Exact solution for the peripheral photoresponse of a *p-n* junction

L. M. Sander
Physics Department, University of Michigan, Ann Arbor, Michigan 48109

(Received 25 June 1984; accepted for publication 24 August 1984)

An exact analytical solution is given for the photoresponse of a *p-n* junction on a thick semiconductor due to lateral diffusion of minority carriers.

I. INTRODUCTION

In a recent series of papers[1–3] Holloway and Brailsford have considered the peripheral response of a photodiode, that is, the effective increase in collecting area of the device due to the lateral diffusion of minority carriers into the depletion region. The geometry being considered is shown in Fig. 1. Clearly, carriers will arrive at the depletion region only if they are generated within a distance of the order of L, the minority carrier diffusion length, of the edge. In fact, if we consider a stripe geometry (with a mesa length much greater than L) and a device whose thickness c is much less than L, the problem can be solved exactly.[1] The number of carriers collected on each side of the diode is QL per unit length of stripe, where Q is the generation rate per unit area of surface. For finite thicknesses c, however, the number of carriers collected can only be found numerically.[2]

Also, Holloway and Brailsford[3] have shown that the solution to this size effect problem for the case where the light generates carriers only at the surface of the *p*-type region yields the finite-size *correction* to the diffusion-limited saturation current of a *p-n* junction if Q is replaced by the Shockly result for the saturation current of an infinite junction. Holloway[4] has very recently given numerical solutions for the peripheral response to line or spot illumination (as from an electron beam) near the edge of a junction. This may be used to infer diffusion lengths.[5] Thus it is of quite significant practical importance to extend our understanding of diffusion problems of this type.

In this paper we present exact analytical expressions for the current collected upon uniform or thin line excitation of the surface of the device in Fig. 1 for the case of a large mesa on a thick device: $c \gg L$ and $w \gg L$. The method we will use is a straightforward application of the Weiner–Hopf technique.[6] In Sec. II we formulate the problem precisely and solve it for uniform illumination. Section III gives our solution for line illumination and Sec. IV summarizes our results.

II. UNIFORM ILLUMINATION

When c and w are large compared to L, our problem reduces to the geometry shown in Fig. 1(b). The depletion region extends from $x = 0$ to $x = -\infty$ for $y = 0$, and the *p*-type semiconductor occupies the lower half of the y plane.

For $y < 0$, the minority carrier concentration n satisfies the steady-state diffusion equation:

$$\nabla^2 n - n/L^2 = 0. \qquad (1)$$

The depletion region absorbs carriers: we may idealize the situation by writing a boundary condition

$$n = 0 \quad (x < 0, y = 0). \qquad (2a)$$

The generation of carriers is represented by

$$\partial n/\partial y = Qe^{-bx} \quad (x > 0, y = 0). \qquad (2b)$$

The convergence factor $\exp(-bx)$ is introduced for convenience. The parameter b will be set to zero eventually. Clearly we must take

$$n \to 0 \quad (y \to -\infty). \qquad (2c)$$

The mixed boundary conditions, Eqs. (2a) and (2b), represent the essential difficulty of the problem. However, the Weiner-Hopf method[6] is designed for problems of just this type. To apply it, write

$$n(x,y) = \int_{-\infty}^{\infty} G(k,y) e^{ikx} dk/2\pi. \qquad (3)$$

Then, using Eq. (1) we find

$$\partial^2 G/\partial y^2 - (k^2 + 1/L^2)G = 0, \qquad (4a)$$

$$G = \Phi(k)\exp(qy), \qquad (4b)$$

where

$$q = (k^2 + l/L^2)^{1/2}. \qquad (4c)$$

Now, using Eq. (2);

