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A PERIODIC RADIOMETER FOR ELIMINATING DRIFTS

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ABSTRACT

It is shown that while the Pfund resonance radiometer reduces the effect of drifts by a factor of several hundred, its advantage over the conventional Moll amplifying system as regards the reduction of the effect of Brownian motion on the accuracy of the readings is only about a factor of three for a resonance radiometer system requiring 140 seconds for a single observation. For a Moll system and a resonant system of equal times of observation, the Moll system is about twice as good as regards Brownian error in the readings. A resonance radiometer loses both its Brownian advantage and its ability to reduce drifts, when designed for a short time of response.

A periodic radiometer particularly suitable for rapid recording is described, drifts being completely eliminated, while the time of response is only six seconds. Two condensers in series with the amplifying circuit effectually stop all drifts while passing the periodic deflections. By averaging visually a considerable number of deflections, the accuracy of the resonance radiometer for equal reading time can be approached so that the same instrument may serve both for rapid exploration of spectra and more careful determination of detail.

Infrared spectra of high resolution have been successfully recorded from a grating spectrometer and in amount of detail shown, closely approach the most carefully manually recorded curves.

Whenever an experimenter attempts to measure a very minute amount of radiant energy by its heating effect, as in infrared spectroscopy, his progress is continually slowed by the presence of drifts as well as Brownian motion and mechanical vibration of the indicating system. Drifts arise from continuous changes of temperature which, by causing a continuous change in the amount of radiation falling on the thermopile, give rise to a steady motion of the indicating galvanometer in one direction. This motion makes visual observations of deflections inaccurate or impossible and in the case of a recording system may cause the spot of light to pass off the record entirely. In the recording system, with drifts present, there is always an uncertainty as to the amount of deflection represented by a given position on the graph because the zero is wandering. When drifts are eliminated, Brownian motion and mechanical vibrations give rise to a zero unsteadiness which limits the accuracy of observations.

Both Brownian motion and drifts may be reduced by the Pfund resonance radiometer in which the radiation to be measured is periodically interrupted at the period of the two slightly damped galvanometers which are used in the amplifying and indicating system. The two galvanometers respond much more readily to impulses of the period to which they are tuned than to the irregular Brownian motion or to the slow drifts. However, the resonance radiometer is so sharply tuned that the steady state of oscillation is not reached till about thirty seconds after the radiation to be measured is applied, thus constituting a system with which observations can be taken but slowly. The writer was particularly interested in applying the resonance radiometer to a recording infrared prism spectrograph which was to be used for rapid exploration of unknown emission and absorption spectra. For such exploratory work, the long time constant of the usual resonance radiometer would not be practical, so an effort was made to determine theoretically how such an instrument of short time constant, say 6 or 8 seconds, might be built without too great sacrifice of its virtues. This theoretical study showed that in regard to the reduction of the influence of both drifts and Brownian motion, the long time constant is essential to the resonance radiometer; that such an instrument, when built with short time of response, possesses no advantages over the conventional Moll type of amplifier. A new type of amplifying system was therefore devised which differs from the resonance radiometer in that the two galvanometers are approximately critically damped, the drifts being entirely removed by the amplifying system which connects the two galvanometers. The time of response is of the order of 6 seconds; the new system suffers twice as much Brownian error as the Moll system of equal time of response but this deficiency can easily be made up by visually averaging the automatic record.

The first part of this paper is devoted to a consideration of the theory of the resonance radiometer as presented by Hardy (J. D. Hardy, R. S. I. Aug. 1930, pp. 429); to certain minor corrections to his paper and to an extension of his theory to cover the case of a rapid instrument such as one might wish to use for recording. In the second part, the periodic radiometer for eliminating drifts is described. It is not necessary to read the first part in order to understand the second.

PART I. THEORY OF THE RESONANCE RADIOMETER

LIST OF SYMBOLS USED

ϕ, angular deflection of a galvanometer coil in radians.
E, emf in emu (electromagnetic units).
r, total resistance of galvanometer and external circuit in emu.
I, moment of inertia of galvanometer coil in gram cm^2.
D, torsion constant of coil suspension in dyne cm per radian.
f, resisting torque per unit angular velocity due to mechanical resistances in dyne cm per radian per sec.
p_0, 2π times the natural frequency of the galvanometer, radians per sec.
t, time to reach 97% of a final deflection, sec.
k, gas constant, 1.372×10^{-16} erg deg.$^{-1}$ per degree of freedom.
T, absolute temperature, deg.
β, torque per unit current in dyne cm per emu of current.
Δ, damping factor, sec^{-1}.
τ, period, sec.

The experimental arrangement of the resonance radiometer is shown in Fig. 1 which is reproduced from Hardy's paper. Radiation from the

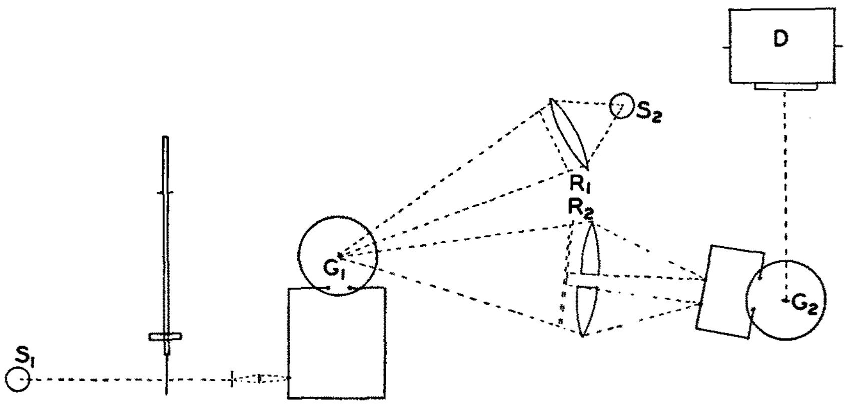

FIG. 1. *The experimental arrangement of the resonance radiometer.*

source S_1, is interrupted periodically by the pendulum, whose period is adjusted to the natural period of the under-damped galvanometer G_1, to which the thermopile is connected. The oscillating deflection of G_1 as built by Hardy is about twenty times as large as the steady deflection which would be produced by the same amount of radiation. Here already is a factor of twenty discrimination against drifts. These deflections of G_1, are now amplified by a beam of light from S_2 which, after passing through appropriate apertures, actuates a double thermocouple arrangement which is connected to the second galvanometer G_2 tuned also to the natural period of G_1. Another factor of twenty discrimination against drifts is achieved in G_2, thus giving the entire instrument a factor of 400 between the deflection produced by a given energy of interrupted radiation and the steady deflection produced by the same energy of steady radiation. Drifts are therefore practically eliminated by this resonance radiometer which requires about 30 seconds to reach 97 percent of its final deflection.

