Poisson's Ratio at High Temperatures

By

F. L. Everett
University of Michigan, Ann Arbor, Michigan

and

Julius Miklowitz
Westinghouse Research Laboratories, East Pittsburgh, Pennsylvania

(Received June 5, 1944)

A method is presented whereby both the modulus of elasticity in tension and compression E and the modulus of elasticity in shear G may be obtained simultaneously from a simple test on a cantilever specimen subjected to combined bending and twist. By means of the relation $\nu=(E/2G)-1$, values of Poisson's ratio at various temperatures from ambient to 1000°F were obtained for five common steels.

I. INTRODUCTION

In recent years, the need for more information on the elastic properties of the ordinary steels at temperatures above ambient conditions has been pointed out. Unfortunately, statically obtained information along these lines exists in only a limited amount. The available data are confined to several references in the technical literature.[1]

In this investigation* an attempt was made to devise a simple test on one specimen which would yield simultaneously both the modulus of elasticity in tension and compression E and the modulus of elasticity in shear G. The moduli were obtained by subjecting a small cantilever rod of uniform circular cross section to combined bending and torsion, and measuring separately the deformations due to each. From the relation $\nu=(E/2G)-1$, a true value of Poisson's ratio would follow. The ultimate objective of the investigation was to determine the variations of Poisson's ratio values of five common steels in the range of temperatures from ambient to 1000°F. The variables of temperature, non-uniformity of the test specimen, manipulation, and others were minimized.

II. SPECIMEN, APPARATUS, AND METHOD

The five S.A.E. steel specimens were rods $\frac{5}{16}$ of an inch in diameter. The composition of each specimen is given in Table I.

The cantilever specimen was both bent and twisted by a single force applied as a dead weight at the end of a 6-inch torque arm which was connected to the free end of the cantilever specimen. A diagrammatic sketch is shown in Fig. 1.

The entire specimen was subjected to combined bending and torsion. The application of a load to

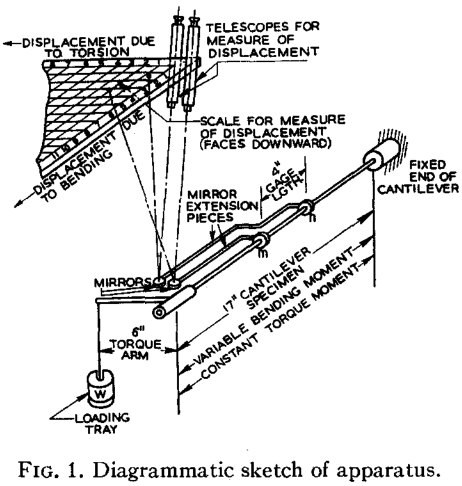

Fig. 1. Diagrammatic sketch of apparatus.

[1] (*a*) F. C. Lea and O. H. Crowther, Engineering **98**, 487 (1914).
(*b*) C. Bach and R. Baumann, ***Festigkeit seigenschaften und Gefügebilder der konstruktionsmaterialien*** (Springer, Berlin, 1921).
(*c*) F. C. Lea, Engineering **43**, 829 (1922).
(*d*) F. L. Everett, Trans. A.S.M.E. **53**, 117 (1931).
(*e*) E. Honegger, The Brown Boveri Review **19**, 143 (1923).
(*f*) G. L. Verse, Trans. A.S.M.E. **57**, 1 (1935).

* The work of this investigation was carried out at the University of Michigan in the year of 1942.

TABLE I. Composition of steels tested.

Type of steel	Composition (%) C	Mn	Si	P	S	Ni	Cr
S.A.E. 1095	0.9 -1.05	0.25–0.50	0.1–0.5	0.04	0.04	—	—
S.A.E. 5140	0.35–0.45	0.6–0.9	—	0.04	0.05	—	0.8 -1.1
S.A.E. 3340	0.35–0.45	0.3–0.6	—	0.04	0.05	3.25–3.75	1.25–1.75
S.A.E. 1020 (hot rolled)	0.15–0.25	0.3–0.6	—	0.045	0.05	—.	—
S.A.E. 1020 (cold rolled)	0.15–0.25	0.3–0.6	—	0.045	0.05	—	—

the end of the torque arm caused relative displacement of two kinds to any two cross sections, m and n. The constant torque moment caused relative rotation around the axis of the specimen and the variable bending moment produced relative rotation about a locus of points[2] outside of the specimen in a plane containing the axis. The object of these experiments was to determine these two relative rotations under various conditions of temperature and amount of loading.

