the scanned object, the latter form of the curve is more favorable than the former because the overlaps between adjacent peaks will be smaller and thus make a general interpretation and, especially, the construction of the equilibrium curve much easier.

From the curves in Figs. 4–6 it follows that the equilibrium curve has to be constructed for the determination of the activity distribution. It is obvious, however, that a rough knowledge of the position of a peak can be obtained directly from the continously recorded curves provided the integrating time is not too long and the scanning speed not too high. The integrating time has a lower limit, which is, of course, set by the statistical errors, and which thus varies with the pulse rate. Such a rough knowledge is in many cases quite satisfactory, e.g., in ion exchange and paper chromatography when the interest is only in the rate of displacement of an active component along the ion exchange column (paper strip). Similarily, half-life determinations may be carried out just by following the activity of a certain point on the continuously recorded curve, preferably the peak value. A more accurate result is obtained by integrating the peak, background subtracted, measured at different times since this decreases the statistical errors. That the latter procedure will give the same result whatever the integrating time or scanning speed, is obvious from the fact that the areas under a continuously recorded curve and the corresponding equilibrium curve are the same as seen from Eq. (2). This method also excludes errors which may result from a peak height decrease due to other phenomena than radioactive decay such as diffusion of the active species in the ion exchange column or the like.

ACKNOWLEDGMENTS

I am greatly indebted to the Head of this Institute, Professor Manne Siegbahn, for his never-failing support and for putting laboratory facilities at my disposal, and to Professor Ole Lamm, Royal Institute of Technology, for his kind interest in this work.

THE REVIEW OF SCIENTIFIC INSTRUMENTS VOLUME 27, NUMBER 6 JUNE, 1956

Transit-Time Accelerometer*

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(Received January 30, 1956)

An omnidirectional transit-time accelerometer, developed for measuring the drag acceleration of spheres dropped from rockets, is described. The ambient density and temperature of air may be calculated from the drag acceleration. In the device, a bobbin is periodically caged and released within a cavity. The time for the bobbin to traverse the distance to the cavity, which distance is the same in any direction, is telemetered and measured. The accelerometer range is about 5×10^{-3} to 5 g. Systematic errors and standard deviations over the range are about 1%. The accelerometer was used successfully in a rocket flight in which the drag acceleration of a 7-in. diam sphere was measured.

I. INTRODUCTION

THE density of the upper atmosphere has been calculated from the measured drag acceleration of spheres dropped from rockets.[1,2] In four flights, the total acceleration of inflated nylon spheres 4 ft in diam was measured by DOVAP[3] (Doppler velocity and position) ground tracking and subtracted from the acceleration of gravity to obtain the drag acceleration. In order to measure drag acceleration directly, an omnidirectional transit-time accelerometer was developed with range and accuracy sufficient to measure the drag acceleration of a rigid 7-in. diam sphere. The sphere was dropped from 344 000 ft and accelerations were measured from 260 000 to 100 000 ft.

The acceleration of spheres might be measured with respect to various reference bodies. In the DOVAP case the earth is used. Free-falling or constrained bodies, either external to the sphere or internal, are possible references. The internal constrained-body type, for example, would correspond to a conventional spring-mass accelerometer. The external free-falling reference system could be realized by a two-body arrangement in which the drag of a large, light sphere is measured by an rf link with respect to a small, dense, nearly free-falling body released from the rocket simultaneously. Considerations of aerodynamic and mechanical simplicity, range of measurement, and accuracy led to the choice of the free-falling, internal-reference-body system, which arrangement constitutes a transit-time accelerometer.

* The research program under which the accelerometer was developed was carried out in the Department of Aeronautical Engineering under Air Force Cambridge Research Center Contract No. AF 19(604)-999 with the Engineering Research Institute of the University of Michigan.

[1] F. L. Bartman *et al.*, J. Atm. and Terrest. Phys., Spec. Suppl. 1 (1954).

[2] Bartman, Chaney, Jones, and Liu, J. Appl. Phys. (to be published).

[3] D. Hoffleit, Sci. Monthly **68**, 172 (1949).

II. OPERATION

Figure 1 is a schematic of the device and Fig. 2 a cut-away view of the construction. In operation, the coils are energized from a battery through an intervalometer switch. The fingers move toward each other, thus picking up the bobbin and centering it in the cavity. When the switch opens, the decay of the field in the field coils sends a large current through the moving coils, thus accelerating them rapidly outward and releasing the bobbin. The bobbin then accelerates freely toward the cavity. The time from the release of the bobbin to the instant of contact with the cavity is a measure of the relative acceleration of the cavity and bobbin, provided the center of rotation of the entire device (sphere) lies on the center of the caged bobbin. Electrical contact to the bobbin is made through a fine "cat's whisker" wire. Both the cavity and bobbin are figures of revolution shaped so that the distance traveled by the bobbin is the same in any direction (0.188 in.). In the rocket flight, the cycling time of the accelerometer was 150 msec for pickup, and 450 msec for maximum transit time. The transit times are telemetered from the sphere to the ground on the 220-mc band by means of a transmitter in the sphere. The sphere was arranged as a sort of dipole antenna by dividing it in two pieces with an equatorial insulating band of Teflon. The accelerometer is seen mounted in the sphere in Fig. 3.

