Effects of pulse-length and emitter area on virtual cathode formation in electron guns[a)]

Ágúst Valfells,[b)] D. W. Feldman, M. Virgo, and P. G. O'Shea
Institute for Research in Electronics and Applied Physics, University of Maryland, College Park, Maryland 20742-3511

Y. Y. Lau
Department of Nuclear Engineering and Radiological Sciences, University of Michigan, Ann Arbor, Michigan 48109-2104

Recent experiments at the University of Maryland using photoemission from a dispenser cathode have yielded some interesting results regarding the effects of the area of emission and of the ratio between the pulse length and the gap transit time on the amount of current that may be drawn from an electron gun before a virtual cathode forms. The experiments show that a much higher current density may be drawn from a short pulse or limited emitter area than is anticipated by the Child–Langmuir limiting current. There is also evidence that the current may be increased even after virtual cathode formation, which leads a distinction between a limiting current density and a current density critical for virtual cathode formation. The experiments have also yielded some interesting results on the longitudinal structure of the current pulse passed through the anode. Some empirical and theoretical scaling laws regarding the formation of virtual cathodes in an electron gun will be presented. This work was motivated by the needs of the University of Maryland Electron Ring (UMER) [P. G. O'Shea, M. Reiser, R. A. Kishek *et al.*, Nucl. Instrum. Methods Phys. Res. A **464**, 646 (2001)] where the goal is to generate pulses that are well-localized in time and space. © *2002 American Institute of Physics.* [DOI: 10.1063/1.1463065]

I. INTRODUCTION

The subject of virtual cathode formation and limiting currents in charged particle beam diodes has been explored for almost a century. The advent of laser-driven electron photoinjectors has created renewed interest in the topic. In conventional electron guns with thermionic cathodes, the normal mode of operation is in the space-charge limited regime. Photocathodes in rf photoinjectors, however, are normally operated in the source-limited regime,[1] i.e., the beam current is limited by the laser illumination intensity rather than by space-charge. Another factor that makes the photoinjector different from a conventional diode is that in a photoinjector the pulse length is usually much less than the transit time of the electrons across the gun, whereas in a conventional diode the opposite is the case. There had been very little experience with operation of photoinjectors in or near the space-charge limited regime. The problem of the behavior of beams in photoinjectors at or near the space-charge limit is of some importance. In principle, one would like to extract as much current as possible from a given photocathode, however, previous work[1,2] has shown that significant degradation in the quality of the beam occurs as the space-charge limit is approached. Therefore, there would appear to be a practical current limit somewhat below the space-charge limit.

The familiar Child–Langmuir[3,4] formula gives the maximum current density that can be transported—without virtual cathode formation—across a planar diode with an anode–cathode spacing of D and potential difference, V. The maximum current density that can be achieved before virtual cathode formation is given by

$$J_{CL} = \frac{4\varepsilon_0}{9D^2}\sqrt{\frac{2q}{m}}V^{3/2}, \tag{1}$$

where q and m are the particle charge and mass, respectively. This law is derived using a planar model of a parallel plate diode of infinite area, and a steady state with the diode gap filled with charge. There is also no consideration of the effects of the initial velocity of the particles injected into the diode nor of magnetic field effects. Naturally, the question arises about the importance of these various assumptions, and how applicable the Child–Langmuir formula is to an actual electron gun. A substantial amount of work has been done on different aspects of this problem. The constraints on the emission velocity have been relaxed to include a Maxwellian distribution for the electrons emitted from the surface,[5] and for monoenergetic injection.[6] These refinements have resulted in a small modification of the limiting current except for the case where the injection energy is not small compared to the diode potential. Other refinements have shown that the 3/2 power law is extendable to cylindrical and spherical diodes and triodes.[5,7] The effects of magnetic field effects have been investigated as well.[6,8]

