Dynamics of phase-locked semiconductor laser arrays

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Time-dependent coupled mode theory is used to investigate the stability of phase-locked semiconductor laser arrays. The output of individual array elements is dynamically unstable and exhibits large amplitude chaotic pulsations. The total output initially exhibits damped relaxation oscillations and then settles down to a quasi-steady state characterized by small amplitude fluctuations. The theory predicts both the pulsation frequency and the phase lock-in time of the array.

Phase-locked semiconductor laser arrays have been demonstrated as sources that can produce high output power in a spatially coherent beam.[1] The spatial mode patterns observed[2] under steady-state conditions are predicted rather well by coupled mode theory which leads to a description of the field profiles in terms of array modes or supermodes.[3,4] While the steady-state behavior of phase-locked arrays has been the subject of numerous investigations, there is relatively little data available on the transient response of these arrays. Recent streak camera measurements have shown that the output of individual elements of the array exhibits irregular, undamped spiking behavior on a 100 ps time scale.[5] The temporal stability of array operation is thus an issue that merits further investigation.

In this letter, we present a theoretical analysis of the temporal behavior of phase-locked semiconductor laser arrays. The analysis is based on semiclassical laser theory[6] modified to include terms that describe evanescent wave coupling between adjacent elements of the array. The numerical results are in excellent qualitative agreement with the available experimental data on the dynamic characteristics of laser arrays.[5] In particular, the theory shows that the output of the individual array elements is dynamically unstable and exhibits chaotic pulsations. The theory also yields the period of the pulsations and the establishment time of phase locking, both of which are inversely proportional to the coupling coefficient between array elements.

An array of N coupled lasers is considered. When isolated from its neighbors, each laser supports a single transverse and longitudinal mode assumed same for all the lasers. The temporal evolution of the carrier density (N_i) and of the amplitude (E_i) and phase (ϕ_i) of the electric field in the ith channel of the array is described by

$$\frac{dE_i}{dt} = \frac{1}{2}\left(\Gamma G(N_i) - \frac{1}{\tau_p}\right)E_i - \frac{\kappa c}{n} \times \left[E_{i+1}\sin(\phi_{i+1}-\phi_i) + E_{i-1}\sin(\phi_{i-1}-\phi_i)\right], \tag{1}$$

$$\frac{d\phi_i}{dt} = \omega(N_{\rm th}) - \omega(N_i) + \frac{\kappa c}{n}\left(\frac{E_{i+1}}{E_i}\cos(\phi_{i+1}-\phi_i) + \frac{E_{i-1}}{E_i}\cos(\phi_{i-1}-\phi_i)\right), \tag{2}$$

$$\frac{dN_i}{dt} = \frac{J}{ed} - \frac{N_i}{\tau_s} - \Gamma G(N_i)|E_i|^2. \tag{3}$$

Here the field amplitude is normalized such that $|E_i|^2$ gives the photon density in the ith channel. The constant parameters in the above equations are the coupling coefficient κ, mode confinement factor Γ, the photon lifetime τ_p, the carrier lifetime τ_s, the injection current density J, the width d of the active region, and the electronic charge e. The gain $G(N_i)$ and the laser frequency $\omega(N_i)$ depend on the carrier density through

$$G(N_i) = G(N_{\rm th}) + \left(\frac{\partial G}{\partial N_i}\right)(N_i - N_{\rm th}), \tag{4}$$

$$\omega(N_i) = \omega(N_{\rm th}) + h\,(N_i - N_{\rm th}), \tag{5}$$

where $N_{\rm th}$ is the threshold carrier density, $h = d\omega/dN_i = (\omega/n)dn/dN_i$, and the derivatives are evaluated at threshold. In writing down these equations, we have neglected a small term representing spontaneous emission into the lasing mode. Its inclusion in our simulations produced negligible effects.

