Vortex pair generation and interaction with a free surface

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Two vertical, rotating flaps are used to generate a vortex pair beneath a free surface. Vortex pair formation, propagation, and interaction with a free surface are described. Numerical simulations for inviscid flow about a constant upwash sheet of vorticity beneath a free surface agree with experiment up to the time that turbulent mixing occurs during interaction with the free surface. The spacing between the vortex pairs then becomes larger than the calculated spacing. The experiments and lines of marked particles included in the simulations show fluid ingestion and transport toward the free surface.

The interaction of vorticity with a free surface is being studied to understand better the surface disturbances produced in the wake of a ship. Sarpkaya and Henderson[1] (and later Sarpkaya[2]) studied the wake of a delta or rectangular wing moving parallel to and at various distances beneath the surface. They identified and described surface disturbances produced by an ascending pair of vortices approaching a free surface.

The primary surface interactions observed by Sarpkaya and Henderson[1] and Sarpkaya[2] were termed striations and scars. The first surface disturbances (the striations) were alternate surface elevations and depressions in the form of narrow bands normal to the direction of motion which grew to a length of order of the vortex pair spacing. The striations appeared when the depth of the vortex pair was approximately equal to the separation distance between the vortices. The surface disturbances termed scars were long, narrow surface depressions outboard of the location of the vortex cores on either side of the wake. The scars appeared when the depth of the vortex pair was approximately 0.6 times the spacing between the vortices. As the vortices approached the surface they moved apart carrying the scars ahead of them and farther apart. This was followed by a breakup of the scar pattern accompanied by a complex and rapid three-dimensional interaction between the vorticity and the free surface.

When the angle of attack of the wing is small, the rollup of the trailing vorticity and the propagation of the resulting vortex pair toward the surface will not be seriously affected by streamwise pressure gradients (with possible vortex breakdown), allowing a two-dimensional study of the process.

A vortex pair generator has been devised which consists of two vertically oriented flaps linked to a pneumatic actuator that causes the upper tips of the flaps to rotate toward each other until they touch and form a triangle with a small apex angle. A sketch of the vortex pair generator is shown in Fig. 1. The flaps were made from 0.188 cm thick stainless steel. The distance from the tip of a flap to the center of rotation was 8.26 cm and the flap span was 61 cm. When the flap tips were touching the included angle between the flaps was 31.3°. The tips of the flaps were 10.2 cm below the surface. The outer side of the tip of each flap was beveled to a 40° included angle, the inner side was flat, and the flaps were mounted between glass endplates.

The final flap configuration (with tips touching) is well below the vortex pair and approximately parallel to the streamlines of the flow. The flaps do not appear to interfere with the rollup process. This scheme was devised after reading the paper by Barker and Crow,[3] who used a piston between two parallel plates to generate a vortex pair and reported that the rollup process was inhibited by the piston. They were able to overcome this difficulty by constructing a mechanism that retracted the piston from the two-dimensional flow region after the piston motion ceased.

Recently, Sarpkaya *et al.*[4] reported on the construction and use of a vortex pair generator with two counter-rotating flaps hinged to a rigid surface beneath and parallel to the free surface. When actuated, the flaps rotated toward each other until they came to rest parallel to the surface on which they were hinged. The vortex pair generated in this fashion is created in the proximity of a rigid surface. The Kelvin oval is very stable, always moves vertically, and the wall (bottom) effect becomes rapidly negligible.

A thin (≈ 1 mm) sheet of blue, argon–ion laser light with a wavelength of 488 nm directed downward through the free surface was used to illuminate a two-dimensional cross section of the flow in the center of the flaps. The fluid motion was visualized with the aid of a solution of fluorescein dye or with a cloud of 40 μm diam titanium dioxide particles injected in the water between and outside the flaps before the vortices were generated. The flow visualized with dye was photographed with a Bolex motion picture camera