All of the previously listed treatments have assumed that the diode (or triode) is filled with charge, and that geometrical end-effects are lacking. Thus, an immediate concern is how well the Child–Langmuir formula applies to situations where these assumptions do not apply, e.g., in a electron gun with a Pierce-type geometry or in a situation where the length of a beamlet is less than the gap spacing of the diode into which it is injected. Virtual cathode formation for short pulse cases has been studied previously.[9] However, this work was done for drift tubes, whereas our work assumes that an applied field is present. There have also been some experimental observations of virtual cathodes in short pulse situations.[2] The problem of virtual cathode formation in a finite emitter has attracted some attention in the past few years both using simulations and analysis.[10–13] These investigations have all looked at a steady state solution of the problem, where the space charge extends between the anode and cathode, but the area of the emitter, in relation to the gap spacing, is variable. The results of the aforementioned work will be discussed in greater detail when compared to the new data to be presented.

An experiment, done at the University of Maryland, on the use of photoemission from dispenser cathodes to generate a perturbation for experiments on space-charge dominated beams drew the authors' attention to the problem of space-charge limited flow in a real gun. The experimental setup, though not tailored to the purpose, proved to be useful to test some aspects of this problem. In this paper results of experiments on virtual cathode formation and space-charge limited flow in an electron gun will be presented. Additionally, we will present some simple theoretical work on how short pulse lengths modify the Child–Langmuir formula. This latter aspect being of importance for photoinjector operation.

II. EXPERIMENT

A. Experimental setup and procedure

In our experiment we show a PAR LN1000 nitrogen laser (337nm wavelength) onto a heated WBaCO-dispenser cathode used as a photocathode in a Pierce-type gun. The output energy of the laser was 1 mJ in each 1.25 ns pulse with a repetition rate of 3 pps. An aperture was used to limit the output laser beam energy to 300 μJ. We could adjust the laser spot size with focusing optics, and the intensity of the laser with plastic sheets, to vary the emitter area and injection current. The cathode was heated to approximately 700 °C, at which point thermionic emission was negligible. The quantum efficiency of the cathode was found to be 7.4 $\times 10^{-5}$ at the laser wavelength. The lifetime of the cathode (defined as the time it takes for photoemission to drop by a factor of e^{-1}) was 30 hours. The cathode could be rejuvenated by heating it to 1020 °C. The diameter of the cathode was 2.54 cm and the anode–cathode gap was 2.54 cm. The accelerating voltage used in the experiment ranged from 1–9 kV. To measure the current from the electron gun, a Bergoz current transformer with a temporal resolution of 200 ps was used. The distance from the cathode to the center of the
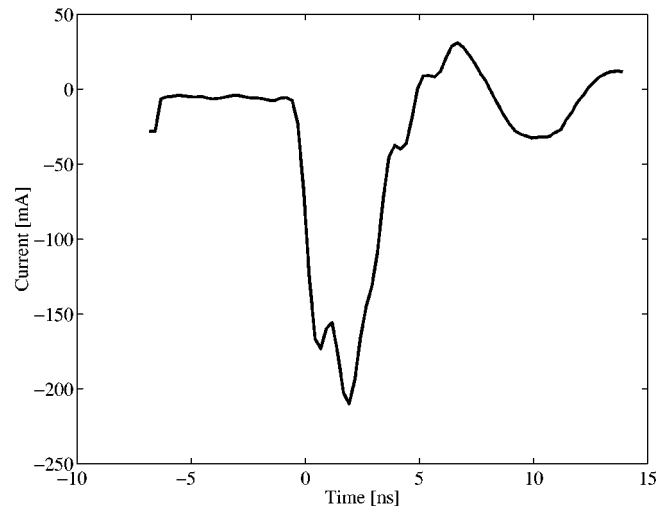

FIG. 1. Example of a current pulse with the virtual cathode manifesting itself through a dip in the pulse.

transformer is 14 cm. The background pressure was approximately 5×10^{-8} Torr.

