2 + \left(\frac{\partial\rho}{\partial c_z}\epsilon c_z\right)^2 + \left(\frac{\partial\rho}{\partial a_z}\epsilon a_z\right)^2 + \left(\frac{\partial\rho}{\partial a_e}\epsilon a_e\right)^2 + \left(\frac{\partial\rho}{\partial a_n}\epsilon a_n\right)^2\right]^{\frac{1}{2}}$$

where $\epsilon(x)$ designates the expected errors in x.
By use of equation (4) this becomes

$$\left(\frac{\epsilon\rho}{\rho}\right)\% = 100\left\{\left(\frac{\epsilon C_d}{C_d}\right)^2 + \left(2\frac{\epsilon c_z}{c_z}\right)^2 + \left[\left(1 + \frac{2B}{(1+B)}\right)\frac{\epsilon a_z}{a_z}\right]^2 + \left[\frac{2a_e}{a_z^2(1+B)}\epsilon a_e\right]^2 + \left[\frac{2a_n}{a_z^2(1+B)}\epsilon a_n\right]^2\right\}^{\frac{1}{2}} \qquad 2.11$$

where

$$B = (a_e/a_z)^2 + (a_n/a_z)^2 \qquad 2.12$$

For errors near the density threshold, the nominal values of the variables (within the proper order of magnitude) may be taken as:

$C_d = 2$

$c_z = 1500$ m/sec

$a_z = 10^{-4}g = 10^{-3}$ m/sec^2

$a_e = 2\times10^{-4}$ m/sec^2 (high value)

$a_n = 2\times10^{-4}$ m/sec^2 (high value)

It is unlikely that c_e and c_n will be larger than 300 m/sec, even if a strong wind blows in the direction opposite to v_e or v_n. From equations 2.2 and 2.3 the above values of a_e and a_n have been determined by the relations

$$a_e = a_z\, c_e/c_z \text{ and } a_n = a_z\, c_n/c_z$$

From Equation 2.7

$$\epsilon(c_z) = \epsilon(v_z) - \epsilon(w_z) \qquad 2.13$$

The state of the art of inertial guidance systems indicates that velocities can be determined to within 0.5 m/sec, thus $\epsilon(v_z) = 0.5$ m/sec. Assuming that the vertical winds, w_z, encountered will not be larger than 10 m/sec, the use of the assumption $c_z = v_z$ implies $\epsilon(c_z) \simeq 10$ m/sec.

The accelerometer errors arise from two sources, the platform misalignment, and the accelerometer itself. For a small error $\epsilon(\phi)$ in platform alignment the error in the z accelerometer is

$$\epsilon_m(a_z) = a_z\frac{(\epsilon\phi)^2}{2} + (a_e^{\,2} + a_n^{\,2})^{\frac{1}{2}}\ \epsilon(\phi) \qquad 2.14$$

The expected platform misalignment during the short flight duration of the experiment will be of the order of $\epsilon(\phi) = 0.002$ radians. Inserting the proper values into equation 2.14 gives an acceleration error due to platform misalignment of

$$\epsilon_m(a_z) = 2\times10^{-9} + 4\sqrt{2} \times 10^{-7} \simeq 4\sqrt{2} \times 10^{-7} \text{ m/sec}^2$$

which is small compared with the accelerometer instrument errors, as discussed in Section 2.2.1 [redacted]. These errors are in the form of threshold and linearity errors. The threshold is about 2x 10^{-5}g [redacted] and the linearity error is on the order of 10^{-4} a. Thus, for the z accelerometer

$$\epsilon_i(a_z) = 2\text{x}10^{-5}g + 10^{-4}\, a_z$$
$$\simeq 2\text{x}10^{-4}\ \text{m/sec}^2 \text{ (for small accelerations)} \qquad [redacted]\ 2.15$$

By applying equations similar to Eqs. 2.14 and 2.15 [redacted] the errors in the n and e accelerometers [redacted] for small accelerations, are found to be:

$$\epsilon(a_n) = \epsilon(a_e) \simeq 2\text{x}10^{-4}\ \text{m/sec}^2$$

~~[redacted]~~. These above values are used in equation 2.11 [redacted] to calculate the error in density at its threshold of measurement.

