equation, (2), and match the singularity at $\mathbf{r}_2 = \mathbf{r}_1$ of the expression on the right-hand side of (28) with the amplitude of the delta function source term in (2). This is achieved by the requirement

$$G(\mathbf{r}_2, \mathbf{r}_1, \omega) \sim -1/4\pi\,|\mathbf{r}_2 - \mathbf{r}_1|, \quad (\mathbf{r}_2 \to \mathbf{r}_1). \tag{29}$$

Now when $\mathbf{r}_2$ approaches $\mathbf{r}_1$, u approaches v, and the determinant in (28) goes over into the Wronskian of the two functions W and $\mathcal{M}$. This Wronskian has the value[12] $1/\Gamma(1 - i\nu)$. Hence the missing numerical factor in (28) is $-\Gamma(1 - i\nu)/4\pi$. Supplying (28) with this factor gives the familiar closed form expression (4) for the coordinate space Coulomb Green's function.

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Time-Dependent Green's Function for a Moving Isotropic Nondispersive Medium*

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The time-dependent Green's function for a moving isotropic nondispersive medium is hereby obtained by taking the ω-integration of the time-harmonic solution which was previously obtained by means of an operational method and by making use of the known result of the two-dimensional Klein–Gordon equation.

INTRODUCTION

THE time-dependent Green's function for a moving isotropic medium was recently found by Compton.[1] He applied a four-fold Fourier transform to the pertinent differential equation to obtain the desired result. In evaluating the reciprocal Fourier transform, he considers the ω-integration first, followed by the $\bar{k}$-integration. The steps involved in the calculation are rather long, comparable to those of Lee and Papas.[2]

In this paper, we show that the time-dependent solution can readily be obtained by taking the ω-integration of the time-harmonic solution[3] which was previously obtained by an operational method without a lengthy calculation.

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[2] K. S. H. Lee and C. H. Papas, J. Math. Phys. **5**, 1668 (1964).

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THE BASIC EQUATION AND ITS SOLUTIONS

The time-dependent Green's function considered by Compton[4] satisfies the differential equation

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{1}{a}\frac{\partial^2}{\partial z^2} - \frac{2\Omega}{a}\frac{\partial^2}{\partial z\partial t} + \left(\frac{\Omega^2}{a} - \frac{n^2 a}{c^2}\right)\frac{\partial^2}{\partial t^2}\right] G(\bar{R}, \bar{R}'; t, t') = -\delta(\bar{R} - \bar{R}')\delta(t - t'), \tag{1}$$

where

$$a = \frac{1 - \beta^2}{1 - n^2\beta^2}, \quad \Omega = \frac{(n^2 - 1)\beta}{(1 - n^2\beta^2)c},$$

$$n = \left(\frac{\mu\epsilon}{\mu_0\epsilon_0}\right)^{\frac{1}{2}}, \quad \beta = \frac{v}{c}, \quad c = (\mu_0\epsilon_0)^{\frac{1}{2}}, \quad \bar{v} = v a_z.$$

We define the Fourier pair

$$F(\omega) = \int_{-\infty}^{\infty} G(t)e^{-j\omega t}\,dt, \tag{2}$$

$$G(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}\,d\omega. \tag{3}$$

[4] Compton defines the Green's function with a positive sign attached to the delta function.

Thus, if Eq. (1) is multiplied by $e^{-j\omega t}$ and integrated with respect to t, we obtain

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{1}{a}\frac{\partial^2}{\partial z^2}-\frac{2j\omega\Omega}{a}\frac{\partial}{\partial z}+\omega^2\left(\frac{n^2a}{c^2}-\frac{\Omega^2}{a}\right)\right]F(\omega)=-e^{-j\omega t'}\delta(\bar{R}-\bar{R}'). \tag{4}$$