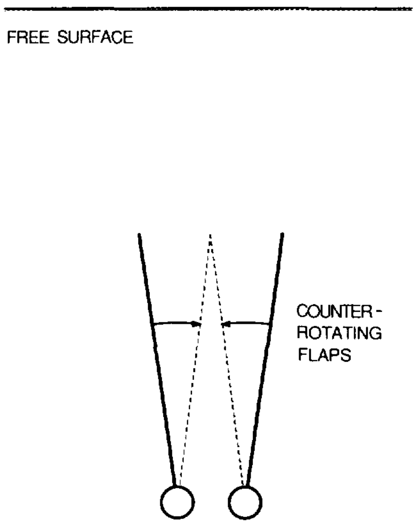

FIG. 1. Sketch of the flap arrangement.

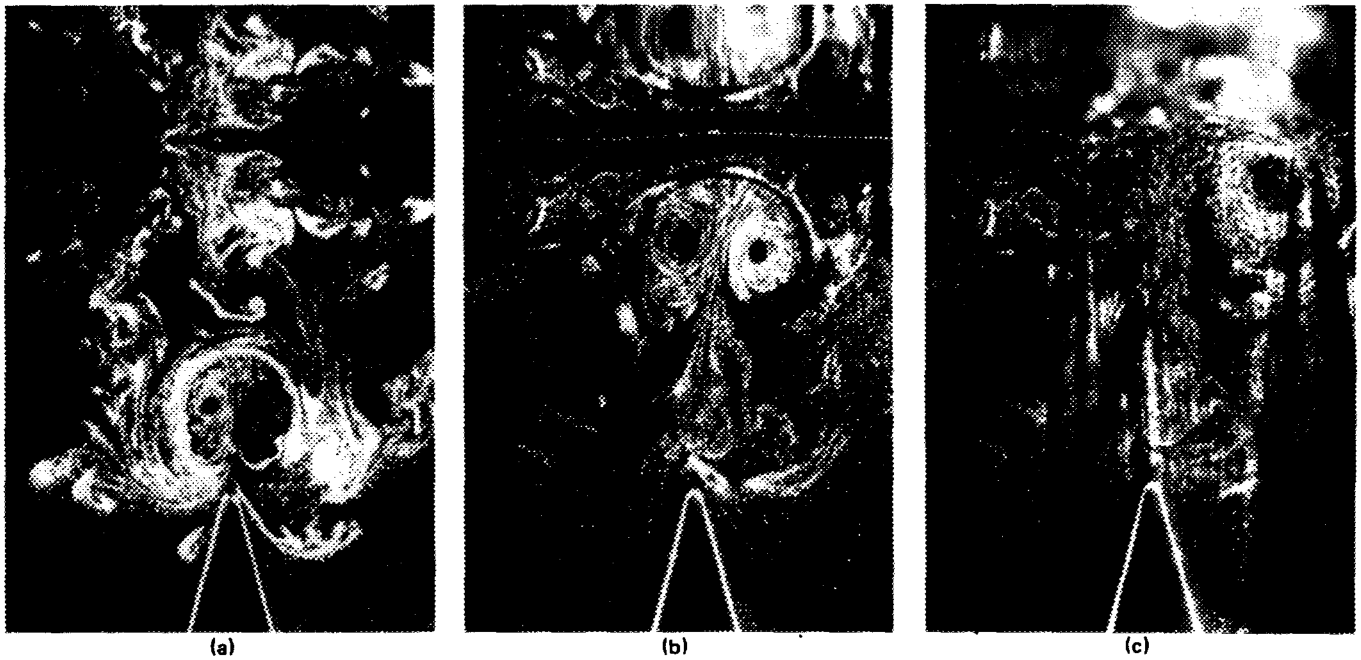

FIG. 2. (a) Vorticity rollup and ingestion of fluid. (b) A vortex pair propagating toward the free surface and the beginning of surface interaction. (c) A vortex pair during surface interaction and deposition of fluid near the surface.

at 64 frames per second with a 75 mm, $f/1.4$ lens using Kodak 4X film. The flow visualized with titanium dioxide particles was photographed at 3.33 frames per second with a motorized, 35 mm, Nikon camera with a 50 mm, $f/1.4$ lens using Kodak Tmax ASA 400 film exposed at 1/500th of a second.

