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LOSS OF PRESTRESS IN PRETENSIONED PRESTRESSED CONCRETE BEAMS

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TABLE OF CONTENTS

Page

ACKNOWLEDGMENT.. ii

LIST OF TABLES.. iv

LIST OF FIGURES... v

LIST OF APPENDICES.. viii

NOMENCLATURE.. ix

I INTRODUCTION.. 1

II OBJECTIVE AND SCOPE... 6

III NATURE OF PRESTRESS LOSSES.................................... 7

Elastic Shortening of the Concrete........................ 7
Shrinkage of Concrete..................................... 10
Creep of Concrete... 10
Creep of the Prestressing Tendon.......................... 13

IV INSTRUMENTATION... 15

Gage Points in Beams...................................... 15
Gage Points in Cylinders.................................. 19
Gage Points on Cables..................................... 20
Tensioning the Strands.................................... 22
Loading the Cylinders..................................... 25
Testing Cylinders... 28
Measuring Deflection...................................... 28
Testing Springs... 28
Calibrating Dynamometers or Load Cells.................... 28

V TEST SPECIMENS.. 31

VI TEST RESULTS AND DISCUSSION................................... 38

Concrete.. 38
Modulus of Elasticity of Concrete Cylinders............... 38
Elastic Loss of Prestress................................. 41
Shrinkage of Concrete..................................... 54
Creep of Concrete... 61
Anchorage Length of Strands and Bond of Strands to Concrete 73
Creep of Strand... 81

VII **CONCLUSIONS AND RECOMMENDATIONS**.............................. 84

BIBLIOGRAPHY.. 87

LIST OF TABLES

Table Page

6.1 Modulus of Elasticity of Concrete.............................. 40

6.2 Measured Average Strain Readings at Transfer in Full Anchorage Zone.. 40

6.3 Concrete Stresses at c.g.s. Immediately After Transfer of Prestress.. 55

6.4 Deflection of Beam at Transfer and Growth of Camber............ 55

6.5 Measured Elastic, Creep and Shrinkage Strains in Beams and Cylinders.. 69

6.6 Ratio (R)* of Measured Creep Strain to Measured Elastic Strain for Beams and Cylinders....................................... 69

6.7 Ratio of (R)* for Beam to (R)* for Cylinders................... 71

LIST OF FIGURES

Figure Page

4.1 Whittemore Gages (Two Inches and Ten Inches Long)...... 16

4.2 Supreme Strand Chuck, Brass Plug, Rubber Cork and Strand with Glued Brass Plug.......................... 16

4.3 Position of Gauge Points on Beams and Cylinders........ 17

4.4 Beams in Steel Frames.................................. 21

4.5 Tensioning Device...................................... 23

4.6 Hydraulic Jack, Load Cell, Shims and Strand Chuck Arrangement Used in Prestressing the Strand........... 24

4.7 Shim.. 25

4.8 Test Rig for Determining Creep of Concrete Under Load.. 26

4.9 Calibration of Dynamometer No. 1...................... 29

4.10 Calibration of Dynamometer No. 3...................... 30

5.1 Modulus of Elasticity - 7/16" Ø - Seven-Wire Strand.... 34

6.1 Compressometer... 39

6.2 Concrete - Modulus of Elasticity - Test Series No. 1 - Age 25-1/2 Hours.. 42

6.3 Concrete - Modulus of Elasticity - Test Series No. 1 - Age 28 Days.. 43

6.4 Concrete - Modulus of Elasticity - Test Series No. 1 - Age 25-1/2 Hours.. 44

6.5 Concrete - Modulus of Elasticity - Test Series No. 1 - Age 28 Days.. 45

6.6 Concrete - Modulus of Elasticity - Test Series No. 2 - Age 27 Hours... 46

6.7 Concrete - Modulus of Elasticity - Test Series No. 2 - Age 28 Days.. 47

6.8 Concrete - Modulus of Elasticity - Test Series No. 2 - Age 27 Hours... 48

LIST OF FIGURES (CONT'D)

Figure Page

6.9 Concrete - Modulus of Elasticity - Test Series No. 2 - Age 28 Days.. 49

6.10 Concrete - Modulus of Elasticity - Test Series No. 3 - Age 26 Hours.. 50

6.11 Concrete - Modulus of Elasticity - Test Series No. 3 - Age 28 Days.. 51

6.12 Concrete - Modulus of Elasticity - Test Series No. 3 - Age 26 Hours.. 52

6.13 Concrete - Modulus of Elasticity - Test Series No. 3 - Age 28 Days.. 53

6.14 Shrinkage Strain vs. Time - Test Series No. 1......... 57

6.15 Shrinkage Strain vs. Time - Test Series No. 2......... 58

6.16 Shrinkage Strain vs. Time - Test Series No. 3......... 59

6.17 Creep Strain vs. Time - Test Series No. 1............. 63

6.18 Creep Strain vs. Time - Test Series No. 1............. 64

6.19 Creep Strain vs. Time - Test Series No. 2............. 65

6.20 Creep Strain vs. Time - Test Series No. 2............. 66

6.21 Creep Strain vs. Time - Test Series No. 3............. 67

6.22 Creep Strain vs. Time - Test Series No. 3............. 68

6.23 Creep Strain vs. Time Curve Using Shank Formula....... 72

6.24 Strand Stress as Computed by "Effective Modulus" Method - Test Series No. 1........................... 74

6.25 Strand Stress as Computed by "Effective Modulus" Method - Test Series No. 2........................... 75

6.26 Strand Stress as Computed by "Effective Modulus" Method - Test Series No. 3........................... 76

6.27 Anchorage of 7/16" Ø Seven-Wire Uncoated Strand - Test Series No. 1.................................... 78

LIST OF FIGURES (CONT'D)

Figure Page

6.28 Anchorage of 7/16" ∅ Seven-Wire Uncoated Strand - Test Series No. 2.................................... 79

6.29 Anchorage of 7/16" ∅ Seven-Wire Uncoated Strand - Test Series No. 3.................................... 80

LIST OF APPENDICES

Appendix — Page

A DESIGN OF BEAM.. 92

B MEASURED VERSUS CALCULATED ELASTIC LOSS OF BEAMS........... 95

C WHITTEMORE GAGE DATA SHEET CREEP AND SHRINKAGE............. 98

D DERIVATION OF FORMULA FOR FINAL STRESS IN STEEL (STRAND) IN BEAM AFTER LOSSES ACCORDING TO "EFFECTIVE MODULUS" METHOD.. 100

E CALCULATIONS FOR FINAL STRESS IN STRAND AS PLOTTED ON FIGURES (6.24) THROUGH (6.26).............................. 102

F CALCULATED STRESSES AND STRAINS IN BEAMS AND CYLINDERS AT TRANSFER OF STRESS... 104

G TYPICAL CALCULATION OF CREEP STRAINS USING SHANK FORMULA... 106

H CALCULATION OF FINAL STRESS AS RECOMMENDED................ 108

NOMENCLATURE

A_c	Net area of concrete section of beam
A_g	Gross area of section of beam
A_s	Area of steel
A_t	Transformed area of section of beam
e_t	Eccentricity of c.g.s. from c.g. of section
E_s	Modulus of elasticity of steel
E_c	Modulus of elasticity of concrete
σ_{ot}	Stress in strand just before transfer
σ_i	Stress in strand just after transfer
n	E_s/E_c
F_o	Total force in strands when tensioned
F_i	Total force in strands just after transfer
f_t	Stress in top fibers of concrete in a beam
f_b	Stress in bottom fibers of concrete in a beam
f_s	Stress in concrete at the c.g.s. in a beam
C_1	Distance between top fiber and neutral axis of beam
C_2	Distance between bottom fiber and neutral axis of beam
p	A_s/A_c
σ	Final stress in strand after losses
F	Final force in strands after losses
s	Shrinkage strain
c	Creep strain
I_t	Moment of inertia of the transformed section

f_i	F_i/A_c
M_g	Moment due to dead load

Others are defined as they appear.

CHAPTER I

INTRODUCTION

In December of 1951, pretensioning of reinforcing steel in concrete was first used in the United States when a 24-foot span bridge was erected in Pennsylvania. Since then the use of pretensioned prestressed concrete members has increased rapidly. As of December 31, 1959, only 16 prestressing plants were older than seven years but 104 had been in operation less than three years. As of August 1, 1960, the total number of plants in the United States was approximately 300. In these plants 33 per cent of the output was in precast prestressed beams.

It is essential that the structural designer should fully know the properties of materials and utilize them in the most effective way possible. As yet the behavior of prestressed concrete members has not yet been fully ascertained. In order to design safe and economical pretensioned prestressed concrete beams, it is necessary to know the prestress losses due to elastic action, creep and shrinkage of concrete and due to creep of the steel. Although many structures have been built in the last ten years, there is no agreement as to the determination of prestress losses.

Pretensioned bonded prestressed concrete beams are studied in this investigation. To determine the effective prestress, four major losses are deducted from the original prestress. They are:

(1) Loss due to elastic shortening of concrete.

(2) Shrinkage of concrete.

(3) Creep of concrete.

(4) Creep of the tendons.

Some of the basic terminology may be defined as follows:

Prestressed concrete member: A prestressed concrete member is a structural member in which a permanent predetermined force system is established in such a manner that the stresses resulting from any anticipated condition of external loading are counteracted to a desired degree.

Tendon: A tendon is a steel (cable, wire or bar) used to impart prestress to the concrete.

Bonded tendons: The tendons are "Bonded tendons" if bonded to the concrete either directly or through grouting throughout its length to the surrounding concrete.

Unbonded tendons: The tendons are "Unbonded tendons" if free to move relative to the surrounding concrete.

Pretensioning: An application of the prestress to the concrete in which the tendons are tensioned before the concrete is placed.

Post-tensioning: An application of the prestress to the concrete in which the tendons are tensioned after the concrete is placed and cured.

Jacking force: A jacking force is the temporary force exerted by the jacking device in prestressing the tendons.

Transfer: Transfer is the operation of transferring the tendon force from the fixed anchorage to the concrete.

Effective prestress: Effective prestress is the stress remaining in the tendons after all losses have occurred, excluding the effect of superimposed loads, but including effect of weight of member.

Loss due to elastic shortening of concrete: At transfer, an elastic shortening of the concrete takes place which results in shortening of the tendon and thus loss in stress in the tendons.

Shrinkage of concrete: Shrinkage of concrete is contraction of concrete due to a great variety of causes other than the application of load.

Creep of concrete: Creep of concrete is the time dependent deformation which occurs in concrete under sustained load as distinguished from the elastic deformation which occurs when the load is first applied.

Creep of tendon: Creep of tendon is an inelastic deformation of the steel under stress.

Much work(6,7,11,16,41,47) has been done on creep and shrinkage of ordinary sand and gravel concrete, normally used for reinforced concrete, under sustained loads. But data have been lacking in creep and shrinkage of concrete normally used for prestressed work. Very few investigations(40,43) of creep of high strength tendons have been made. Most of the work on creep and shrinkage measurements on concrete has been on concentrically loaded cylinders. Some of the investigations reported are by Staley and Peabody(9) in 1946 on prisms, by Magnel(10) in 1947, by Shank(11) in 1949 on cylinders, by Washa and Fluck(12) in 1950 on cylinders, by F. Eugene Seamen(16) in 1957 on cylinders, by Ross(17) in 1958 on cylinders. Inge Lyse(61) reported on creep and shrinkage on full sized beams in 1958. Also Cernica and Charignon(44) studied creep and shrinkage behavior of beams in 1961. Alan Mattock(45) reported on creep and shrinkage studies on precast-prestressed concrete bridges in 1961.

Recommendations for estimating prestress losses differ in different countries and also from time to time. Magnel recommended in 1947 that the loss due to creep and shrinkage of concrete in design be taken as 15 per cent of the initial prestressing force for concrete of high strength (greater than 5000 psi) made from sand and crushed stone.

The ACI-ASCE Joint Committee 323 (1956) Report recommended that the loss in steel stress not including friction may be assumed to be 35,000 psi in the case of pretensioning and 25,000 psi in the case of post-tensioning. Alternatively, the loss in steel stress should be computed from the elastic, creep and shrinkage strains of concrete and the relaxation of steel stress, when individual losses can be predicted with reasonable accuracy.

The 1963 revision of the ACI Building Code requires the designer to deduct losses due to elastic shortening, creep and shrinkage of concrete and relaxation of steel stress from the original prestress to estimate the effective prestress. No guides are set for the designer to estimate losses.

A review prepared by the Prestressed Concrete Development Committee(46) suggested overall losses in the steel, expressed as a per cent of initial stress, be 20 per cent for post-tensioning and 25 to 30 per cent for pretensioning. For pretensioning, they also suggested using a strain of 300×10^{-6} inches/in. for shrinkage and 0.33×10^{-6} inches/in. for creep for each lb/sq.in. of applied stress for concrete of high strength.

Most of the data available for creep and shrinkage of concrete has been derived from tests of concentrically loaded concrete cylinders without steel. Since actual beams contain steel, there is an interaction between the concrete and steel so as to modify the creep and shrinkage behavior of concrete in the beams. The strain measurements are made on the surface of the beams assuming perfect bond of the cables. This has been questioned but data are lacking in the study of difference in strain readings between the cable and surface of the concrete beam. Also, the effect of creep of steel on creep and shrinkage of concrete and vice versa in the beams are not known from the cylinder tests. An investigation is needed to compare the estimated losses as found from cylinder measurements with that found on beams.

CHAPTER II

OBJECTIVES AND SCOPE

The purpose of this investigation is:

(1) to study experimentally the four major factors which contribute to prestress losses in pretensioned eccentrically prestressed concrete beams;

(2) to study experimentally the difference in strain reductions, if any, on cables and the surface of the concrete beams;

(3) to compare the losses as observed for the beams with those found from loading cylinders concentrically;

(4) to compare the losses as observed for the beams with current design practices;

This investigation is limited to:

(1) Members with straight tendons.

(2) Members with one eccentricity of tendons.

(3) One kind of concrete mix.

(4) Loading of cylinders at stress at the level of cable.

Also as the specimens were stored in the laboratory without temperature or humidity control, these effects were not studied.

CHAPTER III

NATURE OF PRESTRESS LOSSES

Four major losses that affect the design of prestressed concrete were defined in the previous chapter and will now be examined in more detail.

Elastic Shortening of the Concrete

When concrete is stressed in compression, it shortens. At transfer of prestress, an elastic shortening of the concrete takes place. Due to bonding of tendon and concrete, the steel also shortens. This results in the loss of stress in the tendon. The loss of tension in the tendons upon transfer is computed upon the assumption that the unit strain in the tendon at the position of the center of gravity of tendons is equal to that in concrete at the same position.

The effectiveness of the actual bond is reported to depend mainly on the following factors: the quality of the concrete, the working of the concrete in the molds, its strength at transfer, the type of curing before and after transfer, the level of prestress of the concrete, and the diameter and the surface properties of the tendons and how they are distributed in the cross section. If slippage occurs in the tendons at transfer, the strain measurements on the surface of beams will not adequately reflect this. The shrinkage in the concrete itself during curing and a small amount of creep may be included in the strain measurements immediately after transfer.

The strain measured on cylinders under concentric loads includes some creep and shrinkage as it is difficult to load cylinders and then measure strain reductions instantaneously.

As concrete shortens under the application of load an amount proportional to the load and the elastic modulus, elastic deformation of the concrete can be calculated from elastic theory. Here it is assumed the bond of concrete to steel is complete. Then the two conditions can be applied: (1) Force in tendons is equal to force in concrete; and (2) strain in concrete at c.g.s. is equal to strain in the tendons at c.g.s.

<u>Condition (1)</u>: Force in Steel = Force in Concrete

$$\sigma_i A_s = \left[\frac{F_i}{A_c} + \frac{F_i e(C_2 - C_1)}{2I_t} + \frac{M_g(C_1 - C_2)}{2I_t} \right] A_c \qquad (3.1)$$

(see Appendix A)

Let

$$\ell_1 = \frac{e(C_2 - C_1)A_c}{2I_t}$$

$$\ell_2 = \frac{M_g(C_1 - C_2)}{2I_t}$$

$$K_1 = 1 + \ell_1$$

$$K_2 = \ell_2/p$$

$$p_1 = p/K_1$$

$$p = A_s/A_c$$

Then

$$\sigma_i A_s = (f_i + f_i \ell_1 + \ell_2)A_c$$

$$= f_i A_c K_1 + \ell_2 A_c$$

Therefore

$$\sigma_i = \frac{f_i}{p} K_1 + K_2$$

$$= \frac{f_i}{p_1} + K_2 \qquad (3.2)$$

Condition (2): Compressive strain in the concrete = change of strain in the steel.

$$\frac{\frac{F_i}{A_c} + \frac{F_i e^2}{I_t} - \frac{M_g e}{I_t}}{E_c} = \frac{\sigma_{ot} - \sigma_i}{E_s} \qquad (3.3)$$

Substituting

$$\ell_3 = 1 + \frac{A_c e^2}{I_t}$$

$$\ell_4 = \frac{M_g e}{I_t} \qquad \text{in Equation (3.3)}$$

$$f_i \cdot \ell_3 \cdot n - n\ell_4 = \sigma_{ot} - \frac{f_i}{p_1} - K_2$$

i.e., $f_i(1 + p_1 n\ell_3) = p_1(\sigma_{ot} - K_2) + n\ell_4 p_1$

or $$f_i = \frac{p_1(\sigma_{ot} - K_2) + p_1 n\ell_4}{1 + nK_3}, \quad K_3 = p_1\ell_3 \qquad (3.4)$$

or $$\sigma_i = \frac{\sigma_{ot} - K_2 + n \cdot \ell_4}{1 + nK_3} + K_2 \qquad (3.5)$$

If we neglect the effect of dead load, $\ell_4 = K_2 = 0$ and

$$\sigma_i = \frac{\sigma_{ot}}{1 + nK_3} \qquad (3.6)$$

For a concentrically prestressed member the eccentricity "e" is zero and, neglecting dead load, Equation (5) reduces to

$$\sigma_i = \frac{\sigma_{ot}}{1 + np} \qquad (3.7)$$

$$\text{Elastic Loss in Cable} = \sigma_{ot} - \sigma_i \qquad (3.8)$$

Equations (3.5), (3.6) and (3.7) provide a direct method of estimating the steel stress in the tendons after the elastic loss. It is necessary to know the stress in the tendons just before the transfer of stress to the concrete, i.e, it is necessary to know loss of prestress due to creep of steel between anchorage and cutting of the strands.

Shrinkage of Concrete

Shrinkage of concrete was previously defined as contraction of concrete due to great variety of causes other than the application of load. The phenomenon of drying shrinkage has been discussed by Davis and Troxell.(13) According to them, shrinkage of concrete is due to the loss of adsorbed water through evaporation from the hardened cement gel. The amount of shrinkage depends on many factors such as water-cement ratio, chemical composition of the cement, maximum size, grading and mineral character of the aggregate, size of the member and rate of drying.

Much experimental data is available on shrinkage of various mixes and the total shrinkage varies from 0 to 800×10^{-6} inches/in. (In design of Beams, for the concrete normally used in prestressed work, values of 200 to 300×10^{-6} inches/in. are used for the shrinkage strain in pretensioning.)

Creep of Concrete

Creep of the concrete is the time-dependent deformation under sustained load. Many theories of creep in concrete have been advanced te explain the mechanism of creep(5,15) but there appears to be no conclusive evidence as to the exact mechanism of creep. The reader is referred to a paper "Theories of Creep in Concrete" by A. M. Neville(14) for an excellent comparison of different theories of creep of concrete. Sustained load causes the concrete to creep, this creep being due to:

(1) Viscous flow of the cement paste (Lorman, Freudenthal, etc.).

(2) Consolidation due to seepage, or flow of adsorbed water from the cement gel due to applied pressure (Lorman and Ross).

(3) "Delayed elasticity" due to the cement paste acting as a restraint on the elastic deformation of the skeleton formed by the aggregate and the cement crystals in the manner of "Kelvin's Sponge" (Cowan, Arnan, Reiner and Teinowitz).(48)

(4) Permanent deformation caused by localized fracture.

Creep of the concrete is reported to depend on many variables such as magnitude of sustained load; age at which the sustained load is applied; size of the member; composition of concrete and fineness of cement used in it; size, grading and mineral character of the aggregates; amount of pozzolans used; water-cement ratio; volume of cement paste; temperature and humidity during the curing period prior to loading; rate of drying during the loading period; and temperature during the loading period.

An article "Creep of Plain and Reinforced Concrete" by Fluck and Washa(47) discusses in general regarding the creep behavior and the factors influencing such behavior of plain and reinforced and prestressed concrete. It lists many references on laboratory and field tests of creep of concrete. Very little information is currently available on creep of concrete under normal prestressed conditions, i.e., with triangular stress distribution and a changing prestress force. This series of tests would yield creep data for one stress distribution.

Many mathematical relationships(40) to predict creep of concrete subjected to constant and also to variable loading have been developed but none have been generally accepted. All require some constants to be determined from laboratory tests and some are too cumbersome to be used in practical design. Also, in all structures in use, stress in the concrete varies with time, and the problem becomes that of predicting the creep under varying stress from the results of tests made under constant stress. Three methods of calculating for the effects of creep and shrinkage, "Effective Modulus," "Rate of Creep" and "Method of Superposition" are discussed by Ross.(17) The "Effective Modulus" Method has been used to predict final stress in the cable. (See Appendix D.)

Creep strain as measured in the present investigation includes flow strains due to the load and whatever difference there may be between shrinkage of non-stressed and stressed specimens.

The rate of creep decreases with time and creep approaches a limit under constant load. The time to approach a limit may vary considerably depending on many variables such as level of stress and type of aggregate.

Also, it is noted(11) that the change in the rate of creep seems to be uniform up to 3/4 or more of the ultimate strength (well through the working range) and that the change is abrupt near the "true" ultimate strength.

In pretensioning, the loss of stress in the tendon resulting from the creep of concrete is normally assumed to be 150% of the total stress loss due to elastic action. But the creep strain may vary from 100% to 300% of the elastic deformation of the concrete, depending on many factors, such as strength of concrete, temperature and humidity of environment, level of stress.

Creep of the Prestressing Tendon

Creep of the tendons is an inelastic deformation of the steel dependent on time and initial prestress. The loss of stress occurs without strain reduction.

The mechanism of creep has been dealt with elsewhere.(5,14) As in creep of concrete, there is no general agreement on one theory. The creep depends on various factors such as stress level of the steel, type of steel, chemical composition, processing and final treatment such as drawn or stress relieved.

Some tests on creep of steel wire have been reported.(40,43) Initially the tests measured creep at constant loads. As the development of prestressed concrete progressed, creep at constant strain was suggested as it was a good duplication of actual conditions prevailing in prestressed concrete.

Normally the strands are tensioned with a stress level of about 70% of the ultimate tensile strength. In an actual prestressed concrete member, the deformation of the concrete reduces the drop in stress attributable to relaxation. Tests made by Niels Thorsen(25) showed stress relaxation in 10 days of 5-8 per cent of the initial prestress for stress levels equal to 70 per cent of the actual ultimate tensile strength for various smooth wires. Relaxation for 7-wire strands are only slightly higher than for smooth wires. The stress-relieved strand would creep considerably more and is not considered desirable. After transfer of prestress subsequent stress relaxation loss is insignificant.

In prestress concrete plants, the transfer of stress from steel to concrete takes place within 24 hours after thetensioning of the strands for pretensioned work in order to make maximum use of the prestress beds. As the stress relaxation after transfer of prestress is much less, it seems that there should be less stress relaxation in steel stress for this kind of work.

CHAPTER IV

INSTRUMENTATION

For strain measurements on beams and cylinders, Whittemore Mechanical Strain Gages, one with a ten inch gage length and the other with a two inch gage length were used (Figure 4.1). For strain measurements on cables used in beams, another ten inch Whittemore Gage was used. Gage point locations on beams and cylinders are as shown in Figure 4.3.

Gage Points in Beams

The gage points consisted of 3/8" dia. brass plugs tapered to 1/4" dia. in 1/4" length as shown in Figure 4.2 for beams and cables. Gage points were located on both sides of the beams at the elevation of the center of gravity of tendons. For prestressed beams, gage points were 2" c./c. for 24" on each end to facilitate in obtaining anchorage length. For the rest of the beam the plugs were 10" c./c. For shrinkage beams, the gage points were 10" c./c. throughout (Figure 4.3). Numbers on the beams represent the average distance from the end of the beam. The gage points on the cables were only near the center of the specimens.

For obtaining reliable and accurate strain readings with gage points it is necessary that the gage point holes in the plugs are perpendicular to the plane of the beam surface and that the gage point holes are reamed to seat the points of the Whittemore Gage properly.

In Test Series 1, the following procedure was followed to obtain the gage points in the beams:

(1) Both the surfaces to be glued - brass plug and wooden form - were cleaned.

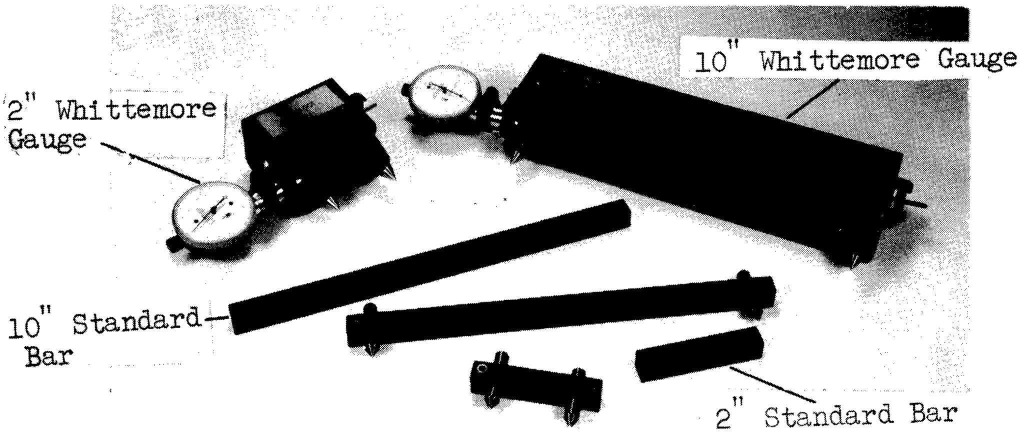


Figure 4.1. Whittemore Gages (Two Inches and Ten Inches Long).