An additional error in the determination of density is caused by the uncertainty in the sphere's altitude. Assuming that the atmospheric temperature is constant over small changes in altitudes, the percent change in density as a function of error in altitude determination is

$$\left(\frac{\epsilon\rho}{\rho}\right)_{\%} = 100 \exp\left(-\epsilon z/H_S\right) - 100 \qquad [redacted]\ 2.16$$

where ϵz is altitude error and H_S is the scale height.

Table II shows the error in density caused by the various factors discussed above. For a typical drag acceleration-altitude profile the vertical drag acceleration will remain small until the sphere reaches the denser atmosphere and then increase to a peak of 5 to 10 g subsequently decreasing to a steady 1g (Figure [redacted] 2). ~~XXXXXXX~~ [redacted] The points at which the errors were calculated in

Table II correspond to a vertical drag acceleration a_z of :

1. Density measurement threshold.
2. Wind determination threshold (discussed in next section).
3. A large drag acceleration on the rise of the peak while c_z is still large.
4. The 1g drag acceleration after the peak where c_z is small.

The value of c_z used for the first three points was 1500 m/sec and a value of 100 m/sec was used for the last point. These values are smaller than would actually be encountered; thus the $\sigma(c_z)$ error in Table II is an upper bound.

The altitude error in Table II was calculated from equation 2.16 [redacted] using a value of $\sigma(z) = 100$ m.

So far nothing has been said about the errors in density determination caused by uncertainty in the drag coefficient C_d. If the C_d term in equation 2.11 [redacted] is not used, the resulting error would be that of the product ρC_d. Table II shows that the error in ρC_d then is quite small with the exception of the high and low altitudes where, respectively, the accelerometer sensitivity and lack of knowledge of vertical wind w_z cause large errors.

Good data on the drag coefficient is lacking for conditions existing during the major portion of this experiment. The drag coefficient is a function of Mach number and Reynolds number. Experimental data is available for C_d in the region of Mach numbers from 0 to 4 and Reynolds numbers from 10 to 10^6 (Jones and Hartman, 1956). Table III shows trajectory data and Mach and Reynolds numbers for a 12 foot diameter, 120 pound sphere dropped from 300 km (no winds). The calculations were made on the IBM 650 computer

Good data on the drag coefficient is lacking for conditions existing during a large portion of this experiment. For satellite velocities and altitudes, a drag coefficient of 2 or 2.3 is usually assumed (Kallmann, 1959). The method used to calculate the drag coefficient depends upon the type of flow across the body. The type of flow is characterized in one of the following three phases:

1. a free molecular flow region, in which the mean free molecular path of the air molecules is large compared to the sphere dimensions,

2. a slip flow region, in which the molecular mean free path is small compared to the boundary layer about the sphere, but not small enough to be neglected, and

3. the continuous flow region, in which the molecular mean free path is very small and the medium can be considered as a continuum. [redacted]

The free molecular flow region can be defined as the region in which $\frac{M}{Re} > 10$ (1.0) where M is Mach ~~XXX~~ number and Re is the Reynolds number. The ratio $\frac{M}{Re}$ is an indication of the ratio of molecular mean free path to sphere diameter. In this region the drag coefficient is a function of the ratio $\frac{c}{c_i}$ where c is the sphere velocity and c_i is the mean molecular velocity (Petersen, 1956). Table III shows trajectory data and Mach and Reynolds numbers for a 12 foot diameter, 120 pound sphere dropped from 300km (no winds); Figure 2 shows acceleration and velocity curves. The calculations were made on the IBM 650 computer at the Willow Run Laboratories using the ARDC Standard Atmosphere. Calculating $\frac{M}{Re}$ from the computer data indicates that the sphere is in the free molecular flow region for altitudes above 145 km. Petersen (1956) shows theoretical results of C_d vs. $\frac{c}{c_i}$. Using values of velocity c from the computed trajectory, the drag coefficient varies from 2.8 at 166 km to 2.4 at 145 km. The error in C_d determination in this region has been taken as 10% for use in Table II. Figure 3 shows a plot of M and R_e versus altitude using the computer data.

