

# Effects of non-uniform charge injection on gain, threshold current, and linewidth enhancement factor for a 1.55 $\mu\text{m}$ InP-based multiple quantum well laser

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In laser structures where the active region consists of several quantum wells, non-uniform charge injection can occur. We examine the consequences of non-uniform charge injection on gain, threshold current, and linewidth enhancement factor. Non-uniform charge injection in a InP-based multiple quantum well laser was considered in order to analyze effects on gain, threshold current, and linewidth enhancement factor. We find that although the best values for gain, threshold current and linewidth enhancement factor occur under uniform charge injection conditions, these parameters do not suffer significant degradation under even highly non-uniform charge injection.

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## I. INTRODUCTION

Multiple quantum well (MQW) and strained MQW lasers show improvements in material gain and differential gain over conventional double heterostructure lasers, leading to improvements in threshold current, modulation bandwidth, and linewidth enhancement factor. In the design of long wavelength lasers (say operating at 1.55  $\mu\text{m}$ ), Auger recombination is quite important at room temperature. Due to the strong carrier density dependence of Auger recombination rate, it is important that the carrier density in the active region,  $n_{th}$ , at lasing be as small as possible. This can be accomplished by using an active region which has a large number of quantum wells. The large number of wells allows a larger optical confinement factor for the laser so that the gain per well needed to overcome losses becomes small. This in turn reduces  $n_{th}$ . As a result MQW-based active regions are widely used in InP-based 1.55  $\mu\text{m}$  systems.

Recently, very high performance 1.55  $\mu\text{m}$  lasers have been demonstrated using an active region with 8 wells. For lasers designed for longer wavelengths, the optimum well number may be even higher. An important issue that has to be addressed in such lasers is the following: since the laser operates in the forward bias mode where electrons are injected from the  $n$ -side and holes from the  $p$ -side, it is expected that the injection density will be non-uniform in the active region when the well number is large. How important are the effects of non-uniform injection? In this paper we address the concern raised above. We examine the effects of non-uniform charge injection on gain, spontaneous emission rate, threshold current density, and linewidth enhancement factor. We find that with increasing non-uniformity, gain and spontaneous emission rate decrease. Additionally, for increasing injected carrier density, the linewidth enhancement factor under non-uniform charge injection conditions becomes greater than that under uniform charge injection. However, the overall degradation of the laser in terms of threshold current density is rather minimal.

The organization of this paper is the following. In the next section we discuss the structure examined and the formalism used. In Section III, we present our results. Finally, we conclude in Section IV.

## II. FORMALISM

In this section we will discuss the laser structure used in our studies, present the phenomenological model used to simulate non-uniform injection and present the formalism used to examine the effect of non-uniform injection on laser performance. Note that this paper does not address the question of how non-uniform injection occurs or what its dependence on the structure chosen, temperature, biasing, etc., is. We confine our studies to answering the question: If non-uniform injection occurs at varying degrees how important is it for the laser operation?

The structure we choose for study is a zero-net strain 8 well 1.55  $\mu\text{m}$  InGaAsP-InP MQW laser, with 80  $\text{\AA}$  wells and 100  $\text{\AA}$  barriers, using 0.38% compressive strain in the wells and 0.303% tensile strain in the barriers.<sup>1</sup> A schematic for the active region is shown in Fig. 1(a). We assume in steady-state the injected electron and hole densities follow exponential profiles which mirror each other with respect to the  $n$ - and  $p$ -injection sides. In our phenomenological model used we represent the carrier densities by a form

$$n_i = N \exp L(i-1)/L_d \quad (1)$$

where  $n_i$  is the number of injected carriers in the  $i^{\text{th}}$  well,  $N$  is the number of injected carriers ( $n$  or  $p$ ) in the first well on the  $n$ - or  $p$ -injection side,  $L$  is the distance between from the beginning of one well to the next, and  $L_d$  is a decay length. For uniform injection  $L_d$  is infinity.

