Geometric interpretation of the possible velocity vectors obtained with multiple-sensor probes

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A geometric interpretation for the operation of probes that use multiple sensors is presented. This interpretation provides a method for visualizing how the individual sensor reponse and the geometrical arrangement of the sensors are related to the measurements made with a given probe. The interpretation also provides a simple explanation for the occurrence of multiple solutions of the nonlinear equations for a probe. It is shown that a measured set of three cooling velocities, using a three-sensor hot-wire probe, can be produced by as many as eight different velocity vectors, only one of which is the correct velocity vector.

I. INTRODUCTION

The use of hot-wire (or hot-film) probes with three or more sensors for measurements of all three time-resolved, velocity-component, fluctuations has been reported by a number of authors.[1–4] We have recently reported[5,6] time-resolved measurements of the velocity in a highly turbulent flow in the atmosphere using a three sensor hot-wire probe. Instantaneous velocity components were determined by simultaneously solving three nonlinear algebraic equations. These three equations use the known sensor response and the geometrical relationship of the sensors to relate the measured sensor signals to the three velocity components.

In general, there is more than one velocity vector that can cause the same three values of the sensor signals, and multiple values of the velocity components will be found. Additional information is then required before the correct velocity vector can be determined. Also, when one or more of the sensor measurements are in error, there may be no velocity vector which is capable of causing the three measured sensor signals. In other words, the simultaneous solution of the equations may result in imaginary values for the velocity components.

During the course of our investigation,[5,6] we had a great deal of difficulty physically understanding the multiple solutions of the nonlinear probe response equations. We have found a relatively simple geometric interpretation for the mode of operation of such multisensor probes and have found it helpful for visualizing and understanding measurements made with such probes. This interpretation is presented in the following paragraphs.

II. ARRAY OF TWO HOT WIRES

Consider the simplest case of a hot-wire with infinite aspect ratio which is exposed to a uniform flow. More complicated sensor reponse characteristics can also be interpreted, as will be explained later. The signal produced by the heated wire depends on the amount of heat transferred to the flow. For a wire with large aspect ratio, in a constant temperature flow, the linearized hot-wire signal is proportional to the magnitude of the component of velocity normal to the wire. There are infinitely many velocity vectors which can produce a given hot-wire signal. If we arrange all these vectors so that the head of each vector touches the wire at one point, the ends (or tails) of the vectors will lie on a cylinder concentric with the wire with radius equal to the magnitude of the velocity component normal to the wire.

Now consider the classic X-wire array with two wires positioned in closely spaced parallel planes at an angle of 45° to the mean flow direction. The array is modeled as two long wires which intersect at an angle of 90°. For simplicity, the necessity for supporting prongs and the occurrence of aerodynamic interference between the wires and/or prongs is completely ignored.

Suppose that the X array is exposed to a uniform flow. In general, the measured signals from the two wires will not be equal. The flow velocity vectors which can produce the signal measured by one of the wires must lie on a cylinder concentric with the wire in question with a radius equal to the magnitude of the component of the flow velocity normal to that wire. The same statement can be made about the other wire. Therefore, the only velocity vectors of the flow that can produce the two signals measured by the X array (i.e., the two normal velocities) must lie on the curves formed by the intersection points of the surfaces of the above two cylinders, as sketched in Fig. 1. Since the cylinders have different diameters, there are two closed curves of intersection formed where the smaller diameter cylinder passes through the larger cylinder. The ends of the velocity vectors which can produce a given *pair* of sensor signals must lie on these two intersection curves. Thus, one can observe that there are infinitely many velocity vectors that can produce a single pair of X-array sensor signals.

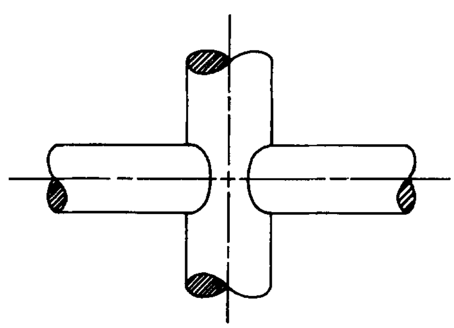

FIG. 1. Intersection of the surfaces of two orthogonal cylinders with unequal diameters.

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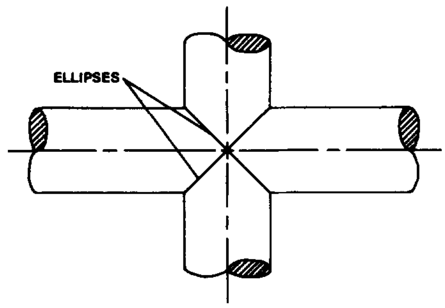

FIG. 2. Intersection of the surfaces of two orthogonal cylinders with equal diameters.