Let us now consider the Brownian motion of the coil of the primary galvanometer G_1, forgetting for the moment the part played by the amplifier and tuned secondary galvanometer G_2. The coil of the primary galvanometer G_1, suffers a Brownian motion due to the combined effect of bombardment by air molecules, thermal motion of the electricity in the circuit, and random fluctuations of temperature of the thermopile junctions due to thermal interchange of energy with their surroundings. It has been shown by de Haas-Lorentz (G. L. de Haas-Lorentz, Die Brownsche Bewegung, Friedr. Wieweg & Sohn, 1913) that the combination of these several causes of Brownian motion results in an amount of motion of the galvanometer coil which is the same as would be produced by the action of any one of them separately. In a system such as a galvanometer coil, which is elastically bound to a position of equilibrium, the amount of the Brownian motion of the coil is such that in the degree of freedom under consideration the mean kinetic energy and mean potential energy are each equal to the mean kinetic energy of thermal agitation of a molecule of air per degree of freedom.

$$\tfrac{1}{2}I\overline{\dot{\phi}^2} = \tfrac{1}{2}D\overline{\phi^2} = \tfrac{1}{2}kT. \tag{1}$$

From this the root mean square deflection of the galvanometer due to Brownian motion, which for brevity we may call the mean Brownian deflection, is seen to be

$$\sqrt{\overline{\phi^2}} = \sqrt{\frac{kT}{D}}\,. \tag{2}$$

This represents a sort of probable error of a single observation since in taking a reading it is equally probable that a Brownian deflection of the above order of magnitude may either add itself to or subtract itself from our reading.

The above equations hold regardless of the amount of damping which may be present. From equation 2 we see that for a given temperature, the mean Brownian deflection depends only on D, the torsion constant of the suspension; therefore, any changes which can be made in the design or operation of the galvanometer which will increase the deflection due to the emf to be measured, without changing D, will correspondingly decrease the error in the measurements due to the Brownian motion. For instance, one may change the moment of inertia of the coil or the strength of the magnetic field and, although the form of the Brownian motion or its frequency characteristic may change, its root mean square value will remain the same so long as D remains unchanged. It will be shown later that by weakening the magnetic field of a galvanometer below its critical damping value and periodically interrupting the actuating emf, a larger double amplitude of oscillating deflection will be obtained than the steady deflection produced by a steady emf of the same value acting on the same galvanometer with its magnetic field adjusted for critical damping. This swinging factor, as it might be called, is about 2.87 for a galvanometer of 3.8 sec period and such small damping that it would require 140 seconds to arrive at 97 percent of its final deflection. (This is less damping than Hardy was able to attain.) Thus, since the swinging deflections are 2.87 times as large as the steady deflection would be under normal damping conditions and since the mean Brownian deflection is the same in both cases, there would be correspondingly less Brownian error in the swinging deflections than in the steady. This constitutes the principal source of the Brownian advantage of the resonance radiometer; the fact that the secondary galvanometer is also sharply tuned and therefore tunes out some of the Brownian motion of the primary galvanometer is only a slight advantage and except for its assistance in reducing drifts, the secondary galvanometer might as well be critically damped.

The above assertions will now be supported by computations. For convenience, let us define the "voltage change" of a pulsating or alternating emf as the difference between the algebraic maximum and minimum values of that emf; thus the voltage change of a sinusoidal alternating emf is twice the maximum value and is $2\sqrt{2}$ times the effective value. (Somewhat inconsistently, the voltage change will be expressed in electro-magnetic units.) In comparing a resonance radiometer system with a steady deflection system it is necessary to remember that if a given amount of radiation falling continuously on a thermopile will cause it to generate 1 microvolt, the same radiation periodically interrupted will give rise to a pulsating voltage whose voltage change will be slightly less than 1 microvolt because of the lag of the thermopile. Furthermore, the fundamental harmonic component of this pulsating voltage wave, this being the only component to which the resonance radiometer responds, will usually have a voltage change somewhat less than the voltage change of the pulsating voltage from which it is derived. However, it will be assumed that in the above case a fundamental harmonic component of 1 microvolt of voltage change will be obtained, this representing a value which will be merely approximated experimentally; the effective or root mean square value of this component would be $\frac{1}{2}\sqrt{2}$ microvolts.

Let us first compute the swinging factor, that is, the ratio between the double amplitude of the oscillating deflection produced by a periodic emf when the magnetic field strength is adjusted to give the largest swing, and the steady deflection which would be produced by a steady emf of equal voltage change acting on the same galvanometer after the magnetic field had been increased so as to give critical damping. Hardy has shown that a resonant galvanometer has its maximum sensitivity to periodic emf's when the magnetic field has been so weakened that the electromagnetic damping is equal to the damping by mechanical causes and in that condition the voltage sensitivity is

$$\frac{\delta\phi}{\delta E} = \frac{1}{p_0}\sqrt{\frac{1}{2rf}} \text{ when } \frac{B^2}{r} = f. \tag{3}$$

In this equation ϕ is the amplitude (half the full swing) measured in radians while E is the effective value of the simple harmonic impressed emf in electromagnetic units. In order that this equation can be compared with the sensitivity to steady deflections let it be rewritten using the double amplitude ϕ_c and the voltage change E_c

$$\frac{\delta\phi_c}{\delta E_c} = \frac{2\delta\phi}{2\sqrt{2}\delta E} = \frac{1}{2p_0}\sqrt{\frac{1}{rf}}\,. \tag{4}$$

This is the double amplitude per unit voltage change of the periodic emf. Considering now the sensitivity of the same galvanometer to steady emf's when the field is adjusted for critical damping

$$\frac{\delta\phi_s'}{\delta E_s} = \frac{B}{rD} = \sqrt{\frac{2}{rDp_0}} \tag{5}$$

since it can be shown that for critical damping

$$B = \sqrt{\frac{2rD}{p_0}}\,.$$

Equation 4 divided by equation 5 gives the swinging factor

$$= \sqrt{\frac{D}{8p_0 f}} \cdot \tag{6}$$

Hardy gives as the constants of his galvanometers $D = 0.0684$ dyne cm rad^{-1}, $B = 3.22 \times 10^3$ dyne cm per emu of current, $r = 20 \times 10^9$ emu, $f = 6.4 \times 10^{-4}$ dyne cm rad^{-1} sec., $p_0 = 1.64$ rad. sec.$^{-1}$. There is probably an error of about a factor of three in one of the above constants since these constants would give a galvanometer having a time of response of 140 sec. instead of the 30 sec. which may be observed in Hardy's Fig. 5 and would give a sharper resonance curve than is indicated in his Fig. 6. Therefore in computing the advantages of the resonance radiometer from the constants given above we will obtain a larger factor of advantage than was actually achieved in his instrument as built. Putting these constants into equation 6 we find the swinging factor to be 2.87. Thus by waiting 140 sec. to take one reading, a roughly three fold greater deflection can be obtained by interrupting the radiation and weakening the field of the galvanometer than by taking steady deflections with the field adjusted for critical damping. Since the Brownian motion would be of the same amount in both cases, this factor of three is the advantage of this resonance radiometer in overcoming the effect of the Brownian motion on the readings, except in so far as the tuned secondary galvanometer may assist, which assistance will be shown to be of but small value.