III. DESIGN AND CONSTRUCTION

Many of the design limits were set by the size of the sphere, which, in turn, was determined by the size of the rocket. The vehicle used was an all-solid-propellent combination consisting of a Nike booster and Deacon sustaining rocket. The Deacon is 6 in. in diam so that its payload nose cone must be about the same. The sphere was finally designed 7 in. in diam, and the accelerometer 6 in. long. In order to provide room within the sphere for batteries, interval switch, transmitter, etc., the accelerometer diameter was limited to 2.25 in. Within the accelerometer in turn, having allowed for coils and other components, the transit distance of 0.188 in. was arrived at. Next, the drag trajectory of a 7-in. sphere dropped from the 350 000-ft predicted peak altitude of the Nike-Deacon was solved on an analog computer.[4] The solution indicated that densities to nearly 300 000 ft could be calculated if accelerations as low as 10^{-2} g were measured. It was also shown that maximum accelerations of about 5 g would be encountered, although 3 g would cover the region above 100 000 ft, which is of primary interest. The transit times corresponding to accelerations of 5, 1, and 10^{-2} g for a transit distance of 0.188 in. are approximately 14, 31, and 312 msec, respectively.

[4] F. L. Bartman, "DC analog computer survey of trajectories of spheres falling through the upper atmosphere," Association for Computing Machinery, Ann Arbor Meeting, June, 1954.

The cavity and magnetic circuit were constructed of soft steel, the bobbin of aluminum with a stainless-steel center bushing. The pickup fingers were made also of stainless steel. The cavity and bobbin were silver plated for electrical conductivity. Contact to the bobbin was made through a silver-wire spring 0.002 in. in diam. The moving coils were wound with 80 turns each of AWG 28 wire. The field coils were taken from commercial relays. In order to minimize magnetic disturbances to the bobbin, the cavity was made of soft steel. In order to minimize turbulent air disturbances to the bobbin, provision was made to evacuate the accelerometer; but this was found to be unnecessary.

IV. ERRORS

Four primary sources of error were anticipated in making and operating the accelerometer. These are initial velocity imparted to the bobbin during release, error in time measured, error in distance traveled, and centrifugal acceleration due to off-center mounting of the bobbin in the spinning sphere. The equation of motion of the bobbin is

$$a = 2(s - v_0 t)/t^2, \tag{1}$$

where a = acceleration, s = distance, v_0 = initial velocity, and t = time. The percentage error in acceleration resulting from v_0 is

$$\delta a_{v_0} = \frac{200 v_0}{at}\left(\frac{v_0}{(v_0^2 + 2as)^{\frac{1}{2}}} - 1\right). \tag{2}$$

The total systematic and probable errors measured in the laboratory at 5, 1, and 0.01 g were about 1%. Even if all of this error were due to v_0, it may be seen by Eq. (2) that $v_0 \ll (2as)^{\frac{1}{2}}$. This leads to the following simple expressions for percent errors in acceleration due

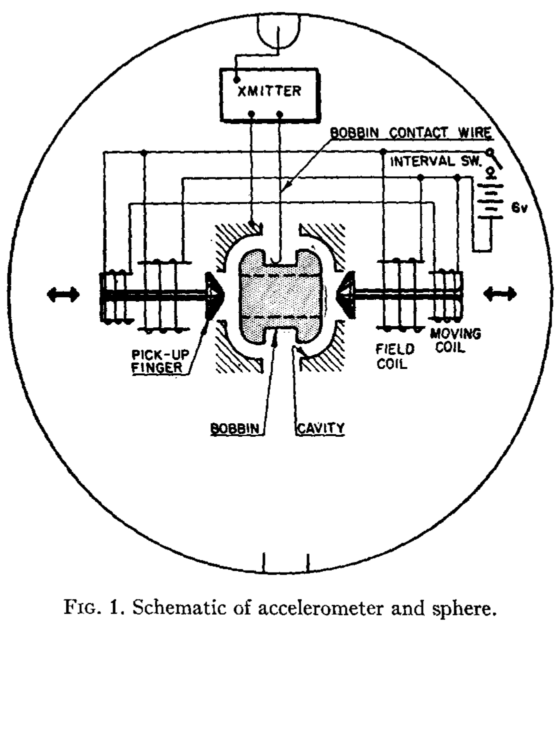

Fig. 1. Schematic of accelerometer and sphere.

