

# Measurement of the mean period between bursts

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The effect of a high pass filter on the measurements of the mean period between bursts is studied. The second mild maximum of the autocorrelation curve of the fluctuating streamwise velocity is not necessarily produced by bursting.

There has been considerable experimental work in the past few years on the process of turbulent production near the wall in a turbulent boundary layer. The visual study of Kim *et al.*<sup>1</sup> gave evidence of the existence of a certain production sequence. Although the details of the production sequence, called a burst by Kim *et al.*, still require further study, the average time interval between bursts has been measured using a number of different techniques. Kim *et al.* measured the mean time interval between bursts using visual observations of hydrogen-bubbles or dye in the fluid. They found that the mean time interval between visually observed bursts was nearly the same as the time lag required to obtain the second mild maximum in the curve of the autocorrelation coefficient of the fluctuating streamwise velocity near the wall. Rao *et al.*<sup>2</sup> measured the mean time interval between bursts using a complex procedure to process a hot-wire signal representing streamwise velocity fluctuations. They collected their own and previous results obtained over a wide range of Reynolds numbers and showed that the mean burst period  $\bar{T}$ , scaled with outer rather than inner boundary-layer flow variables, and that  $U_\infty \bar{T} / \delta^* = 32.2$ .

In addition to the difficulties of definitive identification of bursts in the work of Rao *et al.*<sup>2</sup> (see discussion in Lu and Willmarth,<sup>3</sup> Sec. VA); Rao *et al.* reported only one measurement of  $\bar{T}$  for really high Reynolds number flow ( $Re_\theta = 38\,000$ ). This was obtained from the time lag required to obtain the second maximum in the curve of the autocorrelation coefficient of the streamwise velocity near the wall measured by Tu and Willmarth<sup>4</sup> (see Fig. 6, for  $x_2/\delta^* = 4 \times 10^{-3}$  or  $yu^*/\nu = 6.2$ ). In Ref. 4 and in this research note we refer only to time averages of random variables measured over a long enough time period to allow them to attain a stationary value.

It can be demonstrated that the second maximum in the autocorrelation curve is not necessarily produced by bursting. Tu and Willmarth<sup>4</sup> stated that, in their measurements of pressure-velocity and velocity-velocity space-time correlations, a Krohn-Hite filter had been used to reject all frequencies below  $\omega\delta^*/U_\infty = 0.13$  (see p. 11, Ref. 4). Therefore, a study of the

effect of the high pass filter upon the autocorrelation measurement is desired. This has now been done both analytically and experimentally.

Analytically, the autocorrelation is obtained using the Fourier transform

$$R(T) = \int_0^\infty F(\omega) \cos(\omega T) d\omega, \quad (1)$$

where  $F(\omega)$  is the measured power spectrum. Willmarth and Wooldridge<sup>5</sup> have measured  $F(\omega)$  at  $yu^*/\nu = 162$  down to frequencies of the order of  $\omega\delta^*/U_\infty = 0.04$ , the lower limit of the wave analyzer, without filtering. Their measured power spectrum is shown in Fig. 1. Figure 2 shows the attenuation of the high pass Krohn-Hite filter used for the autocorrelation measurement of Tu and Willmarth<sup>4</sup> when the filter was set to reject frequencies below  $\omega\delta^*/U_\infty = 0.13$ .

The function

$$F(\omega) = B/[1 + (\omega/\omega_h)^2]^{1/2}; \quad (2)$$

with  $B = 2.75$ ,  $\omega_h = 2\pi \times 160 \text{ sec}^{-1}$ , and  $U/\delta^* = 2920 \text{ sec}^{-1}$

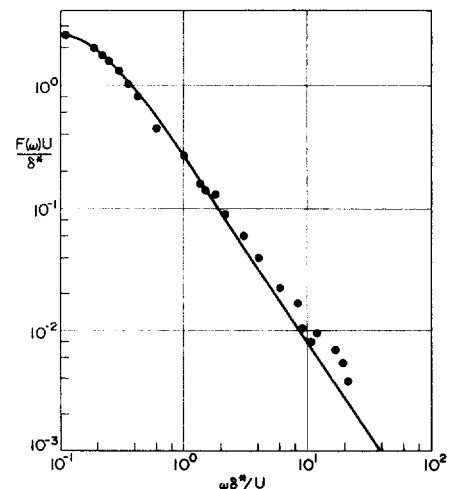


FIG. 1. The power spectrum of streamwise velocity fluctuations; ● as measured by Willmarth and Wooldridge (Ref. 5) at  $yu^*/\nu = 162$ ,  $Re_\theta = 38\,000$ . — as approximated by Eq. (2).

was used to fit the measured streamwise velocity spectrum of Ref. 5 at  $y^*=162$ . The integration, Eq. (1), was performed using Eq. (2). The result is displayed in Fig. 3. As can be seen, there is only one maximum, at  $T=0$ .

