Measurement of the thermal conductivity of helium up to 2100 K by the column method

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(Received 21 December 1971)

The thermal conductivity of helium was measured at atmospheric pressure in the range 800–2100 K by the column method. The data could be correlated by the polynomial

\[ \lambda = 0.635 \times 10^{-1} + 0.310 \times 10^{-3} T - 0.244 \times 10^{-7} T^2, \]

where \( \lambda \) is in watts per meters degrees Kelvin and \( T \) is in degrees Kelvin. The results obtained were compared with previous thermal conductivity measurements. The data of Desmond and Saxena and Saxena agree closely with the present results; the data of Timrov and Umanskii appear to be too low and those of Blais and Mann too high. Values for \( f = \lambda/\rho C_p \), computed using measured thermal conductivities and available viscosity data, were found to agree well with classical results from kinetic theory.

I. INTRODUCTION

High temperature thermal conductivity data for gases are needed in many problems of practical and basic interest. However, owing to experimental difficulties, few data are available at temperatures above \(~600\) K. In this paper thermal conductivity values of helium are reported up to \(2100\) K. The data were obtained by the column method which, in recent years, has been used with success for measuring thermal conductivities of gases at high temperatures (e.g., see the summaries in Refs. 1 and 2).

II. RESULTS AND DISCUSSIONS

The experimental apparatus and procedure were described elsewhere\(^1\) and, consequently, will not be given here in detail. Essentially, in the experiments the gas was contained between two concentric cylinders (the inner one being a filament) maintained at different temperatures, and the heat transfer through the gas was measured. Two apparatus of different outside diameters were used (referred to as the small and large columns) to assess the magnitude of convection effects. The thermal conductivity was calculated from the expression (see the Appendix)

\[ \lambda(T_f) = \left[ \ln\left(\tau_o/\tau_f\right)/2\pi \right] (dQ_s/dT) \tau_f (1 + \lambda' + \lambda''), \]

where \( \tau_f \) and \( \tau_o \) are the radii of the filament and the outer cylinder, respectively, and \( Q_s \) is the heat conducted per unit length from the filament at a temperature \( T_f \). \( \lambda' \) is a correction due to the temperature drop across the outer cylinder of thickness \( w \) and thermal conductivity \( \lambda_w \)

\[ \lambda'' \approx \lambda \left[ (2 - \alpha)/(\alpha)(L/\tau_f) \ln(\tau_o/\tau_f) \right]^{-1}. \]

\( \alpha \) is the thermal accommodation coefficient and \( L \) the mean free path of the gas at \( T_f \).

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The thermal conductivity of helium was determined at \( 760 \) mm Hg, in the range \( 800–2100 \) K. The experimental results are shown in Fig. 1. With the small diameter column data could be obtained only up to \( 1350 \) K since at higher filament temperatures the temperature rise of the water cooling the outer cylinder became excessive. It is noted, however, that up to \( 1350 \) K the measurements made in the small and large columns agree closely, indicating the absence of significant convection effects. As the temperature increases the Rayleigh number (which is an indication of the magnitude of convection effects\(^5\)) decreases from 16 (at \( 1300 \) K) to 12 (at \( 2000 \) K) for the small column, and from 125 (at \( 1300 \) K) to 106 (at \( 2000 \) K) for the large one. At higher temperatures, convection effects would be even less important than below the

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Table I. Estimate of random errors (percent).

<table>
<thead>
<tr>
<th>Temp (°K)</th>
<th>Most probable error</th>
<th>Maximum error</th>
</tr>
</thead>
<tbody>
<tr>
<td>800</td>
<td>2.4</td>
<td>2.9</td>
</tr>
<tr>
<td>1200</td>
<td>2.2</td>
<td>2.6</td>
</tr>
<tr>
<td>1600</td>
<td>2.0</td>
<td>2.4</td>
</tr>
<tr>
<td>2000</td>
<td>1.9</td>
<td>2.3</td>
</tr>
</tbody>
</table>
1350°K limit of the smaller column. The slight crossing over of the two sets of data points is, therefore, more likely due to random errors than to a systematic error caused by convection.