The experimental process is the following: For a given accelerating voltage and spot size, we vary the intensity of the laser beam by attenuating it with transparent slides. We then capture the averaged pulse profile from the current transformer, as shown in Fig. 1. Note the ringing due to the instrumentation. From this we can calculate the total charge in the beam pulse and also look for evidence of virtual cathode formation. In Fig. 1 the dip in the pulse suggests that a virtual cathode has formed, as will be discussed later. This is repeated for various spot sizes and accelerating voltages.

B. Experimental results

When comparing the shape of the current pulse to that of the laser pulse, we see an interesting phenomenon that points to how virtual cathode formation affects the longitudinal profile of the electron beam. Figure 2 shows the normalized current pulse amplitude compared to the normalized laser pulse amplitude, when the laser intensity is 1% of its peak
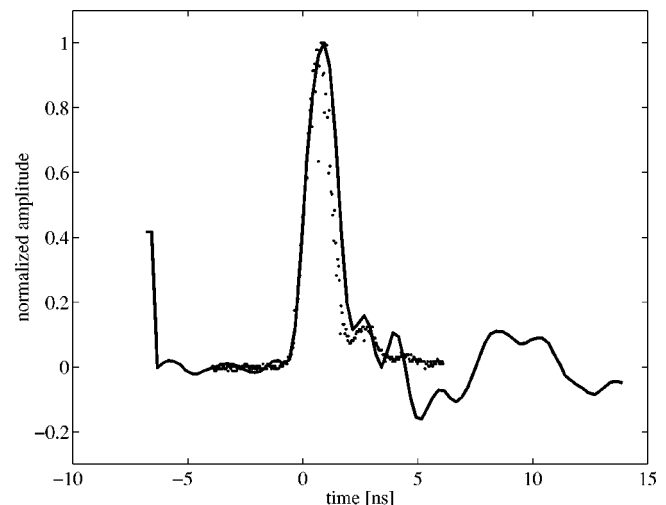

FIG. 2. Shape of the current pulse (solid line) compared with that of the laser pulse (dots) for an accelerating voltage of 9 kV, and the laser attenuated to 1% of its peak intensity.
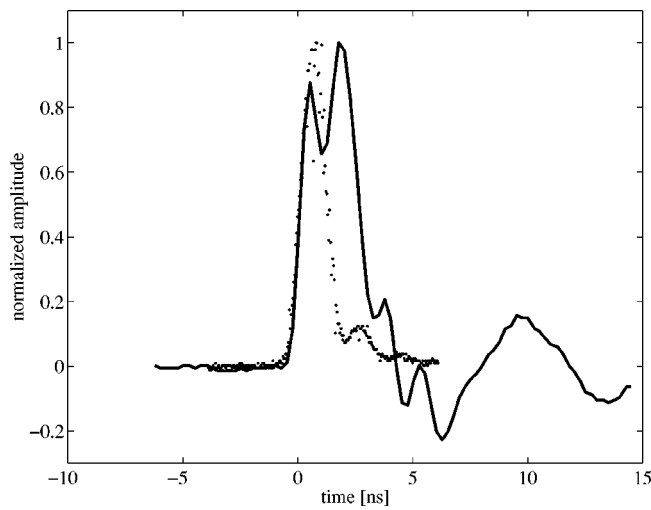
FIG. 3. Shape of the current pulse (solid line) compared with that of the laser pulse (dots) for an accelerating voltage of 9 kV, and the laser unattenuated.

FIG. 5. Total charge in current pulse versus relative laser intensity (relative intensity=1 is equivalent to unattenuated laser). Accelerating voltage is 5 kV, with laser spot covering entire cathode.

value. One sees how well the shape of the current pulse follows that of the laser pulse. By contrast, Fig. 3 shows the normalized current pulse amplitude compared to that of the normalized laser pulse amplitude, when the laser is unattenuated (i.e., the injected current is greater). Here we see that the pulse is extended and that it has a minimum midpulse, which is due to a virtual cathode oscillation. This is similar to behavior observed by Dowell.[2] Supporting evidence of this being a virtual cathode oscillation is shown in Fig. 4, which shows how the total charge in the current pulse, for an unattenuated laser beam, comes close to following the 3/2 power law predicted by Eq. (1), while that for the 99% attenuated laser pulse shows a much weaker dependence, due to the Schottky effect.