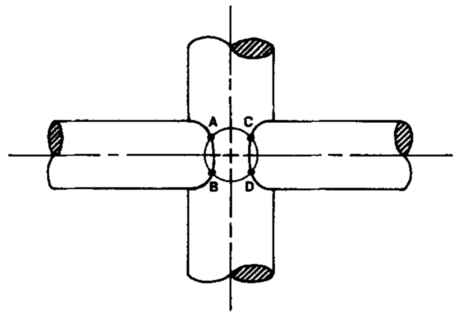

FIG. 3. Intersection of the surfaces of three orthogonal cylinders with unequal diameters occurs at eight points where the curves of intersection of pairs of cylinders intersect. Four points of intersection are located at A, B, C, and D and the other four points are directly beneath these points.

If the two hot-wire signals are equal, these cylinders are of equal diameter. The intersection curves on opposite sides of the largest cylinder "merge," as sketched in Fig. 2, to form two elliptical cuves of intersection lying in planes normal to the plane formed by the wires. The ends of the velocity vectors that produce two equal sensor signals must lie on either of these ellipses.

When the angle between the two wires is different from 90 °, a similar geometric interpretation using the intersection points of cylindrical surfaces can be used to describe all possible velocity vectors which produce a *pair* of hot-wire signals.

III. ARRAY OF THREE HOT WIRES

Now consider an array of three hot wires that are orthogonal to each other. We model the array as three lines parallel to the actual hot wires of the array and intersecting at a point. When exposed to a uniform flow at an arbitrary angle to the array a different signal will, generally, be measured by each wire. A concentric cylinder is then placed about each wire with a radius corresponding to the magnitude of the component of the flow velocity normal to the wire, as indicated by the signal from the wire.

As described before, there will be two closed curves formed by the intersection points of any one pair of the cylinders. Consider the pair of cylinders with the largest radii. There will be two closed curves formed by the intersection points where the surface of the smaller cylinder passes through the surface of the larger cylinder. The surface of the smallest cylinder, whose axis is normal to the axes of the other two cylinders, will also intersect the surface of the largest cylinder and form two more closed curves of intersection. The cylinders and the curves of intersection are sketched in Fig. 3.

One end of the velocity vector of the flow is at the intersection point of the three wires, and the other end must lie on the surface of *each* of the three cylinders and therefore at one of the points where the curves of intersection between pairs of cylinders intersect. Suppose that the tail of the velocity vector of the flow is the point A in Fig. 3. Since the axes of the three cylinders are orthogonal, it can be seen, by the symmetry of the arrangement, that in addition to this one point of intersection, A, that corresponds to the flow velocity vector there are *seven* more points lying on the two curves of intersection of the largest and smallest cylinders where the three cylindrical surfaces intersect. Three of these points are labeled B, C, and D in Fig. 3. The remaining four points are on the other closed curve of intersection between the largest and smallest cylinder, and by the symmetry of the arrangement are directly beneath the above points in Fig. 3. The point directly beneath point D of Fig. 3 corresponds to a flow parallel to the original flow but in the reversed direction. It is concluded that there are, in general, eight velocity vectors that can produce a single set of three different signals from this orthogonal three-sensor probe array.

If the radius of the smallest cylinder is reduced [for example, if the flow direction indicated by the location of point A is changed in the proper way], the intersection curves sketched in Fig. 3 will become tangent to each other at the points where point A merges with point B and point C merges with point D. In this case there will be only four intersection points between the three cylindrical surfaces and only four possible velocity vectors that can produce a single set of three measured signals from the hot wires.

If any two of the signals from these wires are the same, two of the curves of intersection "merge" (see Fig. 2), to form two elliptical intersection curves between the two cylinders of equal diameter. The intersection curves and points are as sketched in Fig. 4. There are still eight possible velocity vectors which can produce the single set of three signals from the wires. If all signals are equal, all the intersection curves are elliptical, and there are still eight possible velocity vectors which could have produced the single set of three equal

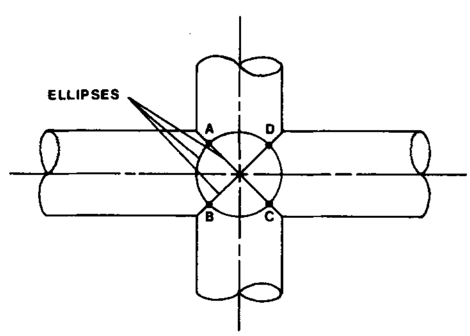

FIG. 4. Intersection of the surfaces of three cylinders with equal diameters also occurs at eight points where the elliptical curves of intersection between pairs of cylinders intersect. Four points of intersection are located A, B, C, and D and the other four points are directly beneath these points.

signals. Thus, for an orthogonal array of three wires, there are, in general, eight possible flows that can produce a single set of three signals from the array, but there are also singular sets of three signals for which only four possible flows can produce the three signals.

For an arbitrary arrangment of the three infinite-aspect-ratio hot wires, the angles between the wires will be different but a similar analysis can be made. In general, there will be eight possible velocity vectors which correspond to a single set of three measured sensor signals. This can be seen by considering a gradual change in the angles between the wires from the orthogonal array until the desired angular arrangement is obtained. The intersection curves between the surfaces will deform and change shape as the angles between the wires are changed. However, the curves of intersection will remain smooth and the intersection points will be well behaved. The maximum number of intersection points of the cylindrical surfaces will be eight, and four intersection points can occur for certain directions of the velocity vector, in a similar fashion to that described above for the orthogonal array.

