equation, (2), and match the singularity at \( r_2 = r_1 \) of the expression on the right-hand side of (28) with the amplitude of the delta function source term in (2). This is achieved by the requirement
\[
G(r_2, r_1, \omega) \sim -1/4\pi |r_2 - r_1|, \quad (r_2 \rightarrow r_1). \tag{29}
\]
Now when \( r_2 \) approaches \( r_1 \), \( u \) approaches \( v \), and the determinant in (28) goes over into the Wronskian of the two functions \( W \) and \( \mathcal{M} \). This Wronskian has the value\(^1\) \( 1/\Gamma(1 - iv) \). Hence the missing numerical factor in (28) is \(-\Gamma(1 - iv)/4\pi\). Supplying (28) with this factor gives the familiar closed form expression (4) for the coordinate space Coulomb Green’s function.

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**Time-Dependent Green’s Function for a Moving Isotropic Nondispersive Medium**

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The time-dependent Green’s function for a moving isotropic nondispersive medium is hereby obtained by taking the \( \omega \)-integration of the time-harmonic solution which was previously obtained by means of an operational method and by making use of the known result of the two-dimensional Klein–Gordon equation.

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**INTRODUCTION**

The time-dependent Green’s function for a moving isotropic medium was recently found by Compton.\(^4\) He applied a four-fold Fourier transform to the pertinent differential equation to obtain the desired result. In evaluating the reciprocal Fourier transform, he considers the \( \omega \)-integration first, followed by the \( \vec{k} \)-integration. The steps involved in the calculation are rather long, comparable to those of Lee and Papas.\(^2\)

In this paper, we show that the time-dependent solution can readily be obtained by taking the \( \omega \)-integration of the time-harmonic solution\(^3\) which was previously obtained by an operational method without a lengthy calculation.

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**THE BASIC EQUATION AND ITS SOLUTIONS**

The time-dependent Green’s function considered by Compton\(^4\) satisfies the differential equation
\[
\left[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{2\Omega}{a} \frac{\partial}{\partial t} + \left\{ \frac{\Omega^2}{a} - \frac{n^2 a}{c^2} \frac{\partial^2}{\partial t^2} \right\} G(\vec{R}, \vec{R'}; t, t') \right] = -\delta(\vec{R} - \vec{R'})\delta(t - t'), \tag{1}
\]
where
\[
an = \left( \frac{\mu_0 \varepsilon_0}{\mu_0 \varepsilon_0} \right)^{1/2}, \quad \beta = \frac{v}{c}, \quad c = (\mu_0 \varepsilon_0)^{1/2}, \quad \vec{v} = va_z.
\]

We define the Fourier pair
\[
F(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(t) e^{-i\omega t} \, dt, \tag{2}
\]
\[
G(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{i\omega t} \, d\omega. \tag{3}
\]

Compton defines the Green’s function with a positive sign attached to the delta function.
Thus, if Eq. (1) is multiplied by $e^{-i\omega t}$ and integrated with respect to $t$, we obtain

$$\left[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{1}{a} \frac{\partial^2}{\partial z^2} - \frac{2\omega \Omega}{a} \frac{\partial}{\partial z} + \omega^2 \left( \frac{n^2 a}{c^2} - \frac{\Omega^2}{a} \right) \right] F(\omega) = -e^{-i\omega t} \delta(\vec{R} - \vec{R}').$$  

(4)

If we introduce an auxiliary function $f(\omega)$ such that

$$F(\omega) = e^{i\omega \Delta t} f(\omega),$$

(5)

then $f(\omega)$ satisfies the following equation:

$$\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{1}{a} \frac{\partial^2}{\partial z^2} + \frac{\omega^2 n^2 a}{c^2} \right) f(\omega) = -e^{-i\omega(t' + \Omega\Delta t)} \delta(\vec{R} - \vec{R}').$$

(6)

Except for the factor $e^{-i\omega(t' + \Omega\Delta t)}$, Eq. (6) is the same as Eq. (26) considered in Ref. 3. Hence, its solution is given by the following:

**Case I: $n\beta < 1$**

$$f(\omega) = \frac{a^{\frac{1}{2}} \exp \left\{-i\omega [t' + \Omega z' + (n/c) a^{\frac{1}{2}} R_a] \right\}}{4\pi R_a},$$

(7)

where

$$R_a = (a^2 + r^2)^{\frac{1}{4}}, \quad r = (x - x')^2 + (y - y')^2, \quad \xi = (z - z').$$

**Case II: $n\beta > 1$**

$$f(\omega) = \begin{cases} 
0, & |a^{\frac{1}{2}} \xi < r \\
|a^{\frac{1}{2}} \exp \left\{ -i\omega (t' + \Omega z') \right\} \cos (\omega n/c) |a^{\frac{1}{2}} R_a' \right\}, & |a^{\frac{1}{2}} \xi > r 
\end{cases}$$

(8)

where

$$R_a' = (|a| \xi^2 - r^2)^{\frac{1}{4}}.$$

In view of Eqs. (3) and (5), we can obtain readily the solutions for $G(t)$; they are the following:

**Case I: $n\beta < 1$**

$$G(t) = \frac{\frac{1}{2} a^{\frac{1}{2}}}{8\pi^2 R_a} \int_{-\infty}^{\infty} \exp \left\{ -i\omega \left( \tau + \Omega \xi - \frac{n}{c} a^{\frac{1}{2}} R_a \right) \right\} d\omega$$

$$= \frac{\frac{1}{2} a^{\frac{1}{2}}}{4\pi R_a} \delta \left( \tau + \Omega \xi - \frac{n}{c} a^{\frac{1}{2}} R_a \right),$$

(9)

where $\tau = t - t'$.  

**Case II: $n\beta > 1$**

$$G(t) = \frac{|a|^{\frac{1}{2}}}{4\pi^2 R_a} \int_{-\infty}^{\infty} \exp \left\{ -i\omega (\tau + \Omega \xi) \right\} \cos \frac{n}{c} |a|^{\frac{1}{2}} R_a' d\omega$$

$$= \frac{|a|^{\frac{1}{2}}}{4\pi R_a} \delta \left( \tau + \Omega \xi - \frac{n}{c} |a|^{\frac{1}{2}} R_a' \right), \quad |a|^{\frac{1}{2}} \xi > r,$$

(10)

$$G(t) = 0, \quad |a|^{\frac{1}{2}} \xi < r.$$

(11)

Our expressions for $G(t)$ appear to be of slightly different form as compared to Compton's, but they are equivalent. In fact, the present ones are simpler in form and also put the time-dependent part explicitly in the delta function.

To discuss the locus of the wave front, we consider, for example, the case corresponding to $n\beta < 1$. The impulsive wave front is described by

$$\tau + \Omega \xi - (n/c) a^{\frac{1}{2}} R_a = 0.$$

(12)

The above equation can be written in the form

$$[(\xi - \xi_c)^2/A^2] + (r^2/B^2) = 1,$$

(13)

where

$$\xi_c = \frac{(n^2 - 1)\beta \tau}{n^2 - \beta^2}, \quad A = \frac{n(1 - \beta^2)\tau}{n^2 - \beta^2},$$

$$B = \left( \frac{1 - \beta^2}{n^2 - \beta^2} \right)^{\frac{1}{4}}.$$

Equation (13) defines the same ellipsoid discussed by Compton. It can be shown that the same algebraic equation applies to the case $n\beta > 1$. For the latter case, $\xi_c$ is numerically smaller than $A$. The detached ellipsoidal wave front is therefore confined within the Mach cone defined by $|a|^{\frac{1}{2}} \xi - r = 0$.  