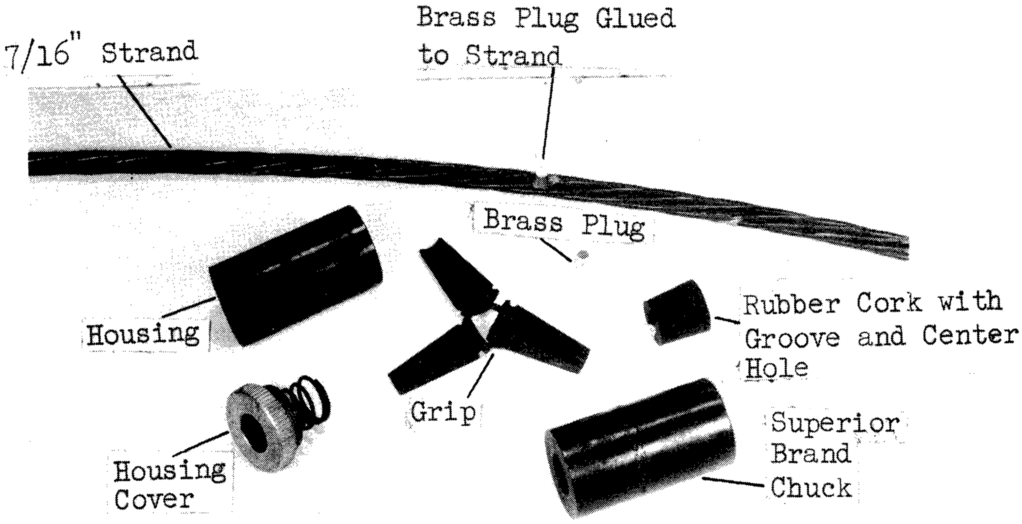
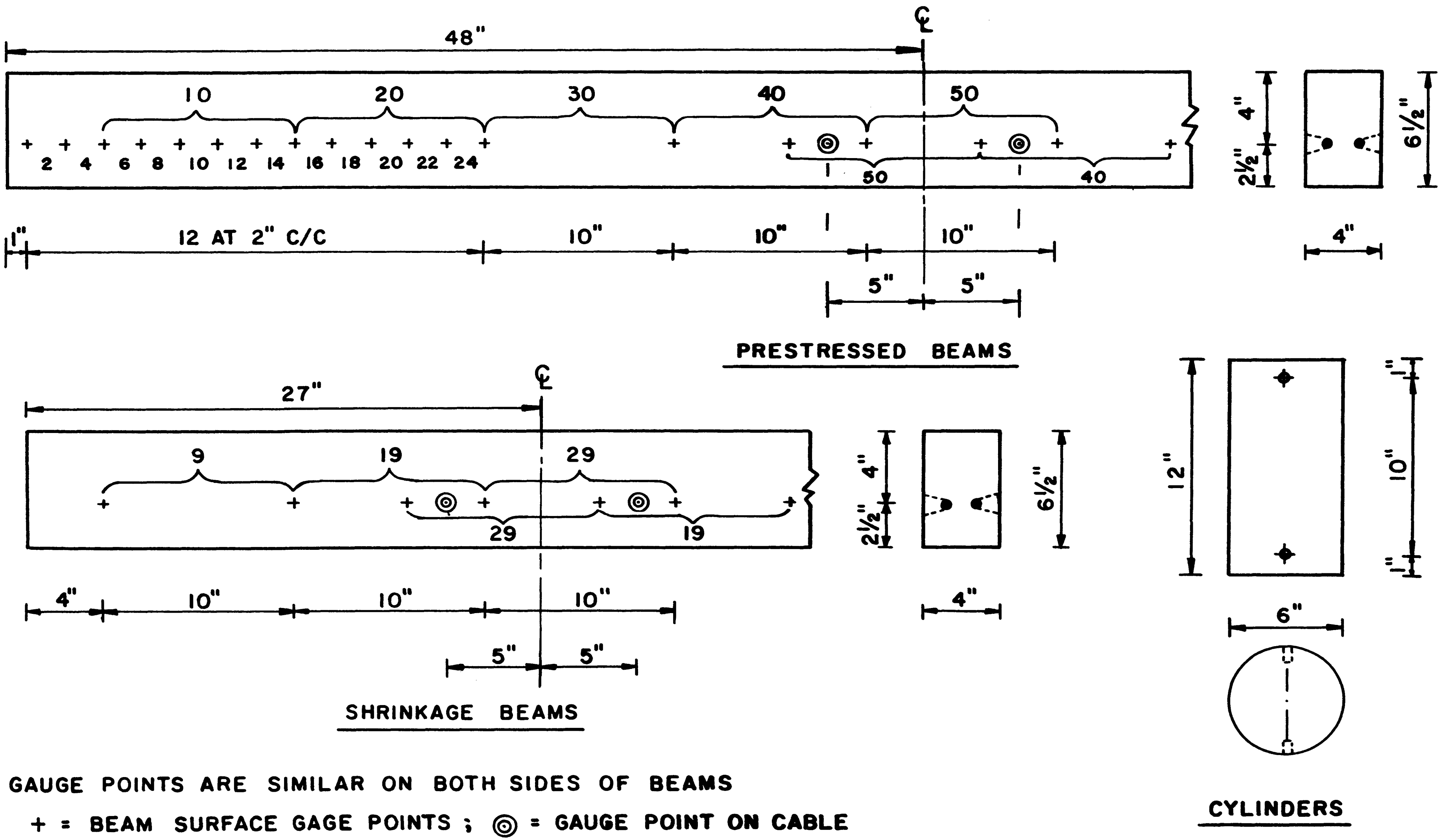


Figure 4.2. Supreme Strand Chuck, Brass Plug, Rubber Cork and Strand with Glued Brass Plug.



GAUGE POINTS ARE SIMILAR ON BOTH SIDES OF BEAMS

+ = BEAM SURFACE GAGE POINTS ; ◎ = GAUGE POINT ON CABLE

Figure 4.3. Position of Gauge Points on Beams and Cylinders.

(2) An initial coat of Duco Cement, of the type used for cementing electrical strain gages, was placed on the previously marked gage point locations on the forms. This coat was allowed to dry for 20-30 minutes.

(3) Another coat of cement was applied to both the wood and brass plugs and the plugs were then set in place. The cement was allowed to dry for at least one day.

(4) The forms were assembled and concreting done. Care was taken not to strike the plugs with the internal vibrator during concrete placement.

(5) The plugs were thus embedded in the concrete at the proper locations. When the forms were removed, the exposed ends of the plugs were punchmarked to the proper gage lengths.

(6) These points were then drilled with drill No. 56 and reamed.

(7) All sets of points were checked with the Whittemore Gage for obtaining consistent readings. Some points were found to be defective and were redrilled and additionally reamed to obtain consistent readings.

This procedure required: (1) careful punchmarking to prevent the loosening of the plug from its bond with the concrete; (2) drilling the holes in the brass plugs with hand drill held approximately at right angles to the surface of the beams; (3) hand reaming the holes keeping the tool at right angles to the surface of the concrete and (4) one hour of time after the forms were stripped and before any readings could be taken.

To save time as well as to get better gage points the following procedure was followed in Test Series 2 and 3:

(1) The brass plugs were drilled and reamed prior to gluing onto the inner form surface. This required very careful positioning of the plugs to insure being within the range of gage lengths readable on the Whittemore Gage. The holes were filled with plastic clay to avoid filling with Duco Cement.

Steps (1) through (4) were done as for Test Series 1.

(5) The plugs were embedded in concrete at their proper locations. After the forms were removed, the clay from the gage points was removed and the plugs were cleaned with acetone to remove any cement that might have entered the points.

(6) All sets of points were checked by Whittemore Gage. All except one performed satisfactorily. Hand drilling and reaming was used to correct the faulty point.

Gage Points in Cylinders

The gage points consisted of brass plugs 1/2" in diameter and about 3/4" long. They were drilled and tapped so that a 1/8" dia. bolt can fit into the hole. Holes of the size of the bolt were made at 10" c./c. in two diametrically opposite vertical rows on the cardboard cylinder molds. The plugs were then held in position by the bolt inserted from outside. After the concrete reached approximately 4000 psi strength as measured on independent test cylinders, the bolts were taken out, and the forms stripped. The exposed surface of the 1/2" diameter plug was used for gage point location. As done on the beams, holes at 10" c./c. were drilled in the brass plugs and reamed to fit the Whittemore Gage points.

Gage Points on Cables

For measuring strain reductions on the cable, brass plugs of the type used in beams were modified to fit the curvature of the strand on one side. Different glues were tried to attach these plugs on the strand. The results obtained by using epoxy-type "Twin Weld" Metal Mender manufactured by Fybrglas Industries, Chicago 18, Illinois, were satisfactory if they were applied as follows: Clean the cable and plugs with carbon tetrachloride. Mix well the two constituents of "Twin Weld" in equal proportions. Wait for two to three minutes. Stretch the cable to a load of 1000 lb. or more. Apply epoxy glue at a desired point enough to fill the gooves of the cable. Apply a thin coat on the plug and attach it on the cable. Use C-clamps to keep plugs in their position. For faster drying use the heat lamp as suggested by Fybrglas Industries for four to five hours. Wait for a week before taking out the clamps.

For taking readings on plugs attached to the cable (Figure 4.2), 1" holes as seen in Figure 4.4 were made by placing 1" diameter tapered rubber corks (Figure 4.2) between the formwork and the cable. The corks were grooved to fit the cable and also the plugs. The rubber corks were waxed for easy removal from the concrete. They were easily pulled out after the formwork was removed.

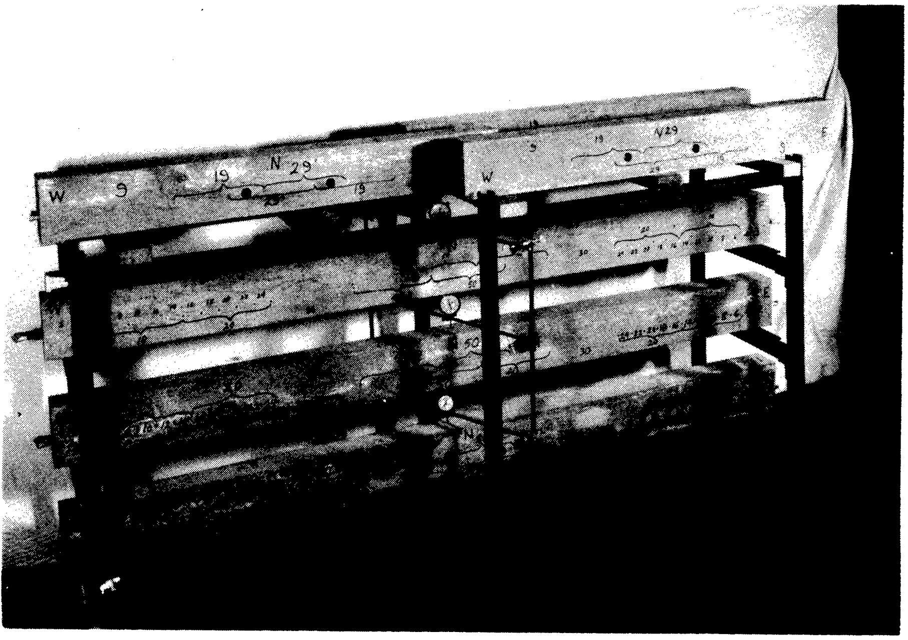


Figure 4.4. Beams in Steel Frames.

In the first test series, where the cable was tensioned after the formwork and cork were placed in position, one of the plugs was dislodged due to the shear force created between the strands and the rubber cork during tensioning of the strands. In Test Series 2 and 3, the cables were tensioned to their design limit first and then the rubber corks and formwork were assembled. No plugs were lost in these series.

In each series, the brass plugs were punchmarked for proper distance, drilled and reamed properly to fit points of the Whittemore Gage.

Tensioning the Strands

Each strand used in the prestressed beams was stressed to a load of 18900 pounds in the self-stressing frame as shown in Figure 4.5. This was accomplished as follows: A 30-ton Simplex hydraulic jack (Figure 4.6) was used to stress the strands individually. The force in the strand was measured by calibrated load cell which had a hole in the center through which the cable passed. This eliminated possible eccentricity on the cell. "Supreme" brand strand chucks were used. The required force on the strand was obtained by setting the calibrated amount of strain on a SR-4 Wheatstone bridge and by balancing the bridge when the strand was stressed. The relative positions of the jack, load cell, the strand chucks and ths shims used are shown in Figure 4.6.

The sequence of operations to obtain desired force was as follows:

(1) After passing the strand through proper holes of the formwork and tensioning frame, the strand was stressed by a jack until the preset Wheatstone bridge was balanced.

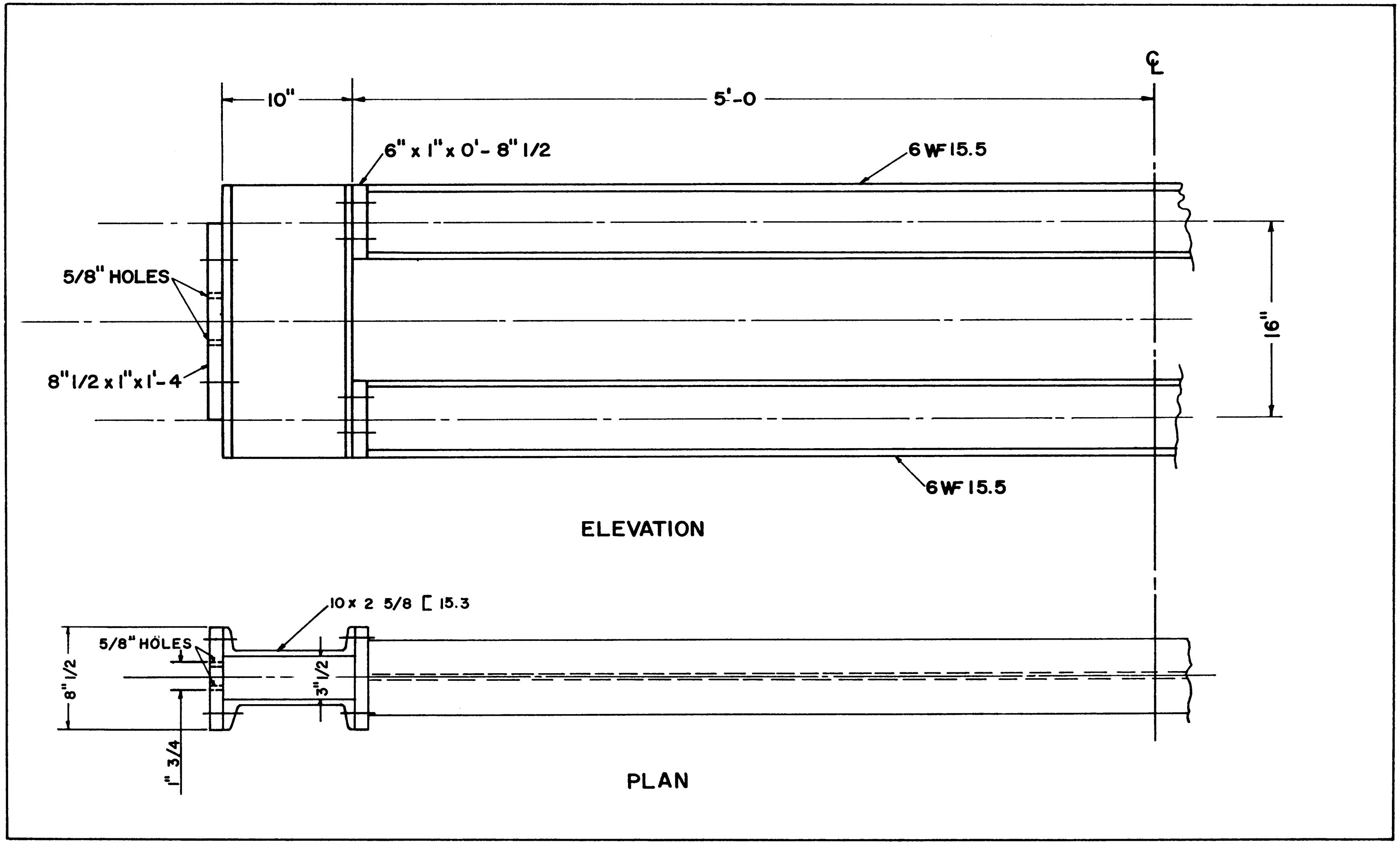


Figure 4.5. Tensioning Device.

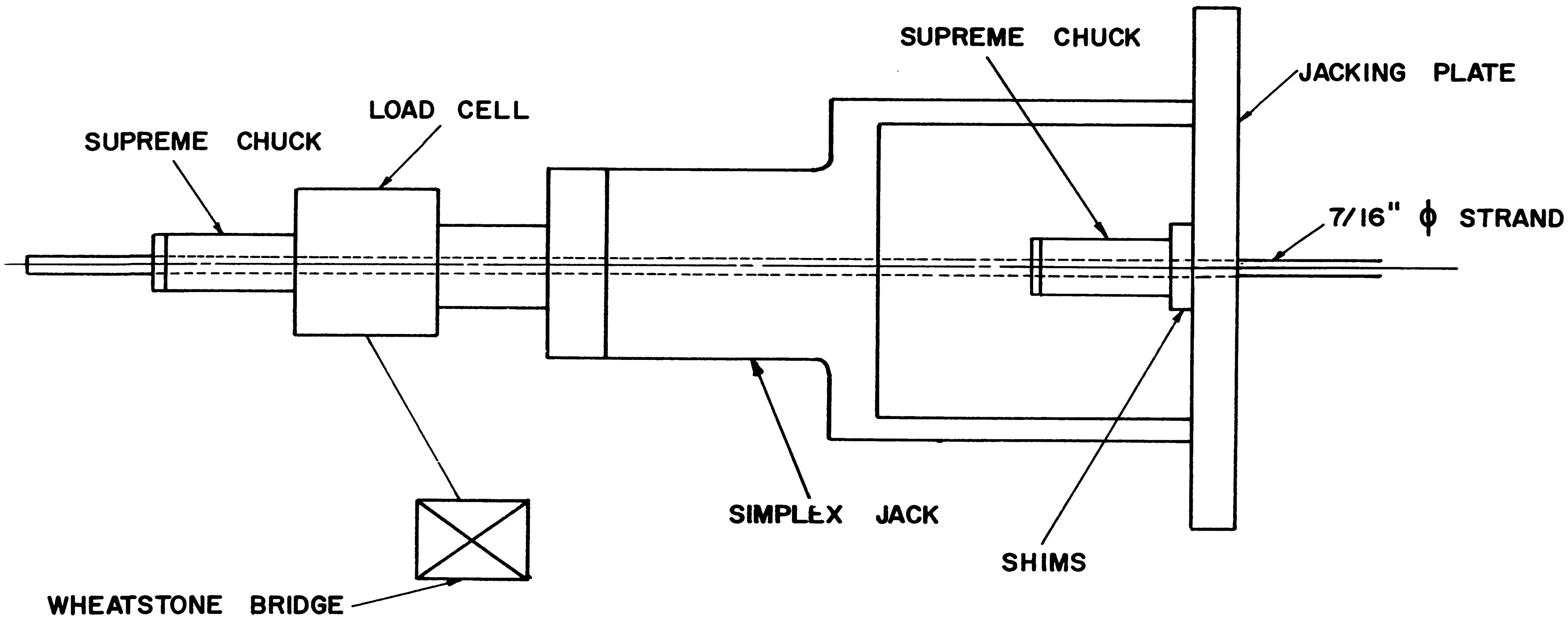


Figure 4.6. Hydraulic Jack, Load Cell, Shims and Strand Chuck Arrangement Used in Prestressing the Strand.

(2) The chuck was slid tightly against the end plate of the stressing frame.

(3) The jack was released; some slippage of the strand occurred resulting from the chucks "seating" on the cable.

(4) The strand was rejacked until the bridge balanced again. This created some gap (approximately 1/8") between the chuck and the end plate.

(5) Shims as shown in Figure 4.7 were inserted in the gap. The thickness of thin shims was 0.007".

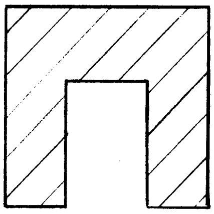


Figure 4.7. Shim.

(6) The strand was tensioned a little more so that one thin shim could be inserted in the gap.

(7) The jack was released.

(8) The strand was rejacked until the bridge balanced again. It was checked to determine whether the shims were loose or not.

(9) In all cases, shims were just loose and so the jack was then released to obtain a desired force on the strand.

Loading the Cylinders

A test rig as shown in Figure 4.8 was used to load the cylinders concentrically to a desired stress. A 30-ton hydraulic Simplex jack was placed between two plates to exert the pressure. The load was measured by two calibrated SR-4 load cells and a Wheatstone Bridge. The sequence was as follows:

Figure 4.8. Test Rig for Determining Creep of Concrete Under Load.

(1) Level the upper base plate by putting shims on the spring tops if required.

(2) Place 2 - 6" x 4" dummy cylinders on top and bottom with 3 - 6" x 12" cylinders in between on the upper base plate in the center of the plate.

(3) Place the lower jack plate on top of the dummy cylinder and position the cylinders to be in the center of the plate.

(4) Place the jack between lower jack plate and upper jack plate.

(5) Place two dynamometers between upper jack plate and top plate.

(6) Level the top plate.

(7) Apply the load slowly, bringing the special nuts down under upper base plate and lower jack plate until the required load is reached.

(8) Tighten the nuts above lower jack plate. Release the jack.

(9) Apply the load again and check to see if the nuts are loose.

(10) Tighten the nuts and release the jack to obtain the necessary load.

An improvement in the above procedure would be to take readings on cylinders after loading 10% of the final load and correcting the cylinder position if an eccentric loading has been obtained. After the satisfactory readings for concentric loading are obtained the final loading can be reached by applying further load.

Testing Cylinders

The cylinders were tested with a 300,000 pound capacity range Tinius Olsen Testing Machine. Strain readings for cylinders under load were obtained using a compressometer (Figure 6.1). One cylinder in each set was preloaded to 1/2 f_c' before taking strain readings.

Measuring Deflection

An Ames' dial reading 1/1000" was used to measure instantaneous as well as creep deflections. The dials were set on the stand as seen in Figure 4.4 for creep deflection measurements.

Testing Springs

Springs used for test rigs were tested for their deflection characteristics. Four springs with nearly the same spring constants were used in each test rig. According to the manufacturer, the springs were not susceptible to creep for static load of up to 12,000 pounds.

Calibrating Dynamometers or Load Cells

The load cells which were used in measuring force on steel strand were carefully calibrated by applying known load and noting the strain readings as observed by a standard SR-4 Wheatstone Bridge. The load vs. strain curves for Dynamometers No. 1 and No. 3 are given in Figures 4.9 and 4.10.

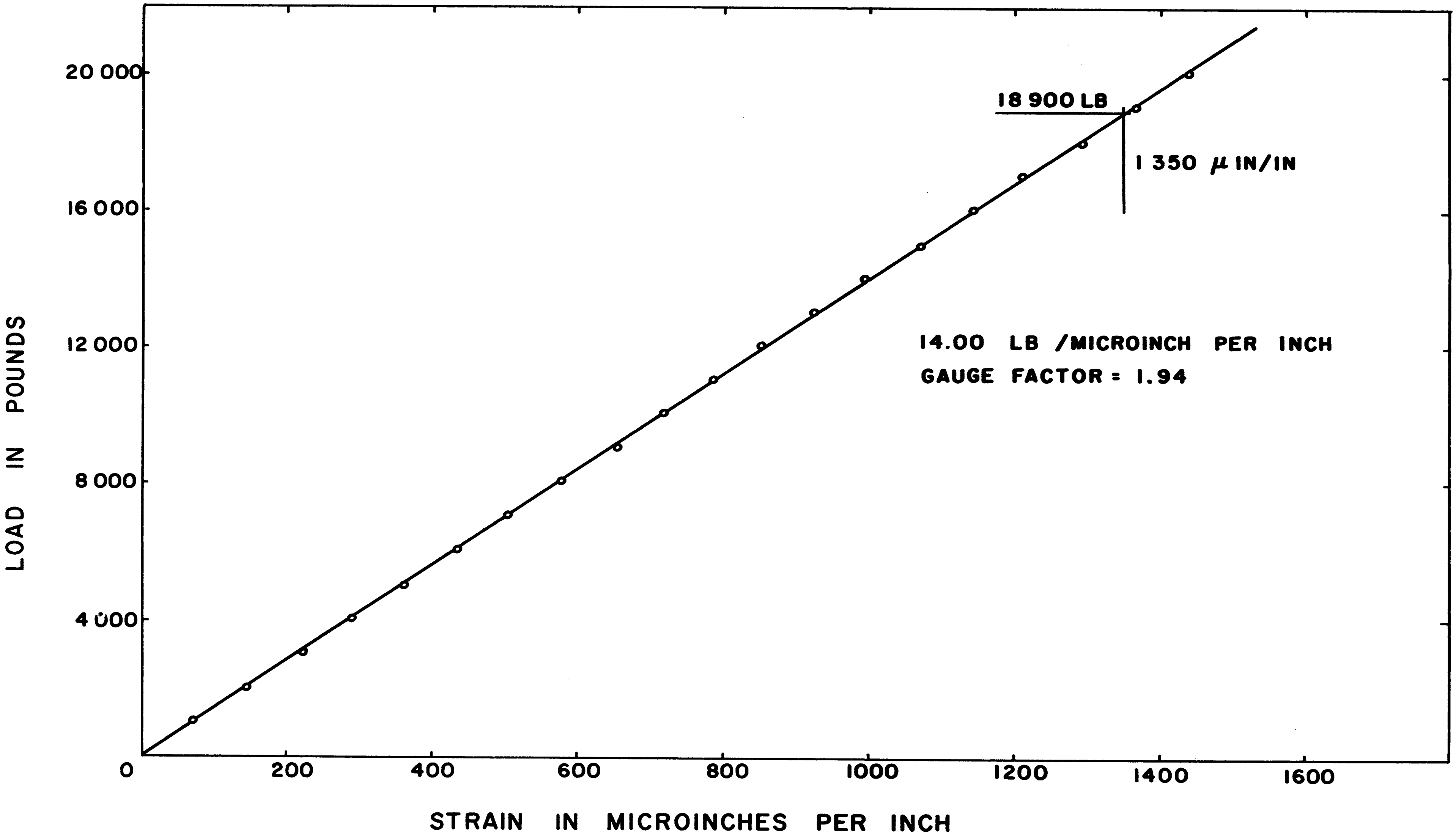


Figure 4.9. Calibration of Dynamometer No. 1.

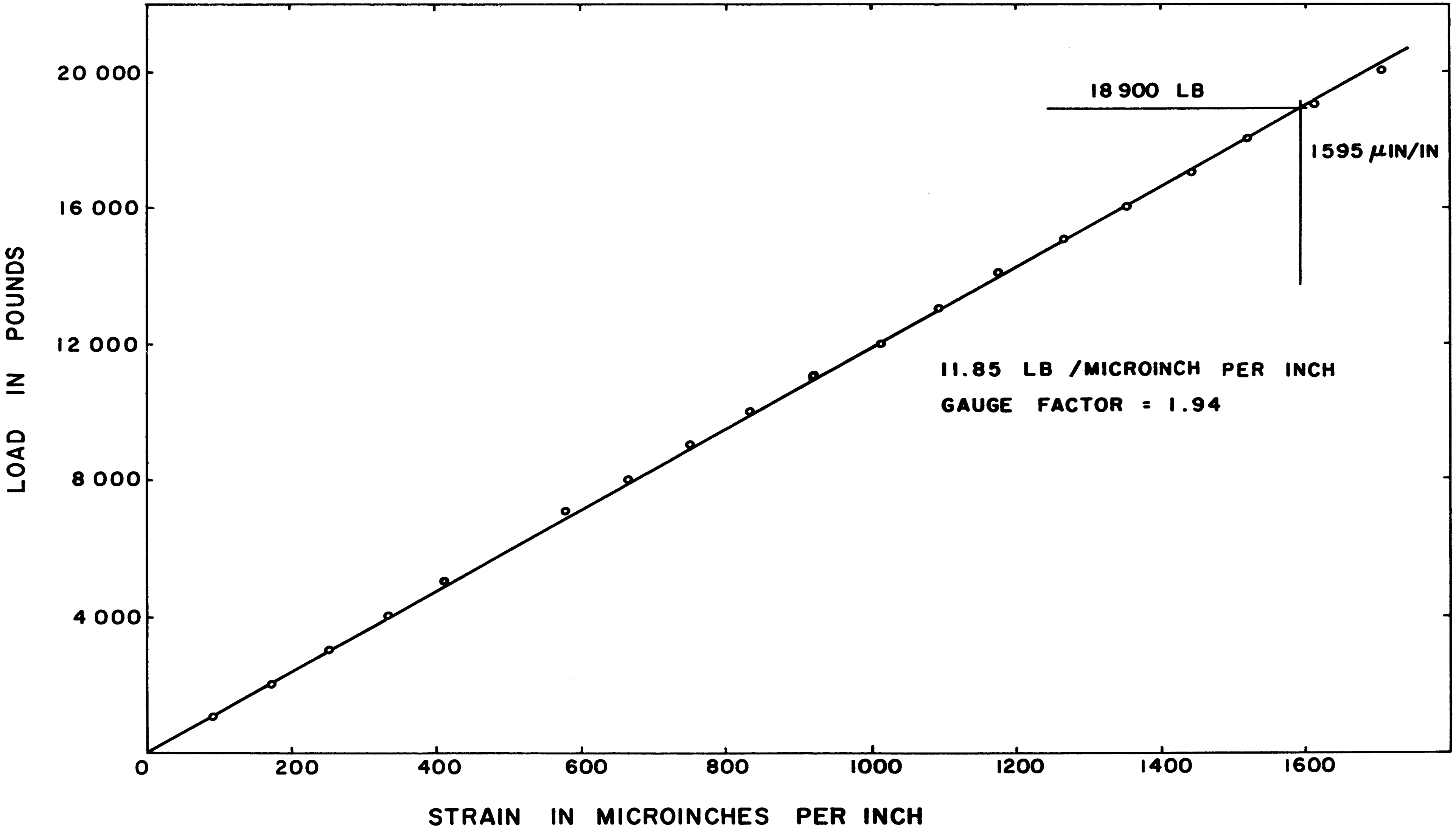


Figure 4.10. Calibration of Dynamometer No. 3.

CHAPTER V

TEST SPECIMENS

In pretensioned prestressed concrete beams, strain measurements give losses due to creep and shrinkage of the concrete and to creep of the steel. To separate these effects, two test beams were cast in each series. One beam was prestressed and the other was a unprestressed pilot beam used to measure shrinkage strain only. An attempt was made to find creep characteristics of strands stressed approximately the same as the strand in the prestressed beam. As most of the steel creep loss takes place in the early stage of tensioning, it was assumed that all creep loss took place before the strands were cut. Thus creep of concrete in beams was obtained by deducting the shrinkage strain of concrete from total strain reduction in the prestressed beams. Similarly, for compressed cylinders, the creep of concrete was separated from shrinkage strain by deducting shrinkage strain of nonstressed cylinders from the total strain reduction of loaded cylinders.

Three identical test series of specimens of concrete were cast for this study. Each series consisted of one prestressed concrete beam, one shrinkage beam, three stressed cylinders, two shrinkage cylinders and seven compression test cylinders.