A detailed error analysis of the data has been made and the results are summarized in Table I. As can be seen the most probable random error ranges from 1.9% to 2.4%, and the maximum random error from 2.3% to 2.9%. A large part (about 2/3) of these errors was estimated to be due to the numerical differentiation procedure used for evaluating $dQ_0/dT$ in Eq. (1). The good agreement between the results of the two test cells suggest that the actual random errors were less than the estimated values. The magnitude of the systematic errors is difficult to assess. A significant systematic error may be introduced by the thermal accommodation coefficient used in calculating $\lambda''$. The accommodation coefficient may vary from 0.01 to 0.5, depending upon the condition of the surface.\(^4\) For “engineering surfaces” $\alpha$ is between 0.2–0.5. Here the value of $\alpha=0.3$ was selected, resulting in $\lambda''$ of ~0.03 (at 800°K) and ~0.07 (at 2100°K). Thus, the systematic error due to $\alpha$ may be as high as 1.5% for the small column and 4% for the large one. As will be shown below, the present results agree very closely with existing thermal conductivity data and with viscosity measurements. Therefore, it is felt that the actual systematic errors were less than these theoret-

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**Fig. 1.** Experimental results.

**Fig. 2.** Comparison between the present result (solid line) and previous data. ○ Blais and Mann, + Desmond, ○ Saxena and Saxena, □ Timrot and Umanskii.
Table II. The parameter \( f = \lambda / (C_v \eta) \).

<table>
<thead>
<tr>
<th>Temp (°K)</th>
<th>( f )</th>
<th>( f^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1100</td>
<td>2.43</td>
<td>2.35</td>
</tr>
<tr>
<td>1200</td>
<td>2.44</td>
<td>2.34</td>
</tr>
<tr>
<td>1300</td>
<td>2.45</td>
<td>2.35</td>
</tr>
<tr>
<td>1400</td>
<td>2.45</td>
<td>2.36</td>
</tr>
<tr>
<td>1500</td>
<td>2.46</td>
<td>2.34</td>
</tr>
<tr>
<td>1600</td>
<td>2.47</td>
<td>2.36</td>
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<tr>
<td>1700</td>
<td>2.48</td>
<td>2.35</td>
</tr>
<tr>
<td>1800</td>
<td>2.48</td>
<td>2.35</td>
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<tr>
<td>1900</td>
<td>2.50</td>
<td>2.34</td>
</tr>
<tr>
<td>2000</td>
<td>2.53</td>
<td>2.36</td>
</tr>
</tbody>
</table>

\( f^* \) is calculated without including \( \lambda'' \) in Eq. (1).

A least square fit of the data points results in the polynomial

\[
\lambda = 0.635 \times 10^{-1} + 0.310 \times 10^{-3} T - 0.244 \times 10^{-7} T^2,
\]

where \( T \) is in degrees Kelvin and \( \lambda \) in watts per meter degrees Kelvin. Equation (4) synthesizes the data within an average absolute deviation of 0.5\%.

The thermal conductivity values given by Eq. (4) are compared to previous data in Fig. 2. The present results agree closely with the measurements of Desmond and Saxena and Saxena up to the limits of their experiments, which were 1100 and 1350°K, respectively. The measurements of Timrot and Umanskii fall consistently below the present data. Timrot and Umanskii's data for argon and krypton are also lower than those reported by other investigators (e.g., see Faubert and Springer), suggesting a possible systematic error in their experiments. The results of Blais and Mann appear to be too high. As was pointed out by Saxena and Agrawal convection effects were not entirely negligible in these experiments and this may have been the cause of the higher thermal conductivity values.