$$\Phi = \int_0^{\infty} n(x,0) e^{-ikx} dx = \Phi_-, \qquad (5a)$$

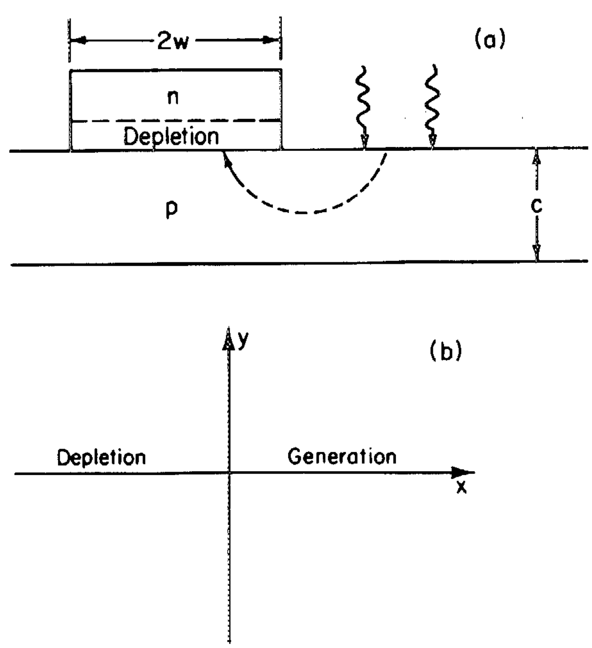

FIG. 1.(a) Geometry of the lateral diffusion problem. (b) Coordinate system used when $w,c \gg L$.

 0021-8979/85/062057-03$02.40

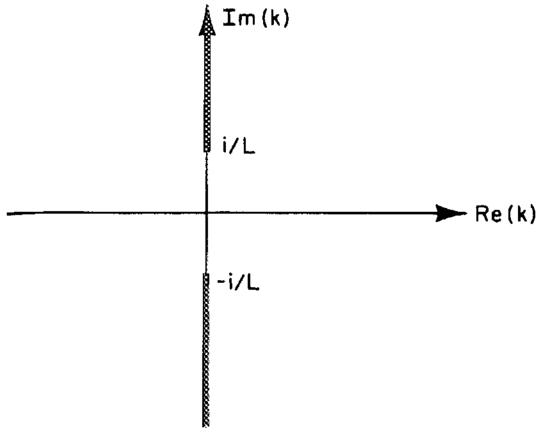

FIG. 2. Branch cuts for the function $q(k)$.

$$q\Phi = \int_{-\infty}^{0} \partial n(x,0)/\partial y e^{-ikx} dx + \int_0^{\infty} Q e^{-bx} e^{-ikx} dx = \psi_+ + Qi/(k - ib). \tag{5b}$$

By inspection of Eq. (5) we see that if we analytically continue k into the complex plane the unknown function Φ_- is analytic and bounded for $\mathrm{Im}(k) \mapsto -\infty$, and Ψ_+ is analytic and bounded for $\mathrm{Im}(k) \mapsto \infty$. Both functions are analytic in a strip containing the real axis. To handle the square root in Eq. (4c) we introduce branch cuts at in Fig. 2, and put

$$\sqrt{k - i/L}\,\Phi_- = \Psi_+/\sqrt{k + i/L} + f, \tag{6a}$$

$$f = Qi/[(k - ib)\sqrt{k + i/L}\,] = f_+ + f_-, \tag{6b}$$

$$f_+ = [Qi/(k - ib)][1/\sqrt{k + i/L} - 1/\sqrt{ib + i/L}\,], \tag{6c}$$

$$f_- = Qi/[(k - ib)\sqrt{ib + i/L}\,]. \tag{6d}$$

Once more, f_- is analytic on the real axis and for $\mathrm{Im}(k) < 0$, and f_+ for $\mathrm{Im}(k) > 0$. Now write

$$\sqrt{k - i/L}\,\Phi_- - f_- = \Psi_+/\sqrt{k + i/L} + f_+. \tag{7}$$

Now the left-hand side of the equation is, by construction, analytic in the lower half plane and near the real axis, but since it is equal to the right-hand side which is analytic near the real axis and in upper half plane, both sides are entire. However, the only entire function which approaches zero as $\mathrm{Im}(k) \mapsto \pm\infty$ is the constant zero. Thus,

$$\Phi = \Phi_- = f_-/\sqrt{k - i/L}. \tag{8}$$

And from Eqs. (3) and (4b),

$$n(x,y) = \int_{-\infty}^{\infty} (f_-/\sqrt{k - i/L}\,) e^{ikx + qy} dk/2\pi. \tag{9}$$

This is the complete solution to the boundary value problem.

