to be

$$V_x=K\cdot(E_0^2/8)[(C_1D\Gamma'+2C_2T_B')^2+(C_1D\Gamma''+2C_2T_B'')^2], \quad (7)$$

where $\Gamma'=\Gamma_0+A\chi''$, and $\Gamma''=-A\chi'$.

This expression can be rewritten as

$$V_x=K\cdot(E_0^2/8)[(4C_2^2|T_B|^2+C_1^2D^2\Gamma_0^2+4C_1C_2D\Gamma_0T_B')+2C_1^2D^2\Gamma_0A\chi''+C_1^2D^2A^2(\chi'^2+\chi''^2)+4C_1C_2DT_B'A\chi''-4C_1C_2DT_B''A\chi']. \quad (8)$$

For $\beta=1$, $A$ is maximum and $\Gamma_0=0$. Therefore

$$V_x=K\cdot(E_0^2/8)[4C_2^2|T_B|^2+4C_1C_2DA(T_B'\chi''-T_B''\chi')+C_1^2D^2A^2(\chi'^2+\chi''^2)]. \quad (9)$$

Neglecting $C_1^2D^2A^2(\chi'^2+\chi''^2)$, the change of output voltage at the terminals of the crystal when passing through resonance is

$$V_r=K\cdot(E_0^2/2)C_1C_2DA(T_B'\chi''-T_B''\chi'). \quad (10)$$

THE REVIEW OF SCIENTIFIC INSTRUMENTS VOLUME 40, NUMBER 4 APRIL 1969

# Attenuation Correction Factor for Scattering from Cylindrical Targets

John M. Carpenter
*Department of Nuclear Engineering, The University of Michigan, Ann Arbor, Michigan 48105*
(Received 22 November 1968)

The target attenuation correction factor is given for the situation in which the attenuation coefficients differ for incident and scattered radiation. The calculation applies for cylindrical targets, with the scattering plane perpendicular to the cylinder axis. The correction factor is expressed in the form of a series expansion which is appropriate for cases of small attenuation.

Measurements of the intensity of scattered radiation (neutrons, x rays) suffer from the effect of attenuation of the incident and emerging beams by the target material. Corrections for this attenuation have been given[1–4] for a number of geometrical cases of interest. Most commonly, those results assume that the attenuation coefficients $\sum_i$ and $\sum_s$ are identical for the incident and scattered radiation. We have found it necessary to compute the target attenuation correction factor for inelastic neutron scattering by cylindrical targets. Under these conditions, $\sum_i$ differs from $\sum_s$ because incident and scattered neutrons have in general different energies, while $\sum$ varies significantly with neutron energy. In other cases, differences due to anisotropy of the material may produce the same effect. The correction factor for this case appears not to have been computed previously.

Figure 1 illustrates the general problem. The emerging total current $J_s$ is related to the incident current per unit area $j_i$

$$J_s=j_i\sum_{is}\int_V dVe^{-\Sigma_il}e^{-\Sigma_sl'}, \quad (1)$$

where $\sum_{is}$ is the macroscopic scattering cross section and $V$ is the target volume. The incident radiation is attenuated as it travels distance $l$ to the point of scattering, and the emerging radiation is similarly attenuated traveling distance $l'$. In Eq. (1) the incident current is assumed constant across the target; $\sum_{is}$ is assumed to be constant in $V$ and the target is assumed to be viewed uniformly by a detector. Incident and emerging radiations are assumed unidirectional.

Writing (1) as

$$\Sigma_{is}=[J_s/(j_iV)]F, \quad (2)$$

the first factor appears as it would in the absence of attenuation, and $F$ is the target attenuation correction factor

$$\frac{1}{F}\equiv\frac{1}{V}\int_V dVe^{-\Sigma_il}e^{-\Sigma_ll'}. \quad (3)$$

In order to express the result in a form calculable independently of $\sum_i$, and $\sum_s$, the integrand is expanded

$$\frac{1}{F}=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}}{m!n!}(\Sigma_i)^m(\Sigma_s)^n\left[\frac{1}{V}\int dV(l)^m(l')^n\right]. \quad (4)$$

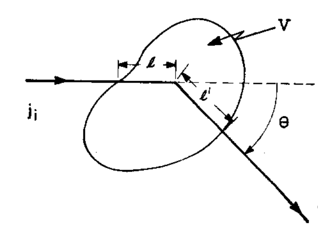


FIG. 1. The general case of target attenuation in a radiation scattering experiment.

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[2] M. Renninger, Z. Phys. **106**, 141 (1937).
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TABLE I. Chebycheff expansion coefficients for calculation of $Z_{mn}(\theta)$.

| $l$ | $c_l^{(1,1)}$ | $c_l^{(1,2)}=c_l^{(2,1)}$ | $c_l^{(1,3)}=c_l^{(3,1)}$ | $c_l^{(1,4)}=c_l^{(4,1)}$ | $c_l^{(2,2)}$ | $c_l^{(2,3)}$ |
|---|---|---|---|---|---|---|
| 0 | 0.730284 | 0.848859 | 1.133129 | 1.641112 | 1.000006 | 1.358113 |
| 1 | −0.249987 | −0.452690 | −0.749962 | −1.241639 | −0.821100 | −1.358076 |
| 2 | 0.019448 | 0.056557 | 0.118245 | 0.226247 | 0.166645 | 0.349199 |
| 3 | −0.000006 | −0.000009 | −0.000018 | −0.000045 | −0.012096 | −0.038817 |
| 4 | 0.000249 | 0.000000 | −0.001345 | −0.004821 | 0.000008 | 0.000022 |
| 5 | −0.000004 | −0.000006 | −0.000012 | −0.000030 | −0.000126 | −0.000021 |

Values of $Z_{mn}(\theta)$ which are constant $Z_{00}=1$; $Z_{01}=8/(3\pi)$; $Z_{02}=1$; $Z_{03}=64/(15\pi)$; $Z_{04}=2$; $Z_{05}=1024/(105\pi)$; $Z_{06}=5$.