Let us now look at how the total charge in the electron beam coming out of the gun is affected by the injected cur-

rent density. Figure 5 shows the total charge in the current pulse as a function of the normalized laser intensity (which is proportional to the injected current) for an accelerating voltage of 5 kV, and with the spot size equal to the size of the cathode. Evidence of virtual cathode formation appears around the point where there is a bend in the curve—as indicated in the figure. Also note that the charge in the current pulse keeps increasing even after formation of the virtual–cathode, though the curve seems to tend to a horizontal asymptote. This curve is akin to the familiar curves showing emitted current versus cathode temperature for a thermionic gun. The onset of the virtual cathode is rather abrupt. For the case shown in Fig. 5 we see that as the laser intensity changes from 7% to 12% of the peak intensity, a virtual cathode forms. This can be seen more clearly in Fig. 6.
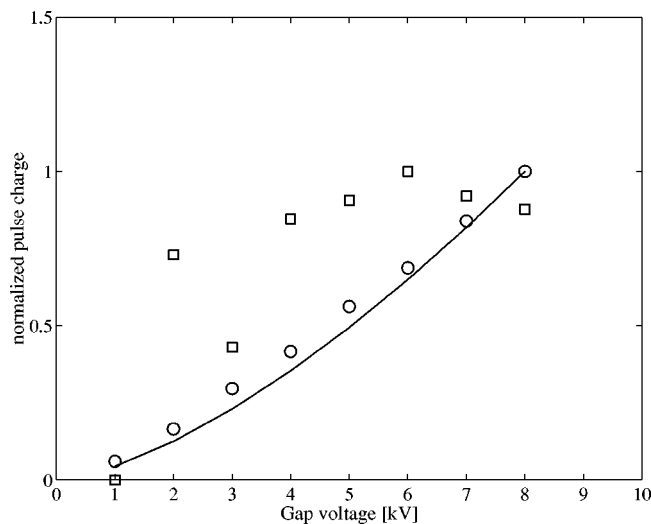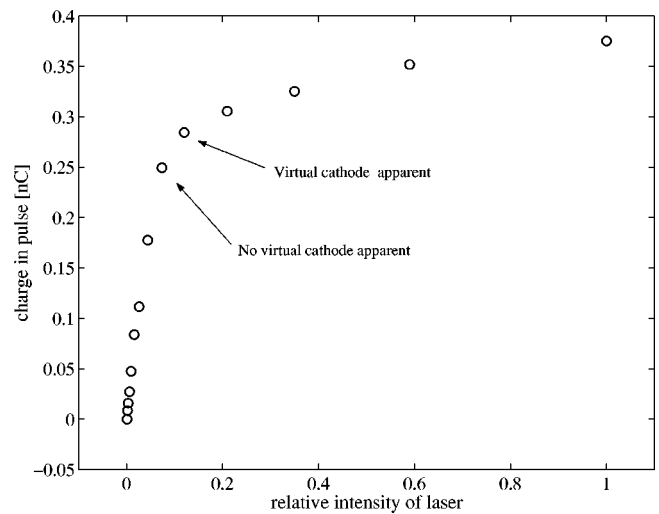