It is of interest to display $n(x,0)$ and $\partial n(x,0)/\partial y$ explicitly. For the concentration itself

$$n(x,0) = \int_{-\infty}^{\infty} [Qi/\sqrt{ib + i/L}\,] 1/[(k - ib)\sqrt{k - i/L}\,] \times e^{ikx} dk/2\pi. \tag{10}$$

For $x < 0$ we may close the contour in the lower half plane, so that $n = 0$, as required. For $x > 0$ we have a simple pole and a branch cut. A bit of algebra gives

$$n(x,0) = QL \,\mathrm{erf}(\sqrt{x/L}\,), \tag{11}$$

where we have put $b = 0$ at the end of the calculation. The current into the depletion region is

$$\partial n(x,0)/\partial y = \int_{-\infty}^{\infty} [Qi/\sqrt{ib + i/L}\,][\sqrt{k + i/L}/(k - ib)] \times e^{ikx} dk/2\pi. \tag{12}$$

For $x > 0$, only the pole at ib contributes to the integral and we recover Eq. (2b). For $x < 0$ we must integrate along a branch cut:

$$\partial n/\partial y = Q e^{x/L} \frac{1}{\pi} \int_0^{\infty} dt \frac{\sqrt{t} e^{tx/L}}{t + 1}, = Q[1 - \mathrm{erf}(\sqrt{-x/L}\,) + 2e^{x/L}/\sqrt{-\pi x/L}\,]. \tag{13}$$

The total current into the junction is

$$J = \int_{-\infty}^{0} (\partial n/\partial y) dx = QL/2, \tag{14}$$

Thus a thick device collects exactly one-half as much laterally diffusing current as a thin one. This remarkably simple solution was previously conjectured on the basis of a numerical calculation.[2] Both Eqs. (11) and (13) agree with the results of Ref. 2 in the proper limit.

III. LINE ILLUMINATION

The photoresponse to a small illuminated spot could be measured by electron-beam or optical excitation, as we mentioned above. Clearly this sort of experiment yields far more information than the response to uniform illumination, as in the last section. As Holloway[4] has shown, by translational invariance along the edge, the response to a spot of length δz is the same as the current collected by δz of the edge of the device due to an illuminated line. Thus we consider the situation illustrated in Fig. 3. We must solve the same problem as in Sec. II except that Eq. (2b) is replaced by

$$\partial n/\partial y = Q\delta(x - x_0) dx, \tag{15}$$

where dx is the (small) width of the illuminated region. Repeating the analysis above we find

$$\Phi = \Phi_- \tag{16a}$$

$$q\Phi = \Psi_+ + Q dx e^{-ikx_0}. \tag{16b}$$

As before we write

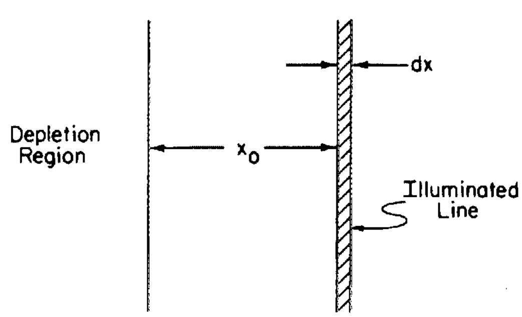

FIG. 3. Geometry for line illumination.

