LETTERS

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# Deformation of a free surface as a result of vortical flows

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The deformation of a free surface caused by the roll up of a vortex sheet below the surface is studied. The large amplitude motion depends on both the strength and depth of the vortex sheet. A distinction is made between three different scenarios of the free-surface motion: a breaking wave, entrainment of air, and the generation of relatively short surface waves.

The free-surface signature of an unsteady, submerged, vortical flow is familiar to any observer of a river or a ship's wake. Nevertheless, little information is available about the detailed interaction of vortical flow and a free surface. The reason, at least partly, is that other phenomena, such as waves and bores, are more dramatic and potentially more damaging to any object on the surface. However, recent advances in remote sensing techniques have led to an interest in the wake region of ships. The most visible surface mark left by moving objects is usually the wave pattern commonly referred to as a Kelvin wave, but observations by remote sensing techniques have also shown a narrower mark (or scar) that persists several hours after the ship is gone. The narrow angle rules out Kelvin wakes, and although it is not completely clear whether subsurface motion or an alteration in the composition of the surface water leaves the detectable mark, the cause is likely to be the fluid motion in the wake. The wake consists of rotational, turbulent, high Reynolds number flow, and coherent motion could last a long time.

Sarpkaya and Henderson[1] recently conducted experimental studies of the surface deformation resulting from the vortex system behind a lifting surface (with a negative angle of attack, so the trailing vortex pair moves upward) and Madnia and Bernal[2] are currently investigating the generation of surface waves as a result of a shear flow by considering a jet below a free surface. The experimental observations suggest that many competing and interacting processes are responsible for the observed patterns. Not only are surface waves generated by the vortical flow, but the waves also radiate energy away from the disturbance region and thereby affect the flow itself. Analytical investigations are, of course, limited to rather simple situations. Linearized solutions exist for a flow over a fixed vortex[3] and for a vortex moving freely under the free surface.[4] For the full nonlinear solutions, however, it is necessary to turn to numerical techniques. Considerable progress has been made in numerical simulations of the nonlinear evolution of large amplitude surface waves during the last decade. The most successful calculations generally use the boundary integral technique pioneered by Longuet-Higgins and Cokelet.[5] These methods now permit relatively routine simulations of two-dimensional large amplitude surface waves.[6,7] However, studies of the interaction of vortical flows with the free surface are just beginning. The motion of a pair of point vortices toward a free surface has recently been simulated numerically by Telste,[8] using a generalized vortex method. A strong vortex pair penetrates the surface, but weak vortices separate much as if the free surface were a rigid wall. Similar calculations have been done by this author and also by Baker, Meiron, and Orszag.[9]

Here we study the generation of surface disturbances by a submerged shear layer. The flow is assumed to be inviscid, incompressible, and two dimensional. To simplify the problem even more, and make the numerical setup easier, we assume periodic boundary conditions. We use a generalized vortex method developed by Baker, Meiron, and Orszag,[10] which is applicable to arbitrary stratification and employs an efficient iterative solution procedure. Both the free surface and the shear layer are modeled as vortex sheets. On the free surface, circulation is generated as prescribed by Bjerkness' theorem, but the circulation of any segment of the regular vortex sheet modeling the shear layer remains constant. The velocity of the free surface and the submerged vortical flow is found by integrating over both sheets. To prevent the regular vortex sheet from forming a singularity and to reduce the growth of short-wave instability, we use vortex blobs to represent the sheet. Krasny[11] has shown that vortex blob methods can produce smooth, well-behaved solutions for any finite blob size ($\delta$), in contrast to simulations that do not employ any regularization. The evolution in the vortex center will depend on $\delta$, but the effect on the large-scale structure is minimal. For further discussions of the vortex blob regularization the reader is referred to Krasny.[11] All our calculations have been thoroughly checked for accuracy and the effect of the blob size. These checks will be reported elsewhere.

The setup for our simulations is an initially flat free surface, and a vortex sheet perturbed slightly by a sine wave of small amplitude (equal to 0.03 in the calculations presented here) at a depth $d$ below the free surface (see Fig. 1). The

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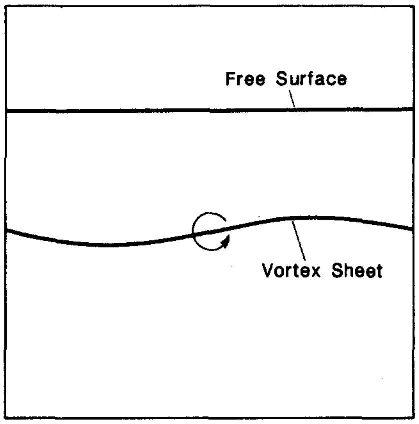


FIG. 1. The initial conditions. The free surface is flat, but the vortex sheet is given a slight perturbation. The flow regimes are specified by the relative depth of the vortex sheet, $d/l$, and $R = gl^3/\Gamma^2$.

flow of the fluid above the vortex sheet is from right to left, and from left to right below the vortex sheet. The average velocity, and the velocity of the vortex sheet, is therefore zero.