FIG. 4. Total charge in the current pulse as a function of the accelerating potential for unattenuated laser beam (circles) and laser beam with 99% attenuation (squares). The solid line shows the $V^{3/2}$ scaling that is anticipated for space-charge limited flow.
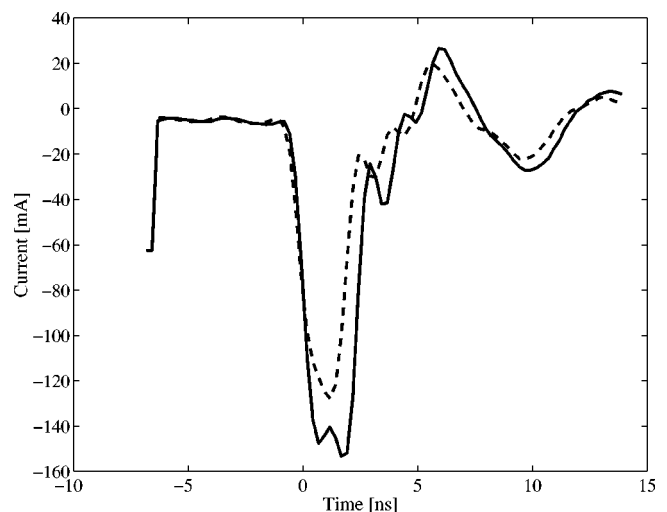

FIG. 6. Appearance of virtual cathode with decreasing laser attenuation. Accelerating voltage is 5 kV, with laser spot covering entire cathode. The solid line shows the current when the laser is at 12% of peak intensity. The dashed line shows the current when the laser is at 7% of the peak intensity.
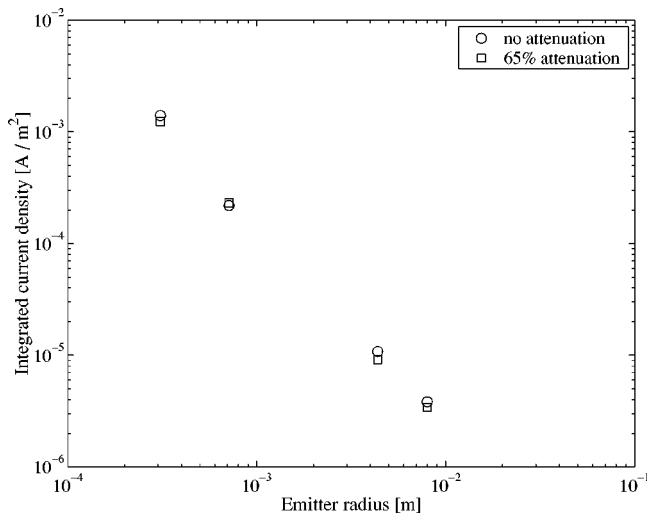
FIG. 7. Variation of integrated current density in the current pulse as a function of the laser spot size. Accelerating voltage is 5 kV.

The fact that the charge increases with increasing laser intensity, after virtual cathode formation, may be attributed to either one, or a combination of the following effects. First the virtual cathode starts forming in front of the center of the cathode while the outer portions of the emitting area can transport more current before forming a virtual cathode, as can be seen in Refs. 10–12. Second, it is possible that this is also due to a transient effect due to the fact that initially the pulse does not extend across the anode–cathode gap and subsequently more charge might be pushed towards the front end of the beam (this aspect will be discussed later in this paper). One should note that, unlike when the source is a thermionic cathode, increasing the injected current (laser intensity) does not change the average emission energy of the electrons (which would lead to a slight increase in the space-charge limiting current using the conventional Child–Langmuir equation).

This effect of the total charge coming from the gun increasing even after virtual cathode formation has taken place suggests that one should distinguish between a critical current for virtual cathode formation, and a limiting current that describes how much current can be pulled out of the gun.