Loading and unloading was accomplished by means of a loading tray which hung from the end of the torque arm. Loads in amounts of 100 and 200 grams were placed on the tray. The maximum total load in any case did not exceed 2400 grams.

The measurements of angular rotations were made by mirrors connected to the end points of the gauge length mn. Extensions were necessary to bring the mirrors outside of the electric furnace

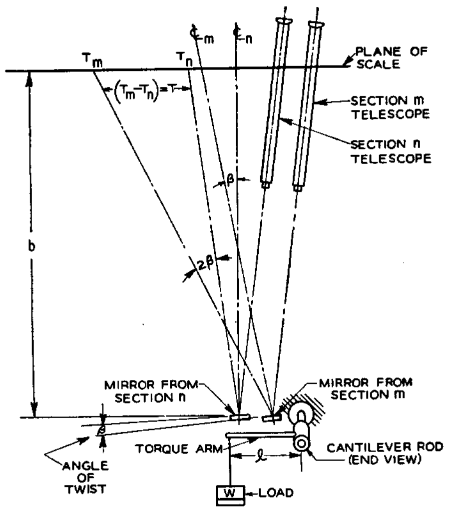

FIG. 2. Torsion problem.

[2] The points are the centers of curvature corresponding to the bending moments within the gauge length mn.

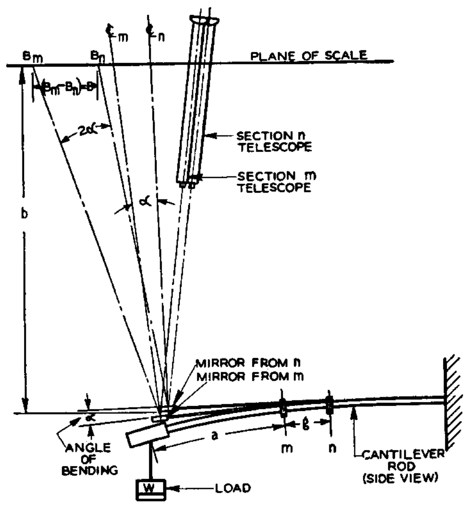

FIG. 3. Bending problem.

which surrounded the specimen and supplied the heat. The temperature was automatically controlled and held constant within ±2°F for one hour before as well as during each experiment. A telescope was focused on each mirror and back on to a special two-dimensional scale consisting of graph paper which was suspended in a horizontal plane approximately $4\frac{1}{2}$ feet above the mirrors. Torsion of the rod caused readings on the graph paper in a direction perpendicular to the axis of the specimen, and bending produced readings in the direction parallel to the axis of the specimen. The torsion permitted the determination of the modulus of elasticity in shear G; and the bending, the modulus of elasticity in tension and compression E.

Dividing the problem into its elementary parts, torsion and bending, a simple sketch may be shown for each, as in Figs. 2 and 3.

The value of the modulus of elasticity in shear is found by use of the principles of the strength of materials. Referring to Fig. 2,

$$\beta = Wlg/GI_p, \tag{1}$$

where β is the angle of twist for gauge length g, l is the torque arm, W the load, I_p the polar moment of inertia of a circular rod, and G the modulus of elasticity in shear. Since l, g, and I_p are all constants, we may write (1) as

$$G = K_1(W/\beta). \tag{2}$$

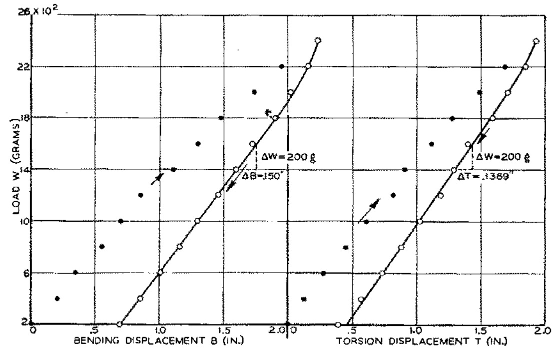

FIG. 4. Load-displacement curves for S.A.E. 1095 steel at 800°F.