In each case of uniform or non-uniform injection, we obtain the gain and spontaneous emission rates for each individual well by first calculating the bandstructure using a  $4 \times 4 \mathbf{k} \cdot \mathbf{p}$  Hamiltonian<sup>2</sup> to describe the confined valence band states; the conduction band is described using the parabolic approximation with an electron of effective mass  $m^*$ .<sup>3,4</sup> The effect of strain due to lattice mismatch is

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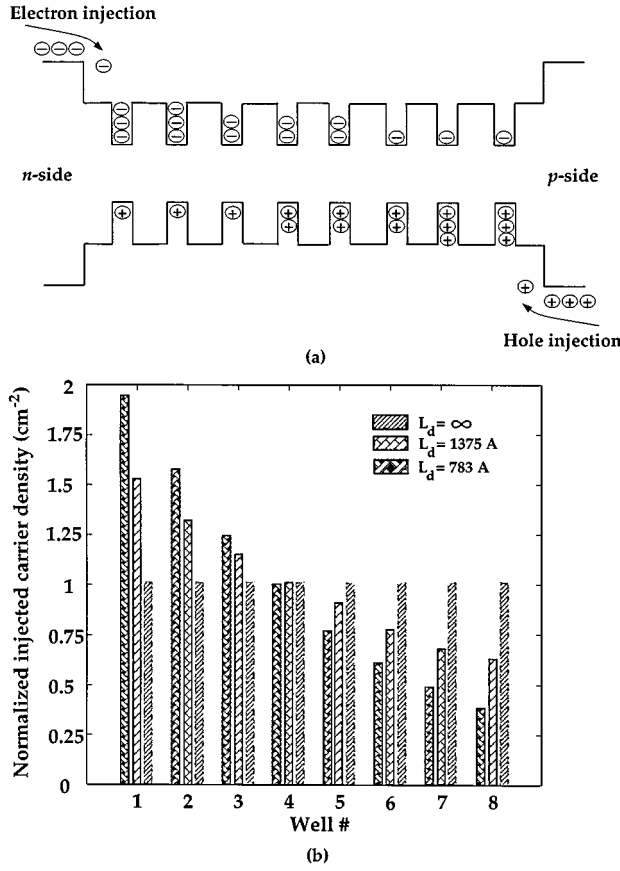


FIG. 1. (a) Visualization of non-uniform charge injection into MQW structure and (b) normalized injected electron density distribution for an 8-well MQW laser for the uniform and non-uniform charge injection cases considered in this study, from the  $n$ -injection side.

included.<sup>4</sup> After calculating both electron and hole density of states functions, the Fermi Golden Rule was applied and for each well the subband-to-subband optical gain and spontaneous emission rates were obtained:<sup>5</sup>

$$g_{nm}(n_i, p_i, E) = \frac{4\pi^2 e^2 \hbar}{n_0 c m_0^2 W E} \frac{1}{(2\pi)^2} \int d\mathbf{k} \times \sum_{\sigma} |\hat{\mathbf{e}} \mathbf{P}_{nm}^{\sigma}(\mathbf{k})|^2 \delta(E_n^e(\mathbf{k}) - E_m^h(\mathbf{k}) - E) \cdot [f^e(E_n^e(\mathbf{k})) + f^h(E_m^h(\mathbf{k})) - 1], \quad (2)$$

$$R_{sp}(n_i, p_i) = \int dE \frac{4e^2 n_0 E}{3m_0^2 c^3 \hbar^2} \frac{1}{(2\pi)^2} \sum_{nm} \int d\mathbf{k} \times \sum_{\sigma} |\mathbf{P}_{nm}^{\sigma}(\mathbf{k})|^2 \delta(E_n^e(\mathbf{k}) - E_m^h(\mathbf{k}) - E) \cdot [f^e(E_n^e(\mathbf{k}))][f^h(E_m^h(\mathbf{k}))], \quad (3)$$

where  $g_{nm}(n_i, p_i, E)$  is the optical gain at photon energy  $E$  from the  $i^{th}$  well with electron density  $n_i$  and hole density  $p_i$  from the  $n^{th}$  electron subband and  $m^{th}$  hole subband,  $R_{sp}(n_i, p_i)$  is the spontaneous emission rate from the  $i^{th}$  well,  $e$  is the electron charge,  $n_0$  is the refractive index of the active region,  $c$  is the vacuum speed of light,  $m_0$  is the free

electron mass,  $W$  is the active region width,  $\hat{\mathbf{e}}$  is the polarization of the light,  $\mathbf{P}_{nm}^{\sigma}$  is the optical matrix element, and  $f^e(E_n^e(\mathbf{k}))$  and  $f^h(E_m^h(\mathbf{k}))$  are the distribution functions for electrons and holes in the  $n^{th}$  and  $m^{th}$  subbands with momentum  $\mathbf{k}$  and energies  $E_n^e(\mathbf{k})$  and  $E_m^h(\mathbf{k})$ , respectively. To get the total gain and spontaneous emission rates for a particular case, then, we assume