Finally we will present some results that show how varying the emitter area affects the average current density. To see this effect, we measure the charge in the current pulse for a fixed accelerating voltage of 9 kV, varying the laser attenuation and spot size. When this is done, we find that the following scaling law for the points in the space-charge limited region (the region where a virtual cathode has formed):

$$\frac{Q}{\pi R^2} \propto R^{-1.8 \pm 0.03}, \tag{2}$$

where R is the emitter radius and Q the charge in the pulse. Figure 7 shows how the pulse charge divided by the area of the laser spot varies with the radius of the laser spot. One should immediately make note of one point that might skew our results somewhat—namely that as the laser is focused down to a smaller spot its intensity increases. Since we know

that the emitted current increases with increased laser intensity in the space-charge limited region, we may assume the exponent in Eq. (2) to be somewhat greater than -1.8 (the absolute value of the exponent should be smaller). It should also be mentioned that the ratio $Q/\pi R^2$ is roughly equivalent to an average value of the current density in the pulse, but not exactly, since the current pulse is slightly lengthened by space-charge forces with increasing laser intensity. It should be safe to state the following: The average value for the current density in space charge limited flow from diode with emitter area R is roughly proportional to $R^{-1.8}$. The fact that the exponent in the scaling law is greater than -2 is physically correct, since if it were not then the total current emitted from the cathode would become infinite as the emitter radius vanished. A practical point for those interested in obtaining bright beams is to note that since total current scales as the square of the emitter radius, the current in a pulse scales as $R^{0.2}$.

III. SHORT PULSE EFFECTS

We now turn our attention to the problem of determining the critical current density for virtual cathode formation in an electron gun where the beam pulse is short in comparison with the transit time through the gun. An example of this is an rf photoinjector. This is a matter of concern for those who wish to be able to obtain bright beams without longitudinal oscillations. We will approach this problem using two different approximations and comparing them to the results of computer simulations.

A. Single sheet model

In this case we assume that we have a diode of infinite area and gap spacing D. The potential difference across the diode is V. We assume the emission velocity of the electrons to be negligible. We will treat the electron pulse as a single sheet. Prior to the injection of the electron sheet, the electric field in the gap is simply given by $E = V/D$. Since the emission velocity is negligible, virtual cathode formation occurs when the electric field drops to zero behind the sheet of charge. The drop in the field strength is given by $\Delta E = \sigma/\epsilon_0$ where σ is the charge density of the electron sheet. Thus the condition for virtual cathode formation becomes $\sigma = V\epsilon_0/D$. Our assumption that the short pulse may be treated as a single sheet can be expressed as

$$\sigma = J\tau_p, \tag{3}$$

where J is the injection current (assumed to be uniform during injection) and τ_p is the pulse duration. From these considerations we get the following simple formula for the maximum current density, J_{crit}, allowed before virtual current formation for a pulse of length τ_p:

$$J_{crit} = \frac{\epsilon_0 V}{D\tau_p}. \tag{4}$$

We now define the vacuum transit time, T_V, as the time it takes for a single electron to cross the unloaded gap. It is given by

$$T_V = 2D \sqrt{\frac{m}{2qV}}. \tag{5}$$

Similarly, we may define the Child–Langmuir transit time, T_{CL}, as the time it takes for an electron to cross the gap under Child–Langmuir current conditions. It can be given by $T_{CL} = \frac{3}{2}T_V$. Thus we may write the equation for the critical current in the following manner:

$$J_{crit} = \frac{9J_{CL}}{8X_V} = \frac{3J_{CL}}{4X_{CL}}, \tag{6}$$

where $X_V = \tau_p/T_V$ and $X_{CL} = \tau_p/T_{CL}$ are normalized pulse lengths, and J_{CL} is the classic Child–Langmuir limiting current.

Rather than stating this problem in terms of gap voltage, V, and spacing, D, which is not applicable to an rf photoinjector for instance, one may rewrite Eq. (4) in terms of the surface electric field as

$$J_{crit} = \frac{\epsilon_0 E}{\tau_p}. \tag{7}$$

From Eq. (7) we can see that if one assumes that a short beamlet is being emitted, at the critical current density, the total charge in the beamlet, $Q = A\sigma$ is constant as the pulse length is varied. This means that the maximum charge one can emit, without virtual cathode formation, in a single pulse is independent of the pulse length under short pulse conditions.