The concept that the signals produced by an array of hot wires is related to the intersection points of surfaces can be used to understand how an error in measurement can result in a set of measured values for which the equations used to determine the components of the velocity vector of the flow cannot be solved. Suppose that the hot wires have infinite aspect ratio. If only two wires are used, there is always a solution for the velocity vector since the cylindrical surfaces about each wire always intersect. If a measurement error occurs, the lines of intersection are incorrect, but there is no way to determine that an error has occurred. If three or more hot wires are used, an error in measurement can result in a situation for which there are no intersection points between all three or more cylinders.

For example, consider Fig. 3, and suppose that the signal measured by the wire corresponding to the smallest cylinder is in error and much smaller than it should have been. Then the cylinder described about this wire may be so small that the intersection points A, B, C, and D do not exist. Instances in which there is no solution will be most likely in actual practice when for a given probe geometry the flow velocity is such that pairs of intersection points are nearly "merged," for example, when a pair of points like A and B or C and D are very close together. Then small errors of measurement may cause difficulty in the solution of the equations used to determine the velocity components.

IV. EXTENSION TO FINITE-ASPECT-RATIO HOT WIRES

One can extend these considerations to the more practical case of finite-aspect-ratio hot wires with prong interference. According to Jorgensen[7] the response of a single sensor can be described by a cooling law expressed as:

$$Q = (U_n^2 + AU_t^2 + BU_b^2)^{1/2}, \tag{1}$$

where Q is the instantaneous, effective cooling velocity of the wire. Here U_n is the velocity component normal to the wire and parallel to the plane of the prongs, U_t is the velocity component tangent to the wire, and U_b is the velocity component normal to the wire and normal to the plane of the prongs (binormal). The constants A and B are determined by calibration of the wire and prong assembly. From Eq. (1), it is apparent that the velocity vectors which can produce a given measured value of cooling velocity, Q, must lie on a surface in the orthogonal, U_n, U_t, U_b space which is described by Eq. (1). Thus, instead of a cylindrical surface about an infinite-aspect-ratio hot wire which represents the locus of the ends of all possible velocity vectors capable of producing a given measured normal velocity, the surface describing the locus of the ends of the velocity vectors for a given measured cooling velocity is that described by Eq. (1). Analysis of the intersection curves for two or more such surfaces can be used to visualize the response characteristics of an array of finite-aspect-ratio hotwires.

We have not attempted such an analysis. However, for small values of the constants A and B and for values of velocity for which no sensor is exposed to only a tangential velocity, we believe that the results will be similar to those given before for infinite-aspect-ratio sensors.

V. COMMENTS ON THE DETERMINATION OF THE CORRECT VELOCITY VECTOR

Without information in addition to the sensor signals from an array of three hot wires, one cannot determine which of the eight possible velocity vectors is the correct vector. In practice, the effects of probe interference make it impractical to measure reversed flows. Thus, an actual probe will usually be used with a large free-steam velocity, and the hot wires will be arranged so that the correct velocity vector is well separated from all the others. Four of the possible vectors, representing reversed flow, may then be eliminated by the requirement that the velocity component in the free-stream direction be positive. For velocity fluctuations of small amplitude relative to the free stream velocity, the single correct velocity vector will be well separated from the remaining incorrect velocity vectors.

If velocity fluctuations of large amplitude occur, there does not appear to be any method that uses only three hot wires to determine the correct velocity vector. It may be necessary to employ additional sensors to help single out the correct velocity vector. When velocity fluctuations occur which are large enough to cause the flow velocity to be parallel to one of the wires in the array, prong interference and the accuracy of the velocity indicated by the sensor in question will present a serious problem.

VI. EXTENSION TO LASER DOPPLER ANEMOMETERS

The concept of the intersection of the surfaces describing the locus of the end of the measured flow velocity vector can be used for arrays using the other types of velocity sensors. For the case of a single laser Doppler anemometer (LDA), a component of the flow velocity in a given direction is measured. This measured velocity component can be produced by any flow velocity vector whose head is in the center of the measurement volume and whose tail lies on a plane normal to the direction of the component of velocity being measured and at a distance from the measuring volume equal to that of velocity component measured. There are two such planes on either side of the measuring volume. If a Bragg cell is used, the flow direction along the direction of the component being measured can be determined and only one plane describes the locus of the end of the flow velocity vector.

For a two-component LDA without Bragg cells, the velocity vectors which can produce a *pair* of measured velocity components must lie on one of the four lines of intersection of the two pairs of parallel planes which correspond to each component measured. If a third component at an arbitrary angle to the first two components is measured there will be eight points of intersection between the above four lines and the pair of parallel planes corresponding to the third component measured. If Bragg cells are used for the measurement of each component, there will be only one plane describing the locus of the end of the velocity vector for each component. Since three planes intersect at a single point, the set of three measured components uniquely determines the flow velocity when Bragg cells are used for each component.

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