$$g_{tot}(n_{tot}, E) = \sum_{i=1}^8 g(n_i, p_i, E), \quad (4)$$

$$R_{sp_{tot}}(n_{tot}) = \sum_{i=1}^8 R_{sp_i}(n_i, p_i), \quad (5)$$

where  $g_{tot}(n_{tot}, E)$  is the total gain from all wells at total carrier density  $n_{tot}$ ,  $g(n_i, p_i, E)$  is the gain from the  $i^{th}$  well after summing over all subbands and convolving with a Lorentzian to account for collision broadening due to carrier-phonon and carrier-carrier scattering, and  $R_{sp_{tot}}$  is the total spontaneous emission rate from all wells.

In these lasers the quantum well width is less than 2 orders of magnitude smaller than the lasing wavelength. Since the light experiences gain only in the well regions, we characterize the laser optical gain by an optical confinement factor per well  $\Gamma_w$ .

We approximate the threshold current density as<sup>6</sup>

$$J_{th} = e[A n_{th} + R_{sp}(n_{th})] + J_A(n_{th}), \quad (6)$$

where  $n_{th}$  is the total three-dimensional threshold carrier density,  $R_{sp}(n_{th})$  is the total spontaneous emission rate at threshold,  $A$  is the Shockley-Read-Hall non-radiative recombination coefficient, and  $J_A(n_{th})$  is the Auger recombination current density at threshold. We assume at threshold stimulated emission is negligible compared to spontaneous emission. In InP-based devices the CHHS process dominates the Auger recombination rate. Hence, Auger recombination takes the form

$$J_A = e C_A \sum_{i=1}^8 n_{th_i} p_{th_i}^2, \quad (7)$$

where  $C_A$  is the Auger recombination coefficient, and  $n_{th_i}$  and  $p_{th_i}$  are the corresponding three-dimensional electron and hole densities, respectively.

The linewidth enhancement factor,  $\alpha$ , via the Kramers-Kronig relations, is a measure of the resonance between the differential gain spectrum and the peak of the gain spectrum, and can be written<sup>7</sup>

$$\alpha = \frac{d\chi_R(n)/dn}{d\chi_I(n)/dn}, \quad (8)$$

where  $\chi_R(n)$  and  $\chi_I(n)$  are the real and imaginary parts of the complex susceptibilities of the active region, respectively. From the Kramers-Kronig relations, their derivatives can be given as<sup>7</sup>

$$\frac{d\chi_I(n)}{dn} = \int dE \frac{dg(n, E)}{dn} \frac{\Delta E/2\pi}{(E - E_l)^2 + (\Delta E/2\pi)^2} \quad (9)$$

$$\frac{d\chi_R(n)}{dn} = \int dE \frac{dg(n, E)}{dn} \frac{(E - E_l)}{(E - E_l)^2 + (\Delta E/2\pi)^2}, \quad (10)$$

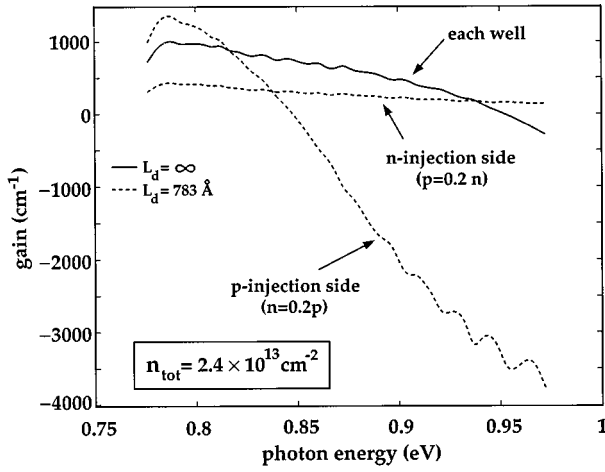


FIG. 2. Single well gain spectrum of the well closest to the  $n$ -injection side and the well closest to the  $p$ -injection side for  $L_d=783 \text{ \AA}$ , and single well gain spectrum of well under uniform injection. The slight irregularities in the gain curves are a consequence of the numerical procedure and have no noticeable effect on the quoted results.

where  $E_l$  is the lasing photon energy,  $\Delta E$  is the collision broadening energy due to carrier-carrier and carrier-phonon interactions, and  $dg(n,E)/dn$  is the differential gain.