It should be noted that even though the isolated laser stripes all have the same single frequency, once they are coupled the oscillating frequency splits into N different values, one for each supermode. In the absence of nonlinearity, these

TABLE I. Parameters used in the numerical simulations.

Parameter	Symbol	Numerical value	Unit
Current density	J	3×10^{6}	A/m^2
Active layer thickness	d	300	Å
Carrier lifetime	τ_s	2	ns
Photon lifetime	τ_p	1.6	ps
Threshold carrier density	$N_{\rm th}$	1.0×10^{24}	m^{-3}
Refractive index	n	3.6	···
Coupling coefficient	κ	317	m^{-1}
Gain at threshold	$G(N_{\rm th})$	1.24×10^{12}	s^{-1}
Differential gain	$\partial G/\partial N$	0.856×10^{-13}	m^3/s
Rate of change of resonant frequency	h	2.14×10^{-12}	m^3/s
Fraction of power in active region	Γ	0.46	···

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supermodes would simply beat against one another leading to a periodic or quasiperiodic output in time. The fundamental beat frequency is given approximately by $(2/\pi)(\kappa c/n)$. It should also be noted that the amplitude and phase equations (1) and (2) are identical to the equations that describe FM laser oscillation and active FM mode locking.[7] Similar multimode behavior is thus expected from the coupled array.

The time-dependent coupled mode equations have been solved numerically by means of a program geared for stiff differential equations. A ten-element array was simulated, which required the solution of 30 coupled nonlinear differential equations. The values of the parameters used are given in Table I and the numerical results for a step input current are presented in Figs. 1–3.

Figure 1 shows the single-shot intensity of an individual array element as measured in the near field. The trace shows strong relaxation oscillations initially and then undamped spiking behavior for the duration of the injection current pulse. The pulsations are chaotic and show no sign of damping out even after times as long as 50 ns, which is 20 times the carrier lifetime. These results agree with the experimental observations of Elliot *et al.*[5] who made single-shot streak camera measurements of the pulsed output of individual array elements.

The total output of the array, also measured in the near field, is shown in Fig. 2. Unlike the output of the individual elements, the total intensity exhibits relaxation oscillations which damp out to a quasi-steady state characterized by a much reduced level of fluctuations.

Figure 3 shows the output of an individual array element averaged over ten shots. The fields were given random initial phases which varied from shot to shot. It is seen that the average output intensity from individual elements exhibits reduced fluctuations. This result also agrees with the experimental observations.

It is clear from these results that even though the total output of a phase-locked laser array may be steady in time, the individual stripe intensities can continue to pulsate in an irregular manner. Such behavior is generic to multimode lasers and is due, in this case, to competition between array supermodes.

The spiking behavior in the first 10 ns is simply the collective relaxation oscillations of the semiconductor laser ensemble. The characteristic frequency of this oscillation is governed by the rate of energy transfer between electrons and photons in a single laser stripe. This frequency is proportional to the square root of the driving current, inversely proportional to the geometric mean of the photon and carrier lifetimes, and independent of the coupling between laser stripes. After the initial relaxation oscillations, the individual array elements enter a new oscillatory regime characterized by the rate of energy transfer between elements. The frequency of this oscillation is proportional to the coupling coefficient $\kappa c/n$. For a coupling constant of $\kappa = 100\ \mathrm{m}^{-1}$, the pulsation frequency is in the 10 GHz range, which is in agreement with the experimental value obtained by Elliott *et al.* Measurement of the pulsation period may provide a means of determining the coupling constant κ.

