

per proved the dislocation clusters to be relatively stable, so that the small changes seen between Fig. 2b and 2d may be attributed to the decoration itself.

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RESOLUTION-RETRIEVING SOURCE-EFFECT COMPENSATION IN HOLOGRAPHY WITH EXTENDED SOURCES

(x-ray diffraction microscopy; image synthesis;
"3-dimensional photography"; T/E)

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In ref. 1 we showed that the resolution "loss" which would result from the use of an extended source T_s , in place of a "point" source, in the recording of a hologram, can be compensated, and the resolution "retrieved", provided that the source $T_{s'}$ used in the reconstruction is such that $(T_{s'} \star T_s^* = \delta)$, a delta function, where \star indicates a correlation¹ and where $T_{s'}$ and T_s are complex amplitude distributions.² In analogy with comparable problems arising in apodisation,³ spectroscopy,⁴⁻⁶ and cases of "matched filtering in noncoherent light",⁵⁻⁷ we also noted¹ that there should exist a class of "structured" source functions, satisfying the preceding correlation condition, in high-resolution holography with *coherent* light.¹

It is of a particular interest to show that a "structured" source, suitable for holography, may indeed be realized by interference, which may be necessary in microscopy, and notably at x-ray wavelengths. It has been known for some time, in connection with filtering in noncoherent light,⁶⁻⁸ that Fresnel-zone plates, and Fresnel-zone half-plates (the portion on one side of a diameter of a Fresnel-zone plate) may have the suitable correlation property^{1,5} and that they can be produced by interference.^{9,10}

Figure 1 shows the hologram obtained with a Fresnel-zone half-plate (itself previously obtained by interference), and Fig. 2 shows the well-compensated reconstruction of the side-band image¹¹ $[T_0 \otimes (T_{s'} \star T_s^*)]_{\xi-a}$ of the object T_0 (where $T_{s'} = T_s$, in this case, and where \otimes indicates a convolution;¹ a is the "offset" between the object and the source¹¹ in the object plane ξ).

The mathematical proof of the compensation property of "structured" sources such as the Fresnel-zone half-plate is straightforward.¹ However, the physical principles, as they apply to holography, may be readily demonstrated by noting the analogy of a Fresnel-zone plate with a lens of two principal foci (a real one and a virtual one).^{6,9} A Fresnel-zone

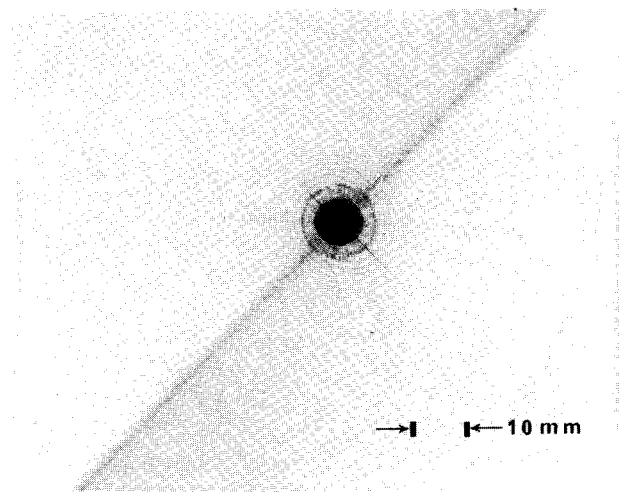


Fig. 1. Fourier-transform hologram (with focal length $f = 600$ mm)¹¹ obtained with a "structured" 19-mm "diameter" "Fresnel-zone half-plate" source, used in place of the conventional "point" source, in the recording. Note that the two halves of the hologram (appearing separated by a "diagonal" in the figure) were each illuminated by one-half of a spherical wave, respectively originating from the real and virtual foci of the zone plate. Alternate equivalent nonstructured extended sources (e.g. two half - "field lenses"), used in place of the structured Fresnel half-plate source may also be suitable, as are full Fresnel-zone plates.

half-plate produces, from the two foci, two separated "halves" of spherical waves, of different radii, each wave covering one half of the hologram (see Fig. 1). In the reconstruction, illumination of the hologram with the same spherical half-waves, produces, by transmission through each appropriate half of the hologram, precisely the desired compensation, through a ($t_s t_s^* = 1$) equation, over the hologram, equivalent to the required ($T_s T_s^* = \delta$) condition, mentioned in our ref. 1 and above (t_s is the Fourier transform of T_s).

General classes of functions, which may be used to determine suitable source structures for compensation in holography, have been investigated by Jacquinet and Roizen-Dossier in connection with "a posteriori apodisation",³ with which we wish to note the analogies,¹ as we do with Gabor's method of suppressing the "twin" image, in holographic microscopy, by the use of strongly aberrated wavefronts.¹²

Both the recording of the hologram (Fig. 1) and

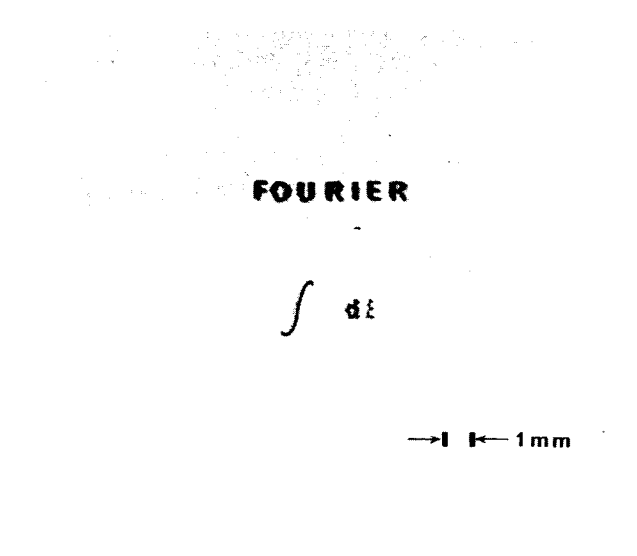


Fig. 2. Compensated image, reconstructed by Fourier transformation,^{1,11} by illuminating the hologram of Fig. 1 with the same extended source, as used in the recording of Fig. 1.

the reconstruction of the image (Fig. 2) were carried out in 6328-Å laser light on Polaroid P/N film in the Fourier-transform holography arrangements,^{1,11,13-17} of which the high-resolution advantages were previously pointed out^{7,11,13-17} and the "lensless" form in our refs. 7, 11, and 15. The Fresnel-zone plate (with a "fringe" interval of about 5 μ at the 19-mm diam) was recorded by interference on a high-resolution Kodak 649F plate.

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ERRATUM

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