We may also write as a close approximation

$$\beta = T/2b, \tag{3}$$

where T is the torsion scale reading corresponding to an angle of twist β, and b is the average vertical distance from the mirrors to the graphic scale.

The small change in b due to the deflection of the mirrors may be neglected. Therefore, considering b a constant and substituting (3) in (2), we have

$$G = K_2(W/T), \tag{4}$$

which may be written as

$$G = K_2(\Delta W/\Delta T). \tag{5}$$

From the load-torsion displacement curves a value for the modulus of elasticity in shear can be obtained with the use of Eq. (5).

The modulus of elasticity in tension and compression is found similarly (see Fig. 3). We have

$$\alpha = \frac{W(a+g)^2}{2EI} - \frac{Wa^2}{2EI} = \frac{Wg(g+2a)}{2EI}, \tag{6}$$

where α is the angle of bending for gauge length g, a the forelength of rod, W the load, I the moment of inertia about the neutral axis of the cantilever specimen, and E the modulus of elasticity in tension and compression. Since a, g, and I are all constants, we may write (6) as

$$E = K_3(W/\alpha). \tag{7}$$

By former reasoning we arrive at the following:

$$E = K_4(\Delta W/\Delta B), \tag{8}$$

where B is the bending scale reading corresponding to an angle of rotation due to bending, α. From the load-bending displacement curves a value for the modulus of elasticity in tension and compression can be obtained with the use of Eq. (8).

At constant temperatures of ambient, 200, 400, 600, 800, and 1000 degrees Fahrenheit, tests were run on each of the five different steels to obtain moduli values. The 800° and 1000°F tests were made by increasing the load on the specimen by equal increments and observing the readings of the rotation due to twist and that due to bending until noticeable creep occurred. The rod was then unloaded using the same small increments as before. Upon unloading, creep persisted for several readings only, and then became negligible, as evidenced by the fact that, upon plotting the data, a straight line resulted. This method of utilizing the data on unloading rather than on loading so as to eliminate the serious effect of

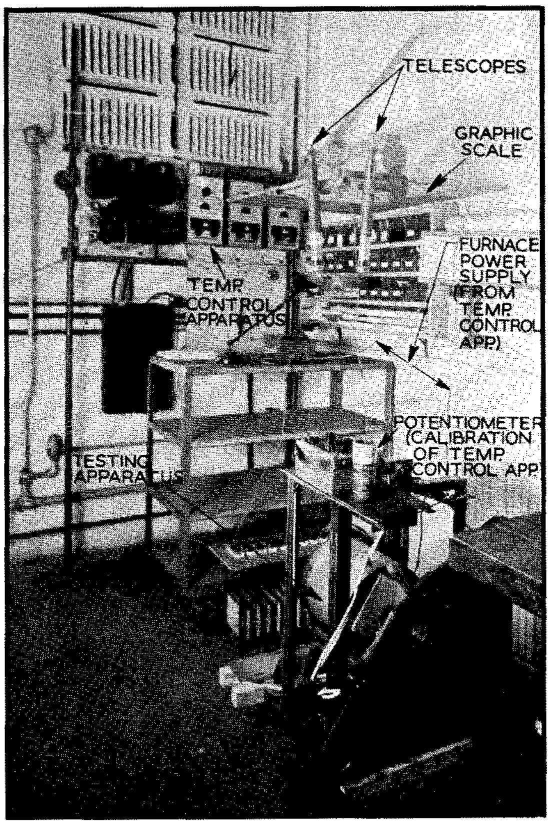

FIG. 5. Entire apparatus.