The dimensions of the concrete beams were as follows:

Prestressed Beam: 4" wide, 6-1/2" deep, 8'-0" long.

Shrinkage Beam: 4" wide, 6-1/2" deep, 4'-6" long.

All the cylinders used were of standard size - 6" diameter, 12" long. The two beams and twelve cylinders of each series were cast at the same time. The size of the prestressed concrete beams was partly governed by the readily available prestressing frame. Shrinkage beams were made shorter as they were found satisfactory for this purpose in earlier studies.

The concrete mix was selected from the recommended mixes for use in prestressed concrete bridges by the Michigan State Highway Department. The batch weights required per cubic yard of concrete are:

Cement - Type III	658 lbs.
Sand	1180 lbs.
Gravel (No. 4 to one inch)	1850 lbs.
Water	235 lbs.
Pozzolith No. 8 (Improved)	1.82 lbs.
Vinsol (.012% by Wt. of Cement)	0.079 lbs.

The sand and gravel used was from Killins Gravel Company, Ann Arbor, Michigan.

The concrete was mixed in a 5-1/2 cu. ft. capacity pug-mill type mixer. An internal vibrator with a 1" diameter head was used to facilitate the placing of concrete. When the concrete had initially set, after about five to six hours, the beams were covered with wet burlap. The cylinders were covered with steel plate to avoid moisture loss. All the specimens were cured for approximately 24 hours under wet burlap. When the average cylinder strength was approximately 4000 psi, the forms for beams and cylinders were stripped. The concrete work sheets and test cylinders data are included on pages 35 through 37. At all times, beams and cylinders were cured and stored under similar conditions.

Two straight, stress relieved, 7-wire, cold drawn, high carbon, uncoated strands of Roebling System 7/16" diameter were used in prestressed beams and shrinkage beams. The strands used in shrinkage beams were not stressed whereas the tension in each strand used in the prestressed beams during anchorage was 18900 lb. The stress was transferred to the beams by bond of concrete to the strands. No other reinforcing was used in the beams. According to the manufacturer, the properties of the cable were as follows:

Ultimate strength	- 248,000 psi
Modulus of Elasticity	- 27×10^6 psi
Recommended Tensioning Load/Cable	- 18900 lb.

Stress-strain curves as obtained for two strands 60' long are shown on Figure 5.1. An attempt was made to measure relaxation of steel stress on these cables.

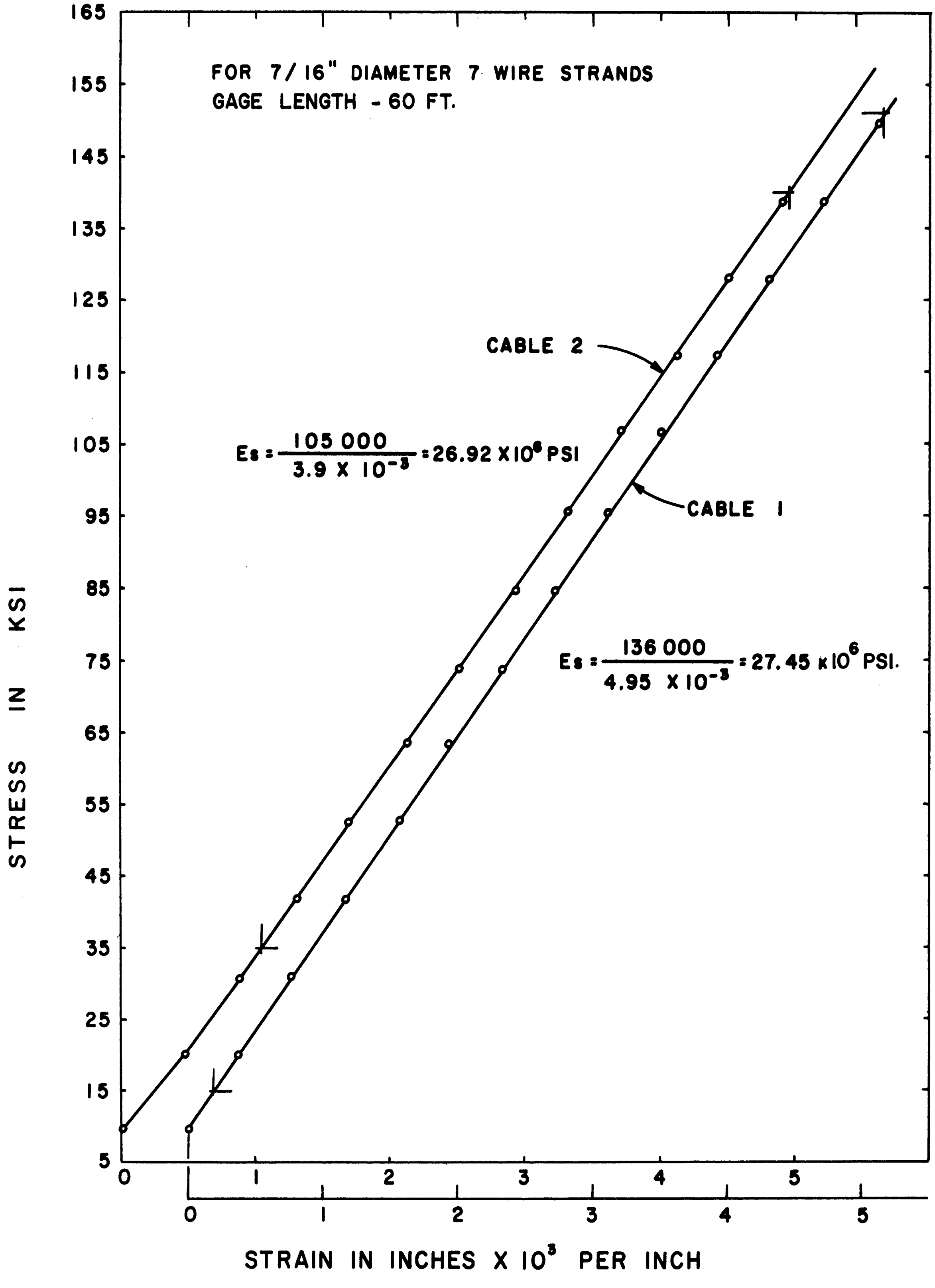


Figure 5.1. Modulus of Elasticity.

CONCRETE MIX WORK SHEET

Series No.: One   Date: April 3, 1961   Time: 9:30 A.M.

Temperatures: 70°F   Humidity:

Estimated batch weights required per cubic yard of concrete are:

Cement = 658 lb. (7 Sk/cyd)
Sand (Dry) = 1180 lb.
Gravel (Dry) = 1850 lb.
Water (Net) = 235 lb.
Pozzolith No. 8 = 1.82 lb.
Vinsol resin = 0.079 lb. (.012% by wt. of cement)

Estimated batch weights required for these tests for 5 cu. ft. of concrete including 10% surplus are:

Material	Sand	Gravel	Water	Cement	Pozzolith	Vinsol
Dry Weight	218.5 lb.	338.50 lb.	42.25 lb.	121.85 lb.	.337 lb.	.0169 lb.

Required 337 c.c. of Pozzolith No. 8 at a concentration .001 lb/c.c.

Required 42.25 c.c. of Vinsol resin at a concentration .0004 lb/c.c.

Water at Mixer:

Initial weight = 42.25 lb.
Added Water = 14.75 lb.
Total added = 57.00 lb.
Surplus = 2.50 lb.
Net Water used = 54.50 lb.
W/C = .447

Slump = 2-1/2 in.

Unit Weight:

Tare +Conc. = 92.75 lb.
Tare = 18.125 lb.
Wt. of Conc. = 74.625 lb.
Volume of Tare = .499 c.ft.
Unit weight = 149.55 lb/c.ft.

Air Content = 3.8%

Remarks:

Mix 1 minute dry, 3 minutes with water, wait 2 minutes then mix 2 minutes.

Age of Concrete Cylinders	Date	No.	Load in lb.	Strength in psi	Avg. Strength of Cyl. in psi
25 hours	4-4-61	1	114,000	4040	4160
		2	120,000	4230	
		3	119,000	4200	
28 days	5-1-61	1	182,500	6450	6490
		2	185,000	6525	

CONCRETE MIX WORK SHEET

Series No.: Two Date: May 9, 1961 Time: 10:15 A.M.

Temperatures: 69°; 58.5° Humidity: 49%

Estimated batch weights required per cubic yard of concrete are:

Cement Type III = 658 lb. (7 Sk/cyd)
Sand (Dry) = 1180 lb.
Gravel (Dry) = 1850 lb.
Water (Net) = 235 lb.
Pozzolith No. 8 = 1.82 lb.
Vinsol resin = 0.079 lb. (.012% by wt. of cement)

Estimated batch weights required for these tests for 5-1/2 cu.ft. of concrete including 10% surplus are:

Material	Sand	Gravel	Water	Cement	Pozzolith	Vinsol
Dry Weight	240 lb.	372.75 lb.	47.85 lb.	134 lb.	.3707 lb.	.01608 lb.

Required 370.7 c.c. of Pozzolith No. 8 at a concentration .001 lb/c.c.

Required 40.2 c.c. of Vinsol resin at a concentration .0004 lb/c.c.

Water a Mixer:

Initial weight = 47.85 lb.
Added Water = 10.15 lb.
Total added = 58.00 lb.
Surplus = .00 lb.
Net Water used = 58.00 lb.
W/C = .433

Slump = 2-3/4 in.

Unit Weight:

Tare +Conc. = 92.400 lb.
Tare = 18.125 lb.
Wt. of Conc. = 74.275 lb.
Volume of Tare = .499 c.ft.
Unit weight = 148.9 lb/c.ft.

Air Content = 3.5%

Remarks:

Mix 1 minute dry, 3 minutes with water, wait 2 minutes then mix 2 minutes.

Age of Concrete Cylinders	Date	No.	Load in lb.	Strength in psi	Avg. Strength of Cyl. in psi
27 hours	5-10-61	1	110,000	3890	3890
		2	105,500	3730	
		3	114,500	4040	
28 days	6-6-61	1	159,000	5610	5810
		2	165,000	5830	
		3	169,500	5990	

CONCRETE MIX WORK SHEET

Series No.: Three Date: May 25, 1961 Time: 9:30 A.M.

Temperatures: 78°; 61° Humidity: 38%

Estimated batch weights required per cubic yard of concrete are:

Cement = 658 lb. (7 Sk/cyd)
Sand (Dry) = 1180 lb.
Gravel (Dry) = 1850 lb.
Water (Net) = 235 lb.
Pozzolith No. 8 = 1.82 lb.
Vinsol resin = 0.079 lb. (.012% by wt. of cement)

Estimated batch weights required for these tests for 5-1/2 cu.ft. of concrete including 10% surplus are:

Material	Sand	Gravel	Water	Cement	Pozzolith	Vinsol
Dry Weight	240 lb.	372 lb.	47.85 lb.	134 lb.	.3707 lb.	.01608 lb.

Required 370.7 c.c. of Pozzolith No. 8 at a concentration .001 lb/c.c.

Required 40.2 c.c. of Vinsol resin at a concentration .0004 lb/c.c.

Water at Mixer:

Initial weight = 47.85 lb.
Added Water = 13.15 lb.
Total added = 61.00 lb.
Surplus = 0.50 lb.
Net Water used = 60.50 lb.
W/C = .450

Slump = 3 in.

Unit Weight:

Tare +Conc. = 91.625 lb.
Tare = 18.125 lb.
Wt. of Conc. = 73.50 lb.
Volume of Tare = .499 c.ft.
Unit weight = 147.5 lb/c.ft.

Air Content = 4.5%

Remarks:

Mix 1 minute dry, 3 minutes with water, wait 2 minutes then mix 2 minutes.

Age of Concrete Cylinders	Date	No.	Load in lb.	Strength in psi	Avg. Strength of Cyl. in psi
26 hours	5-26-61	1	112,500	3970	4080
		2	114,000	4015	
		3	117,500	4150	
28 days	6-23-61	1	157,500	5560	5760
		2	165,000	5830	
		3	167,000	5900	

CHAPTER VI

TEST RESULTS AND DISCUSSION

Concrete

An attempt was made to keep the quality of concrete the same for three test series. Cement was procured in sufficient quantity to make concrete for the three test series. Sand and gravel were used in dry condition to avoid discrepancies in the w/c ratio and were obtained from the same source. The average strength obtained at transfer for the three test series varied by less than 6-1/2 per cent and strength of cylinders in the same mix varied less than 5 per cent. This can be considered as good control for concrete mixes. The air content obtained for the three mixes was less than the planned 5 per cent air. Slump was between 2-1/2" to 3". The average strength at 28 days for the three mixes varied by less than 10 per cent and strength of cylinders in the same mix varied by about 5 per cent. All cylinders and beams were kept wet until the average strength of concrete was approximately 4000 psi and then they were air dried.

Modulus of Elasticity of Concrete Cylinders

Modulus of Elasticity for concrete was found by loading concrete cylinders. Two different procedures were employed in obtaining the modulus. In one procedure, the cylinder was preloaded to 1/2 its ultimate strength twice and then measurements for strains corresponding to certain loads were taken with the use of the compressometer as shown in Figure 6.1. Stress-strain curves for the other procedure were found without preloading the cylinder. Although the first procedure is recommended by ASTM in

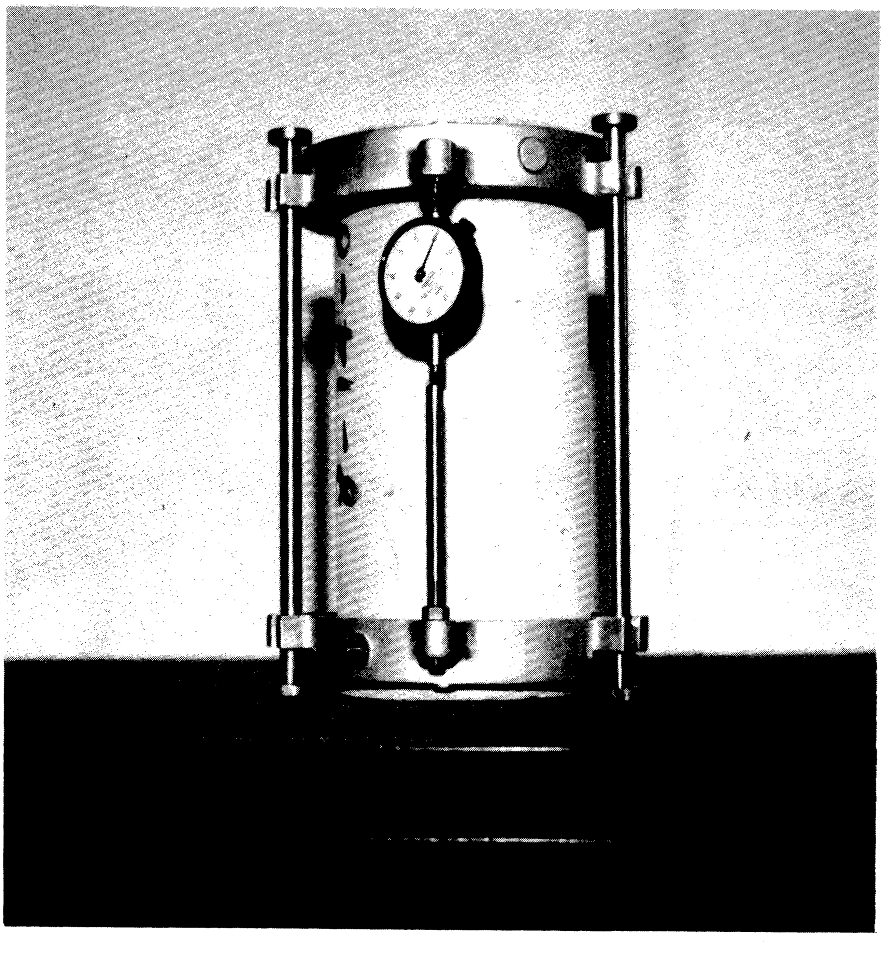


Figure 6.1. Compressometer

TABLE 6.1

MODULUS OF ELASTICITY OF CONCRETE

Test Series	Age in Days	f'_c in psi	Measured Secant Modulus @ 1/2 f'_c in psi	ACI Formula(1) $E_C = 33\sqrt{w^3 f'_c}$ in psi	Remarks
1	1	4240(2)	3.906 x 10^6	3.751 x 10^6	
	1	4200(3)	3.78 x 10^6	3.734 x 10^6	
	28	6525(2)	5.034 x 10^6	4.667 x 10^6	
	28	6450(3)	4.54 x 10^6	4.627 x 10^6	
	1	4160	-	3.716 x 10^6	f'_c - Avg.(4)
	28	6490	-	4.64 x 10^6	f'_c - Avg.(4)
2	1	4045(2)	3.894 x 10^6	3.664 x 10^6	
	1	3730(3)	3.943 x 10^6	3.526 x 10^6	
	28	5990(2)	4.65 x 10^6	4.46 x 10^6	
	28	5610(3)	4.675 x 10^6	4.321 x 10^6	
	1	3890	-	3.595 x 10^6	f'_c - Avg.(4)
	28	5810	-	4.394 x 10^6	f'_c - Avg.(4)
3	1	4015(2)	3.86 x 10^6	3.653 x 10^6	
	1	4150(3)	3.878 x 10^6	3.711 x 10^6	
	28	5560(2)	4.649 x 10^6	4.293 x 10^6	
	28	5900(3)	4.214 x 10^6	4.425 x 10^6	
	1	4085	-	3.682 x 10^6	f'_c - Avg.(4)
	28	5760	-	4.374 x 10^6	f'_c - Avg.(4)

(1) Value of w in ACI(62) Formula assumed as 145

(2) Cylinders loaded to 1/2 f'_c twice before obtaining stress-strain curve

(3) Cylinders not preloaded

(4) Avg. values of f'_c - See Concrete Mix Work Sheet

TABLE 6.2

MEASURED AVERAGE STRAIN READINGS AT TRANSFER IN FULL ANCHORAGE ZONE

Test Series No.	Avg. Beam Surface Strain in μin./in.	Avg. Strain as Measured on Cable in μin./in.
1	370	375
2	346	350
3	380	390

obtaining stress-strain curves, it was felt that to obtain elastic loss in pretensioned prestressed concrete, the second procedure might be more realistic. Secant modulus at 1/2 f'_c did not vary by more than 2-1/2 per cent for concrete cylinders for the two procedures at transfer of prestress and thus it appears preloading of cylinders to 1/2 f'_c is unnecessary. Modulus of Elasticity as found by the empirical formula $E_c = 33 \sqrt{w3f'_c}$ and by two previous procedures is tabulated in Table 6.1. There appears to be good agreement between the test values and the empirical values although the test values represent only two cylinders for each series. From the stress-strain curves (Figures 6.2 to 6.13) it is seen that for the stress of 1600 psi at c.g.s. or less, secant modulus at 1/2 f'_c would be a good approximation for modulus of elasticity of concrete. It should also be noted from stress-strain curves that there is a linear relationship between the stress and strain for the stress levels used in the beam.

Elastic Loss of Prestress

Elastic loss of prestress is the reduction of force in the cables caused by the elastic shortening of the concrete when the prestress force is applied (at transfer). The concrete is considered to be elastic. The strain readings for each test were recorded immediately before and after transfer of prestress on beams and cylinders. The difference between the two readings gave shortening of concrete due to stress imposed by the strand. Strain readings, obtained in the region of full anchorage on the surface of the concrete beams and on the strands at transfer of prestress, were very nearly equal. (See Table 6.2.) Thus the assumption, that the

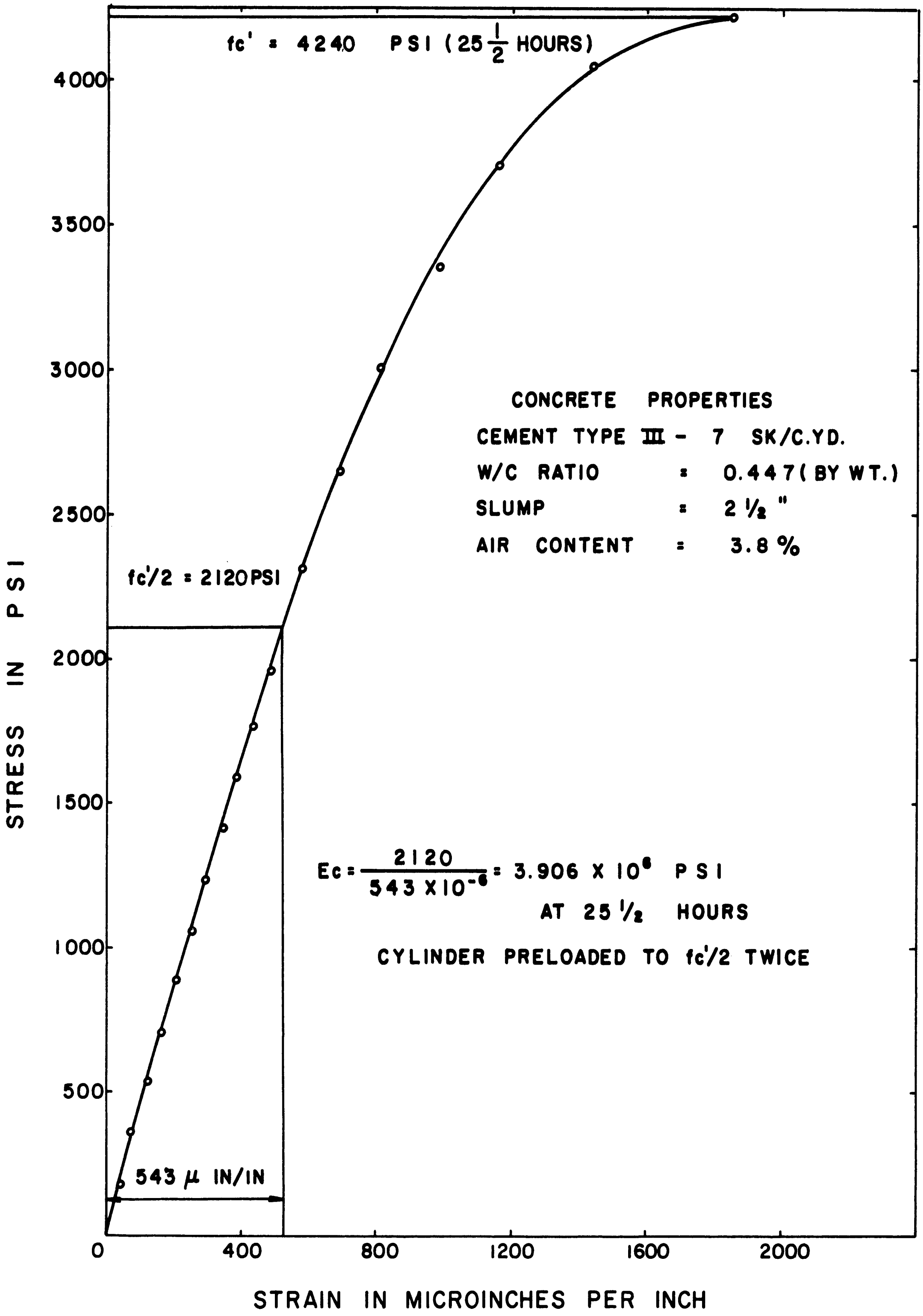


Figure 6.2. Concrete - Modulus of Elasticity - Test Series No. 1 - Age 25-1/2 Hours.

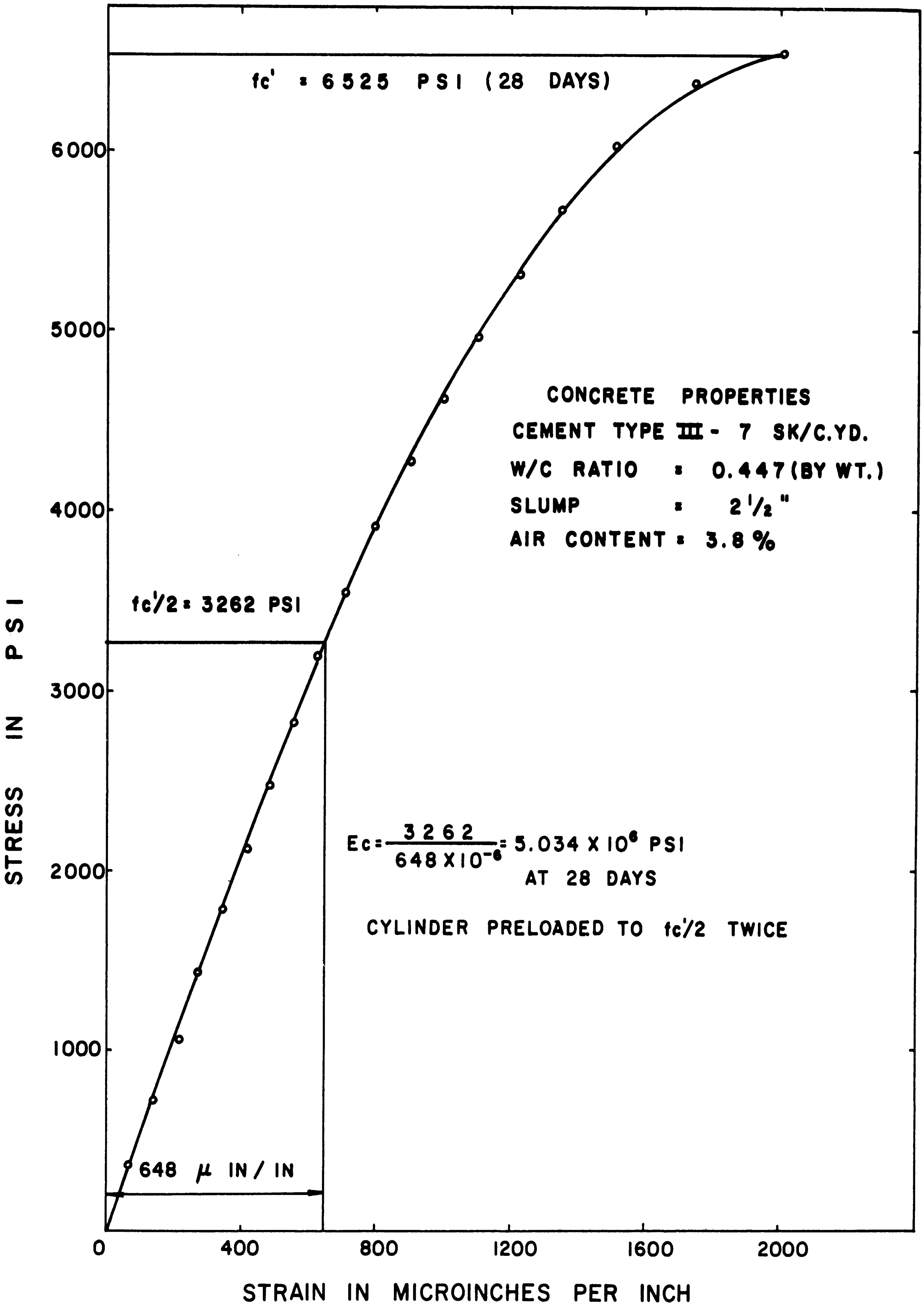


Figure 6.3. Concrete - Modulus of Elasticity - Test Series No 1 - Age 28 days.