Since the Brownian advantage of the resonance radiometer comes at the expense of an increase in time of observation, let us compute the Brownian limit of the resonant primary galvanometer G_1, in terms of its time of observation and compare it with the Brownian limit of the usual critically damped galvanometer also expressed in terms of its time of observation. When the periodic emf is applied to a resonant galvanometer, the final amplitude of swing is arrived at exponentially and Wenner (F. W. Wenner, Bull. Bureau Stand., *6*, 347; 1909) has shown that the time required for the amplitude to build up to $(1 - e^{-1})$ or 63 percent of its final value is

$$t_i = \frac{2I}{f + \frac{B^2}{r}} \cdot$$

For the case of maximum sensitivity, $B^2/r = f$, and since $p_0^2 = D/I$ this becomes

$$t_i = \frac{D}{p_0^2 f}.$$

If we take as the time of observation t, the time required to reach 97 percent of the final deflection, we find by referring to a table of exponentials that

$$t = 3.5t_i = \frac{3.5D}{p_0^2 f}. \tag{7}$$

The root mean square deflection due to Brownian motion is

$$\sqrt{\overline{\phi^2}} = \sqrt{\frac{kT}{D}}$$

and the double amplitude of a simple harmonic deflection having the same energy as the Brownian motion would be

$$2\sqrt{2}\sqrt{\overline{\phi^2}} = 2\sqrt{\frac{2kT}{D}} \cdot \tag{8}$$

Taking the above deflection of equation 8 in conjunction with equation 4 for the sensitivity of the resonant galvanometer, we obtain the Brownian voltage E_B, that is, the voltage change of the periodic emf required to produce a periodic deflection of the galvanometer having an energy equal to the energy of the Brownian deflections.

$$E_B = 4p_0\sqrt{\frac{2kTrf}{D}} \cdot \tag{9}$$

Now bringing in the time of observation and eliminating f from the above equation with the aid of equation 7

$$E_B = 4\sqrt{\frac{7kTr}{t}}. \tag{10}$$

This assumes that the condition of maximum sensitivity, $B^2/r = f$ has been met. E_B is the order of magnitude of the least voltage change that can be detected. It is thus seen to vary as $1/\sqrt{t}$. Let us now make a similar computation for a critically damped galvanometer. The root mean square Brownian motion will of course be the same as before, $\sqrt{kT/D}$ and we assume that to be equally observable, the steady deflection of the critically damped galvanometer must equal the double amplitude of the swing of the resonant galvanometer as given in equation 8. Using the deflection of equation 8 along with the sensitivity of a critically damped galvanometer as given in equation 5 we find for the Brownian voltage in this case

$$E_B' = 2\sqrt{kTrp_0}\,. \tag{11}$$

For a critically damped galvanometer 97 percent of the deflection has been attained in a time equal to the undamped period; consequently the time of observation is[1]

$$t' = \frac{2\pi}{p_0}\,. \tag{12}$$

Eliminating p_0 from 11 with the aid of 12

$$E_B' = 2\sqrt{\frac{2\pi kTr}{t'}}\,. \tag{13}$$

This is the steady emf required to produce a steady deflection equal to the double amplitude that would be produced by the Brownian motion if it were simple harmonic in character. E_B' is of the order of the least steady voltage which can be detected by a critically damped galvanometer by a single observation; it, also, varies as $1/\sqrt{t'}$. From equations 10 and 13 we can find the Brownian advantage of the resonant galvanometer over the critically damped type in terms of their times of observation.

$$\frac{E_B'}{E_B} = 0.47\sqrt{\frac{t}{t'}}\,. \tag{14}$$

From this it is seen that for equal times of observation[2] the Brownian motion introduces only about one half as much error into the measurement of small voltage with the critically damped system as with the resonant system; the resonance radiometer is actually at a disadvantage as regards Brownian error. Thus to surpass the resonance radiometer, one merely adds sufficient moment of inertia to the galvanometer coil to secure the desired slowness of response and then strengthens the mag-

[1] While in all these computations it is assumed that the time of an observation with a critically damped galvanometer is equal to its period, practically the observation time is longer than the period. In any case where energy is so scarce that Brownian motion must be considered, drifts will be encountered of such magnitude that the zero must be checked both before and after each reading, so the time of observation is twice the period. With small energy, drifts make any continuous recording of energy with a critically damped steady deflection system impossible.

[2] See previous footnote.

netic field to obtain critical damping. This strengthening of the magnetic field will increase the sensitivity by a greater amount than the increase on passing over to the resonance principle and will therefore aid in riding over the Brownian motion which is of the same amount in both cases. However, as the time of observation of a critically damped system is increased, the influence of drifts becomes correspondingly greater so that experimentally the resonance radiometer might be better even though Brownian motion disturbs it twice as much.

Those who have used the resonance radiometer say that although the zero unsteadiness due to the Brownian motion may be 10 mm, successive adjacent swings on a fairly large deflection will repeat to 1 mm. The writer believes this to be only an illusory accuracy as there are slow changes in the amplitude of large swings which will probably be as large as 10 mm; these slow changes have been ascribed to lack of exact tuning of the galvanometers though the theory would indicate that a steady state would soon be reached for any condition of tuning. The writer believes these slow variations of amplitude to be the Brownian motion; it seems reasonable that if it takes a considerable time for an applied periodic voltage to build up a considerable amplitude of swing it would take the same order of time for the Brownian forces to produce a modification of the amplitude.

As to the factor by which the resonance radiometer reduces drifts, the sensitivity of the resonant galvanometer to steady emf when adjusted to maximum sensitivity to periodic emf's is, since $B^2/r = f$

$$\frac{\delta\phi_s}{\delta E_s} = \frac{B}{rD} = \frac{1}{D}\sqrt{\frac{f}{r}}\,. \tag{15}$$

The best sensitivity to periodic emf's as given by equation 4, divided by the sensitivity to steady emf's under the same damping condition as given by equation 15, makes the drift reduction factor equal $D/2p_0f$ per galvanometer. A galvanometer of constants as given above would have a drift reduction factor 32.5 while Hardy observed but 20, this being additional evidence of an error in the constants as given. The primary and secondary galvanometer together would reduce drifts about a factor of 900.

Turning now to the consideration of that part of the resonance radiometer system which follows the primary galvanometer, this amplifying part serves the double function of increasing the size of the original deflections so as to make them easily readable and of tuning out a small amount of the Brownian motion of the primary galvanometer; the amount of this latter effect is to be evaluated. While the amount of Brownian motion passing through the tuned amplifying system may be computed analytically as was done by Hardy, the physical situation is more easily comprehended by the use of a graphical method of solution similar to that used by Hull and Williams, (A. W. Hull and N. H. Williams, Phys. Rev. *25*, 147; 1925) graphical accuracy being sufficient for our purpose. It has been shown by Uhlenbeck and Goudsmit (G. E. Uhlenbeck and S. Goudsmit, Phys. Rev. *34*, 145; 1929) that the irregular forces which give rise to the Brownian motion may be analyzed into a Fourier series in which the amplitudes of all the terms are equal, that is, the originating force is equally potent at all frequencies. The mean potential energy of the resulting motion of the primary galvanom-

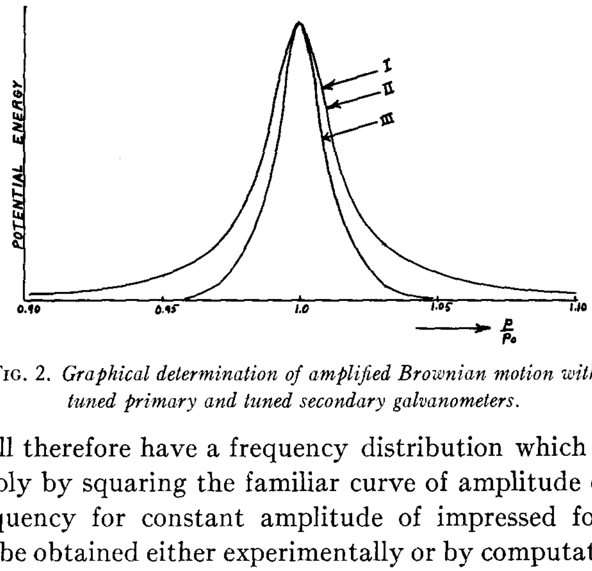

FIG. 2. *Graphical determination of amplified Brownian motion with tuned primary and tuned secondary galvanometers.*