### III. RESULTS

In this section we will present the results obtained for various degrees of non-uniform injection.

The decay length,  $L_d$ , is used as a variable in our study to introduce a nonuniform charge distribution. In this study, we chose the decay length value such that the number of holes in the  $n$ -injection side well (or the number of electrons in the  $p$ -injection side well) is 20% ( $L_d=783 \text{ \AA}$ ) of, 40% ( $L_d=1375 \text{ \AA}$ ) of, or equal to ( $L_d=\infty$ ) the number of electrons in the  $n$ -injection side well. We show the carrier distribution profile from the  $n$ -injection side in Fig. 1(b). We have found that the exponential profile appears to be a simple, yet good, approximation when compared to more rigorous simulations involving the carrier transport equations and laser rate equations.<sup>8,9</sup>

Under non-uniform injection, in some wells the electron density is larger than the hole density while in others the hole density is larger than the electron density. Since the hole effective mass is much larger (in this material system) than the electron effective mass, an increase in the number of holes in a well will have a larger effect on the Fermi hole distribution function than an equal increase in the number of electrons, due to the positioning of the (hole or electron) quasi-Fermi energy level via the density-of-states function. Therefore, under non-uniform injection, we expect for a given total carrier density the  $p$ -injection side well contributes significantly more to the total peak gain than the  $n$ -injection side wells. In Fig. 2, we show the gain-energy curves for the uniform injection case and for a non-uniform injection case ( $L_d=783 \text{ \AA}$ ). In the non-uniform case we show the gain-energy relation for the well closest to the  $n$ -injection side (where  $p=0.2n$ ) and for the well closest to the  $p$ -injection side ( $n=0.2p$ ). The criteria for positive gain,

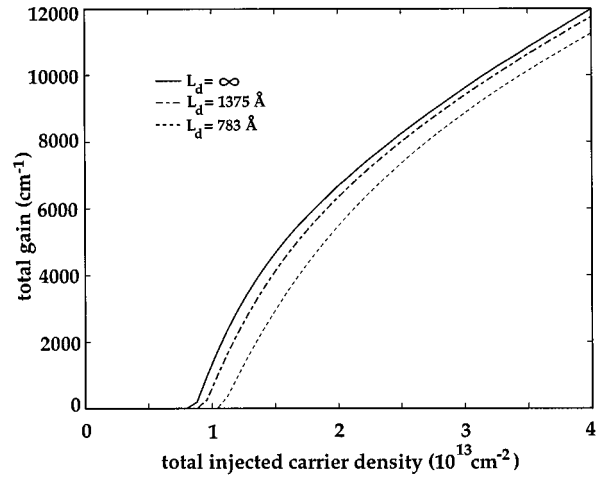


FIG. 3. Total peak gain versus total injected carrier density for an 8-well quantum well laser for the uniform and non-uniform charge injection cases considered in this study.

$$E_{fn} - E_{fp} > E, \quad (11)$$

is satisfied for greater photon energies at the  $n$ -injection side well than at the  $p$ -injection side well with  $L_d=783 \text{ \AA}$ , since the electron density-of-states is smaller. This is also shown in Fig. 2.

Using Eq. (4), we calculate the total peak gain versus total carrier density from all wells for the distributions plotted in Fig. 1(b). We show the results in Fig. 3. We find that the total peak gain decreases as decay length decreases, although the variation in total peak gain between the different decay lengths is only  $\sim 10\%$ . This can be explained via our phenomenological model. Following the discussion of the preceding paragraph, we expect the gain at the lasing energy  $E_l$  of the  $i^{\text{th}}$  well to be primarily dependent on the hole density of the  $i^{\text{th}}$  well. Our calculations show

$$g(n_i, p_i, E_l, L_d \neq 0) + g(n_{8-i+1}, p_{8-i+1}, E_l, L_d \neq 0) < 2g(L_d=0), \quad (12)$$

i.e., at  $E_l$  the gain from any well  $i$  and its complementary well  $8-i+1$  (since the electron and hole distributions mirror each other) under non-uniform injection is less than the gain from any 2 wells under uniform injection. Hence, the total peak gain under uniform injection is always larger. We expect the difference between the uniform injection and non-uniform injection cases to remain small even if the electron and hole distributions do not mirror each other (from their respective injection sides), as  $L_d=783 \text{ \AA}$  incorporates a large degree of nonuniformity.