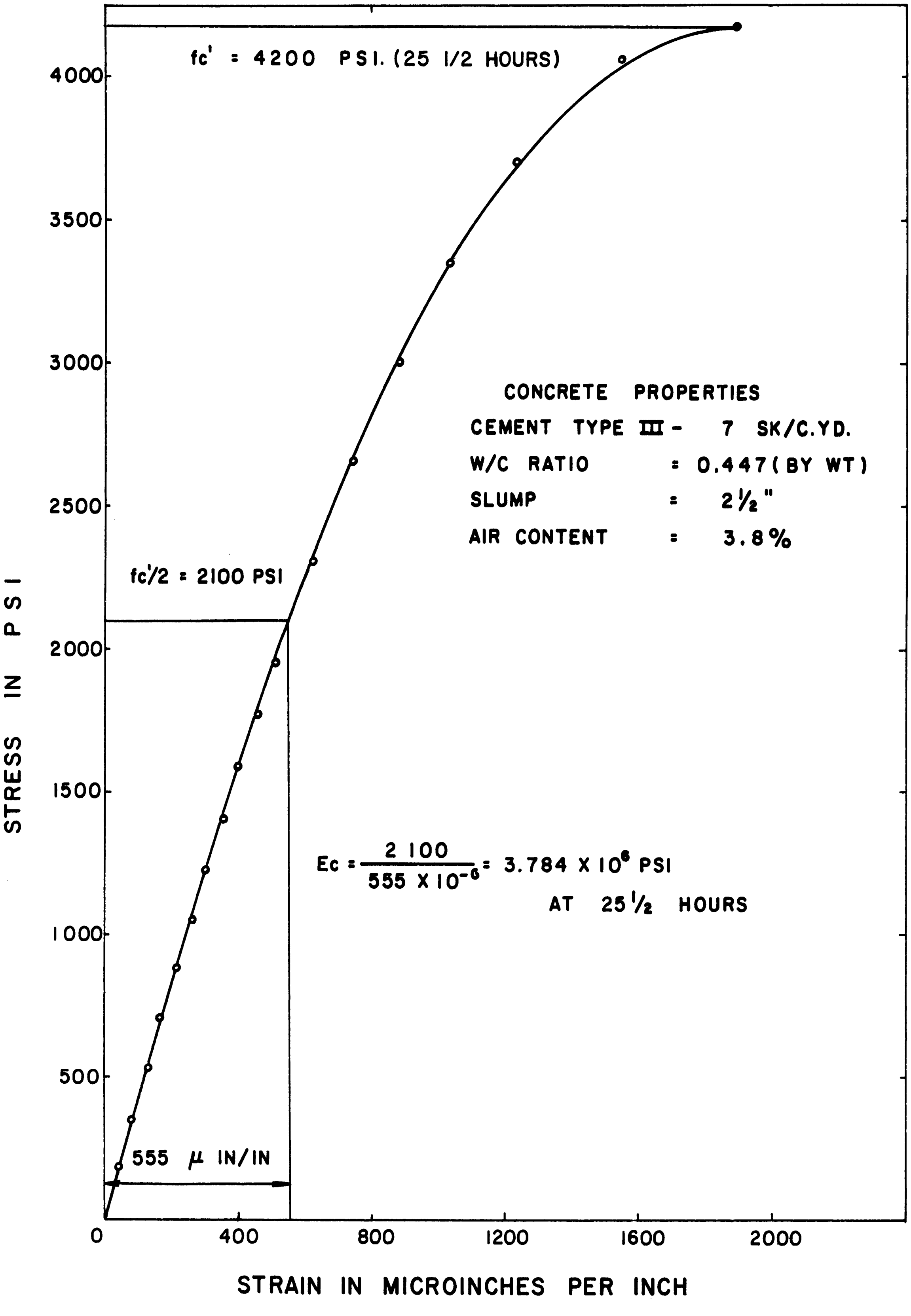


Figure 6.4. Concrete - Modulus of Elasticity - Test Series No. 1 - Age 25-1/2 Hours.

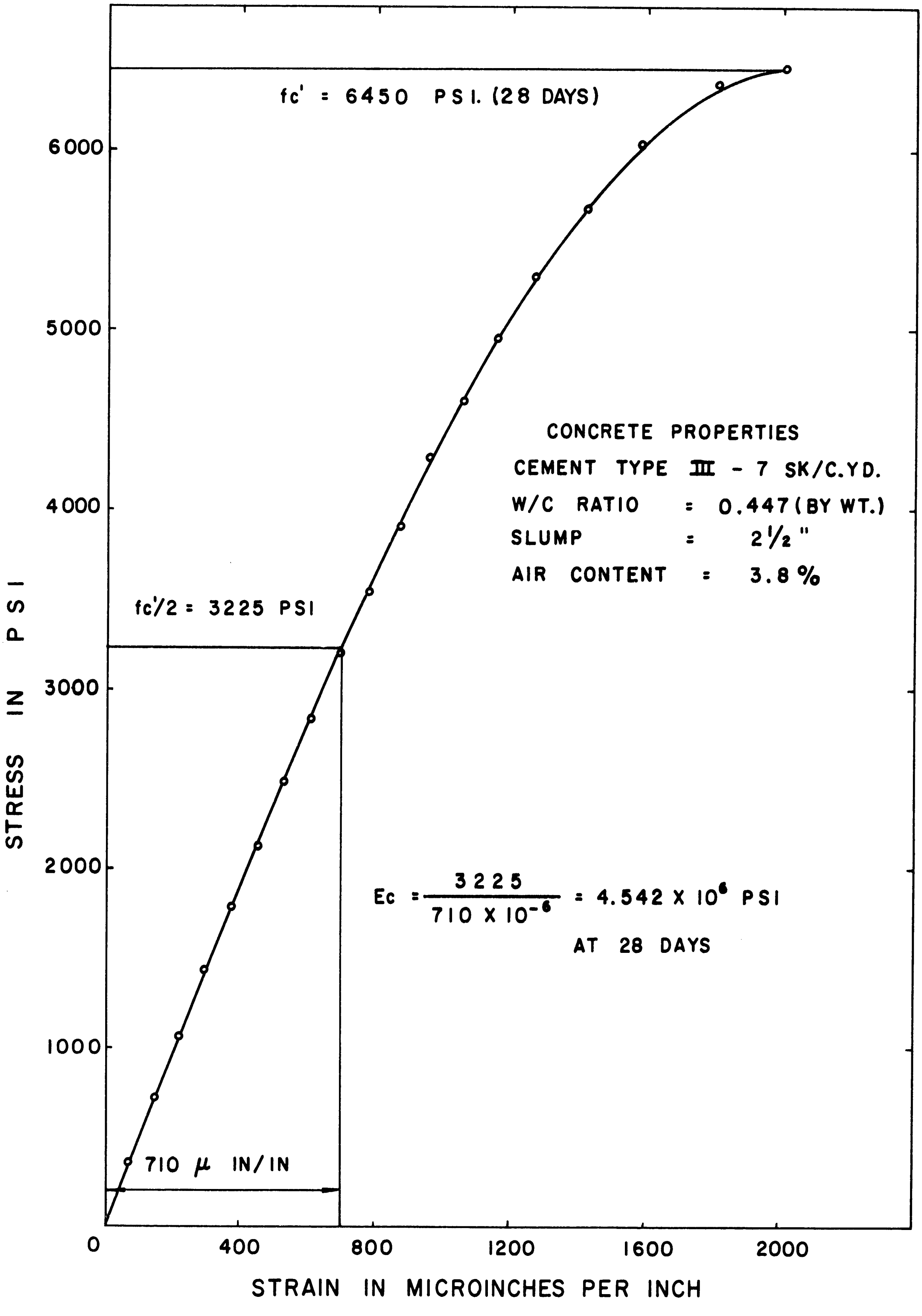


Figure 6.5. Concrete - Modulus of Elasticity - Test Series No. 1 - Age 28 Days.

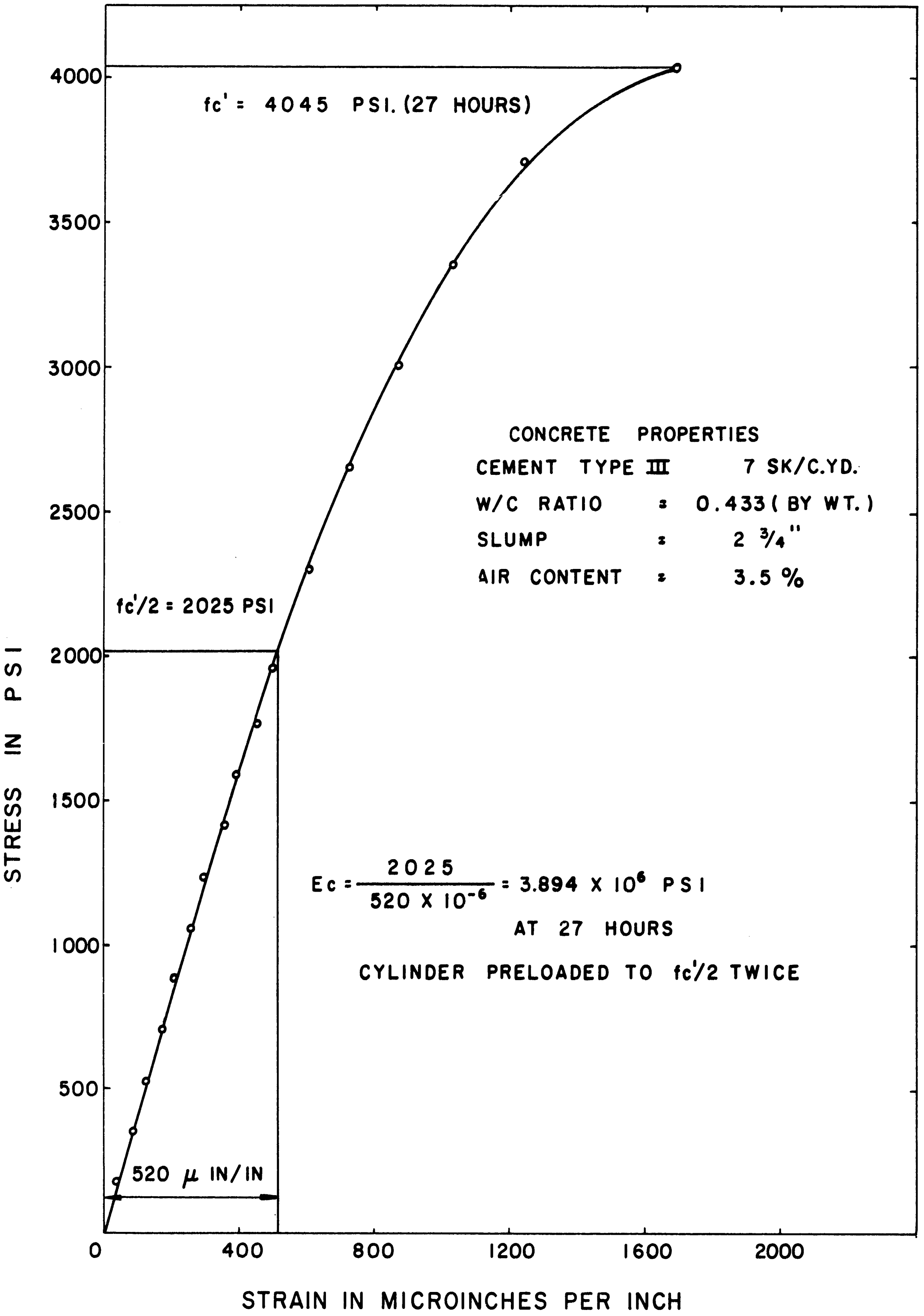


Figure 6.6. Concrete - Modulus of Elasticity - Test Series No. 2 - Age 27 Hours.

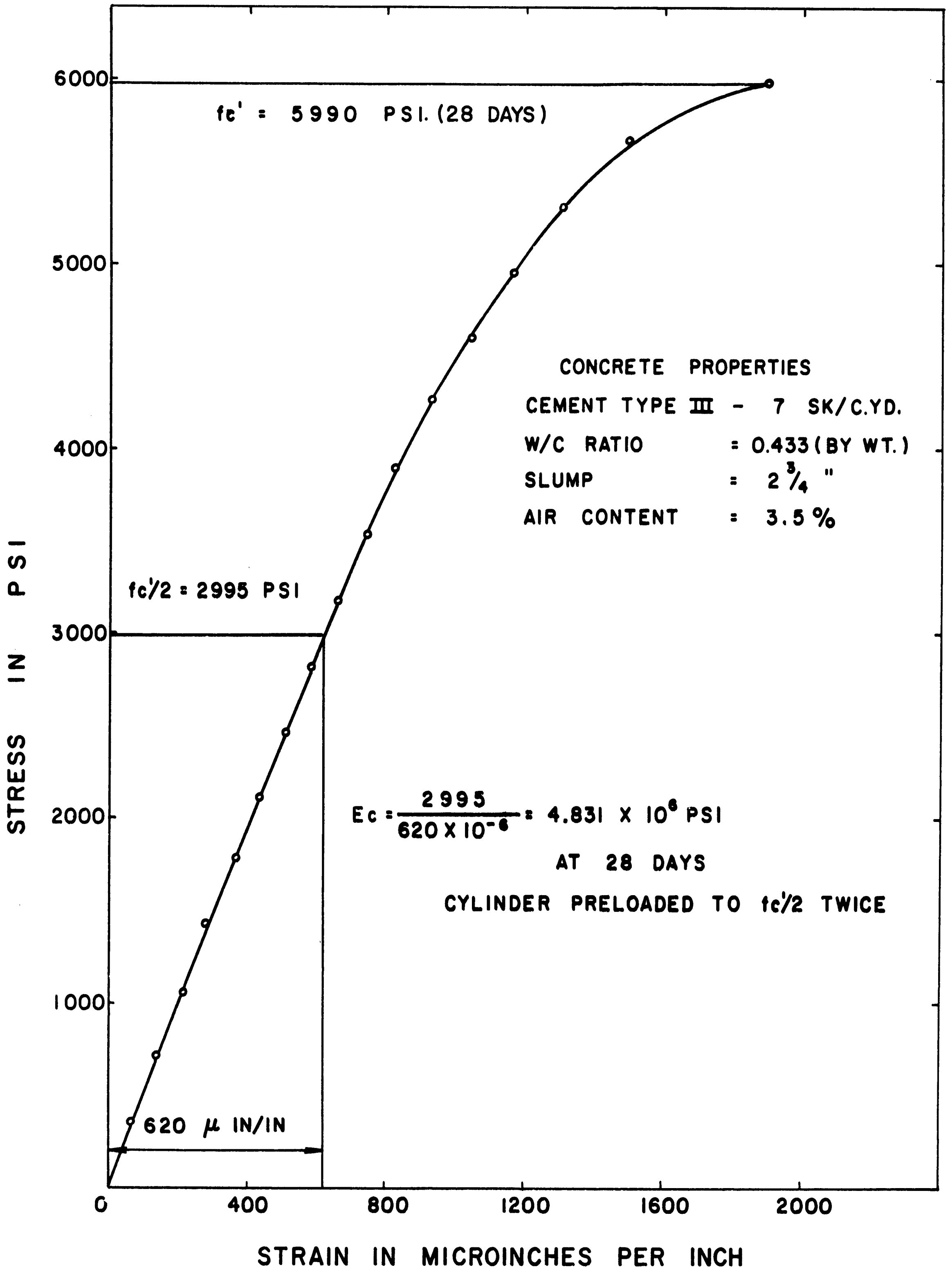


Figure 6.7. Concrete - Modulus of Elasticity - Test Series No. 2 - Age 28 Days.

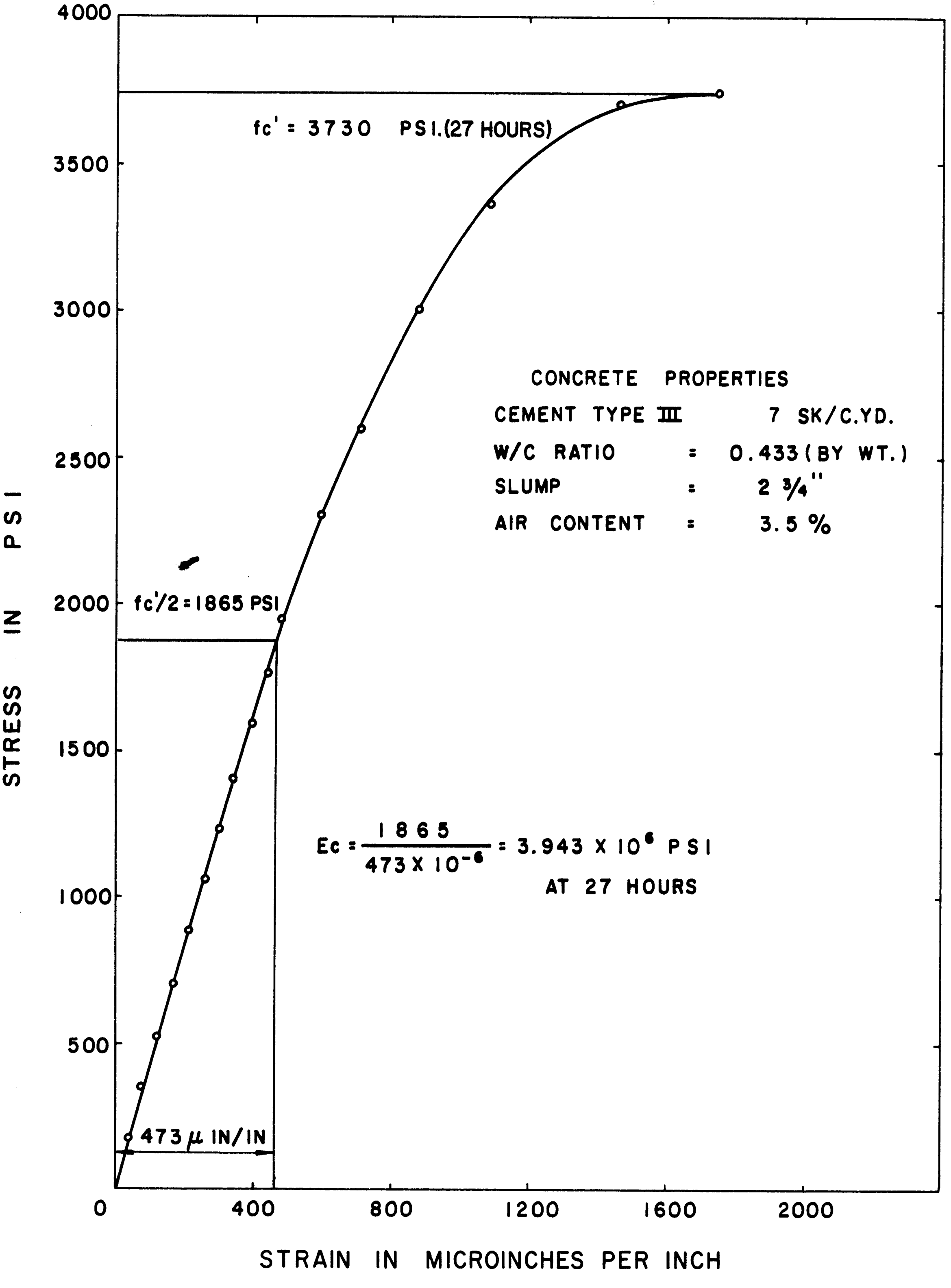


Figure 6.8. Concrete - Modulus of Elasticity - Test Series No. 2 - Age 27 Hours.

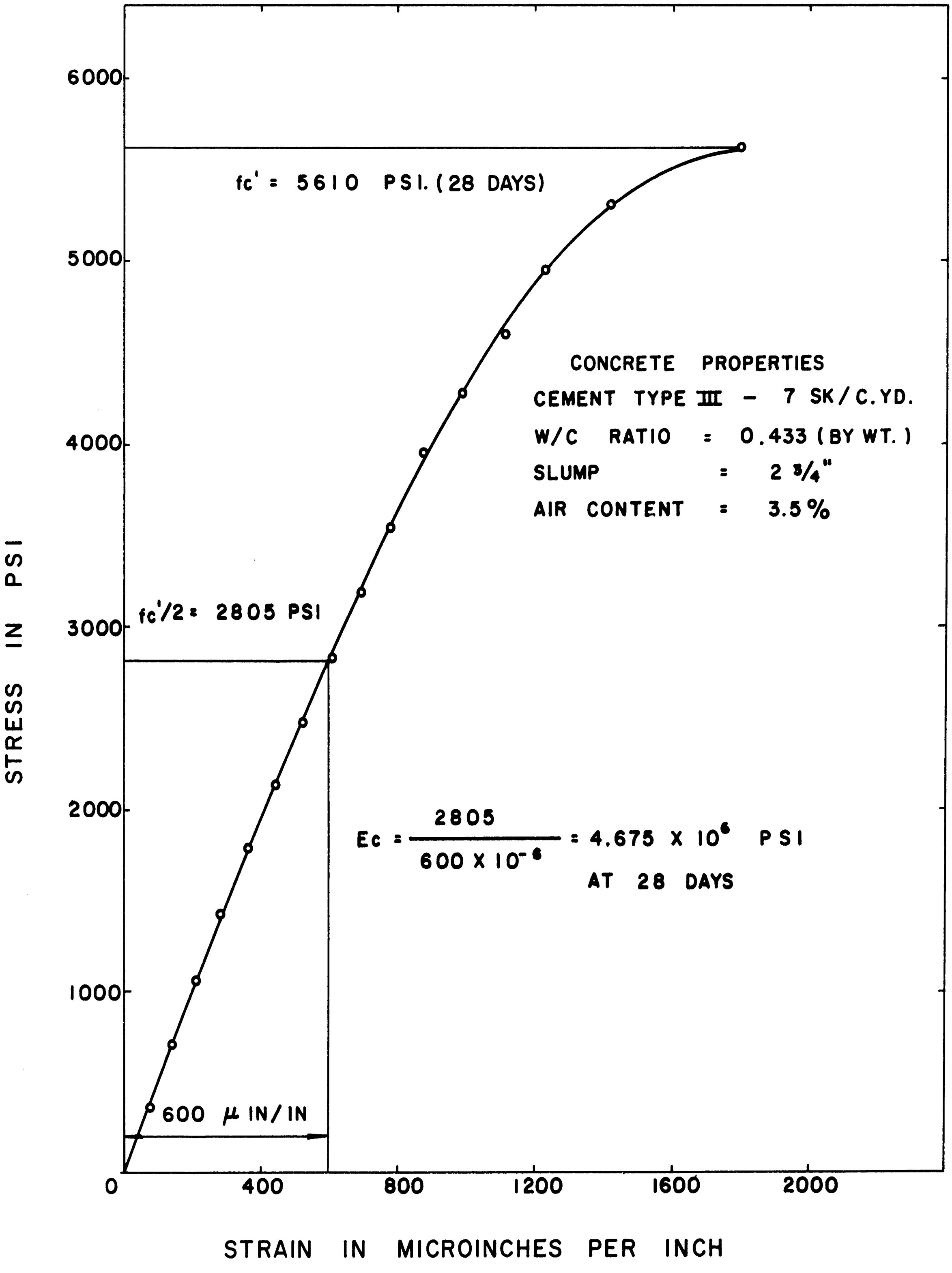


Figure 6.9. Concrete - Modulus of Elasticity - Test Series No. 2 - Age 28 Days.

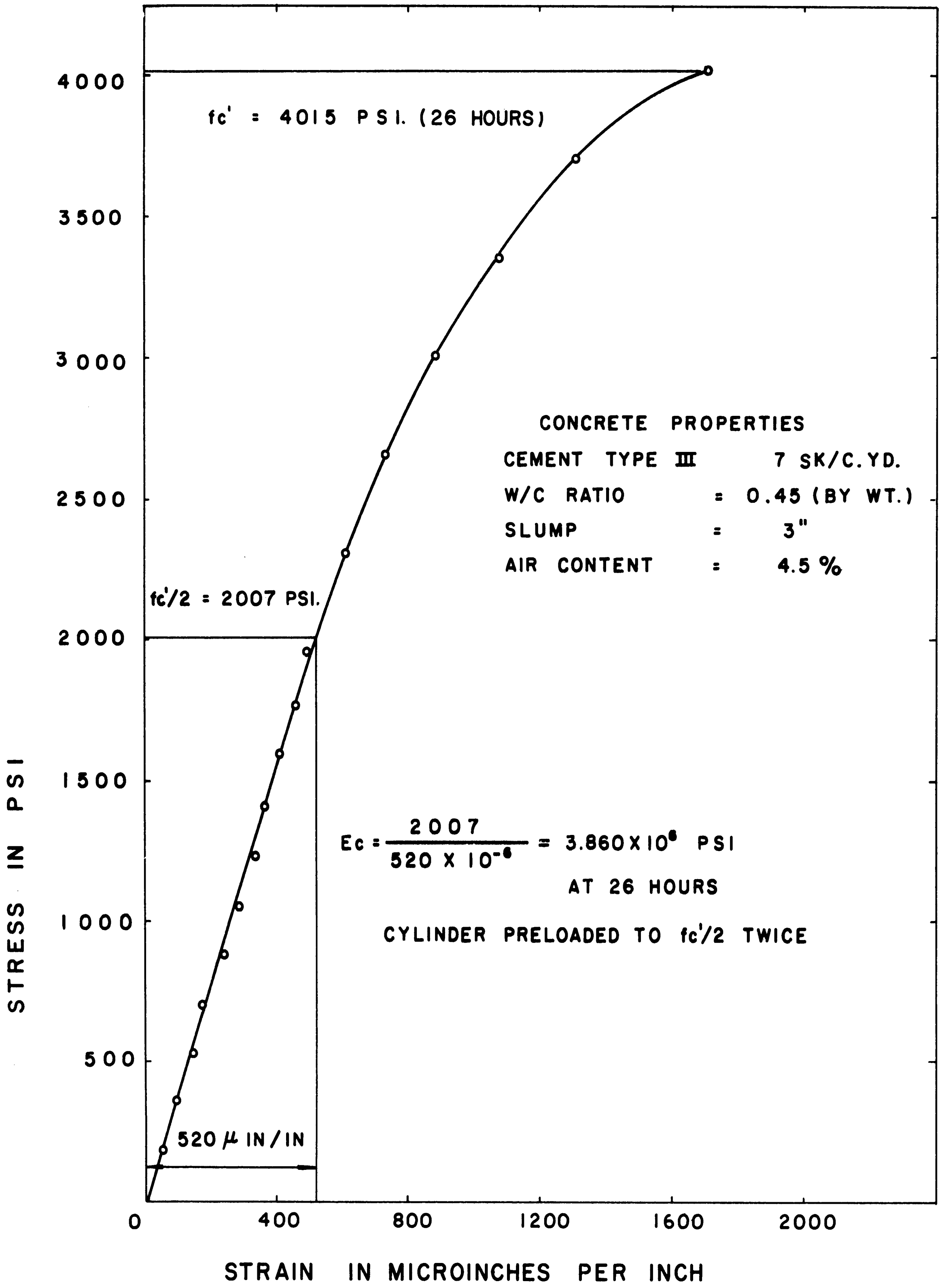


Figure 6.10. Concrete - Modulus of Elasticity - Test Series No. 3 - Age 28 Hours.

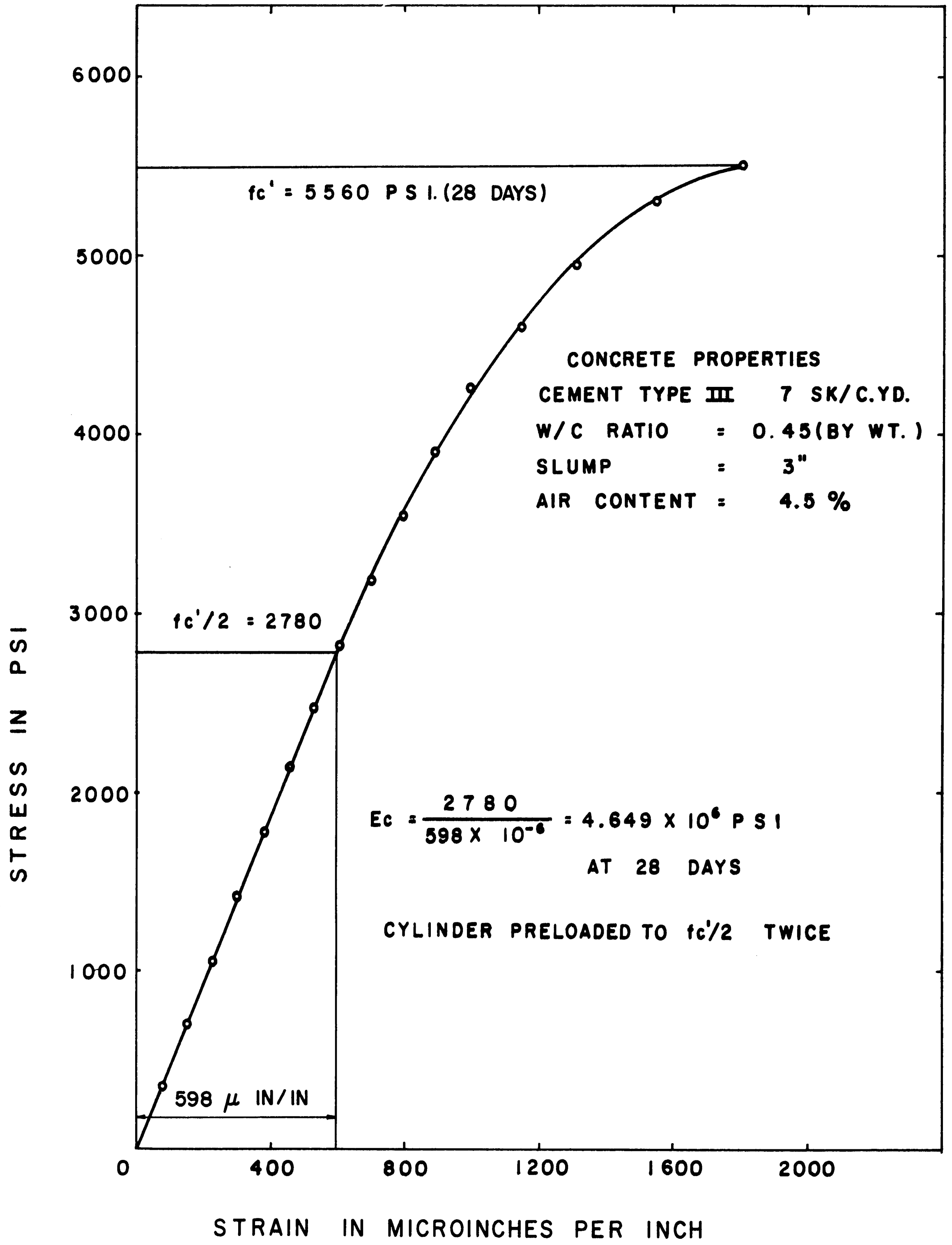


Figure 6.11. Concrete - Modulus of Elasticity - Test Series No. 3 - Age 28 Days.