Calculations are reported here for cylindrical targets with the plane of scattering perpendicular to the cylinder axis, as in Fig. 2.

For this case,

$$l=R[\rho\cos\xi+(1-\rho^2\sin^2\xi)]$$
$$l'=R[-\rho\cos(\xi+\theta)+[1-\rho^2\sin^2(\xi+\theta)]^{\frac{1}{2}}], \quad (5)$$

where $\rho=r/R$.

The integrals of Eq. (4) are conveniently expressed in terms of the functions

$$Z_{mn}(\theta)=\frac{1}{\pi}\int_0^1\rho d\rho\int_0^{2\pi}d\xi\left(\frac{l}{R}\right)^m\left(\frac{l'}{R}\right)^n. \quad (6)$$

Several properties of the $Z_{mn}$ are useful:

$$Z_{0k}(\theta)=\left[2^{k+1}\Gamma\left(\frac{k+1}{2}\right)\right]\bigg/\left[\sqrt{\pi}(k+2)\Gamma\left(\frac{k}{2}+1\right)\right]$$

$$\text{(constant with } \theta) \quad (7)$$

$$Z_{mn}(\theta)\leqslant Z_{0(m+n)} \quad (8)$$

$$Z_{mn}(\theta=180°)=Z_{0(m+n)} \quad (9)$$

$$Z_{mn}(\theta)=Z_{nm}(\theta). \quad (10)$$

In terms of the $Z_{mn}(\theta)$,

$$\frac{1}{F}=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}}{m!n!}(\Sigma_i R)^m(\Sigma_s R)^n Z_{mn}(\theta). \quad (11)$$

It is easily shown that $1/F$ computed by including all terms such that $(m+n)\leqslant N$ is in error by less than

$$[(\Sigma_i+\Sigma_s)R]^{N+1}Z_{0(N+1)}/(N+1)!$$

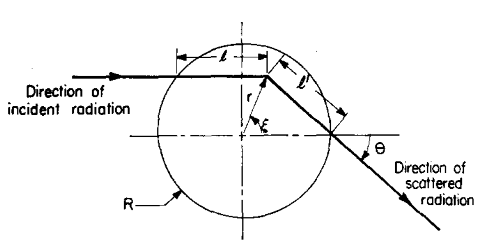


FIG. 2. Target attenuation in the case with radiation perpendicular to the axis of a cylindrical target.

due to truncation. The $Z_{mn}$'s were computed to within a fractional error less than $\epsilon=10^{-4}$ on the University of Michigan's IBM 360 system for $(m+n)\leqslant 5$. Error in $1/F$ due to inaccuracies in the computed $Z_{mn}$'s can be shown to be less than $5\epsilon e^{(\Sigma_i+\Sigma_s)R}$ for the series including all $(m+n)\leqslant 5$. Uncertainty in $1/F$ due to uncertainties in $\Sigma_i$ and $\Sigma_s$ will probably always be dominant and is conveniently estimated by computing $1/F$ for different values of $\Sigma_i$ and $\Sigma_s$.

A lengthy table of the $Z_{mn}(\theta)$ is avoided and calculations are simplified for arbitrary $\theta$ by expressing the $Z_{mn}$ as functions of $\theta$ in the form of an expansion

$$Z_{mn}(\theta)=\sum_{l=0} c_l^{(m,n)}T_l(\theta), \quad (12)$$

where the $T_l$ are the Chebycheff polynomials (in $\cos\theta$)

$$T_l(\theta)=\cos l\theta. \quad (13)$$

The coefficients $c_l^{(m,n)}$ were determined by least square fitting the computed functions $Z_{mn}(\theta)$ at 21 equally spaced values of $\cos\theta$ between $+1$ and $-1$. Table I contains the coefficients. Six terms in Chebycheff series are adequate to reproduce the $Z_{mn}(\theta)$ to within the accuracy with which they were computed. The $c_l^{(m,n)}$ are tabulated to six places even though the $Z_{mn}$'s were obtained only to within $10^{-4}$, since the fitting procedure tends to smooth out the random calculation errors. The smallest coefficients are probably insignificant.

$1/F$ is easily and quickly computed from the given data. For the desired angle $\theta$, compute the $Z_{mn}$ using Eq. (12) and the Chebycheff expansion coefficients of Table I. Using the attenuation coefficients $\Sigma_i$ and $\Sigma_s$ and the target radius, compute $1/F$ from Eq. (11). The results of such a calculation were compared with the correction factor computed from Ref. 4, for the case with $\Sigma_i R=\Sigma_s R=0.6$, and satisfactory agreement was found, the present calculation giving $1/F=0.3946$, the calculation using the tables of Ref. 4 giving $1/F=0.3999$, which is within the error estimate cited above due to truncation. It is thus to be emphasized that the present calculation applies with accuracy only for small $\Sigma_i$ and $\Sigma_s$.

The author expresses his gratitude to N. A. Lurie for his help in some of the calculations.