Equations (3) and (5) enable us to calculate the total spontaneous emission rates for the different decay lengths. We show our results in Fig. 4. We find that the spontaneous emission rate also is larger under uniform injection than non-uniform injection, which follows from the discussion given in the preceding paragraphs. Although we find that the total peak gain under uniform injection is larger, this result implies a small difference in threshold current density for different decay lengths.

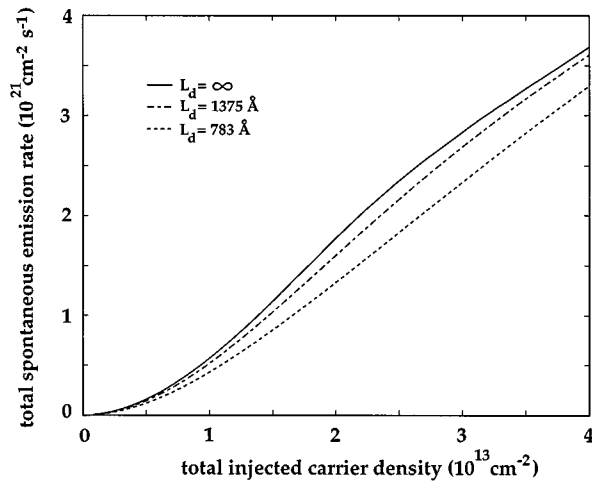


FIG. 4. Total spontaneous emission rate versus total injected carrier density for an 8-well quantum well for the uniform and non-uniform charge injection cases considered in this study.

From Eqs. (6) and (7) we calculate the threshold current density. We assume typical values for  $A$  and  $C$  of  $10^8 \text{ s}^{-1}$  and  $10^{-28} \text{ cm}^6/\text{s}$ , respectively. For the structure presented in Section II we calculate  $\Gamma_w = 0.0144$ . In Fig. 5 we show the threshold current density,  $J_{th}$ , versus the cavity gain,  $\Gamma_w g_{tot}$ , for both  $L_d = \infty$  and  $L_d = 783 \text{ \AA}$ . We find that the difference in  $J_{th}$  between non-uniform and uniform injection is small, with uniform injection having slightly lower threshold current density. At lower levels of injection, with  $n_{tot}$  slightly greater than the transparency carrier density, we find that under uniform injection  $J_{th}$  will be smaller, because the gain under uniform injection is greater while the spontaneous emission rates are approximately the same, with Auger recombination not as significant as at high levels of injection. At higher levels of injection, however,  $J_{th}$  becomes nearly the same for the two cases, due to the influence of (1) the  $n_i p_i^2$  term in Eq. (7) which effectively lowers the Auger rate for non-uniform injection, and (2) the increase in the uniform injection spontaneous emission rate.

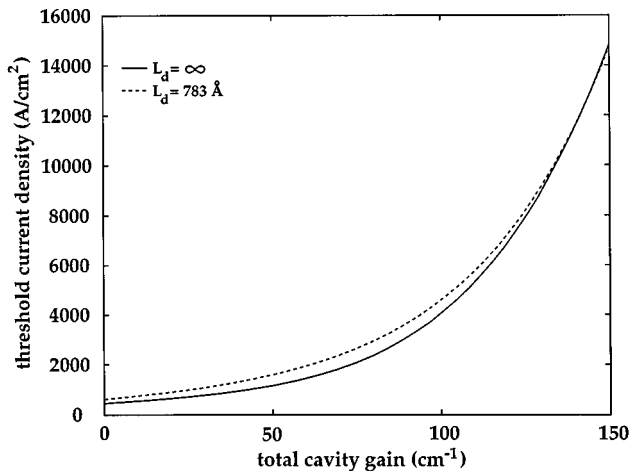


FIG. 5. Threshold current density versus total cavity gain for an 8-well quantum well structure for the  $L_d = 783 \text{ \AA}$  and uniform injection cases.

