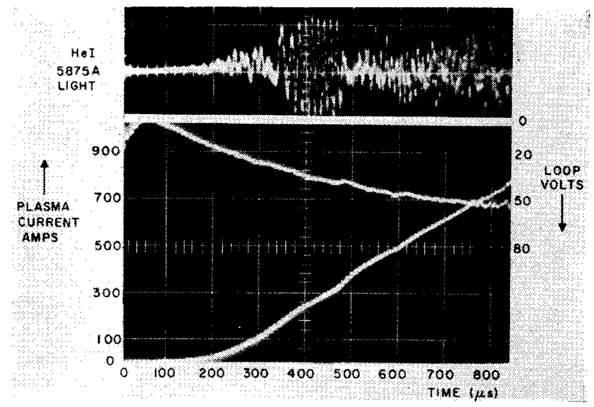

FIG. 1. Fluctuation signal in the He I 5875 A light signal observed during the current rise.

finite distortions of the plasma may result in "stable" oscillations.

It may be observed that at the lower limit of axial current for which the mth mode in stability occurs, the effective rotational transform of the axial magnetic field is $2\pi/m$. Thus any local disturbance of the magnetic field will, in propagating around the stellarator, tend to induce disturbances of an m-fold symmetry in the plasma column cross section. Experimentally, the m-fold distortions may tend to occur over a range of currents rather than at the single value for which the degeneracy occurs. We have not been able to determine whether the regular oscillations are the result of instability, or whether they are related simply to the magnetic field degeneracy.

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On the "Escape Speed" of a Conducting Fluid in a Transverse Magnetic Field

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IN a constant area duct, a rapidly accelerated piston will generate a rarefaction wave. From the Riemann invariants of one-dimensional nonsteady flow of an ideal fluid, one can obtain the well known result that fluid cannot follow a piston that is moving with a velocity greater than $2a_0/(\gamma - 1)$, where a_0 denotes the speed of sound where the fluid is at rest. For a perfectly electrically conducting fluid in a transverse magnetic field, one can show that the "escape speed" will depend not only on the speed of sound where the fluid is at rest, but also on the ratio of magnetic pressure to hydrodynamic pressure initially present $(B_0^2/8\pi p_0)$. From the generalized Riemann invariants,[1] it can be shown that, in the limit, the "escape speed" will equal

$$u_{\max} = 2a_0 \int_0^1 \{1 + [2B_0^2/(8\pi p_0\gamma)] \cdot x^{(4-2\gamma)/(\gamma-1)}\}^{\frac{1}{2}}\, dx/(\gamma - 1).$$

When the magnetic field vanishes, one obtains the known result already mentioned. For $(B_0^2/8\pi p_0) \gg 1$, the foregoing expression is approximately equal to

$$u_{\max} \simeq 2a_0(2B_0^2/8\pi p_0\gamma)^{\frac{1}{2}}.$$

For $(B_0^2/8\pi p_0) \ll 1$, it is approximately

$$u_{\max} \simeq 2a_0[1 + (B_0^2/8\pi p_0) \cdot (\gamma - 1)/(3\gamma - \gamma^2)]/(\gamma - 1).$$

If $\gamma = 5/3$, one readily can evaluate the foregoing integral and find

$$u_{\max} = 2a_0(20\pi p_0/3B_0^2)\,[(1 + 3B_0^2/20\pi p_0)^{\frac{1}{2}} - 1].$$

To illustrate the effect of a transverse magnetic field on the "escape speed" in a conducting fluid, let $B^2/8\pi p = 10$. With $\gamma = 5/3$ one finds $u_{\max}/a_0 = 7.6$. In the absence of the field one finds $u_{\max}/a_0 = 3.0$.

The "escape speed" in one-dimensional flow of a perfectly electrically conducting fluid in a transverse magnetic field will exceed the value of the "escape speed" given by ideal fluid flow theory of a nonconducting fluid for otherwise identical conditions.

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Stagnation Point Fluctuations on a Body of Revolution*

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HOT-wire measurements of velocity fluctuations near the noses (but outside the boundary layers) of three blunt-nosed bodies of revolution over a wide speed range indicate that, for subsonic speeds, (1) the rms values of the fluctuations are considerably higher than those in the free stream, and (2) most of the turbulent energy is in the frequency range 0–5 cps. Measurements at *both* subsonic and supersonic speeds indicate that the stagnation point wanders in a random fashion about its equilibrium position.