For flow regions other than free molecular flow, the drag coefficient has been determined mainly by experimental methods. Experimental data on C_d, to about 2% accuracy, is available for Mach numbers up to 4.0 and Reynolds numbers ranging from 10 to 10^6 (Jones and Bartman, ~~1956~~ 1956b; May, 1957). As shown in ~~Table~~ Figure 3 ~~II~~, the Mach numbers of the sphere are above 4 until the sphere has descended to an altitude of about 50 km. ~~The experimental data on C_d can be repeated to about 2% accuracy.~~ For use in Table II the error in C_d has been taken as 2% for altitudes below 50 km and as 10% for altitudes between 50 km and 145 km where data on C_d ~~has~~ have not yet been found.

the [redacted] atmosphere. [redacted] results [redacted] shown that [redacted] the Reynolds number is [redacted] the Mach [redacted] is greater [redacted]

[redacted] usu[redacted] and [redacted] the region [redacted] soci[redacted] high altitudes a value [redacted] is used [redacted] be [redacted]

2.4 Wind Determination

[redacted]

2.4.1 Wind Error Analysis

Equation 2.5 will be now analyzed with respect to the expected errors in w_e determination. The same discussion will hold for w_n.

$$\epsilon(w_e) = \epsilon(v_e) - \frac{a_e}{a_z}\,\epsilon(c_z) - c_z\,\epsilon\left(\frac{a_e}{a_z}\right) \qquad 2.17$$

On a deterministic basis; or on a probabilistic basis;

$$\epsilon w_e = \left[(\epsilon v_e)^2 + \left(\frac{a_e}{a_z}\,\epsilon c_z\right)^2 + \left(c_z\,\epsilon\,\frac{a_e}{a_z}\right)^2\right]^{\frac{1}{2}} \qquad 2.18$$

on the assumption that the errors are independent.

The error terms $\epsilon(v_e)$ and $\epsilon(c_z)$ will be taken as 0.5 m/sec and 10 m/sec respectively, for reasons given in the preceding section. The second term in equations 2.17 and 2.18 contains the ratio $\frac{a_e}{a_z}$ which will be on the order of 0.2 or smaller (section 2.3). Thus, the total contribution of this term will be about 2 m/sec.

The third term, $c_z\,\epsilon\left(\frac{a_e}{a_z}\right)$, is caused by two sources of error, platform misalignment, $\epsilon_m\left(\frac{a_e}{a_z}\right)$, and the accelerometer instrument errors, $\epsilon_i\left(\frac{a_e}{a_z}\right)$. Limiting the discussion to the east-up plane; if the accelerometers are perfect instruments and the platform is in perfect alignment then

$$\frac{a_e}{a_z} = \tan \phi \qquad 2.19$$

where ϕ is the angle the drag acceleration vector makes with the vertical. When the platform is misaligned by the angle $\epsilon(\phi)$, the ratio read by perfect accelerometers will be

$$\frac{a_e}{a_z} + \epsilon_m\left(\frac{a_e}{a_z}\right) = \tan(\phi + \epsilon\phi) \qquad 2.20$$

Solving for the error term we have

$$\epsilon_m\left(\frac{a_e}{a_z}\right) = \frac{a_e/a_z + \tan \epsilon\phi}{1 - \frac{a_e}{a_z}\tan \epsilon\phi} - \frac{a_e}{a_z} \qquad 2.21$$

For $\epsilon(\phi) \simeq$ 0.002 radians (see Section 2.3.2)

$$\epsilon_m\left(\frac{a_e}{a_z}\right) \simeq 0.002$$

Assuming $c_z \simeq$ 1500 m/sec, the error due to platform misalignment will be of the order of 3m/sec.

The crucial factor in determining the altitude limit of wind measurement is the instrument error caused by the accelerometers; namely the term $c_z\, \epsilon_i\left(\frac{a_e}{a_z}\right)$. In section 2.3 it was shown that for small accelerations the accelerometer error is approximately 2×10^{-5}g. Since $\frac{a_e}{a_z} \ll 1$, the uncertainty in the numerator will be the dominant factor and the uncertainty in the denominator could be neglected. The accelerometer instrument error will then be

$$c_z\, \epsilon_i\left(\frac{a_e}{a_z}\right) = c_z \frac{2\times10^{-5}g}{a_z} \qquad 2.22$$