creep in obtaining the true values of the moduli was originally reported by one of the authors in an earlier paper[3] and was utilized in further experiments by G. L. Verse.[4]

In the 600°, 400°, 200°F and ambient temperature tests a similar procedure was used. Considerably less creep was evidenced; however, data on unloading only were considered to be reliable for establishing the values of the moduli. At each temperature the data for both moduli of a specific steel were plotted. Figure 4 shows a plot of load-displacement data of the 800°F test made on the S.A.E. 1095 steel. These curves are typical plots of the load-displacement data of this investigation. The slopes of the unloading curves in all cases, multiplied by the appropriate constants as given by Eqs. (5) and (8) above, gave the values of the two moduli.

From the values of the moduli a value for Poisson's ratio at each temperature was derived from the equation $\nu = (E/2G) - 1$.

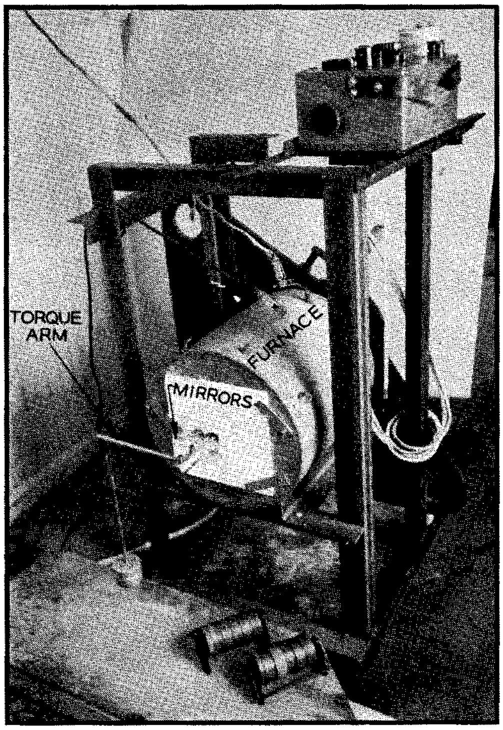

FIG. 6. Testing apparatus.

[3] See reference 1(*d*).
[4] See reference 1(*f*).

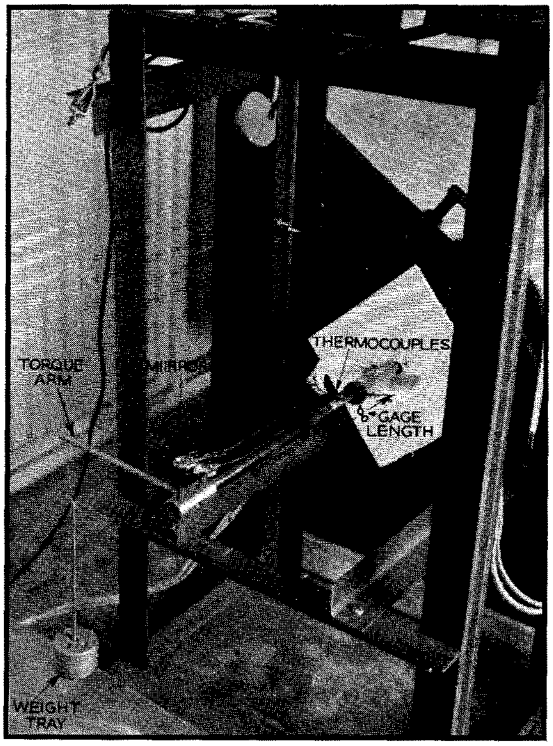

FIG. 7. The cantilever specimen.

The accuracy of obtaining Poisson's ratio by this method has been considered. When the relation

$$\nu = (E/2G) - 1 = (E - 2G)/2G \tag{9}$$

is differentiated we have

$$d\nu = (dE/2G) - (EdG/2G^2). \tag{10}$$

Dividing (10) by (9) we have

$$\frac{d\nu}{\nu} = \frac{E}{E-2G}\left[\frac{dE}{E} - \frac{dG}{G}\right]. \tag{11}$$

From relation (11) it is seen that errors of like sign in measuring both E and G tend to cancel each other, while errors of unlike sign add, and when multiplied by the factor $E/(E-2G)$ might make an appreciable error in ν. There is no reason to believe that errors in measuring E and G were of unlike sign. The large magnification of the rotations due to bending and twist helps to make the readings accurate; the distance of the mirrors from the graphic scale was approximately 53 inches. The authors believe that the values for

Poisson's ratio found in this investigation are reliable within a fair degree of accuracy.