The existence of velocity fluctuations in the vicinity of the stagnation point of a two-dimensional body was observed by Piercy and Richardson.[2] They found in a tunnel of high turbulence that the amplitude of the fluctuations near the nose of a streamline strut was about

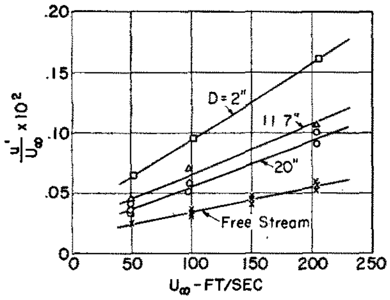

FIG. 1. Turbulence outside boundary layer 7° from stagnation point.

4.5 times that in the free stream and that the region of increased turbulence extended about ¼ chord ahead of the body. The current investigation was carried out on bodies of revolution in a low-turbulence tunnel and in a supersonic tunnel.

Figure 1 shows measurements of relative rms velocity fluctuations, u'/U_0, in the tunnel airstream and near the body at 7° from the stagnation point for the three bodies. The bodies had hemispherical noses with diameters 2 in., 11.7 in., and 20 in., and fineness ratios 17, 6.3, and 5.2, respectively. The measurements were made in the 5 ft. × 7 ft. wind tunnel at the University of Michigan at Reynolds numbers based on body diameter of 5×10^4 to 2×10^6 for the results shown. Typical spectra of the fluctuations, with constant band width, are shown in Fig. 2. In all cases the hot-wires were well outside of the boundary layer; h/δ, where h is the distance from the surface to the hot-wire, and δ is the boundary layer thickness, had values of 4.8, 5.9, and 7.6 for the 2 in., 11.7 in., and 20 in. models, respectively.

Examination of Figs. 1 and 2 indicates that the low frequency components in the free stream fluctuations are amplified strongly in the region near the stagnation point. While the electronic amplification available was not sufficient to measure the spectrum of the free stream fluctuations, Schubauer and Skramstad[3] conclude that sound waves make up a substantial part of the fluctuation energy in a low-turbulence tunnel. Since these sound waves originate at the fan and from vibration of the tunnel parts, their frequencies would be high compared with those shown in Fig. 2. Then the low-frequency end of the spectrum is attributed to vorticity fluctuations. Accordingly, Figs. 1 and 2 indicate the presence of a mechanism causing amplification of vorticity fluctuations in the flow near the stagnation point.

In an attempt to learn more about the nature of the fluctuations, hot wires were mounted at ±7° from the nose and spatial correlation coefficients, $R=\overline{u_1u_2}/u_1'u_2'$, were measured (subscripts 1 and 2 refer to the two wire positions). The values of h/δ for the two wires on each model were the same as those given above for the single wires. Sample numerical results are given in Table I. For all subsonic speeds, large negative correlation coefficients were obtained indicating a random motion of the stagnation point. Peterson and Horton[4] give a similar interpretation for the fluctuations in pressure measured on either side of the stagnation point of a 10-ft-diam sphere. Donaldson[5] also observed an unsteadiness near

TABLE I.

Diameter (in.)	U_∞ (ft/sec)	R
2.0	125	−0.90
11.7	94	−0.84
20.0	98	−0.75.

the stagnation point on the basis of smoke photographs at low speeds on several bodies of revolution.

In addition to the subsonic tests, a measurement of the correlation coefficient was made on the 2-in.-diam model at a Mach number of 2.44 in the supersonic tunnel at the University. When the hot-wire response over the range 10 to 40 000 cps was used the correlation factor was near zero. However, when those components with frequencies above 50 cps were filtered out the correlation factor was −0.4. Comparison of these two results indicates the presence in the supersonic airstream of relatively high-frequency positively-correlated fluctuations, probably sound waves.[6] The measurements indicate further that at supersonic as well as at subsonic speeds a random wandering of the stagnation point occurs. Measurements by J. M. Kendall, Jr., at the Jet Propulsion Laboratory[7] show high turbulence behind the bow shock of a 3.5-in.-diam disk at $M=3.5$. No correlations or spectra were measured but these features are probably similar to those reported here.

At the subsonic speed, attempts were made to determine whether the fluctuations were associated with flow unsteadiness along the surface or in the wake of the body. These tests were conducted on the 20-in.-diam body (the one with the lowest fineness ratio) and comprised boundary layer trips near the nose, and shrouds around the aft end. None of these devices altered the magnitude or spectra of the fluctuations near the nose. These tests and, of course, those at supersonic speed indicate then that we could not trace the fluctuations near the nose to unsteady flow over the body or in the wake.