It will be assumed arbitrarily that the wind threshold will occur when the error in the horizontal wind component is 10 m/sec. Assuming again the vertical velocity to be 1500 m/sec, the threshold of wind measurement will occur approximately where

$$a_z = 3\times10^{-3}g \qquad 2.23$$

Remembering that the density measurement was assumed to occur when $a_z = 10^{-4}$g, the drag has to increase by a factor of about 30 before winds can be measured. From table I for the Model A atmosphere the density measurement threshold occurs at approximately 166 km (assuming c_z= 1500 m/sec), where the density is 2.3×10^{-9} kg/m^3; the wind measurement threshold then will occur at about 120 km, where the density is 6.9×10^{-8} kg/m^3. Assuming the ARDC standard atmosphere the density threshold occurs at about 147 km (ρ= 2.3×10^{-9} kg/m^3) and the wind threshold occurs at about 116 km (ρ = 6.9×10^{-8} kg/m^3). The computed trajectory in table III shows that the thresholds occur at 150 km and 117 km respectively, which is in good agreement.

Table IV shows the errors in wind measurement. It should be noted that the error $c_z \in_i \left(\frac{a_e}{a_z}\right)$ which determines the threshold becomes insignificant as the altitude decreases. The remaining errors are constant for the points calculated because of the assumption that c_z and the ratio $\frac{c_e}{c_z}$ (therefore the ratio $\frac{a_e}{a_z}$) remain constant. The value of c_e (300 m/sec) used is believed to be an upper limit and will be lower in most cases with a proportionate decrease in the first and third error terms of table IV.

It should be pointed out that the error caused by aerodynamic forces related to the spin of the sphere are believed small, provided that the spin rate stays within the limitations of the inertial platform.

