Measurement of the thermal conductivity of helium up to 2100°K by the column method

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The thermal conductivity of helium was measured at atmospheric pressure in the range 800–2100°K by the column method. The data could be correlated by the polynomial $\lambda = 0.635 \times 10^{-1} + 0.310 \times 10^{-3} T - 0.244 \times 10^{-7} T^2$, where λ is in watts per meters degrees Kelvin and T is in degrees Kelvin. The results obtained were compared with previous thermal conductivity measurements. The data of Desmond and Saxena and Saxena agree closely with the present results; the data of Timrot and Umanskii appear to be too low and those of Blais and Mann too high. Values for $f = \lambda/\eta C_v$, computed using measured thermal conductivities and available viscosity data, were found to agree well with classical results from kinetic theory.

I. INTRODUCTION

High temperature thermal conductivity data for gases are needed in many problems of practical and basic interest. However, owing to experimental difficulties, few data are available at temperatures above ~600°K. In this paper thermal conductivity values of helium are reported up to 2100°K. The data were obtained by the column method which, in recent years, has been used with success for measuring thermal conductivities of gases at high temperatures (e.g., see the summaries in Refs. 1 and 2).

II. RESULTS AND DISCUSSIONS

The experimental apparatus and procedure were described elsewhere[1–3] and, consequently, will not be given here in detail. Essentially, in the experiments the gas was contained between two concentric cylinders (the inner one being a filament) maintained at different temperatures, and the heat transfer through the gas was measured. Two apparatus of different outside diameters were used (referred to as the small and large columns) to assess the magnitude of convection effects. The thermal conductivity was calculated from the expression (see the Appendix)

$$\lambda(T_f) = [\ln(r_o/r_f)/2\pi](dQ_\lambda/dT)_{T_f}(1+\lambda'+\lambda''), \quad (1)$$

where r_f and r_o are the radii of the filament and the outer cylinder, respectively, and Q_λ is the heat conducted per unit length from the filament at a temperature T_f. λ' is a correction due to the temperature drop across the outer cylinder of thickness w and thermal conductivity λ_w

$$\lambda' \cong \frac{\lambda(T_b)}{\lambda_w} \frac{\ln[1+(w/r_o)]}{\ln(r_o/r_f)}, \quad (2)$$

where T_b is the temperature of the water bath surrounding the outer cylinder. λ' is about 0.009 for the large column and 0.0104 for the small one. λ'' is a correction due to the temperature jump at the filament surface

$$\lambda'' \cong \tfrac{15}{8}[(2-\alpha)/\alpha](L/r_f)[\ln(r_o/r_f)]^{-1}. \quad (3)$$

α is the thermal accommodation coefficient and L the mean free path of the gas at T_f.

The thermal conductivity of helium was determined at 760 mm Hg, in the range 800–2100°K. The experimental results are shown in Fig. 1. With the small diameter column data could be obtained only up to 1350°K since at higher filament temperatures the temperature rise of the water cooling the outer cylinder became excessive. It is noted, however, that up to 1350°K the measurements made in the small and large columns agree closely, indicating the absence of significant convection effects. As the temperature increases the Rayleigh number (which is an indication of the magnitude of convection effects[2]) decreases from 16 (at 1300°K) to 12 (at 2000°K) for the small column, and from 125 (at 1300°K) to 106 (at 2000°K) for the large one. At higher temperatures, convection effects would be even less important than below the

TABLE I. Estimate of random errors (percent).

Temp (°K)	Most probable error	Maximum error
800	2.4	2.9
1200	2.2	2.6
1600	2.0	2.4
2000	1.9	2.3

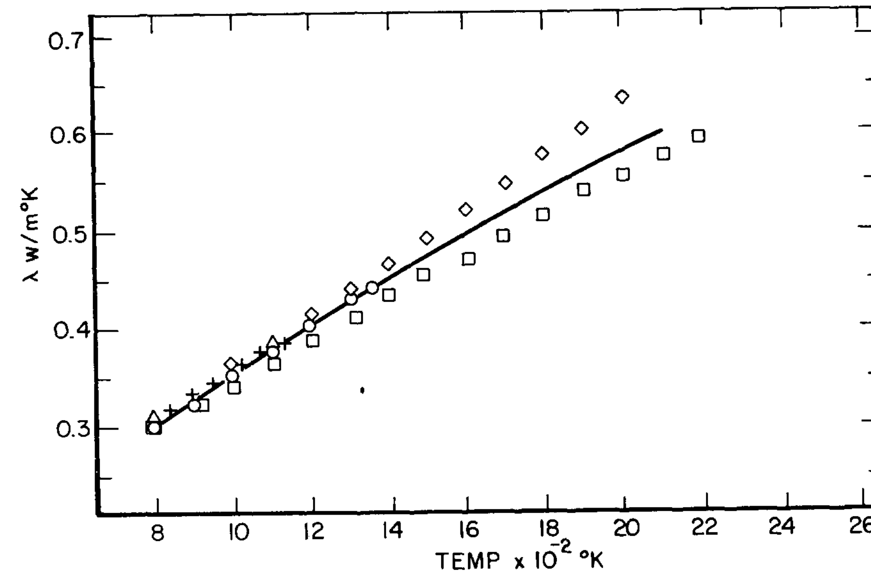

FIG. 1. Experimental results.