To assess the effect of the high pass filter the attenuation of the filter is approximated by

$$A(\omega) = 1/[1 + (\omega_1/\omega)^4]. \quad (3)$$

This is displayed in Fig. 2 (solid line). Then,  $R(T)$  was computed using

$$R(T) = \int_0^\infty A(\omega)F(\omega) \cos(\omega T) d\omega / \int_0^\infty A(\omega)F(\omega) d\omega. \quad (4)$$

and  $B^{-1}A(\omega)F(\omega)$  is shown in Fig. 2 (dashed line). The result is included in Fig. 3. The second maximum occurs at a time lag of  $U_\infty T/\delta^* \approx 35$ . This value is very close to the value of 32.2 for the mean burst period as suggested by Rao, *et al.*<sup>2</sup>

In addition, an fm tape recording of the streamwise velocity near the wall,  $yu^*/\nu = 6.2$ ,  $Re_\theta = 38\,000$ , from the work of Ref. 4, was still available. This was used to provide an unfiltered signal for which the autocorrelation coefficient could be measured using a Princeton Applied Research correlation function computer model 101. The results are also included in Fig. 3. It is seen that, without filtering, there is no second maximum

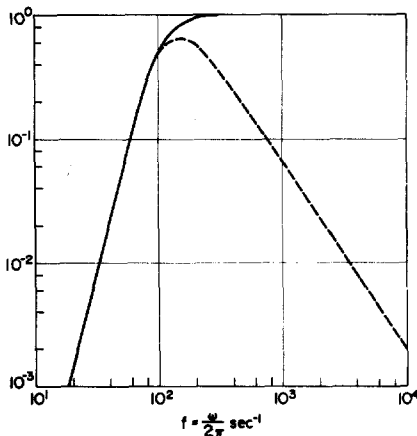


FIG. 2. The effect of the high pass attenuation on the power spectrum; — attenuation of Krohn-Hite high pass filter, Eq. (3),  $A(\omega)$ . --- power spectrum of Eq. (2) attenuated by the filter, Eq. (3),  $B^{-1}A(\omega)F(\omega)$ .

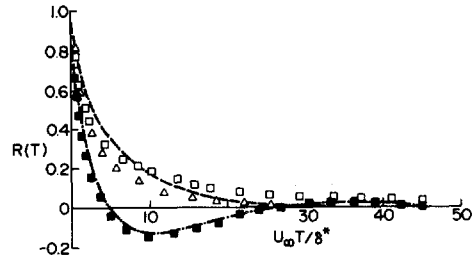


FIG. 3. The autocorrelation coefficients of the fluctuating streamwise velocity; --- computed from Eq. (1) using Eq. (2) (no filter); -- computed from Eq. (1) using Eq. (2) and Eq. (3) (with filter);  $\square$  measured at  $Re_\theta = 38\,000$ ,  $yu^*/\nu = 6.2$  (no filter);  $\blacksquare$  measured at  $Re_\theta = 38\,000$ ,  $yu^*/\nu = 6.2$  (with filter, high pass filter half-power point at  $\omega\delta^*/U_\infty = 0.13$ );  $\triangle$  measured at  $Re_\theta = 4230$ ,  $yu^*/\nu = 14$  (no filter).

in the autocorrelation curve. With a high pass filter, the same one used by Tu and Willmarth,<sup>4</sup> the second maximum does appear. Furthermore, the time interval required to obtain the second mild maximum has a value of  $U_\infty T/\delta^* \approx 35$ . Note that both analytical and experimental results are very close to each other for the case of filtered signal. Thus, the validity of the mean burst period at  $Re_\theta = 38\,000$  as quoted by Rao *et al.*<sup>2</sup> is doubtful.

Also included in Fig. 3 is an autocorrelation curve for a lower Reynolds number boundary layer flow ( $yu^*/\nu = 14$ ,  $Re_\theta = 4230$ ). The second maximum is not observed. Thus, it is doubtful whether the mean burst period can be inferred from the autocorrelation curve of the fluctuating streamwise velocity.

We are grateful to the referee for pointing out that the correlation function measured by Kim *et al.*<sup>1</sup> was measured using a relatively short averaging time.

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<sup>2</sup>K. N. Rao, R. Narasimha, and M. A. Badri Narayanan, *J. Fluid Mech.* **48**, 339 (1971).

<sup>3</sup>S. S. Lu and W. W. Willmarth, *J. Fluid Mech.* **60**, 481 (1973).

<sup>4</sup>B. J. Tu and W. W. Willmarth, Department of Aerospace Engineering, The University of Michigan Technical Report 02920-3-T (1966).

<sup>5</sup>W. W. Willmarth and C. E. Wooldridge, AGARD-NATO Technical Report 456 (1963).