The numerical simulation was done using a generalized vortex method, similar to the method discussed in Tryggvason.[5] The only difference is that here we represent the free surface by a distribution of dipoles instead of a vortex sheet. This generally leads to more stable calculations, as observed by Baker, Meiron, and Orszag.[6]

In preliminary trials of the vortex pair generation mechanism the flaps were approximately parallel to the surface with a gap of a few centimeters between the tips. When rotated rapidly through an angle approximately equal to 10°, so that the flap tips moved downward, a vortex pair propagating toward the surface was produced. The initial flap motion produced a downward surface motion that modified the free-surface motion caused by the interaction of the vortex pair with the free surface.

The vertically oriented flaps produced negligible initial motion of the free surface when rotated toward each other through a small ($\approx 15°$) angle. The vortex pair produced with a given flap rotation rate was considerably stronger than with an initially horizontal flap orientation. At high flap rotation rates or for large values of the depth-to-spacing ratio, the flow became turbulent before the vortex pair arrived at the free surface.

For this initial study the flaps were rotated through an angle of approximately 9° until the tips were touching at a rate which produced a vortex pair that was not turbulent when surface deformation was first observed. Figures 2(a)–2(c) are photographs of the flow visualized with titanium

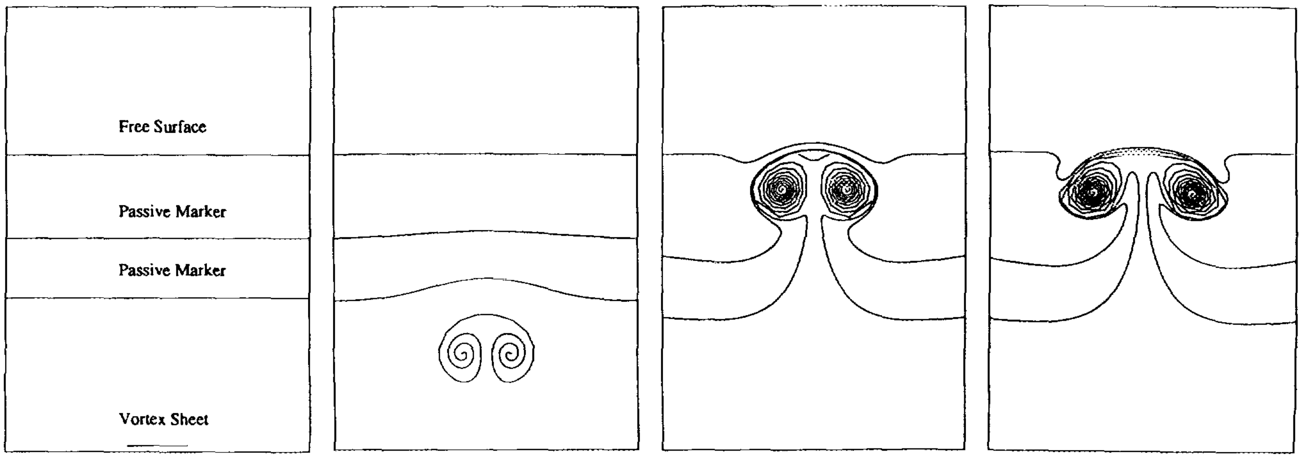

FIG. 3. Numerical simulations of the formation of a vortex pair and the subsequent interaction of the pair with the free surface. The first two frames are at early times and the last two frames are at late times.