1350°K limit of the smaller column. The slight crossing over of the two sets of data points is, therefore, more likely due to random errors than to a systematic error caused by convection.

A detailed error analyis of the data has been made[1] and the results are summarized in Table I. As can be seen the most probable random error ranges from 1.9% to 2.4%, and the maximum random error from 2.3% to 2.9%. A large part (about 2/3) of these errors was estimated to be due to the numerical differentiation procedure used for evaluating dQ_λ/dT in Eq. (1). The good agreement between the results of the two test cells suggest that the actual random errors were less than the estimated values. The magnitude of the systematic errors is difficult to assess. A significant systematic error may be introduced by the thermal accommodation coefficient used in calculating λ''. The accommodation coefficient may vary from 0.01 to 0.5, depending upon the condition of the surface.[4] For "engineering surfaces" α is between 0.2–0.5. Here the value of $\alpha \simeq 0.3$ was selected, resulting in λ'' of $\sim$0.03 (at 800°K) and $\sim$0.07 (at 2100°K). Thus, the systematic error due to α may be as high as 1.5% for the small column and 4% for the large one. As will be shown below, the present results agree very closely with existing thermal conductivity data and with viscosity measurements. Therefore, it is felt that the actual systematic errors were less than these theoret-

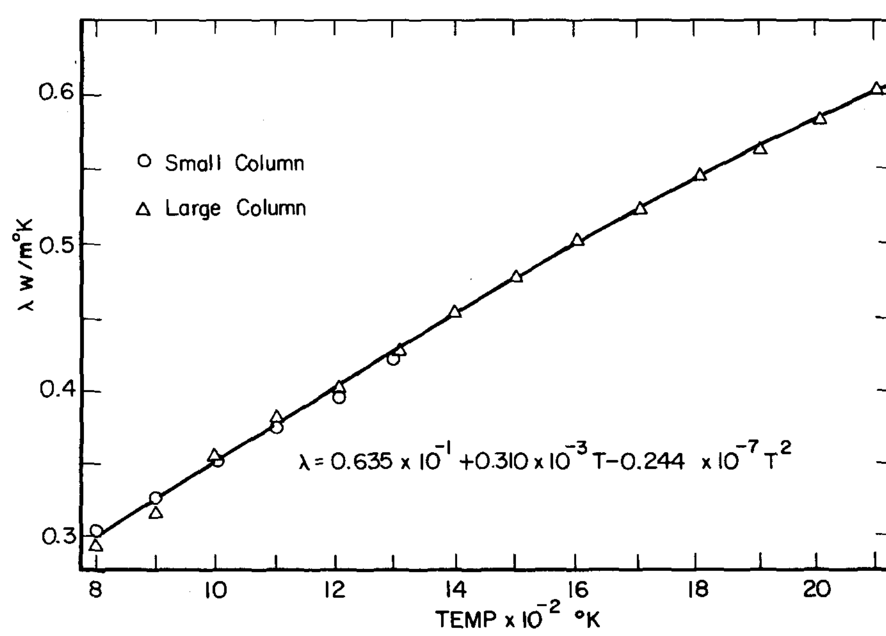

FIG. 2. Comparison between the present result (solid line) and previous data. ◇ Blais and Mann, + Desmond, ○ Saxena and Saxena, □ Timrot and Umanskii.

TABLE II. The parameter $f=\lambda/(C_v\eta)$.

Temp (°K)	f	f^{*a}
1100	2.43	2.35
1200	2.44	2.34
1300	2.45	2.35
1400	2.45	2.36
1500	2.46	2.34
1600	2.47	2.36
1700	2.48	2.35
1800	2.48	2.35
1900	2.50	2.34
2000	2.53	2.36

[a] f^* is calculated without including λ'' in Eq. (1).

ically possible values, and caused at most an additional uncertainty of 1.0%.

A least square fit of the data points results in the polynomial

$$\lambda=0.635\times10^{-1}+0.310\times10^{-3}T-0.244\times10^{-7}T^2, \quad (4)$$

where T is in degrees Kelvin and λ in watts per meter degrees Kelvin. Equation (4) synthesizes the data within an average absolute deviation of 0.5%.