The accuracy of the thermal conductivity data can be assessed well by the use of the parameter

\[
f = \lambda / (C_v \eta),
\]

where \( C_v \) is the constant volume specific heat and \( \eta \) is the viscosity. For a monatomic gas such as helium the value of \( f \) should be very nearly 2.5. \( f \) values calculated using Eq. (4) for \( \lambda \) and the viscosity data of Guevara et al. are shown in Table II.

The results show that \( f \) is close to 2.5, its value rising slightly as the temperature increases. This rise may be due to a change in the condition of the filament surface and a corresponding decrease in the thermal accommodation coefficient. This is supported by the observation that \( f \) values calculated without the contribution of the temperature jump (i.e., neglecting \( \lambda'' \) in Eq. (1) do not show a systematic increase with temperature (Table II).

Presently, further efforts are made in our laboratory to extend measurements to higher temperatures.

**Acknowledgment**

This work was supported by the National Science Foundation under Grant No. GK-14006.

**Appendix: Derivation of Eq. (1)**

Let us assume that the filament is at a uniform temperature, \( T_f \), and that the heat transferred from the filament by conduction, \( \bar{Q}_s \), is known as a function of \( T_f \). The thermal conductivity of the gas \( \lambda \) can be calculated then as follows. The Fourier heat conduction equation is

\[
\frac{\bar{Q}_s}{S} = -2\pi \lambda \frac{dT}{dr},
\]

where \( r \) is the radial coordinate, \( S \) is the filament length, and \( T \) is the temperature at any point in the gas. Equation (1) may be integrated to yield

\[
\frac{\bar{Q}_s}{2\pi} \ln \frac{r_o}{r_f} = \int_{T(r_f)}^{T(t_o)} \lambda dT,
\]

where \( r_f \) and \( r_o \) are the radii of the filament and the inner surface of the outer tube, respectively.\( T_b \) and \( T_f \) are the temperatures of the water bath around the outer cylinder and the filament, respectively. Differentiation of both sides with respect to \( T_f \) yields

\[
\frac{\ln(r_o/r_f)}{2\pi} \left( \frac{d\bar{Q}_s}{dT_f} \right) = \lambda(T_f) - \frac{d}{dT_f} \int_{T(r_f)}^{T(t_o)} \lambda dT
\]

Replacing in the second integral \( \lambda \) by its value evaluated at \( T_b \) and \( T_o \) \( \lambda = \text{const} \), and applying Eq. (A1) across the outer cylinder (wall thickness \( w \), conductivity \( \lambda_w \)) we obtain

\[
\frac{d}{dT_f} \int_{T(r_f)}^{T(t_o)} \lambda dT = \frac{\lambda(T_b) \ln(1 + (w/r_o))}{\lambda_w} \left( \frac{d\bar{Q}_s}{dT_f} \right).
\]

In the third integral of Eq. (A4) we replace \( \lambda \) by its value evaluated at \( T_f \) \( \lambda = \text{const} \), i.e.,

\[
\int_{T(r_f)}^{T(t_o)} \lambda dT \equiv \lambda(T_f)[T_f - T(r_f)].
\]
The above temperature difference can be expressed as

$$T_f - T(r_f) = -g(dT/dr)_r$$

(A7)

where $g$ is the temperature jump distance

$$g \approx (15/8)[(2-\alpha)/\alpha]L.$$  

(A8)

$\alpha$ is the thermal accommodation coefficient, and $L$ is the mean free path. Equations (A6)-(A8), together with Eq. (A1) yield

$$\frac{d}{dT_f} \int_{T(r_f)}^{T_f} \lambda dT \approx \frac{15}{8} \frac{2-\alpha}{\alpha} (2\pi)^{-1} \frac{L}{r_f} \left(\frac{dQ}{dT}\right)_{r_f}.$$  

(A9)

Substitution of Eqs. (A5) and (A9) into Eq. (A4) gives Eq. (1) in the text.