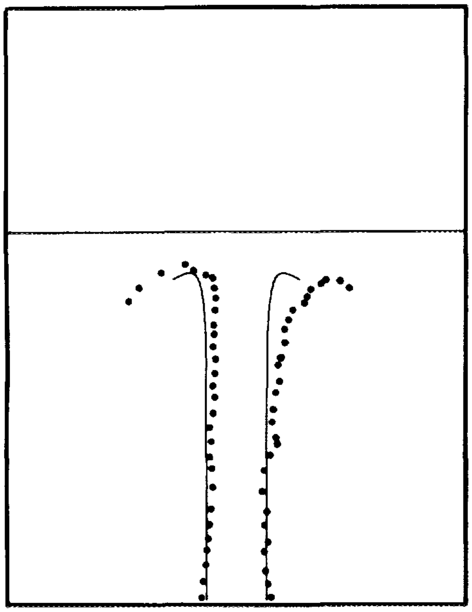

FIG. 4. A comparison of the path of the vortices as predicted numerically and observed experimentally: solid line, numerical results; dots, experimental observations.

dioxide particles injected into the water before the flaps were rotated. In Fig. 2(a) the flaps have just touched and the vortex pair is ingesting fluid during the process of formation. The free surface can be observed as a horizontal line. In Fig. 2(b) the vortex pair is fully formed and the initial deformation of the free surface can be observed. The fluid containing titanium dioxide particles ingested in the region of formation beneath the surface has been transported toward the surface within the Kelvin oval about the vortex pair. Figure 2(c) shows the deposition of this fluid on the surface. The fluid motion is turbulent and three-dimensional surface deformation is evident, especially on the left-hand side of the photograph. The illuminating sheet of laser light entered the water through the free surface, which acts as a mirror and reflects a portion of the illuminated sheet into the field of view of the camera. In Fig. 2(a) the reflected image is clear and the free surface is flat, but in Figs. 2(b) and 2(c) the reflected image is blurred (indicating three-dimensional surface motion), especially in Fig. 2(c). The curvature of the free surface also acts as a lens and this is the cause of the vertically oriented light and dark regions visible in the water beneath the surface in Fig. 2(c).

A motion picture of the flow produced with the same initial flap motion was projected on a large sheet of paper. The location of each vortex center, the Kelvin oval, and the free surface were traced on the paper for every other frame during the formation, propagation, and surface interaction phases of the flow. The ratio of the depth of the vortex pair below the free surface to the vortex spacing was $h/D = 5.4$ when the formation phase was complete, and $h/D = 3.5$ when the vortex pair spacing began to increase and surface motion was first observed. The circulation was estimated (assuming that the vortices were point vortices) from the average speed of propagation ($U = 17$ cm/sec) and the average distance ($D = 1.9$ cm) between the centers of the vortices before surface motion began. The circulation was approximately $\Gamma = 203$ cm^2/sec. The Froude number based on the propagation speed and spacing of this vortex pair was $F = (\Gamma/D)/(gD)^{1/2} = 2.47$.

Numerical simulations modeling the experimental observations are presented in Fig. 3. The initial condition is a constant upwash vortex sheet (wake of a wing with elliptic loading) at the same depth as the tip of the flaps (in units of vortex separation) with the same Froude number as the experiment. The vortex sheet rolls up into two counter-rotating vortices that propagate toward the free surface. As the vortices encounter the surface they are deflected in a way that is akin to vortices encountering a rigid surface. On the free surface, depressions are formed outward of the vortices that increase in depth with time, and move outward. Two horizontal lines of passive material particles were included in the calculations to show how the vortex pair carries fluid from below up to the surface. The predicted spacing between the vortices and the shape of the fluid carried with the vortex pair agrees rather well with the experimental results, up to the time when the vortices become turbulent. Turbulent mixing in the vortical flow alters the vortex pair configuration and the path of the vortices deviates significantly from the calculated path. Figure 4 compares the calculated path with the experimental one.

It should be noted that these initial results are somewhat lacking in precision because the flap generation apparatus was a preliminary model. The vortex pairs were not of equal strength since the path of the pair was not exactly vertical and the vortex interaction with the free surface was not symmetrical [see Fig. 2(c)]. The flap rotation rates were not equal because the mechanism transmitting torque to the flaps was not perfectly symmetrical. An improved vortex pair generation mechanism has been designed and will be used in further investigations. We plan a more detailed study of the flow and surface interaction for a wide variety of Froude numbers.

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