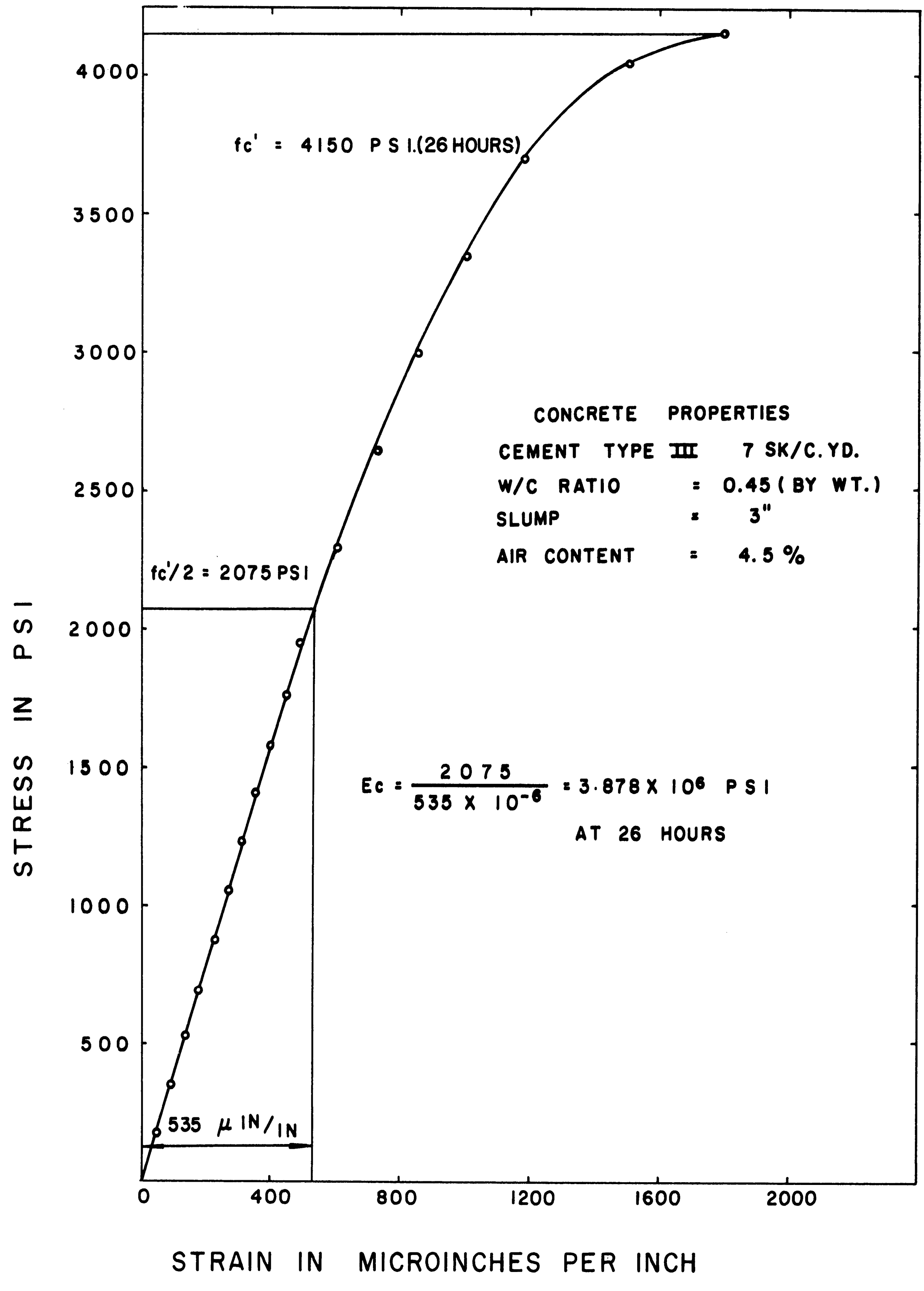


Figure 6.12. Concrete - Modulus of Elasticity - Test Series No. 3 - Age 26 Hours.

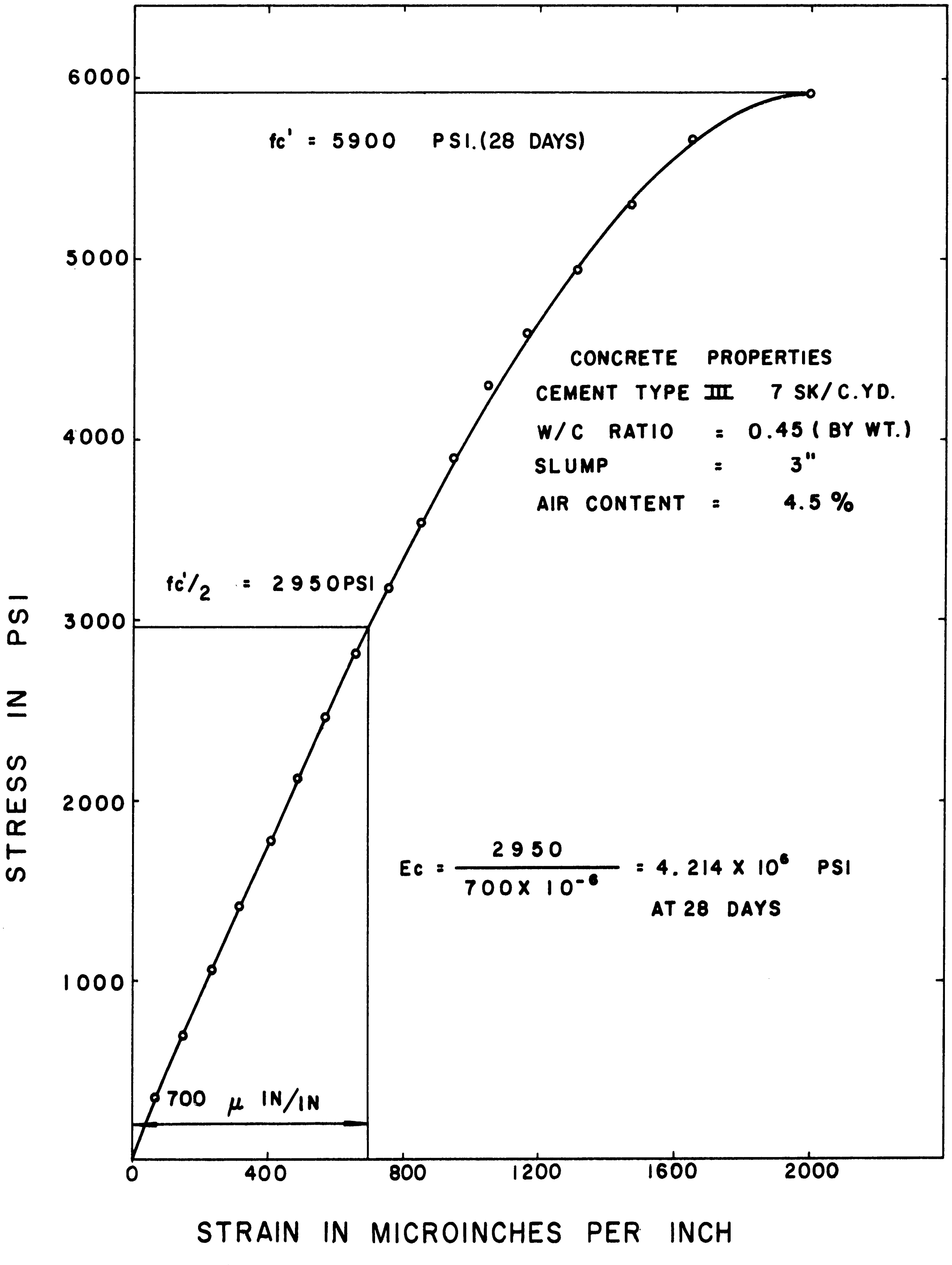


Figure 6.13. Concrete - Modulus of Elasticity - Test Series No. 3 - Age 28 Days.

unit strain in the tendons at the position of the center of gravity of tendons is equal to that in concrete at the same position at transfer in the region of full transfer, seems to be reasonable.

An expression for the elastic losses was developed in Chapter III. Concrete stresses at c.g.s. immediately after transfer of prestress are calculated for the middle section of beams (Table 6.3). This agrees fairly well with the measured concrete stresses obtained by multiplying strain and elastic modulus of concrete for test series 1 and 3 (Table 6.3). For test series 2, the elastic strain and thus, the elastic stress loss in beam, is less than the value found from the formula. This could happen if the modulus of elasticity of concrete in the beam is higher than the cylinder or the creep loss in steel is much higher than normal before transfer of prestress. This is in part confirmed by the variation in instantaneous deflections observed for beams 2 and 3 (Table 6.4).

Thus, elastic loss of prestress can be computed fairly well using the equation as developed in Chapter III.

It is believed that some creep strain might have been included in values of elastic strains for later parts of the readings.

Shrinkage of Concrete

Shrinkage of concrete was measured on 4'-0 long shrinkage beams as well as two 6 x 12 cylinders. Also strain in the cable due to shrinkage in concrete was noted on the shrinkage beam. The cables were embedded in a shrinkage beam to simulate the effect of cables in pretensioned beams on shrinkage of concrete. The shrinkage beams were supported at several points to avoid bending stress effect. Shrinkage versus time curves for

TABLE 6.3

CONCRETE STRESSES AT c.g.s. IMMEDIATELY AFTER TRANSFER OF PRESTRESS

Test Series No.	Measured Avg. Elastic Strain in μin./in. = ϵ_c	Measured E_c, psi	Stress at c.g.s. = $\epsilon_c E_c$, psi	Calculated stress* at c.g.s., psi
1	380	3.78×10^6	1436	1424
2	346	3.943×10^6	1364	1421
3	370	3.878×10^6	1432	1424

* See Appendix F for calculations.

TABLE 6.4

DEFLECTION OF BEAM AT TRANSFER AND GROWTH OF CAMBER

Test Series No.	Deflection at ℄ of Beam at Transfer	Growth of Camber
2	.081 in.	.0439 in. (294 days)
3	.112 in.	.0686 in. (278 days)

the above three are plotted on Figures 6.14 - 6.16. Also on the same sheet values of humidity and temperature are plotted. This was desirable to observe the effects of changes in temperature and humidity on shrinkage. On Figures 6.15 and 6.16, shrinkage of beams is corrected for temperature using a value of coefficient of expansion of 5.5 μin/in. per °F. It is clearly seen from these curves that whenever the humidity increased there was reduction in total shrinkage value. Thus uncorrected shrinkage curves include the simultaneous effect of temperature and humidity and so fluctuate according to atmospheric conditions. The shrinkage value used in these curves was the average of 12 readings on shrinkage beams, the average of four readings on cylinders and the average of two readings on the effect of concrete shrinkage on cable strain.

From Figures 6.14 - 6.16 it appears that shrinkage in beams is more than cylinders in the earlier stage but later cylinder strain surpasses beam strain and becomes greater. This is due to the steel cable resisting the shrinkage after a certain shrinkage has occurred. Cylinders without reinforcement shrunk more than beams due to lack of restraint of steel. The maximum average value for cylinders appears to be .00047 in./in. while for the beams the value is 0.00043 in./in. after correcting for temperature at the end of one year. The ultimate value for shrinkage of the beam would reach 0.0005 in./in. considering other investigators' reports on long term shrinkage. Magnel(10) reports that for air cured concrete the generally acceptable value for the ultimate shrinkage is 400 μin./in. out of which 200 μin./in. occurs during the first 28 days.

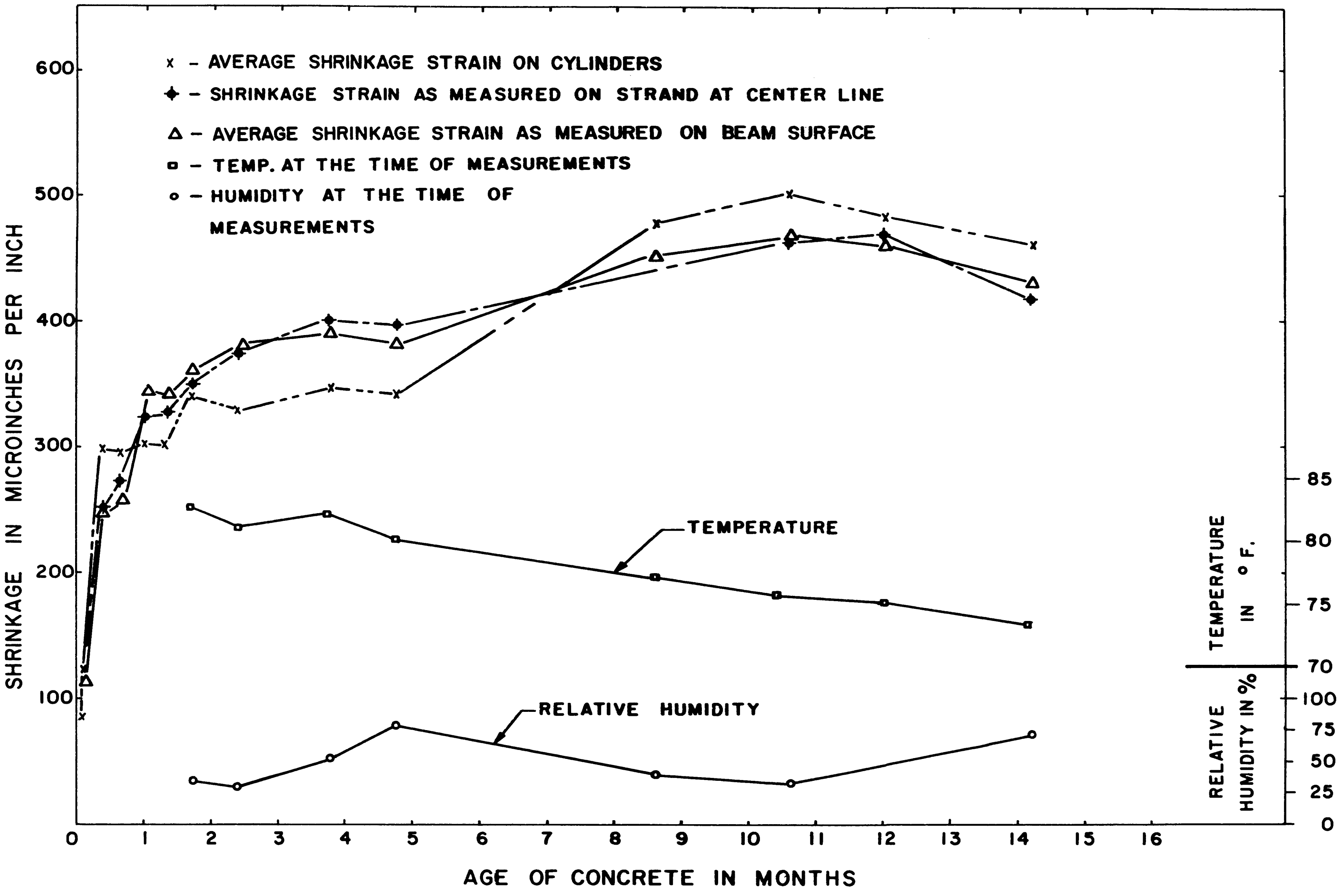


Figure 6.14. Shrinkage Strain vs. Time - Test Series No. 1.

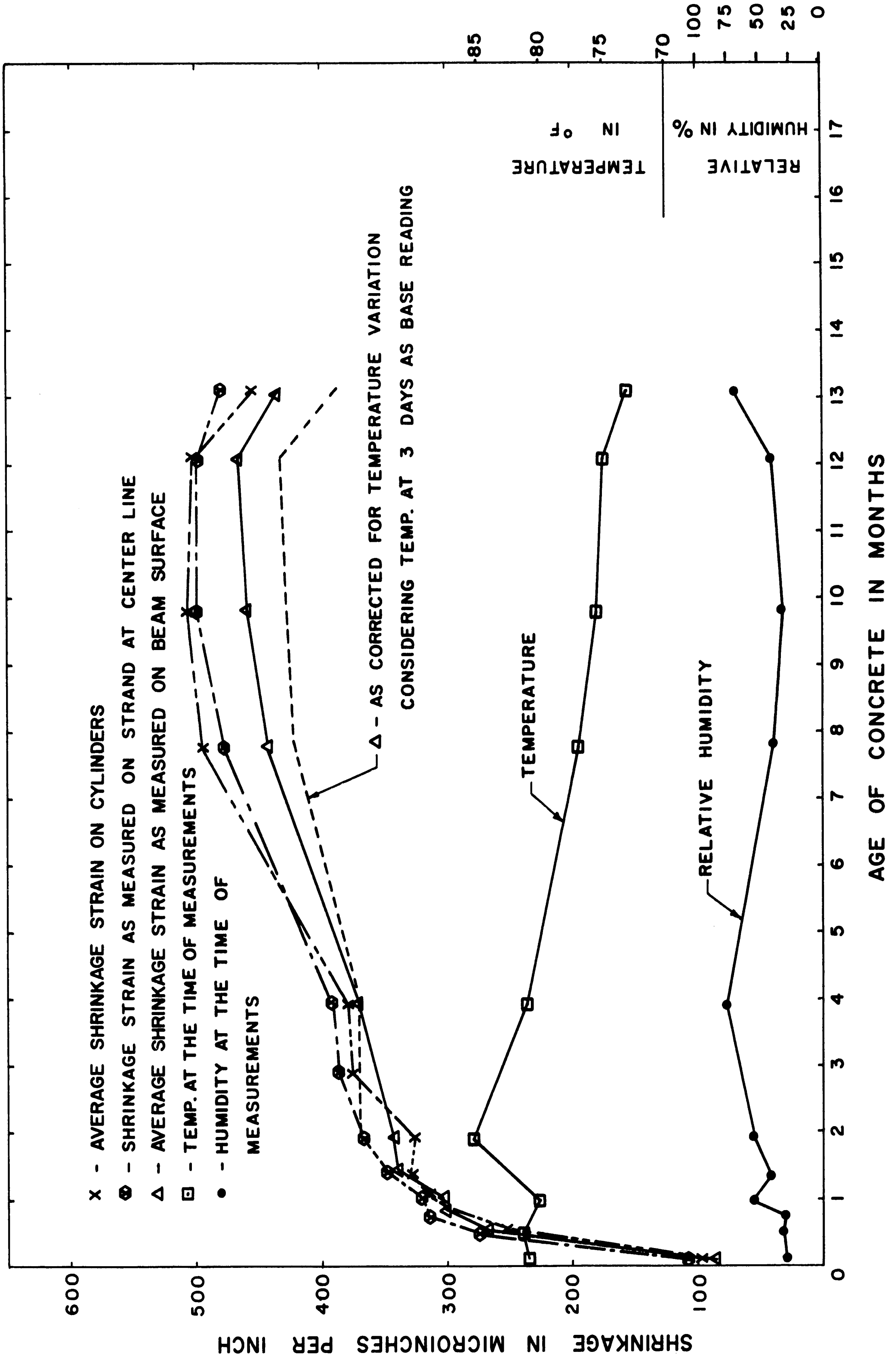


Figure 6.15. Shrinkage Strain vs. Time - Test Series No. 2.

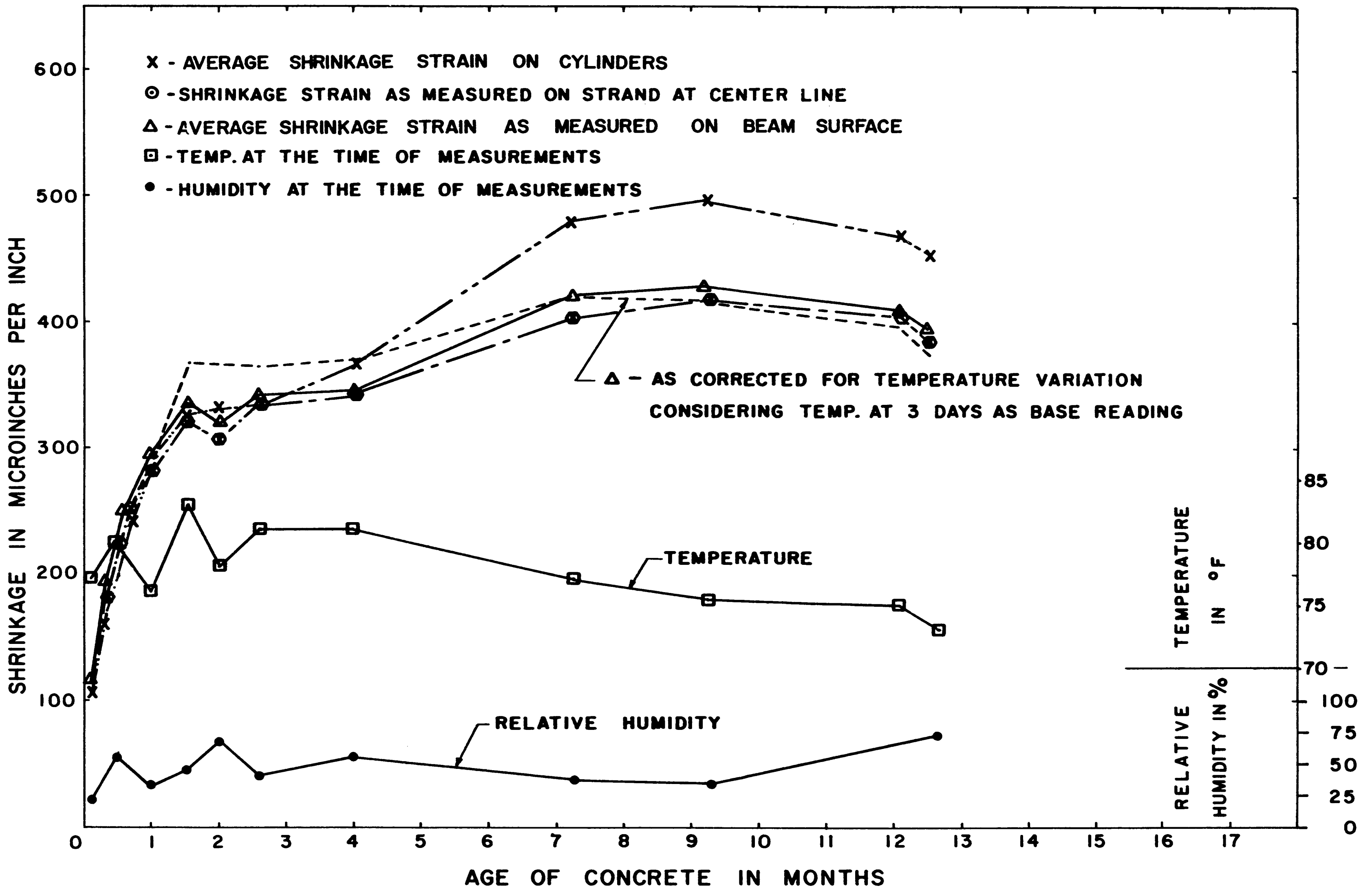


Figure 6.16. Shrinkage Strain vs. Time - Test Series No. 3.

In our case, at 28 days, shrinkage is approximately 300 μin./in. both for beams and cylinders.

The values of 0.00047 in./in. for cylinders and of 0.00043 in./in. for beams at the end of one year are shrinkage strains measured after transfer of prestress in beams and cylinders. Before this the concrete was kept moist and it is felt that shrinkage, if it occurred, would be very small. Also its effect on prestress loss until the bond is developed would be very little. In prestressing plants, where steam curing is employed and transfer of prestress takes place in about 18 hours, the effect of shrinkage before transfer would be still less.

The new ACI Code (1963) removed the recommended value of 0.0002 in./in. to 0.0003 in./in. for shrinkage as given in the Tentative Recommendation for Prestressed Concrete published by ACI-ASCE Joint Committee 323 in 1956.

Zuidema(49) even with a w/c ratio of around .25 got shrinkage of about .00032 in four months. In this investigation, as well as in another investigation(50) the shrinkage of .0005 in./in. has been obtained at the end of twelve months. This points out that the value of shrinkage losses should be decided from the type of mix to be used for the member to be designed and the way it is to be cured. Previous investigations(52) show that steam cured concrete shrinks less than air cured concrete. Also concrete with a low w/c ratio shrinks less. Thus, if the precasting plants use a very stiff mix and use the optimum steam curing procedure recommended by PCI, it is probable that the value of shrinkage may be reduced to less than .0003 in./in., as was recommended for use in design by ACI-ASCE Joint Committee 323 in their Tentative Recommendations for Prestressed Concrete.

In the absence of a stiff mix and optimum steam curing for manufacture of members, a higher value for shrinkage losses is recommended for design purposes - possibly a value of about 0.00045 in./in to 0.0005 in./in. for normal weight concrete with strength of about 4000 psi at transfer and 5000 psi at 28 days.

Creep of Concrete

In order to separate the effect of creep of concrete from other effects, two beams, one prestressed and another without prestress, i.e., shrinkage beam, with identical cross sections but shorter in length, were used. Also, three 6 x 12 cylinders were stressed and two 6 x 12 cylinders were used as shrinkage cylinders. The cylinders were stressed at the same level of stress at which concrete near c.g.s. in the beam was stressed at transfer. A slightly higher load was applied in cylinders to take care of loss when the load is transferred through nuts to the steel bars.

The creep of concrete was obtained in the following manner: The readings immediately after transfer were considered as base readings for the prestressed beam and shrinkage beam. The change in strain at time t in the prestressed beam accounted for creep of concrete, shrinkage of concrete, temperature and humidity effects on concrete. The change in strain at time t in the shrinkage beam accounted for shrinkage of concrete plus temperature and humidity effects on concrete. Thus differences in readings on the prestressed beam and shrinkage beam gave the creep of concrete at time t.

Creep of cylinders was obtained in the same manner as previously described for beams. Appendix C gives a typical strain data sheet for prestressed beams, shrinkage beams, and cylinders. For typical calculation of creep and shrinkage of concrete for beams and cylinders, see Appendix C.

Creep of concrete versus time for three sets of beams and cylinders are plotted in Figures 6.17 through 6.22. Also Table 6.5 summarizes the average creep strains, shrinkage strains and elastic strains as measured on beams and cylinders at the end of one year.

It appears from Figures 6.18, 6.20 and 6.22 that creep in cylinders has not reached its ultimate value but in beams it appears that it is closer to the ultimate value. It should be noted that cylinders are stressed at a uniformly higher level of stress than beams, which have a distribution of stress of a low value on top to a higher value on the bottom. Also the cylinders were kept at approximately constant load while in the beams the load was decreasing due to time-dependent losses. It has been proved(56) that there is not much reduction of load on cylinders when they are spring loaded. Also, the load was increased to its original level, first within six weeks and again at about one year. From this experiment the maximum reduction at one year was less than 1200 lbs. or about 3 per cent.