If we introduce an auxiliary function $f(\omega)$ such that

$$F(\omega)=e^{j\omega\Omega z}f(\omega), \tag{5}$$

then $f(\omega)$ satisfies the following equation:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{1}{a}\frac{\partial^2}{\partial z^2}+\frac{\omega^2n^2a}{c^2}\right)f(\omega)=-e^{-j\omega(t'+\Omega z')}\delta(\bar{R}-\bar{R}'). \tag{6}$$

Except for the factor $e^{-j\omega(t'+\Omega z')}$, Eq. (6) is the same as Eq. (26) considered in Ref. 3. Hence, its solution is given by the following:

Case I: $n\beta < 1$

$$f(\omega)=\frac{a^{\frac{1}{2}}\exp\{-j\omega[t'+\Omega z'+(n/c)a^{\frac{1}{2}}R_a]\}}{4\pi R_a}, \tag{7}$$

where

$$R_a=(a\xi^2+r^2)^{\frac{1}{2}},\quad r^2=(x-x')^2+(y-y')^2,\quad \xi=(z-z').$$

Case II: $n\beta > 1$

$$f(\omega)=\begin{Bmatrix}0, & |a|^{\frac{1}{2}}\xi<r\\ \dfrac{|a|^{\frac{1}{2}}\exp[-j\omega(t'+\Omega z')]\cos(\omega n/c)\,|a|^{\frac{1}{2}}R_a'}{2\pi R_a'}, & |a|^{\frac{1}{2}}\xi>r\end{Bmatrix}, \tag{8}$$

where

$$R_a'=(|a|\,\xi^2-r^2)^{\frac{1}{2}}.$$

In view of Eqs. (3) and (5), we can obtain readily the solutions for $G(t)$; they are the following:

Case I: $n\beta < 1$

$$G(t)=\frac{a^{\frac{1}{2}}}{8\pi^2R_a}\int_{-\infty}^{\infty}\exp\left[j\omega\left(\tau+\Omega\xi-\frac{n}{c}a^{\frac{1}{2}}R_a\right)\right]d\omega=\frac{a^{\frac{1}{2}}}{4\pi R_a}\delta\left(\tau+\Omega\xi-\frac{n}{c}a^{\frac{1}{2}}R_a\right), \tag{9}$$

where $\tau=t-t'$.

Case II: $n\beta > 1$

$$G(t)=\frac{|a|^{\frac{1}{2}}}{4\pi^2R_a'}\int_{-\infty}^{\infty}\exp[j\omega(\tau+\Omega\xi)]\cos\frac{n}{c}|a|^{\frac{1}{2}}R_a'\,d\omega=\frac{|a|^{\frac{1}{2}}}{4\pi R_a'}\delta\left(\tau+\Omega\xi-\frac{n}{c}|a|^{\frac{1}{2}}R_a'\right),\quad |a|^{\frac{1}{2}}\xi>r, \tag{10}$$

$$G(t)=0,\quad |a|^{\frac{1}{2}}\xi<r. \tag{11}$$

Our expressions for $G(t)$ appear to be of slightly different form as compared to Compton's, but they are equivalent. In fact, the present ones are simpler in form and also put the time-dependent part explicitly in the delta function.

To discuss the locus of the wave front, we consider, for example, the case corresponding to $n\beta < 1$. The impulsive wave front is described by

$$\tau+\Omega\xi-(n/c)a^{\frac{1}{2}}R_a=0. \tag{12}$$

The above equation can be written in the form

$$[(\xi-\xi_c)^2/A^2]+(r^2/B^2)=1, \tag{13}$$

where

$$\xi_c=\frac{(n^2-1)\beta c\tau}{n^2-\beta^2},\quad A=\frac{n(1-\beta^2)c\tau}{n^2-\beta^2},\quad B=\left(\frac{1-\beta^2}{n^2-\beta^2}\right)^{\frac{1}{2}}c\tau.$$

Equation (13) defines the same ellipsoid discussed by Compton. It can be shown that the same algebraic equation applies to the case $n\beta > 1$. For the latter case, ξ_c is numerically smaller than A. The detached ellipsoidal wave front is therefore confined within the Mach cone defined by $|a|^{\frac{1}{2}}\xi-r=0$.