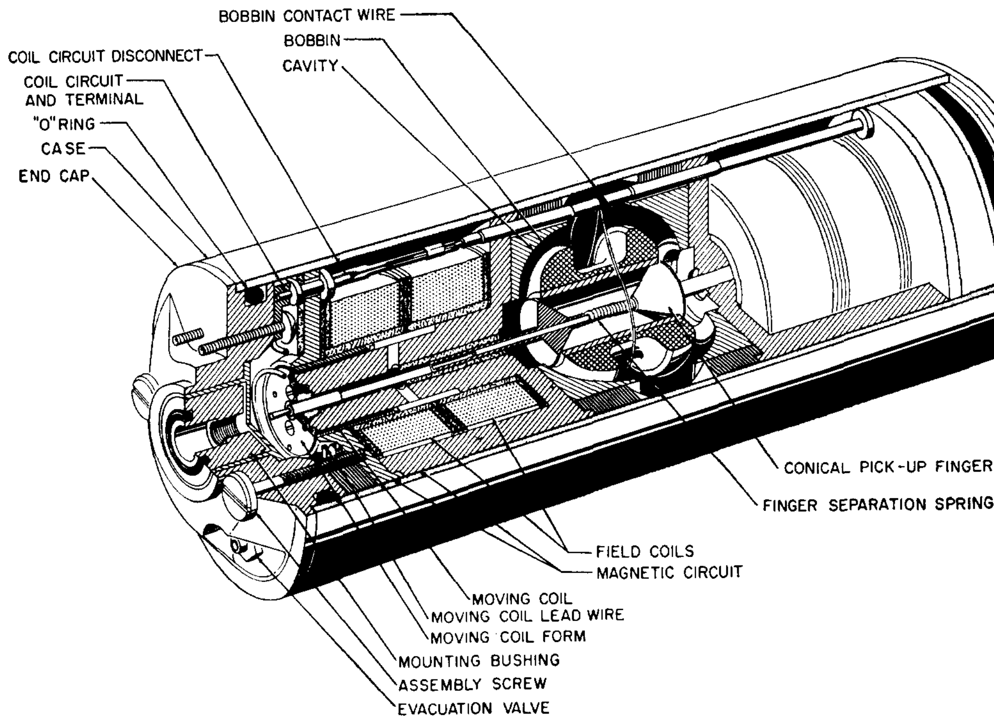

FIG. 2. Construction of accelerometer.

to v_0 and errors in t and s:

$$\delta a_{v0} = 200 v_0 / at, \quad (3)$$

$$\delta a_t = 200 \Delta t / t, \quad (4)$$

$$\delta a_s = 100 \Delta s / s. \quad (5)$$

Actual values of v_0 were not determined directly, but the magnitude of v_0 required to cause a 1% error in acceleration is seen from Eq. (3) to be

Acceleration	v_0
5 g	0.13 in./sec
1 g	0.060 in./sec
0.01 g	0.006 in./sec.

The transit times were measured with an electronic counter[5] capable of measuring time intervals to $\pm 10^{-5}$ sec. From Eq. (4), the percent errors in acceleration due to an error of 10^{-5} sec in time are

Acceleration	Percent error
5 g	0.14
1 g	0.064
0.01 g	0.006.

The accelerometer was constructed with shop tolerances of ± 0.001 in. or better on critical dimensions. It is reasonable to say that it could be assembled and adjusted so that the bobbin transit distance was within ± 0.002 in. of the design value. This would cause a 1% error at any acceleration.

[5] Hewlett-Packard Model 522B.

If the center of the caged boggin does not lie on the center of rotation (CG) of the spinning, tumbling sphere, an error due to centrifugal acceleration will be present. A rocket is said to spin about its longitudinal axis and tumble about a transverse axis through the CG. These motions are conserved by the sphere if the ejection is gentle. The sphere was mounted in the rocket in such a way that rocket spin would cause the sphere to spin about the longitudinal axis of the accelerometer and tumble about a transverse axis. Tumble periods are very large and may be ignored. Spin periods for the Nike-Deacon are of the order of 10 sec. In order that centrifugal accelerations be 1% or less of the 0.01 g lower limit of measurement, it was necessary to locate the CG of the sphere within 0.1 in. of the common geometrical centers of the sphere and caged bobbin. Measurements with a theodolite on the sphere suspended on a fine wire showed that the CG was within 0.05 in. of any diameter. This could have been improved with balancing masses, but it was not thought necessary to do so.

V. LABORATORY TESTS

The accelerometer is an absolute device which requires no calibration once it has been shown to be working correctly. Tests at 1.0 g showed that the omnidirectional characteristic was good.