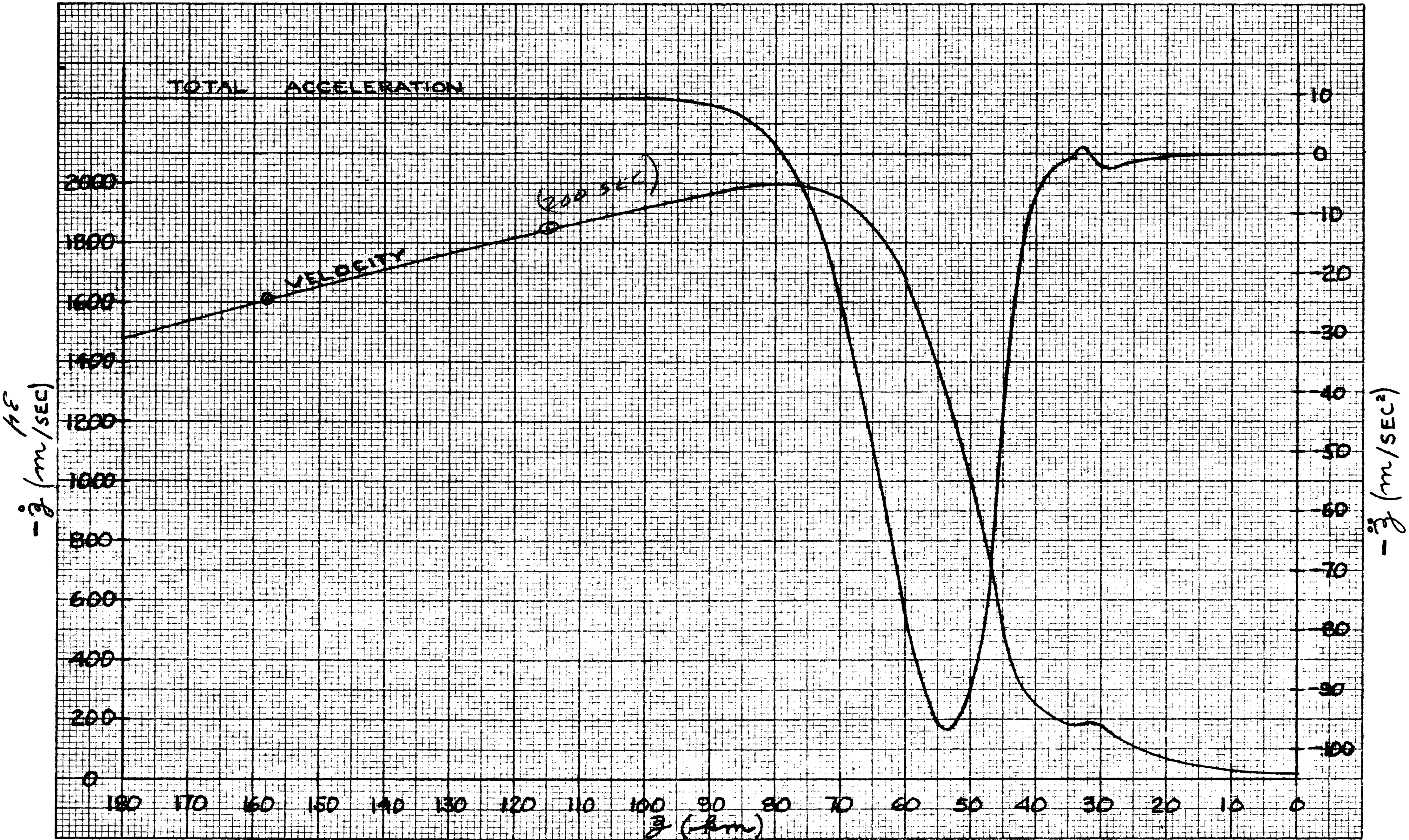

FIG. 2 ACCELERATION & VELOCITY OF A 12 FOOT DIA., 120 POUND SPHERE DROPPED FROM 300 km.