Figures 5–7 are pictures of the apparatus.

III. TEST RESULTS

Table II presents the final results of the moduli

TABLE II. Tabulated results.

Temperature °F	Steel	E	G	P.R.(ν)
	S.A.E. 5140	30.35*	11.20*	0.355
	3340	29.75	11.30	.315
Ambient	1020 h.r.	29.80	11.33	.313
	1020 c.r.	29.20	11.35	.286
	1095	29.90	11.55	.295
	5140	29.90	11.06	.353
	3340	29.40	11.12	.320
200	1020 h.r.	29.40	10.90	.348
	1020 c.r.	29.18	11.00	.326
	1095	29.65	11.45	.297
	5140	30.40	10.50	.447
	3340	28.75	10.07	.425
400	1020 h.r.	28.95	10.45	.384
	1020 c.r.	28.60	10.36	.380
	1095	29.90	10.97	.362
	5140	29.40	9.72	.510**
	3340	28.35	9.65	.470
600	1020 h.r.	26.85	9.89	.358
	1020 c.r.	27.87	9.97	.398
	1095	28.20	10.11	.394
	5140	28.80	9.46	.520**
	3340	24.97	9.07	.376
800	1020 h.r.	26.15	9.33	.400
	1020 c.r.	26.30	9.53	.380
	1095	23.95	8.62	.389
	5140	24.90	8.23	.513**
	3340	23.30	8.34	.397
1000	1020 h.r.	22.90	7.90	.450
	1020 c.r.	18.76	6.50	.440
	1095	17.95	5.98	.500

* $\times 10^6$ lb./in.²
** Slightly above maximum value for Poisson's ratio.

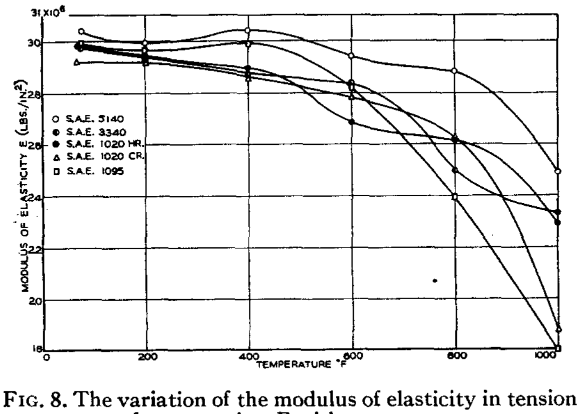

FIG. 8. The variation of the modulus of elasticity in tension and compression E with temperature.

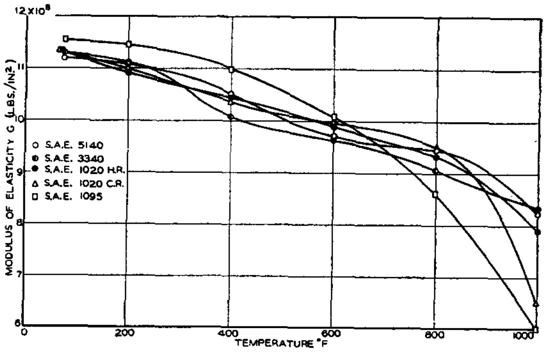

FIG. 9. The variation of the modulus of elasticity in shear G with temperature.

of tension and compression and of shear, and the derived values of Poisson's ratio. As stated previously, only data on unloading were used in calculating the moduli since creep may be considered negligible over the major portion of the unloading curve.