The thermal conductivity values given by Eq. (4) are compared to previous data in Fig. 2. The present results agree closely with the measurements of Desmond[5] and Saxena and Saxena[6] up to the limits of their experiments, which were 1100 and 1350°K, respectively. The measurements of Timrot and Umanskii[7] fall consistently below the present data. Timrot and Umanskii's data for argon and krypton are also lower than those reported by other investigators (e.g., see Faubert and Springer[2]), suggesting a possible systematic error in their experiments. The results of Blais and Mann[8] appear to be too high. As was pointed out by Saxena and Agrawal[9] convection effects were not entirely negligible in these experiments and this may have been the cause of the higher thermal conductivity values.

The accuracy of the thermal conductivity data can be assessed well by the use of the parameter

$$f=\lambda/C_v\eta, \quad (5)$$

where C_v is the constant volume specific heat and η is the viscosity. For a monatomic gas such as helium the value of f should be very nearly 2.5. f values calculated using Eq. (4) for λ and the viscosity data of Guevara *et al.*[10] are shown in Table II.

The results show that f is close to 2.5, its value rising slightly as the temperature increases. This rise may be due to a change in the condition of the filament surface and a corresponding decrease in the thermal accommodation coefficient. This is supported by the observation that f values calculated without the contribution of the temperature jump (i.e., neglecting λ'' in Eq. (1) do not show a systematic increase with temperature (Table II).

Presently, further efforts are made in our laboratory to extend measurements to higher temperatures.

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APPENDIX: DERIVATION OF EQ. (1)

Let us assume that the filament is at a uniform temperature, T_f, and that the heat transferred from the filament by conduction, $\bar{Q}_\lambda$, is known as a function of T_f. The thermal conductivity of the gas λ can be calculated then as follows. The Fourier heat conduction equation is

$$Q_\lambda\equiv\bar{Q}_\lambda/S=-2\pi r\lambda(dT/dr), \quad (A1)$$

where r is the radial coordinate, S is the filament length, and T is the temperature at any point in the gas. Equation (1) may be integrated to yield

$$\frac{Q_\lambda}{2\pi}\ln\frac{r_o}{r_f}=\int_{T(r_o)}^{T(r_f)}\lambda dT, \quad (A2)$$

where r_f and r_o are the radii of the filament and the inner surface of the outer tube, respectively. $T(r_f)$ and $T(r_o)$ are the gas temperatures at r_f and r_o. Neglecting temperature jump at r_o, Eq. (A2) may be written as

$$\frac{Q_\lambda}{2\pi}\ln\frac{r_o}{r_f}=\int_{T_b}^{T_f}\lambda dT-\int_{T_b}^{T(r_o)}\lambda dT-\int_{T(r_f)}^{T_f}\lambda dT. \quad (A3)$$

T_b and T_f are the temperatures of the water bath around the outer cylinder and the filament, respectively. Differentiation of both sides with respect to T_f yields

$$\frac{\ln(r_o/r_f)}{2\pi}\left(\frac{dQ_\lambda}{dT}\right)_{T_f}=\lambda(T_f)-\frac{d}{dT_f}\int_{T_b}^{T(r_o)}\lambda dT-\frac{d}{dT_f}\int_{T(r_f)}^{T_f}\lambda dT. \quad (A4)$$

Replacing in the second integral λ by its value evaluated at $T_b[\lambda(T_b)=\text{const}]$ and applying Eq. (A1) across the outer cylinder (wall thickness w, conductivity λ_w) we obtain

$$\frac{d}{dT_f}\int_{T_b}^{T(r_o)}\lambda dT\cong\frac{\lambda(T_b)}{\lambda_w}\frac{\ln[1+(w/r_o)]}{2\pi}\left(\frac{dQ_\lambda}{dT}\right)_{T_f}. \quad (A5)$$

In the third integral of Eq. (A4) we replace λ by its value evaluated at T_f, $[\lambda(T_f)=\text{const}]$, i.e.,

$$\int_{T(r_f)}^{T_f}\lambda dT\cong\lambda(T_f)[T_f-T(r_f)]. \quad (A6)$$

The above temperature difference can be expressed as

$$T_f - T(r_f) = -g(dT/dr)_{r_f}, \tag{A7}$$

where g is the temperature jump distance

$$g \cong (15/8)[(2-\alpha)/\alpha]L. \tag{A8}$$

α is the thermal accommodation coefficient, and L is the mean free path. Equations (A6)–(A8), together with Eq. (A1) yield

$$\frac{d}{dT_f}\int_{T(r_f)}^{T_f} \lambda dT \cong \frac{15}{8}\,\frac{2-\alpha}{\alpha}\,(2\pi)^{-1}\,\frac{L}{r_f}\left(\frac{dQ_\lambda}{dT}\right)_{T_f}. \tag{A9}$$

Substitution of Eqs. (A5) and (A9) into Eq. (A4) gives Eq. (1) in the text.

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