B. Equivalent diode approximation

In this case we will make the same assumptions as those listed in the preceding section, except that we will treat the pulse not as a single sheet, but as a pulse of finite length. It is assumed that over a time period $0 < t < \tau_p$ a continuous injection of current, of density J_0, takes place. It is furthermore assumed that the magnitude of the injected current is such that, at the time $t = \tau_p$ the electric field at the cathode goes to zero. At this time the pulse extends a length $x = \xi$ into the anode–cathode gap (x being zero at the cathode and equal to D at the anode). We denote the potential at the front of the beam as ϕ_ξ, and the electric field at the front of the beam as E_ξ. One may immediately realize the following relations:

$$E_\xi = -\frac{J_0 \tau_p}{\epsilon_0} \tag{8}$$

and

$$E_\xi = \frac{\phi_\xi - V}{D - \xi}. \tag{9}$$

Next, we assume that one may look at the pulse as if it were filling an equivalent diode with space-charge limited current. The anode–cathode gap of this equivalent diode is equal to ξ, and the potential difference equal to ϕ_ξ. In other words the density profile in the beam is the same as it would be in the steady state for a diode with gap spacing ξ and potential

ϕ_ξ when the current density is that given by the Child–Langmuir formula. Using these parameters with Eq. (1) one may relate the parameters J_0, ϕ_ξ, and ξ. One may also use the well-known formula for the field strength at the equivalent anode

$$E_\xi = \frac{-4\phi_\xi}{3\xi}. \tag{10}$$

Using this information one may construct a second order equation in ϕ_ξ that has the physically meaningful solution

$$\phi_\xi^{1/2} = \frac{6D}{r\tau_p} + \sqrt{\frac{36D^2}{r^2\tau_p^2} - 3V}, \tag{11}$$

where $r = \sqrt{2q/m}$. One may write the following equation for the critical injection current density for a pulse length of τ_p in a diode of width D and potential difference V:

$$J_{crit} = 2\frac{1 - \sqrt{1 - \frac{3}{4}X_{CL}^2}}{X_{CL}^3}J_{CL}, \tag{12}$$

where $X_{CL} = \tau_p/T_{CL}$ is a normalized pulse length as before. For small values of the normalized pulse length Eq. (12) reduces to the single sheet solution, whereas it reverts to the traditional Child–Langmuir solution when $X_{CL} = 1$.

C. Computer simulations

We have used the XPDP1 particle-in-cell code[14] to model the short pulse problem described above. This code has one spatial dimension and two velocity dimensions. It is electrostatic and nonrelativistic. We use a model with an open diode, where we may vary the gap voltage, gap spacing, and injected current density. For given values of these parameters, we let the code run and monitor the number of particles in the simulation. As long as the beam front has not traversed the gap the number of particles in the simulation increases with time up to a point when a virtual cathode forms and the particles are returned to the cathode. In this fashion it is possible to determine the critical current density as a function of gap voltage, spacing, and pulse length. Results from these simulations are shown in Fig. 8, and compared to the short pulse and steady state approximations for the critical density as a function of normalized pulse length.