In the absence of frequency-pulling effects (i.e, $h = 0$), the coupling pulsations would be periodic, or at worst, quasiperiodic. The dependence of the resonance frequency on the excitation level within each laser stripe ($h \neq 0$) causes the array to behave as a set of coupled nonlinear oscillators. For moderate values of the coupling strength, the pulsations are chaotic. A fast detector that can resolve the output of individual array members will thus measure behavior akin to mode partition noise: individual modes show random behavior in time while the total intensity is steady. The difference

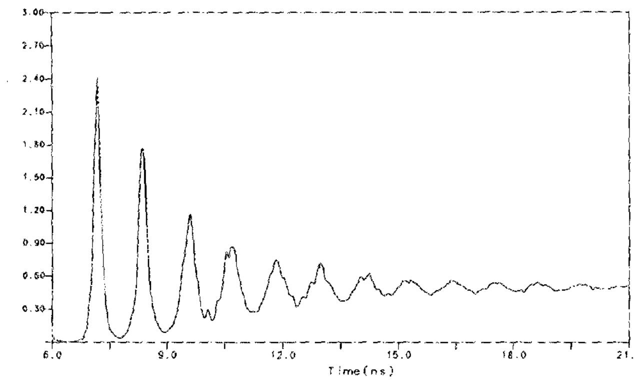

FIG. 2. Time evolution of the total intensity (near field) emitted in a single pulse by a ten-element array.

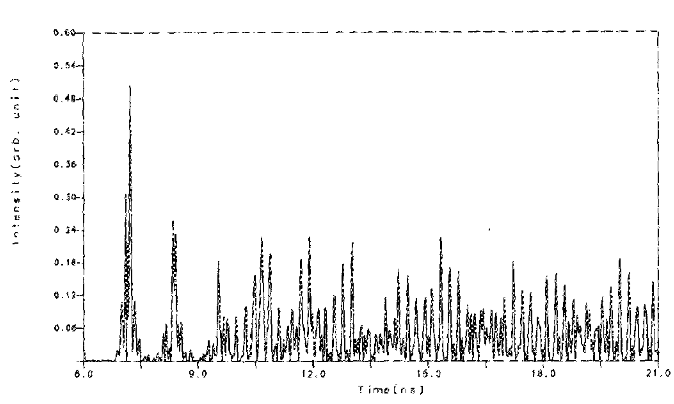

FIG. 1. Single-shot intensity of light emitted by an individual laser in the array (stripe No. 1).

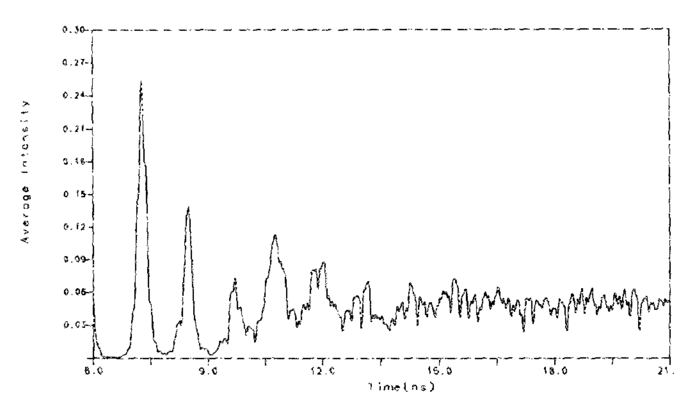

FIG. 3. Ten-pulse average of the intensity emitted by a single element in the array (stripe No. 1).

here however, is that while mode partition noise arises from stochastic fluctuations, the behavior observed in this simulation is completely deterministic.[8]

An important question concerning phase-locked arrays is how long it takes for phase locking to be established after the initiation of lasing. That question can be answered by following procedures similar to Adler's analysis of injection locking. An approximate solution of the phase equation (2) shows that, to first order, locking is established with a time $(\kappa c/n)^{-1}$ following the initiation of lasing. Details of these calculations and of the evolution of the far-field patterns will be presented in a future publication.

The results presented here have important implications for the use of semiconductor laser arrays in applications where temporal stability is important. Laser arrays are intrinsically unstable. The dynamic instability seen here will adversely affect the modulation behavior of these arrays. On a more fundamental note, laser arrays offer the possibility for direct observation of spatio-temporal chaos in a system of coupled nonlinear oscillators.

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