$$\sqrt{k-i/L}\,\Phi_- = \Psi_+/\sqrt{k+i/L} + f, \quad (17a)$$

$$f = Qdxe^{-ikx_0}/\sqrt{k+i/L}. \quad (17b)$$

However, the decomposition of f into $f_\pm$ is less trivial than before. There is a general solution[6] to the problem, however

$$f_\pm = \pm \int_{\infty \mp i\delta}^{\infty \mp i\delta} f(k')dk'/[2\pi i(k'-k)], \quad (17c)$$

where δ is positive and less than $1/L$. Equation (9) is still valid with Eq. (17c) for f_-.

Assembling these results we can write an explicit expression for the collected current:

$$\partial n(x,0)/\partial y = [iQdx/(2\pi)^2] \int_{-\infty}^{\infty} dk \int_{\infty + i\delta}^{\infty + i\delta} dk' \times \frac{\sqrt{k+i/L}\,e^{ikx}e^{-ik'x_0}}{\sqrt{k'+i/L}\,(k'-k)}. \quad (18)$$

Now the physically relevant quantity is the total current collected:

$$dJ = \int_{-\infty}^{0} dx \partial n(x,0)/\partial y e^{bx}$$

$$= \frac{Qdx}{2\pi i} \int_{-\infty + i\delta}^{\infty + i\delta} dk' \frac{e^{-ik'x_0}}{k'-ib}\left(1 - \frac{\sqrt{i/L+ib}}{\sqrt{k'+i/L}}\right). \quad (19)$$

Once more, we have used a convergence factor. Note that the pole at $k' - ib$ has zero residue. Converting the expression to a real integral along the branch cut we have

$$dJ = 2Qdx/\pi \int_0^\infty du e^{-(u^2+1)x_0/L}(u^2+1)$$

$$= Qdx[1 - \mathrm{erf}(\sqrt{x_0/L})]. \quad (20)$$

This result appears to agree with the results of Holloway[4] in the proper limit.

IV. SUMMARY

In this paper we have given exact solutions to the peripheral photoresponse problem for a thick semiconductor. It is of interest to explicitly compare the thin[1,4] ($c \ll L$) and thick ($c \gg L$) limits:

For uniform illumination

$$J = QL \quad \text{(thin)}$$
$$= QL/2 \quad \text{(thick)};$$

for line or spot illumination[4]

$$dJ = Qdxe^{-x_0/L} \quad \text{(thin)}$$
$$= Qdx[1 - \mathrm{erf}(\sqrt{x_0/L})] \quad \text{(thick)}.$$

It is natural to ask whether the techniques used here can be extended to the case of $c \sim L$. Though a formal answer can be written down, the expression, which involves an infinite number of poles spaced by roughly c/L (rather than a branch cut), does not seem very tractable. It is probably preferable to solve the case of $c \sim L$ numerically as was done in Refs. 1–4.

ACKNOWLEDGMENTS

I would like to thank H. Holloway who introduced me to this problem and checked my results against his numerical solutions. This work was supported by NSF Grant No. DMR-82-03698 and ARO Grant No. DAAG-29-83-K0131.

[1]H. Holloway, J. Appl. Phys. **49**, 4264 (1978).
[2]H. Holloway and A. D. Brailsford, J. Appl. Phys. **54**, 4641 (1983).
[3]H. Holloway and A. D. Brailsford, J. Appl. Phys. **55**, 446 (1984).
[4]H. Holloway, J. Appl. Phys. **55**, 3669 (1984).
[5]J. I. Hanoka and R. O. Bell, Annu. Rev. Mater. Sci. **11**, 353 (1981).
[6]B. Noble, *Methods Based on the Weiner–Hopf Technique* (Pergamon, New York, 1958); J. Mathews and R. Walker, *Mathematical Methods of Physics*, 2nd edition (Benjamin, New York, 1970), pp. 245–252.
[7]H. Holloway (private communication).