The creep in cylinders is more than that in beams as was expected (Table 6.5), the cylinders being loaded at a higher average stress level than beams as losses occurred in the beams. The elastic strain in the cylinders in Test Series 2 seems to be somewhat high. For the prestressed beam in Test Series 2, the elastic strain is less than would be

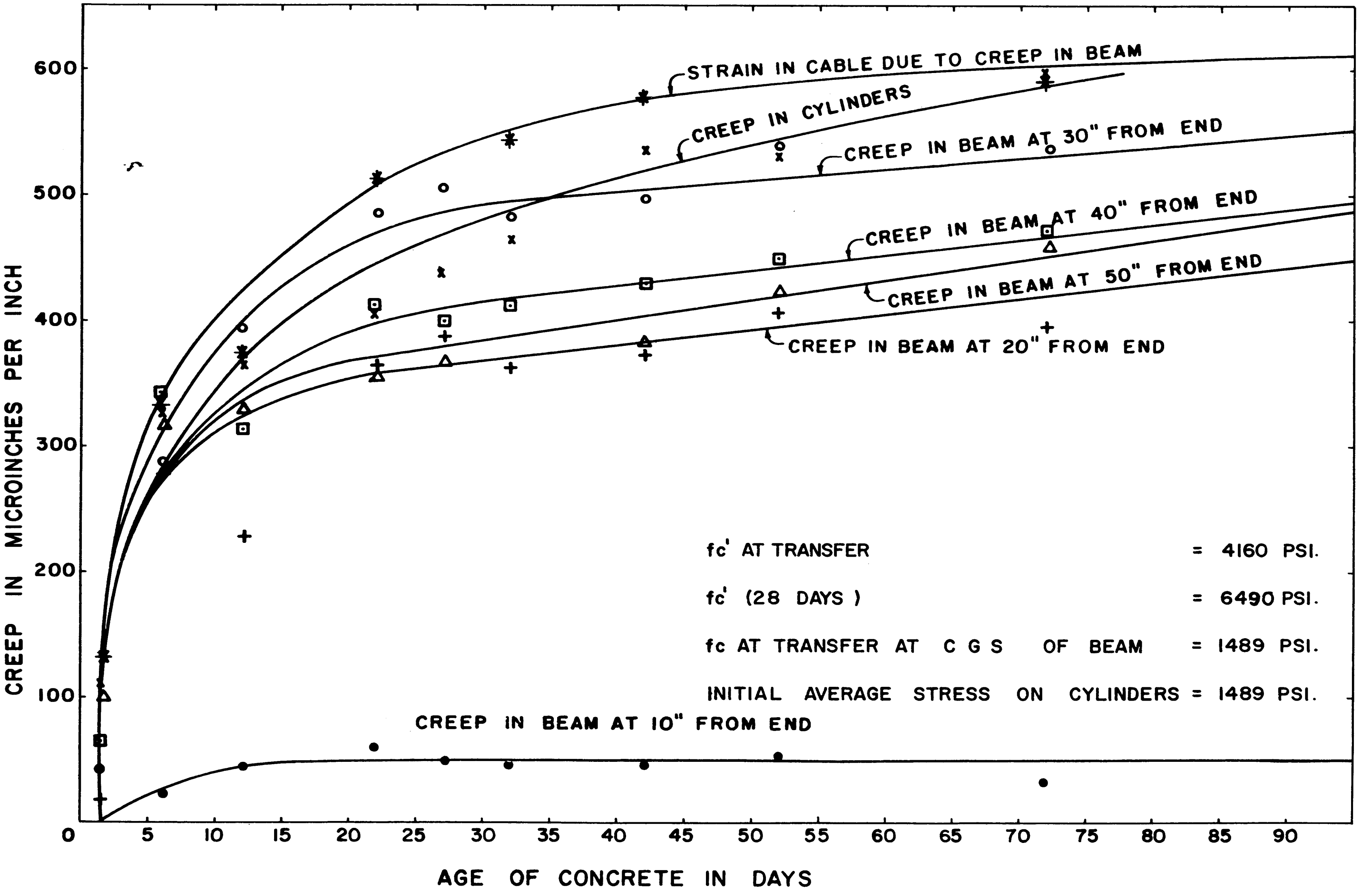


Figure 6.17. Creep Strain vs. Time - Test Series No. 1.

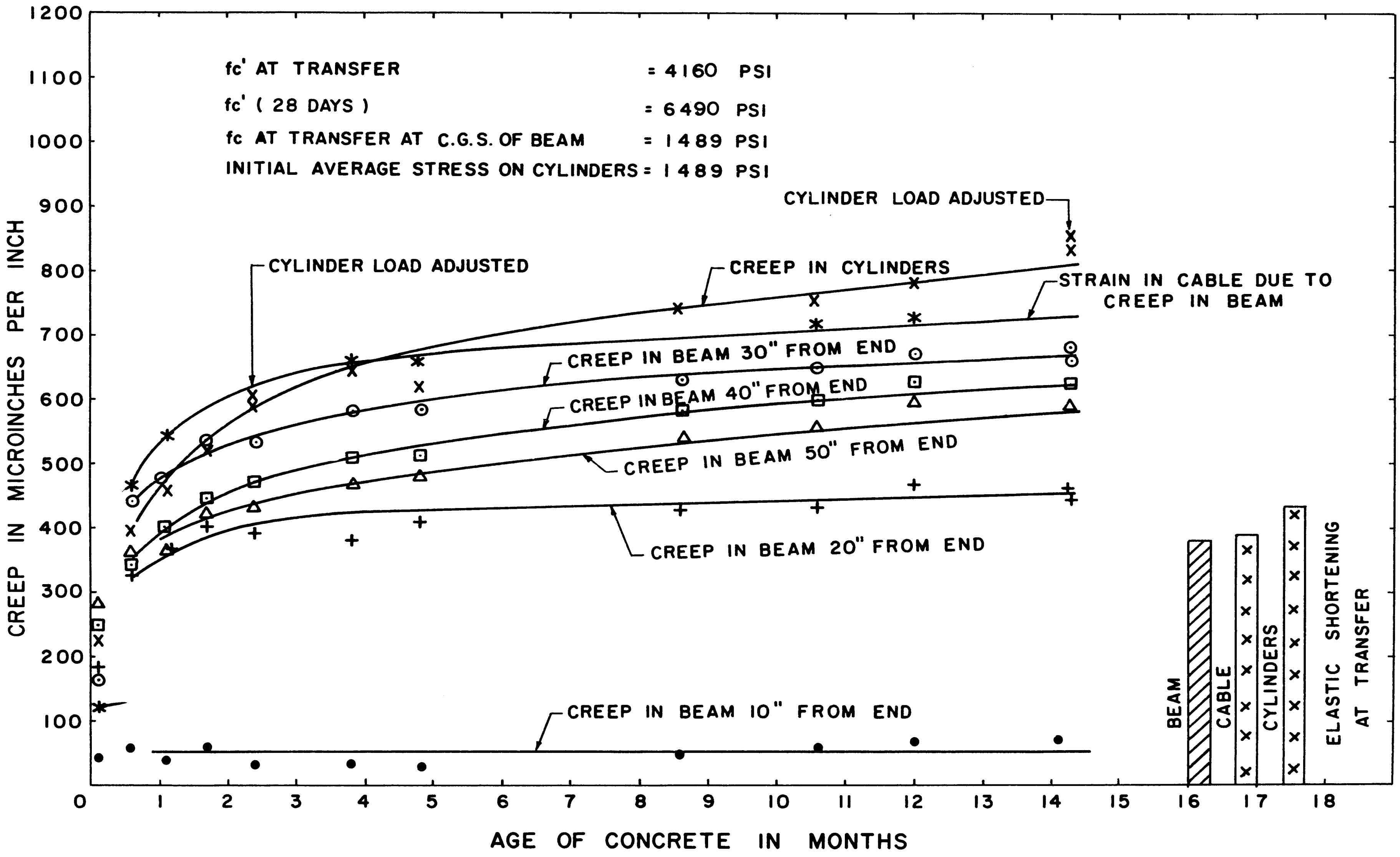


Figure 6.18. Creep Strain vs. Time - Test Series No. 1.

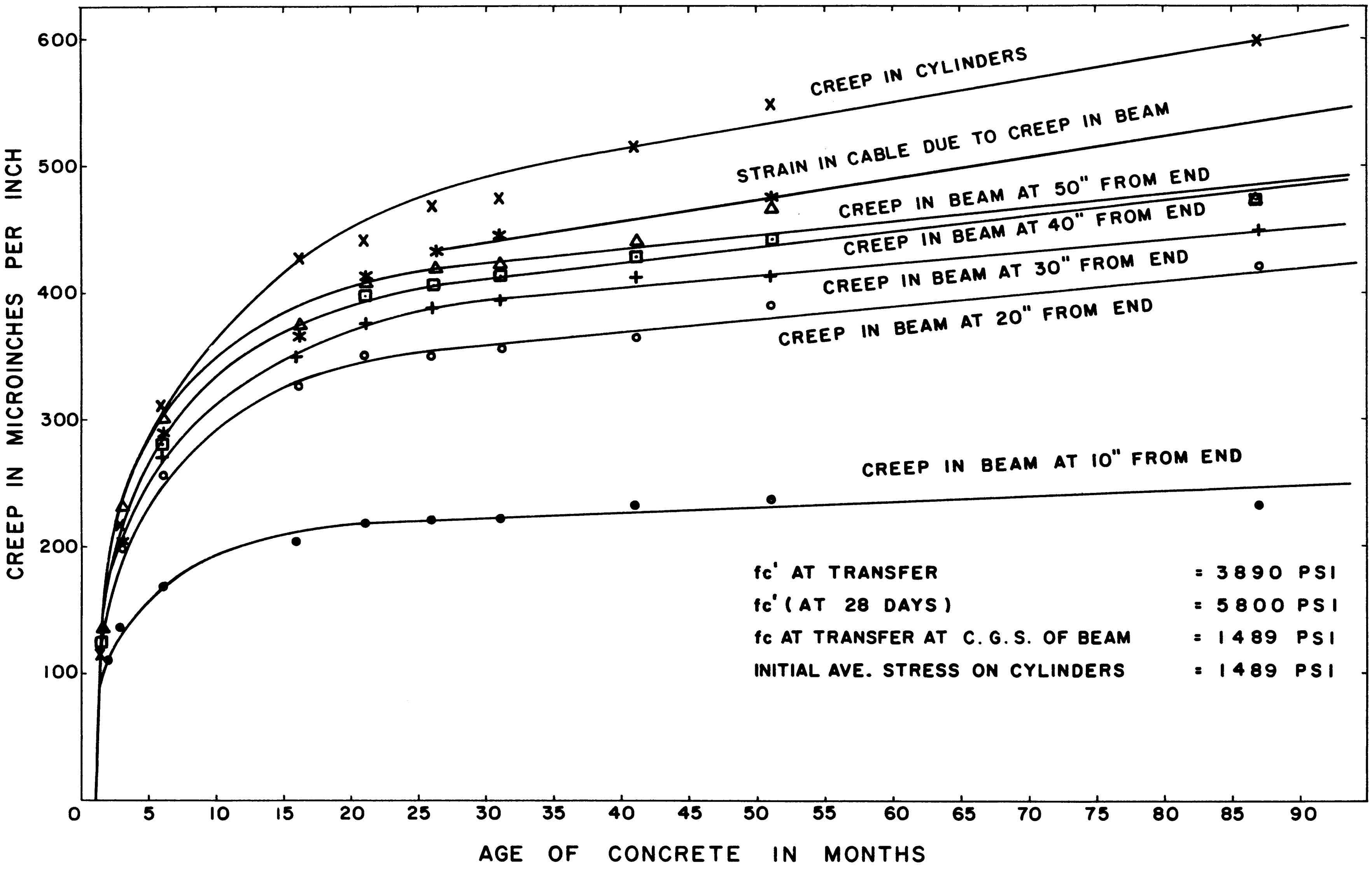


Figure 6.19. Creep Strain vs. Time - Test Series No. 2.

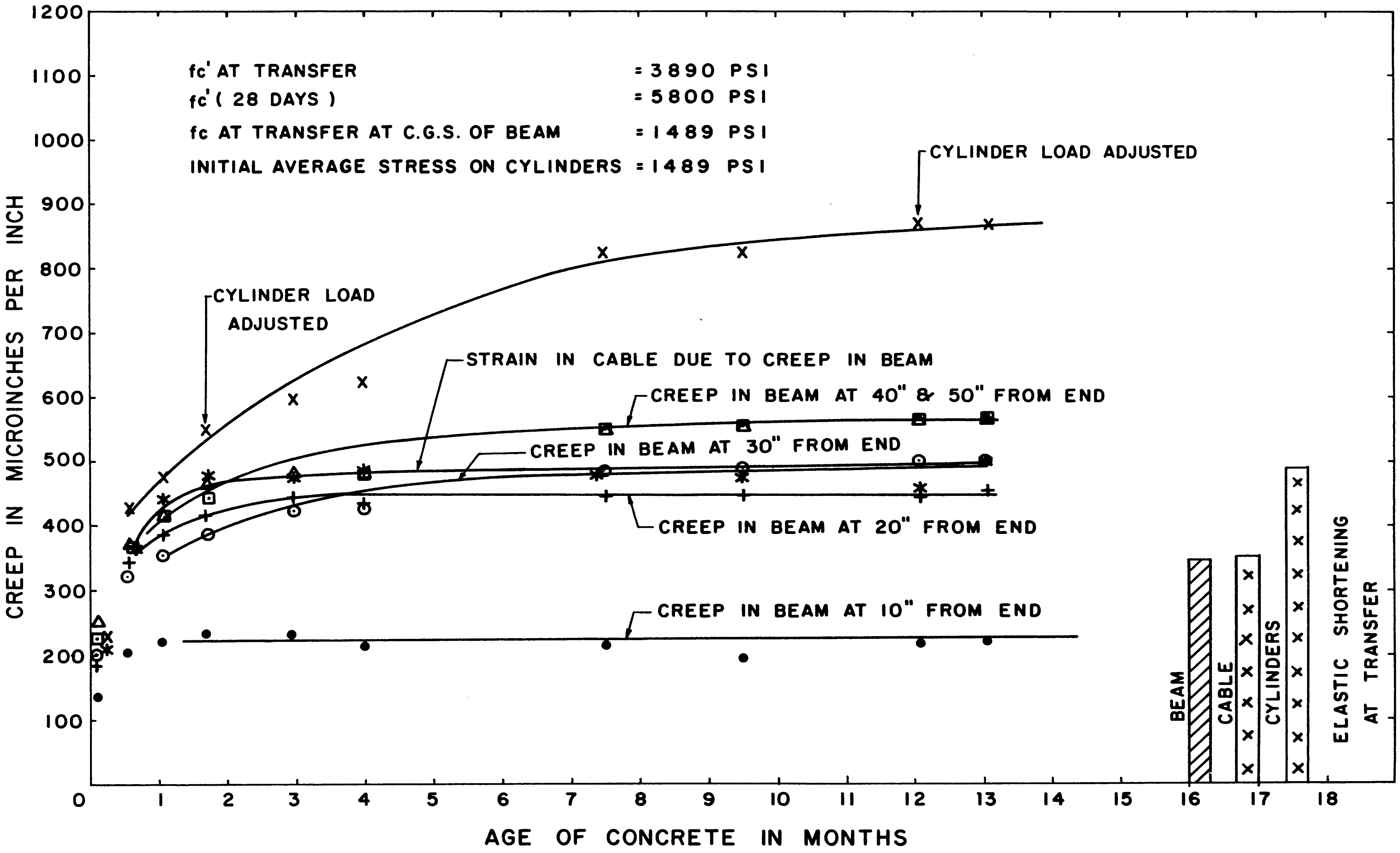


Figure 6.20. Creep Strain vs. Time - Test Series No. 2.

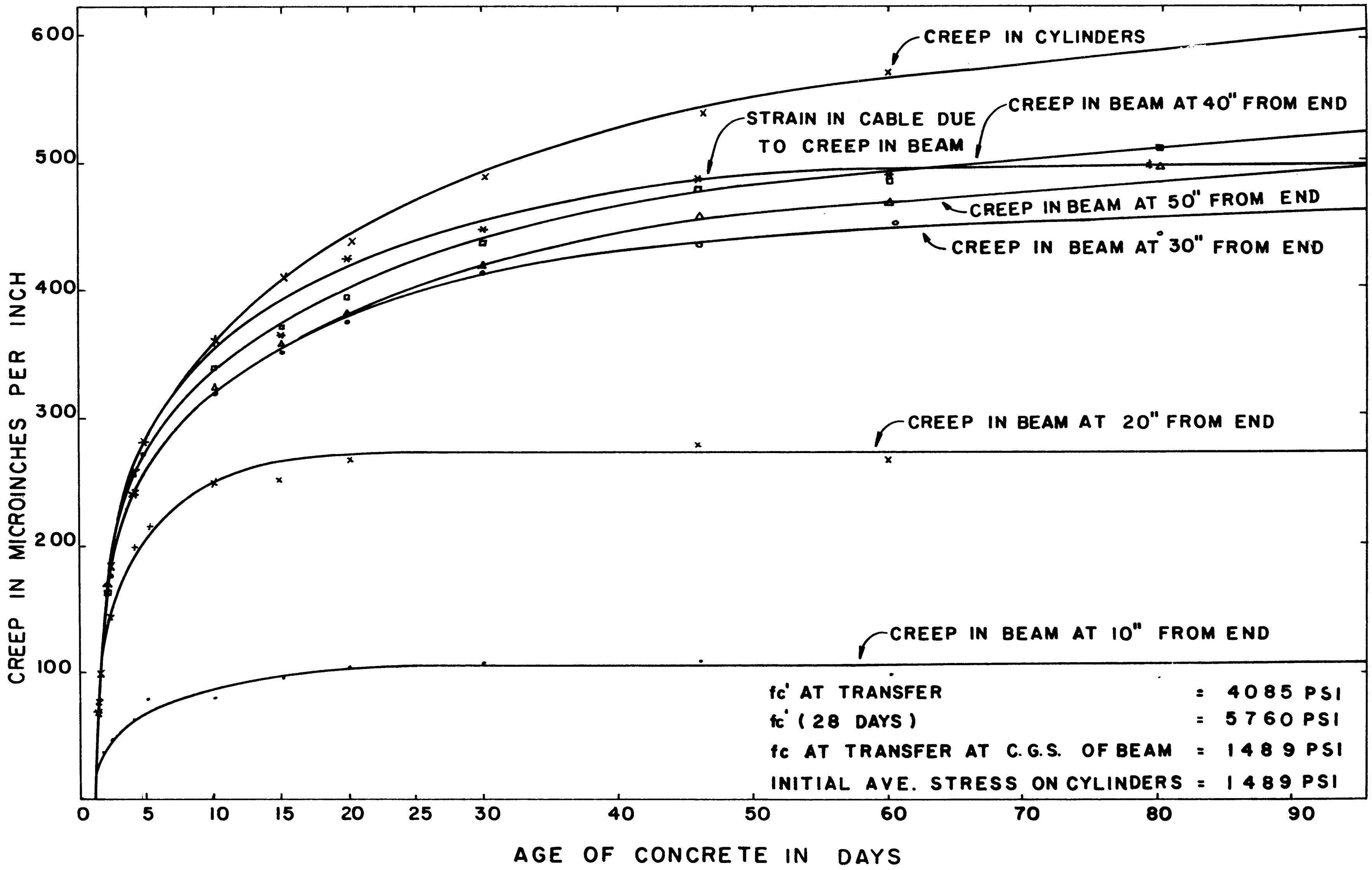


Figure 6.21. Creep Strain vs. Time - Test Series No. 3.

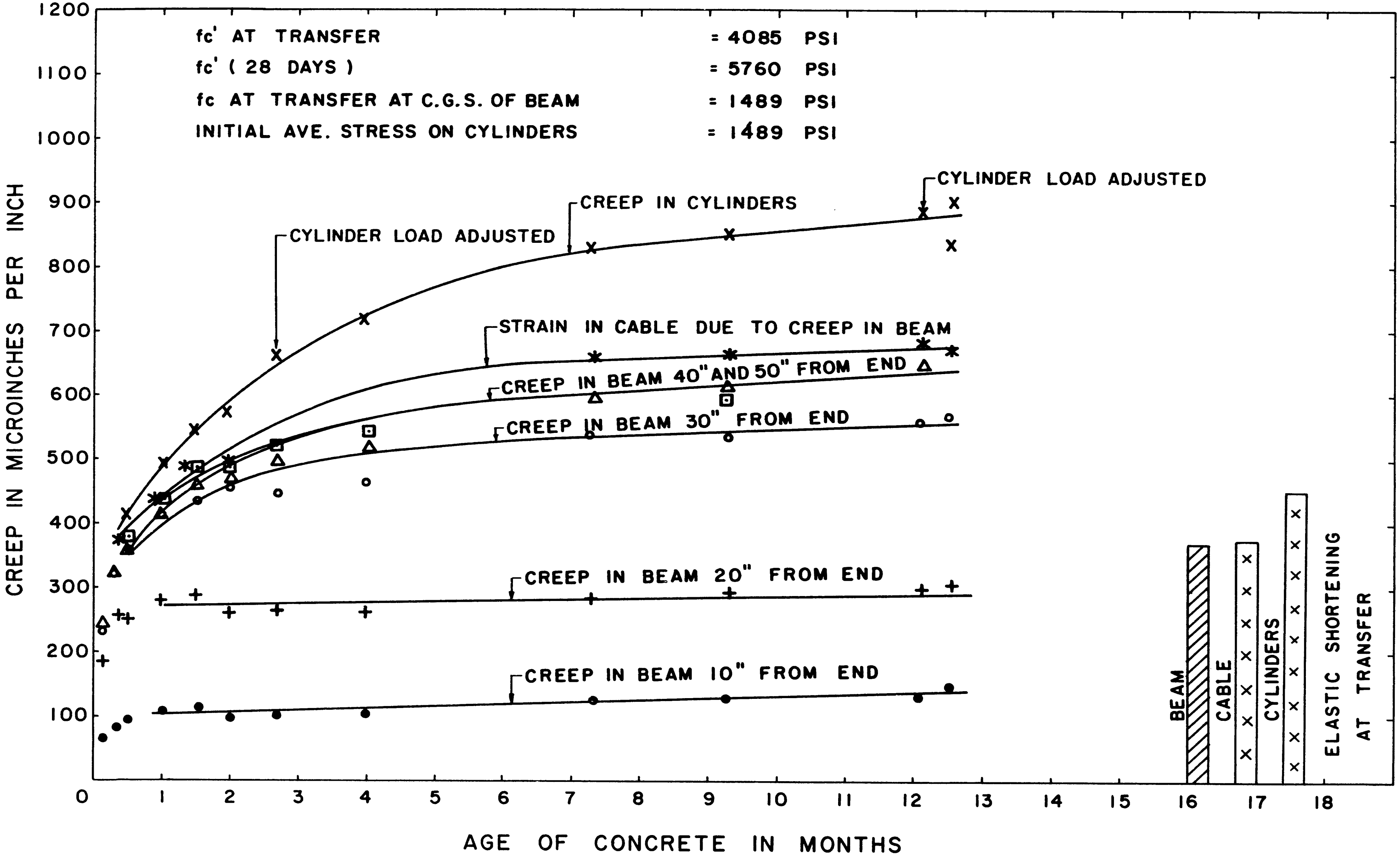


Figure 6.22. Creep Strain vs. Time - Test Series No. 3.

TABLE 6.5

MEASURED ELASTIC, CREEP AND SHRINKAGE STRAINS IN BEAMS AND CYLINDERS

	Beams									
	Beam Surface Readings			Cable Readings			Cylinders			
	Test Series 1	Test Series 2	Test Series 3	Test Series 1	Test Series 2	Test Series 3	Test Series 1	Test Series 2	Test Series 3	Remarks
Elastic Strain in μin./in.	380	346	370	390	350	375	432	490	450	At transfer
Creep Strain in μin./in.	633	560	640	718	500	675	780	860	875	Values @ one year
Shrinkage Strain in μin./in.	465	465	435	470	500	420	500	505	495	Max^{m} Value in one year

TABLE 6.6

(R)* = RATIO OF MEASURED CREEP TO MEASURED ELASTIC STRAIN FOR BEAMS AND CYLINDERS

	Beams									
	Beam Surface Readings			Cable Readings			Cylinders			
	Test Series 1	Test Series 2	Test Series 3	Test Series 1	Test Series 2	Test Series 3	Test Series 1	Test Series 2	Test Series 3	Remarks
$\frac{\text{Measured Creep Strain}}{\text{Measured Elastic Strain}} = (R)*$	1.665	1.62	1.73	1.835	1.43	1.80	1.80	1.752	1.94	Values @ end of one year

expected. Due to these opposite effects on beams and cylinders, the ratio between the two creep strains became much higher than in the other two series. To include this variation effect, it was decided to compare ratios $\frac{\text{Creep Loss}}{\text{Elastic Loss}}$ (R)* of the beams to $\frac{\text{Creep Loss}}{\text{Elastic Loss}}$ (R)* of the cylinders for the three test series (Tables 6.6 and 6.7). At the end of one year this ratio for the three series has an average of .91. As the creep in cylinders is still increasing at a faster rate than the beams, the above ratio will decrease. If more experiments are made with different stress levels and different stress distribution on beams, it is believed that some kind of relation can be established between creep of cylinders and creep of beam.

According to a previous investigation(53) about 75 per cent of 7-year creep in beams was obtained in the first year. Other investigators have reported 80 per cent of 20-year creep in cylinders during the first year. In this investigation the ratio of creep loss to elastic loss at the end of one year is an average of 1.67 for beams. By projecting the creep curves of beams in Figures 6.18, 6.20, and 6.22 the ultimate creep loss for this investigation would reach a value of about 1.80 times elastic loss. This is in agreement with the report(55) by Subcommittee 5, ACI Committee 435.

Many mathematical models(40) have been suggested to represent creep curve of concrete. An equation as suggested by Shank(51) is used here to draw creep curve of concrete. It is recognized that this curve could fit only a limited range of test data and that it does not satisfy the requirement that at infinite time there is a finite value of creep per psi. There are other models which can overcome the deficiency as discussed above but they are cumbersome to calculate. Almost all require some constant or constants to be found from experiment.

TABLE 6.7

RATIO OF (R)* FOR BEAM TO (R)* FOR CYLINDER

	Test Series 1	Test Series 2	Test Series 3	Remarks
$\frac{(R)^* \text{ for Beam}}{(R)^* \text{ for Cylinder}}$	.925	.925	.892	At the end of one year

(R)* - From Table 6.6

An equation as suggested by Shank is of the form $y = c'(x)^q$ where y is creep in μin./in. per psi, x is number of days after loading and c', q are constants to be found from experiment. Values of c' and q are found to be .1715 and .202 respectively. (For calculations see Appendix G.) From Figure 6.23 it can be seen that the curve of creep obtained from $y = .1715(x)^{.202}$ fits very well in the envelope of creep data for cylinders. Using the creep per psi as obtained from cylinder creep, the curve of creep for beams was drawn on the same figure. The beam creep data for the three test series was plotted and an envelope of data was drawn. The curve of creep for beam obtained from cylinder creep per psi fitted nicely in the above envelope of beam creep data. From this it can be concluded that creep of concrete is dependent on stress level but specific creep (or creep per psi) is very nearly constant, at least for variation in stress in the beam due to loss of prestress due to shrinkage and creep of concrete.

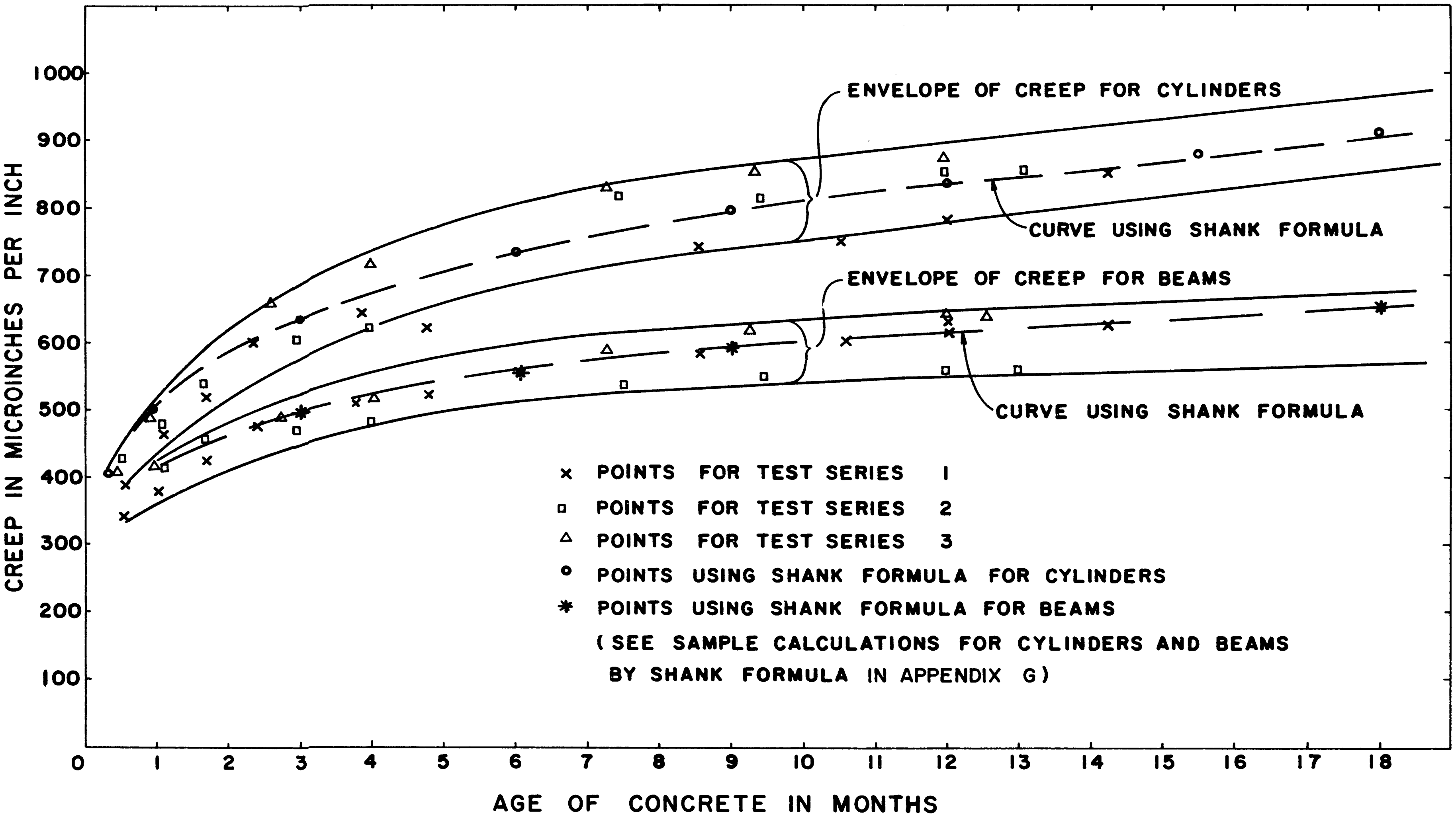


Figure 6.23. Creep Strain vs. Time Curve Using Shank Formula.