The governing nondimensional groups for this problem can be taken as the relative depth of the vortex sheet $d/l$ and a parameter related to the vortex sheet strength, $R = gl^3/\Gamma^2$, where $g$ is the gravity acceleration, $l$ is the wavelength of the basic perturbation, and $\Gamma$ is the circulation of one period of the vortex sheet. We have selected the characteristic velocity of the vortex sheet evolution as a velocity scale and the length of the vortex sheet perturbation as a length scale. A time scale is then given by $l^2/\Gamma$. The parameter $R$ can be related to a Froude number.

The evolution of the free surface depends on both the depth of the vortex sheet and its strength. In Fig. 2 the evolution is shown for $R = 4$ and $d/l = 0.3$. As the vortex sheet rolls up, a depression appears on the free surface, roughly above the vortex, and moves slightly downstream as the amplitude increases. At the same time the wave becomes steeper and eventually forms a sharp corner suggesting a breaking wave. Breaking waves of this type, where the breaking takes place below the crest of the wave, are sometimes referred to as "collapsing breakers" in the literature.[12] The calculation is terminated when the curvature of the wave crest becomes large compared to the resolution and the iterative solution technique fails. The possibility that the calculation could be continued by slightly smoothing the free surface has not been investigated. However, when the breaking is not too severe, a slight modification of the free surface at the breaking point might have a negligible influence on the rest of the solution and thus lead to acceptable (or useful) results. (The somewhat "jagged" appearance of the vortex sheet just outside the vortex center is caused by the fact that we have connected the points by straight lines. It does not indicate irregular motion as typically results from insufficient resolution.) For stronger and shallower vortex sheets the large amplitude motion of the free surface is somewhat different, in addition to developing faster relative to the time scale of the vortical motion. Figure 3 shows the motion for $R = 0.5$ and $d/l = 0.2$. The depression moves downstream faster, and instead of developing a steep upwind facing slope, the depression increases rapidly in depth and the vortex entrains the top fluid into the interior of the flow. We therefore refer to this as entrainment flow. For weak and deep vortex sheets neither entrainment nor wave breaking seems to take place. Figure 4 shows the free surface for different times and $R = 8$ and $d/l = 0.4$. Since the amplitude of the surface disturbance is much smaller in this case, the vertical dimension is amplified ten times. The evolution of the vortex sheet

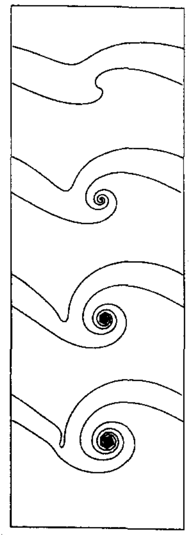

FIG. 3. Same setup as in Fig. 2, but $d/l = 0.2$ and $R = 0.5$. The times are 0.75, 1.0, 1.25, 1.45. We refer to this as entraining flow.

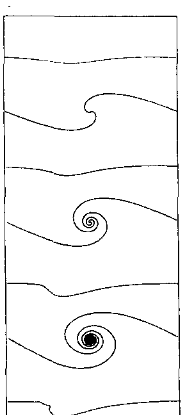

FIG. 2. The evolution of a vortex sheet below a free surface. Here $d/l = 0.3$ and $R = 4$. The nondimensional times are 0.75, 1.0, 1.25, 1.5. This flow regime is characterized by the breaking of the wave.

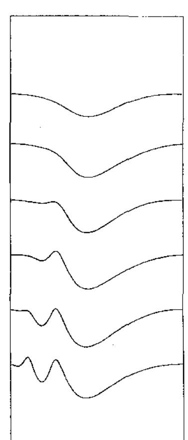

FIG. 4. The evolution of a free surface for $d/l = 0.4$, $R = 8$. The nondimensional times shown are 1.0, 1.25, 1.5, 1.75, 2.0, 2.25. The vertical dimension is amplified ten times. Here the evolution is governed by the formation of waves shorter than the period of the vortex sheet perturbation.