Tests at 0.01 g were made by retarding the free fall of the entire accelerometer very slightly by means of a pulley. The accelerometer was attached to a massive

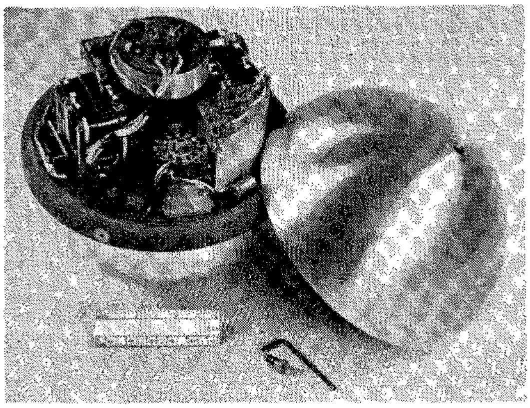

FIG. 3. Construction of sphere showing accelerometer in the center, transmitter to the right forward.

bullet, which was dropped through a distance of 4 ft. The air drag of the combination was calculated to be negligible. A thread attached to the accelerometer was wound around the pulley so that its moment of inertia provided a small retarding acceleration. The moment of inertia of the pulley was measured accurately by timing the fall rates of various light masses attached to the thread. The pulley friction was estimated by measuring the angular deceleration during spinning. The friction was found to be negligible.

Prior to flight, no tests were made at accelerations greater than 1 g except to substitute a steel bobbin for the aluminum one to show that the fingers would cage the tripled weight, and hence be capable of caging the aluminum bobbin under a drag acceleration of 3 g. The flight test showed the desirability of insuring operation up to 5 g. Consequently, a centrifuge was built for extending the test accelerations. Tests showed that it was extremely important that the battery and interval switch be in excellent condition for good operation of the accelerometer up to 5 g. Under these conditions, the errors are about the same as at 1 g and 0.01 g. In summary, the laboratory tests on two accelerometers showed that systematic errors and standard deviations (in 10 runs) are $\pm 1\%$ or better between 5 g and 0.01 g.

VI. FLIGHT RESULTS

One accelerometer was operated in a sphere carried in a Nike-Deacon rocket fired at the NACA site, Wallops Island, Virginia, on June 24, 1955. The sphere was ejected at 202 000 ft on the upleg and reached a

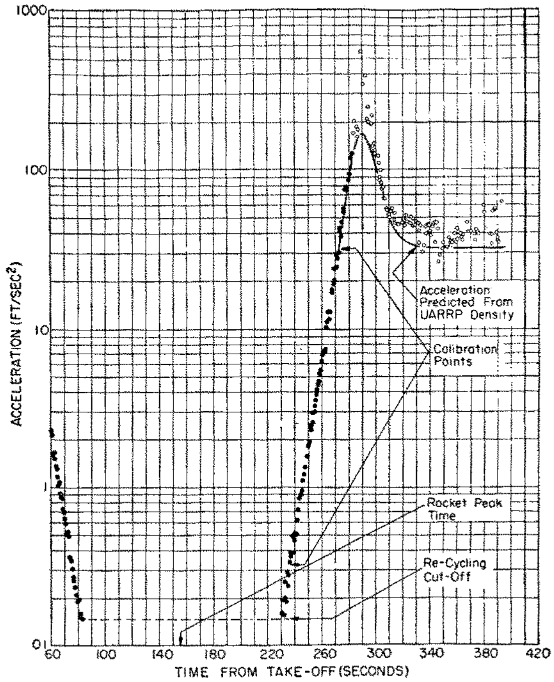

FIG. 4. Acceleration *vs* time of a 7-in. diam sphere ejected from a Nike-Deacon rocket at Wallops Island, Virginia, June 24, 1955, at 13:04 EST. Peak altitude of sphere, 344 000 MSL.

peak altitude of 344 000 ft. The accelerations measured are shown in Fig. 4. The cutoff at the low-acceleration end is caused by the recycling of the accelerometer. At this point, the accelerometer was successfully measuring 5×10^{-3} g. At 284 sec, the accelerations became impossibly high, perhaps because of exhausted batteries and/or defective switch contacts caused by continued arcing. Predicted accelerations for the region beyond 284 sec, based on the UARRP[6] density, are shown in the solid line. The lower envelope of actual points is approximately correct. This result is not unexpected, as the result of failure to completely cage the bobbin would be a short transit time, and hence apparent high acceleration. In any case, the valid measurements up to 284 sec covered the range 260 000 to 100 000 ft. The corresponding densities and temperatures are in excellent agreement with other rocket results and will be reported.[7]

[6] The Rocket Panel, Phys. Rev. 88 (1952).

[7] Bartman, Jones, Hansen, and Schaefer, "A new sphere method for upper-air density and temperature," J. Appl. Phys. (to be published).