Stresses in the steel strands of beams were found using specific creep of cylinders by the "Effective Modulus" method.(17) (See Figures 6.24 - 6.26, Appendix G.) Stresses in strands, as obtained from using measured strains on the concrete beam surface as well as the strand surface, were plotted in Figures 6.24 - 6.26. The agreement between these curves was fairly good indicating that the "Effective Modulus" method can be used for estimating strand stress if specific creep and shrinkage of beams are known. Ultimate magnitude of specific creep of plain concrete can range from 0.2 to 2.0 millionths in terms of length, but is ordinarily about one millionth or less. For this investigation, specific creep for stressed cylinders is approximately .58 millionths at the end of one year. The ultimate value for specific creep would reach 0.7 millionths.

Stress in steel strand, using computed elastic loss, estimated creep loss from formula as suggested by El-Darwish,(50) and assumed shrinkage strain of .00050 in./in., is computed in Appendix H and shown in Figures 6.24 - 6.26. Final stress in steel agrees fairly well with the value obtained using the Effective Modulus method.

Anchorage Length of Strands and Bond of Strands to Concrete

In pretensioned prestressed concrete no end anchorages are used and thus the transfer of prestress takes place through bond of the strand to the concrete. Hanson and Kaar(54) attributed the transfer to these three factors:

(1) Adhesive bond of the concrete to the steel.

(2) Friction between the steel and concrete. This is due to Poisson's Effect. A reduction in stress of strand

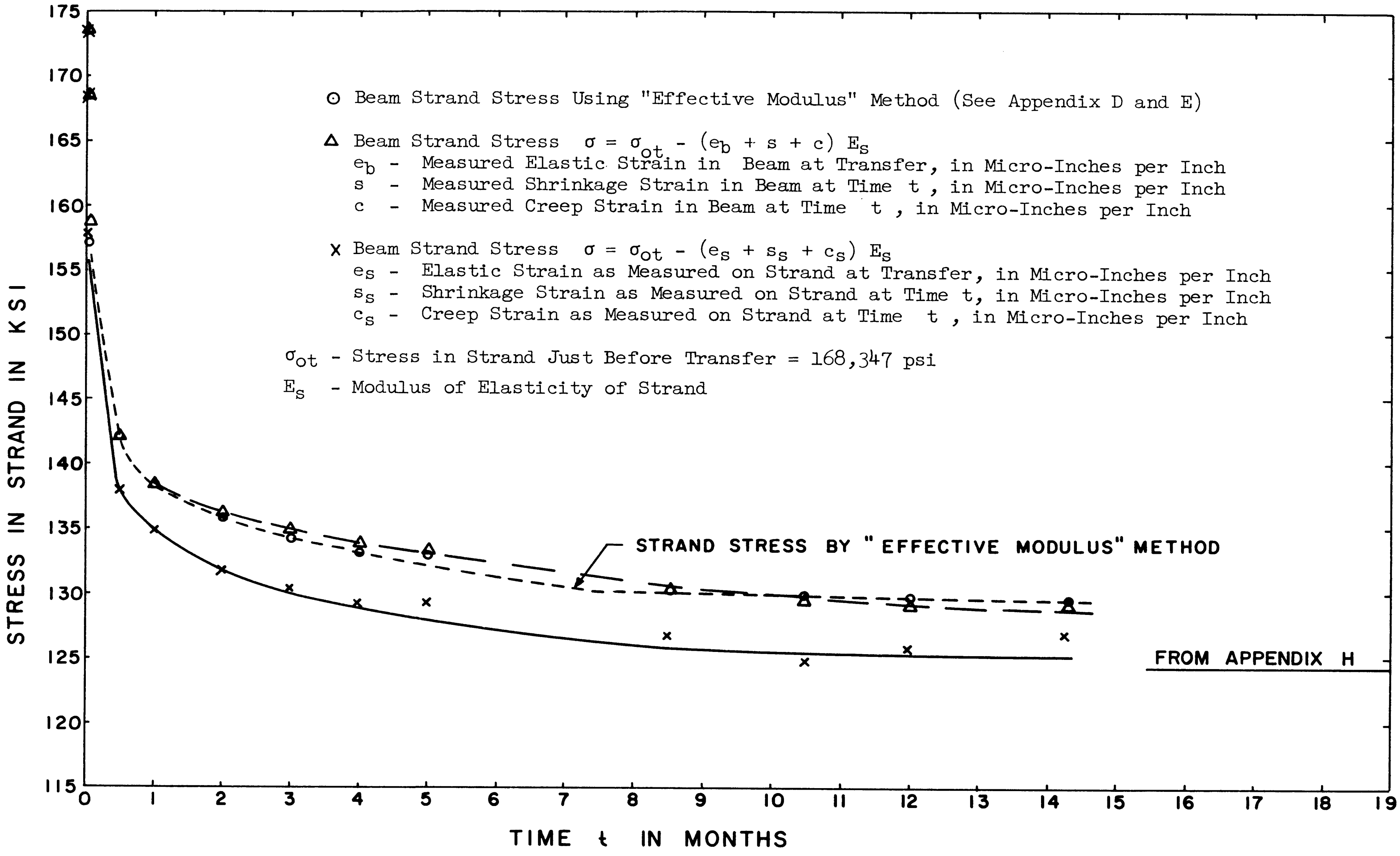


Figure 6.24. Strand Stress as Computed by "Effective Modulus" Method - Test Series No. 1.

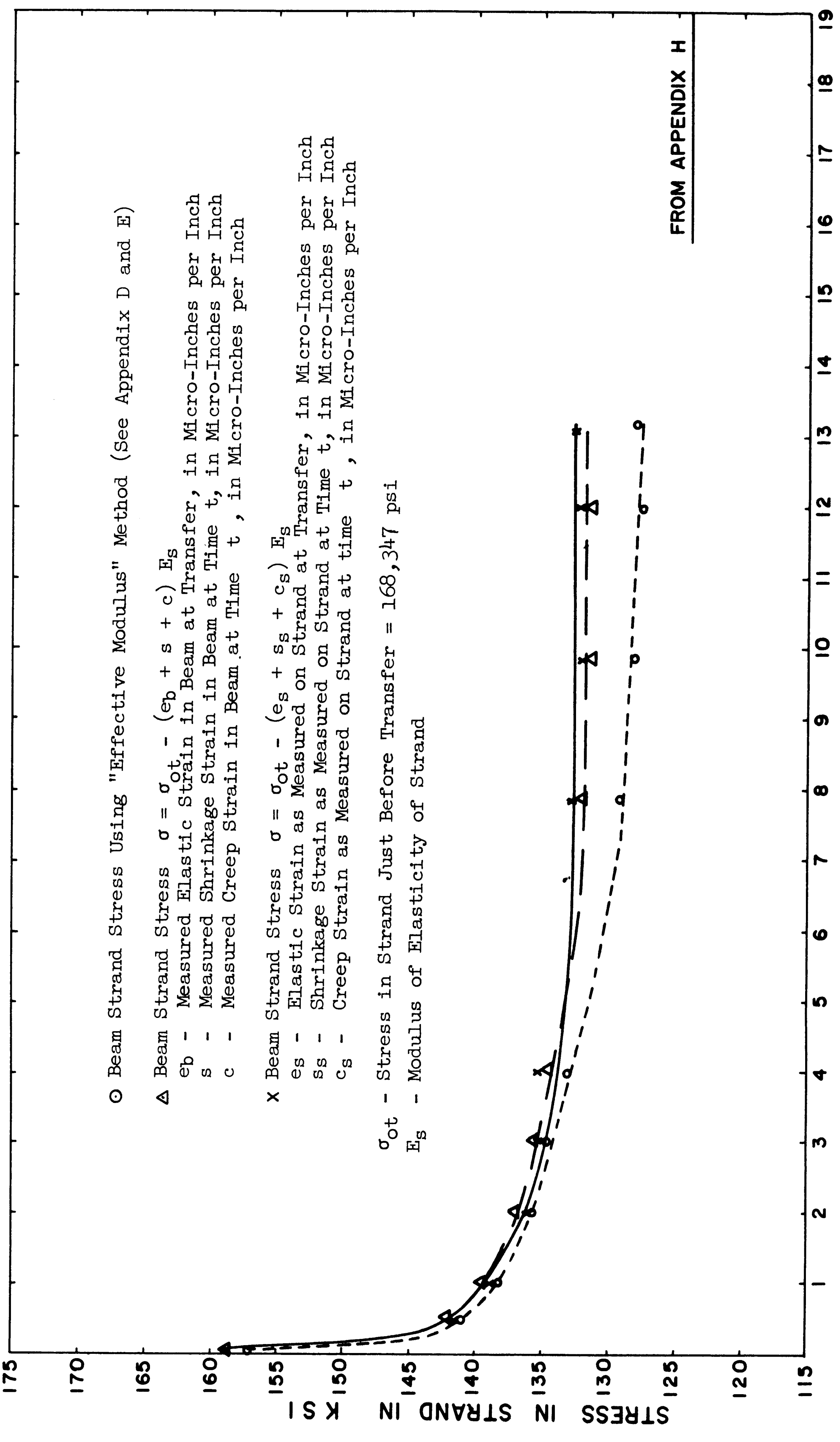


Figure 6.25. Strand Stress as Computed by "Effective Modulus" Method - Test Series No. 2.

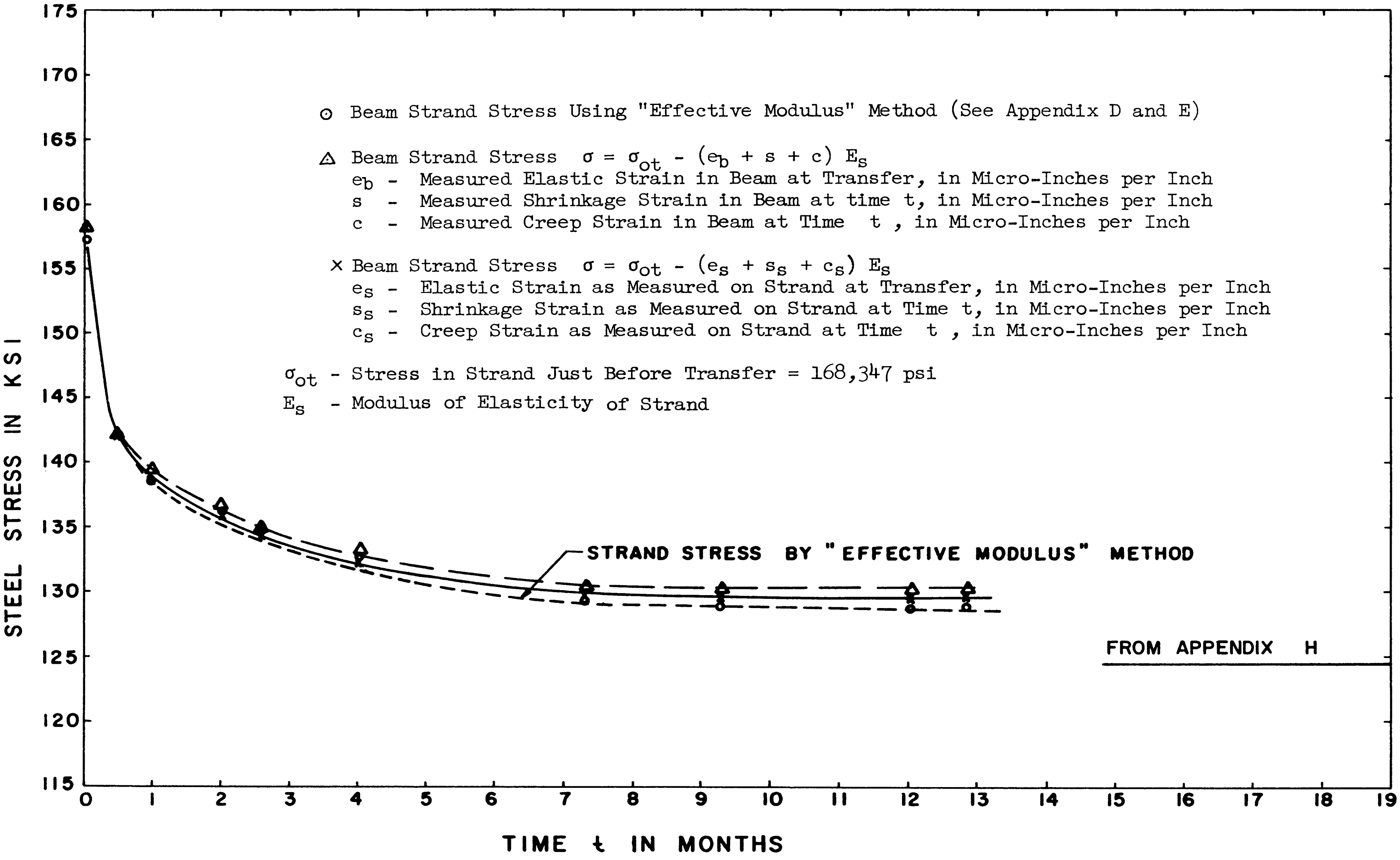


Figure 6.26. Strand Stress as Computed by "Effective Modulus" Method - Test Series No. 3.

due to elastic loss causes an increase in the diameter of individual wires and thus the increase in the diameter of the strand.

(3) Mechanical anchorage due to the lay of the six outer wires.

Surface conditions of all strands used in these tests were almost identical. There were no rust spots and all strands were cleaned with carbon tetrachloride before concreting. From Figures 6.27 - 6.29 it can be seen that the anchorage length for the 7/16" - 7 wire strand for the mix as used and at concrete strength at transfer of around 4000 psi is from 22 in. to 27 in. One could also notice that there is no transfer of prestress in the end region of 2 in. or so. This might be due to sudden transfer of stress due to burning of cables. This could result in the breaking of bond and slippage could occur resulting in zero stress in the cable.

In a series of tests conducted at the University of Michigan, it was found that the surface conditions of the strands play an important role in transfer of prestress to concrete.

For 7/16 in. strands the anchorage lengths have been reported(49,50,57) between 12 in. to 50 in. by various investigations, lower values for the rusted strands and higher values for smoother and shiny strands. A value of 25 in.(49) has been reported for normal weight concrete and non-rusty strands. This investigation confirms the findings of previous works, as this value of transfer length is 22 in. to 27 in.

From Table 6.2, it is seen that in the complete anchorage zone there is no slippage during transfer.

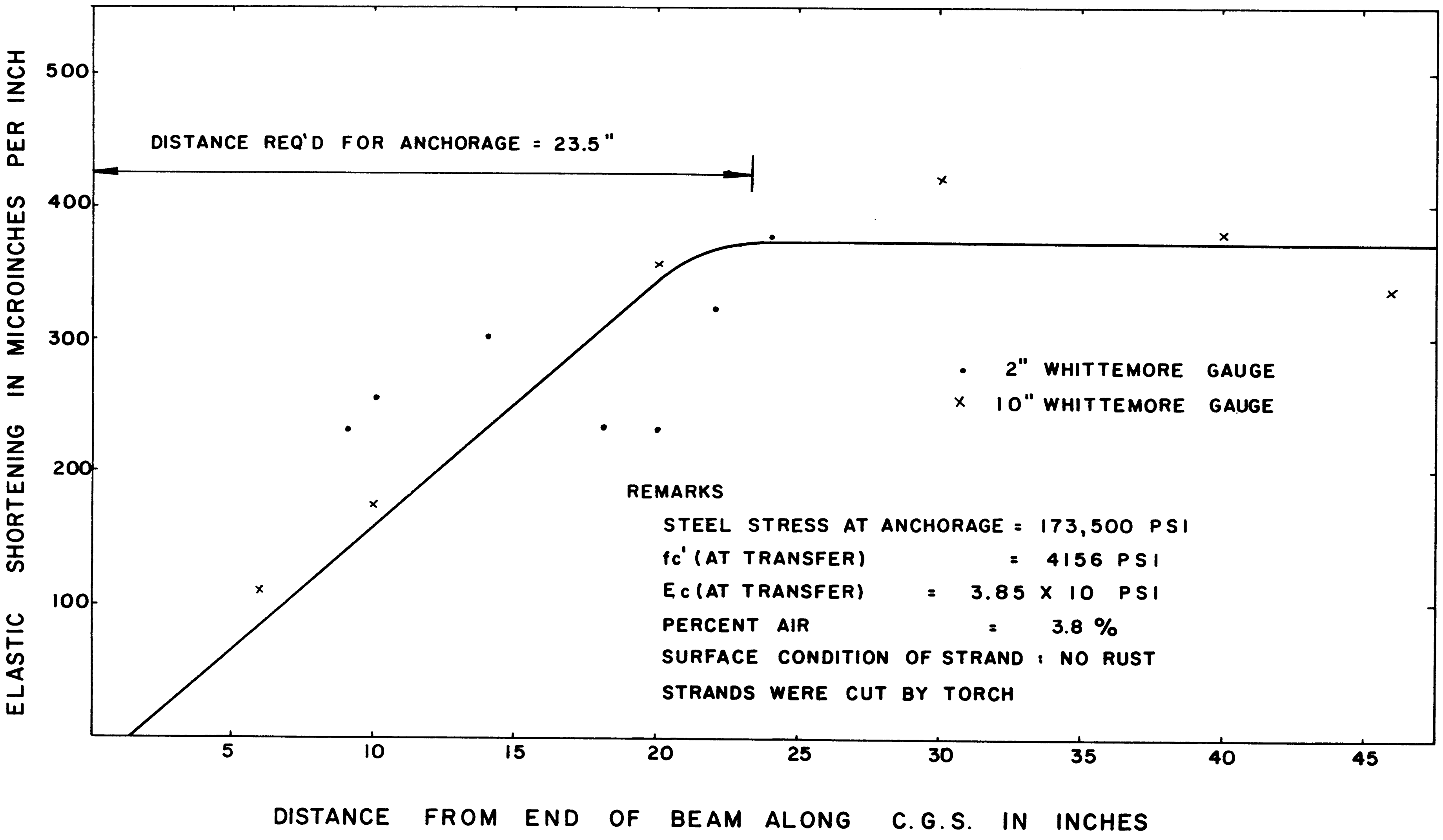


Figure 6.27. Anchorage of 7/16" ∅ Seven-Wire Uncoated Strand. Test Series 1.

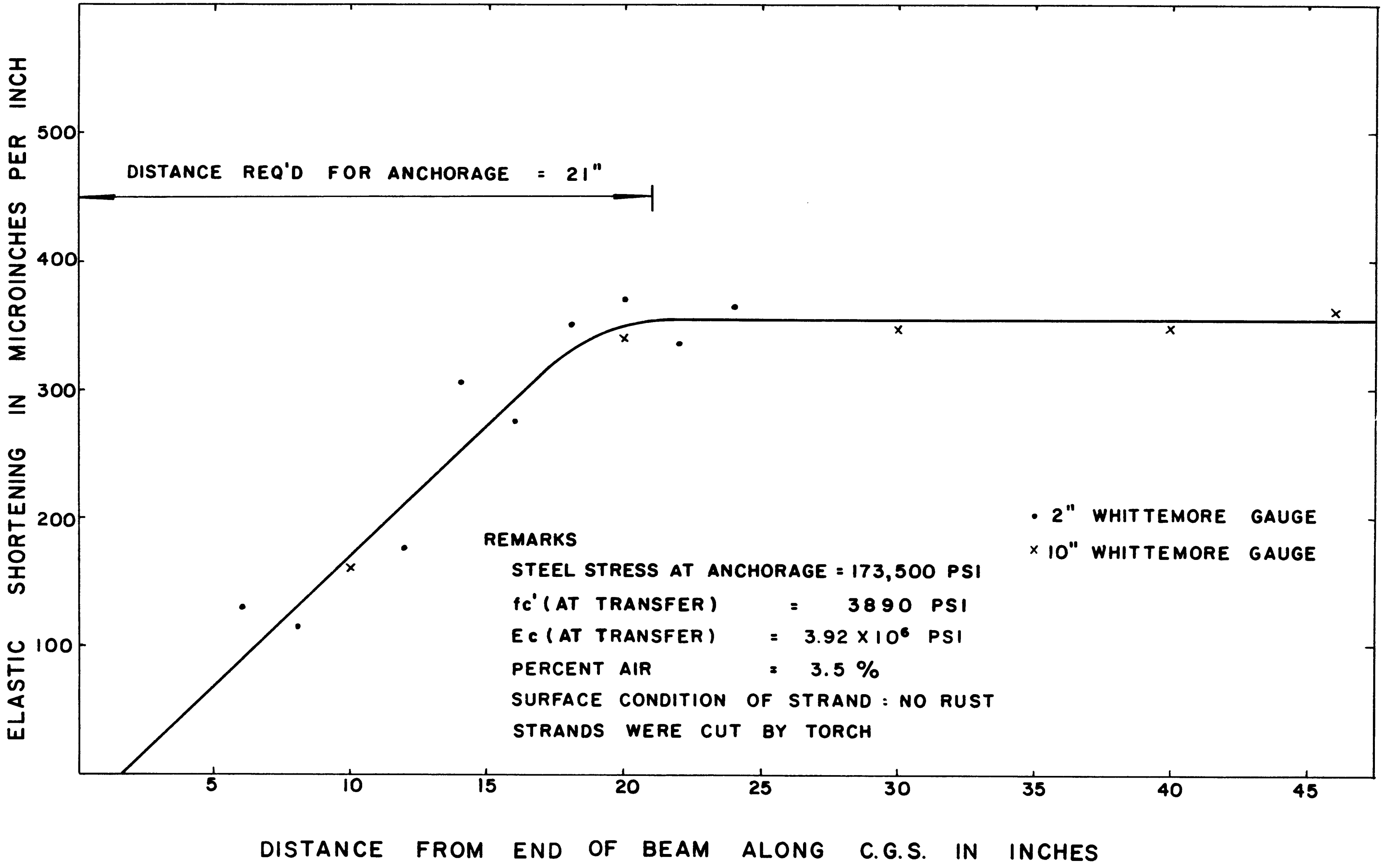


Figure 6.28. Anchorage of 7/16" ∅ Seven-Wire Uncoated Strand. Test Series 2.

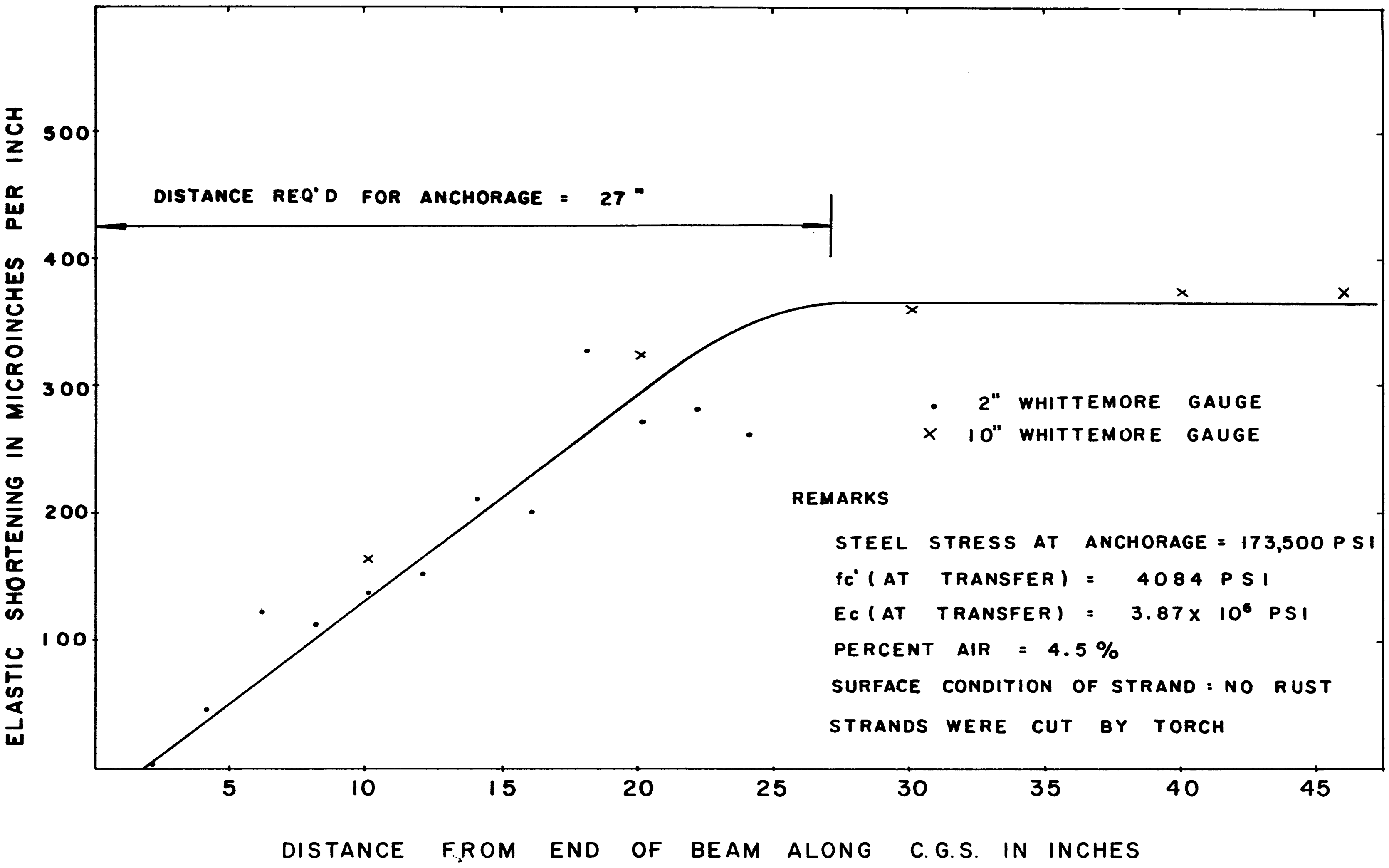


Figure 6.29. Anchorage of 7/16" ∅ Seven-Wire Uncoated Strand. Test Series 3.

From Figures 6.24 - 6.26 it can be seen that strand stress as obtained by measuring strains on the beam surface and strand surface agrees fairly well for Test Series 2 and 3. In Test Series 1, only one strain reading on the strand surface was available as one of the plugs was lost during tensioning of the strand. The value of steel stress, using the average of four strain readings on a concrete surface, was higher than the value of steel stress using one strain reading on a strand surface.

Creep of Strand

An attempt was made to find the difference in relaxation of strands stressed under two different conditions. One strand was stressed between two supports at a constant length. The other strand was stressed between two supports such that measured reduction in length could be made at one end. This was done by putting a known thickness of shims between the end plate and anchorage as shown in Figure 4.6. Due to the availability of only one dynamometer and jack for both the strands, difficulties were encountered in removing strand chucks from the cable and the investigation had to be stopped after 10 days. There was no temperature control in the room. As the force in the cable was measured when the shims became slightly loose, a possible personal error in judgment was involved in the readings. In spite of the above difficulties the results obtained were quite indicative of what would normally happen. The maximum stress loss as observed in strand at constant length was 5.95% in 10 days while it was 4-1/2% for the variable length strand. The first value of 5.95% loss agrees fairly well with results obtained for a stress-relieved 3/8" strand stressed at $0.7\ f_s'$ as reported in (40). As these tests were made after the concrete beams were cast, elastic, creep and shrinkage strains as observed for the beams were used in reducing the length of the strand at the respective times. This in effect duplicated the condition of a prestressed concrete beam in steps instead of continuous reduction due to effects of creep and shrinkage of concrete. It is felt from the above experiment that in prestressing plants where cables are cut in approximately 18-19 hours after pretensioning, the creep would be about 2-1/2% before the cutting of the cables. After cutting the cables, elastic loss occurs followed by creep and shrinkage losses in concrete which occur at a rapid rate in the beginning. This reduces the tension in the strand from the initial stress of 70% of ultimate strength down to 55% of ultimate value after which the creep in the cable is found to be very little. Thus the average value of 4% for creep of steel loss is a good average for strands tensioned at 70% of ultimate strength. This is true for stress-relieved, 7-wire, cold-drawn, high carbon, uncoated Roebling System strands as used in this experiment. The characteristics of other strands may differ from the above values.

As the strands were cut 26-28 hours after tensioning, 3.0% creep loss is assumed in computation of stress after transfer of beams. The total loss would probably reach 5% of the initial stress. The 3% creep loss was arrived at by using information from Reference 40 and using their formula

$$\Delta_r = gf_i \left[\frac{f_i}{f_s'}\right]^d \quad t_1^b$$

where

Δ_r = relaxation stress loss at time t_1

f_i = initial stress

f'_s = ultimate strength of steel

t_1 = time from application of initial stress in hours

g, b and d are constants to be found from tests.