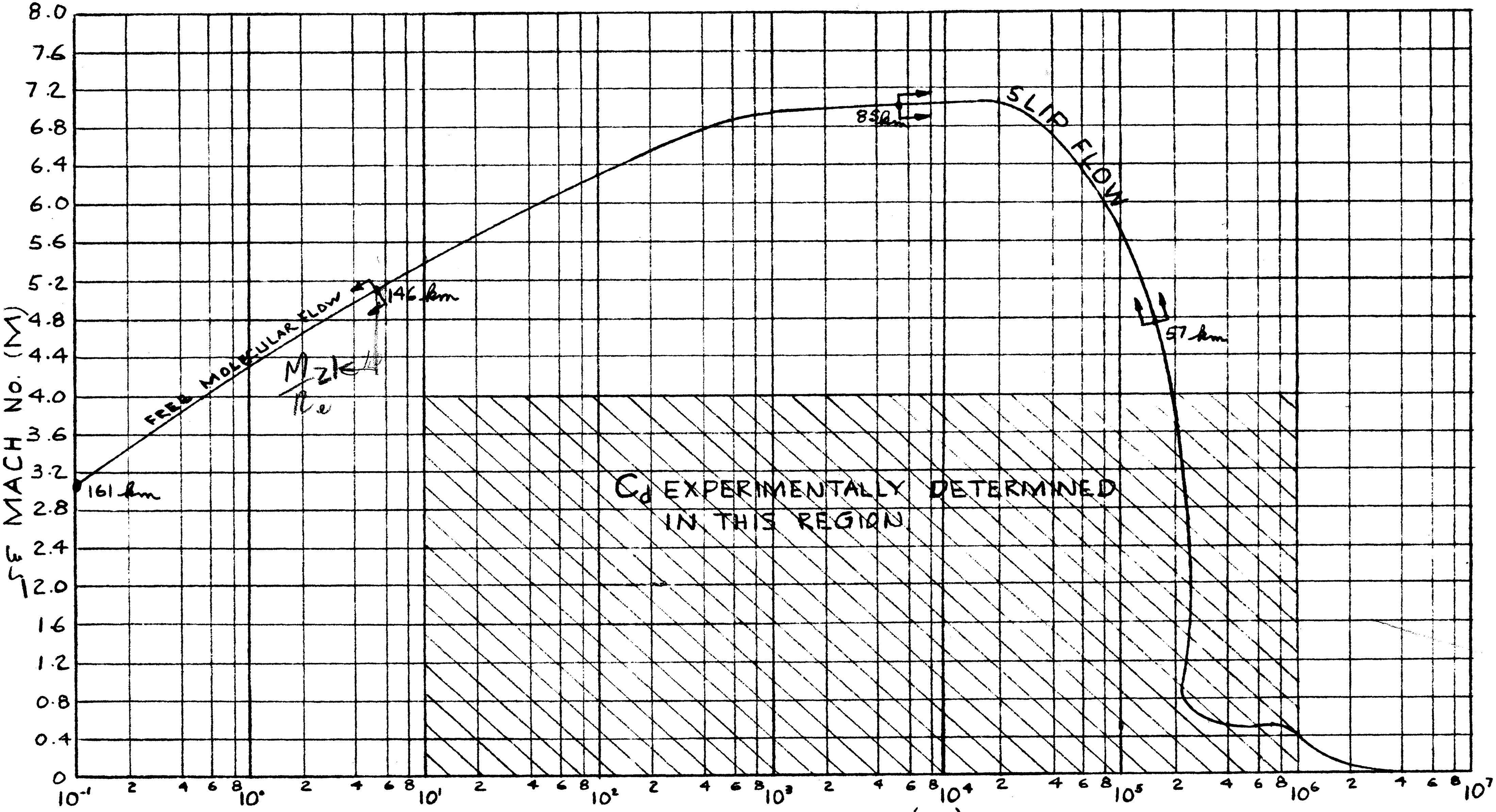

FIG. 3 M & Re vs ALTITUDE, z.

Altitude	ARDC Atmosphere			Model A Atmosphere		
	Velocity	Equiv. free-fall distance	Density	Velocity	Equiv. free-fall distance	Density
z (km)	v (m/sec.)	h (km)	ρ (kg/m^3)	v (m/sec.)	h (km)	ρ (kg/m^3)
130	630	21	1.3×10^{-8}	446	11	2.6×10^{-8}
140	1090	64	4.3×10^{-9}	657	24	1.2×10^{-8}
147	1500	120	2.3×10^{-9}	830	74	7.5×10^{-9}
150	1695	150	1.8×10^{-9}	920	48	6.1×10^{-9}
160	2465	342	8.5×10^{-10}	1270	88	3.2×10^{-9}
166				1500	120	2.3×10^{-9}
170				1650	177	1.9×10^{-9}
180				2077	300	1.2×10^{-9}

TABLE I - ~~xxx~~ v, and h required to give a 12 foot dia., 120 pound sphere, a drag acceleration of 1×10^{-4} g at various altitudes

Drag Acc. a_z	Approx. Alt. z (km)	Scale Height H_s (km)	% error in density measurement caused by corresponding terms in eq. (11)				Altitude Error $\epsilon(z)$	% error in ρ (rms)
			$\epsilon(c_d)$	$\epsilon(\dot{c}_z)$	$\epsilon(a_z)$	$\epsilon(a_e, \epsilon a_n)$		
1×10^{-4}g	165	37.8	10%	1.3%	20%	4%	0.4%	23%
3×10^{-3}g	120	14.9	10	1.3	0.5	0.1	0.8	10
5g	70	6.6	10	1.3	0.04	0.02	1.5	10
1g	30	6.8	2	20	0.8	0.04	1.5	20

TABLE II - Error in density measurement caused by various factors (Model A atmosphere).

50 288.5 462 m/sec

100 253.8 924

150 196 1386

175 158 1618

Altitude z (km)	Velocity v (m/sec.)	Drag. Acc. a_z (m/sec.^{2})	Mach. No.	Reynolds No.	Time t (sec.)
150	1660	$1x10^{-3}$	3.6	0.27	181
117	1835	$3x10^{-2}$	5.4	9.6	200
100	1920	$2.5x10^{-1}$	6.3	104	208 206
65	1859	50	6.0	$8.5x10^4$	226
55	1400	105	4.2	$1.7x10^5$	232
45	531	40	1.6	$2.3x10^5$	243
30	181	9.7	.59	$7.3x10^5$	306
0	15	9.8	.04	$3.7x10^6$	1166

TABLE III - Trajectory of a 12 foot dia., 120 pound sphere dropped from 300 km (no winds, ARDC standard atmosphere).

Approx. Altitude z (km)	Drag Acc. a_z (g)	Error in wind measurement caused by corresponding terms of equation (17). $\frac{a_e}{a_z}\epsilon(c_z)$	$c_z \epsilon_i\left(\frac{a_e}{a_z}\right)$	$c_z \epsilon_m\left(\frac{a_e}{a_z}\right)$	$\epsilon(v_e)$	Wind Error $\epsilon(w_e)$ (rms)
120	$3\text{x}10^{-3}$	2 m/sec.	10 m/sec.	3 m/sec.	0.5 m/sec	10.7 m/sec
100	$3\text{x}10^{-2}$	2	1	3	0.5	3.8
70	5	2	0.03	3	0.5	3.6

TABLE IV - Errors in wind determination. Approximate altitudes are for Model A atmosphere.