eter coil will therefore have a frequency distribution which can be obtained simply by squaring the familiar curve of amplitude of response versus frequency for constant amplitude of impressed force, which curve may be obtained either experimentally or by computation. Fig. 2, curve *I*, shows such an energy curve computed, for a galvanometer of the constants given above, from the expression

$$\text{potential energy} \propto \frac{1}{\left\{1 - \left(\frac{p}{p_0}\right)^2\right\}^2 + \frac{4p^2f^2}{D^2}\left(\frac{p}{p_0}\right)^2}$$

which comes directly from the usual resonance considerations assuming that $B^2/r = f$ so that the total damping is $2f$. Ordinates on curve *I* are proportional to the mean potential energy of the Brownian motion of the galvanometer coil in that frequency region. Therefore, the total area under curve I, most of which area lies in the vicinity of the resonance frequency, represents the total mean potential energy of the Brownian motion produced, which we know to be equal to $kT/2$. This composite motion of the primary galvanometer coil actuates the thermoelectric amplifier, whose frequency characteristic may be neglected, and thence is applied to the resonant secondary galvanometer. For ease of computation we may assume that the amplification factor of this final system is unity at the resonant frequency. At other frequencies, the energy amplification factor of this secondary system will be found by squaring the curve of amplitude response versus frequency for constant impressed force, thereby obtaining curve II which happens to coincide with curve I since primary and secondary galvanometers are alike. Now multiplying the primary galvanometer energy curve I by the secondary galvanometer energy amplification curve II we obtain curve III, whose area, when compared with the area under curve I, gives the fraction of the original Brownian energy $kT/2$ which has passed through the tuned amplifying system. By plotting these curves on coordinate paper and determining the area under them by counting squares, it was found that the amplified Brownian energy was 0.56 of the primary galvanometer energy assuming unity amplification factor for the resonant frequency. Since the mean square deflection varies as the square root of the mean potential energy it will be reduced by the tuned amplifying system to $\sqrt{0.56}=0.75$ of its value in the primary galvanometer. This proves our previous assertion that the Brownian motion of the primary galvanometer is not materially reduced relative to the deflections by the use of the tuned secondary galvanometer. Even the 25% reduction comes at the expense of increased time of observation due to the lag of the secondary galvanometer. The primary galvanometer causes most of the Brownian motion to lie in the neighborhood of its resonance frequency so that it passes well through the tuned secondary galvanometer, there being but little energy at those frequencies which the secondary galvanometer might effectually stop.

This suggests that it might be an advantage to use a critically damped primary galvanometer, whose Brownian energy would be well distributed in frequency, and a sharply tuned secondary galvanometer which would tune out most of the Brownian energy of the primary. A graphical solution for this case is shown in Fig. 3 where curve I is the frequency distribution of the Brownian energy of a critically damped primary galvanometer, curve II is the energy amplification curve of a tuned secondary galvanometer of the same constants as before, and curve III

is the product of I and II. The area under curve III represents the amount of the primary Brownian energy which passes through the secondary galvanometer and by counting squares on coordinate paper it was found to be but 0.0130 of the area under curve I which is $kT/2$. Since the root mean square deflection of the secondary galvanometer due to the Brownian motion of the primary is proportional to the square root of the energy of the secondary's motion, the latter will have a root mean square deflection only $\sqrt{0.0130}=0.114$ that of the primary, a very decided advantage. However, the sensitivity of this critically

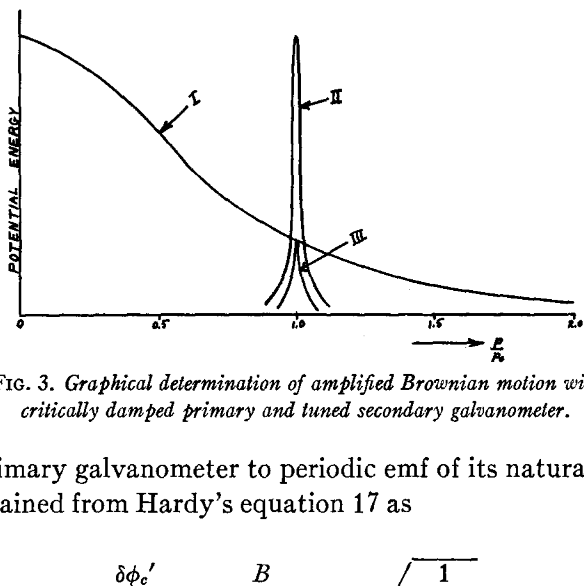

FIG. 3. *Graphical determination of amplified Brownian motion with critically damped primary and tuned secondary galvanometer.*

damped primary galvanometer to periodic emf of its natural frequency may be obtained from Hardy's equation 17 as

$$\frac{\delta\phi_c'}{\delta E_c}=\frac{B}{p_0(rf+B^2)}=\sqrt{\frac{1}{2rDp_0}}$$

since for critical damping $rf \ll B^2$ and

$$B=\sqrt{\frac{2rD}{p_0}},$$

while its sensitivity when adjusted to maximum by making $B^2/r=f$ is given by our equation 4. Dividing the tuned sensitivity by the critically damped sensitivity to periodic emf we obtain $\sqrt{D/2\ p_0 f}$ which with the same constants as before is equal to 5.70. Thus while the system with critically damped primary galvanometer passes less Brownian motion by a factor of $0.75/0.114=6.6$, the system with both galvanometers resonant is 5.7 times as sensitive to the periodic deflections which it is desired to measure, so that they are about equally good as regards Brownian error in the readings. The advantage, if any, is in favor of the critically damped primary galvanometer system especially since it would be somewhat faster, having only one tuned galvanometer to set into motion. The above computations assume that all the galvanometers considered were of the same period. One might use a very slow critically damped primary galvanometer along with a tuned secondary galvanometer of the usual period which we have considered, the radiation being interrupted at this latter period. But a similar computation shows that even in this case the same amount of Brownian error will appear in the readings as in the previous cases. The fact that much the same limit is reached in these various arrangements may indicate that there is a limit of accuracy prescribed by thermodynamic laws which cannot be surpassed with any system of given time of observation.

While the influence of the amplifying system on the Brownian error can be determined graphically as shown above, the Brownian error of the primary galvanometer alone under any conditions of damping can be simply computed from the formulas given below which are easily derived from Hardy's equation (17). These formulas give the root mean square value of the periodic emf of the natural frequency of the galvanometer, which will produce a periodic deflection whose root mean square value is equal to the root mean square Brownian deflection, $\sqrt{\overline{\phi^2}}$. All quantities are measured in emu except as noted.

$$E_{Brms} = \sqrt{\frac{kT}{D}}\,\frac{p_0(rf+B^2)}{B}.$$

If

$$rf = B^2,$$

$$E_{Brms} = 2p_0B\sqrt{\frac{kT}{D}} = 2p_0\sqrt{\frac{rfkT}{D}}.$$

If $rf \ll B^2$, as when near critical damping,

$$E_{Brms} = p_0B\sqrt{\frac{kT}{D}} = \sqrt{2rkT\Delta}$$

where the damping factor

$$\Delta = \frac{B^2}{2rI} = \frac{p_0{}^2B^2}{2rD} \text{ as in } e^{-\Delta t}.$$

Putting in the value of kT and changing to practical units we have when not too far from critical damping

$$E_{Brms}(\text{volts}) = 0.896 \times 10^{-10}\sqrt{r\,(\text{ohms})\,\Delta\,(\text{sec}^{-1})}. \qquad (16)$$

At critical damping $\Delta = p_0 = 2\pi/\tau$ where τ is the natural period, then

$$E_{Brms}(\text{volts}) = 2.24 \times 10^{-10}\sqrt{\frac{r\,(\text{ohms})}{\tau\ \text{sec}}}.$$

This last equation is seen to be consistent with that developed by Ising for steady deflections (Lond. Phil. Mag. 1, 826; 1926), when it is remembered that a given voltage at zero frequency produces twice as much deflection of a critically damped system as the same voltage at the natural frequency would produce. Ising's equation had a numerical coefficient of 1.12 instead of 2.24 so we see that a periodic radiometer which uses critically damped galvanometers has twice the Brownian error of a Moll system of equal time of response.[1]

The conclusions relative to the resonance radiometer are given in the abstract above.