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

From this investigation it can be concluded that:

1. Empirical formula for Modulus of Elasticity - $E_c = 33\sqrt{w^3 f'_c}$ is confirmed from this investigation.
2. Modulus of Elasticity of concrete is almost equal to secant modulus at half the ultimate strength with the range of concrete stresses in the beam. (Maximum stress $0.6\ f'_c$.)
3. Strains as measured on the beam surface agree fairly well with strains measured on the strands in the middle section of the beam. This indicates perfect bond between concrete and strands and hence possibility of slip is excluded for creep mechanism. Thus the use of plugs on the surface is satisfactory for shrinkage and creep studies of beams.
4. An ultimate shrinkage strain of .0005 in./in. is estimated for beams of which .0003 in./in. occurs during the first 28 days.
5. The creep and shrinkage as measured on cylinders should be used with caution. The ratio between maximum average cylinder shrinkage to maximum average beam shrinkage at the end of one year is 1.093.
6. The creep in cylinders, concentrically loaded at approximately constant load, is greater than creep in beams, stressed initially at the same stress at the c.g.s. of beam.
7. The average ratio of creep strain to elastic strain for cylinders is 1.83 at the end of one year. The ultimate creep loss would reach about 2.25 times elastic loss.

8. For beams, consider loss due to creep of concrete and creep of steel after transfer as "Creep Loss." This "creep loss" is 1.67 times the elastic loss at the end of one year. On the basis of projection of data, the ultimate creep loss would reach about 1.8 times elastic loss. This ratio of creep loss to elastic loss is for:

   (1) Initial level of stress in concrete at c.g.s. of 1420 psi.

   (2) Concrete strength at transfer of about 4000 psi.

   (3) Stress on top fiber of about 450 psi and bottom fiber of about 2100 psi in the beam.

   For other conditions the ratio of creep loss to elastic loss may differ.

9. The ratio of $\frac{\text{Creep Loss}}{\text{Elastic Loss}}$ for beams to $\frac{\text{Creep Loss}}{\text{Elastic Loss}}$ for cylinders is an average value of .91 at the end of one year.

10. Specific creep is very nearly constant. This is verified for stress range from initial stress level in beams to the final stress level in beams at the end of one year.

11. "Effective Modulus" method can be used for estimating strand stress if specific creep and shrinkage of beams are known (Chapter VI).

12. The anchorage length for the 7/16" 7-wire strand is from 22" to 27" (for 4000 psi concrete at transfer) when the strand is cut by torch at transfer. The strength of concrete at transfer is an important factor in the anchorage length.

Recommendations

To find the final stress in the tendons of the pretensioned prestressed concrete beams, the following is recommended to evaluate various losses:

1. Calculate the loss of prestress due to creep of steel for the time elapsed between tensioning of tendons and transfer of prestress using creep formula for steel (Chapter VI, page 82). This value may be around 2-1/2 to 3% for 7-wire strands stressed to .7 times their ultimate strength and where the transfer takes place in 16 to 20 hours.

2. Calculate the elastic loss of prestress using Equation (3.5), (3.6) or (3.7) as suggested in Chapter III. Modulus of Elasticity of concrete can be found using the empirical equation as suggested by the ACI. Modulus of Elasticity of steel can be used as suggested by the manufacturer.

3. (a) "Effective Modulus" method can be used for estimating strand stress in beams using specific creep obtained from stressed cylinders.

   (b) Equation $r = \frac{28.5}{\sqrt[3]{f'_c}}$ (50), in which r is the ratio of ultimate creep strain to elastic strain at transfer, can be used to predict the concrete creep strain of prestressed concrete beams. This equation was derived from test beams protected from weather and without control of temperature and relative humidity.

4. Shrinkage strain of .0003 in./in. is recommended for normal weight concrete of 5000 psi strength with low w/c ratio and optimum steam curing. In the absence of stiff mix (low w/c ratio) and optimum steam curing for manufacture of members, a value of .0005 in./in. for shrinkage is recommended for normal weight concrete with 4000 psi strength at transfer and 5000 psi at 28 days.

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APPENDIX A

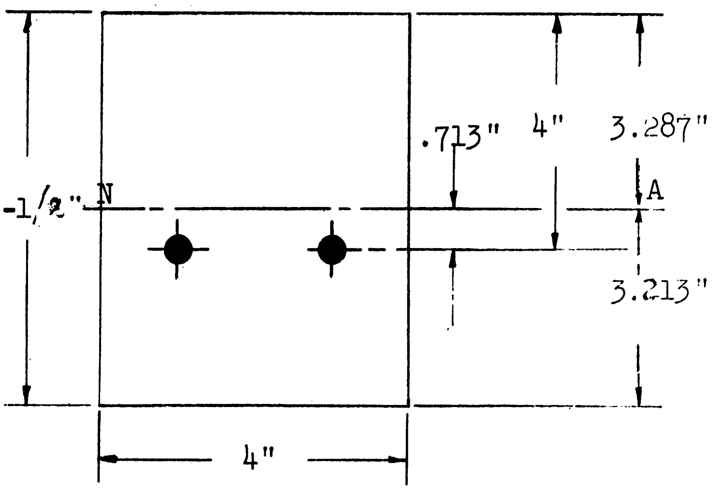
DESIGN OF BEAM

		At Transfer	At 28 Days
Concrete Strength	-	4000 psi	5000 psi
Limiting Stresses	-	2400 psi	2250 psi

Load from 2 - 7/16" Roebling cables = 37,800 lbs = 173,554 psi

$E_s = 27 \times 10^6$ psi

$E_c = 1,800,000 + 5000 \times 4000 = 3.8 \times 10^6$ psi



Section Properties:

$A_c = 25.7812$ in.^{2}

$A_t = 27.33$ in.^{2}

$C_1 = 3.287"$; $C_2 = 3.213"$

$e_t = .713"$

$I_t = 92.24$ in.^{4}

$A_s = .2178$ in.^{2}

w = 27.08 lb /'; Span = 7' - 6"; $M_g = 2284"$lb

$F_o = 37800$ lb ; n = 27/3.8 = 7.1

$$(1)\quad F_i = \frac{F_o + \dfrac{M_g e_t n A_s}{I_t}}{1 + nA_s\left(\dfrac{1}{A_t} + \dfrac{e_t^2}{I_t}\right)} = \frac{37800 + \dfrac{2284 \times .713 \times 7.1 \times .2178}{92.24}}{1 + 7.1 \times .2178\left(\dfrac{1}{27.33} + \dfrac{.713^2}{92.24}\right)}$$

$$= 35515.5 \text{ lb}$$

Elastic Loss = 37800 - 35515.5 = 2284.5 lb

$$= \frac{2284.5}{.2178} = 10489 \text{ psi}$$

$$\% \text{ Elastic Loss} = \frac{10489 \times 100}{173{,}554} = \text{---------------------------- } 6.04\%$$

(2) Due to Shrinkage of Concrete - Assume .00025"/"

Loss = $.00025 \times 27 \times 10^6 = 6750$ psi

$$\% \text{ Loss} = \frac{6750}{173{,}554} \times 100 \quad \text{---------------------------- } 3.89\%$$

(3) Due to Creep of Concrete

Loss = 1.5 x 10489 = 15733 psi

% Loss = 1.5 x 6.04 ---------------------------- 9.06%

(4) Due to Creep of Steel - Assume 4%

Loss = .04 x 175,554 = 6942 psi

% Loss = 4% ---------------------------- 4.00%

Total Losses = 39914 psi

$$\% \text{ Losses} = \frac{39914 \times 100}{173{,}554} = 22.99\%, \quad \text{say, } 23\%$$

F = 37800 x .73 = 27594 lb

Concrete Stresses at Transfer:

$$f_t = -\frac{35515.5}{27.33} + \frac{35515.5 \times .713 \times 3.287}{92.24} - \frac{2284 \times 3.287}{92.24}$$

$$= -1299.5 + 902.3 - 81.4 = -478.6 \text{ psi}$$

$$f_b = -1299.5 - \frac{35515.5 \times .713 \times 3.213}{92.24} + \frac{2284 \times 3.213}{92.24}$$

$$= -1299.5 - 882 + 79.5 = -2102 \text{ psi}$$

$$f_s = -1299.5 - \frac{35515.5 \times .713^2}{92.24} + \frac{2284 \times .713}{92.24}$$

$$= -1299.5 - 195.7 + 17.6 = 1477.6 \text{ psi}$$

Load to be put on Cylinder:

$$= 1477.6 \times 28.3 = 41816 \text{ lbs}$$

Final Concrete Stresses after all Losses Occur:

$$f_t = -\frac{27594}{27.33} + \frac{27594 \times .713 \times 3.287}{92.24} - \frac{2284 \times 3.287}{92.24}$$

$$= -1009.6 + 701.1 - 81.4 = -389.9 \text{ psi}$$

$$f_b = -\frac{27594}{27.33} - \frac{27594 \times .713 \times 3.213}{92.24} + \frac{2284 \times 3.213}{92.24}$$

$$= -1009.6 - 685.3 + 79.5 = -1615.4 \text{ psi}$$

$$f_s = -\frac{27594}{27.33} - \frac{27594 \times .713^2}{92.24} + \frac{2284 \times .713}{92.24}$$

$$= -1009.6 - 152.1 + 17.65 = -1144.1 \text{ psi}$$

APPENDIX B

MEASURED VERSUS CALCULATED ELASTIC LOSS OF BEAMS

σ_o = Stress in Steel at Anchorage = 173,554 psi

Use 3% Loss in Steel due to Creep of Steel

$$\sigma_{ot} = 173{,}554 \times .97 = 168347 \text{ psi}$$

See Appendix A for Beam Properties.

Use formula as given in Chapter III for finding stress after cutting cables.

$$\sigma_i = \frac{\sigma_{ot} - K_2 + n\ell_4}{1 + nK_3} + K_2$$

$$E_s = 27 \times 10^6 \text{ psi}$$

$$\ell_1 = \frac{e(C_2 - C_1)A_c}{2I_t} = \frac{.713(3.213 - 3.287)25.7812}{2 \times 92.24} = - .007373$$

$$M_g = \frac{w\ell^2}{8} \times 12 = 27.08 \times 7.5^2 \times 1.5 = 2284'' \text{ lbs}$$

$$\ell_2 = \frac{M_g(C_1 - C_2)}{2I_t} = \frac{2284 \times .074}{2 \times 92.24} = \frac{169.016}{184.48} = .91617$$

$$p = \frac{.2178}{25.7812} = .008448$$

$$K_1 = 1 + \ell_1 = 1 - .007373 = .992727$$

$$K_2 = \frac{.91617}{.008448} = 108.45$$

$$\ell_3 = 1 + \frac{A_c e^2}{I_t} = 1 + .1421 = 1.1421$$

$$p_1 = \frac{p}{K_1} = \frac{.008448}{.992727} = .008509$$

$$K_3 = p_1\ell_3 = .008509 \times 1.1421 = .009718$$

$$\ell_4 = \frac{M_g e}{I_t} = \frac{2284 \times .713}{92.24} = 17.65$$

Test Series 1

$$n = \frac{27}{3.716} = 7.2658 \; ; \quad nK_3 = .070609 \; ; \quad n\ell_4 = 128.2$$

$$\sigma_i = \frac{168{,}347 - 108.5 + 128.2}{1.070609} = 157{,}262 \text{ psi}$$

Elastic Loss = $\sigma_{ot} - \sigma_i$ = 168,347 - 157,262 = 11,085 psi

% Loss = $\frac{11{,}085}{173{,}554}$ x 100 = ------------------- 6.39%

Measured Loss = 380 x 10^{-6} x 27 x 10^{6} = 10,260 psi

% Loss = $\frac{10{,}260}{173{,}554}$ x 100 = ------------------- 5.91%

Test Series 2

$$n = \frac{27}{3.595} = 7.511 \; ; \quad nK_3 = .072992 \; ; \quad n\ell_4 = 128.2$$

$$\sigma_i = \frac{168{,}347.3}{1.072992} = 156{,}913 \text{ psi}$$

Elastic Loss = 168,347 - 156,913 = 11,434 psi

% Loss = $\frac{11{,}434}{173{,}554}$ x 100 ------------------- 6.58%

Measured Loss = 346 x 10^{-6} x 27 x 10^{6} = 9,342 psi

% Loss = $\frac{9342}{173{,}554}$ x 100 = ------------------- 5.38%

Test Series 3

$$n = \frac{27}{3.682} = 7.333 \; ; \quad nK_3 = .071265 \; ; \quad n\ell_4 = 128.2$$

$$\sigma_i = \frac{168{,}367.3}{1.071265} = 157{,}166 \text{ psi}$$

Elastic Loss = 168,347 - 157,166 = 11,181 psi

% Loss = $\frac{11,181}{173,554}$ x 100 = --------------- 6.44%

Measured Loss = $370 \times 10^{-6} \times 27 \times 10^{6}$ = 9,990 psi

% Loss = $\frac{9,990}{173,554}$ x 100 = -------------- 5.76%

APPENDIX C

WHITTEMORE GAGE DATA SHEET

SHRINKAGE

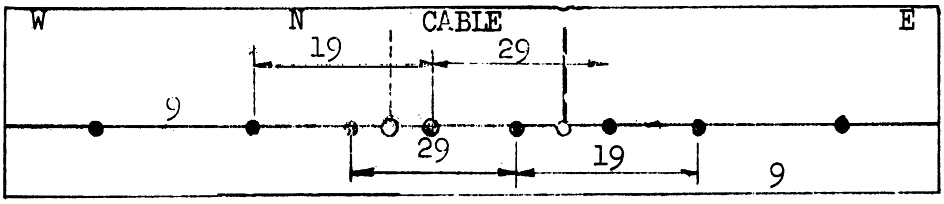
Conc. Date	May 25,1961	Date	Feb 26,1962	Age	278 days
Time	9.30 a.m.	Time	-	Humidity	34%
Temps.	78° - 61°	Temps.	75.5° - 58°	Std. Bars	

Std. Bars
1 10.05580 in.
(#1 gage-#2 bar, Beam)
2 10.052000 in.
(#2 gage-#2 bar-Cable)
10.04780 in.
(#1 gage-#1 bar-Cylinders)

Beam :- S-3

Cylinders:- S-3-1; S-3-2

Gage No.	Initial Reading	Final Reading	Shortening	Average Shrinkage in μin/in.	Average Shrinkage in μin/in.	Remarks
9 NE	3660	3265	395	430		
NW	4685	4240	445			
SE	5140	4725	415			
SW	5015	4550	465			
19 NE	2600	2145	455	442	426	
NW	5585	5140	445			
SE	2900	2450	450			
SW	3990	3570	420			
29 NE	3500	3095	405	405		29 SW Reading Discarded
NW	3540	3130	410			
SE	3770	3370	400			
SW	5615	5300				
Cable N	2590	2150	440	417	417	
S	4485	4090	395			
S-3 1N	7275	6770	505	498	498	
1S	7380	6890	490			
2N	7640	7105	535			
2S	6790	6330	460			



WHITTEMORE GAGE DATA SHEET

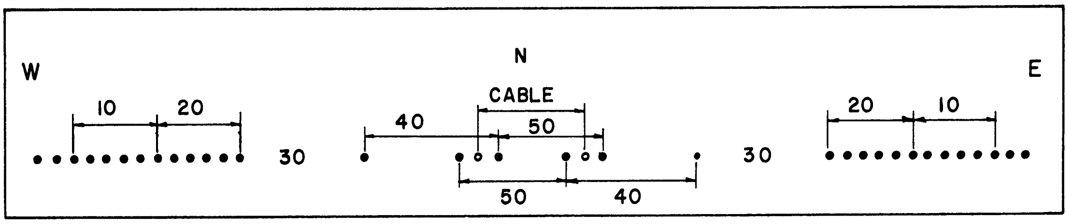
CREEP

Beam : P-3

Cylinders: C-3

Age of Concrete 278 days

Gage No.		Initial Reading	Final Reading	Shortening	Average Creep and Shrinkage in μ in/in.	Average Shrinkage in μ in/in.	Average Creep in μ in/in.	Remarks
10	NE	8220	7665	555	556	426	130	
	NW	5820	5225	595				
	SE	4510	3945	565				
	SW	3560	3050	510				
20	NE	7230	6595	635	725	426	299	
	NW	3645	2730	915				
	SE	4480	3600	880				
	SW	5350	4880	470				
30	NE	4710	3790	920	957	426	531	
	NW	6700	5495	1205				
	SE	5145	4145	1000				
	SW	5715	5010	705				
40	NE	5735	4670	1065	1032	426	606	
	NW	2780	1740	940				
	SE	6425	5325	1100				
	SW	3470	2445	1025				
50	NE	5035	4010	1025	1047	426	621	
	NW	5050	4045	1005				
	SE	1800	690	1110				
	SW	5810	4760	1050				
Cable	N	5875	4810	1065	1080	417	663	
	S	4950 2310	3855 1165	1095 1145				
Cylinders	1N	5665	4490	1175	1346	498	848	
	1S	6735	5270	1465				
	2N	5835	4780	1055				
	2S	5535	3825	1710				
	3N	6440	5315	1125				
	3S	6715	5170	1545				



APPENDIX D

DERIVATION OF FORMULA FOR FINAL STRESS IN STEEL (STRAND) IN BEAM AFTER LOSSES ACCORDING TO "EFFECTIVE MODULUS" METHOD(17)

1. Force in Steel = Force in Concrete

i.e.,

$$\sigma A_s = \left[\frac{F}{A_c} + \frac{Fe(C_2 - C_1)}{2I_t} + \frac{M_g(C_1 - C_2)}{A_t} \right] A_c$$

Similar to solution on pages 8 and 10, Chapter III.

$$\sigma = \frac{f}{p_1} + K_2$$

2. Strain in Concrete at C.G.S. = Change of Strain in Steel at c.g.s.

$$\frac{\frac{F}{A_c} + \frac{Fe^2}{I_t} - \frac{M_g e}{I_t}}{E_c'} + s = \frac{\sigma_{ot} - \sigma}{E_s}$$

"Effective Modulus" E_c' is defined as

$$E_c' = \frac{E_c}{1 + c_1 E_c}$$

where

c_1 = specific creep under 1 psi at time t (see page 73, Chapter VI)

E_c = Elastic Modulus

Similar to solution in Chapter III for finding Elastic Loss,

$$fn'\ell_3 - \ell_4 n' + sE_s = \sigma_{ot} - \frac{f}{p_1} - K_2$$

i.e.,

$$f = \frac{p_1(\sigma_{ot} - sE_s - K_2 + n'\ell_4)}{1 + n'K_3} \qquad n' = \frac{E_s}{E_c'}$$

i.e.,

$$\sigma = \frac{\sigma_{ot} - sE_s - K_2 + n'\ell_4}{1 + n'K_3} + K_2$$

Thus if the shrinkage and creep per psi are known, the stress in steel can be found.

APPENDIX E

CALCULATIONS FOR FINAL STRESS IN STRAND AS PLOTTED ON FIGURES (6.24) THROUGH (6.26)

Final Stress $$\sigma = \frac{\sigma_{ot} - sE_s - K_2 + n'\ell_4}{1 + n'K_3} + K_2 \quad \text{(Appendix D)}$$

Values for σ calculated for Test Series 1, 2, and 3 in Tables E1, E2, and E3, respectively.

Values of σ_{ot} , E_s , K_3 , ℓ_4 and K_2 used from Appendix B.

TABLE E1

E @ TRANSFER = 3.716 x 10^6 PSI; n = 7.2658

Days	Cylinder Creep in μin./in. c	Creep per psi = $\frac{c}{1491} = c_1$	$1 + c_1E$	n' = n x (4)	$1 + n'K_3$	Shrinkage of Beam in μin./in. s	sE_s	$n'\ell_4$	$\sigma_{ot} - sE_s - K_2 + n'\ell_4$	$\frac{(10)}{(6)}$	Final Stress $\sigma = (11) + K_2$ in psi
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
15	400	2683	1.997	14.51	1.141	250	6750	256	161,745	141,757	141,865
30	460	.3085	2.146	15.59	1.152	340	9180	275	159,334	138,310	138,418
60	560	.3756	2.396	17.41	1.169	365	9855	307	158,691	135,749	135,857
90	620	.4158	2.545	18.49	1.180	385	10395	326	158,170	134,042	134,149
120	655	.4393	2.632	19.12	1.186	390	10530	337	158,046	133,259	133,367
150	680	.4561	2.695	19.58	1.190	375	10125	346	158,460	133,159	133,267
255	740	.4963	2.844	20.66	1.201	450	12150	365	156,454	130,269	130,377
315	760	.5097	2.894	21.03	1.204	465	12555	371	156,055	129,613	129,721
365	780	.5231	2.944	21.39	1.208	460	12420	376	156,195	129,300	129,408
430	810	.5433	3.019	21.94	1.213	430	11610	387	157,016	129,444	129,552

TABLE E2

E @ TRANSFER = 3.595 x 10^6 PSI; n = 7.511

15	415	.2783	2.000	15.02	1.146	265	7155	265	161,349	140,793	140,901
30	490	.3286	2.181	16.38	1.159	305	8235	289	160,293	138,302	138,410
60	560	.3756	2.350	17.65	1.172	345	9315	312	159,236	135,866	135,974
90	625	.4192	2.507	18.83	1.183	355	9585	332	158,986	134,392	134,500
120	680	.4561	2.640	19.83	1.193	370	9990	350	158,599	132,941	133,049
235	815	.5466	2.965	22.27	1.216	440	11880	393	156,752	128,907	129,015
295	840	.5634	3.025	22.72	1.221	460	12420	401	156,220	127,944	128,052
365	860	.5768	3.074	23.09	1.224	465	12555	408	156,092	127,526	127,634
400	865	.5801	3.085	23.17	1.225	435	11745	409	156,903	128,084	128,192

TABLE E3

E @ TRANSFER = 3.682 x 10^6 PSI; n = 7.333

15	415	.2783	2.025	14.85	1.144	230	6210	262	162,291	141,862	141,970
30	495	.3320	2.222	16.29	1.158	299	8073	287	160,453	138,560	138,668
60	590	.3957	2.457	18.02	1.175	315	8505	318	160,052	136,214	136,322
78	640	.4292	2.580	18.92	1.184	345	9315	334	159,258	134,508	134,616
120	720	.4829	2.778	20.37	1.198	350	9450	360	159,149	132,845	132,953
220	830	.5567	3.050	22.37	1.217	420	11340	395	157,294	129,247	129,355
278	850	.5701	3.099	22.72	1.221	430	11610	401	157,030	128,607	128,715
365	880	.5902	3.173	23.27	1.226	410	11070	411	157,580	128,531	128,639
380	885	.5936	3.186	23.36	1.227	395	10665	412	157,986	128,757	128,865

APPENDIX F

CALCULATED STRESSES AND STRAINS IN BEAMS AND CYLINDERS AT TRANSFER OF STRESS

Test Series 1

$F_i = 157{,}262^* \times .2178 = 34251.6$ lbs

$$f_s = -\frac{34251.6}{27.33} - \frac{34251.6 \times .713^2}{92.24} + 17.65$$

$$= -1253.3 - 188.7 + 17.6 = -1424.4 \text{ at c.g.s. of Beam}$$

Calculated Strain in Beam $= \dfrac{f_s}{E_c} = \dfrac{1424.4}{3.716 \times 10^6} = 383 \times 10^{-6}$ in./in.

Cylinders: Stress $= \dfrac{42200}{28.3} = 1491$ psi

Calculated Average Strain in Cylinders $= \dfrac{1491}{3.716 \times 10^6} = 401 \times 10^{-6}$ in./in.

Test Series 2

$F_i = 156{,}913^* \times .2178 = 34175.6$ lb

$$f_s = -\frac{34175.6}{27.33} - \frac{34175.6 \times .713^2}{92.24} + 17.65$$

$$= -1250.5 - 188.3 + 17.6 = -1421.2 \text{ at c.g.s. of Beam}$$

Calculated Strain in Beam $= \dfrac{1421.2}{3.595 \times 10^6} = 395 \times 10^{-6}$ in./in.

Cylinders: Stress = 1491 psi

Calculated Average Strain in Cylinders $= \dfrac{1491}{3.595 \times 10^6} = 414.7 \times 10^{-6}$ in./in.

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* From Appendix B.

Test Series 3

$$F_i = 157{,}166^{*} \times .2178 = 34230.7 \text{ lbs}$$

$$f_s = -\frac{34230.7}{27.33} = \frac{34230.7 \times .713^2}{92.24} + 17.65$$

$$= -1252.5 - 188.6 + 17.6 = -1423.5 \text{ at c.g.s. of Beam}$$

Calculated Strain in Beam $= \dfrac{1423.5}{3.682 \times 10^6} = 386.6 \times 10^{-6}$ in./in.

Cylinders: Stress = 1491 psi

Calculated Average Strain in Cylinders $= \dfrac{1491}{3.682 \times 10^6} = 405 \times 10^{-6}$ in./in.

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* From Appendix B.

APPENDIX G

TYPICAL CALCULATION OF CREEP STRAINS USING SHANK FORMULA

Shank Formula:

$$y = c'(x)^q$$

where

y = creep in μ in./in. per psi

x = number of days after loading

c', q = constants to be determined from experiment

Days	Creep of Cylinders in μ in./in.			Ave. Creep in μ in./in.
	Test Series 1	Test Series 2	Test Series 3	
90	620	625	655	633
360	780	860	875	838

Average Cylinder Stress = 1491

$$\text{Creep per psi} = \frac{633}{1491} = .424 \text{ (at 90 days)} = y_{90}$$

$$\text{Creep per psi} = \frac{838}{1491} = .562 \text{ (at 360 days)} = y_{360}$$

$$\frac{.562}{.424} = \frac{y_{360}}{y_{90}} = \frac{(360)^q}{(90)^q} = 4^q$$

$q = .202$

$$360^{.202} = 3.28 \qquad \therefore\ c' = \frac{.562}{3.28} = .1715$$

Equation is $y = .1715\,(x)^{.202}$

To Find Creep Strain in μ in./in. for Cylinder and Beam at 180 Days.

Cylinder: $y = .1715\,(180)^{.202} = .49$

Creep strain at 180 days = 1491 x .49 = 731 μ in./in.

Beam: We need stress in concrete at c.g.s. to find creep strains at c.g.s. Using σ_i - average value from Appendix B - equal to 157,000 psi and average shrinkage, s, of Beam and creep strain, c_2, as obtained from formula for Cylinder (above), stress at c.g.s. is calculated as follows:

$$\sigma_a = \text{stress in strand} = \sigma_i - (s + c_2)E_s$$

$$= 157{,}000 - (425 + 731) \times 27$$

$$= 125{,}800 \text{ psi}$$

$$f_s = -\frac{125{,}800 \times .2178}{25.7812} - \frac{125{,}800 \times .2178 \times .713^2}{92.24} + \frac{2284 \times .713}{92.24}$$

(at c.g.s. of Beam)

$$= -1134 \text{ psi}$$

Estimated Creep Strain in Beam at c.g.s. = .49 x 1134 = 556 μ in./in.

APPENDIX H

CALCULATION FOR FINAL STRESS IN STRAND AS RECOMMENDED

1. $\sigma_o = 173,554$ psi

Say 3% Loss in Strand due to Creep of Steel before Transfer of Stress.

$\sigma_{ot} = 173,554 \times .97 = 168,347$

2. At Transfer $f'_c = 4000$ psi (Design)

$$E_c = 33 \sqrt{145^3} \times 4000 = 3.645 \times 10^6 \text{ psi}$$

$$n = \frac{27}{3.645} = 7.4074 \; ; \; nK_3 = .009718 \times 7.4074 = .07198$$

$$\sigma_i = \frac{168,347 - 108.5 + 128.2}{1.07198} = \frac{168,367}{1.07198} = 157,061 \text{ psi}$$

Elastic Loss: 168,367 - 157,061 = 11,306 psi

% Elastic Loss: $\frac{11,306}{173,554} \times 100$ --------------- 6.51%

3. $$r = \frac{28.5^*}{\sqrt[3]{f'_c}} = \frac{28.5}{\sqrt[3]{5000}} = \frac{28.5}{17.1} = 1.67$$

Creep Loss: 11,306 x 1.67 = 18,881 psi

% Creep Loss: $\frac{18,881}{173,554} \times 100$ -------------- 10.87%

4. Shrinkage Loss: $.0005 \times 27 \times 10^6 = 13,500$ psi

% Shrinkage Loss: --------------- 7.77%

$= \sigma_i$ - Creep Loss -Shrinkage Loss = 157,061 - 18,881 - 13,500

= 124,680 psi

Total Loss = 173,554 - 124,680 = 48,974 psi

% Loss = $\frac{48,974}{173,554} \times 100$ ---------------28.21%

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* From Reference 50.