The values of the modulus of elasticity in tension and compression are plotted against their respective temperatures in Fig. 8. The major changes in E values occur after 400°F, the S.A.E. 1095 steel showing an 18×10^6 lb./in.² modulus at 1000°F. The S.A.E. 5140 steel maintains its rigidity better than the other steels throughout the temperature range. This may be expected since the 5140 steel is commonly known as a high temperature alloy. The S.A.E. 1020 hot-rolled steel maintains its rigidity surprisingly well at the higher temperatures; its modulus drops to 22.9×10^6 lb./in.² at 1000°F. The modulus of the S.A.E. 3340 steel drops suddenly at 600°F, but the drop is cut at 800°F; the final value (at 1000°F) being 23.3×10^6 lb./in.².

The values of the modulus of elasticity in shear are plotted against their respective temperatures in Fig. 9. The curves exhibit many similar trends. Immediate drop of the modulus G occurs with an increase of temperature. The S.A.E. 1095 and cold-rolled steel again show the lowest values at 1000°F. The hot-rolled steel again demonstrates its ability to keep its rigidity at high temperatures. The high temperature steels (S.A.E. 3340 and S.A.E. 5140) have the highest values of G at 1000°F: 8.3 and 8.2×10^6 lb./in.², respectively.

Figure 10 shows the variation of the derived Poisson's ratio with temperature of each steel

TABLE III. Comparison of modulus E values.

Temp.	S.A.E. 1020 hot-rolled	Bach & Baumann[a] mild steel	Lea[b] mild steel	Verse[c] 0.43% carbon steel	S.A.E. 1095	Bach & Baumann[a] carbon steel	Lea[b] high (0.95%) carbon steel
20 °C	29.8*	29.9	29.3	29.9	29.9	29.5	28.4
100	29.4	29.4	—	29.0	29.6	29.0	26.3
200	29.0	28.0	28.2	28.0	29.9	28.3	23.4
300	27.1	27.2	27.2	26.5	28.6	27.2	21.6
400	26.2	26.5	—	24.6	25.0	26.1	17.8
500	24.6	(20.1)	18.7	21.8	20.1	(21.8)	

[a] See reference 1(b).
[b] See reference 1(c).
[c] See reference 1(f).
* $\times 10^6$ lb./in.^{2}.

TABLE IV. Comparison of modulus G values.

Temp.	S.A.E. 1020 hot-rolled	S.A.E. 1020 cold-rolled	Everett[a] 0.34% C steel	Verse[b] 0.34% C steel	Lea[c] mild steel
70°F	11.3*	11.4	11.5	11.5	12.4
200	10.9	11.0	11.5	11.4	12.1
400	10.5	10.4	11.4	11.3	11.8
600	9.9	10.0	11.0	10.8	11.3
800	9.3	9.5	9.7	9.6	10.5
1000	7.9	6.5	6.2**	6.6**	9.0**

[a] See reference 1(d).
[b] See reference 1(f).
[c] See reference 1(c).
* $\times 10^6$ lb./in.^{2}.
** Extrapolated from curves.

tested. The resulting curves are irregular but show many similar trends. There is a definite increase of Poisson's ratio at the higher temperatures in all cases: This means that as the temperature rises the percentage decrease of the modulus in shear is greater than that of the modulus in tension and compression. There is no appreciable change in Poisson's ratio between ambient temperature and 200°F, for the S.A.E. 5140, S.A.E. 3340, and S.A.E. 1095 steels. The two S.A.E. 1020 specimens show a slight increase in Poisson's ratio over the same range. From 200° to 400°F there is a large increase for all steels; and, with the exception of the S.A.E. 1020 hot-rolled steel, Poisson's ratio continues to increase up to 600°F. From 600° to 800°F, the S.A.E. 3340, S.A.E. 1020 cold-rolled, and S.A.E. 1095 steels show a decrease in Poisson's ratio; the same steels show an increase from 800° to 1000°F. In the case of the two S.A.E. 1020 specimens and the S.A.E. 1095, Poisson's ratio reaches a maximum value at 1000°F; for the S.A.E. 3340 and S.A.E. 5140 steels, the maximum values are at 600° and 800°F, respectively.