PART II. THE PERIODIC RADIOMETER FOR ELIMINATING DRIFTS

The periodic radiometer differs from the resonance radiometer in that both primary and secondary galvanometers are critically damped, thereby securing rapid response, and the amplifying part is photoelectric instead of thermoelectric, using a single stage of vacuum tube amplification. It is similar to the resonance radiometer in that the radiation is periodically interrupted. Theoretically, one can eliminate the drifts from any radiometer system merely by periodically interrupting the radiation and connecting a large condenser in series with the galvanometer. Practically, however, any ordinary size of condenser to be found in the laboratory will have so much impedance at this low frequency that its presence in the galvanometer circuit would seriously impair the sensitivity. By using a photoelectric cell and vacuum tube amplifier, circuits of sufficiently high resistance are encountered so that the impedances of the series condensers at the working frequency are not of serious consequence, while their presence effectually bars the drifts.

A diagram of the arrangement of the periodic radiometer is given in Fig. 4. The electromagnetically maintained pendulum P actuates the electromagnetic shutter S which thus periodically interrupts the radia-

[1] Again see the previous footnote.

tion from the source s as it enters the recording spectrograph H. After being dispersed by the optical system, a certain wave length is brought to a focus on the thermopile T which is connected to the primary galvanometer G_1. G_1 is a high sensitivity low resistance galvanometer of 3.6 sec. period adjusted approximately to critical damping. Light from the 32 candle power headlight bulb B passes through the lens l_1 and the one inch square aperture A_1 and falls on the mirror of the galvanom-

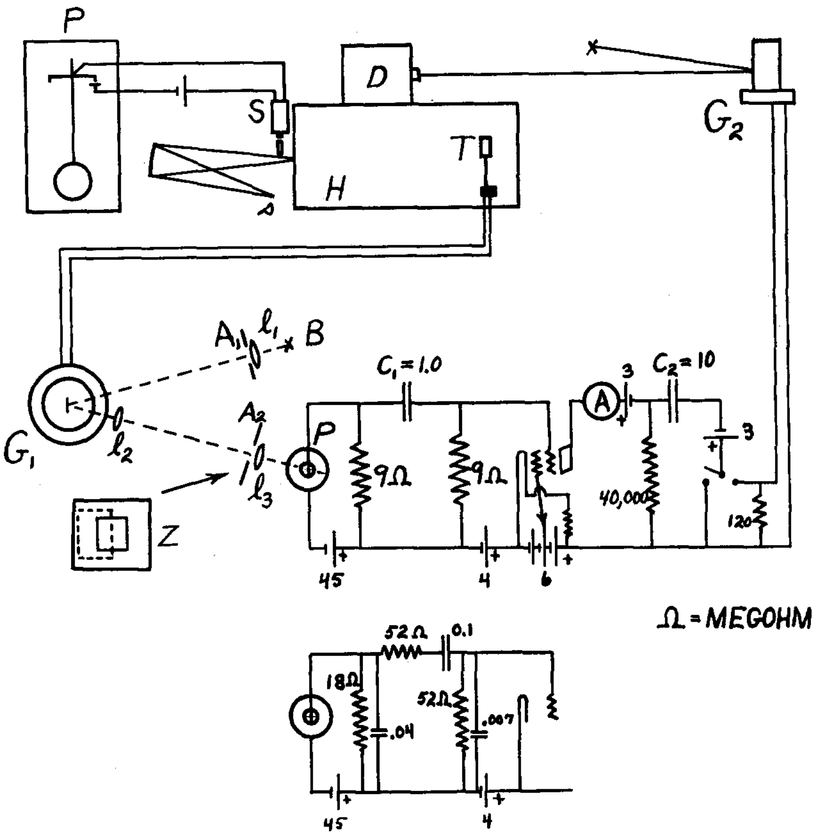

FIG. 4. *Diagram of periodic radiometer showing alternative input circuit.*

eter G_1. The lens l_2 forms an image of A_1 on the 5/8 inch square aperture A_2, the mirror of the galvanometer being so turned that the edge of the image of A_1 falls about on the center of the aperture in A_2 as shown at Z, so that as the galvanometer mirror turns, the amount of light passing through the lens l_3 to the photocell P, varies linearly. The lens l_1 forms an image of the filament B on the galvanometer mirror, l_2 forms an image of A_1 on A_2, and l_3 forms an image of the filament image at the mirror, on the active surface of the photocell so that as the mirror turns, the brightness of the image in the photocell varies but its position is unchanged, this giving uniform sensitivity.

The amplifying circuit is understandable from the diagram. The photocell is a vacuum type. Condensers C_1 and C_2 were chosen to have high dc resistances, the 10 mfd C_2 having about 300 megohms. The vacuum tube is the General Electric FP-54 which was designed for measuring very small currents. Its advantage for the present purpose is that having very low plate voltage (9 volts), the plate current is extremely steady so that no serious unsteadiness is produced in the high sensitivity (though low resistance) critically damped galvanometer G_2 (1.0 sec. period) which makes the record on the drum D. According to Bennett (R. D. Bennett, Rev. Sci. Inst., 1, 446; 1930) a conventional UX-222 screen grid tube may be used in a similar manner. As the plate resistance of the FP-54 is 40,000 ohms, the 10 mfd condenser C_2, which at a frequency of 1/3.6 cycles per second has an impedance of 57,000 ohms, does not seriously reduce the alternating current through the secondary galvanometer G_2 although it effectively stops the flow of direct current. The battery in series with C_2 serves to reduce leakage and may not be necessary. The switch at the output discharges C_2 and serves to protect the galvanometer G_2 when the amplifier is turned on or off. All connecting wires to the galvanometers are run in a grounded copper sheath and the amplifier is enclosed in a metal shield. The light path to and from the primary galvanometer must be enclosed so as to prevent the circulation of air currents as these produce scintillations which result in an unsteadiness of the secondary galvanometer. Also, for steadiness, the current through the lamp B must be very constant.

If G_1 drifts at a constant rate, the charge on C_1 will vary at a constant rate, causing the plate current of the tube to remain fixed at a value slightly different from its normal value, but on account of C_2, the secondary galvanometer G_2 will not move from its normal position. If the rate of drift of G_1 is changing at a constant rate, G_2 will be slightly displaced from its normal position but will not be in motion. The zero position of G_2 has never been observed to move more than a few millimeters in the course of a day even though G_1 may be drifting badly. Practically the only way in which a drift can affect the readings is by causing a change of amplification as the normal position of the image of A_1 moves across A_2. Constancy of amplification requires uniformity of luminous flux in various parts of the image of A_1 and linearity of response of the photocell and its circuit. These conditions are not difficult to fulfill to a considerable accuracy.