Piercy and Richardson[2] mention the possibility that the relatively high fluctuation level near the nose is caused by amplification of three-dimensional disturbances during their motion along the concave streamlines near the stagnation point. The instability would be analogous to the instability of viscous flow between rotating cylinders.[8] Recent theoretical work[9] indicates that the flow near the stagnation point of a two-dimensional body is indeed unstable to three-dimensional disturbances. However, this type of instability, being confined largely to the boundary layer, cannot account for the relatively high fluctuations at a distance from the surface of 30 to 50 times the boundary layer thickness.[1,2] Another source of amplification is the stretching of vortex filaments in the diverging flow near the stagnation point.

Both of the foregoing effects could contribute to the fluctuation level near the stagnation point and since the disturbances are in general non-axially-symmetric a random displacement of the stagnation point will result. The larger the scale of a given disturbance the greater the displacement will be. In addition, the random vorticity gradients associated with the larger scale dis-

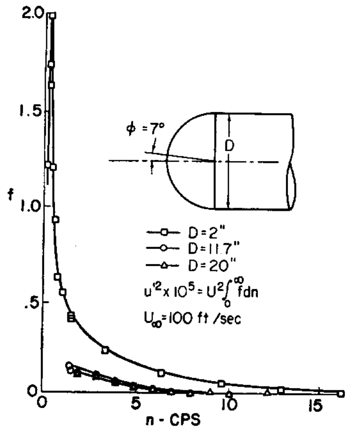

FIG. 2. Spectrum of turbulence outside boundary layer 7° from stagnation point.

turbances will in themselves contribute to a wandering of the stagnation point.[10]

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* This study was made as part of an investigation of the stability of the flow in the boundary layer near the nose of a body of revolution. The program was supported in part by the Air Force Office of Scientific Research under Contract No. AF 49-638-33. A more complete exposition of the work will be published in reference 1.

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Streamlines in Bénard Convection Cells

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IN a recent paper dealing with thermal instability in a viscous fluid heated from below,[1] the form of the cell pattern which occurs at the onset of instability was discussed for the first even mode of instability (which corresponds to the case of two rigid bounding surfaces) when the cell pattern is assumed to have a hexagonal plan form. That discussion was limited to a determination of the streamline patterns in the two planes of symmetry. The results given there (Fig. 4 of reference 1) are in error, however, because of the incorrect assumption of two-dimensional motion in the planes of symmetry. Since the motion is steady, the streamlines still can be obtained by integrating the equations for the particle paths and, in the planes of symmetry, this can be done without difficulty since the streamlines themselves then lie in the planes of symmetry.

Consider first the determination of the streamline pattern in the plane $\eta = 0$. This is a plane of symmetry on which $v = 0$ but, since $\partial v/\partial\eta \neq 0$, the motion is not two-dimensional. The streamlines must be obtained, therefore, by integrating the equation for the particle paths

$$d\xi/u = d\zeta/w, \tag{1}$$

where

$$u = -\frac{\kappa}{d}\frac{1}{3} f_0 \frac{\sqrt{3}}{a} \sin \tfrac{1}{2}\sqrt{3}a\xi\, W(\zeta), \tag{2}$$

and

$$w = +\frac{\kappa}{d}\frac{1}{3} f_0\{2 \cos \tfrac{1}{2}\sqrt{3}a\xi + 1\} W'(\zeta). \tag{3}$$

With u and w given by Eqs. (2) and (3), Eq. (1) can be integrated to give

$$\left[\frac{f(\frac{1}{2}\sqrt{3}a\xi)}{3\sqrt{3}/4}\right]^{2/3} \frac{W(\zeta)}{W(0)} = \text{const}, \tag{4}$$

where

$$f(x) = \sin x(1 - \cos x). \tag{5}$$

Since the maximum value of $f(x)$ is $3\sqrt{3}/4$ for $x = 2\pi/3$, the constant in Eq. (4), which serves to identify a particular streamline, takes on values between 0 and 1.

Similarly, in the plane $\xi = 0$ which is also a plane of symmetry on which u, but not $\partial u/\partial\xi$, vanishes, the streamlines are obtained by integrating the equation

$$d\eta/v = d\zeta/w, \tag{6}$$

where

$$v = -\frac{\kappa}{d}\frac{1}{3} f_0 \frac{1}{a}\{\sin \tfrac{1}{2}a\eta + \sin a\eta\} W'(\zeta), \tag{7}$$

and

$$w = +\frac{\kappa}{d}\frac{1}{3} f_0\{2 \cos \tfrac{1}{2}a\eta + \cos a\eta\} W(\zeta). \tag{8}$$

With v and w given by Eqs. (7) and (8), Eq. (6) can be integrated to give

$$\frac{g^2(\frac{1}{2}a\eta)}{3(2\sqrt{3} - 3)} \frac{W(\zeta)}{W(0)} = \text{const}, \tag{9}$$