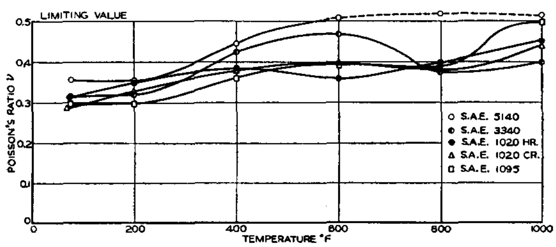

FIG. 10. The variation of Poisson's ratio ν with temperature.

From the strength of materials it can be shown that 0.5 is the maximum value of Poisson's ratio for isotropic materials. The dotted portion of the S.A.E. 5140 steel curve has been so designated since it lies above the limiting value of Poisson's ratio.

From 800°F on, the curves approach the limiting value which seems logical since in the plastic state Poisson's ratio is 0.5.

For the purpose of comparison, values taken from some of the works quoted in an earlier part of this paper[5] have been collected and are presented in Tables III and IV. With the exception of the results shown in the last columns

[5] See reference 1.

of Tables III and IV, reasonably good agreement between the values given by the various experimenters is evidenced.

The encouragement given by Dr. A. Nadai of the Westinghouse Research Laboratories and by the Horace A. Rackham School for Graduate Studies, University of Michigan, is duly acknowledged. The authors also appreciate the help received at the Westinghouse Research Laboratories in the preparation of this paper.

A Numerical Method in the Theory of Vibrating Bodies

ANDREW VAZSONYI
Graduate School of Engineering, Harvard University, Cambridge, Massachusetts
(Received May 25, 1944)

In this paper a numerical method is presented for the determination of the characteristic numbers and modes of vibrating (conservative) systems. The method for continuous bodies is based on a finite difference approximation and is a "relaxation method" in the same sense as the term is used by R. V. Southwell. No claim is made of theoretical advancement in the subject; the paper purports to present a thoroughly practical method of obtaining numerical answers speedily. Detailed computations are carried out for the transverse vibrations of a quadrangular elastic membrane. The nature of the method is such that it can be easily extended to other vibrating systems of finite or infinite number of degrees of freedom. It is to be emphasized that the quadrangular shape is just a simple example; the real strength of the method lies in the fact that numerical results can be obtained for bodies with more complicated and irregular boundaries.

INTRODUCTION

THE two most successful procedures for obtaining numerical results in the study of vibrating bodies are the variational methods and the perturbation methods. These widely used "analytical" computational procedures are very powerful indeed so long as the bodies are simple in their shape, but fail in the case of complicated shapes because of the prohibitive amount of numerical work involved. To meet these circumstances the method of finite differences was developed by several authors.[1–4] The practical value of these attractive "non-analytical" computational procedures was strongly limited by the amount of numerical work involved in the solution of the difference equations. Recently this situation was remedied by the entirely new outlook of R. V. Southwell,[5] whose relaxation technique transforms the method of finite differences into a very powerful and thoroughly practical numerical procedure. Among other problems, Southwell has successfully attacked the equations of vibrating bodies with finite number of degrees of freedom, and also of one-dimensional continuous bodies.

Southwell's method could be easily extended to two-dimensional bodies also, but we prefer to present an entirely different technique which seems superior to that of Southwell's, both for systems of finite or of infinite number of degrees of freedom. It should not be at all surprising that such alternatives exist within the frame of the relaxation method, and a comparison is made between the two procedures when treating forced vibrations (see Section 8).

[1] R. von Mises and H. P. Geiringer, Zeits. f. angew. Math. u. Mech. 9, 58–77, 152–164 (1929).
[2] F. Bleich and E. Melan, *Die Gewöhnlichen und Partiellen Differenzengleichungen der Baustatik* (Julius Springer, Berlin and Wien, 1927).
[3] G. E. Kimball and G. H. Shortley, Phys. Rev. 45, 815–820 (1934).
[4] R. H. Bolt, Phys. Rev. 57, 1057A (1940).
[5] R. V. Southwell, *Relaxation Methods in Engineering Science* (The Clarendon Press, Oxford, 1940).