Clearly the match between the approximate theory and the simulation is quite good at shorter distances. In fact, the simple relation given in Eq. (7) should prove useful in most practical instances. One should note that simulations reveal that the time it takes for the beam front to traverse the anode–cathode gap is equal to the vacuum transit time for a single electron. One can clearly see that even well after the beam extends all the way across the anode–cathode gap, at a time equal to the vacuum transit time $X_{CL} = 2/3$, the critical current remains somewhat higher than the Child–Langmuir current. This is an indication of the time it takes for the system to reach the steady state. In fact, from the simulations it appears that it takes the system $4-6$ times the vacuum transit time to reach equilibrium.
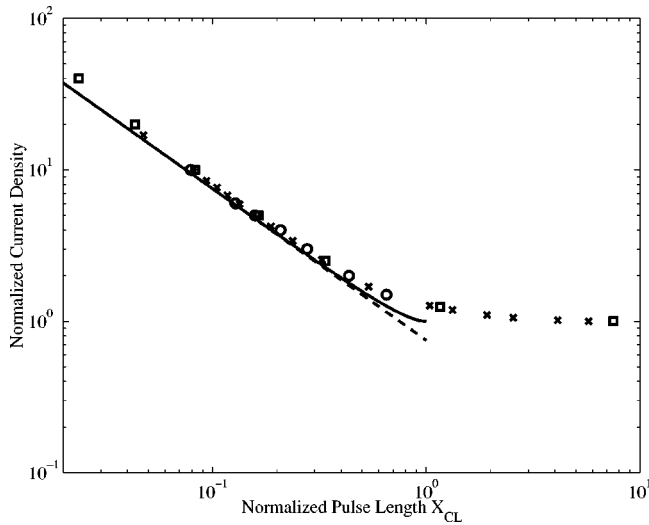
FIG. 8. Normalized critical current density, J/J_{CL} as a function of the normalized pulse length, X_{CL}. Comparison of two approximate models [dashed line from Eq. (6), solid line from Eq. (12) with XPDP1 simulation (circles for voltage=100 V, spacing=10 cm; crosses for voltage=100 V, spacing=5 cm; squares for voltage=1 kV, spacing=2.5 cm].

IV. COMMENTS

Let us now summarize the main findings presented in this paper, and look at where there is room for improvement. We have sought to better understand the conditions for virtual cathode formation in electron guns, as the virtual cathode may affect the longitudinal structure of the beam (most often in adverse fashion).

First, we have shown experimentally that virtual cathode formation occurs when the injected current reaches a critical current density. At this point we may surmise that the current emitted from the gun (beam current) is equal to the current emitted from the cathode (injected current). As the injected current is increased the beam current may increase asymptotically up to a limiting current which may be as much as 30% greater than the critical current. If one desires to generate a clean beam pulse it is necessary to operate below the critical current density. We have also observed that in the space-charge dominated region, i.e., when a virtual cathode is present, the integrated beam current density scales approximately as $R^{-1.8}$, where R is the radius of the emitter area. This means that the total charge drawn from an emitter is weakly dependent upon its area. Due to limitations of our experimental setup we have not been able to measure the critical current density accurately enough to test the theoretical result obtained by Lau and Luginsland that the critical current density is given by[10,11]

$$J_{crit} \approx \left(1 + \frac{C}{L/D}\right) J_{CL}, \tag{13}$$

where C is a constant depending on the shape of the emitter, L is the characteristic length of the emitter, and D the gap spacing of the diode.

We have devised reasonably accurate (compared to computer simulations) and, above all, simple scaling laws to de-

termine how the critical current density behaves in short pulse situations. For most practical cases one may use Eq. (7) to determine how the critical current density behaves as a function of the strength of the surface electric field, and of the pulse length. From these considerations one may anticipate that the maximum amount of charge that may be generated without virtual cathode formation, in a short pulse, is independent of the pulse duration. We have not been able to conduct an experiment on the short pulse effect, since signal to noise considerations limit us to using an accelerating voltage no less than 1 kV. This corresponds to a vacuum transit time of 2.7 ns which is comparable to the pulse duration.

Another issue, which needs to be investigated further, is whether the short pulse and limited emission area effects are independent of each other or not. If they are independent one might assume that the equation for the critical current, for short pulses, would be of the form

$$J_{critical} \propto \frac{E}{\tau_p R^n}, \tag{14}$$

where E is the surface electric field, τ_p the pulse duration, R the radius of the emitter area, and n ranging from 0 to 2. Simulations of transient effects in a finite area diode are underway, and it is hoped that they will shed some light on the question of independence of the two effects.

The reader should also note that virtual cathode formation in a gun is inherently a nonrelativistic effect. For instance, in a typical rf photoinjector the head of the beam will have an energy on the order of 10 keV when emission is terminated.

ACKNOWLEDGMENTS

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