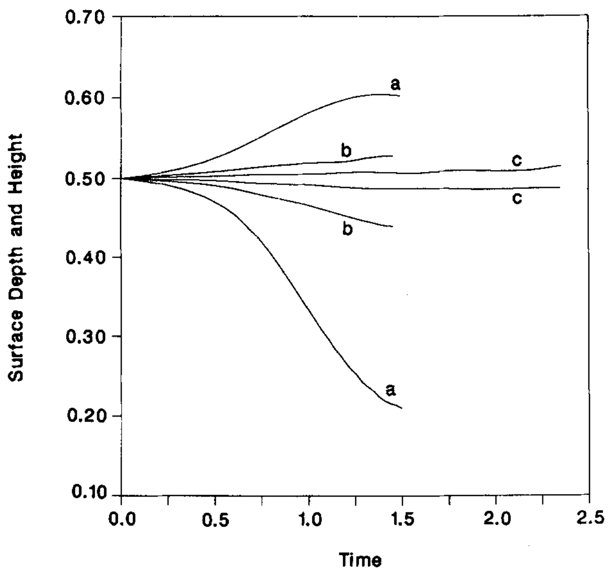


FIG. 5. The maximum and minimum elevation of the free surface for the runs in Figs. 2–4. The lines correspond to the following flows: a, entraining flow, Fig. 3; b, breaking-wave flow, Fig. 2; and c, short-wave flow, Fig. 4.

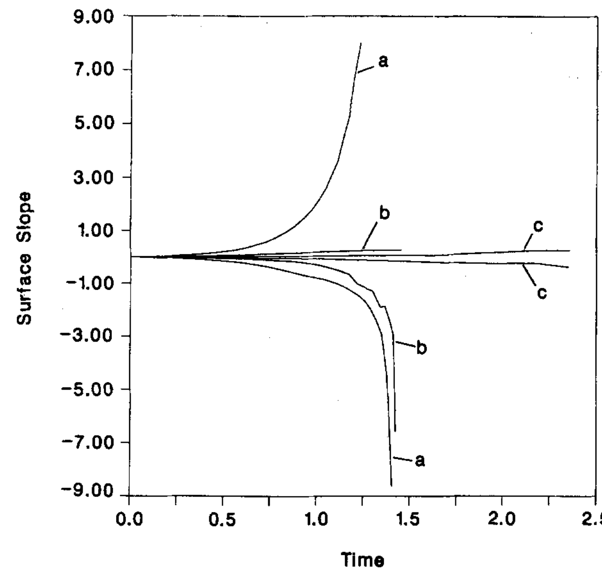


FIG. 6. The maximum and minimum slope of the free surface for the runs in Figs. 2–4. The letters identifying each curve are as in Fig. 5.

(which is not shown) is essentially identical to the one in Fig. 3. As in the previous calculations the vortical flow causes a depression on the free surface, but here the maximum depression remains almost stationary. At large times the single-wave characteristic of the previous runs is changed by the formation of smaller waves of shorter amplitude downstream. This might therefore be called the wave-generating flow or the short-wave regime. To quantify the evolution, the maximum and minimum elevation of the free surface is plotted versus time for these three cases in Fig. 5. As apparent from Figs. 2–4, the amplitude grows fastest for the entrainment case, and slowest for the short-wave case. When entrainment occurs, the depth of the trough increases considerably more rapidly than the amplitude of the crest and, while the crest amplitude levels off at later times, the trough depth continues to grow. In the breaking-wave case the growth of the crest and the trough are more similar, although the trough depth increases somewhat faster. For the short-wave case the growth is even more symmetric and, instead of being monotone as in the previous case, it is slightly oscillatory. Another diagnostic is presented in Fig. 6, where the maximum positive and negative slope of the interface is shown. For the entrainment flow both the slopes grow without bound as both sides of the trench become vertical; for the breaking-wave case the slope of the upstream facing side increases rapidly, but the slope of the other side levels off. For the short-wave regime both slopes increase only very gradually.

The examples shown here have been selected to emphasize the difference between the various scenarios. A denser sampling of the parameter space shows that although there are ranges of the parameter values where the evolution is essentially similar to those shown here, there is also considerably large overlap between those regions, and the transition between them is gradual. These transition zones and further diagnostics for entraining, breaking-wave, and short-wave flows, as well as details about the numerics, are forthcoming.

The results presented here are our initial attempt to understand the relation between unsteady vortical flow and its surface signature, and certainly fall short of fully explaining the surface signature of a ship's wake, which is the motivation for this study. One of the fundamental questions for this problem is how the length and time scales of the free-surface flow are related to the scales of the submerged flow. Although intuition and this study suggest that the length scale of the surface motion is representative of the scales of the vortex motion, our two-dimensional calculations of a single simple flow are not sufficient to establish the general relationship.

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