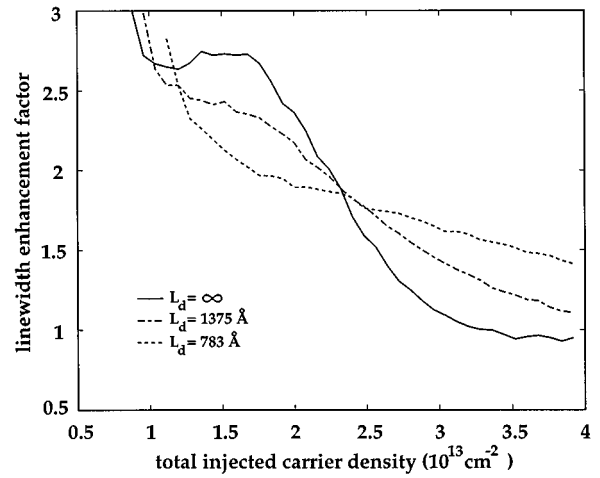


FIG. 6. Linewidth enhancement factor versus total injected carrier density for an 8-well quantum well structure for the uniform and non-uniform charge injection cases considered in this study.

Figure 6 shows the dependence of the linewidth enhancement factor,  $\alpha$ , on total carrier density,  $n_{tot}$  for the distributions shown in Fig. 1. It is seen that at low injection that  $\alpha$  increases as  $L_d$  increases, while at higher levels of injection  $\alpha$  increases as  $L_d$  decreases. Since our calculations show the lasing energy (shown on Fig. 7),  $E_l$ , shifts only very slightly with increasing carrier density (for the range considered in this study), by Eqs. (10) and (11), the change in linewidth enhancement factor must result from a change in the differential gain spectrum.

Figure 7 shows differential gain spectrums for  $n_{tot} = 1.8 \times 10^{13} \text{ cm}^{-2}$  ( $\alpha$  larger for uniform injection) and  $n_{tot} = 3.8 \times 10^{13} \text{ cm}^{-2}$  ( $\alpha$  smaller for uniform injection). We see that at lower carrier densities the area under the uniform injection differential gain spectrum is greater than that of the

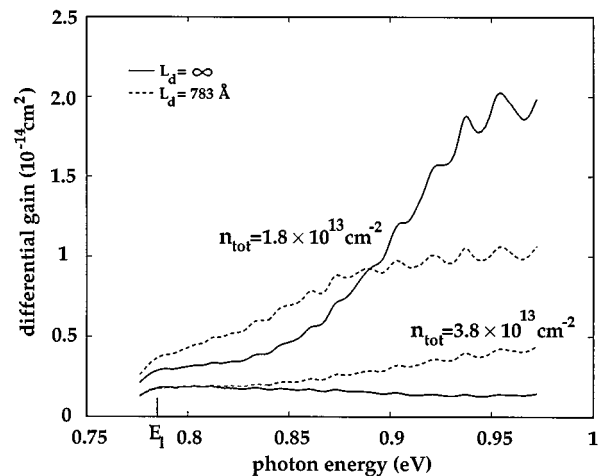


FIG. 7. Total differential gain spectrum for an 8-well quantum well structure for the  $L_d = 783 \text{ \AA}$  and uniform injection cases, at two different injected carrier densities. The slight irregularities in the differential gain curves are a consequence of the numerical procedure and have no noticeable effect on the quoted results.

non-uniform injection; thus, under uniform injection,  $\alpha$  is greater. By the same reasoning, at higher injection levels under uniform injection  $\alpha$  is less.

#### IV. CONCLUSIONS

In this paper we have used a phenomenological model to represent non-uniform injection in a MQW laser. The effects of non-uniform charge injection on gain, threshold current density, and linewidth enhancement factor have been investigated. Non-uniform charge injection is seen to produce slightly smaller gain due to the reduction in hole density far away from the  $p$ -injection side. This reduction in gain causes a moderate increase in threshold current density. The linewidth enhancement factor is seen to be smaller under uniform injection assuming large losses, which is typical of InP-based systems. However, although uniform injection is seen to provide the most favorable results using typical parameters, highly non-uniform injection results do not seem to significantly differ from those obtained assuming uniform injection. Since in long wavelength lasers, there are significant benefits of using a large number of quantum wells, our

studies show that concerns of non-uniform injection are not serious.

#### ACKNOWLEDGMENTS

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