The time constant of the grid condenser C_1 and its associated resistances is so long that the slow transients in this circuit are not repeated through C_2 and therefore this input circuit does not appreciably delay the attainment of the steady state in the deflections. The time constant of C_2 and its circuit is so short that it does not appreciably delay the readings. The steady state is reached in about six seconds.

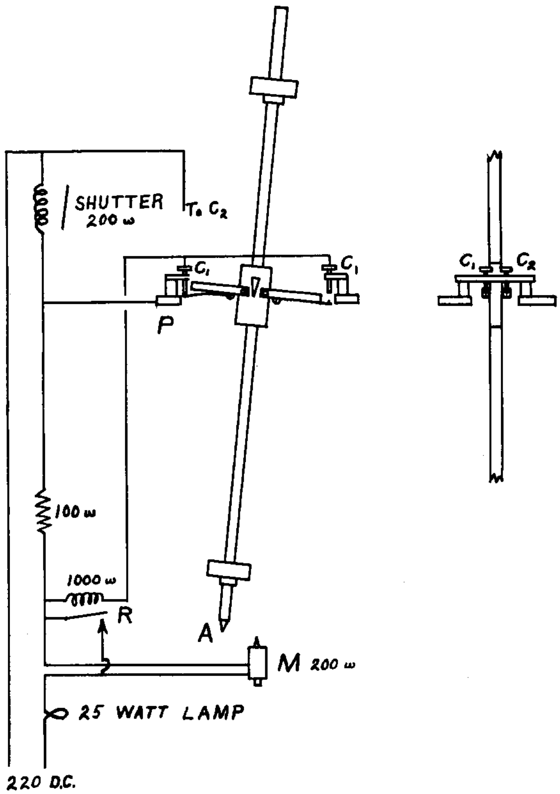

FIG. 5. *Maintained pendulum for periodically operating the electromagnetic shutter.*

Due to mechanical vibrations, the coil of G_1 swings as a pendulum with a period of 0.55 sec. while its natural period of torsional deflection is 3.6 sec. Due to asymmetry of the coil this pendular swing deflects the light beam and is thus amplified into a zero unsteadiness of G_2. By using the input circuit shown in the lower part of Fig. 4 it was possible to discriminate against this rapid pendular frequency by a factor of 25 and thus reduce its effect in G_2 to negligible proportions.

A convenient style of pendulum for controlling the periodic shutter is shown in Fig. 5. The motion of the pendulum is maintained, without greatly influencing its period, by means of the magnets *M* which impel the pendulum as it passes through its equilibrium position. Current flows through these magnets continuously except that when the armature *A* approaches to within about 1/4 inch of the magnets, they are shorted by the relay *R*, the short being removed when the armature has passed beyond the magnets about 1/4 inch. Due to a slight lag in the operation of the relay, the magnets give a stronger pull on the armature as it approaches than as it recedes, thereby maintaining the motion. The relay is controlled by the pair of contacts C_1 on the pendulum; the contacts C_2 actuate the shutter. Current enters the pendulum rod through the metal plate *P* on which the knife edge rests and then flows out through the contract springs to the insulated contact screws. The pendulum is in a box to protect it from air currents. Of course, a motor driven contactor might be used to replace the pendulum, in fact the same motor which drives the spectrograph may operate the contactor.

It is obvious that if the Brownian error is to be kept to a minimum, the amplifier should have its maximum amplification at the working frequency. The frequency characteristic of the amplifier was determined by removing the primary galvanometer G_1, and placing at a suitable distance in front of l_2 a circular white card which was illuminated by diffuse light and rotated somewhat eccentrically by a motor and reduction gear. An image of the edge of the card fell on the aperture A_2 thus producing a sinusoidal illumination of the photocell whose frequency could be varied by controlling the speed of the motor. When the input circuit of the lower part of Fig. 4 was used, the following results were obtained.

Frequency characteristic of amplifying part of system

Frequency, cycles per second		Deflection of G2
.050		21 mm
.071		27
.125		40
.185		48
.26	working frequency	51
.35		47
.48		41
.69		27
.85		18
1.04		10
1.82	frequency of pendular swing of pri. galv.	2

Fortuitously, this circuit, which was designed to attenuate the frequency of pendular swing of the primary galvanometer, gave with this short period secondary galvanometer, a maximum amplification at the working frequency. If this had not been the case it would have been important to so change the constants of the circuit that a suitable frequency characteristic be obtained.

On setting up the apparatus a considerable amount of zero unsteadiness was observed and it was desirable to see how much was due to the unavoidable Brownian motion and how much to other causes such as mechanical vibration of G_1, or unsteadiness of the amplifier. The first step was the determination of the overall sensitivity of the system. This was done by applying in series with the thermopile a known small periodic voltage of rectangular wave form and of the working frequency, 0.26 cycles per second. From Fourier analysis it is found that the rms value of the fundamental in a rectangular wave is 0.45 times the voltage change of the rectangular wave. Higher harmonics are of but small importance as the sensitivity of the system falls off rapidly at the higher frequencies. In this way it was found that at the working frequency a sinusoidal voltage of rms value 2.62×10^{-10} volts would produce a sinusoidal swing of rms value of 1 mm at the photographing distance of 5 meters from G_2. Under these conditions the zero unsteadiness was photographed for two minutes, two such records being made in the afternoon, and two at three A.M. The rms values of these curves were then determined with an Amsler integrator and the average of the two day curves checked the average of the two night curves within one percent, this being considerable evidence that mechanical vibration of G_1 is not serious even in the daytime now that the pendular motion of the galvanometer coil has been tuned out. (In our reinforced concrete building we fasten the primary galvanometer rigidly to one of the supporting columns with heavy cast iron brackets, having found the heavy building to be more steady than our lighter piers which rest on their own foundations.) The average rms zero unsteadiness was 1.8 mm which therefore corresponds to an rms voltage of the working frequency of 4.7×10^{-10}, which is a satisfactory check with the observed value of 4.7×10^{-10}, that is, this rms voltage would produce a swing of G_2 having a mean energy equal to the mean energy of the zero unsteadiness.

To compare with the above observed value we may obtain a computed value of the Brownian voltage of the primary galvanometer only from equation (16). The primary galvanometer swings slightly, being not quite critically damped, so Δ was found, by comparing the amplitude of successive swings, to be 0.65, while the resistance of the galvanometer, 26 ohms, plus the resistance of the thermopile, 23 ohms, make r equal 49 ohms. These values placed in equation (16) give E_{Brms} equal to 5.05×10^{-10} volts. By the graphical method outlined in Part I it was determined that the frequency response characteristic of the amplifier, as shown in the above table, would cut down the effect of the Brownian motion to 0.87 of its value at the primary galvanometer. The computed effective E_{Brms} at the secondary galvanometer is therefore 4.4×10^{-10} which is a satisfactory check with the observed value of 4.7×10^{-10}. We are therefore justified in saying that the observed zero unsteadiness is mainly due to Brownian motion and that no serious unsteadiness is due to the amplifier itself.

Theoretically, small deflections due to Brownian motion are more probable than large ones although very large deflections have a certain small probability. In the four two-minute curves of zero unsteadiness which were taken, the maximum excursion to the right plus the maximum excursion to the left bore a constant ratio of about 5.3 to the rms value of the curve. Therefore, a simple approximate measurement of the rms zero unsteadiness can be made by merely dividing the maximum double amplitude of the unsteadiness by 5.3, no Amsler integrator being required. From theoretical considerations based on the relative probability of large and small displacements as given by Einstein (Annalen der Physik, (4), 19, 1906, pp. 371.) I have found that the ratio of that double amplitude which is exceeded only one percent of the time, to the rms value is 5.18.