SYMBOLS

A	Cross-section area of sphere
B	Term defined in Eq. 2.12
C_d	Coefficient of drag
c_i	Mean molecular velocity
$\bar{D}$	Drag force vector
g	Acceleration of gravity
h	Free-fall distance
H_ζ	Scale height
m	Mass of sphere
M	Mach number
Re	Reynolds number
z	Altitude
$\epsilon(x)$	Expected error in x
ϵ_m	Error due to platform misalignment
ϵ_i	Error due to accelerometer
ρ	Atmospheric density
ϵ ϕ	Angle of platform misalignment

Accelerations

$\bar{a}$	Drag and thrust acceleration vector in the earth-coordinate system
a_e; a_n; a_z	East, North, and Up components of $\bar{a}$

Velocities and Winds

$\bar{v}$	Velocity vector of sphere in the earth-coordinate system
v_e; v_n; v_z	East, North, and Up components of $\bar{v}$
$\bar{c}$	Velocity vector of sphere relative to the ambient air in the earth-coordinate system.

SYMBOLS (continued)

c_e; c_n; c_z East, North, and Up components of $\bar{c}$

$\bar{w}$ Velocity vector of wind relative to the earth-coordinate system

w_e; w_n; w_z East, North, and Up components of $\bar{w}$

* Refers to downleg of trajectory only

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$$\left(a_e^2+a_m^2+a_z^2\right)^{1/2} = |a| = \frac{C_d\,\rho A}{2m}\left(c_e^2+c_m^2+c_z^2\right)^{1/2} \qquad 2.1$$

$$\left[\frac{\epsilon(\rho)}{\rho}\right]_{\%} = \frac{100}{\rho}\left[\left(\frac{\partial\rho}{\partial C_d}\epsilon(C_d)\right)^2+\left(\frac{\partial\rho}{\partial C_z}\epsilon(C_z)\right)^2+\left(\frac{\partial\rho}{\partial a_z}\epsilon(a_z)\right)^2 + \left(\frac{\partial\rho}{\partial a_e}\epsilon(a_e)\right)^2+\left(\frac{\partial\rho}{\partial a_m}\epsilon(a_m)\right)^2\right]^{1/2}$$

no number

$$\left[\frac{\epsilon(\rho)}{\rho}\right]_{\%} = 100\left\{\left(\frac{\epsilon(C_d)}{C_d}\right)^2+\left(\frac{2\,\epsilon(C_z)}{C_z}\right)^2+\left[\left(1+\frac{2B}{(1+B)}\right)\frac{\epsilon(a_z)}{a_z}\right]^2 + \left[\frac{2a_e}{a_z^2(1+B)}\epsilon(a_e)\right]^2+\left[\frac{2a_m}{a_z^2(1+B)}\epsilon(a_m)\right]^2\right\}^{1/2} \qquad 2.11$$

$$\epsilon_m(a_z) = a_z\frac{[\epsilon(\phi)]^2}{2} + \left(a_e^2+a_m^2\right)^{1/2}\epsilon(\phi) \qquad 2.14$$

$$\left[\frac{\epsilon(\rho)}{\rho}\right]_{\%} = 100\ \exp\left(-\epsilon z/H_S\right)-100 \qquad 2.16$$

$$\epsilon(w_e) = \epsilon(V_e)-\frac{a_e}{a_z}\epsilon(C_z) - C_z\,\epsilon\left(\frac{a_e}{a_z}\right) \qquad 2.17$$

$$\epsilon(w_e) = \left\{\left[\epsilon(V_e)\right]^2+\left[\frac{a_e}{a_z}\epsilon(C_z)\right]^2+\left[C_z\,\epsilon\left(\frac{a_e}{a_z}\right)\right]^2\right\}^{1/2} \qquad 2.18$$

$$\epsilon_m\left(\frac{a_\rho}{a_z}\right) = \frac{\frac{a_\rho}{a_z} + \tan\epsilon(\phi)}{1 - \frac{a_\rho}{a_z}\tan\epsilon(\phi)} - \frac{a_\rho}{a_z} \qquad 2.21$$