With the sensitivity as quoted above, the maximum double amplitude of the zero unsteadiness was about 9.5 mm. With the light turned off, the unsteadiness due to the amplifier tube only was 0.2 mm. When the light was turned on but the primary galvanometer coil clamped and turned to such an angle that only one millimeter of the edge of the image was falling into the aperture A_2, the unsteadiness increased to 0.5 mm due to unsteadiness of the light beam or of the photocell response. When the image completely covered the aperture A_2, the unsteadiness due to the light increased to 7 mm. This difficulty is minimized in normal operation by using as small an overlap of image and aperture as will accommodate the swings and drifts, though it might possibly be eliminated through further investigation of its cause.

TYPICAL RESULTS

Fig. 6 shows an absorption spectrum of ammonia from 14μ to 21μ obtained with this apparatus by Weber and Randall (paper soon to be

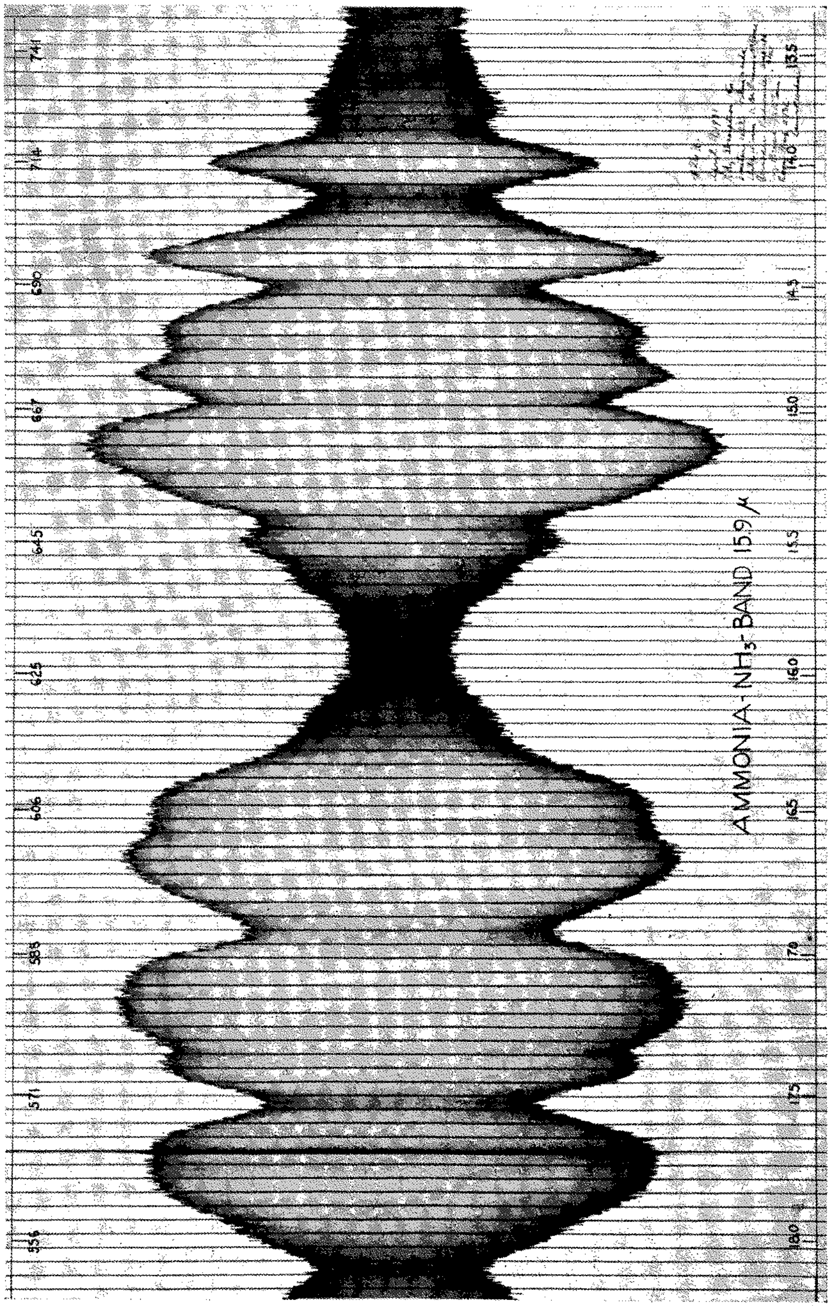

FIG. 6. *Absorption spectrum of ammonia from 14μ to 21μ recorded from a prism spectrometer with the periodic radiometer. See also Fig. 6a.*

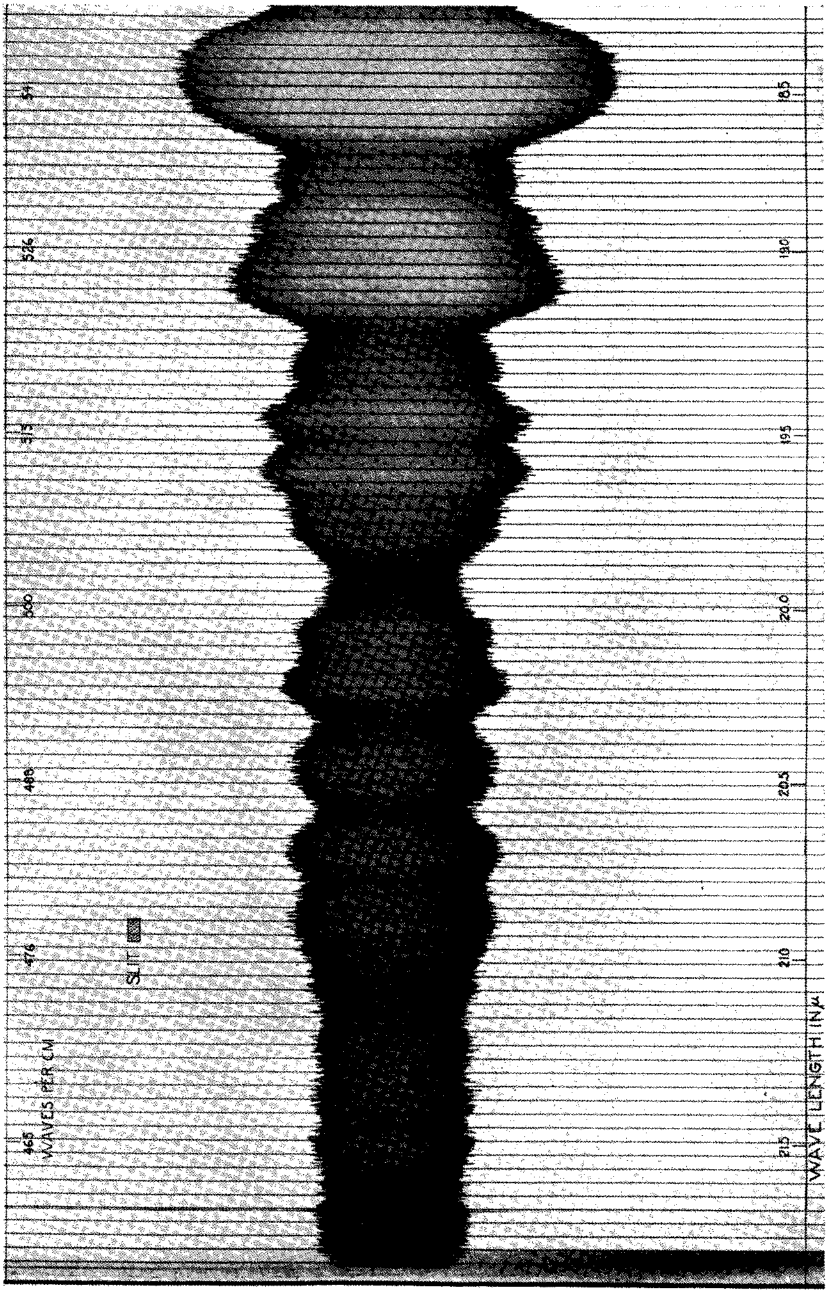

Fig. 6a. Other half of the record shown in Fig. 6.

published in the Physical Review) on the prism spectrograph described by Randall and Strong (R.S.I. *2*, 585, Oct. 1931). The slight irregularities in the envelope were produced by Brownian motion. There is obviously no apparent drift although it required about three hours to take the record, in which time the primary galvanometer drifted several times as far as the size of its deflections.

In the past, many experimenters have automatically recorded infrared spectra with prism spectrometers where energy is relatively abundant, but so far as the author is aware no records of grating spectra have been taken. With the small amount of energy available in a grating spectrum with narrow slits, a large amount of amplification must be used, with consequent increase of trouble from drifts which are always present under these conditions. For this reason all grating spectra have been mapped by the laborious and fatiguing process of visual observation of the galvanometer deflections. Fig. 7, however, shows a water vapor absorption spectrum from 6μ to 7μ recorded with the periodic radiometer from a grating spectrometer by P. E. Martin and E. F. Barker. This record was obtained as a by-product of their experiments as they were trying to eliminate water vapor from their apparatus preliminary to making a series of records of carbon dioxide absorption (paper soon to appear in the Physical Review). In Fig. 8 is a manually determined absorption curve of water vapor in this same region taken by E. K. Plyler and W. W. Sleator (Phys. Rev. *37*, 1493; June 1, 1931), on the same spectrometer. On comparing these two curves by referring to the arbitrary line numbers, it is seen that the automatic record, which was made in ninety minutes, has almost as much detail as the manual curve which required weeks of work. By adjusting the amount of water vapor and the width of the slits, a better automatic record could probably be made. On careful records, the observer watches in the reading microscope the passage of the lines on the grating circle and presses a key which flashes a lamp to give an angle coordinate on the record. If the circle has a carefully designed driving system, then the driving system itself may periodically flash a lamp to mark out the coordinates, as in Fig. 6.

As suggested by Professor H. M. Randall, it is possible to have the periodic shutter S be transparent to certain regions thereby obtaining the usual advantages of partial reduction of the effects of stray radiation or overlapping orders. It has been demonstrated by others that the sensitivity of a thermopile is increased several fold on operating it at liquid air temperature. This has not previously been practical as large

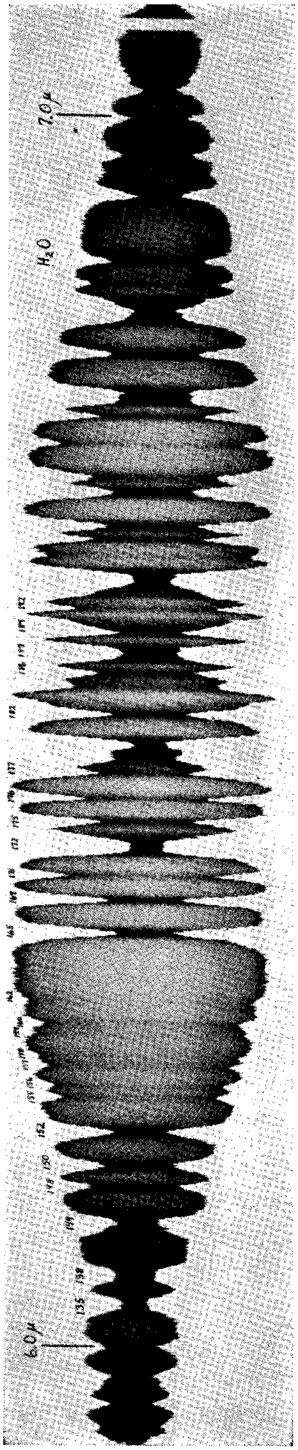

FIG. 7. *Absorption spectrum of water vapor from 6μ to 7μ recorded from a grating spectrometer with the periodic radiometer in 90 minutes.*

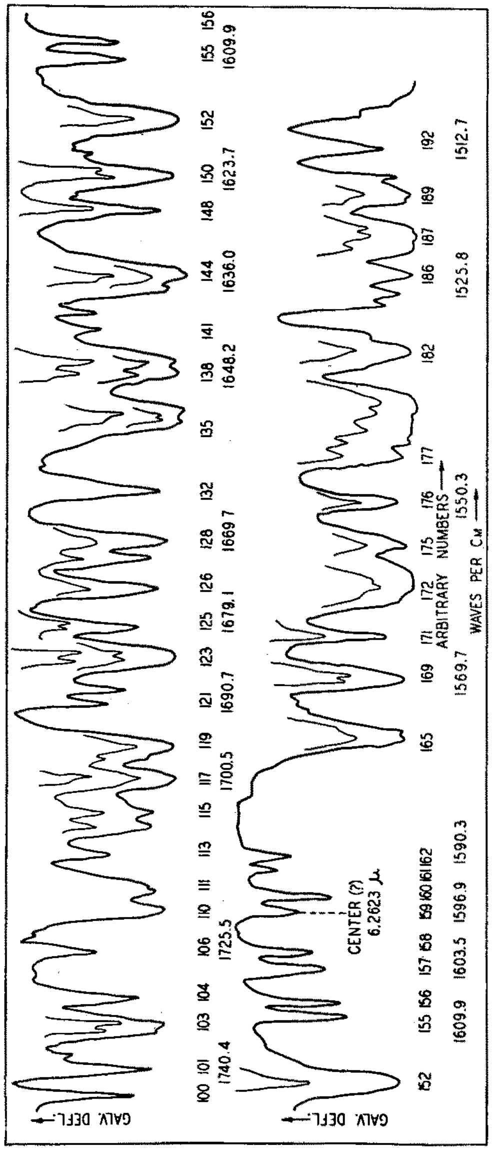

FIG. 8. *The same region of the water vapor spectrum as manually recorded by Plyler and Sleator. Refer to the arbitrary line numbers for comparison with Fig. 7.*

drifts were encountered which destroyed the advantages of increased sensitivity. With the periodic radiometer eliminating the drifts, the low temperature operation of the thermopile should be practical. Of course, any increase in the sensitivity of the thermopile reduces the effective Brownian error in measurements of amount of radiation.

If the period of interruption of the beam is too short, the thermopile will not have an opportunity to attain its full voltage. The 3.6 sec. period used in the instrument described above is somewhat shorter than would give best results with the present thermopile. A period of 6 sec. is recommended for use with most thermopiles.

Since the Brownian motion which was computed is a kind of probable error of a single observation, the accuracy of a determination may be increased by averaging a large number of observations. The error in the average varies inversely as the square root of the number of observations, that is, inversely as the square root of the total time spent in taking deflections, as was true also for the resonance radiometer (equation 10). This suggests that the rapid periodic radiometer may approach the accuracy of the slower systems, when desired, by running the recorder very slowly and then drawing an average envelope through the increased number of turning points thus obtained. In this manner the same instrument is useful both for rapid exploration and for the slower determination of fine detail.

I wish to acknowledge the help which I have received from consultation with Dr. J. D. Hardy and Dr. G. A. Van Lear.